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SURVEY

A Literature Survey on Sideslip Angle Estimation Using Vehicle Dynamics Based Methods

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ABSTRACT The vehicle sideslip angle or lateral velocity is a measure both for driving stability and for occupant's subjective perception of safety. With the introduction of vehicle dynamics control systems and automated driving functions, knowledge of this vehicle motion state is required for many control strategies. This article gives an overview on the state of the art on sideslip angle estimation. In contrast to other literature studies on this topic, it focuses on vehicle dynamics based algorithms. The following types of observers are discussed: Kalman Filter-type, recursive least squares (RLS), sliding mode observers (SMO) or nonlinear observers (NLO). Eventually, cascaded observers are used that first estimate some states, which then act as input to the sideslip angle estimator. Since the choice of an observer strategy always depends on the application, this article provides a brief insight into the work of selected research groups that have studied the topic. These examples will help to clarify the presence of many different approaches in the literature. A detailed discussion on vehicle and tire models is not included but referenced to other sources. Finally, this article provides recommendations for two main target groups: First, researchers and engineers that plan to design an algorithm for sideslip angle estimation using deterministic vehicle dynamics based approaches. Second, researchers and engineers planning to include an existing algorithm in an automated driving function that want to learn about advantages and limitations of these types of algorithms.

INDEX TERMS Lateral velocity estimation, literature survey, slip angle estimation, side-slip angle estimation, sideslip angle estimation, state estimation, vehicle motion state estimation, vehicle state estimation.

I. INTRODUCTION

Driving functions of all SAE automation levels require information on vehicle and environment states. Some of the required states are already measured in series-production vehicles, for example vehicles with Electronic Stability Control use measured information on the vehicle yaw rate, lateral acceleration and wheel speeds, see e.g. [1]. Not all relevant states like the vehicle sideslip angle are measured directly, either because it is too costly or existing sensors are not robust and reliable enough. The authors in [2] identified vehicle motion state estimation as one of the most challenging areas in automated driving concerning the topic of vehicle dynamics. According to the authors, main requirements are availability of estimates with high confidence, robustness and fault tolerance over the whole operation range of the vehicle.

The aim of this article is to give an overview on the state of the art on the estimation of the vehicle sideslip angle and the lateral velocity in the vehicle center of gravity (COG). This research area has a considerable size and can be viewed from different angles, many state of the art articles are available. The present article is an extension of the following articles:

- **Grip et al. 2009, [3]** discuss design, implementation and experimental validation of longitudinal velocity and

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sideslip angle estimation. In the course of their paper, they discuss the state of the art and challenges in sideslip angle estimation, such as influence of road inclination, body orientation and unknown surface conditions.

- **Chindamo et al. 2018, [4]** present a literature review on sideslip angle estimation and distinguish between observer-based methods and neural network-based approaches. Concerning observer-based methods, Luenberger observer, sliding-mode observer, and different variants of Kalman filters are discussed. Furthermore, methods using GPS measurements in addition to vehicle dynamics sensors are discussed.
- **Guo et al. 2018, [5]** show a literature review on vehicle dynamic state estimation and focus on the states vehicle velocity, sideslip angle, yaw rate and roll angle. Typical vehicle models, observer structures and sensor configurations are discussed.
- **Singh et al. 2018, [6]** The authors show a literature review on dynamic state estimation and focus on the following vehicle and tire states: tire forces, road profile and slope, vehicle velocity, roll angle, tire cornering stiffness, friction coefficient, and vertical states for active suspension controls such as the motion of sprung and unsprung masses, as well as wheel speed signal analysis. Intelligent tires and wheel bearing for load sensing are touched upon.
- **Jin et al. 2019, [7]** show literature review on dynamic state estimation and distinguish in model-based estimation and in data-driven based estimation, which include neural network approaches. Model-based estimation is distinguished between filter-based and observer-based methods. In that article, most cited literature treats sideslip angle estimation, but also literature on friction coefficient, longitudinal force, horizontal tire forces, cornering stiffness, and the roll motion of the vehicle is also included. Vehicle models, sensor configuration and estimation techniques are discussed.

Unlike the articles mentioned above, this article distinguishes as follows: first, it classifies based on the types of observers used. Second, only vehicle dynamics based approaches are considered, meaning that methods e.g. only relying on optical sensors are not included because they are too expensive for series application and/or not accurate and reliable enough for safety-critical applications. Third, artificial intelligence or neural networks are not considered as the main observer strategy. While the excluded approaches show a lot of potential, the remaining state of the art is still very large. In addition, it is assumed that future observer strategies using sensor fusion will include a vehicle dynamics-based method. Unlike the aforementioned articles, this article will not go into detail about typical vehicle and tire models. While the choice of these models is important for the further observer strategy, the state of the art on its description is comprehensive. Concerning tire models for online estimation purposes, the interested reader is referred to [8], [9], [10], and [11]. Concerning vehicle models, see e.g. [5], [7], and [11]. With few exceptions, this article focuses on the literature since 2011.

Sideslip angle estimation is also often linked to tire-road friction estimation, as its influence on vehicle dynamics cannot be separated. Further investigations show that this also applies to the road banking angle, see [12]. Existing approaches therefore make assumptions about road conditions and therefore apply, for example, only to dry roads with high grip. Others estimate these influencing conditions, mostly with cascaded observer strategies, where several non-measurable driving conditions are estimated step by step. This article does not go into detail about maximum tire-road friction and road slope estimation because of the large scope, but mentions where sideslip angle estimation methods are affected.

The remainder of this article is structured as follows: Section I-A discusses the relevance of the sideslip angle for vehicle dynamics and how the sideslip angle depends on other quantities that are often not measured or unknown during the operation. Then, an overview of the evolution of states is given for selected research groups in the field of sideslip angle estimation. It shows the variety of approaches existing and gives an indication on the application field of the sideslip estimates aimed by the respective authors. In Section II, an overview on lateral velocity and sideslip angle estimation using model-based observers is given. The majority of published methods use a Kalman Filter-type observer. Alternatively, recursive least squares (RLS), sliding mode observers (SMO), nonlinear observers (NLO) or cascaded observer strategies are used. Finally, this article concludes with recommendations for two main target groups of readers: Researchers and engineers that plan to design an algorithm for sideslip angle estimation using deterministic vehicle dynamics based approaches; Researchers and engineers planning to include an existing algorithm in an automated driving function that want to learn about advantages and limitations of these types of algorithms.

A. RELEVANCE OF SIDESLIP ANGLE AND INFLUENCING FACTORS

The sideslip angle β for the vehicle is defined as the angle between the vehicle center plane and the velocity vector and shown positive in Fig. 1 according to ISO 8855. The sideslip angle can be expressed as a function of the longitudinal velocity v_x and the lateral velocity v_y by

$$\beta = \arctan \frac{v_y}{v_x}. \tag{1}$$

Taking this into account, sideslip angle estimation is equivalent to lateral velocity estimation.

