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RESEARCH ARTICLE

Improved COPRAS Method With Unknown Weights Under *p, q*-Quasirung Orthopair Fuzzy Environment: Application to Green Supplier Selection

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ABSTRACT In the last few decades, there has been a significant increase in the importance of assessing social and ecological implications within industrial product supply chains. This tendency has given rise to the notion of supplier sustainability, which entails meeting the economic, environmental, and social demands of all suppliers. The supplier selections are typically based on experts' opinion, which are then reflected in supplier ratings. When examining sustainability indicators, experts may not be fully aware of all aspects of suppliers' economic, social, and environmental qualities. In the Decision Making (DM) approaches such as Pythagorean Fuzzy (PF) and q-Rung Orthopair Fuzzy (q-ROF) sets, experts are unable to use different powers for membership and non-membership grades simultaneously. In real-life DM problems, experts may require different power levels for membership and non-membership grades. In this paper, we propose a new extension of the COPRAS method under *p, q*-Quasirung orthopair fuzzy ($p, q-$QOF) sets, which allow decision-makers to use different power levels for membership and non-membership grades by incorporating parameters p and q. In the proposed approach, it is assumed that in addition to compiling the expert scores for suppliers, the issue analyst evaluates each expert's degree of expertise for each criterion. The best choice is then determined by combining the data using the COPRAS method. The Inter-Criteria Correlation (CRITIC) method is employed to determine the unknown criteria weights. To demonstrate the effectiveness of our proposed approach, we apply it to a Multi-Criteria Group Decision-Making (MCGDM) problem that is focused on green supplier selection. Finally, we conduct a comparative study with existing approaches to demonstrate the practicality and applicability of the proposed DM method.

INDEX TERMS $p, q-$quasirung orthopair fuzzy sets, COPRAS method, MCGDM, green supplier selection.

I. INTRODUCTION

Green Supply Chain Management (GSCM) entails integrating environmental concerns across various business functions. This evolving approach embeds environmental factors throughout the supply chain, encompassing areas such as product design, material sourcing, manufacturing, distribution, and waste management [1], [2], [3]. Supplier selection stands out as a pivotal decision for businesses. While Multi-Criteria Decision-Making (MCDM) methods traditionally emphasize financial aspects, there is a growing recognition of the need to incorporate sustainability considerations into

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all phases of the supply chain, given mounting concerns about global warming and other environmental impacts. This underscores the significance of supplier selection processes. The Green Supplier Selection (GSS) problem is a complex issue that is sometimes made more challenging by the mismatch between organizational objectives and environmentally sound criteria. Usually, choices are made using the professional opinions of different suppliers. Throughout the GSS process, critical data may not always be provided indisputably, which leads to ambiguity and uncertainty in specific data points. Experts use linguistic variables to express the imprecision inherent in their judgements. The fuzzy model allows linguistic expressions to be transformed into Intuitionistic Fuzzy (IF) [4], Pythagorean Fuzzy (PF) [5], $q-$Rung Orthopair Fuzzy ($q-$ROF) [6], or $p, q-$Quasirung Orthopair Fuzzy ($p, q-$QOF) numbers [7]. Furthermore, while evaluating sustainability measures, it is critical to recognize that these experts may not be equally proficient in each of the fields (economic, social, and environmental). This study presents a novel way for GSS using the COPRAS method [8] under the $p, q-$QOF environment. The COPRAS approach, recognized for its low complexity, efficiency, and resilience, is a ranking system whose computations do not need complex mathematical procedures, making it more accessible and applicable. The proposed approach considers the influence of optimizing favorable indicators and minimizing unfavorable indicators on evaluation results. It evaluates the competing circumstances of the criteria, evaluating them suitably in order to compare ideas and set priorities. The proposed technique requires the presence of an analyst beside the experts, who not only collects scores but also analyses their expertise. The analyst then combines the data to determine the best decision.

A. LITERATURE REVIEW

Over the past two decades, researchers have made significant strides in expanding and refining the Multi-Criteria Group Decision-Making (MCGDM) process to address various decision-making and selection challenges. The primary aim of these efforts is to enhance decision-making processes, leading to more informed outcomes. One notably practical MCGDM method is the method of COPRAS, introduced by Zavadskas and Kaklauskas [8] in 1996. COPRAS focuses on establishing proportional and direct relationships between the importance and utility of prospective candidates, particularly in scenarios involving competing criteria. It evaluates candidates' performance across multiple criteria while considering assigned weights to each criterion. By examining both positive and negative ideal solutions, COPRAS enables informed judgments. Its key advantage lies in its relative simplicity and ability to generate clear and distinct preference orders [9], [10]. In recent years, COPRAS has found effective applications in various domains, such as selecting Low-E windows for enhancing public buildings [11], supervisor selection [12] and material selection of sustainable composites [13]. Agarwal and Tayal [14] proposed an enhanced model of the fuzzy COPRAS technique, which serves as an improved multicriteria decision-making method for identifying and prioritizing relevant indicators, thereby aiding institutions in enhancing accreditation data. Li et al. [15] developed a MCDM method utilizing Interval-Valued Fuzzy Numbers (IVFNs), extended DEMATEL and entropy methods with IVFNs for uncertain indicator weights, and employed the extended COPRAS approach with IVFNs to assess the rock burst hazard level in mining engineering. Erdebilli et al. [16] integrated the AHP and COPRAS supplier selection techniques with Interval-Valued PF (IVPF) logic, providing a comprehensive review and analysis of the effectiveness of these assessments in supplier selection.

