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RESEARCH ARTICLE

Radiation From a Multiple Dipole Antenna With Phased Excitation in a Magnetoplasma

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ABSTRACT Radiation characteristics of a multiple dipole antenna immersed in a cold homogeneous magnetoplasma are considered. The antenna consists of phased electric dipoles which have a common geometric center and make a certain angle to one another. Each dipole is perpendicular to an external static magnetic field and assumed to be electrically short, so that the current of each has a triangular distribution. Closed-form expressions are obtained for the total radiation resistance of such an antenna and its partial radiation resistances which describe the excitation efficiency of different azimuthal harmonics of the radiated field. It is found that the antenna with appropriately phased currents in the dipoles is capable of selectively exciting twisted electromagnetic waves with given azimuthal indices in a magnetoplasma such as exists in the Earth's ionosphere.

INDEX TERMS Azimuthal field harmonics, magnetoplasma, multiple dipole antenna, phased excitation, radiation resistance, twisted electromagnetic waves.

I. INTRODUCTION

Antennas that are capable of exciting twisted electromagnetic waves in a magnetoplasma have been the subject of intense experimental and theoretical studies in the past decade (see [1], [2], [3], [4], [5], [6], and references therein). Such waves have helical phase fronts [7] and demonstrate some interesting properties during the propagation in magnetized plasmas [8], [9]. The most remarkable feature of the twisted waves is that they carry orbital angular momentum (OAM), the projection of which onto the propagation direction takes an infinite number of values [10], [11]. This fact makes it possible to increase the amount of information transmitted at a given frequency through the OAM-based data encoding [7]. Such a possibility seems especially interesting with application to very low-frequency (VLF) satellite antenna systems that can be used for both the wave excitation experiments and communication purposes in the ionospheric plasma [3].

To excite twisted electromagnetic waves in a magnetoplasma, phased antenna arrays have been proposed. The simplest configurations of such arrays, which have been used in the corresponding laboratory experiments, are crossed magnetic loop antennas with quadrature-phased currents [1], [12]. The predominant contribution to the total radiated power of such sources comes from the field harmonic with the azimuthal index equal to 1 or -1, depending on the sign of the phase shift of the currents in the loops [4], [5], [12]. The desire to excite higher azimuthal harmonics requires the use of phased arrays containing a greater number of antennas [2], [6], [13]. This circumstance can significantly complicate the possible application of such antenna systems in future satellite experiments. The interest in the related experiments is motivated by the need to develop new techniques for space plasma diagnostics and OAM-based multiplexing communication in the ionosphere. Hence, it becomes topical

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to develop antenna devices which, on the one hand, should be as simple as possible and, on the other hand, would be capable of selectively exciting at least a limited number of azimuthal field harmonics in a magnetoplasma. One such system having the form of a multigap loop antenna with phased excitation has recently been proposed and discussed in [14].

It is evident that the use of phased arrays consisting of electric dipole antennas for excitation of twisted waves in a magnetoplasma is of no less interest. The simplicity of realization of antenna systems on the basis of electric dipoles makes them attractive in comparison with other types of antennas. Despite the recent upsurge of interest in the characteristics of electric dipole antennas and their arrays in a magnetoplasma [15], [16], [17], [18], [19], [20], only one theoretical work [21] has been published to date on the radiation characteristics of a phased electric-dipole array capable of exciting twisted waves under ionospheric conditions, as far as we are aware. The analysis in [21] is primarily focused on the input impedance of a turnstile antenna in the form of two orthogonal electric dipoles with quadrature-phased currents in the VLF band. However, the efficiency of excitation of different azimuthal field harmonics by such an antenna was not considered in [21].

It is the purpose of this article to study the radiation characteristics of a multiple dipole antenna immersed in a magnetoplasma. It is assumed that such a source consists of appropriately phased electric dipoles which are placed perpendicular to an external static magnetic field at a certain angle to one another. The choice of this geometry is explained by that it seems most suitable for excitation of waves having helical phase fronts and, at the same time, is sufficiently simple for realization in practice. We will thus consider a more general case compared with that of a turnstile antenna consisting of only two quadrature-phased orthogonal dipoles. Specifically, we will find the total radiation resistance of the multiple dipole antenna and its partial radiation resistances which describe the excitation efficiency of different azimuthal harmonics of the radiated field. The analysis of these radiation characteristics will be aimed at determining conditions under which the considered antenna can selectively excite twisted VLF waves with given azimuthal indices in a magnetoplasma modeled upon the Earth's ionosphere. Since such analysis is very difficult in the general case, the emphasis will be placed on the case of electrically short dipoles, whose current distribution is known to have a triangular shape.

It should be noted that the most convenient technique for determining the partial powers going to different azimuthal harmonics, as well as the corresponding partial radiation resistances, is by using an eigenfunction representation of the antenna field [4], [5]. However, determination of the total radiated power and, hence, the total radiation resistance in some frequency intervals of the VLF band may be easier when employing the Fourier transform technique [22], [23]. In this article, we apply both these techniques, which also makes it possible to validate the results by comparing the antenna characteristics obtained by different methods. Another feature of our study is that we will be using an analytical approach as much as possible. Clearly, such an approach may have significant advantages because it not only provides an insight into the physical factors determining the antenna characteristics but can also serve to check the purely numerical results.

Our article is organized as follows. In Section II, we formulate the problem and describe the model of the antenna. Section III deals with the derivation of the total radiated power of the antenna and its partial powers going to different azimuthal harmonics of the excited field. The analytical and numerical results for the radiation resistances corresponding to these powers, with the emphasis placed on selective excitation of different azimuthal field harmonics in the VLF band, are presented in Section IV. Our conclusions are summarized in Section V. The radiation resistance of a single electric dipole in a magnetoplasma is obtained in the Appendix.

II. FORMULATION OF THE PROBLEM

Consider a multiple dipole antenna which is immersed in a cold collisionless homogeneous magnetoplasma. Assuming that an external static magnetic field $\mathbf{B}_0$, which is superimposed on the plasma, is parallel to the z-axis of a Cartesian coordinate system (x, y, z), we can describe the plasma medium by the permittivity tensor

$$\boldsymbol{\varepsilon} = \epsilon_0 \begin{pmatrix} \varepsilon & -jg & 0 \\ jg & \varepsilon & 0 \\ 0 & 0 & \eta \end{pmatrix} \tag{1}$$

where ϵ_0 is the permittivity of free space. Taking the time factor in the form $\exp(+j\omega t)$, we will consider the case where the angular frequency ω is much higher than the ion gyrofrequency Ω_H. Under such conditions, the quantities ε, g, and η are given by [24]

$$\varepsilon = \left(1 + \frac{\omega_p^2}{\omega_H^2 - \omega^2}\right)\left(1 - \frac{\omega_{\rm LH}^2}{\omega^2}\right)$$
$$g = -\frac{\omega_p^2 \omega_H}{(\omega_H^2 - \omega^2)\omega}, \quad \eta = 1 - \frac{\omega_p^2}{\omega^2}. \tag{2}$$

Here, ω_p and ω_H are the plasma frequency and the gyrofrequency of electrons, respectively, and $\omega_{\rm LH}$ is the lower hybrid resonance frequency. Note that if $\omega_p^2 \gg \omega_H^2$, which is typical of the ionospheric plasma, the lower hybrid resonance frequency can be written as $\omega_{\rm LH} = (\omega_H \Omega_H)^{1/2}$ [23].