Reference [1] shows how vehicle stability in lateral direction depends on β by using the β-method that was developed by [13]. To drive a vehicle around a curve, the steering system is usually actuated, which generates lateral tire forces and thus a yaw moment on the vehicle. At large slip angles, however, the steering angle can hardly change

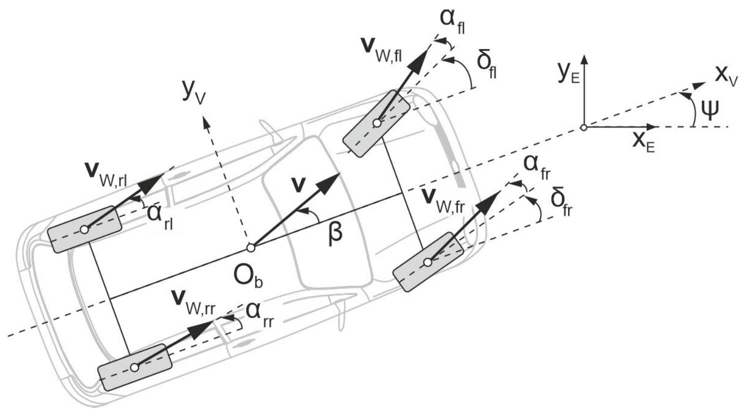

FIGURE 1. Definition of the sideslip angle β and the wheel slip angles α_i. All angles defined positive in accordance to ISO 8855 and figure based on [124].

the yaw moment anymore. According to [1], the ability to steer a vehicle is almost lost at the physical limits, which is about $\beta = \pm 12°$ on dry asphalt roads and $\beta = \pm 2°$ on ice, depending on the vehicle velocity, e.g. previous considerations are not valid in low speed maneuvers like in parking conditions. The range of the stable vehicle motion not only depends on β, but also on the maximum friction coefficient $\mu^{\max}$ between tire and road. [14, p. 81-82] showed that both when using the $(\beta, \dot{\psi})$-phase plane and the $(\beta, \dot{\beta})$-phase plane for determining the stable vehicle motion, the stable area reduces with decreasing $\mu^{\max}$. With β being an important indicator for vehicle stability, it is extremely important to measure or estimate it to properly control the car to avoid instability.

Strategies to estimate β often rely on knowledge or estimation of other states, or make simplifications for their conditions. Typical examples are the kinematic states: road slope, road banking angle as well as pitch and roll motion of the chassis. These quantities can affect β states estimation, especially when the estimation strategy depends on either of the following effects:

- **Maximum tire-road friction coefficient $\mu^{\max}$:** The transmissible force of a tire depends on both $\mu^{\max}$ and the slip angle. The same force can be achieved with different combinations of these two quantities. Separating these two influences is challenging but necessary when using the measured lateral acceleration and a tire model for sideslip estimation. Some approaches therefore use a constant road condition, typically dry and high grip roads, or assume to get the road condition as an input to the sideslip angle estimation algorithm. Approaches to separate both quantities exist, for example, see e.g. [15], where both quantities are estimated simultaneously: while at small slip angles the influence of $\mu^{\max}$ is negligible, at very high slip angles the force depends more on $\mu^{\max}$ and less on the slip angle.
- **Measurements of vehicle horizontal accelerations:** On a horizontal road surface, the dominant forces acting on a vehicle in lateral direction are the tire forces while other forces, such as wind and air resistance, can often be neglected. Thus, using Newton's second law and having an assumption on the vehicle mass, the tire forces can be determined directly from horizontal acceleration measurements [3]. However, a gravitational component is added to the horizontal acceleration measurements both due to road slope/banking and chassis pitch/roll relative to the road plane. The difficulty of separating the influence of a low friction surface and road banking angle is discussed in [1] for the application of sideslip angle estimation within ESC. These findings were confirmed by [3] who could show that the estimation problem is poorly conditioned when mainly relying on vehicle accelerations as measurement input. Thus, estimator stability cannot be guaranteed for combined estimation of sideslip angle, maximum tire-road friction coefficient and road banking angle. For a detailed discussion on this topic, interested readers are referred to [12].
- **Dynamic wheel loads:** The change in wheel load is influenced by vehicle accelerations, pitch and roll, road inclinations or road irregularities. For an overview on estimation techniques for vertical forces see [10] and typical models used see [11].

Whether and to which these states influence β estimation, however, strongly depends on the specific estimation strategy. Different strategies from different research groups will be discussed in the next section.

B. EVOLUTION OF RESEARCH FOCUS IN SIDESLIP ANGLE ESTIMATION

In the following paragraph an overview of the research activities of selected research groups in the field of sideslip angle estimation and related topics will be given. In addition to the estimated states, sensor configurations, models, and observer strategies used will be discussed. This section will show the evolution of researcher's strategies over time, especially on the observers used and the assumptions made on the vehicle and the tires.

The research group gathered around **Charara** from Université de Technologie de Compiégne, France, has its main focus on sideslip angle estimation (see [16], [18], [20], [21], [22], [23], [77], [80]), on longitudinal and lateral tire-road forces ([17], [18], [20], [21], [22], [23], [77], [80]), on vertical forces ([19], [20], [21]) and on friction coefficient estimation ([18], [22], [77]). According to these articles, more accurate estimation is obtained with precise knowledge of quantities like vehicle mass, load transfer, and tire cornering stiffness. To achieve this, most articles use so-called cascading observers as in [18], [19], [20], [21], and [23]. Cascading observers are multi-step estimation processes where there are multiple states estimated beforehand to make the final estimate as accurate as possible. The authors use both single track and four wheel vehicle models and explored several different tire models in the observer strategies: linear, Dugoff, Burckhardt, Pacejka, adaptive linear, and adaptive Bruckhardt/Kiencke model. Various types of observers are implemented and tested: linear Kalman filter (KF), extended Kalman filter (EKF), unscented Kalman filter (UKF), particle filter (PF), linear Luenberger observer (LO), extended Luenberger observer and sliding mode observer (SMO).

Similar to Charara, the research group around **Cheli and Sabbioni** from Politecnico di Milano, Italy, focuses on sideslip angle estimation. In contrast to the cascaded observers with many individual unmeasured quantities, a simple and robust model seems to be the choice here. The researchers combine a kinematic and a model-based approach using fuzzy logic [24]. While the previous research group used only sensors from series production vehicles, the addition of GPS measurements for sideslip angle estimation is explored here [25], [26]. In [27], authors use tires equipped with sensors (smart tires) to improve the performance of the EKF-based observer.

Gerdes from Stanford University, USA, and his group based their research on improving simple sideslip angle observers that use KF via additional measurements. Said measurements are either GPS-obtained velocities in [28], [29], and [30] or steering torque in [30] and [31]. Similar to Cheli and Sabbioni, in [30] authors combine simple kinematic and physical model approaches. However, in this case, the measurements are reinforced with GPS data. The use of steering torque information is proposed to improve estimation, considering that the signal is never lost and is inexpensive contrary to GPS [31], [32]. Findings show that the combination of steering angle, yaw rate, and steering torque measurements is comparable to GPS while estimating sideslip angle.

Gadola and Chindamo, from University of Brescia in Italy, proposed an EKF to estimate vehicle sideslip angle and lateral tire forces [91]. Instead of a mathematical tire model, the authors use a two-dimensional lookup table based on Pacejka curves. In [33], a neural network is proposed for sideslip angle estimation. Furthermore, a comparison between EKF and neural network approach was done in [34]. The neural network approach demands rigorous training and it might not work well in unpredicted and untrained situations. On the other hand, the EKF requires accurate models and knowledge on several constantly changing vehicle parameters.