The current COPRAS techniques impose limitations on the degree of non-membership (Φ) and membership Ψ in the IF environment; in the PF environment, the sum of the squares of Φ and Ψ is restricted to be less than or equal to 1; and in the q-ROF environment, the sum of the q^{th} powers of Φ and Ψ is restricted to be less than or equal to 1. Furthermore, decision-makers must use the same values of q for both Φ and Ψ when applying the COPRAS approach in the q-ROF context. These limitations can significantly impact decision outcomes. The objective of this study is to overcome these restrictions and explore decision-making scenarios without such constraints.

B. $p, q-$ QUASIRUNG ORTHOPAIR FUZZY SETS

DM is an important process to select the best-suited alternative from among those available. Several researchers have presented a variety of theories to make the best decisions. In the past, judgements were made based on sharply numbered data sets, but this practice produced insufficient outcomes that were less applicable to actual operating scenarios. However, as time passes and the complications of the system expand, it becomes more challenging for the decision-makers to handle the uncertainties in the data, and hence the traditional techniques are unable to determine the optimal choice. Later on, Zadeh [17] proposed the Fuzzy Set (FS) theory, which is used to describe the fuzzy and uncertain information of objective things with applications in various fields. For example, Lu et al. [18] used fuzzy change-point algorithm for shift detection in control charts. Poulik et al. [19] introduced and examined the properties of the Randic index for fuzzy graphs and fuzzy subgraphs. Das et al. [20] introduced the concepts of picture fuzzy $\phi-$tolerance competition graphs, which introduce additional uncertainties to the fuzzy $\phi-$tolerance competition graphs. Poulik and Ghorai [21] proposed the concept of the bipolar fuzzy incidence graph along with its matrix representation. A theoretical foundation for handling uncertain information is provided by FS theory, which also makes it possible to convert MD information from linguistic to quantitative form. Also, the FS theory addresses the flaws of the traditional decision-information process and introduces a new technique to represent imprecise and uncertain data. FS only has a degree membership Φ with $\Phi \in [0, 1]$) that describes how much an element belongs to a given fixed set. To address the shortcoming of FS, Atanassov [4] introduced the Intuitionistic FS (IFS), which has both membership Φ and non-membership degree Ψ such that $0 \leq \Phi+\Psi \leq 1$. The IFS has been extensively researched in recent decades, with several accomplishments documented. For example, Hwang and Yang [22] gave some similarity measures between IFSs based on lower, upper and middle FSs. Yang et al. [23] constructed a belief-plausibility TOPSIS for IFSs. Mahanta and Panda [24] introduced the distance function for IFSs which is a quantitative tool to measure the difference between two intuitionistic fuzzy numbers. Ding et al. [25] provided a comprehensive review of research advancements in three-way decision methods within the generalized IF environment. However, in real-world DM problems, decision-makers may face situations in which the sum of Φ and Ψ by decision-makers is bigger than one (i.e. $\Phi + \Psi > 1$) while their square sum is less or equal to one. For example, if decision makers provide their assessment about the object (alternative) such that $\Phi = 0.6$ and $\Psi = 0.7$, then it is clear that $0.6 + 0.7 = 1.30 > 1$. To address the limitations of IFSs, Yager [5] proposed PF Sets (PFSs) under the condition $\Phi^2 + \Psi^2 \leq 1$ which are the expansion of IFSs. Several authors have introduced various methods in the context of PFSs. Zhang [26] proposed a hierarchical QUALIFLEX approach based on PFSs. Rahim et al. [27] presented an extension of Bonferroni mean. Yager [6] introduced $q-$ROF Sets ($q-$ROFSs) which is the general class of IFSs and PFSs. Wei et al. [28] presented a series of aggregation operators based on the Heronian mean under a $q-$ROF environment. Yang et al. [29] and Zhang et al. [30] gave a three-way decision on $q-$ROFSs. Zulqarnain et al. [31] devised geometric aggregation operators to effectively combine q-rung orthopair fuzzy information. Seikh and Mandal [32] presented q-rung orthopair fuzzy Frank aggregation operators to deal with MADM problems. Zhang et al. [33] proposed a MAGDM method under incomplete $q-$ROF information systems.

From the above discussion, it is evident that in IFSs, decision-makers are required to provide assessments for alternatives under the condition that the degrees of membership (Φ) and non-membership (Ψ) should be with $0 \leq \Phi+\Psi \leq 1$. Similarly, in PFSs, decision-makers must ensure that the sum of the squares of the Φ and Ψ is less than or equal to 1(i.e. $0 \leq \Phi^2 + \Psi^2 \leq 1$). In the context of $q-$ROFSs, decision-makers are constrained to set their assessments in a manner such that the sum of the q^{th} powers of the Φ and the degree of Ψ should be with $0 \leq \Phi^q + \Psi^q \leq 1$. This means that decision-makers are obligated to establish uniform degrees for both membership and non-membership. To overcome these constraints, Seikh and Mandal [7] introduced $p, q-$QOF Sets ($p, q-$QOFSs) which extend the concept of $q-$ROFSs. In $p, q-$QOFSs, decision-makers are empowered to set distinct powers for membership and non-membership degrees by introducing the parameters p and q. Through the incorporation of these parameters, $p, q-$QOFSs offer a more adaptable framework for managing the membership and non-membership degrees of an element. There are several applications of $p,q-$QOFSs in the literature, such as in [34], [35], and [36].