Let the antenna consist of K electric dipoles having the form of narrow straight strips of half-length L and half-width d each, with a common geometric center and axes lying in the xy plane, such that the angle between the kth dipole and the x-axis is equal to ϕ_k, as shown in Fig. 1(a). Here, $k = 1, 2, \ldots, K$ and $0 \le \phi_1 < \ldots < \phi_K < \pi$. The current I_k of the kth dipole is represented as $I_k = |I_k| \exp(j\psi_k)$, where $|I_k|$ and ψ_k are the magnitude and phase of this current, respectively. As an example, Fig. 1(b) shows a two-dipole

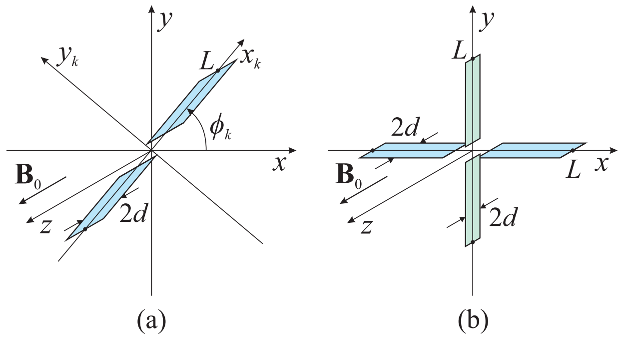

FIGURE 1. (a) Location of the *k*th dipole and (b) geometry of the problem in the special case where the antenna system consists of two orthogonal dipoles.

array, for which $K = 2$, $\phi_1 = 0$, and $\phi_2 = \pi/2$. It is worth noting that under the condition $\psi_2 - \psi_1 = \pm\pi/2$, such a system is commonly known as a turnstile antenna. Arrays with a greater number of elements can be obtained by inserting additional dipoles and are not shown here for brevity.

The current density $\mathbf{J}(\mathbf{r})$ of the described source is written in the form

$$\mathbf{J}(\mathbf{r}) = \sum_{k=1}^{K} \mathbf{J}_k(\mathbf{r}) \tag{3}$$

where $\mathbf{r}$ is the radius vector and $\mathbf{J}_k(\mathbf{r})$ is the current density of the kth dipole. We specify the current distributions along and across the strip conductors on the assumption that each strip dipole is centrally excited by a delta-gap source and satisfies the conditions $d \ll L$ and $k_0 L|\varepsilon\eta|^{1/4} \ll 1$, where k_0 is the wavenumber in free space. The latter inequality corresponds to the case of an electrically short dipole having a triangular current distribution. In this case, using the results of [25], [26], and [27], we can write the components $J_{x,k}(\mathbf{r})$ and $J_{y,k}(\mathbf{r})$ of $\mathbf{J}_k(\mathbf{r})$ along the x- and y-axes as follows:

$$\begin{bmatrix} J_{x,k}(\mathbf{r}) \\ J_{y,k}(\mathbf{r}) \end{bmatrix} = \frac{I_k}{\pi} \begin{bmatrix} \cos\phi_k \\ \sin\phi_k \end{bmatrix} \left(1 - \frac{|x_k|}{L}\right) \delta(y_k) \frac{1}{\sqrt{d^2 - z^2}} \tag{4}$$

where $\delta(\zeta)$ is the Dirac function, $|x_k| < L$, $|z| < d$, and the coordinates x_k and y_k are related to the x_k- and y_k-axes, obtained by rotating the x- and y-axes through an angle ϕ_k around the z-axis, as shown in Fig. 1(a). Hence

$$x_k = x\cos\phi_k + y\sin\phi_k, \quad y_k = -x\sin\phi_k + y\cos\phi_k. \tag{5}$$

For $|x_k| > L$ and $|z| > d$, the quantities $J_{x,k}(\mathbf{r})$ and $J_{y,k}(\mathbf{r})$ in (4) are zero. The choice of the dipole conductors in the form of electrically short narrow strips is explained by that the current distribution used for them has rigorously been derived in the works indicated above. Note that this distribution accounts for the well-known current singularity at the edge of an infinitesimally thin, perfectly conducting strip (see [27] and references therein), and integration of the current density over the cross section of each strip yields the total current along the corresponding dipole.

In what follows, we will also need a representation of $\mathbf{J}_k(\mathbf{r})$ in a cylindrical coordinate system (ρ, ϕ, z). Applying the relations $x = \rho\cos\phi$ and $y = \rho\sin\phi$, we have, from (5), $x_k = \rho\cos(\phi - \phi_k)$ and $y_k = \rho\sin(\phi - \phi_k)$. We substitute these expressions into (4) and make use of the relationship

$$\delta\big(\rho\sin(\phi - \phi_k)\big) = \frac{1}{\rho}\left[\delta(\phi - \phi_k) + \delta(\phi - \phi_k - \pi)\right]. \tag{6}$$

Taking into account (6) and the properties of the Dirac function, it is not difficult to obtain from (4) that only the radial component $J_{\rho,k}(\mathbf{r})$ of $\mathbf{J}_k(\mathbf{r})$ is nonzero for $\rho < L$ and $|z| < d$ in the cylindrical coordinates. Noting that $J_{\rho,k}(\mathbf{r}) = J_{x,k}(\mathbf{r})\cos\phi + J_{y,k}(\mathbf{r})\sin\phi$, we get

$$J_{\rho,k}(\mathbf{r}) = \frac{I_k}{\pi\rho}\left(1 - \frac{\rho}{L}\right) \frac{\delta(\phi - \phi_k) - \delta(\phi - \phi_k - \pi)}{\sqrt{d^2 - z^2}}. \tag{7}$$

In the present article, our main concern is to determine the radiated power of the described array of phased dipoles. In what follows, this problem will be solved using the eigenfunction expansion method and the Fourier transform technique. As in the case of an isotropic medium [28], both approaches enable one to derive the dyadic Green's function for a magnetoplasma [29]. However, we will show below that they make it possible to find the radiated power directly, not applying the conventional Green's function technique.

III. POWER RADIATED

We now determine the total power radiated from the considered source and the partial powers going from it to individual azimuthal harmonics of the excited field. The phases of such harmonics have the form $\omega t - m\phi - k_0 p_{s,\alpha}(q) z$, where m is the azimuthal index ($m = 0, \pm 1, \pm 2, \ldots$), $p_{s,\alpha}(q)$ is the function relating the normalized (to k_0) longitudinal wavenumber p to the normalized transverse wavenumber q for the ordinary (α = o) and extraordinary (α = e) normal waves of the magnetoplasma, and the subscript s denotes the waves transferring energy in the positive ($s = +$) and negative ($s = -$) directions of the z-axis. The quantities $p_{s,\mathrm{o}}(q)$ and $p_{s,\mathrm{e}}(q)$ satisfy the relation $p_{\pm,\alpha}(q) = \pm p_\alpha(q)$ and are defined by [23], [29]

$$p_\alpha(q) = \left[\varepsilon - \frac{1}{2}\left(1 + \frac{\varepsilon}{\eta}\right) q^2 + \chi_\alpha R(q)\right]^{1/2} \tag{8}$$

where

$$R(q) = \left[\frac{1}{4}\left(1 - \frac{\varepsilon}{\eta}\right)^2 q^4 - \frac{g^2}{\eta} q^2 + g^2\right]^{1/2} \tag{9}$$

and $\chi_\mathrm{e} = -\chi_\mathrm{o} = \mathrm{sgn}\,(1 - \varepsilon/\eta)$. It is also assumed that at least in the limit of vanishing losses in the plasma, $\mathrm{Re}\,[R(q)] > 0$ and $\mathrm{Im}\,[p_\alpha(q)] < 0$. Note that the latter inequality corresponds to the radiation condition for the excited field.

A. EIGENFUNCTION EXPANSION METHOD

The field of an antenna with the given current can be represented in terms of cylindrical vector eigenfunctions of a homogeneous magnetoplasma [4], [5], [14]. These eigenfunctions are the solutions of the source-free field equations, written in cylindrical coordinates for such a medium, and correspond to the real positive transverse wavenumbers constituting the complete continuous eigenvalue spectrum. Integration of the eigenfunctions, multiplied by their expansion coefficients, over these eigenvalues yields the total source-excited field satisfying the radiation condition at infinity [29]. At $|z| > d$, the eigenfunction expansion for the electric field component $E_\rho(\mathbf{r})$, which will be needed in what follows, is known to have the form [4]

$$E_\rho(\mathbf{r}) = \sum_{m=-\infty}^{\infty} \sum_{\alpha=\mathrm{o}}^{\mathrm{e}} \int_0^\infty a_{m,s,\alpha}(q) E_{\rho;m,s,\alpha}(\mathbf{r}, q)\, dq \quad (10)$$

where $s = +$ and $s = -$ for $z > d$ and $z < -d$, respectively, the summation over the index α accounts for the contributions from the ordinary and extraordinary normal waves, $a_{m,s,\alpha}(q)$ stands for the expansion coefficients, and

$$E_{\rho;m,s,\alpha}(\mathbf{r}, q) = E_{\rho;m,s,\alpha}(\rho, q) \exp[-jm\phi - jk_0 p_{s,\alpha}(q) z]. \quad (11)$$