The research group gathered around **Fujimoto and Hori** from The University of Tokyo, Japan, employs Recursive Least Squares (RLS) method to estimate sideslip angle, tire-road forces, and cornering stiffnesses [35], [36]. Non-standard sensors are used for the improvement of estimation accuracy. For example, in [83] authors use sensors for lateral tire forces. Also, they introduce a combination of linear single track and camera-based vision models. Furthermore, the Multirate Kalman filter is introduced, which is useful when there is a mismatch (disparity) between sampling frequencies between measurements [37], [38], for example, when GPS measurements are combined with sensor measurements from the vehicle.

A research group around **Diaz and Boada** from Universidad Carlos III de Madrid, Spain, based their work on combining neural networks with traditional observers. The benefits of both neural networks and fuzzy logic are incorporated in ANFIS (adaptive neuro-fuzzy inference system) [39], which authors use extensively. Specifically, they explore a combination of neural network and unscented Kalman filter for sideslip angle estimation [110], and linear Kalman filter for roll angle estimation [40]. Furthermore, authors propose a fuzzy logic method that uses smart tires to estimate slip angle and tire working conditions [41].

II. LATERAL VELOCITY AND SIDESLIP ANGLE ESTIMATION

Over the last few decades, different techniques have been developed to tackle the problem of sideslip angle estimation. In the following, a review of observer-based methods frequently used for sideslip angle estimation is given. These include Luenberger observer (LO), recursive least squares (RLS) estimation, sliding mode observer (SMO), different variants of Kalman filter (KF) and cascaded observers. In this publication, the methods are referred to as cascaded observer, which consider multi-step observation schemes with several secondary states being estimated before a primary state is finally determined. Besides traditional state observers, Artificial neural networks (ANN) are also often utilized in cascading structures. Other methods, like first-order Stirling's interpolation filter (DD1), also appear in the literature.

Used vehicle models are usually categorized as either dynamic (physical) or kinematic. Both groups have certain drawbacks. Dynamic models depend on knowledge of vehicle and tire parameters as well as road conditions. Some of these parameters are time-varying and dependent on external factors. This makes it hard to predict their values. The kinematic model works independently of said parameters but is sensitive to sensor noise and bias and generally produces a noisier estimate as shown by [78], [79], and [98]. Dynamic approaches often use single track (or bicycle) model and four wheel (or two track) model, with many variants of both.

Most methods use measurements available in series production cars such as steering angle, yaw rate, longitudinal acceleration, lateral acceleration, and wheel speeds. Longitudinal velocity is often calculated using wheel speeds or also estimated. Some researchers utilize GPS to improve estimation. The most used GPS measurements are course angle, velocities, and position. Some approaches estimate tire forces to improve primary states, see [10], and some use force sensors from smart tires (sensor-equipped tires), see e.g. [123].

Researchers in [78], [79], [81], [98], and [105] have shown that observability problems occur when there is no lateral excitation. To avoid this problem, authors in [82] proposed heuristic scheduling that combines the kinematic model and empirical information. Researchers in [78] merge a kinematic approach and dynamic formulation when observability is lost. In [105], lateral forces and sideslip angle are set to zero either when the steering angle or when v_x is zero. Authors of [98] use a measure of the degree of nonlinearity to provide a smooth transition between operating conditions.

A. LUENBERGER OBSERVER

Luenberger state observer (LO) [44] is a deterministic, closed feedback loop observer that uses the error between predicted and measured states to improve estimation quality. In the linear case, it can show fast convergence, [45], but exhibits a strong dependency on the used mathematical model and is sensitive to variations in vehicle parameters, nonlinearities, and uncertainties. This limits its application to parameter-invariant systems according to [45] and [46]. Eigenvalue assignment is used to tune the observer gain. The linear form of LO applies to linear, time-invariant systems [44], but can be extended for time-variant and nonlinear systems, [47]. An LO variant is the extended Luenberger observer (ELO), which is based on an extended Jacobian linearization of the error dynamics, [48].

LO is often used for system monitoring and regulation [46]. The authors of [50] did a comparative study of observers for sensorless vector control of induction motor drives. They found that LO is ideal for steady-state performance and low-speed operation. Concerning sideslip angle estimation, the few applications are shown in Table 1. This table compares the observer models used with regard to estimated states, model inputs, measurements and type of vehicle model (VM) and tire model (TM) used. The sideslip angle is either estimated together with the longitudinal velocity v_x or the yaw rate $\dot{\psi}$. Typical inputs and/or measurements are the steering angle δ, $\dot{\psi}$, v_x and horizontal accelerations a_x and a_y are used. Simple single track vehicle models or kinematic approaches are applied. The last column in Table 1 provides information on whether the authors validated their method with measurements (M) or simulations (S).

ELO shows problems with convergence with initial states that are not optimal, [80]. States are unobservable when there is no lateral excitation, that is when the yaw rate is zero as shown by [79] and [81]. In [82], heuristic scheduling is used to prevent this.

B. RECURSIVE LEAST SQUARES ESTIMATION

Recursive Least Squares is an adaptive parameter estimation technique that minimizes a quadratic cost function. The algorithm is updated at each step with new data points [42]. RLS converges fast, however, its computational complexity is high [43]. Table 2 shows the application of RLS for sideslip angle estimation and compares estimated states, measurements, as well as vehicle and tire models used in the observer. Only the article [83] used RLS for v_y estimation. The measurements and models used are similar to LO. More often it is used to estimate secondary states or parameters needed for sideslip angle estimation, like cornering stiffness [107], [108] or vehicle mass, [117].

C. SLIDING MODE OBSERVER

The sliding mode observer is also a deterministic, closed loop feedback observer. Different from other observers, the SMO has a sign function of error in the correction term or gain. This discontinuous element forces the convergence of the observer in finite time which represents an advantage in comparison with other observers, see [49], [50], and [53]. A high level of robustness against parameter variations, modeling errors, uncertainties, and disturbances is one of the main advantages of SMO, see [46], [49], [52], [54], and [55]. SMO has weaker observability demands in comparison with other observers, [49]. SMO exhibits high sensitivity to a choice of observer gain as a consequence of the so-called chattering phenomena, see [46] and [55]. Adequate gain must be chosen, so that trade-off between the stability and robustness of the observer is taken into account, [55]. SMO has linear and nonlinear forms.

Considering its property of robustness, it is no surprise that the sliding mode observer is often used in fault detection and fault reconstruction, see [49] and [56]. In a comparative study of observers concerning the sensorless vector control of induction motor drives in [50], the authors concluded that the general performance of a SMO is similar to the performance of a Luenberger observer. In the field of sideslip angle estimation, the advantages of sliding mode observer hold. An overview of approaches is given in Table 2. Estimated states, measurements, vehicle and tire models, as well as validation with simulations or measurements are again compared in this table. Compared to LO and RLS, it has to be mentioned that the vehicle models are more complex and show a higher degree of freedom; Dugoff tire model is often applied because of its simpler equation and low computational load compared to Pacejka MF tire model. Estimated states and measurements used are typically similar to LO and RLS methods we have seen so far.

The observer is shown to be robust and to have a fast convergence according to [58], [59], and [57]. However, a chattering (oscillation) problem occurs as shown by [57].

