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RESEARCH ARTICLE

Can Ridesharing Improve the Reserve Capacity of Transportation Network?

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ABSTRACT Ride-sharing is one of the effective method to reduce car ownership, thus it may have a profound impact on the reserve capacity of road network. However, it's unclear the relationships among users' ridesharing behaviors, travel demand pattern and the reserve capacity of road network. To this end, this paper builds a ridesharing trip-assignment model which considers users' ridesharing choice, destination choice and path choice, and further proposes a bi-level programming for reserve capacity of road network with ride-sharing. The bi-level programming is then converted into an equivalent single-layer optimization problem by a conventional relaxation scheme. Finally, numerical experiments are conducted to provide valuable insights and examine the effectiveness of the proposed model. The results show that subsidizing ridesharing drivers can improve almost as much reserve capacity of road network as expanding link capacity without ridesharing. However, retrofitting High-Occupancy Toll (HOT) lane has limited impact on improving reserve capacity of road network.

INDEX TERMS Reserve capacity, ride-sharing, user equilibrium, bi-level programming.

I. INTRODUCTION

Urbanization has spurred a significant increase of car usage in cities [1]. In addition, with the increase of urban population, the land use of urban road construction has gradually shrunk [2]. As a result, traffic congestion becomes a serious urban disease globally, which are threatening sustainable mobility for our future. For years, the direct solution to mitigate traffic congestion has been adding new capacity to the road network, such as building new streets or expanding existing streets [3]. However, adding new capacity to the road network is restricted by limited urban spatiality and financial constraints [2], [3]. Hence, tapping the reserve capacity of road network to meet the growing urban travel demand has attracted widespread attention of scholars and traffic managers. Scholars have proposed methods such as optimizing signal control, optimizing lane allocation to improved the reserve capacity of road network [4], [5], [6].

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These methods actually change the distribution of traffic flow by influencing users' travel behavior, as discovered by Wang et al. [7] and Lu et al. [8] that there is a certain connection between user's travel behavior and the reserve capacity of road network.

With technological advances in GPS, smartphones and mobile internet, many ridesharing platforms operated by commercial companies have been rapidly growing, such as Uber, DiDi, and Grab. Ridesharing becomes a popular travel mode [9], which has been gradually changing users' travel behavior. Ride-sharing allows individual traveler to share unoccupied rides with other travelers who have similar itineraries and time schedules, and travelers who participate in ridesharing can share their travel cost [10]. Therefore, rational travelers are willing to participate in ridesharing, as long as the mechanism is properly designed. Besides, ridesharing may reduce car ownership on road thus release road capacity to accommodate more vehicles on road network. Thereby, apart from prohibiting driving, ridesharing may currently be the most effective way to alleviate traffic congestion. Ridesharing may have a profound impact on the reserve capacity of road network. However, few literatures have studied how ridesharing affects the reserve capacity of road network, although the potential advantages of ridesharing on travel cost reduction, traffic congestion mitigation, and energy consumption lessening have gained extensive attention [11], [12].

Calculating reserve capacity of road network is a typical bi-level programming problem. The upper level model is maximizing the reserve capacity of road network with some practical constraints, while the lower level model is usually a user equilibrium model [3], [5], [6], [13], [14], [15], [16]. To predict the reserve capacity of road network in the context of ridesharing, the lower level model should be replaced by a ridesharing user equilibrium model. Although there have been proposed some ridesharing user equilibrium models, such as Xu et al. [17], Di et al. [18], [19], Li et al. [20], Ma et al. [21] and Li et al. [22], these models may not be truly suitable for predicting the reserve capacity of road network in the context of ridesharing, since these models assume that the Origin-Destination(OD) matrix is fixed. However, the reserve capacity of road network reflects the number of potential travelers that the road network can accommodate in the future [23], thus it cannot assumes that the OD matrix of these potential travelers is invariant. Besides, travelers in ridesharing can choose different travel modes, i.e., solo driver, ridesharing driver and ridesharing passenger (or called rider), and the modes selections base on the travel cost of each mode [17], [18], [19], [20], [21], [22]. Thus, travelers' destination choice will inevitably affect their mode selections. For instance, some central business districts (CBDs) are usually difficult to park, thus travelers who choose these destinations may be more inclined to choose passenger mode. Overall, in order to accurately calculate the reserve capacity of road network in the context of ridesharing, there need to build a combined trip distribution and assignment model with ridesharing, which is one of the main contributions of this article.

In a nutshell, the goal of this paper is to investigate the relationships among travelers' ridesharing behaviors, travel demand pattern and the reserve capacity of road network. Accordingly, this paper proposes a bi-level programming for calculating the reserve capacity of road network with ridesharing. The main contributions and conclusions include as follows,

- This research proposes a combined trip-assignment model with considering user's ridesharing choice, route choice and destination choice simultaneously, which relaxes the fixed OD demand matrix assumption of existed RUE models. On this basis, this research further proposes a bi-level programming model for calculating the reserve capacity of road network in the context of ridesharing. The numerical results suggest that travelers' destination choices have a significant impact on the reserve capacity of road network in the presence of ridesharing.
- This research explores how much reserve capacity can be improved by ridesahring compared to expanding link capacity without ridesharing. The numerical results show that subsidizing ridesharing drivers with ridesharing can improve almost as much reserve capacity of road network as expanding link capacity without ridesharing.
- This research extends the proposed model to retrofitting HOT lane and answering the question that whether incentivizing ridesharing necessarily improves the reserve capacity of road network. The numerical results indicate that although retrofitting HOT lane can encourage travelers to participate in ride-sharing, it has limited impact on improving reserve capacity of road network.

The remainder of the paper is organized as follows. Section II reviews the related literatures, Section III presents the problem statement and formulates the network reserve capacity model with ride-sharing. Section IV introduces a relaxation algorithm to solve the proposed model and performs a few numerical examples. Section V extends the proposed model to retrofitting HOT lane and Section VI concludes the paper.

II. LITERATURE REVIEW

The concept of reserve capacity was first extended to road network by Wong and Yang [6] and is measured by a greatest common multiplier of existing OD demand flows that can be accommodated subject to the approach of capacity constraints, cycle time, minimum green constraints and others. Yang and Bell [14] then simplified the reserve capacity for a road network to find a largest multiplier which can be applied to a given basic OD matrix subject to the flow on each link not exceeding its capacity. Since then, numerous studies have extended this research area. These studies mainly focus on approaches to improve reserve capacity of road network. These approaches can be generally classified as strategic (e.g.,building new streets or enhancing the capacity of existed streets) [5], [13], [14], [15], [16], tactical (e.g., determining the orientation of one-way streets or the allocation of lanes in two-way streets) [5], and operational ones (e.g.,scheduling traffic lights) [6], [6], [13], [15]. The above studies assume that travelers follow user equilibrium (UE) principle. Only a few literatures studied how travelers' choice behavior affects the reserve capacity of road network. Wang et al. [7] proposed a reserve capacity model with logit-based stochastic user equilibrium and found that the quality of travelers' information significantly affects the network reserve capacity. Lu et al. [8] compared the network reserve capacity with probit-based and logit-based stochastic user equilibrium and found that the maximum reserve capacity would be same under a certain-level quality of users' information. Xiao et al. [23] proposed a reserve capacity model with logit-based destination choice based on deterministic user equilibrium. Han and Cheng [3] applied a tradable credit scheme in maximizing network reserve capacity with stochastic user equilibrium. However, these literatures base on the classical assumption that every traveler directly occupies road facilities and drives alone to complete his/her trip. Nevertheless, things have changed when ride-sharing emerges. Travelers can give up driving and take rides as passengers and thus may not need to directly occupy road facilities. In addition, travelers can also choose to share their unoccupied ride sources as ride-sharing drivers in stead of driving alone. Up to now, few attentions have focused on the relationship between the reserve capacity of road network and travelers' ride-sharing behaviors, which is the main gap filled by this paper.