Here,

$$E_{\rho;m,s,\alpha}(\rho, q) = j^m \left[(1+u_\alpha) J_{m+1}(k_0 q\rho) - u_\alpha m \frac{J_m(k_0 q \rho)}{k_0 q \rho} \right] \quad (12)$$

where J_m is the Bessel function of the first kind of order m and

$$u_\alpha = g^{-1} \left(q^2 + p_\alpha^2 - \varepsilon \right) - 1. \quad (13)$$

In the case considered, the expansion coefficients in (10) are given by [5]

$$a_{m,\pm,\alpha}(q) = \frac{1}{N_{m,\alpha}(q)} \sum_{k=1}^{K} \int J_{\rho,k}(\mathbf{r}) E^{(\mathrm{T})}_{\rho;-m,\mp,\alpha}(\mathbf{r}, q) d\mathbf{r} \quad (14)$$

where the integration is performed over the source volume, the superscript (T) indicates that the corresponding field is taken in an auxiliary ("transposed") medium described by the transposed dielectric tensor $\boldsymbol{\varepsilon}^{\mathrm{T}}$, and

$$N_{m,\alpha}(q) = \frac{4\pi(-1)^{m+1}}{Z_0 k_0^2 p'_\alpha(q)} \left(1 + \eta^{-1} n_{s,\alpha}^2 \right). \quad (15)$$

Here, Z_0 is the impedance of free space, the prime indicates the derivative with respect to the argument, and

$$n_{s,\alpha} = \left[\varepsilon^2 - g^2 - \varepsilon(q^2 + p_\alpha^2) \right] (g p_{s,\alpha})^{-1}. \quad (16)$$

The derivation of the normalization quantity (15) is given in [5]. Substituting (7) into (14) and making use of the fact that $E^{(\mathrm{T})}_{\rho;-m,-s,\alpha}(\rho, q) = -E_{\rho;m,s,\alpha}(\rho, q)$, we obtain

$$a_{m,s,\alpha}(q) = -2j^m N_{m,\alpha}^{-1}(q) J_0(k_0 d p_\alpha) F_{m,\alpha}(q) \tilde{\Phi}_m \delta_m \quad (17)$$

where $\delta_m = [1 - (-1)^m]/2$, and the quantities $F_{m,\alpha}(q)$ and $\tilde{\Phi}_m$ are written as follows:

$$F_{m,\alpha}(q) = \int_0^L \left[\left(1 - \frac{\rho}{L} \right) J_{m+1}(k_0 q \rho) - u_\alpha \frac{J_m(k_0 q \rho)}{k_0 q L} \right] d\rho \quad (18)$$

$$\tilde{\Phi}_m = \sum_{k=1}^{K} |I_k| \exp[j(m\phi_k + \psi_k)]. \quad (19)$$

It is evident from (17) that the coefficients $a_{m,s,\alpha}$ are nonzero only for the odd indices m. This means that the considered source can excite only the odd azimuthal field harmonics. Among them, those that are propagating, i.e., have real values of $p_\alpha(q)$ for real q, contribute to the radiated power.

The total radiated power may be written as [4]

$$P_\Sigma = \sum_{m=-\infty}^{\infty} \sum_{s=-}^{+} \sum_{\alpha=\mathrm{o}}^{\mathrm{e}} \int_{Q_\alpha} |a_{m,s,\alpha}(q)|^2 \mathcal{P}_{m,\alpha}(q)\, dq \quad (20)$$

where the symbol Q_α denotes the regions Q_o and Q_e of integration over the positive real q values for which the functions $p_\mathrm{o}(q)$ and $p_\mathrm{e}(q)$, respectively, are purely real, and $\mathcal{P}_{m,\alpha}(q)$ is the power normalization quantity, which is expressed as $\mathcal{P}_{m,\alpha}(q) = (-1)^m N_{m,\alpha}(q)/4$. Substituting expansion coefficients (17) into (20), with allowance for the fact that they are independent of s, and denoting the partial contribution of the mth harmonic to P_Σ as P_m, we get

$$P_\Sigma = \sum_{m=-\infty}^{\infty} P_m = \sum_{m=-\infty}^{\infty} Z_0 \frac{k_0^2}{2\pi} |\tilde{\Phi}_m|^2 \delta_m \sum_{\alpha=\mathrm{o}}^{\mathrm{e}} \int_{Q_\alpha} \frac{(-\eta)}{n_{+,\alpha}^2 + \eta} \times F_{m,\alpha}^2(q) J_0^2(k_0 d p_\alpha) p'_\alpha(q) dq. \quad (21)$$

If, regardless of the value of k,

$$|I_k| = |I_0|, \quad \phi_{k+1} - \phi_k = \Delta\phi, \quad \psi_{k+1} - \psi_k = \Delta\psi \quad (22)$$

then the quantity (19) reduces to

$$\tilde{\Phi}_m = |I_0| \Phi_m \times \exp\left\{ j \left[m\phi_1 + \psi_1 + \frac{K-1}{2}(m\Delta\phi + \Delta\psi) \right] \right\} \quad (23)$$

with the array factor

$$\Phi_m = \frac{\sin[K(m\Delta\phi + \Delta\psi)/2]}{\sin[(m\Delta\phi + \Delta\psi)/2]}. \quad (24)$$

It follows from (23) and (24) that $|\tilde{\Phi}_m|^2 = |I_0|^2 \Phi_m^2$. Substituting this relationship into (21), we can introduce the total radiation resistance $R_\Sigma = 2P_\Sigma/|I_0|^2$ and the partial radiation resistances $R_m = 2P_m/|I_0|^2$ of the multiple dipole antenna. In the case of a single dipole where $K = 1$ and $\Phi_m = 1$, these radiation resistances will be denoted as $R_\Sigma^{(s)}$ and $R_m^{(s)}$.

Note that the factor Φ_m^2 takes the maximum value equal to K^2 under the condition

$$m\Delta\phi + \Delta\psi = 2\pi l \tag{25}$$

where $l = 0, \pm 1, \pm 2, \ldots$ Below we will consider the case where the dipoles are positioned evenly over the azimuth, so that $\Delta\phi = \pi/K$. Then from (25) one has $\Delta\psi = \pi(2l - mK^{-1})$. To ensure the maximum value of R_m for the given odd index $m = \tilde{m}$ ($\tilde{m} = \pm 1, \pm 3, \ldots$), it suffices to put $\Delta\psi = \Delta\psi_{\tilde{m}} = -\tilde{m}\pi/K$. Under this condition, $\Phi_m^2 = K^2\delta_{m,\tilde{m}+2lK}$, where $\delta_{m,n}$ is the Kronecker delta. Then, from (21), the total radiation resistance is derived in the form

$$\begin{aligned} R_\Sigma &= \sum_{l=-\infty}^{\infty} R_{\tilde{m}+2lK} \\ &= \sum_{l=-\infty}^{\infty} Z_0 \frac{k_0^2}{\pi} K^2 \sum_{\alpha=\text{o}}^{\text{e}} \int_{Q_\alpha} \frac{(-\eta)}{n_{+,\alpha}^2 + \eta} \\ &\quad \times F_{\tilde{m}+2lK,\alpha}^2(q) J_0^2(k_0 d p_\alpha) p'_\alpha(q) dq. \end{aligned} \tag{26}$$

Since the quantities $F_{m,\alpha}$ decrease in magnitude with increasing $|m|$, the radiation resistance $R_{\tilde{m}}$ will be maximum among the terms summed in (26), provided that $K > |\tilde{m}|$.

Although expressions (21) and (26) are very convenient for determining the individual contributions of the azimuthal field harmonics to the radiated power, the series over azimuthal indices in these expressions can be slowly convergent in certain frequency ranges. In such a situation, the total radiated power can be calculated more easily using the Fourier transform technique, as will be demonstrated below.