D. KALMAN FILTER AND VARIANTS

Kalman filter is a widely used method with application in almost all engineering fields according to [61]. It is an

TABLE 1. Luenberger observers used for sideslip angle estimation. Abbreviations: VM: Vehicle model, TM: Tire model, S: Simulations, M: Measurements.

Type	*Paper*	*Estimated states*	*Inputs*	*Meas.*	*Model*	*Val.*
LO	[79]	v_x, v_y	a_x, a_y	v_x	kinematic VM	S
		$v_y, \dot{\psi}$	δ	$a_y, \dot{\psi}$	single track VM	
	[80]	$\beta, \dot{\psi}$	δ	$\dot{\psi}$	single track VM, linear TM	S, M
	[82]	v_x, v_y	a_x, a_y	v_x	kinematic VM	M
ELO	[80]	$v, \beta, \dot{\psi}, F_{x1}, \dot{F}_{x1}, F_{x2}, \dot{F}_{x2}$	δ	$\dot{\psi}$ v $v, \dot{\psi}$	single track VM, lat. TM linear, long. force random walk	S, M

TABLE 2. Sideslip angle estimation approaches using recursive least square estimation (RLS), sliding mode observers (SMO) and nonlinear observer (NLO). Abbreviations: VM: Vehicle model, TM: Tire model, S: Simulations, M: Measurements.

Type	*Paper*	*Estimated states*	*Meas.*	*Model*	*Val.*
RLS	[83]	v_y	$\delta, v_x, \dot{\psi}, F_{yij}$	single track VM, linear TM	M
SMO	[80]	$\beta, v, \dot{\psi}, F_{x1}, \dot{F}_{x1}, F_{x2}, \dot{F}_{x2}$	$\delta, v, \dot{\psi}$	single track VM, long. force random walk, lat. TM linear	M
	[84]	v_x, v_y	$\delta, v_x, a_x, a_y, \dot{\psi}$	7 DOF kinematic VM	S
	[85]	$v_x, v_y, \omega_{ij}, \dot{\psi}$	$\delta, v_x, \omega_{ij}, \dot{\psi}$	5 DOF four wheel VM	S, M
	[86]	$v_x, v_y, \dot{\psi}$	$a_x, a_y, \dot{\psi}$	3 DOF four wheel VM, Dugoff TM	S
NLO	[87]	$v_x, v_y, \dot{\psi}$	$\delta, \omega_{ij}, \dot{\psi}, a_x, a_y$	3 DOF four wheel VM, Dugoff TM	S

algorithm based on recursive Bayesian estimation framework that estimates the states using propagation of mean and covariance through time and was developed originally by [60]. A detailed description is given by [61] and [62]. In contrast to deterministic LO and SMO, the Kalman filter utilizes system and measurement noise and uncertainties, see also [61]. In [50], a comparison of LO, SMO, and KF in the application of sensorless vector control of induction motor drives is analyzed. They concluded that while the KF is superior in its robustness to noise, it is also the most complex observer to implement and tune.

The most popular extensions of the Kalman filter for nonlinear systems are the extended Kalman filter (EKF) and the unscented Kalman filter (UKF). EKF is based on the linearization of nonlinear models around the current estimate, see [61] and [63]. Despite being simple and computationally efficient for nonlinear systems as shown by [64], problems appear if the linearizations are not a good approximation of a nonlinear model or if the Jacobian matrix does not exist. Furthermore, even if Jacobian exists, it can be difficult and computationally expensive to calculate, see [63] and [65]. Unscented Kalman filter (UKF) represents another extension of KF created to avoid the limitations of the extended Kalman filter as discussed by [63]. It is based on unscented transformation, which uses a set of sample points (called sigma points) that can appropriately capture real means and covariances of probability distributions. UKF improves the performance of EKF in a variety of applications as shown by [61], [66], and [67]. Sometimes, however, EKF is shown to be superior to UKF concerning speed and thus computational time, see [68], [69], [70], and [71].

It is not surprising that KF is the most used in vehicle sideslip angle estimation. A linear Kalman filter is used for lateral velocity estimation in [76] and [79]. Authors in [51] tested four different observers (KF, EKF, SUKF, and SCKF) with three tire models (linear, Dugoff, and Magic Formula). They found that even though nonlinear observers outperform the linear one, there is no significant difference between nonlinear observers themselves. Tables 3 and 4 show a recent development of sideslip angle estimation based on KF approaches. The estimated states often include lateral tire forces or cornering stiffnesses, and sometimes vertical tire forces. Inputs and measurements are similar to LO, RLS and SMO. Very often, wheel speeds are additionally used and in some cases information on the wheel torques. The vehicle models used vary from kinematic over single track to different four wheel vehicle models with up to 8 degrees of freedom. Tire models used range from linear over random walk force models to Dugoff and Magic Formula.

Further adaptations and extensions of the KF are used. Adaptive EKF (AEKF) represents EKF that is modified in some way. For example, in [98] covariance matrices are adaptive and depend on the current behavior of the vehicle. Multirate Kalman filter (MRKF) is used when there is a disparity between sampling frequencies between measurements, see [38]. If vehicle model adaptation with disturbance accommodation is added to MRKF, the disturbance accommodating multirate Kalman filter (DAMRKF) is achieved, see [89]. Interacting multiple model (IMM) combines two models of vehicle-road interaction to enhance accuracy. IMM-EKF/UKF filter combines IMM with EKF/UKF, see [99]. Cubature Kalman filter (CKF) works on a principle of spherical-radial cubature rule, that provides easier calculation of integrals in the nonlinear Bayesian filter. There are different variations of CKF. Square-root cubature Kalman filter (SCKF) represents a square-root extension of the regular cubature Kalman filter. Square-root cubature based receding horizon Kalman filter (SCHRKF) estimates are based on a finite number of measurements over a moving horizon. This filter has an FIR structure as shown by [74]. Fuzzy Adaptive Robust Cubature Kalman Filter (RCKF) combines fuzzy logic with cubature Kalman filter, see [117]. Double Cubature Kalman filter (DCKF) utilizes single-value decomposition to improve the standard cubature Kalman filter, see [75]. UKF filter that takes into account constraints on states is called Constrained UKF (CUKF), see [103]. Event-triggered Kalman filter (ETKF) utilizes an event-triggered mechanism and Kalman filter to eliminate integration errors caused by noise and sensor bias during straight-line driving as shown by [119].

Strong tracking Kalman filter (STKF) represents a of the Kalman filter that utilizes time-varying suboptimal fading factors to predict covariance. To predict these factors, fuzzy adaptive strong tracking Kalman filter (FASTKF) can utilize fuzzy logic, as shown by [73].

Tables 3 and 4 show a recent development of sideslip angle estimation based on KF approaches.

E. PARTICLE FILTER AND OTHER NONLINEAR OBSERVERS

The particle filter (PF) is a probability-based, nonlinear state estimation method. Similar to the Kalman filter, it is based on Bayesian state estimators, see e.g. [61, p. 462-466]. A random set of points is generated, which are called particles and from which the filter takes its name. These point represent a posterior distribution. Similarly to UKF, these points are transformed to obtain the mean and the covariance of the estimate. PF works better than a Kalman filter for systems that are highly nonlinear. Its downside is that it is computationally expensive. For a more detailed description of particle filters, see [61]. Publication [96] examines application of particle filter on the sideslip angle estimation problem. Detailed description can be found in Table 3.