C. GREEN SUPPLIER SELECTION

Green Supplier Selection (GSS) is the process of evaluating and selecting suppliers based on their environmental performance and sustainability practices [37]. This can include considerations such as the supplier's energy and resource efficiency, waste reduction efforts, and compliance with environmental regulations. The goal of green supplier selection is to minimize the environmental impact of a company's supply chain and to support the adoption of sustainable practices throughout the industry. In the green supplier selection process, a company may consider a variety of factors, such as the supplier's greenhouse gas emissions, water usage, and waste generation. The company may also evaluate the supplier's environmental policies and initiatives, as well as its track record of environmental compliance. By choosing suppliers that are environmentally responsible, a company can reduce its environmental footprint and support the transition to a more sustainable economy. There are several key factors that companies may consider when evaluating and selecting green suppliers. These can include:

- **Metrics for Environmental Performance**

  i. Includes the production of garbage, the consumption of energy and water, and greenhouse gas emissions.
  ii. Businesses look for vendors who will gradually improve their environmental performance.

- **Environmental Mechanisms**

Assessment of supplier policies, including the use of renewable energy, recycling initiatives, and sustainable sourcing.

- **Implementation of Sustainability Policies**

Taking into account the supplier's compliance with pertinent environmental regulations and guidelines.

- **Reliability and Performance Background**

Evaluation of a supplier's reputation and performance history about sustainability and environmental performance.

Due to rising consumption levels and the negative effects of industrial advancements, green consideration has become one of the most critical challenges for environmental conservation. Supplier selection may be considered a crucial factor of green supply chain management for mitigating the negative consequences of industrial operations. A green supplier selection is the operational management system and optimization process used to decrease the environmental effect of a green product throughout its life cycle. Supply chains choose suppliers based on a variety of analytical, numerical, psychological, and objective parameters [38]. Green Supply Chain Management (GSCM) is the practice of integrating environmental sustainability into the design, management, and improvement of a company's supply chain. This can involve a variety of practices, such as reducing the environmental impact of transportation, optimizing the use of resources and energy, and promoting the use of environmentally friendly products and materials. One common application of GSCM is in the manufacturing industry, where companies may implement sustainable practices, such as reducing waste, conserving energy, and water, and using environmentally friendly materials. Other industries, such as retail and logistics, may also adopt GSCM practices to reduce their environmental impact and support the adoption of sustainable practices throughout their supply chain. By implementing GSCM practices, companies can improve their environmental performance, reduce costs, and enhance their reputation and competitiveness. It is increasingly being recognized as an important part of a company's overall sustainability strategy. Min and Kim [39] were the first to define GSCM as the biologically responsive management of supply chain processes throughout the whole chain. According to Mohanty and Prakash [40], GSCM is a method that may be used to improve sustainability and economic efficiency for the entire socio-economic growth. Akcan and Taş [41] noted that GSS differs from traditional supplier selection in that it places equal emphasis on environmental issues such as carbon dioxide and carbon monoxide emissions, resource waste, and social responsibility in addition to profitability, customer satisfaction and quality. In other words, GSS is capable of balancing ecological and financial advantages. Therefore, choosing an appropriate green supplier for enterprises may be seen as a crucial strategy and the importance of scientific and logical decision criteria and processes is critical. Several factors need to be considered when evaluating green suppliers, including qualitative and quantitative data. Also, during the GSS process, enterprises frequently vote at meetings to evaluate prospective suppliers.

D. MOTIVATIONS

Currently, available COPRAS method extensions are subject to certain limitations. For example, in a Pythagorean fuzzy environment, it is limited by the requirement $\Phi^2 + \Psi^2 \leq 1$. Similarly, in a $q-$ROF environment, the constraint is $\Phi^q + \Psi^q \leq 1$ $(q > 1)$. The necessity for the identical value of q for both membership degrees in $q-$ROF numbers presents a barrier, as there are cases where separate powers are required for membership and non-membership degrees. The current COPRAS technique cannot handle decision-makers with varying levels of authority membership.

To solve these restrictions while optimizing the benefits of $p, q-$QOFSs, this work offers an adaptation of the COPRAS approach for the $p, q-$QOF environment. This improved COPRAS technique empowers decision-makers by allowing them to employ distinct parameters, p and q, to control the effect of membership degrees independently during the decision-making process. The main objectives of the proposed framework are summarized as follows:

1. In the first phase, we extend the fuzzy COPRAS approach based on cubic PFSs which provide a relaxed and fixable environment for decision-makers to avail themselves of their desirable alternatives (suppliers).
2. An assessment approach for the GSS is proposed, which can assist enterprises in efficiently selecting the appropriate supplier.
3. Even though the strategy has been proposed for decades, there is a lack of relevant studies in the available literature. As a result, our study, which develops a $p, q-$QOF COPRAS ($p, q-$QOF-COPRAS) method, can cover a research need.
4. Several criteria have different weights in the supplier selection process. It is challenging to give the criterion weights exactly because the decision-makers may be bound by their expertise and experience. To calculate weight values, an objective weight determination technique based on the CRITIC approach is described in this article.
5. The MCGDM problem is provided to demonstrate the proposed approach. Also, the results of the proposed approach are compared with some existing approaches which validate the stability of the approach with respect to state-of-art.