B. FOURIER TRANSFORM METHOD

We define the Fourier transforms $\mathbf{J}(\mathbf{n})$ and $\mathbf{E}(\mathbf{n})$ of the current $\mathbf{J}(\mathbf{r})$ and the electric field $\mathbf{E}(\mathbf{r})$ due to this current by

$$\begin{bmatrix} \mathbf{J}(\mathbf{n}) \\ \mathbf{E}(\mathbf{n}) \end{bmatrix} = \int \begin{bmatrix} \mathbf{J}(\mathbf{r}) \\ \mathbf{E}(\mathbf{r}) \end{bmatrix} \exp(jk_0 \mathbf{n}\cdot\mathbf{r}) d\mathbf{r} \tag{27}$$

where $\mathbf{n}$ is a vector with the components n_x, n_y, and n_z.

The Fourier transforms of the components $J_x(\mathbf{r})$ and $J_y(\mathbf{r})$ of the current (3) are readily found from (4):

$$\begin{aligned} \begin{bmatrix} J_x(\mathbf{n}) \\ J_y(\mathbf{n}) \end{bmatrix} &= \sum_{k=1}^{K} \begin{bmatrix} J_{x,k}(\mathbf{n}) \\ J_{y,k}(\mathbf{n}) \end{bmatrix} \\ &= \frac{4J_0(k_0 d n_z)}{k_0^2 L} \sum_{k=1}^{K} I_k f_k \begin{bmatrix} \cos\phi_k \\ \sin\phi_k \end{bmatrix} \end{aligned} \tag{28}$$

where

$$f_k = \frac{\sin^2[k_0 L(n_x\cos\phi_k + n_y\sin\phi_k)/2]}{(n_x\cos\phi_k + n_y\sin\phi_k)^2}. \tag{29}$$

When integrating over x and y in (27) to obtain (28) and (29), we passed to the variables x_k and y_k according to (5) for each term $\mathbf{J}_k(\mathbf{r})$ with its components (4).

From the Maxwell equations for the Fourier-transformed field, we get the following system of equations [23], [29]:

$$(n^2\delta_{\gamma,\gamma'} - n_\gamma n_{\gamma'} - \tilde{\varepsilon}_{\gamma,\gamma'})E_{\gamma'}(\mathbf{n}) = -jk_0^{-1} Z_0 J_\gamma(\mathbf{n}). \tag{30}$$

Here, $n^2 = n_x^2 + n_y^2 + n_z^2$, the subscripts γ and γ' denote projections onto the x-, y-, or z-axis, $\tilde{\varepsilon}_{\gamma,\gamma'}$ stands for the elements of the relative plasma dielectric tensor $\epsilon_0^{-1}\boldsymbol{\varepsilon}$, and the repeated subscripts are summed over. The determinant D of the system in (30) is

$$\begin{aligned} D &= \det|n^2\delta_{\gamma,\gamma'} - n_\gamma n_{\gamma'} - \tilde{\varepsilon}_{\gamma,\gamma'}| \\ &= -\eta(n_z^2 - p_\text{o}^2)(n_z^2 - p_\text{e}^2) \end{aligned} \tag{31}$$

where the quantities p_o and p_e are defined by (8). From (30), one can obtain the components $E_x(\mathbf{n})$ and $E_y(\mathbf{n})$ due to the current with the components $J_x(\mathbf{n})$ and $J_y(\mathbf{n})$ as follows:

$$\begin{aligned} E_x(\mathbf{n}) &= -jk_0^{-1} Z_0[C_{xx}J_x(\mathbf{n}) + C_{xy}J_y(\mathbf{n})]/D \\ E_y(\mathbf{n}) &= -jk_0^{-1} Z_0[C_{yx}J_x(\mathbf{n}) + C_{yy}J_y(\mathbf{n})]/D. \end{aligned} \tag{32}$$

Here,

$$\begin{aligned} C_{xx,yy} &= (n_{x,y}^2 - \varepsilon)(q^2 - \eta) + (n_{x,y}^2 - \eta)n_z^2 \\ C_{xy,yx} &= (q^2 + n_z^2 - \eta)n_x n_y - jg(q^2 - \eta)\sigma_{xy,yx} \end{aligned} \tag{33}$$

where q is related to n_x and n_y by $q = (n_x^2 + n_y^2)^{1/2}$, and $\sigma_{xy} = -\sigma_{yx} = 1$.

The total radiated power is written as [22], [23]

$$P_\Sigma = -\frac{k_0^3}{16\pi^3}\text{Re}\int \mathbf{J}^*(\mathbf{n})\cdot\mathbf{E}(\mathbf{n}) d\mathbf{n} \tag{34}$$

where the asterisk denotes complex conjugation. We substitute (28) and (32) into (34), with allowance for (33), and integrate the result over n_z. This leads to integration of quantities having the form $jF(n_z)/(n_z \mp p_\alpha)$, where $F(n_z)$ stands for a certain real-valued function with a power-law decrease at $|n_z| \to \infty$. The contribution to the total radiated power comes only from the propagating normal waves, for which both the quantities p_o and p_e, or either of them, should be purely real in the case of a collisionless magnetoplasma. The integration is then performed by making the replacement $p_\alpha \to p_\alpha - j0$. Here, the symbol $j0$ means that one should introduce a minor loss in the plasma and, after integration over n_z, put this loss equal to zero. This integration can be performed via the residue theorem [30] or simpler, using the Sokhotski–Plemelj formula [23]. An application of this formula gives

$$\text{Re}\int_{-\infty}^{\infty} \frac{jF(n_z)}{n_z \mp p_\alpha \pm j0} dn_z = \pm\pi F(\pm p_\alpha). \tag{35}$$

When applying (35) to evaluating the integral over n_z in (34), one should calculate the quantities (33) at $n_z = \pm p_\alpha$. Noting that these quantities, taken at $n_z = \pm p_\alpha$, are independent of the sign of p_α and denoting them with the additional subscript α, we find

$$\begin{aligned} C_{xx,\alpha} &= \Lambda_{x,\alpha}\tilde{\Lambda}_{x,\alpha}C_\alpha, \quad C_{yy,\alpha} = \Lambda_{y,\alpha}\tilde{\Lambda}_{y,\alpha}C_\alpha \\ C_{xy,\alpha} &= \tilde{\Lambda}_{x,\alpha}\Lambda_{y,\alpha}C_\alpha, \quad C_{yx,\alpha} = \Lambda_{x,\alpha}\tilde{\Lambda}_{y,\alpha}C_\alpha \end{aligned} \tag{36}$$

where

$$\Lambda_{x,\alpha} = n_x + \frac{jgn_y}{q^2 + p_\alpha^2 - \varepsilon}, \quad \Lambda_{y,\alpha} = n_y - \frac{jgn_x}{q^2 + p_\alpha^2 - \varepsilon}$$

$$C_\alpha = q^{-2}(q^2 + p_\alpha^2 - \varepsilon)(q^2 - \eta). \tag{37}$$

The quantities $\tilde{\Lambda}_{x,\alpha}$ and $\tilde{\Lambda}_{y,\alpha}$ in (36) are obtained from $\Lambda_{x,\alpha}$ and $\Lambda_{y,\alpha}$, respectively, by the replacement $j \to -j$. In deriving (36) and (37), use was made of the identity [23]

$$\eta p_\alpha^2(q^2 + p_\alpha^2 - \varepsilon) = (q^2 - \eta)[\varepsilon^2 - g^2 - \varepsilon(q^2 + p_\alpha^2)] \tag{38}$$

which is obtained from (8) by straightforward manipulation. Note that for propagating normal waves in a collisionless magnetoplasma, we have $\tilde{\Lambda}_{x,\alpha} = \Lambda_{x,\alpha}^*$, $\tilde{\Lambda}_{y,\alpha} = \Lambda_{y,\alpha}^*$, and $C_{xy,\alpha} = C_{yx,\alpha}^*$. By making use of these relations, along with formulas (32) and (35)–(37), evaluation of the integral over n_z in (34) yields

$$P_\Sigma = -\frac{Z_0 k_0^2}{32\pi^2\eta}\sum_{\alpha=\mathrm{o}}^{\mathrm{e}}\chi_\alpha \int_{\Xi_\alpha}\frac{(q^2+p_\alpha^2-\varepsilon)(q^2-\eta)}{q^2 p_\alpha R(q)} \times \Big\{|J_x(\mathbf{n}_\alpha)\Lambda_{x,\alpha}|^2 + |J_y(\mathbf{n}_\alpha)\Lambda_{y,\alpha}|^2 + 2\mathrm{Re}\left[J_x(\mathbf{n}_\alpha)J_y^*(\mathbf{n}_\alpha)\Lambda_{x,\alpha}\Lambda_{y,\alpha}^*\right]\Big\}\, dn_x dn_y \tag{39}$$

where $J_{x,y}(\mathbf{n}_\alpha) = J_{x,y}(n_x, n_y, n_z = p_\alpha)$, $R(q)$ is given by (9), and Ξ_α denotes the area of real values of p_α in the plane of the variables n_x and n_y.