Another aspect of researches related to this article is studies of ridesharing user equilibrium (RUE). A lot of attention has been paid to the relationship between ridesharing and traffic congestion [11], [12]. And there have been proposed bits of user equilibrium models with ridesharing considering different real situations. Xu et al. [24] first incorporated ride-sharing into a static user equilibrium model in a network to analyze how ride-sharing and traffic congestion interact and how people can be motivated to participate in ride-sharing. But only drivers' congestion cost and utility are considered in the objective function and travelers are assumed to car-pool in the same origin-destination (OD) pair in [24]. Xu et al. [17] further developed a new complementarity traffic equilibrium model with ride-sharing, where congestion costs of both driver and rider were considered and ride-sharing may occur in different OD pairs. However, travelers who choose to be riders have to transfer to multiple vehicles to complete their journeys in [17], which may be not realistic in the real situation. Thereupon, Di et al. [18], Li et al. [25] and Li and Bai [26] proposed path-based ride-sharing user equilibrium (RUE) models where ridesharing occurs in the same OD pairs and the different OD pairs, respectively. These path-based ride-sharing equilibrium model ensures that a rider only needs to get on one vehicle to complete his/her trip. But these models require path enumeration, which makes model calculation non-trivial. Although Li et al. [20] and Ma et al. [21] proposed path filtering algorithm and projection algorithm, their algorithms require that the RUE models should satisfy certain properties. To facilitate calculation, Di et al. [19] proposed a link-node based ride-sharing user equilibrium model, which does not require path enumeration. Based on Di et al. [19], Chen and Di [27] incorporated the matching cost between drivers and riders at each node into a link-node-based RUE model, where drivers are restrained to frequent pick-up and drop off riders. A more detailed literature review on RUE can be seen in [28]. The above models postulated that the OD demand matrix is fixed, which may not completely guarantee the consistency in the travelers' choices. Thus, these models can not be suitable for predicting reserve capacity of road network in the context of ridesharing. This paper will relaxe the fixed OD demand matrix assumption of existed RUE models and propose a combined trip-assignment model with considering user's ridesharing choice, route choice and destination choice simultaneously.

III. MATHEMATICAL FORMULATION

A. DEFINITION OF ZONE-LEVEL RESERVE CAPACITY OF ROAD NETWORK

The definition of reserve capacity of road network proposed by Wong and Yang [6] is finding a greatest common demand multiplier μ when given the OD demand matrix, which seems not in line with the realistic [13]. In fact, the actual conditions between each OD pair in urban network are different, e.g., the travel demand, the income of travelers, the number of paths and the number of signal light on each path, so the demand multiplier μ may be different for each OD pair [13]. As a result, Gao and Song [13] extended the concept that all demand multipliers are different and the OD demand multipliers are denoted by a vector $\boldsymbol{\mu}$. However, urban travel demand would increase due to land-use in certain zones (such as building new residential quarters, new commercial plazas, factory, etc.), and these increased demands may not be necessarily distributing on the original OD pairs [29]. Therefore, Yang et al. [30] proposed the concept of origin-based reserve capacity or zonal reserve capacity, in which travel demands are assumed to be generated from original zones increased by reasonable configuration management. Xiao et al. [23] further defined zonal reserve capacity systematically and proposed origin-based model for reserve capacity of road network in Definition 1.

Definition 1 [23]: Let $\mathbf{Q}$ denote vector of demand flows beginning in origin zones O, where $o \in O$ is a specific origin zone. Suppose that flow beginning in origin zone o is multiplied by a factor μ_o that can be accommodated subject to the approach of capacity constraints, cycle time, minimum green constraints and others. Then the multiplied trip generation $\mu_o Q_o$ is zonal reserve capacity and the origin-based reserve capacity of road network is calculating by $(\boldsymbol{\mu} - \mathbf{1})\boldsymbol{Q}$, where $\boldsymbol{\mu} = \{\mu_o\}_{o \in O}$.

From **Definition 1**, it can be found that the value of the demand multiplier μ_o actually reflects the size of the zonal reserve capacity. **Definition 1** covers the shortage of Gao and Song [13], but user's ride-sharing behavior is not considered. Moveover, Yang and Bell [14] simplified the reserve capacity of road network to find a largest multiplier which can be applied to a given basic OD matrix subject to the flow on each link not exceeding its capacity. Combined with the definition of Yang and Bell [14] and Xiao et al. [23], the zonal reserve capacity in the context of ride-sharing is redefined as follows,

Definition 2: Considering travelers' ride-sharing choices, route choices, and destination choice behaviors, the potential travel demand that can be accommodated in each origin zone may increase by encouraging ride-sharing activities. The increased potential travel demand in each origin zone is zonal reserve capacity, and the reserve capacity of road network is the summation of all zonal reserve capacities in the road network.

B. UPPER-LEVEL MODEL OF MAXIMIZING THE SUMMATION OF ZONAL RESERVE CAPACITY

Consider a connected network $G_0 = (N_0, A_0)$, where N_0 is the set of nodes and A_0 is the set of links. A link $a_{ij} \in A_0$ is connected by two adjacent nodes $i, j \in N$ (i.e., $a_{ij} = (i, j)$). Let O_0 denote the set of origin zones, where $o \in O_0$ is a specific zone. Let $\boldsymbol{Q} = \{Q_o\}_{o \in O_0}$ be vector of trips generated in origin zones, $\boldsymbol{\mu} = \{\mu_o\}_{o \in O_0}$ be vector of demand multipliers of origin zones. Then, according to **Definition 2**, the reserve capacities of road network can be obtained by the following programming problem,

$$\max_{\Omega} \boldsymbol{\mu Q} \tag{1}$$

where Ω is constraint domain composed of Equation (2) and (3) as follows,

$$x_a(\boldsymbol{\mu}) \leq p_a C_a, a \in A_0 \tag{2}$$

$$\mu_i \geq \mu_0, \forall i \in O \tag{3}$$

In Equation (2), $x_a(\boldsymbol{\mu})$ is equilibrium vehicular flows on link $a \in A_0$, p_a is the maximum acceptable degree of saturation for link $a \in A_0$. Equation (2) indicates that vehicular flows on each links should not exceed a prescribed maximum acceptable value, which means that the queues and delays on these special links are accepted by travelers and traffic managers [6], [13]. If Equation (2) is violated, it indicates that each road can accommodate vehicles infinitely, so the reserve capacity of road network is also infinite, which is obviously not in line with the reality. Therefore, Equation (2) has a greater impact on the upper-level model and has practical modeling significance. In Equation (3), μ_0 is a predefined minimum demand multiplier by the manager, which represents that the demand multiplier should greater than that the traffic manager expected, as defined in [13]. The value of the prescribed multiplier μ_0 is usually greater than or equal to 1. Hence, Equation (3) has practical significance.

Note that $x_a(\boldsymbol{\mu})$ is a reactive function depending on $\boldsymbol{\mu}$ and obtained by solving a ridesharing trip-assignment model which will be detailed in next subsection.

C. LOWER-LEVEL MODEL OF RIDESHARING TRIP-ASSIGNMENT MODEL

As analyzed in the Introduction, travelers' destination choice may affect their ridesharing choice, thus the existed ridesharing user equilibrium models ([17], [18], [19], [20], [21], [22]) should be reformulated as ridesharing trip-assignment model for calculating the reserve capacity of road network. To facilitate calculation, Di et al. model [19] is rebuilt as the lower-lever model.

Consistent with Di et al. research [19], it is also assumed that each traveler has a vehicle and has the right to select his/her travel mode according his/her travel cost, and there are three travel modes available for selection, i.e., solo driver, ridesharing driver and ridesharing passenger (or rider), as well. In order to predict flows of each mode on each link, the original network $G_0 = (N_0, A_0)$ is extended as in [19] shown in Figure 1. As can been seen in Figure 1, each node $i \in N_0$ is copied two times which represent vehicular node $i(vu)$ and passenger node $i(w)$, respectively. This implies that once travelers choose to be passengers then they always need to give up driving. Besides, each link $a = ij$ is copied three times, where link $a(v)$ traveled by solo driver, link $a(u)$ traveled by ride-sharing driver and link $a(w)$ traveled by ridesharing passenger, respectively. Connecting link $a(v)$ and link $a(u)$ by vehicular node $i(vu)$ means that each driver is free to switch his/her drive mode, i.e., either driving alone or sharing ride with passengers. As a result, the extended network $G = (N, A)$ is formed by coupling the vehicular network and the passenger network, where $N = N(vu) \bigcup N(w), A = A(v) \bigcup A(u) \bigcup A(w)$. The node set can be further divided into three subsets, the origin-node set $O \in N$, the intermediate-node set $I \in N$, and the destination-node set $D \in N$. Since the number of travelers choosing each mode is endogenously determined by the travel cost, virtual links $(i_0, i(vu)) \in A^v$ and $(i_0, i(w)) \in A^v$ are introduced at virtual source node $i_0, i_0 \in O^v$ to connect the actual origin node $i(vu), i \in O(vu)$ in the vehicular network and the actual origin node $i(w), i \in O(w)$ in passenger network, respectively. Then the node set of the ultimate extended network is $N^e = N \bigcup O^v$ and the link set of the ultimate extended network is $A^e = A \bigcup A^v$. Let $q^d_{i_0(vu)}, i_0 \in O^v, d \in D$ denote the number of endogenous drivers traveled on link $(i_0, i(vu))$, $q^d_{i_0(w)}, i_0 \in O^v$ denote the number of endogenous passengers traveled on link $(i_0, i(w))$. Such simplification will reduce the total number of variables in model formulation but will not affect model solution [19]. Note that it is not necessary to actually extend the original network but define the corresponding extended link variables. Details on network extensions can be found in [19].