E. THE SCREENING PROCESS FOR IDENTIFYING GREEN SUPPLIERS

Multiple factors have been used in the GSS process. It is challenging for an enterprise to let a single decision-maker choose the appropriate sustainable supplier in a complex environment. Because the decision information provided by the decision-maker is limited. The quantity of information he/she has will influence how much he/she knows about these suppliers. Furthermore, his understanding of the relevant issue and his background in the field also had a role in the selection he/she made. When he/she examines these suppliers based on several criteria, these factors might cause him to make errors. Therefore, enterprises are more likely to assemble a team of professionals to manage these multi-criteria MD problems. Up until today, there have previously been MCDM approaches to pick the appropriate green supplier from the set of alternatives (suppliers) such as AHP, ANP and TOPSIS method. For example, Kilic and Yalcin [42] developed a strategy to address GSS concerns by combining the IF-TOPSIS approach with a modified two-phase fuzzy goal management model. Pischulov et al. [43] proposed the VAHP approach to assist associated managers in selecting suppliers. Wei et al. [44] dealt with GSS challenges using BWM and VIKOR approaches in an interval type-2 fuzzy context. Xu et al. [45] established the AHP-Sort-2 method using IT2FSs to promote the evolution of long-term supplier selection. Fei et al. [46] developed an MCDM strategy for improving GSS accuracy by integrating ELECTRE and DST methodologies. Kaya and Yet [47] used the DEMATEL approach, which was formerly known as the MCDM approach, to Bayesian Networks to provide a methodology for GSS. Liao et al. [48] applied SPAN and ANP approach to a hesitant fuzzy linguistic context to design a new framework for selecting the appropriate low-carbon supplier.

F. CRITIC METHOD

Various methods have been employed in the literature to assess criteria weights such as Entropy and FANMA [49], [50]. Every method has advantages and disadvantages for example, for some DM scenarios, the Entropy technique offers a simple and neutral way to establish criterion weights. Its application in more complicated decision situations may be limited by its sensitivity to data quality and failure to identify relationships between criteria. By considering both positive and negative feedback, FANMA provides a useful expansion to the ANP paradigm [51], offering a more realistic examination of complicated MD scenarios. However, while using the approach in practice, it should be carefully considered for its increased complexity, subjectivity in preference modelling, and data needs.

On the other hand, for DM issues, the CRITIC approach [52]is crucial in calculating the weights of the criteria. CRITIC more precisely determines the most important criteria by considering inter-criteria correlations. It improves the accuracy of the criterion weighting by explicitly taking these connections into account. Additionally, CRITIC shows resilience in coping with data fluctuation and uncertainty in criterion values. CRITIC uses data-driven analysis and correlations to provide a more objective method to criteria weighting, decreasing subjectivity. Its attractiveness as a practical DM tool is further enhanced by its simplicity and ease of application. Numerous researchers have used this technique in a variety of settings. Numerous researchers have used this technique in a variety of settings. For example, Pamucar et al. [53]modified CRITIC method using fuzzy rough numbers. The study uses CRITIC to determine criteria weights because it is resilient in the context of data volatility and criterion value uncertainty. CRITIC uses data-driven analysis and correlations to provide a more unbiased approach to criteria weighting, decreasing subjectivity.

G. PAPER OUTLINE

The reminder for this article is as follows. Section II includes an overview of related literature. In Section II, basic concepts and definition of $p, q-$QOFSs are provided. Section III presents a new extension of COPRAS method by integrating the traditional COPRAS method and $p, q-$QOFSs, and provides simple computing algorithms. Section IV includes an example of green supplier selection to demonstrate the benefits of the proposed method. In the end, we provide a detailed conclusion of the presented study. The layout of the article is presented in Figure 1.

II. PRELIMINARIES

In this section, we review some definitions and operational principles of $p, q-$QOFSs that are related to our proposed methods.

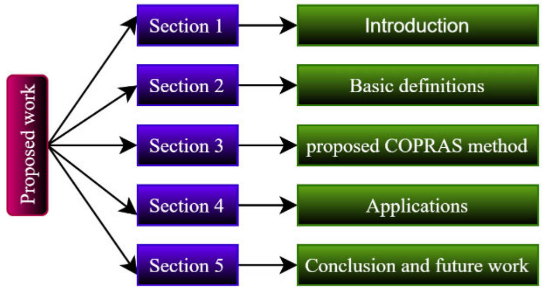

FIGURE 1. Paper layout.

Definition 1: [7] For a non-empty fixed set G, the $p, q-$QOF set is defined as:

$$A = \{g,\ \Phi_A(g),\ \Psi_A(t)\,|g \in G\} \tag{1}$$

where $\Phi_A : G \to [0, 1]$ describes the MD and $\Psi_A : G \to [0,\ 1]$ describes the NMD of the component $g \in G$ respectively, satisfying the condition $\Phi_A^p(g) + \Psi_A^q(g) \leq 1 (q \leq p$ or $p < q)$ for every element $t \in S$. For the sake of simplicity $(\Phi_A(g), \Psi_A(g))$ is referred to as a $p, q-$ QOFN, which can be represented as $\alpha = (\Phi_\alpha,\ \Psi_\alpha)$, satisfying the condition $\Phi_\alpha^p + \Psi_\alpha^q \leq 1$.