In some cases, researchers propose an estimation scheme constructed to directly deal with nonlinear vehicle models. These observers do not fit the other observer types discussed and are referred to as nonlinear observers (NLO), see e.g. [87]. Newer developments of nonlinear observers are shown in Table 2.

F. CASCADED OBSERVERS

Previously mentioned methods require knowledge of observer and vehicle parameters that are often hard to obtain in real time. Also, it is sometimes more convenient to estimate some unmeasurable states and treat them as a measurement to a sideslip angle estimator. Even if the state is measured, sensor noise can present a problem for some observers. Consequently, researchers developed several techniques that estimate mentioned parameters or states and use them as input for sideslip angle estimation. The most commonly estimated variables are cornering stiffnesses, and lateral and vertical forces. In this paper, methods that contain multiple observers are called cascaded. Tables 5 and 6 provide insight into the recent work based on cascaded techniques. Different to the methods discussed so far, the observer strategies involve several steps. Often, cornering stiffnesses or lateral tire forces are estimated in a prior step. Some methods also require the vertical tire force prior. In some cases, the longitudinal velocity is also estimated a priori. Different steps require different measurements or estimates from the prior step. Tire and vehicle models are similar to those for LO, RLS and SMO. In comparison to KF-type observers, the vehicle models show a lower number of degrees of freedom. This can be explained by the fact that the different steps in the cascade use different model assumptions, not requiring one model to account for all relevant effects. Unfortunately, a high number of articles only showed simulations to validate their methodology, thus not allowing to analyze their applicability to the operation with real measurements.

III. DISCUSSION AND RECOMMENDATIONS

This article aims to draw conclusions for further development from the comprehensive and large state of the art on sideslip angle and lateral velocity estimation. As shown in the preceding sections, around 100 articles were published on this topic since 2011, even when excluding those using artifical neural networks. It is hard to keep track on the results of these investigations. Thus, in this section, conclusions and recommendations from previous analysis are drawn. Findings concerning observers used, sensor configuration and states to be included in the observer strategy are distinguished.

A. OBSERVERS FOR SIDESLIP ANGLE ESTIMATION

The majority of published approaches for sideslip angle estimation are based on Kalman Filter-type observers (statistic estimation methods). For sideslip angle estimation, nonlinear Kalman filter types such as EKF or UKF can be quite accurate. In addition, to reach accurate results, Q and R matrices have to be adaptive in dependence of the current driving maneuver or in which range of the tire force-slip-characteristics the tires are currently operated. This can, for example, be done with adaptation of a (extended) Kalman filter. Cubature Kalman filters allow a simple calculation of nonlinear integrals, with different extensions for faster convergence being published. The advantage of Kalman filter and its variants is their optimal/sub-optimal performance compared to the other algorithms while maintaining simple implementation and structure. However, they can be

TABLE 3. Sideslip angle estimation appraoches using Kalman filter (KF) and its variants, as well as particle filter (PF). Abbreviations: VM: Vehicle model, TM: Tire model, S: Simulations, M: Measurements.

Type	*Paper*	*Estimated states*	*Inputs*	*Meas.*	*Model*	*Val.*
KF	[88]	$\beta, \dot{\psi}, \psi, b_{gyro}$	δ	$\dot{\psi} + b_{gyro}, \nu_{GPS}$	Single track VM, linear TM	S, M
DAMRKF	[89]	$\beta, \dot{\psi}, \psi, d_1, d_2$	δ, M_z	$\dot{\psi}, \nu_{GPS}$	single track VM, linear TM	M
EKF	[90]	v_x, v_y	a_x, a_y	v_x, v_y	Kinematic VM	S
	[91]	$\beta, \dot{\psi}, F_{y1}, F_{y2}$	δ, v_x	$a_y, \dot{\psi}$	Single track VM, look-up table using Pacejka TM	S, M
	[92]	$v_x, v_y, \omega_1, \omega_2, F_{x1}, F_{x2}, F_{y1}, F_{y2}$	δ, T_{B1}, T_{B2}	$\omega_1, \omega_2, v_x, a_x, a_y, \dot{\psi}$	Single track VM, Random walk force model	M
	[93]	$x, y, v_x, v_y, \psi, \dot{\psi}, F_{y1}, F_{y2}$	δ, F_{xij}	$x_{GPS}, y_{GPS}, v_{x,GPS}, v_{y,GPS}, a_x, a_y, \psi, \dot{\psi}, \beta_{GPS}$	Four wheel VM	S
	[94]	$\beta, \dot{\beta}, \dot{\psi}, \ddot{\psi}, C_{y1}, C_{y2}$	δ	$a_y, \dot{\psi}$	Single track VM, lin. adaptive TM	S
	[95]	$\beta, \dot{\beta}, \dot{\psi}, \ddot{\psi}, c_{1i}, c_{2i}$	δ, v_x	$a_y, \dot{\psi}$	Single track VM, rational TM	M
	[96]	$v_x, v_y, \dot{\psi}, F_x, F_{y1}, F_{y2}$	δ	$a_x, a_y, \dot{\psi}, v_x$	Single track VM, Random walk force model	S
AEKF	[97]	$v_x, v_y, \dot{\psi}, \theta_{F_x}, \theta_{F_y}$	$\delta, \omega_{ij}, a_x, a_y$	$v, a_x, a_y, \dot{\psi}, \theta_{F_x}, \theta_{F_y}$	Four wheel VM, Magic Formula TM	M
	[98]	$v_x, v_y, \dot{\psi}, C_{y1}, C_{y2}$	δ, a_x	$v, a_y, \dot{\psi}$	Single track VM, lin. adaptive TM	M
SCKF	[96]	$v_x, v_y, \dot{\psi}, F_x, F_{y1}, F_{y2}$	δ	$a_x, a_y, \dot{\psi}, v_x$	Single track VM, Random walk force model	S
IMM-EKF	[99]	$\beta, v, \dot{\psi}, \phi, \dot{\phi}, F_{xij}, F_{yij}$	δ, ω_{ij}	$a_x, a_y, \dot{\psi}, \dot{\phi}, \omega_{ij}$	8 DOF four wheel VM, Dugoff and lin. TM compared	S
PF	[96]	$v_x, v_y, \dot{\psi}, F_x, F_{y1}, F_{y2}$	δ	$a_x, a_y, \dot{\psi}, v_x$	Single track VM, Random walk force model	S

TABLE 4. Sideslip angle estimation approaches using unscented Kalman filter (UKF) and hybrid filter. Abbreviations: VM: Vehicle model, TM: Tire model, S: Simulations, M: Measurements.