In the case where $|I_k| = |I_0|$ for all k in (28), we have the following result for the total radiation resistance from (39):

$$R_\Sigma = 2P_\Sigma/|I_0|^2 = KR_\Sigma^{(s)} + \Delta R_\Sigma \tag{40}$$

where $R_\Sigma^{(s)}$ is the total radiation resistance of a single isolated dipole. Since the quantity $R_\Sigma^{(s)}$ is independent of the angle between such a dipole and the x-axis [see Fig. 1(a)], we can put this angle equal to zero to obtain

$$R_\Sigma^{(s)} = -\frac{Z_0}{\pi^2(k_0L)^2\eta}\sum_{\alpha=\mathrm{o}}^{\mathrm{e}}\chi_\alpha\int_{\Xi_\alpha}\frac{(q^2+p_\alpha^2-\varepsilon)(q^2-\eta)}{q^2p_\alpha R(q)} \times \frac{\sin^4(k_0Ln_x/2)}{n_x^4}|\Lambda_{x,\alpha}|^2 J_0^2(k_0dp_\alpha)dn_x dn_y. \tag{41}$$

The term ΔR_Σ in (40) is expressed as

$$\Delta R_\Sigma = -\frac{Z_0}{\pi^2(k_0L)^2\eta}\sum_{\alpha=\mathrm{o}}^{\mathrm{e}}\chi_\alpha\int_{\Xi_\alpha}\frac{(q^2+p_\alpha^2-\varepsilon)(q^2-\eta)}{q^2p_\alpha R(q)} \times \Big[F_c|\Lambda_{x,\alpha}|^2 + F_s|\Lambda_{y,\alpha}|^2 + G_c\mathrm{Re}\,(\Lambda_{x,\alpha}\Lambda_{y,\alpha}^*) + G_s\mathrm{Im}\,(\Lambda_{x,\alpha}\Lambda_{y,\alpha}^*)\Big] J_0^2(k_0dp_\alpha)dn_x dn_y \tag{42}$$

where

$$\begin{bmatrix}F_c\\F_s\end{bmatrix} = \sum_{k=1}^{K}\sum_{n=1}^{K}(1-\delta_{k,n})f_kf_n\begin{bmatrix}\cos\phi_k\cos\phi_n\\ \sin\phi_k\sin\phi_n\end{bmatrix} \times\cos(\psi_k-\psi_n)$$

$$G_c = \sum_{k=1}^{K}\sum_{n=1}^{K}(1-\delta_{k,n})f_kf_n\sin(\phi_k+\phi_n) \times\cos(\psi_k-\psi_n)$$

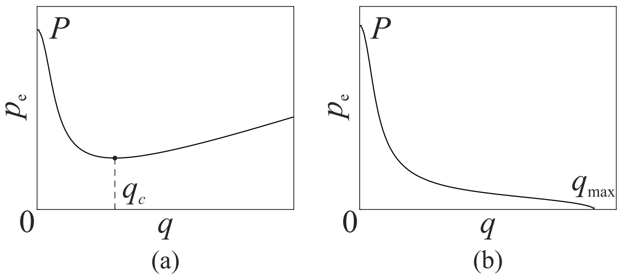

FIGURE 2. Typical whistler-mode refractive index surfaces (not to scale) in the frequency ranges (a) $\omega_{\mathrm{LH}} < \omega < \omega_H/2 < \omega_p$ and (b) $\Omega_H \ll \omega < \omega_{\mathrm{LH}}$.

$$G_s = \sum_{k=1}^{K}\sum_{n=1}^{K}f_kf_n\sin(\phi_k-\phi_n)\sin(\psi_k-\psi_n). \tag{43}$$

Here, the quantities f_n are obtained from f_k by making the replacement $\phi_k \to \phi_n$ in (29).

IV. SELECTIVE EXCITATION OF AZIMUTHAL HARMONICS

We now apply the above formulation to the analysis of the excitation efficiency of different azimuthal harmonics in the VLF band. In the ionospheric plasma, this band is important for many applications and corresponds to the whistler range [3], [23]

$$\Omega_H \ll \omega < \omega_H < \omega_p. \tag{44}$$

At these frequencies, the ordinary wave is evanescent and does not contribute to the radiated power. At the same time, the extraordinary, or whistler-mode wave is propagating. Hence, only the $\alpha = \mathrm{e}$ term should be taken into account in (20), (21), (26), (39), (41), and (42) in the frequency range (44).

Fig. 2 shows typical examples of the whistler-mode refractive index surface, which is described by the function $p = p_{\mathrm{e}}(q)$. The surface is open in the case $\omega > \omega_{\mathrm{LH}}$ where the relation $\operatorname{sgn}\varepsilon \neq \operatorname{sgn}\eta$ takes place. This case, illustrated in Fig. 2(a), corresponds to the resonant part of the whistler range [23] in which the propagation region Q_{e} is defined by $0 < q < \infty$, and the area Ξ_{e} becomes the entire plane of the variables n_x and n_y. The quantity $q = q_c$ shown in Fig. 2(a) indicates the point of minimum of the function $p_{\mathrm{e}}(q)$. In the nonresonant part of the whistler range, where $\omega < \omega_{\mathrm{LH}}$ and $\operatorname{sgn}\varepsilon = \operatorname{sgn}\eta$, the whistler-mode refractive index surface is closed, as seen in Fig. 2(b). At these frequencies, the region Q_{e} is given by $0 < q < q_{\max}$, and the area Ξ_{e} is defined by $(n_x^2 + n_y^2)^{1/2} \le q_{\max}$, where $q_{\max} = [(\varepsilon^2 - g^2)/\varepsilon]^{1/2}$. The quantity P on the plots of Fig. 2 is determined by $P = p_{\mathrm{e}}(0) = (\varepsilon - g)^{1/2}$.

A. RESONANT PART OF THE WHISTLER RANGE

We begin with analysis of the radiation from a single dipole with current I_0. In this case $K = 1$ and $|\tilde{\Phi}_m| = |I_1| \equiv |I_0|$. For the frequency interval considered, it is known [23], [30] that under the conditions

$$d|\varepsilon/\eta|^{1/2} \ll L, \quad k_0L|g|^{1/2} \ll 1 \tag{45}$$

most contributions to the integral with $\alpha = \mathrm{e}$ in (41) for $R_{\Sigma}^{(s)}$ come from the region $q \gg |\eta|^{1/2}$. Interestingly, the second condition in (45) coincides with that of applicability of a triangular current distribution in the range (44) if $\omega_{\mathrm{LH}} \ll \omega \ll \omega_H \ll \omega_p$, when $|g| \approx |\varepsilon\eta|^{1/2}$. A close examination shows that the same region of q predominantly determines the partial radiation resistances $R_m^{(s)}$ of a single dipole. In this region of q under the indicated conditions for the frequencies, we can approximately write

$$p_{\mathrm{e}} = |\varepsilon/\eta|^{1/2} q, \quad n_{+,\mathrm{e}} = -g^{-1}|\varepsilon\eta|^{1/2}(1-\varepsilon/\eta)q$$
$$u_{\mathrm{e}} = g^{-1}(1-\varepsilon/\eta)q^2, \quad \chi_{\mathrm{e}} R(q) = (1-\varepsilon/\eta)q^2/2. \tag{46}$$

Moreover, it is found in this case that when calculating $F_{m,\mathrm{e}}$, the contribution from the first term in the brackets of the integrand in (18) to the integration result can be neglected. Then the radiation resistance $R_m^{(s)}$ is obtained from (21) as follows:

$$R_m^{(s)} = \frac{Z_0}{\pi (k_0 L)^2 |\varepsilon\eta|^{1/2}} \delta_m \int_0^\infty \frac{1}{q^2} \left(\int_0^{k_0 L q} J_m(\xi) d\xi \right)^2 dq. \tag{47}$$

Here, we also made use of the fact that under the first condition in (45), the quantity $J_0(k_0 d p_{\mathrm{e}})$ can be replaced by unity during the integration, without significantly affecting the result in (47) for moderate values of m. Evaluation of the integrals in (47) is somewhat involved but straightforward. Using table integrals [31], it can be shown after some lengthy algebra that

$$R_m^{(s)} = \frac{Z_0}{\pi k_0 L |\varepsilon\eta|^{1/2}} 2\delta_m \left(\frac{1}{|m|} - \frac{\beta_{|m|}}{\pi} \right) \tag{48}$$

where $\beta_{|m|}$ is a numerical coefficient, which vanishes in the limit $|m| \to \infty$. As an example, we present the values of this coefficient for the first three odd azimuthal indices: $\beta_1 = 2$, $\beta_3 = 26/45 \approx 0.58$, and $\beta_5 = 526/1575 \approx 0.33$.