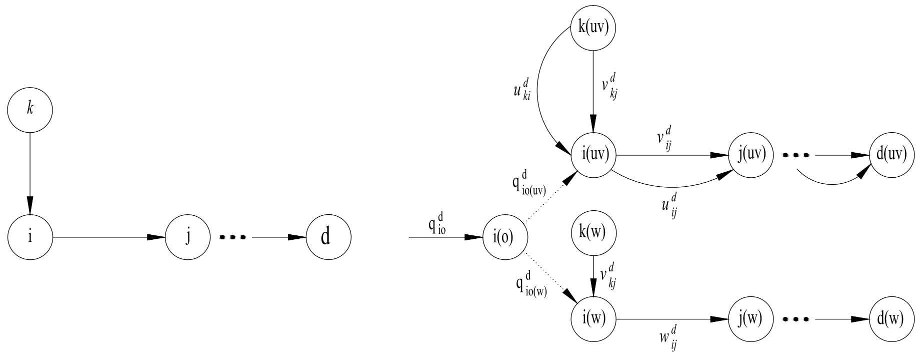
1) TRIP GENERATION CHOICE

As stated in the literature review, the RUE model of Di et al. [19] does not require path enumeration. However, Di's model did not consider travelers' destination choice which inevitably affect travelers' travel mode choices. This paper considers travelers' ridesharing choices and destination choices simultaneously. Let Q_i denote the travel demand generated from virtual origin-node $i \in O^v$, Q^d_i denote the trip distribution from origin-node $i \in O^v$ to destination $d \in D$. Then the trip distribution should satisfy the following relationship,

$$\sum_{d \in D} Q^d_i = Q_i, \quad \forall i \in O^v \tag{4}$$

Equation (4) represents that the sum of the traffic demand from a certain origin-node to each destinations is equal to the traffic demand generated at that origin-node, where Q_i is given exogenously and Q^d_i is an endogenous variable.

Equation (4) hold naturally but it does not actually give the principle of travelers to choose their destination. Travelers' trip distributions may be variable by attraction of large supermarkets, parking lots or large entertainment venues.



(a) Origin network (b) Extended Network

FIGURE 1. **Original network (a) and extended network (b).**

It is generally assumed that travelers always considers two factors at the same time when choosing their destinations, i.e., the greatest attraction and the least resistance [29]. And the logit-based share model is frequently used to describe travelers' destination choices as follows [23],

$$Q_i^d = Q_i \frac{e^{-\gamma(\pi_i^d - M_d)}}{\sum_{k \in D} e^{-\gamma(\pi_i^k - M_k)}}, \quad \forall i \in O^v, d \in D \tag{5}$$

where π_i^d is the least resistance from origin node i to destination node $d \in D$ (i.e., the least travel cost between virtual origin node i to node $d \in D$), M_d is the greatest attraction of the destination node $d \in D$, γ is a correction parameter. It is easy to verify that Equation (5) satisfies Equation (4), and Equation (5) is usually used for calculation.

2) FLOW AND DEMAND CONVERSATION WITH TRIP CHOICE

Combined with the destination choice condition (5), the demand conservation for ride-sharing traffic assignment becomes as follows,

$$\sum_{j:(i,j)\in A^v} (q_i^d(vu)) + q_i^d(w)) - Q_i^d = 0, \ \forall i \in O^v, d \in D \tag{6}$$

where $q_i^d(vu)$, $q_i^d(w)$) are endogenous demands of drivers and of riders respectively. Equation (6) indicates that the sum of travelers who choose to be drivers and travelers who choose to be riders between a certain OD pair should be equal to the travel demand between that OD pair. Note that the OD demand Q_i^d here is determined by Equation (5), which is distinguish with that in Di et al. [19] model where it has been given.

Except for the demand conservation (6), flows should be conserved at all intermediate nodes in the extended network [19]. Let v_{ij}^d denote link flow of solo drivers on link ij whose destination is $d \in D$, u_{ij}^d denote link flow of ride-sharing drivers on link ij whose destination is $d \in D$, w_{ij}^d denote link flow of riders (passengers) on link ij whose destination is $d \in D$. Then, the link-node-based flow conservation at each intermediate node is as below,

$$\sum_{j:(i,j)\in A(vu)} (v_{ij}^d + u_{ij}^d) - \sum_{k:(k,i)\in A(vu)\cup A^v} (v_{ki}^d + u_{ki}^d + q_k^d(vu)) = 0,$$
$$\forall i \in N(vu), d \in D \tag{7}$$

$$\sum_{j:(i,j)\in A(w)} w_{ij}^d - \sum_{k:(k,i)\in A(w)\cup A^v} (w_{ki}^d + q_k^d(w)) = 0,$$
$$\forall i \in N(w), d \in D \tag{8}$$

Equation (7) indicates that the number of drivers (including solo drivers and ridesharing drivers) toward a certain destination flowing into a specific node is equal to those flowing out of that node. Likewise, Equation (8) indicates that the number of riders toward a certain destination flowing into a specific node is equal to those flowing out of that node.

In traditional travel pattern, one unit of vehicular flow serves for only one unit of traveler, while it may serves for several units of travelers in the presence of ride-sharing. Because, travelers may give up driving as passengers (riders) in ride-sharing [17], [19]. Thus, travelers flows are different with vehicular flows in the context of ridesharing, where vehicular flows do not include rider/passenger flows, i.e., $x_a(\boldsymbol{\mu}) = \sum_{d\in D}(v_a^d(\boldsymbol{\mu}) + u_a^d(\boldsymbol{\mu}))$.

3) RIDE-SHARING MATCHING

It is worth noting that travelers who give up driving do not contribute to congestion but are bound by ride-sharing drivers [17], [19]. We assume that vehicle types are uniform. Then, if carpooling occurs, flows of ride-sharing driver and rider should satisfy the capacity constraints as follows,

$$w_{ij} - u_{ij} \geq 0, \quad \forall ij = a_{ij} \in A \tag{9}$$

$$Cap \cdot u_{ij} - w_{ij} \geq 0, \quad \forall ij = a_{ij} \in A \tag{10}$$

where $v_{ij} = \sum_{d\in D} v_{ij}^d$, $u_{ij} = \sum_{d\in D} u_{ij}^d$, $w_{ij} = \sum_{d\in D} w_{ij}^d$; Cap is vehicle capacity, i.e., the maximum units of passengers carried by one unit of ride-sharing driver. Equation (9) indicates that the number of passengers should be greater than the number of ridesharing drivers on any links, ensuring that each driver can pick up and drop off at least one passenger;

Equation (10) indicates the number of rides offered by ridesharing drivers should be greater than the number of passengers on any links, ensuring that each rider can ride at least one vehicle.

Due to the existence of Equation (9) and (10), a special two-sided market has actually formed on each link [31]. In this market, ridesharing participants (namely ridesharing drivers and riders) charge each other a certain compensation cost based on the supply and demand. It is assumed that the ridesharing platform does not charge any fees to participants. Denote η_{ij}^{+}, η_{ij}^{-} as the multipliers for capacity constraints in Eq. (9)-(10) respectively, which can be explained as compensation on link a_{ij} [19]. If the non-negativity of constraint (9) is strictly satisfied, each ride-sharing driver may take on at most one passenger on link a_{ij}. Hence, in this case, ride-sharing driver has to take out η_{ij}^{+} unit of money to attract a rider (passenger). On the other hand, if the non-negativity of constraint (10) is strictly satisfied, each passenger may acquire ride from at most one ride-sharing driver on link a_{ij}. Consequently, in this case, passengers have to pay extra η_{ij}^{-} unit of money to attract a ride-sharing driver. η_{ij}^{+}, η_{ij}^{-} actually ensure the supply-demand balance of the two-sided market market of ridesharing.