Type	*Paper*	*Estimated states*	*Inputs*	*Meas.*	*Model*	*Val.*
UKF	[101]	$v_x, v_y, \dot{\psi}$	δ	$a_y, \dot{\psi}$	3 DOF four wheel VM, Dugoff TM	S
	[100]	$v_x, v_y, \dot{\psi}, \omega_{ij}$	$\delta_{FL}, \delta_{FR}, T_{Tij}, T_{Bij}$	$a_y, \dot{\psi}, \omega_{ij}, \mu_{max}$	Four wheel VM, Magic Formula TM	M
	[102]	$v_x, v_y, \dot{\psi}$	δ, ω_{ij}	a_x, a_y	3 DOF four wheel VM, piecewise linear TM	S
IMM-UKF	[99]	$\beta, v, \dot{\psi}, \phi, \dot{\phi}, F_{xij}, F_{yij}$	δ, ω_{ij}	$a_x, a_y, \dot{\psi}, \dot{\phi}, \omega_{ij}$	8 DOF four wheel VM, Dugoff and lin. TM compared	S
CUKF	[103]	$\beta, \dot{\psi}, F_{y1}, \dot{F}_{y1}, F_{y2}, \dot{F}_{y2}$	δ, v_x	$a_y, \dot{\psi}$	Single track VM Random walk force model	S, M

computationally expensive and tuning process can demand great efforts due to model nonlinearity and model uncertainty, especially uncertainty from tire model and the position of the center of gravity, inclination angle etc. Besides, EKF, UKF etc. cannot mathematically guarantee stability of the estimation results.

TABLE 5. Part 1 of cascading observers for sideslip angle estimation. Abbreviations: VM: Vehicle model, TM: Tire model, S: Simulations, M: Measurements. Numbers in parentheses in the Meas. column refer to the cascade step they are identified in.

Paper	*Cascade*	*Meas.*	*Model*	*Val.*
[104]	1. Parameter identification for C_{y1}, C_{y2} 2. Algebraic relation for β	1. $\delta, v_x, a_y, \dot{\psi}$ 2. C_{y1} (1), C_{y2} (1)	single track VM, linear TM	S, M
[105]	1. KF for F_{zij}, a_y 2. UKF for β, F_{yij}	1. $\Delta_{ij}, a_x, a_y, \dot{\phi}$ 2. $\delta, a_x, a_y, \dot{\psi}, \omega_{ij}, F_{zij}$ (1)	four wheel VM, Dugoff TM	M
[106]	1. F_{y1}, F_{y2} calculation using single TM 2. Fuzzy logic for nonlinear factor identifier (e) 3. α_1, α_2 calculation 4. C_{y1}, C_{y2} calculation using linear TM 5. Fuzzy logic for C_{y1}, C_{y2} 6. F_{y1}, F_{y2} using linear TM 7. Fuzzy logic for F_{y1}, F_{y2} 8. EKF for β	1. $a_y, \dot{\psi}$ 2. $a_y, \dot{\psi}$ 3. $\delta, v_x, \dot{\psi}, \beta$ (8) 4. α_1 (3), α_2 (3), F_{y1} (1), F_{y2} (1) 5. e (2), initial C_{y1}, initial C_{y2}, C_{y1} (4), C_{y2} (4) 6. α_1 (3), α_2 (3), C_{y1} (5), C_{y2} (5) 7. e (2), F_{y1} (1), F_{y2} (1), F_{y1} (6), F_{y2} (6) 8. $\delta, v_x, a_y, \dot{\psi}, F_{y1}$ (7), F_{y2} (7)	single track VM, lin. adaptive TM	S
[107]	1. RLS for C_{y1}, C_{y2} 2. EKF for v_x, v_y 3. Fuzzy logic for w 4. EKF for v_x, v_y	1. $\delta, v_x, a_y, \dot{\psi}$ 2. $\delta, v_x, a_x, \dot{\psi}$, C_{y1} (2), C_{y2} (2) 3. $v_x, \dot{\psi}$ 4. $v_{GPS}, a_x, a_y, \dot{\psi}, \nu_{GPS}$, v_x (2), v_y (2), w (3)	Step 2: Single track VM, lin. adaptive TM Step 4: Kinematic VM	S, M
[108]	1. $a_y, \dot{\psi}, F_{y1}, F_{y2}$ calculation 2. RLS for C_{y1}, C_{y2} 3. DD1 for β	1. δ, T_{eij} 2. $\delta, \dot{\psi}$ (1), F_{y1} (1), F_{y2} (1) 3. $\delta, F_{yij}, \dot{\psi}$, C_{y1} (2), C_{y2} (2)	single track VM, linear TM	S
[74]	1.SCKF for β 2. SCRHKF for β 3. IMM for β	1. $\delta, \omega_{ij}, a_x, a_y, \dot{\psi}$ 2. $\delta, \omega_{ij}, a_x, a_y, \dot{\psi}$ 3. $\beta(1), \beta(2)$	four wheel VM, Magic Formula TM	M
[109]	1. KF for F_{zij}, ϕ 2. EKF for β	1. a_x, a_y, F_{zij} 2. $\delta, v_x, a_x, a_y, \dot{\psi}$	four wheel VM, Magic Formula TM	S
[110]	1. ANFIS for β 2. UKF for v_y	1. $\delta, v_x, a_y, \dot{\psi}$ 2. $\delta, \dot{\psi}, \beta$ (1)	2 DOF four wheel VM, Magic Formula TM	S
[111]	1. Calculation of $\dot{\beta}$ 2. Heuristic scheduling for stabilizing term 3. Adaptive observer for v_x, v_y	1. $v_x, a_y, \dot{\psi}$ 2. $\delta, \dot{\psi}, \dot{\beta}$ (1) 3. $v_x, a_x, a_y, \dot{\psi}$, stab. term (2)	Step 3: kinematic VM	M
[112]	1. ANN for β 2. v_x observer based on direct integration 3. UKF for β	1. $a_x, a_y, a_z, \omega_x, \omega_y, \omega_z$ 2. $v_x, a_x, \dot{\psi}$ 3. v_x (2), $a_x, a_y, \dot{\psi}, \beta$ (1)	kinematic VM, four wheel VM	M

Alternative methods (deterministic estimation methods) like sliding mode observers, nonlinear adaptive observers show similar performance like extended Luenberger observers(ELO). Normally, sliding mode observers and nonlinear observers can offer estimation property of stability and robustness. But, since the gains for these observers are usually pre-defined, optimal performance is difficult to guarantee as the driving conditions vary. Recursive least squares methods are seldom used for sideslip angle estimation, because they are suitable for constant or slow varying parameters.

Data driven based methods normally estimate sideslip angle by utilizing measured signals such as steering angle, IMU data etc. as input. Due to offline training with previous data, such methods can obtain very accurate sideslip angle estimation most of the time without necessity of an accurate vehicle model as shown by [126]. But, these black box methods cannot guarantee robustness and stability of estimation. Meanwhile, it is difficult to interpret why it works sometimes and sometimes not. Hence, some researchers combine model-based methods with data driven methods to enhance the estimation stability and robustness as well as interpretability, see e.g. [127], [128], and [129].

Hence, a combination of above-mentioned observers may be a future route for sideslip angle estimation. For example, inspired by [130], deterministic methods such as sliding mode observers/nonlinear adaptive observers, can be utilized to offer a pre-estimation of sideslip angle. With such an estimation as a measurement, Kalman filter types methods can then be applied for a better sideslip angle estimation.

TABLE 6. Part 2 of cascading observers for sideslip angle estimation. Abbreviations: VM: Vehicle model, TM: Tire model, S: Simulations, M: Measurements. Numbers in parentheses in the Meas. column refer to the cascade step they are identified in.