Summing the terms $R_m^{(s)}$ yields the total radiation resistance $R_{\Sigma}^{(s)}$. To make the resulting series convergent, one should calculate these terms for high magnitudes of m with allowance for the difference of the function $J_0(k_0 d p_{\mathrm{e}})$ from unity. Then the quantities $R_m^{(s)}$ decrease slightly faster than $1/|m|$ in the limit $|m| \to \infty$. Hence, a series of the terms $R_m^{(s)}$ turns out to be slowly convergent, which makes it more expedient to use formula (41) for determining $R_{\Sigma}^{(s)}$.

To obtain $R_{\Sigma}^{(s)}$ in closed form, we use the same assumptions in (41) as those employed when deriving (48). This gives

$$R_{\Sigma}^{(s)} = \frac{8 Z_0}{\pi^2 (k_0 L)^2 |\varepsilon\eta|^{1/2}} \int_0^\infty dn_x \int_0^\infty \frac{\sin^4(k_0 L n_x/2)}{n_x^2} \times \frac{J_0^2(k_0 d |\varepsilon/\eta|^{1/2} q)}{q} dn_y. \tag{49}$$

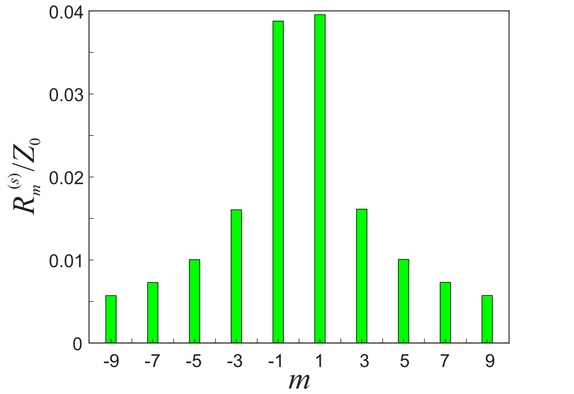

FIGURE 3. **Partial radiation resistances $R_m^{(s)}$ of a single dipole antenna with $L = 5$ m and $d = 1$ cm at the frequency $\omega = 1.9 \times 10^5$ s^{-1} for $\omega_{\mathrm{LH}} = 5.1 \times 10^4$ s^{-1}, $\omega_H = 8.8 \times 10^6$ s^{-1}, and $\omega_p = 5.6 \times 10^7$ s^{-1}.**

Evaluating the integrals in (49) as described in the Appendix, we arrive at the result

$$R_{\Sigma}^{(s)} = \frac{Z_0}{\pi k_0 L |\varepsilon\eta|^{1/2}} \left[\ln\left(\frac{2L}{d} \sqrt{\left|\frac{\eta}{\varepsilon}\right|} \right) - 1 \right]. \tag{50}$$

For a logarithmically thin dipole, when $\ln(2L|\eta/\varepsilon|^{1/2}/d) \simeq \ln(2L/d)$, formula (50) reduces to the result obtained in [30].

Fig. 3 presents numerical results for the normalized (to Z_0) partial radiation resistances of a dipole antenna with $L = 5$ m and $d = 1$ cm at the frequency $\omega = 1.9 \times 10^5$ s^{-1}. Hereafter, we used plasma parameters typical of the F-layer maximum in the daytime Earth's ionosphere, namely, the external static magnetic field $B_0 = 0.05$ mT and the plasma density $N = 10^{12}$ m^{-3}. With these values, the plasma had $\omega_H = 8.8 \times 10^6$ s^{-1} and $\omega_p = 5.6 \times 10^7$ s^{-1}. The lower hybrid resonance frequency was taken equal to $\omega_{\mathrm{LH}} = 5.1 \times 10^4$ s^{-1}, which corresponded to the O^+ ions with the gyrofrequency $\Omega_H = 300$ s^{-1}.

It is seen in Fig. 3 that there exist slight differences between the partial radiation resistances $R_m^{(s)}$ for the positive and negative indices m. This occurs due to violation of the mirror symmetry in a magnetoplasma [32] and is related to the nonzero value of the quantity g in (1). A close examination shows that the presence of the quantity g in the expressions for the partial radiation resistances leads to different contributions of the integration region $q < q_c$ [see Fig. 2(a)] to these resistances for the opposite signs of m. However, the indicated differences are insignificant here because most contributions to the partial radiation resistances come from waves with the transverse wavenumbers $q > q_c$. The properties of these waves are almost independent of the plasma gyrotropy accounted for by the term g. Therefore, despite the fact that the above-mentioned differences were not taken into account in the approximate expression (48), it describes fairly accurately the numerical results shown in Fig. 3. This can be verified for, e.g., $|m| \le 5$, by making use of (48) with the β_m values presented above. The normalized total radiation resistance, which has been determined numerically

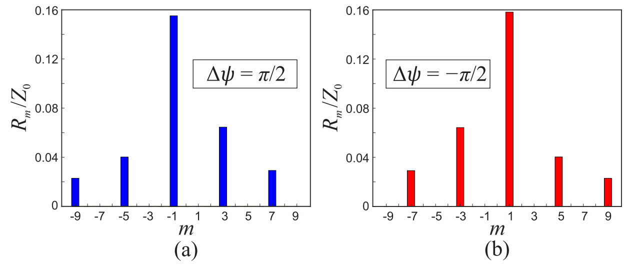

FIGURE 4. Partial radiation resistances R_m of a turnstile antenna with $K = 2$ and $\Delta\phi = \pi/2$ for (a) $\Delta\psi = \pi/2$ and (b) $\Delta\psi = -\pi/2$. The same values of L, d, ω, and the plasma parameters as in Fig. 3.

from (41) for the used parameters, is equal to $R_\Sigma^{(s)}/Z_0 = 0.496$. The same value of the total radiation resistance is obtained by summing the partial resistances $R_m^{(s)}$, but an acceptable accuracy for $R_\Sigma^{(s)}$ is achieved if no less than 4×10^3 terms are summed in the corresponding series because of its slow convergence. Hence, the use of the Fourier transform technique is indeed more expedient when determining the total radiation resistance in the resonant part of the whistler range. The approximate expression (50) gives a close result $R_\Sigma^{(s)}/Z_0 = 0.53$.

The presence of a great number of azimuthal harmonics radiated from a single dipole makes it possible to excite the required harmonics selectively by using several appropriately phased dipoles. In what follows, this will be demonstrated in the case (22) using the results obtained for a single dipole. Note that a predominant contribution of a certain azimuthal harmonic to the radiation from a multiple dipole antenna leads to dominance of the corresponding helicity type in the field structure, i.e., to the excitation of a twisted wave field.

To excite selectively the (-1)st or the 1st azimuthal harmonic, one can use a turnstile antenna with two orthogonal dipoles, when $K = 2$ and $\Delta\phi = \pi/2$ [see Fig. 1(b)], if the phase shift of the currents in the dipoles amounts to $\Delta\psi = \pi/2$ or $\Delta\psi = -\pi/2$, respectively. The results for R_m of such an antenna are shown in Fig. 4. In this case, according to (26), the excited harmonics satisfy the condition $R_{\tilde{m}+4l} = 4R_{\tilde{m}+4l}^{(s)}$, where $\tilde{m} = -1$ or $\tilde{m} = 1$, depending on the sign of $\Delta\psi$ for the phased dipoles. Hence, the differences between the bar charts in two panels of Fig. 4 and onwards are explained by the opposite phase-shift signs of the dipole currents. At the same time, the total radiation resistance is only slightly affected by the sign of $\Delta\psi$. Indeed, (40) yields $R_\Sigma/Z_0 = 0.99$ at $\Delta\psi = \pi/2$ and $R_\Sigma/Z_0 = 0.993$ at $\Delta\psi = -\pi/2$.