4) COMPLEMENTARY EQUILIBRIUM CONDITIONS OF ROUTE AND MODE CHOICE

In the context of ridesharing, travelers choose travel modes and links based on their travel costs. Similar to the existed researches [17], [18], [19], [20], [21], [25], [27], it is likewise assumed that travelers who choose to be solo drivers only experience congestion cost, while those who choose to be ride-sharing drivers endure congestion cost, inconvenience cost, income and compensation cost. In addition, travelers who choose to be passengers experience congestion cost, inconvenience cost, pay fees for the ride and compensation cost. Then, the link cost for each travel mode are non-separable as follows,

$$c_{ij}^{v} = \underbrace{t_{ij}^{0}(1+\alpha(x_{ij}/(C_{ij}))^{\beta})}_{travel\quad time} \tag{11}$$

$$\begin{aligned} c_{ij}^{u} = &\underbrace{t_{ij}^{0}(1+\alpha(x_{ij}/(C_{ij}))^{\beta})}_{travel\quad time}+\underbrace{(-\chi^{u}\cdot u_{ij}+\gamma^{u}\cdot w_{ij})}_{inconvenience\quad cost}\\ &-\underbrace{(\frac{w_{ij}}{u_{ij}}\cdot(\mu\cdot t_{ij}^{0}-\xi\cdot u_{ij}+\psi\cdot w_{ij}))}_{income}\\ &+(\eta_{ij}^{+}-Cap\cdot\eta_{ij}^{-})-S_{ij} \end{aligned} \tag{12}$$

$$\begin{aligned} c_{ij}^{w} = &\rho\cdot\underbrace{t_{ij}^{0}(1+\alpha(x_{ij}/(C_{ij}))^{\beta})}_{travel\quad time}+\underbrace{(-\chi^{w}\cdot u_{ij}+\gamma^{w}\cdot w_{ij})}_{inconvenience\quad cost}\\ &+\underbrace{(\mu\cdot t_{ij}^{0}-\xi\cdot u_{ij}+\psi\cdot w_{ij})}_{ride\quad fee}-(\eta_{ij}^{+}-\eta_{ij}^{-}) \end{aligned} \tag{13}$$

where, c_{ij}^{v}, c_{ij}^{u}, c_{ij}^{w} are travel cost for solo driver, ride-sharing driver and passenger on link ij, respectively, and t_{ij}^{0} is the free flow time on link ij. $x_{ij} = v_{ij} + u_{ij}$ is vehicular flows on link ij. ρ is a parameter representing the time compensation for riders since they need not to pay attention for the traffic during their trips and can do their own business, such as reading or listening to music [21]. α, β are parameters related to congestion. C_{ij} denotes capacity of link ij. The inconvenience cost functions directly adopts that in [32], which means the inconvenience cost increases when the number of ride-sharing drivers decreases or passengers increases, where χ^{u}, γ^{u}, χ^{w}, γ^{w} are parameters related to inconvenience cost. The ride fee function and income function adopts that in [17], [32], which means the amount of ride fee increases as the shared distance increases, or the number of ride-sharing drivers decreases or passengers increases, where μ, ξ, ψ are parameters related to ride fee (income). $S_{ij} = \sigma * t_{ij}^{0}$ is the subsidy for ride-sharing driver from the government [25], where σ is parameters related to the subsidy.

As is known to all, traffic assignment problem with non-separable link cost can be formulated as Variational Inequalities (VI) or Complementary Problem (CP) [33], e.g., models in [17], [18], [19], [20], [21], [25], and [27]. It is easy to find that the link cost functions shown in Eq. (11)-(13) are obviously non-separable. Therefore, the equilibrium conditions in this paper are also complementary. The Wardropian's equilibrium condition can be paraphrased as, at equilibrium, routes that have flow are the shortest routes, or no traveler can reduce travel cost by unilaterally changing routes, or links that have flows for a given destination are on the shortest path [34]. Di et al. [18] has confirmed that the ride-sharing traveler equilibrium holds for generalized travel costs shown in Equation (11-13).

Let $\pi_{i(vu)}^{d}$ denote the minimum travel cost from nodes $i \in N(vu)$ to destination nodes $d \in D(vu)$ for drivers, $\pi_{i(w)}^{d}$ denote the minimum travel cost from nodes $i \in N(w)$ to destination node $d \in D(w)$ for riders. Then the following are the link-node based equilibrium conditions related to link and mode choice on copied links,

$$\begin{aligned} &0 \le \left[\pi_{j(vu)}^{d} + c_{ij}^{v} - \pi_{i(vu)}^{d}\right] \perp v_{ij}^{d} \ge 0,\\ &\qquad \forall ij \in A(vu), i, j \in N(vu),\ d \in D \end{aligned} \tag{14}$$

$$\begin{aligned} &0 \le \left[\pi_{j(vu)}^{d} + c_{ij}^{u} - \pi_{i(vu)}^{d}\right] \perp u_{ij}^{d} \ge 0,\\ &\qquad \forall ij \in A(vu), i, j \in N(vu),\ d \in D \end{aligned} \tag{15}$$

$$\begin{aligned} &0 \le \left[\pi_{j(w)}^{d} + c_{ij}^{w} - \pi_{i(w)}^{d}\right] \perp w_{ij}^{d} \ge 0,\\ &\qquad \forall ij \in A(w), i, j \in N(w),\ d \in D \end{aligned} \tag{16}$$

Equations (14∼ 16) are complementary equilibrium conditions on each link for solo drivers, ridesharing drivers, and passengers, respectively.

Note that this paper has added virtual links on the extended network(Fig. 1). These virtual links connecting drivers' network and passengers' networks but do not carry any information about travel mode. It is assumed that virtual links have zero travel cost. Thus the complementary equilibrium conditions on these virtual link are as follows,

$$0 \leq \left[\pi^d_{j(vu)} - \pi^d_i\right] \perp q^d_i(vu) \geq 0, \quad \forall ij \in A^v, i \in O^v, \ j \in N(vu), d \in D(vu) \tag{17}$$

$$0 \leq \left[\pi^d_{j(w)} - \pi^d_i\right] \perp q^d_i(w) \geq 0, \quad \forall ij \in A^v, i \in O^v, \ j \in N(w), d \in D(w) \tag{18}$$

where π^d_i denote the minimum travel cost from nodes $i \in O^v$ to destination nodes $d \in D$.

Summarizing Equations (5-10) and Equations (14-18), the combined trip-assignment model with ride-sharing can be formulated as a mixed complementary problem (MCP).

D. BI-LEVEL PROGRAMMING FOR RESERVE CAPACITY OF ROAD NETWORK WITH RIDE-SHARING

In the mathematical programming literature, the bi-level programming problem is also frequently referred to as a mathematical program with equilibrium constraints [35]. Based on the above considerations, the reserve capacity of road network with ride-sharing can be calculated by solving the following mathematical programming with complementary constraints (MPCC),

$$\max_{\boldsymbol{x},\boldsymbol{\mu}} \sum_{i \in O^v} \mu_i Q_i \tag{19}$$

subject to

$$x_a(\boldsymbol{\mu}) \leq p_a C_a, \quad a \in A_0 \tag{20}$$

$$\mu_i \geq \mu_0, \quad \forall i \in O \tag{21}$$

where $\boldsymbol{\mu}, x_a(\boldsymbol{\mu}) = \sum_{d \in D}(v^d_a(\boldsymbol{\mu}) + u^d_a(\boldsymbol{\mu}))$ solves the following nonlinear complementary problem,

$$0 \leq \left[Q^d_i \sum_{k \in D} e^{-\gamma(\pi^k_i - M_k)} - \mu_i Q_i e^{-\gamma(\pi^d_i - M_d)}\right] \perp Q^d_i \geq 0, \quad \forall i \in O^v, d \in D \tag{22}$$

$$0 \leq \left[\sum_{j:(i,j) \in A^v} (q^d_i(vu)) + q^d_i(w)) - Q^d_i\right] \perp \pi^d_i \geq 0, \quad \forall i \in O^v, d \in D \tag{23}$$

$$0 \leq \left[\sum_{j:(i,j) \in A(vu)} (v^d_{ij} + u^d_{ij}) - \sum_{k:(k,i) \in A(vu) \cup A^v} (v^d_{ki} + u^d_{ki} + q^d_k(vu))\right] \perp \pi^d_{i(vu)} \geq 0, \forall i \in N(vu), d \in D(vu) \tag{24}$$

$$0 \leq \left[\sum_{j:(i,j) \in A(w)} w^d_{ij} - \sum_{k:(k,i) \in A(w) \cup A^v} (w^d_{ki} + q^d_k(w))\right] \perp \pi^d_{i(w)} \geq 0, \ \forall i \in N(w), \ d \in D(w) \tag{25}$$

$$0 \leq \left[w_{ij} - u_{ij}\right] \perp \eta^+_{ij} \geq 0, \quad \forall ij = a_{ij} \in A_0 \tag{26}$$

$$0 \leq \left[Cap \cdot u_{ij} - w_{ij}\right] \perp \eta^-_{ij} \geq 0, \quad \forall ij = a_{ij} \in A_0 \tag{27}$$

$$0 \leq \left[\pi^d_{j(vu)} + c^v_{ij} - \pi^d_{i(vu)}\right] \perp v^d_{ij} \geq 0, \quad \forall ij \in A(vu), i, j \in N(vu), d \in D \tag{28}$$

$$0 \leq \left[\pi^d_{j(vu)} + c^u_{ij} - \pi^d_{i(vu)}\right] \perp u^d_{ij} \geq 0, \quad \forall ij \in A(vu), i, j \in N(vu), d \in D \tag{29}$$

$$0 \leq \left[\pi^d_{j(w)} + c^w_{ij} - \pi^d_{i(w)}\right] \perp w^d_{ij} \geq 0, \quad \forall ij \in A(w), i, j \in N(w), d \in D \tag{30}$$

$$0 \leq \left[\pi^d_{j(vu)} - \pi^d_i\right] \perp q^d_i(vu) \geq 0, \quad \forall ij \in A^v, i \in O^v, j \in N(vu), d \in D(vu) \tag{31}$$

$$0 \leq \left[\pi^d_{j(w)} - \pi^d_i\right] \perp q^d_i(w) \geq 0, \quad \forall ij \in A^v, i \in O^v, j \in N(w), d \in D(w) \tag{32}$$