Paper	*Cascade*	*Meas.*	*Model*	*Val.*
[113]	1. Parameter adaptation for m, I_z, l_1, l_2 2. EKF/UKF for F_{y1}, F_{y2} 3. C_{y1}, C_{y2} adaptation via Magic Formula TM 4. Algebraic relation for β	1. F_{zij} 2. a_y, $\dot{\psi}$ 3. p_{ij}, T_{ij}, F_{zij}, d_t 4. v, $\dot{\psi}$, F_{y1} (2), F_{y2} (2), C_{y1} (3), C_{y2} (3)	single track VM, random walk force model	M
[114]	1. EKF for v_x, v_y 2. UKF for β, v_x, $\dot{\psi}$ 3. Algebraic equation for β	1. a_x, a_y, $\dot{\psi}$, v_x 2. δ, a_x, a_y, $\dot{\psi}$ 3. β (1), β (2)	Step 1: kinematic VM Step 2: 3 DOF four wheel VM, Magic Formula TM	S
[115]	1. MRKF for β 2. SCKF for θ, ϕ 3. F_{z1}, F_{z2} calculation 4. KF for β	1. δ, v_x, $\dot{\psi}$, d_l 2. δ, a_x, a_y, $\dot{\psi}$, $\dot{\theta}$, $\dot{\phi}$ 3. θ (2), ϕ (2) 4. δ, β (1), F_{z1} (3), F_{z2} (3)	single track VM, brush TM	M
[116]	1. EKF for β 2. Algebraic equation for β 3. Empirical fusion strategy for β	1. a_x, a_y, $\dot{\psi}$ 2. δ, v_x, a_y 3. a_y, $\dot{\psi}$, β (1), β (2)	Step 1: kinematic VM Step 2: 3 DOF VM, linear TM	S, M
[72]	1. Deep ensemble NN for β, σ 2. EKF/UKF for for β	1. δ, v_x, a_y, $\dot{\psi}$ 2. δ, v_x, a_x, a_y, $\dot{\psi}$, β (1), σ (1)	Step 2: single track VM, linear adaptive TM	S, M
[117]	1. RLS for m 2. T-S fuzzy system to update process noise 3. RCKF for β, C_{y1}, C_{y2}	1. δ, v_x, a_y, $\dot{\psi}$ 2. a_y 3. δ, v_x, a_y, $\dot{\psi}$	single track VM, linear adaptive TM	S, M
[118]	1. Linear observer for β, v_x 2. UKF for β, $\dot{\psi}$ 3. Fuzzy logic for β	1. v_x, a_x, a_y, $\dot{\psi}$ (2) 2. δ, v_x (1), a_x, a_y, $\dot{\psi}$ 3. β (1), β (2), a_y	Step 1: kinematic VM Step 2: four wheel VM, modified Dugoff TM	M
[119]	1. ETKF for v_x, v_y 2. ETKF for ϕ 3. a_y correction 4. Multi-sensor fusion based observer for v_x 5. Multi-step KF for v_y	1. $\dot{\psi}$, γ_{GPS} 2. $\dot{\phi}$, $\ddot{\phi}$ 3. a_y, ϕ (2), $\ddot{\phi}$, $\dot{\psi}$ 4. v_x (1) 5. v_y (1), a_y	kinematic VM	S, M
[120]	1. Smart tire meas. for F_{zij}, F_{xij} 2. UKF for v_x, v_y, $\dot{\psi}$, ω_{ij}, μ_{max}	2. δ_{FL}, δ_{FL}, a_x, a_y, $\dot{\psi}$, ω_{ij}, T_T, T_B F_{xij} (1), F_{zij} (1)	7 DOF four wheel VM, Magic Formula TM	M
[121]	1. F_{yij}, F_{zij} calculation 2. ANN for C_{y1}, C_{y2} 3. KF for β	1. a_x, a_y, $\dot{\psi}$ 2. F_{yij} (1), F_{zij} (1) 3. δ, v_x, a_y, $\dot{\psi}$, C_{y1} (2), C_{y2} (2)	Step 1: four wheel VM Step 3: single track VM, linear adaptive TM	S
[122]	1. Adaptive observer for β, v_x, μ 2. Classifier for μ	1. δ, a_x, a_y, $\dot{\psi}$, ω_{ij}, μ (2) 2. δ, a_x, a_y, $\dot{\psi}$, ω_{ij}, T_{Tij}, T_{Bij}, v_x (1), β (1)	Step 1: kinematic VM, single track VM, linear adaptive TM	M

Such strategies can guarantee estimation stability, robustness as well as optimality/sub-optimality. However, model uncertainty will still produce estimation error of sideslip angle. Thus, in an increasing number of recent articles, the slip angle estimation is a part of a larger estimation strategy. These larger estimation strategies are summarized under the term cascaded observer in this article. Besides, such estimation strategy should also be integrated with data driven methods, to achieve more accurate estimation results while keep the nice property from deterministic and statistic estimation methods, which is actually still an open question.

B. SENSOR CONFIGURATION FOR SIDESLIP ANGLE ESTIMATION

With introduction of Electronic Stability Control, many sensors are available in series-production cars that help to reproduce the vehicle's dynamic state. These sensor signals include the vehicle's steering angle, yaw rate, lateral acceleration and wheel speeds. It is the basis for all approaches discussed in this article. Information about forces and motion closer at the wheel would be helpful, e.g. using in-wheel or tire sensors. However, from today's perspective, it is not likely that these will be available in series production.

TABLE 7. List of vehicle parameters with respective units.

Symbol	*Variable*	*Unit*
m	mass of the vehicle	kg
l_1	distance from the COG to the front axle	m
l_2	distance from the COG to the rear axle	m
θ_{F_x}	model uncertainty of F_x	-
θ_{F_y}	model uncertainty of F_y	-
c_{1i}	rational tire model parameter ($i = 1, 2$)	rad^2
c_{2i}	rational tire model parameter ($i = 1, 2$)	$\frac{\mathrm{N}}{\mathrm{rad}}$
d_1	external disturbances & cornering stiffness variation	$\frac{1}{\mathrm{s}}$
d_2	external disturbances & cornering stiffness variation	$\frac{1}{\mathrm{s}^2}$
w	adaptive weight coefficient	-
d_l	lateral distance: camera's preview point to right lane	m
d_t	tire tread depth	m
p_{ij}	tire inflation pressure ($i, j \in \{1, 2\}$)	psi
T_{ij}	tire temperature ($i, j \in \{1, 2\}$)	°C
Δ_{ij}	suspension deflections	m
M_z	yaw moment	Nm
M_{zw}	self aligning torque	Nm
T_T	traction torque	Nm
T_B	braking torque	Nm
T_{eij}	in-wheel (electric) motor torque ($i, j \in \{1, 2\}$)	Nm
b_{gyro}	gyro bias	$\frac{\mathrm{rad}}{\mathrm{s}}$

TABLE 8. List of variables with their respective units.