Definition 2: [7] Let $\alpha = (\Phi_\alpha,\ \Psi_\alpha)$, $\alpha_1 = \left(\Phi_{\alpha_1},\ \Psi_{\alpha_1}\right)$ and $\alpha_2 = \left(\Phi_{\alpha_1},\ \Psi_{\alpha_1}\right)$ be any three $p, q-$QOFN and $\mu > 0$, then

i. $\alpha_1 \oplus \alpha_2 = \left(\sqrt[p]{\begin{matrix}\left(\Phi_{\alpha_1}\right)^p + \left(\Phi_{\alpha_2}\right)^p \\ -\left(\Phi_{\alpha_1}\right)^p\left(\Phi_{\alpha_2}\right)^p\end{matrix}}, \Psi_{\alpha_1}\Psi_{\alpha_2}\right)$,

ii. $\alpha_1 \otimes \alpha_2 = \left(\Phi_{\alpha_1}\Phi_{\alpha_2}, \sqrt[q]{\begin{matrix}\left(\Psi_{\alpha_1}\right)^q + \left(\Psi_{\alpha_2}\right)^q \\ -\left(\Psi_{\alpha_1}\right)^q\left(\Psi_{\alpha_2}\right)^q\end{matrix}}\right)$,

iii. $\mu\alpha = \left(\sqrt[p]{1-\left(1-\Phi_\alpha^p\right)^\mu}, \Psi_P^\mu\right)$,

iv. $\alpha^\mu = \left(\Phi_P^\mu, \sqrt[q]{1-\left(1-\Psi_\alpha^q\right)^\mu}\right)$.

For the comparison of various $p, q-$QOFNs in tackling a decision making scenario, the score function, along with the accuracy function, is pivotal and formulated in $p, q-$QOF information systems [7] as follows.

Definition 3: [7] Let $\alpha = (\Phi_\alpha,\ \Psi_\alpha)$ be a $p, q-$QOFN. The score function of α is defined as follows:

$$sc(\alpha) = \frac{1 + (\Phi_\alpha)^p - (\Psi_\alpha)^q}{2} \tag{2}$$

where $0 \leq sc(\alpha) \leq 1$.

Definition 4: [7] Let α be a $p, q-$QOFN. The accuracy function of α is defined as follows:

$$ac(\alpha) = \Phi_\alpha^p + \Psi_\alpha^q \tag{3}$$

where $0 \leq sc(\alpha) \leq 1$.

Aggregation operators are essential tools for simplifying and summarizing data, facilitating analysis, and aiding decision-making processes. Seikh and Mandal [7] also gave a series of aggregation operators for $p, q-$QOFNs.

Definition 5: [7] Let $\alpha_j = \left(\Phi_{\alpha_j}, \Psi_{\alpha_j}\right)$ $(j = 1, 2, \ldots, n)$ be any collection of $p, q-$QOFNs. The structure of $p, q-$QOF weighted arithmetic ($p, q-$QOFWA) operator is defined as:

$$p, q - QOFWA\left(\alpha_1, \alpha_2, \ldots, \alpha_n\right) = \left(\begin{array}{c}\sqrt[p]{1 - \prod_{j=1}^{n}\left(1 - \left(\Phi_{\alpha_j}\right)^p\right)^{\omega_j}}, \\ \prod_{j=1}^{n}\left(\Psi_{\alpha_j}\right)^{\omega_j}\end{array}\right) \quad (4)$$

where ω_j is a weight vector such that $0 \leq \omega_j \leq 1$ and $\sum_{k=1}^{n}\omega_j = 1$.

Definition 6: [7] Let $\alpha_j = \left(\Phi_{\alpha_j}, \Psi_{\alpha_j}\right)$ $(j = 1, 2, \ldots, n)$ be any collection of $p, q-$QOFNs. The structure of $p, q-$QOF weighted geometric ($p, q-$QOFWG) operator is defined as:

$$p, q - QOFWG\left(\alpha_1, \alpha_2, \ldots, \alpha_n\right) = \left(\begin{array}{c}\prod_{j=1}^{n}\left(\Phi_{\alpha_j}\right)^{\omega_j}, \\ \sqrt[q]{1 - \prod_{j=1}^{n}\left(1 - \left(\Psi_{\alpha_j}\right)^q\right)^{\omega_j}}\end{array}\right) \quad (5)$$

where ω_j is a weight vector such that $0 \leq \omega_j \leq 1$ and $\sum_{k=1}^{n}\omega_j = 1$.

III. THE PROPOSED COPRAS METHOD

The existing fuzzy COPRAS systems are based on membership degrees, however they lack flexibility to specific circumstances and needs since the effects of membership cannot be properly operated. In response, we propose a unique addition to the COPRAS technique that allows for dynamic management of membership degrees for each element (alternative) based on the decision context. This modification adds the parameters p and q, allowing for more subtle adjustments to the effect of membership based on the decision-making situation. The following sections provide a detailed overview of the proposed method's phases.

Suppose we have a set of m alternatives $\{S_1, S_2, \ldots, S_m\}$ with n criteria $\{C_1, C_2, \ldots, C_n\}$ and t experts $\{x_1, x_2, \ldots, x_t\}$. Let $(\phi_1, \phi_2, \ldots, \phi_n)$ and $(\phi_1, \phi_2, \ldots, \phi_t)$ be the weighting vector of criteria and experts respectively which meet $\varphi_i, \phi_k \in [0, 1]$, and $\sum_{i=1}^{n}\phi_i = \sum_{k=1}^{t}\varphi_k = 1$. To discover the appropriate green supplier, the presented COPRAS process has been divided into the following phases.

Phases 1. Collect the assessment of each decision-maker and build the assessing matrix $\mathcal{D}^{(t)} = \left(d_{ij}^k\right)_{m\times n} = \left(\Phi_{\alpha_{ij}}, \Psi_{\alpha_{ij}}\right)$ such that $\left(\Phi_{\alpha_{ij}}\right)^p + \left(\Psi_{\alpha_{ij}}\right)^q \leq 1$.