Note that Equation (5) can easily be rewritten into its complementary form (Equation (22)) based on [36].

IV. NUMERICAL EXPERIMENTS

A. SOLUTION PROCEDURE

Bi-level programming is generally difficult to tackle. Bialas and Karwan [37] shows that even if the upper-level and lower-level problems are both linear, a bi-level programming is still a non-convex programming problem. Jeroslow [38] points out that the bi-level linear programming is NP-hard. A commonly used method for solving bi-level programming is to convert a bi-level programming into a single-level nonlinear programming (NLP), and then solve the converted single-level nonlinear programming [39]. This conversion method has better convergence thus is adopted here. The key of the conversion method is to replace the complementary constraints in the lower-level problem with equivalent complementary relaxation conditions. However, the converted NLP formulation is non-convex and violates the Mangasarian Fromovitz Constraint Qualification (MFCQ) [40]. To using the existing NLP solution algorithms, an auxiliary parameter $\varepsilon > 0$ is introduced to relax the slackness conditions. Under certain conditions, the relaxation scheme turns out to be a prefect method to generate an optimal solution [40]. The relaxed complementarity slackness conditions of Equations (22∼32) are as follows,

$$\left[Q^d_i \sum_{k \in D} e^{-\gamma(\pi^k_i - M_k)} - \mu_i Q_i e^{-\gamma(\pi^d_i - M_d)}\right] \cdot Q^d_i \leq \varepsilon, \quad \forall i \in O^v, d \in D \tag{33}$$

$$\left[\sum_{j:(i,j) \in A^v} (q^d_i(vu)) + q^d_i(w)) - Q^d_i\right] \cdot \pi^d_i \leq \varepsilon, \forall i \in O^v, d \in D \tag{34}$$

$$\left[\sum_{j:(i,j) \in A(vu)} (v^d_{ij} + u^d_{ij}) - \sum_{k:(k,i) \in A(vu) \cup A^v} (v^d_{ki} + u^d_{ki} + q^d_k(vu))\right.$$

$$\cdot \pi^d_{i(vu)} \leq \varepsilon, \forall i \in N(vu), d \in D(vu) \quad (35)$$

$$\left[\sum_{j:(i,j)\in A(w)} w^d_{ij} - \sum_{k:(k,i)\in A(w)\cup A^v} (w^d_{ki} + q^d_k(w)) \right] \cdot \pi^d_{i(w)} \leq \varepsilon, \forall i \in N(w), d \in D(w) \quad (36)$$

$$[w_{ij} - u_{ij}] \cdot \eta^+_{ij} \leq \varepsilon, \forall ij = a_{ij} \in A_0 \quad (37)$$

$$[Cap \cdot u_{ij} - w_{ij}] \cdot \eta^-_{ij} \leq \varepsilon, \forall ij = a_{ij} \in A_0 \quad (38)$$

$$\left[\pi^d_{j(vu)} + c^v_{ij} - \pi^d_{i(vu)}\right] \cdot v^d_{ij} \leq \varepsilon, \forall ij \in A(vu), i, j \in N(vu), d \in D \quad (39)$$

$$\left[\pi^d_{j(vu)} + c^u_{ij} - \pi^d_{i(vu)}\right] \cdot u^d_{ij} \leq \varepsilon, \forall ij \in A(vu), i, j \in N(vu), d \in D \quad (40)$$

$$\left[\pi^d_{j(w)} + c^w_{ij} - \pi^d_{i(w)}\right] \cdot w^d_{ij} \leq \varepsilon, \forall ij \in A(w), i, j \in N(w), d \in D \quad (41)$$

$$\left[\pi^d_{j(vu)} - \pi^d_i\right] \cdot q^d_i(vu) \leq \varepsilon, \forall ij \in A^v, i \in O^v, j \in N(vu), d \in D(vu) \quad (42)$$

$$\left[\pi^d_{j(w)} - \pi^d_i\right] \cdot q^d_i(w) \leq \varepsilon, \forall ij \in A^v, i \in O^v, j \in N(w), d \in D(w) \quad (43)$$

An iterative algorithm proposed by Ban et al. [40] is adopt in the calculation as follows,

- Step 1 Initialization. Choose an initial auxiliary parameter ε^k. Set the iteration limit M, update factor $0 < \lambda < 1$ and $k = 0$.
- Step 2 Major Iteration.
  - Step 2.1 Solve the current relaxed single-level NLP. Use ε^k as the auxiliary parameter
  - Step 2.2 Update and Move. If $k \leq M$, set $\varepsilon^{(k+1)} = \lambda\varepsilon^k$, $k = k + 1$ and go to Step 2.1; otherwise, go to Step 3.
- Step 3 Final Solve. Solve the exact single-level NLP. If it is successful, it can obtain an exact solution for the be-level problem with ride-sharing; otherwise, an approximate solution is achieved from the last run of Step 2.2.

The converted single-level non-linear programming can be solved repeatedly by constantly reducing the value of ε. The auxiliary parameter ε can also be chosen for different values according to the characteristics of each complementary condition, see the research of Ferris et al. [39]. Besides, Ban et al. [40] has performed extensive numerical experiments and found that on small and medium-sized networks, the value of M is generally between 5-20, which can obtain a better solution.

It should be noted that the algorithm adopted in this research can obtain a true solution for most of the cases. Even if the initial solution deviates significantly from the true solution, a larger auxiliary parameter ε can always ensure that equations (33)-(43) hold. Then, by adjusting the update factor λ and iteration parameter M, a sufficiently small auxiliary parameter ε^M will be obtained eventually. Besides, this paper adopts the NLPEC (nonlinear program with equilibrium constraints) solver by Ferris et al. [39] on NEOS. The NLPEC program uses the solution from the previous iteration as the initial solution for the next iteration, which ensures that the exact solution by Step 3 obtained for most of the cases. Note that all numerical examples in this article have obtained exact solutions through the final solve in Step 3. However, for larger networks, the algorithm cannot always guarantee to get a true solution.

B. ANALYSIS OF THE RESERVE CAPACITY OF ROAD NETWORK WITH DEMAND PATTERN CHANGES

In the Introduction, it analyzes that travelers' destination choice will affect the reserve capacity of road network in the presence of ridesharing. This numerical example explores how does the reserve capacity of road network change with travelers' destination choice.

The classical network used in [6], [13], [14], and [23] is chosen for the test network, shown in Figure 2. Suppose that trips generate at zonal A and C and attract at destinations B and D. The parameters settings and related functions are list in Table 1. According to Equation (5), the larger the attractor of a certain destination node, the more travelers will choose to reach that destination node. Let M_B denote the attractor of destination node B and M_D denote the attractor of destination node D. $|M_B - M_D|$ is the absolute value of the difference in attractiveness between the two destination nodes. When $M_B - M_D > 0$, it indicates that more travelers are attracted to node B, and more travelers are attracted to node D when $M_B - M_D < 0$. Therefore, changes of the demand pattern can be described by the change of the difference in attractiveness which reflects travelers' destination choice.

In addition, a zone has reserve capacity, which means that the land use of the zone has the potential for development [30]. Therefore, three situations are considered in this test. Only zonal A has a plan to develop their land use (situation 1), that is, trips generate at zonal A is variable, while trips generate at zonal C remains unchanged ($\mu(C) = 1$). Only zonal C has a plan to develop their land use (situation 2), that is, trips generate at zonal C is variable, while trips generate at zonal A remains unchanged ($\mu(A) = 1$). Both zonal A and zonal C have plans to develop their land use (situation 3), i.e., trips generate at zonal A and zonal C are both variable.