Note that the total radiation resistance of a turnstile antenna admits the approximation $R_\Sigma \approx 2R_\Sigma^{(s)}$, irrespective of the sign of $\Delta\psi$. This result implies the fulfillment of the condition $|\Delta R_\Sigma| \ll R_\Sigma^{(s)}$ in (40), which is satisfied with a sufficient margin for a turnstile antenna. The negligibly small contribution of ΔR_Σ to R_Σ agrees with the calculations in [21] and is explained by that all terms in the brackets of (42), except for the last one containing G_s, are zero for such an antenna. Thus, our results for the total radiation resistance of a turnstile antenna are in line with those obtained earlier in [21].

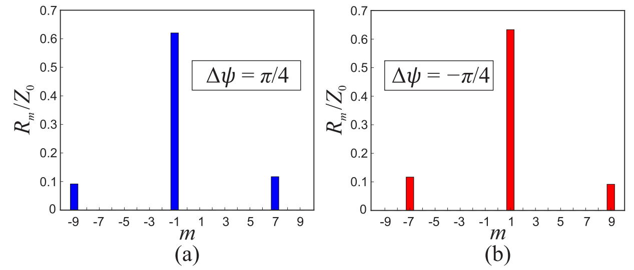

FIGURE 5. Partial radiation resistances R_m of a multiple dipole antenna with $K = 4$ and $\Delta\phi = \pi/4$ for (a) $\Delta\psi = \pi/4$ and (b) $\Delta\psi = -\pi/4$. The same values of L, d, ω, and the plasma parameters as in Fig. 3.

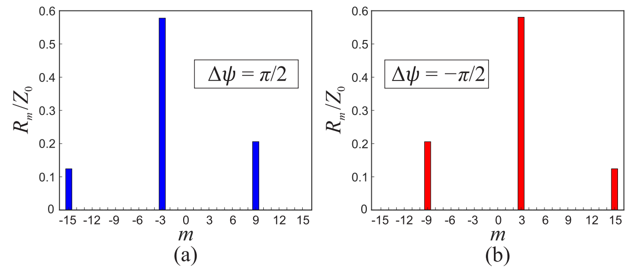

FIGURE 6. Partial radiation resistances R_m of a multiple dipole antenna with $K = 6$ and $\Delta\phi = \pi/6$ for (a) $\Delta\psi = \pi/2$ and (b) $\Delta\psi = -\pi/2$. The same values of L, d, ω, and the plasma parameters as in Fig. 3.

Since selective excitation of the harmonic with $m = -1$ or $m = 1$ does not seem well-pronounced in Fig. 4, we can improve the selectivity by increasing the number of appropriately phased dipoles. This is illustrated by Fig. 5 for the case where $K = 4$, $\Delta\phi = \pi/4$, and $\Delta\psi = \pi/4$ or $\Delta\psi = -\pi/4$, so that the partial radiation resistances of the excited harmonics are given by $R_{\tilde{m}+8l} = 16R_{\tilde{m}+8l}^{(s)}$ for $\tilde{m} = -1$ or $\tilde{m} = 1$. In this case, the quantity ΔR_Σ is no more negligible. Then, with allowance for ΔR_Σ, the total radiation resistance is found numerically from (40) to be equal to $R_\Sigma/Z_0 = 2.145$ and $R_\Sigma/Z_0 = 2.157$ at $\Delta\psi = \pi/4$ and $\Delta\psi = -\pi/4$, respectively.

The selective excitation of the harmonics with higher indices m requires the further increase in the number of phased dipoles. This is demonstrated in Fig. 6 for the case where the number of dipoles is increased up to $K = 6$ and the partial radiation resistance R_{-3} or R_3 is maximized by choosing $\Delta\psi = \pi/2$ or $\Delta\psi = -\pi/2$, respectively. For such an antenna system, from (40) we have $R_\Sigma/Z_0 = 2.759$ at $\Delta\psi = \pi/2$ and $R_\Sigma/Z_0 = 2.762$ at $\Delta\psi = -\pi/2$.

Thus, it may be inferred that in the resonant part of the whistler range, the capabilities of increasing both the partial radiation resistance for a particular harmonic and its excitation selectivity are apparently limited by engineering restrictions imposed on the number of phased dipole elements in the analyzed system. It is evident that for selective excitation of harmonics with moderate magnitudes of m, such restrictions will not be severe.

B. NONRESONANT PART OF THE WHISTLER RANGE

In the nonresonant part of the whistler range, we may put $d = 0$ since this does not result in divergence of the total radiation resistance of a dipole, in contrast to the resonant part of the frequency range (44). Assuming the dipoles to be relatively short such that

$$k_0 L q_{\max} \ll 1 \tag{51}$$

it can be shown that the total radiation resistance $R_\Sigma^{(s)}$ of a single dipole is predominantly determined by the contribution of the $m = -1$ and $m = 1$ harmonics. This is easily verified by evaluating the quantity $F_{m,\mathrm{e}}$ for different m values if the Bessel functions in (18) are replaced by their small-argument approximations under the condition (51). In this case, the total radiation resistance $R_\Sigma^{(s)}$ is adequately approximated by

$$R_\Sigma^{(s)} = R_{-1}^{(s)} + R_1^{(s)} \tag{52}$$

where

$$R_{\pm 1}^{(s)} = -Z_0 \frac{(k_0 L)^2 \eta}{16\pi g^2} \int_0^{q_{\max}} \frac{(q^2 + p_\mathrm{e}^2 - \varepsilon \mp g)^2}{n_{+,\mathrm{e}}^2 + \eta} p_\mathrm{e}'(q) dq. \tag{53}$$

It is worth noting that the result given by (52) and (53) coincides with that yielded by the Fourier transform method in the case $d = 0$ if we put $\sin(k_0 L n_x/2) \approx k_0 L n_x/2$ in (41) for the $\alpha = \mathrm{e}$ term, which is possible under the condition (51). This can be verified using the identity (33) given in [4]. Thus, in the nonresonant part of the whistler range, it suffices to employ the eigenfunction expansion approach since the partial radiation resistances fairly rapidly decay with $|m|$. Therefore, no difficulties appear during their summation when determining the total radiation resistance at such frequencies.

If the frequency ω is not too close to ω_{LH}, the integral in (53) is readily evaluated explicitly. For example, in the frequency interval $\Omega_H \ll \omega < \omega_{\mathrm{LH}}/2$, we have $|\eta| \gg n_{+,\mathrm{e}}^2$ and $|\eta| \gg q_{\max}^2$ [6]. This makes it possible to put formally $\eta = -\infty$ in (53). In this limiting case, it follows from (8) that

$$q^2 = \frac{g^2}{p_\mathrm{e}^2 - \varepsilon} - p_\mathrm{e}^2 + \varepsilon. \tag{54}$$

Then, replacing the integration variable q in (53) by p_e with allowance for (54) and omitting the subscript e in the interests of brevity, we transform (53) to

$$R_{\pm 1}^{(s)} = Z_0 \frac{(k_0 L)^2}{16\pi} \int_0^P \left(1 \mp \frac{g}{p^2 - \varepsilon}\right)^2 dp. \tag{55}$$

From (55), we obtain

$$R_{\pm 1}^{(s)} = Z_0 \frac{(k_0 L)^2}{64} \frac{g^2}{|\varepsilon|^{3/2}} \left(1 + \frac{2}{\pi} \frac{|\varepsilon|^{1/2}}{|g|^{1/2}} \pm 4 \frac{|\varepsilon|}{|g|}\right). \tag{56}$$

In deriving (56), use was made of the relations $|\varepsilon| \ll |g|$ and $\arctan \sqrt{(|g| - |\varepsilon|)/|\varepsilon|} \approx \pi/2$, which are valid in the nonresonant part of the frequency range (44).