In real-life decision-making scenarios, decision-makers may require different levels of powers for membership grades. Therefore, the extended COPRAS method addresses this issue by enabling decision-makers to utilize different power levels for membership grades through the inclusion of parameters p and q. In the proposed framework, the parameters p and q are positive integers, satisfying the conditions of $p > q, p = q$, or $p < q$. The flexibility of this model lies in the adjustability of these parameters, allowing for customization based on the specific requirements of the decision-making problems at hand.

$$\mathcal{D}^{(k)} = \left(d_{ij}^k\right)_{m\times n} = \begin{pmatrix} d_{11}^k & d_{12}^k \cdots & d_{1n}^k \\ d_{21}^k & d_{22}^k \cdots & d_{2n}^k \\ \vdots & \vdots \ddots & \vdots \\ d_{m1}^k & d_{m2}^k \cdots & d_{\Updownarrow\backslash}^k \end{pmatrix} \quad (6)$$

$$\mathcal{D} = \left(d_{ij}\right)_{m\times n} = \begin{pmatrix} d_{11} & d_{12} \cdots & d_{1n} \\ d_{21} & d_{22} \cdots & d_{2n} \\ \vdots & \vdots \ddots & \vdots \\ d_{m1} & d_{m2} \cdots & d_{\Updownarrow\backslash} \end{pmatrix} \quad (7)$$

$$d_{ij} = \left(\begin{array}{c}\sqrt{1 - \prod_{j=1}^{n}\left(1 - \left(\Phi_{\alpha_{ij}^k}\right)^p\right)^{\omega_j}} \\ , \prod_{j=1}^{n}\left(\Psi_{\alpha_{ij}^k}\right)^{\omega_j}\end{array}\right) \quad (8)$$

where d_{ij}^k is the assessment values of alternative S_i with respect to the criterion C_j and decision-maker S_k.

Phases 2. Using the CRITIC approach, determine the weighted structured matrix.

Calculating the criterion weight is seen as a key phase in dealing with MCGDM complications. Different criteria may be counterweights, which might produce different results. Decision-makers find it challenging and impartial weight values from the actual information. Because the process of obtaining weight values is complex, therefore, the expertise and preconceptions of experts may impact the whole decision process. As a result, the Criteria Importance Through Inter-Criteria Correlation (CRITIC) [54] technique is a superb option for avoiding these concerns as an objective method. In this process, it is possible to determine the weights of various criteria based on how much data they include, resulting in objective criteria weights and avoiding subjective assessment. Following that, we shall go into the strategy's mathematical methods.

Step 1. The coefficient of correlation matrix $r = \left(\Im_{j\eta}\right)_{n\times n}$ $(j, \eta = 1, 2, \ldots, n)$ is constructedbased on the average $p, q-$ QOF decision matrix $\mathcal{D} = \left(d_{ij}\right)_{m\times n} = \sum_{k=1}^{t}\left(\omega_k d_{ij}^k\right)$ by determining the correlation coefficient between
criteria.

$$\Im_{j\eta} = \frac{\sum_{i=1}^{m}\begin{array}{c}\left[sc\left(d_{ij}\right) - sc\left(d_j\right)\right] \\ \times\left[sc\left(d_{i\eta}\right) - sc\left(d_\eta\right)\right]\end{array}}{\left(\sqrt{\sum_{i=1}^{m}\left[\begin{array}{c}sc\left(d_{ij}\right) \\ -sc\left(d_j\right)\end{array}\right]^2}\right)\left(\sqrt{\sum_{i=1}^{m}\left[\begin{array}{c}sc\left(d_{i\eta}\right) \\ -sc\left(d_\eta\right)\end{array}\right]^2}\right)} \quad (9)$$

where $sc\left(d_j\right) = \frac{1}{m}\sum_{i=1}^{m} sc\left(d_{ij}\right)$ and $sc\left(d_\eta\right) = \frac{1}{m}\sum_{i=1}^{m} sc\left(d_{i\eta}\right)$.

Step 2. Calculate the standard deviation of each criterionas follows:

$$\mathfrak{M}_j = \left(\sqrt{\sum_{i=1}^{m} \frac{1}{m}\left[sc\left(d_{ij}\right) - \mathcal{S}\left(d_j\right)\right]^2}\right) \tag{10}$$

where $sc\left(d_j\right) = \frac{1}{m}\sum_{i=1}^{m} sc\left(d_{ij}\right)$.

Step 3. Calculate the weight of each criterionas follows:

$$q_j = \frac{\mathfrak{M}_j \sum_{\eta=1}^{n}\left(1 - \mathfrak{I}_{i\eta}\right)}{\sum_{j=1}^{n}\left(\mathfrak{M}_j \sum_{\eta=1}^{n}\left(1 - \mathfrak{I}_{i\eta}\right)\right)} \tag{11}$$

where $q_j \in [0, 1]$and $\sum_{j=1}^{n} q_j = 1$.