Figure 3 plots the change of demand multipliers and total demand with $|M_B - M_D|$ changes in different situations. It can be found that as the attractiveness of destination node B increases, the demand multiplier $\mu(A)$ in both situation 1 and situation 3 increases while the demand multipliers $\mu(C)$ in both situation 2 and situation 3 decreases. Besides, for total potential demand (i.e., the reserve capacity of road network), as the attractiveness of destination node B increases, the dotal potential demand in situation 2 decreases, while the total potential demand increases in both situation 1 and situation 3.

TABLE 1. Input data for the test network and parameter settings.

Link number	1	2	3	4	5	6	7
Free-flow time t_a^0 (Min)	2.0	1.0	2.0	3.0	1.0	2.0	1.0
Link capacity C_a (Veh/Min)	24.0	30.0	30.0	35.0	24.0	30.0	30.0
Acceptable link saturation degree p_a	0.9	0.9	0.9	0.9	1	1	1
Parameters of cost function	$\alpha = 0.5$	$\beta = 2$	$\rho = 0.8$				
Ride-sharing cost function	$\chi^u = 0.1$	$\gamma^u = 0.2$	$\chi^w = 0.1$	$\gamma^w = 0.2$	$\mu = 0.5$	$\xi = 0.1$	$\psi = 0.2$
Investment cost function	$G_a(y_a) = 3 * (y_a)^2$		$S_a = \sigma * t_a^0$				
Trip generation	$Q_A = 18$ Veh/min	$Q_C = 6$ Veh/min					

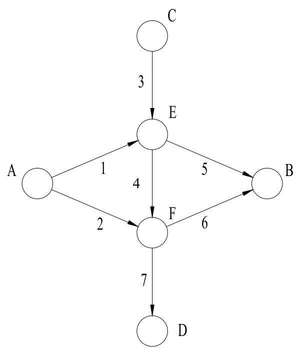


FIGURE 2. Small classical network.

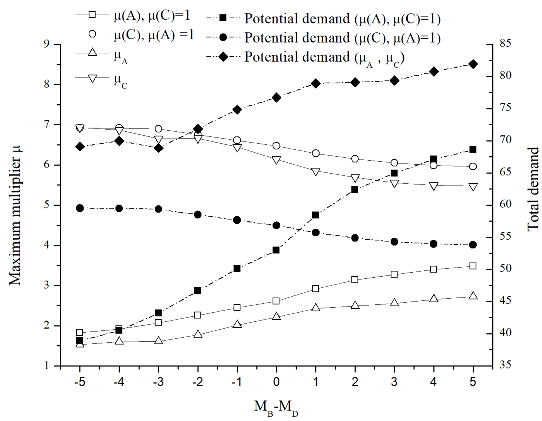


FIGURE 3. The reserve capacity changes as demand pattern varies.

These results suggest that travelers' destination choices have a significant impact on the zonal reserve capacity and the reserve capacity of road network in the presence of ridesharing.

Furthermore, it can also be found from Figure 3 that the total potential demand in situation 3 is always larger than that in both situation 1 and situation 2, which implies that developing multiple zones has more reserve capacity of road network than that by developing a single zone in the presence of ridesharing. This result is contrary to that of Xiao et al. [23] who investigates the reserve capacity of road network without ridesharing and finds that the total potential demand in situation 3 is always smaller than that in both situation 1 and situation 2. The reason for this opposite result in the presence of ridesharing may be that the model proposed in this research allows travelers with different zones to share ride, thus developing multiple zones helps to expand the size of the two-sided ride-sharing market (i.e., an increase in the number of ridesharing drivers and riders).

C. SUBSIDIZING RIDE-SHARING DRIVERS VERSUS ENHANCING LINK CAPACITY WITHOUT RIDESHARING

As stated in the Introduction, adding new capacity to the road network can significantly increase the reserve capacity of road network, such as building new streets or expanding existing streets. However, these methods are usually restricted by limited urban spatiality and financial constraints [2], [3]. This example explores how much reserve capacity can be improved by ridesharing compared to research of Gao and Song [13] who investigated the the reserve capacity of road network by expanding selected link capacity without ridesharing.

The *Model I* in [13] is chosen to calculate the reserve capacity of road network without ridesharing, in which the signal-controlled constraints at the upper level model are removed. To be consistent with the *Model I*, the proposed model in subsection III-D (hereafter referred to as *Model S*) will not consider the travelers' destination choice(i.e., Equation (22) needs to be removed in *Model S*) and the upper-level objective function (19) and the demand conservation (23) should be modified as follows,

$$\max_{\boldsymbol{x},\boldsymbol{\mu}} \sum_{i \in O^v, d \in D} \mu Q_i^d \tag{44}$$

$$0 \leq \left[\sum_{j:(i,j) \in A} (q_i^d(xy)) + q_i^d(z)) - \mu Q_i^d \right] \perp \pi_i^d \geq 0, \quad \forall i \in O, d \in D \tag{45}$$

where Q_i^d is given.

Parameter θ in *Model I* is a weight coefficient of link capacity investment which reflects the stress degree of planner on the two conflicting principles/objects of maximum reserve capacity and minimum investment [13]. Decrease in θ implies that the link capacity investment are more abundant. Meanwhile, parameter σ in *Model S* represents the government's subsidy level for ridesharing drivers. Increase in σ implies there is more fund to subsidy ride-sharing drivers. Since these two parameters both represent the level of financial support of the government, some performance indicators of the road network are compared under the changes of these two parameters, including the demand multiplier, the congestion time cost and the investment cost.

The test network is shown in Figure 2 as used by Gao and Song [13]. The parameter settings are listed in Table 1. The comparison results are listed in Table 2.

As can be seen from Equation (44), the value of the demand multiplier actually reflects the size of the reserve capacity of road network. From the change of the demand multiplier μ in Table 2, it can be found that when $\theta \leq 50$, the demand multiplier μ in *Model I* start to increase, which is consistent with the results in Gao and Song [13] and indicates that the calculation results is correct. Besides, as σ increase, the demand multiplier μ in *Model S* becomes larger, which indicates that subsidizing ride-sharing drivers can indeed improve the reserve capacity of road network. What's more, the demand multiplier μ in *Model S* is almost equal to that in *Model I* when $\sigma = 1$ and $\theta = 0.02$, which may suggest that subsidizing ride-sharing driver could increase as much reserve capacity of road network as enhancing link capacity without ridesharing.

From the change of the total congestion time cost in Table 2, it can be found that the total congestion time decreases as σ increase in *Model S*, while it increases as θ decreases in *Model I*. This result suggests that subsidizing ride-sharing drivers can reduce the total congestion time. The reason for this result is that more travelers choose to be ride-sharing drivers as σ increase, which in turn attracts more travelers to be riders who give up driving. Thus, ridesharing generates positive network externalities. However, in the absence of ridesharing, the potential travel demand will increase as θ decreases in *Model I*. In other words, the number of vehicles in the road network increases as θ decreases in *Model I*, so the total congestion time increases.

From the change of the total investment cost in Table 2, it can be found that the total investment cost increases exponentially with θ decrease in *Model I* while it increases linearly with σ increase in *Model S*. This result may imply that subsiding ride-sharing drivers is cost-effective than expanding link capacity without ridesharing. Note that this comparison only relate to numerical trends but not to numerical dimension. It is a very valuable research comparing the cost-effectiveness of infrastructure investments without ridesharing and subsidies with ridesharing, which should be studied in the future.

V. EXTENSION: THE IMPACT OF RETROFITTING HOT LANE ON THE RESERVE CAPACITY OF ROAD NETWORK

The results in subsection IV-C show that subsidizing ride-sharing drivers can improve the reserve capacity of road network. A natural question is whether incentivizing ridesharing necessarily increase the reserve capacity of road network. Previous studies have shown that retrofitting HOT lane can promote the participation of travelers in ridesharing [18], [19], [25], [26]. This section explores whether retrofitting HOT lane can increase the reserve capacity of road network since it doesn't require much investment.

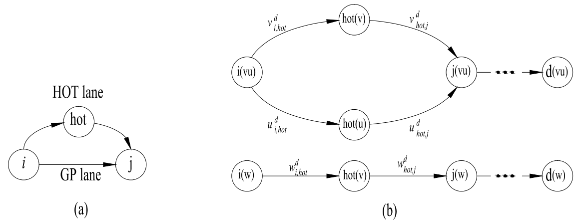


FIGURE 4. HOT lane representation in the extended network.