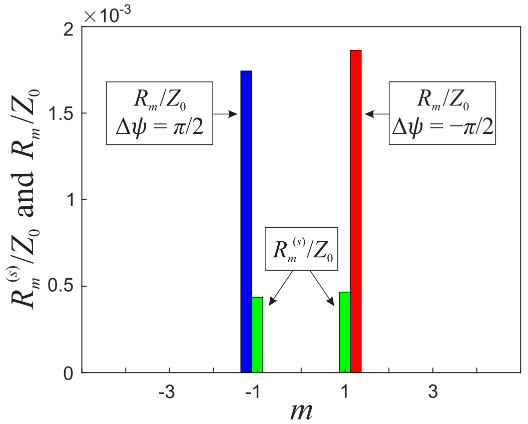

FIGURE 7. Partial radiation resistances $R_m^{(s)}$ and R_m of a single dipole and a turnstile antenna, respectively, for $L = 5$ m at the frequency $\omega = 2.55 \times 10^4$ s^{-1}. The same values of the plasma parameters as in Fig. 3. Two shorter bars refer to a single dipole. The leftmost and rightmost bars are for a turnstile antenna at $\Delta\psi = \pi/2$ and $\Delta\psi = -\pi/2$, respectively.

Since the total radiation resistance of a relatively short dipole is predominantly determined by two harmonics with $m = \pm 1$ in the frequency interval considered, only one of them can selectively be excited by a multiple dipole antenna with phased excitation. To this end, it suffices to use a turnstile antenna, for which $K = 2$, $\Delta\phi = \pi/2$, and $\Delta\psi = \pm\pi/2$. In this case, we have as a good approximation that $R_\Sigma = R_{-1}$ at $\Delta\psi = \pi/2$, and $R_\Sigma = R_1$ at $\Delta\psi = -\pi/2$, where $R_{\pm 1} = 4R_{\pm 1}^{(s)}$. These results yield the approximate relation $R_\Sigma \approx 2R_\Sigma^{(s)}$, which, upon comparison with (40), implies that here $|\Delta R_\Sigma| \ll R_\Sigma^{(s)}$, as for a similar turnstile antenna in the resonant part of the whistler range.

The features described above are illustrated in Fig. 7 for a single dipole and a turnstile antenna in the case where $L = 5$ m, $d = 0$, $\omega = 0.5\omega_{\mathrm{LH}} = 2.55 \times 10^4$ s^{-1}, and the other parameters are the same as in Fig. 3. For the used parameters, $k_0 L q_{\max} = 0.54$ and the inequality (51) is no longer satisfied. However, even in this case, the above closed-form relations can still be used as reasonable approximations. Moreover, at the chosen frequency, it is found that for the dipole half-lengths up to $L = 20$ m, the contributions of the harmonics with $|m| > 1$ to the total radiation resistance are so small that the corresponding partial resistances cannot be shown using the scale adopted for Fig. 7. Thus, electrically short dipoles cannot be used for excitation of the higher azimuthal harmonics in the nonresonant part of the whistler range. A further increase in L leads to the necessity to consider a current distribution differing from a triangular one, which falls beyond the scope of the present article.

V. CONCLUSION

In this article, we have determined the radiation characteristics of a multiple dipole antenna with phased excitation that is immersed in a cold collisionless magnetoplasma.

Assuming that electrically short dipoles have a common geometric center and are placed perpendicular to an external static magnetic field at a certain angle to one another, it has been shown that they can selectively excite twisted (OAM-carrying) waves possessing different azimuthal indices with at least moderate magnitudes in the resonant part of the whistler range, provided that the number and phases of the dipoles are appropriately chosen. In the nonresonant part of this range, such phased dipoles are capable of selectively exciting only the (−1)st or the 1st harmonic. An independent excitation of field harmonics with different azimuthal indices enables one to apply such antenna systems for performing parallel data transmission via the OAM channels corresponding to these harmonics at the same frequency in a magnetoplasma. This possibility is the main difference of the considered multiple dipole antenna with phased excitation from a conventional dipole source.

The validity of the presented conclusions is confirmed by that the studied radiation characteristics, which were obtained by both the eigenfunction expansion method and the Fourier transform technique, are found to coincide. The closed-form expressions of this work are in good agreement with numerical results and believed to be a useful tool for the assessment of the operation features of such antennas in a magnetoplasma as well as the planning of the corresponding satellite experiments in the ionosphere.

Throughout the article, our consideration has been limited to the case of electrically short dipoles with a triangular current distribution. This has made it possible to obtain exact integral representations for the analyzed radiation characteristics of the multiple dipole antenna. The use of more complicated antenna geometries than those discussed herein, including the case of longer dipoles, will require application of direct numerical simulation, which is to be done in future works. Further work should also be focused on determining the input impedances and the radiation patterns of the corresponding antennas in a magnetoplasma. Thus, significant efforts are still to be made toward understanding all factors affecting the performance of such phased antenna systems.

Finally, we note that although we have focused on the antenna radiation in the whistler range, our formulation can be applied to any frequency interval of a magnetoplasma. Therefore, the consideration made in this article may be looked upon as the basis for a wider use of the developed theory.

APPENDIX. RADIATION RESISTANCE OF A SINGLE DIPOLE

To determine $R_\Sigma^{(s)}$, we at first perform integration over n_y in (49). To this end, we introduce the new integration variable q via the relation $n_y = (q^2 - n_x^2)^{1/2}$ and use the identity [31]

$$J_0^2(k_0 d|\varepsilon/\eta|^{1/2} q) = \frac{1}{\pi}\int_0^\pi J_0(2k_0 d|\varepsilon/\eta|^{1/2} q \sin\zeta)\,d\zeta. \tag{57}$$

Denoting the integral over n_y in (49) as $\hat{I}$, we can write [31]

$$\begin{aligned}\hat{I} &= \int_0^\infty \frac{J_0^2(k_0\tilde{d}q)}{q}dn_y \\ &= \frac{1}{\pi}\int_0^\pi d\zeta \int_{n_x}^\infty \frac{J_0(2k_0\tilde{d}q\sin\zeta)}{\sqrt{q^2-n_x^2}}dq \\ &= -\frac{1}{2}\int_0^\pi J_0(k_0\tilde{d}n_x\sin\zeta)Y_0(k_0\tilde{d}n_x\sin\zeta)d\zeta \end{aligned}\tag{58}$$

where Y_0 is the Bessel function of the second kind of order zero and $\tilde{d} = d|\varepsilon/\eta|^{1/2}$. Since most contributions to the remaining integral over n_x in (49) come from the region $n_x < 2\pi/(k_0 L)$, we can use small-argument approximations for the Bessel functions in (58) under the first condition in (45). This yields

$$\begin{aligned}&J_0(k_0\tilde{d}n_x\sin\zeta)Y_0(k_0\tilde{d}n_x\sin\zeta) \\ &\qquad\approx (2/\pi)\big[\ln(k_0\tilde{d}n_x/2) + \ln\sin\zeta + \gamma\big]\end{aligned}\tag{59}$$

where $\gamma = 0.5772\ldots$ is Euler's constant. Using (59) and the integral [31]

$$\int_0^\pi \ln\sin\zeta\,d\zeta = -\pi\ln 2$$

we transform (58) into $\hat{I} = 2\ln 2 - \gamma - \ln(k_0\tilde{d}n_x)$. Then

$$\begin{aligned}R_\Sigma^{(s)} &= \frac{8Z_0}{\pi^2(k_0L)^2|\varepsilon\eta|^{1/2}}\int_0^\infty \frac{\sin^4(k_0Ln_x/2)}{n_x^2} \\ &\quad\times\big[2\ln 2 - \gamma - \ln(k_0\tilde{d}n_x)\big]dn_x.\end{aligned}\tag{60}$$

The integration in (60) can be performed by using that [31]

$$\int_0^\infty \frac{\sin^4(a\xi)}{\xi^2}\Psi_{1,2}(\xi)d\xi = \frac{\pi a}{4}(1-\psi_{1,2})$$

where $\psi_1 = 0$ if $\Psi_1(\xi) = 1$ and $\psi_2 = \ln a + \gamma$ if $\Psi_2(\xi) = \ln\xi$. As a result, we arrive at (50) for $R_\Sigma^{(s)}$.

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