Phase 3. Calculate the normalized decision matrix. The normalized weighted matrix is calculated as:

$$\mathcal{W} = q_j d_{ij} \tag{12}$$

Phase 4. For the benefit sum $\mathcal{R}_i^+$ all the values of the criteria. Let $\{C_1, C_2, \ldots, C_n\}$ be the set of criteria the higher values of which is appropriate. For each alternative $\mathcal{R}_i^+$ calculate the following index.

$$\mathcal{R}_i^+ = \sum_{j=1}^{n} \mathcal{W}_{ij}^+ \tag{13}$$

Phase 5. Sum the values $\mathcal{R}_i^-$ of cost criteria. Let $\{C_1{}^*, C_2{}^*, \ldots, C_n{}^*\}$ be the set of criteria the lower values of which are appropriate. Then, for each alternative $\mathcal{R}_i^-$ calculate the following index.

$$\mathcal{R}_i^- = \sum_{j=1}^{n} \mathcal{W}_{ij}^- \tag{14}$$

Phase 6. Compute the smallest value of $\mathcal{R}_i$ as follows:

$$\mathcal{R}_{min_i} = min\left(\mathcal{R}_i^-\right) \tag{15}$$

Phase 7. Calculate the ratings of each alternative based on benefit and cost criteria as follows:

$$\mathcal{Q}_i = sc\left(\mathcal{R}_i^+\right) + \frac{sc\left(\mathcal{R}_{min}^-\right)\sum_{i=1}^{m}\mathcal{R}_i^-}{sc\left(\mathcal{R}_i^-\right)\sum_{i=1}^{m}\frac{sc\left(\mathcal{R}_{min}^-\right)}{sc\left(\mathcal{R}_i^-\right)}} \tag{16}$$

Phase 8. Calculate optimality criterion $\mathcal{Q}_{max}$ as follows:

$$\mathcal{Q}_{max} = max\left(\mathcal{Q}_i\right) \tag{17}$$

Phase 9. The degree of efficacy for each alternative $\mathfrak{R}_i$ calculated by comparing the other alternative with the best alternative. The values of the degree of utility ranged from 0% to 100% between the worst and best alternatives. The efficacy degree $\mathcal{N}_j$ for each alternative j^{th} is determined as follows:

$$\mathcal{N}_j = \frac{\mathcal{Q}_i}{\mathcal{Q}_{max}} \times 100\% \tag{18}$$

The systemic flowchart of the proposed CPF-COPRAS method is summarized in Figure 2.

Thus, an algorithm for the pseudocode representation of the proposed COPRAS method can be summarized as follows:

Algorithm 1 Pseudocode Representation of the Proposed COPRAS Method

Input: $m-$ number of alternatives, $n-$ number of criteria, and $t-$ number of experts.
*Output:*Green supplier selection.
Begin
*Step 1:*Construct the decision-matrix and weight value of each expert in the form of $p, q-$ QOFNs.
*Step 2:*For $j = 1$ to n
  Compute standard deviation of each criterion $\mathfrak{M}_j$ using Eq. (10).
 Endfor
Step 3: For $j = 1$ to n
  Calculate the weight of each criterion using Eq. (11).
 Endfor
Step 4: For $j = 1$ to n and $i = 1$ to m
 Calculate the normalized decision matrix using Eq. (12).
 Endfor
Step 5: For benefit sum cost sum using the indexes listed in Eqs. (13) and (14)
Step 6: For $i = 1$ to m
  Compute the smallest value of $\mathcal{R}_i$, using Eq. (15)
Endfor
Step 7: for $i = 1$ to m
  Calculate the rating of each alternative based on benefit and cost criteria using Eq. (16)
 Endfor
Step 8: Calculate optimality criterion $\mathcal{Q}_{max}$ using Eq. (17).
*Step 9.*For $i = 1$ to m
  Calculate the efficacy degree $\mathcal{N}_j$ using Eq. (18).
 Endfor
End

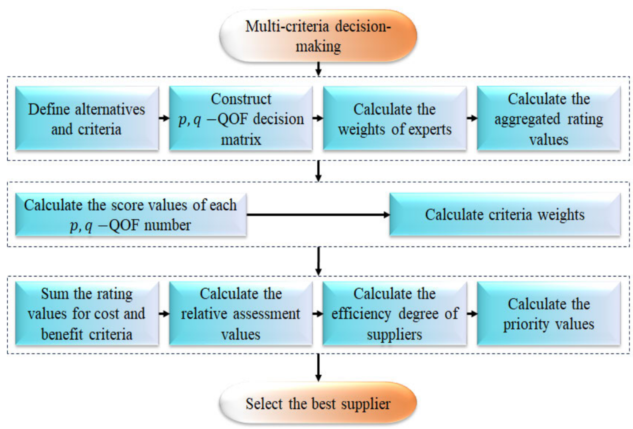

FIGURE 2. Subsequent phases of the $p, q-$QOF-COPRAS method.

IV. APPLICATION

Supply chain strategy forms the foundation of green supply chain management, a pivotal element in promoting environmentally friendly practices. The efficiency of the entire supply network directly hinges on supplier capabilities. Hence, finding swift and efficient solutions for selecting appropriate

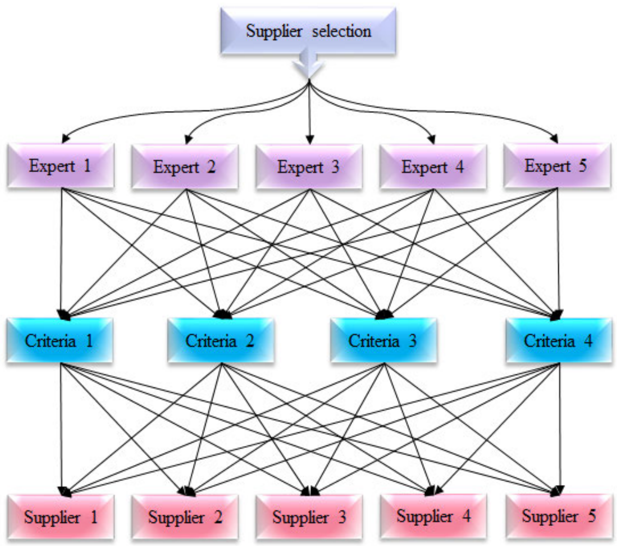

FIGURE 3. The flowchart of the proposed model.