To answer such question, it needs to accommodate HOT lanes to the lower RUE model in Subsection III-D. There are two approaches to accommodate HOT lanes in RUE models. One is to introducing new flow variables and cost functions on HOT lanes, as in [18], [25], and [26]. The other is to add extra nodes and links into the original network to represent HOT lanes, as in [19] and [27]. The latter approach actually increases the nodes and links of HOT lane rather than the number of flow types of variables. Actually, these two approaches result in equivalent equilibrium flow patterns [19]. This paper adopts the second approach to accommodate HOT lanes.

Assume a link (i, j) is retrofitted into one HOT lane and one GP lane in the original network, and the performance of the HOT lane and the GP lane has not changed except that the capacity is half of the original link (i.e., $C_{ij}^{HOT} = C_{ij}^{GP} = \frac{1}{2}C_{ij}$) [19]. Let add a new artificial node *hot* between node i and j and two new links (i, hot) and (hot, j) in the original network to represent the retrofitted HOT lane, as shown in subgraph (a) of Figure 4. The new links,(i, hot) and (hot, j), together represent the retrofitted HOT lane, whereas the original link (i, j) represents the parallel GP lane. This is to differentiate the link identification (i.e., the GP lane and the HOT lane), but does not alter the properties of the problem. To improve the utilization of HOT lanes, solo drivers are usually allowed to use HOT lanes with a toll, whereas ride-sharing drivers and riders are free for charge [18], [19]. In order to distinguish the travel mode flows on HOT lanes, the artificial node *hot* is coped three times for each mode (i.e., solo driver, ride-sharing driver, and rider), since solo drivers and ride-sharing drivers are not permitted to switch roles at the artificial node *hot*(shown in subgraph (b) of Figure 4). Thus, the links (i, hot), (hot, j) are also coped three times as shown in subgraph (b) of Figure 4. Denote A^{HOT} as the set of copied new links of the HOT lanes in the extended network, N^{HOT} as the set of copied new artificial nodes on the HOT lanes in the extended network. Denote $0 < \ell < 1$ as the length proportion of link (i, hot) in the total length of link (i, j), then the length proportion of link (hot, j) is $1-\ell$. Denote y_{ij} as the toll on HOT lane for solo drivers, then the general travel costs along HOT lane (i, hot) are $\tilde{c}_{i,hot}^{v,HOT} = \ell(\tilde{c}_{ij}^{v} + y_{ij})$, $\tilde{c}_{i,hot}^{u,HOT} = \ell\tilde{c}_{ij}^{u}$, $\tilde{c}_{i,hot}^{r,HOT} = \ell\tilde{c}_{ij}^{r}$ for solo drivers, ride-sharing drivers and riders, respectively. And the general travel costs along HOT lane (hot, j) are $\tilde{c}_{hot,j}^{v,HOT} = (1-\ell)(\tilde{c}_{ij}^{v} + y_{ij})$, $\tilde{c}_{hot,j}^{u,HOT} = (1-\ell)\tilde{c}_{ij}^{u}$, $\tilde{c}_{hot,j}^{r,HOT} = (1-\ell)\tilde{c}_{ij}^{r}$ for solo drivers, ride-sharing drivers and riders, respectively. Consequently,

TABLE 2. Comparison of Gao and this model.

Parameters variation		Demand multiplier μ		Total congestion time cost		Total investment cost	
Model I(θ)	*Model S*(σ)	*Model I*	*Model S*	*Model I*	*Model S*	*Model I*	*Model S*
0.02	1	3.687	3.657	473.427	274.508	595.081	108.776
0.03	0.9	3.356	3.589	415.978	277.685	263.256	90.823
0.04	0.8	3.191	3.519	388.957	280.900	147.626	74.431
0.05	0.7	3.092	3.451	373.271	284.148	94.277	59.596
0.1	0.6	2.895	3.381	343.053	287.429	23.448	46.324
0.5	0.5	2.739	3.311	319.964	290.741	0.933	34.618
1	0.4	2.719	3.241	317.145	294.084	0.233	24.485
10	0.3	2.702	3.169	314.621	297.457	0.002	15.933
50	0.2	2.700	3.097	314.397	300.858	9.32×10^{-5}	8.979
100	0.1	2.700	3.017	314.368	304.067	2.33×10^{-5}	3.598

the complementarity condition for link choice on HOT lanes are as follows,

$$0 \le \left[\pi^d_{j(m)} + \tilde{c}^{m,HOT}_{i,j} - \pi^d_{i(m)}\right] \perp m^d_{ij} \ge 0,$$
$$\forall ij \in A^{HOT}, i \in N(m), j \in N^{HOT}, d \in D, m = \{v, u, w\} \quad (46)$$

$$0 \le \left[\pi^d_{j(m)} + \tilde{c}^{m,HOT}_{i,j} - \pi^d_{i(m)}\right] \perp m^d_{ij} \ge 0,$$
$$\forall ij \in A^{HOT}, i \in N^{HOT}, j \in N(m), d \in D, m = \{v, u, w\} \quad (47)$$

In addition, flow conservation should be also satisfied at the artificial node $i \in N^{HOT}$ as follows

$$0 = m^d_{ij} - m^d_{ki} \perp \pi^d_{i(m)} \ge 0,$$
$$\forall i \in N^{HOT}, k, j \in N, d \in D, m = \{v, u, w\} \quad (48)$$

Equation (48) implies that solo drivers and ride-sharing drivers are not permitted to switch roles at the artificial node $i \in N^{HOT}$.

Adding Equation (46∼48) into the lower-level RUE model in Subsection III-D along with ride-sharing match conditions on HOT lanes, it can get RUE model which accommodates HOT lanes.

This subsection executes the model of reserve capacity of road network with HOT lane on the above classical test network shown in Figure 2. Each link is successively retrofitted into one GP lane and one HOT lane one by one. In order to facilitate calculation and comparison, trip choice conditions (i.e., Equation (22)) are removed. The parameter settings remain unchanged. Figure 5 and 6 plot the number of times choosing to be ride-sharing drivers and the number of riders changes with toll increases, separately. From Figure 5, it can be seen that no matter which link is retrofitted into HOT lane, the number of times choosing to be ride-sharing drivers will increase as toll increases. Besides, the number of riders also increase as toll increases shown in Figure 6. These results indicate that retrofitting HOT lanes could incentivize travelers participating in ride-sharing, which is in line with the results of [18], [25], and [26]. Particularly, after links (F, D), (A, F) and (E, B) are respectively retrofitted into HOT lanes, the number of times choosing to be ride-sharing drivers is large within low charges, which tells us that retrofitting

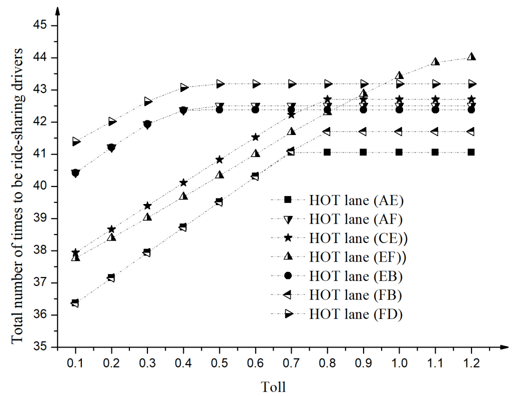


FIGURE 5. The number of times to be ride-sharing drivers changes as toll varies.

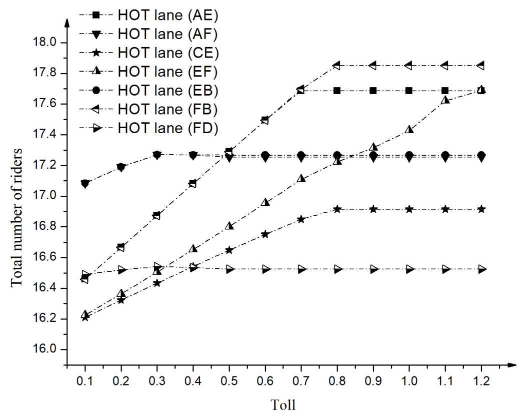


FIGURE 6. The number of riders changes as toll varies.

these links will encourage more travelers to share rides. Moreover, it can also be found that curves in Figure 5 and 6 gradually rise to a maximum and tend to be stable as the toll increases, which tells us that it is not possible to continuously increase the toll of the HOT lane to encourage more travelers participating in ride-sharing activities all the time.