Symbol	*Variable*	*Unit*
δ	steering angle	rad
δ_{FL}	front left steering angle	rad
δ_{FR}	front right steering angle	rad
β	sideslip angle	rad
α_1	front slip angle	rad
α_2	rear slip angle	rad
v_x	longitudinal velocity	$\frac{\mathrm{m}}{\mathrm{s}}$
v_y	lateral velocity	$\frac{\mathrm{m}}{\mathrm{s}}$
v	velocity at the center of gravity	$\frac{\mathrm{m}}{\mathrm{s}}$
ω_x	longitudinal rotational velocity	$\frac{\mathrm{rad}}{\mathrm{s}}$
ω_y	lateral rotational velocity	$\frac{\mathrm{rad}}{\mathrm{s}}$
ω_z	vertical rotational velocity	$\frac{\mathrm{rad}}{\mathrm{s}}$
a_x	longitudinal acceleration	$\frac{\mathrm{m}}{\mathrm{s}^2}$
a_y	lateral acceleration	$\frac{\mathrm{m}}{\mathrm{s}^2}$
a_z	vertical acceleration	$\frac{\mathrm{m}}{\mathrm{s}^2}$
ν	course angle	rad
γ	heading angle	rad
θ	pitch angle	rad
ϕ	roll angle	rad
$\dot{\phi}$	roll rate	$\frac{\mathrm{rad}}{\mathrm{s}}$
ψ	yaw angle	rad
$\dot{\psi}$	yaw rate	$\frac{\mathrm{rad}}{\mathrm{s}}$
$\ddot{\psi}$	yaw acceleration	$\frac{\mathrm{rad}}{\mathrm{s}^2}$
ω_1	front angular velocity	$\frac{\mathrm{rad}}{\mathrm{s}}$
ω_2	rear angular velocity	$\frac{\mathrm{rad}}{\mathrm{s}}$
ω_{ij}	angular velocity of a wheel ($i, j \in \{1, 2\}$)	$\frac{\mathrm{rad}}{\mathrm{s}}$
x	longitudinal position	m
y	lateral position	m
C_{y1}	front lateral cornering stiffness	$\frac{\mathrm{N}}{\mathrm{rad}}$
C_{y2}	rear lateral cornering stiffness	$\frac{\mathrm{N}}{\mathrm{rad}}$
F_{x1}	front longitudinal force	N
F_{x2}	rear longitudinal force	N
F_{xij}	tire longitudinal force ($i, j \in \{1, 2\}$)	N
F_{y1}	front lateral force	N
F_{y2}	front lateral force	N
F_{yij}	tire lateral force ($i, j \in \{1, 2\}$)	N
F_{zij}	tire vertical force ($i, j \in \{1, 2\}$)	N
$\mu^{\max}$	maximal friction coefficient	-

The reasons to not use these sensors are typically the high costs and the lack of reliability in the wide range of operating conditions of a vehicle. However, the use of GPS and optical sensors to improve the slip angle estimate based on vehicle dynamics approaches are promising as shown e.g. by [125]. Optical sensors are now also installed in vehicles with other driving functions, such as to detect the position within one's own lane for active lane-keeping systems. GPS systems are becoming less expensive, but there are large differences in signal quality and available sampling rate. Also, the availability depends on direct line-of-sight contact with satellites, which is not consistently available in forested areas or tunnels, for example. Depending on the type of model used, sideslip angle estimation is more or less affected by the used sensor configuration as will be discussed in the next section.

C. EFFECTS OF MEASUREMENTS, EXCITATION AND CRITICAL PARAMETERS ON SIDESLIP ANGLE ESTIMATION

The sensitivity of a sideslip angle estimator on the measurements differs for different models used. Kinematic model based approaches depend highly on sensor configurations and measurement quality. Based on sensors from current production vehicles, which only consider steering angle and IMU with wheel velocity information, it is very difficult to obtain good sideslip angle estimation due to sensor noise, bias and gravity influence. This type of model does not require tire or vehicle parameters. In contrast, dynamics model based approaches show much better results when facing measurement problems. However, these approaches are more sensitive with regard to sideslip angle estimation concerning tire/vehicle parameters such as cornering stiffness, road friction coefficient and position of the center of gravity.

In terms of dynamic excitation required to deliver accurate results, dynamics based methods require persistent and moderate/large excitation. Otherwise, different road friction and inaccuracies of other vehicle/tire parameter have a high influence on the accuracy of the sideslip angle estimation, since these effects are coupled with the sideslip angle estimation. Kinematics approaches have no requirements on excitation. The sideslip angle can be accurately estimated with accurate measurements.

As discussed, tire/vehicle parameters are important for sideslip angle estimation using dynamics based models. The main parameters are cornering stiffness, tire-road friction coefficient and position of center of gravity. These parameters are either roughly approximated or unknown, and they can also change during driving. Cornering stiffness and tire-road friction coefficient affect the lateral force against slip

TABLE 9. List of abbreviations used in article.

Abbreviation	*Description*
ABS	anti-lock braking system
ACC	automated cruise control
AEB	automated emergency braking
AEFK	Adaptive extended Kalman filter
AFS	active front steering system
ANN	artificial neural network
ANFIS	adaptive neuro-fuzzy inference system
CKF	cubature Kalman filter
CUKF	constrained unscented Kalman filter
DAMRKF	disturbance accommodating multirate KF
DCKF	double cubature Kalman filter
DD1	first-order Stirling's interpolation filter
EKF	extended Kalman filter
ETKF	event-triggered Kalman filter
ELO	extended Luenberger observer
ESC	electronic stability control
FASTKF	fuzzy adaptive strong tracking Kalman filter
IMM	interacting multiple model
KF	linear Kalman filter
LO	Luenberger observer
M	measurement
MRKF	multirate Kalman filter
NLO	Nonlinear observer
PF	particle filter
RCKF	fuzzy adaptive robust cubature Kalman filter
RLS	recursive least squares
S	simulation
SAT	self-aligning torque
SMO	sliding mode observer
SCKF	square-root cubature Kalman filter
SCHRKF	square-root cubature based receding horizon KF
STKF	strong tracking Kalman filter
SUKF	square-root unscented Kalman filter
TM	tire model
UIO	unknown input observer
UKF	unscented Kalman filter
VM	vehicle model

angle characteristics. The accuracy of the estimated slip angle depends on these conditions to a large extent, especially for extreme conditions such as driving on low friction surfaces. The maximum tire-road friction coefficient is a heavily researched topics, but a series-production solution is not yet in sight, see e.g. [2]. Position of center of gravity affects the vehicle lateral states such as sideslip angle and yaw rate. Its error strongly affects sideslip angle estimation. Apart from these parameters, tire transient characteristics and bank angle will also influence the estimation accuracy of sideslip angle.

IV. SUMMARY

As the degree of automation of driving functions increases, so do the requirements for knowledge of vehicle motion states. This article is intended to help researchers that work on or require sideslip angle or lateral velocity estimates for automated driving strategies of different levels to get an overview of the state of the art, in particular what methods and sensor configurations have been used. Although this article focuses exclusively on observer-based methods that follow kinematic or model-based vehicle dynamics approaches, the number of publications in this area is enormous. Therefore, the focus is set on observer-based estimation approaches using on-board vehicle sensors that measure the dynamic response of the vehicle. With few exceptions, the literature since 2011 is considered. In the field of sideslip angle and lateral velocity estimation, many observer types are presented in the existing state of the art. A large number of articles use Kalman filter-based approaches. Methods are increasingly cascaded and require elaborate strategies for other non-measured states that affect the sideslip angle estimates. Finally, recommendations for sideslip angle estimation are given as the result of the analysis of the state of the art.

A. NOMENCLATURE AND NOTATION

See Tables 7–9.

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