Figure 7 plots the maximum travel demand changes as the toll increases, which reflects the change of reserve capacity of road network. It can be found that the reserve capacity decrease as the toll increases except for retrofitting the link (E, F) into HOT lane. For retrofitting the link (E, F), the reserve capacity of road network remains unchange when the

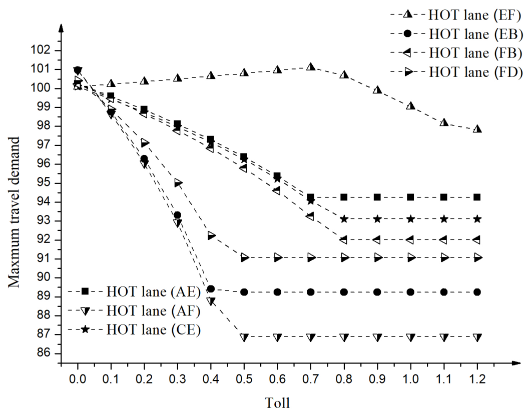


FIGURE 7. The maximum reserve capacity changes as toll varies.

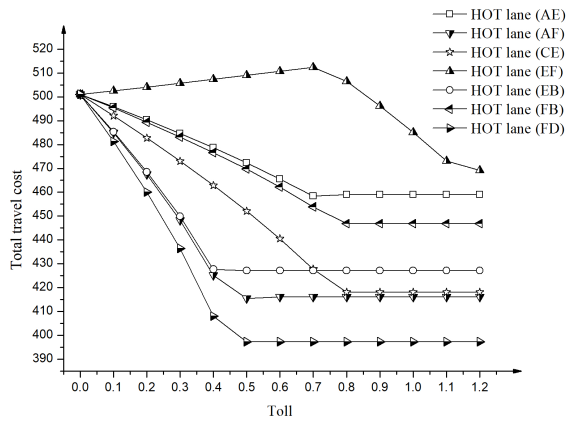


FIGURE 8. The total cost changes as toll varies.

toll is small and it decreases when the toll is large. These results indicate that retrofitting HOT lanes would not improve the reserve capacity of road network. This phenomenon seems to be unexpected, but in fact it makes sense. This phenomenon can be explained from the perspective of user equilibrium. When the toll is low, there are still a large number of solo drivers using the HOT lane. Suppose that the reserve capacity of road network remains unchanged when the toll increases. Then, the travel cost of solo drivers using the HOT lane increase while the travel cost of solo drivers using the GP lane remains unchanged, when the toll increases. As a result, user equilibrium is violated. Thereby, these solo drivers using HOT lane either turn to using the GP lane, or switch modes and become ridesharing participants, to achieve a new equilibrium. However, if they turn to using the GP lane, it would violate capacity constraint(i.e.,Equation(2)). Therefore, in this case, they seem to have to become ridesharing participants. Nevertheless, due to the inconvenience cost and the cost of balancing supply and demand in Equations (12-13), it cannot expect all these solo drivers switch to ridesharing participants. Overall, in order to achieve a new equilibrium, except for a small number of solo drivers using HOT lane who can switch to ridesharing participants, as shown in Figure 5 and Figure 6, the remaining solo drivers can only disappear, becoming a reduction of the reserve capacity of road network, as shown in Figure7.

Figure 8 plots the total travel cost changes as the toll increases. It can be found that the total travel cost decrease as the toll increases except for retrofitting the link (E, F) into HOT lane. For the link (E, F), the total travel cost almost remains unchange when the toll is small and it decreases when the toll is large. The trend of curves in Figure 8 is similar to that in Figure 7, which implies that maximizing the reserve capacity of road network may be same as minimizing system travel cost under some circumstances, as found in [16].

To examine the robustness of the proposed model, this subsection also executes the proposed model on a larger network. Note that the solution procedure introduced in subsection IV-A is better for small and medium-sized network,as Ban et al. [40] points out. For larger networks,

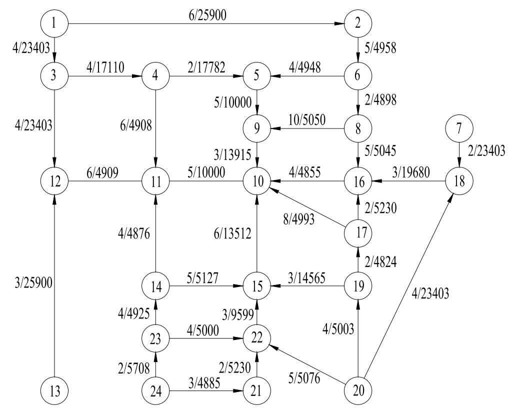


FIGURE 9. Sioux-Falls network.

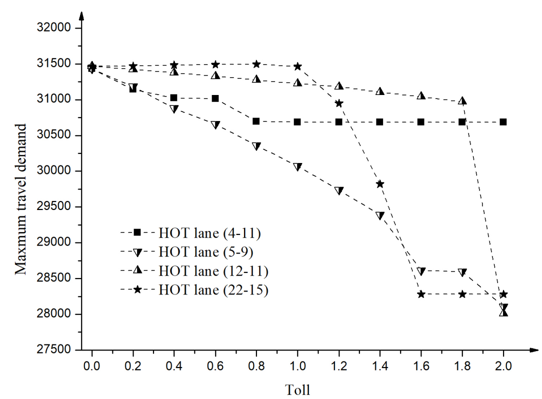


FIGURE 10. The reserve capacity changes as toll varies on Sioux-Falls network.

it cannot always guarantee to get a true solution. Fortunately, after extensive calculations, we still found the true solutions for retrofitting some links in the pruned Sioux-Falls network. The pruned Sioux-Fulls network was used in [32] shown in Figure 9, where links (4, 11), (5, 9), (12, 11) and (22, 15) are retrofitted successively into HOT lanes, separately. The link performance parameters have been marked in Figure 9 where the number before the symbol '/' is free-flow time and the number after the symbol '/' is the link capacity.

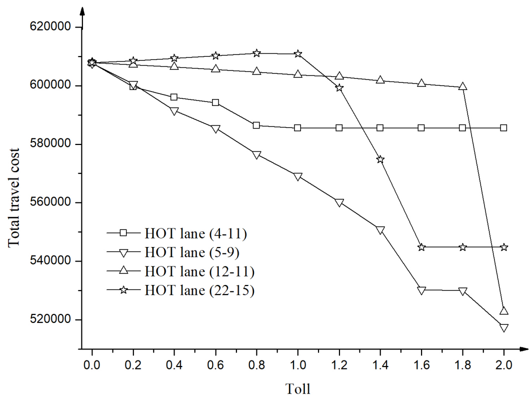


FIGURE 11. The total travel cost changes as toll varies on Sioux-Falls network.

Besides, suppose the travel demands generate from nodes 1, 2, 7, 13, 20 and 24 are 1300 veh/min, 600 veh/min, 1900 veh/min, 1900 veh/min, 2500 veh/min, 800 veh/min, respectively, and are attracted to node 10. The results are plot in Figure 10 and 11. It can be found that the reserve capacity of road network and total travel cost decrease as the toll increases. These results are identical to those in the small network (Figure 2), which indicates that the proposed model are robust.

VI. CONCLUSION

Against the backdrop of increasing urban population and decreasing land use of urban road construction, ridesharing may be the most effective way to improve the reserve capacity of road network to alleviate traffic congestion, apart from prohibiting driving. This paper investigates how ride-sharing affects the reserve capacity of road network. To this end, this research proposes a bi-level programming for reserve capacity of road network with ride-sharing which considers travelers' destination choices, ridesharing choice and path choice simultaneously. A conventional relaxation scheme is introduced to convert the proposed bi-level model to a non-linear programming which is solved by the NLPEC solver. The numerical experiments indicate that the demand pattern has a significant impact on zonal reserve capacity and the reserve capacity of road network in the context of ridesharing. Besides, this paper also compared the reserve capacity of road network by subsidizing ride-sharing drivers versus enhancing link capacity without ridesharing. The numerical results show that subsidizing ride-sharing drivers can improve as much reserve capacity of road network as that by enhancing link capacity without ridehsharing. Finally, this paper examines the impact of retrofitting HOT lanes on reserve capacity of road network. The numerical results show that retrofitting HOT lanes have limited impact on the network reserve capacity, although it may encourage more travelers to participate in ride-sharing. Therefore, it requires finding appropriate methods to incentivize travelers to participate in ridesharing to improve the reserve capacity of road network.

Future works should be extended in the following ways. Firstly, an algorithm for the proposed bi-level model should be developed so that it can be executed on a large-scale network. Secondly, the ridesharing trip-assignment model should limits the number of transfer times of passengers, which is more realistic. Finally, it should further research the reserve capacity of road network with users' multiple travel choices, such as the connection between ridesharing and public transportation.

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