suppliers is crucial for companies. Scholars emphasize the importance of considering both product quality and service standards when choosing suppliers, regardless of whether they are conventional or green. The quality of items provided by suppliers significantly influences enterprise processing, manufacturing, market efficiency, and product quality. The supplier's effectiveness in issue resolution and product delivery is reflected in the service level, a crucial benchmark for ensuring successful raw material supply. In selecting green suppliers, businesses must also consider factors like environmental protection approach and green energy quality. The identification of green suppliers is a well-known MAGDM issue [44], [55]. Building on a fundamental investigation and survey regarding green supplier selection, management has chosen four criteria to review and pick appropriate green suppliers:

1. C_1 is the team's skill.
2. C_2 is the green degree of design and production.
3. C_3 is the expenditure of waste disposal.
4. C_4 is the service level.

We engaged five experts (Environmental Scientist (x_1), Supply Chain Manager (x_2), Sustainability Consultant (x_3), Procurement Specialist (x_4), and Corporate Social Responsibility Manager (x_5)) to individually assess the five green suppliers (S_1, S_2, S_3, S_4, and S_5) based on these criteria, constructing five $p, q-$ QOF decision matrices presented in Tables 1-5. For the following steps we fixed the parameters p and q at 2. The step-by-step path way of the proposed model is presented in Figure 3.

TABLE 1. $p, q-$QOF decision matrix provided by expert x_1.

S_i	C_1	C_2	C_3	C_4
S_1	(0.5,0.4)	(0.3,0.4)	(0.1,0.6)	(0.7,0.2)
S_2	(0.2,0.5)	(0.4,0.2)	(0.3,0.6)	(0.3,0.5)
S_3	(0.6,0.4)	(0.2,0.7)	(0.1,0.4)	(0.1,0.3)
S_4	(0.1,0.3)	(0.2,0.8)	(0.4,0.6)	(0.5,0.6)
S_5	(0.5,0.4)	(0.5,0.2)	(0.6,0.3)	(0.3,0.8)

TABLE 2. $p, q-$QOF decision matrix provided by expert x_2.

S_i	C_1	C_2	C_3	C_4
S_1	(0.3,0.5)	(0.5,0.8)	(0.4,0.3)	(0.3,0.6)
S_2	(0.4,0.6)	(0.1,0.2)	(0.6,0.4)	(0.7,0.4)
S_3	(0.5,0.7)	(0.6,0.4)	(0.3,0.7)	(0.4,0.1)
S_4	(0.3,0.5)	(0.8,0.1)	(0.4,0.3)	(0.2,0.5)
S_5	(0.6,0.3)	(0.3,0.6)	(0.4,0.2)	(0.5,0.4)

TABLE 3. $p, q-$QOF decision matrix provided by expert x_3.

S_i	C_1	C_2	C_3	C_4
S_1	(0.2,0.7)	(0.6,0.3)	(0.4,0.5)	(0.5,0.2)
S_2	(0.1,0.8)	(0.4,0.1)	(0.3,0.6)	(0.4,0.2)
S_3	(0.2,0.1)	(0.3,0.4)	(0.2,0.8)	(0.5,0.6)
S_4	(0.5,0.4)	(0.5,0.7)	(0.4,0.5)	(0.3,0.4)
S_5	(0.3,0.7)	(0.2,0.6)	(0.2,0.4)	(0.3,0.1)

TABLE 4. $p, q-$QOF decision matrix provided by expert x_4.

S_i	C_1	C_2	C_3	C_4
S_1	(0.4,0.5)	(0.4,0.2)	(0.4,0.2)	(0.3,0.4)
S_2	(0.3,0.5)	(0.6,0.2)	(0.6,0.1)	(0.4,0.5)
S_3	(0.7,0.3)	(0.3,0.5)	(0.3,0.4)	(0.6,0.3)
S_4	(0.3,0.2)	(0.1,0.8)	(0.7,0.4)	(0.5,0.3)
S_5	(0.1,0.4)	(0.4,0.5)	(0.3,0.5)	(0.4,0.1)

TABLE 5. $p, q-$QOF decision matrix provided by expert x_5.

S_i	C_1	C_2	C_3	C_4
S_1	(0.4,0.6)	(0.5,0.4)	(0.3,0.7)	(0.4,0.5)
S_2	(0.1,0.2)	(0.1,0.5)	(0.2,0.3)	(0.6,0.5)
S_3	(0.6,0.4)	(0.2,0.7)	(0.4,0.6)	(0.2,0.6)
S_4	(0.2,0.6)	(0.6,0.5)	(0.6,0.4)	(0.3,0.5)
S_5	(0.5,0.4)	(0.1,0.2)	(0.3,0.4)	(0.4,0.3)

Step 1. Using Equations (4) and (5) to calculate aggregated weighted normalized decision matrix. Initially, we assume the weight criterion (factor) as $\omega = (0.15, 0.35, 0.20, 0.30)$. The weighted normalized decision matrices are summarized in Tables 6 and 7. Note that the weight vector for each criterion can be changed according to the situation.

Phase 2. From Table 6, calculate the average cubic PF decision matrix by using Equation (8) as follows:

Phase 3. Compute the individual criteria weights q_j using Equations (9) to (11). The results are summarized in Table 9. From Table 9, we can see that $\sum_{j=1}^{n} q_j = 1$.

TABLE 6. $p, q-$QOF weighted averaging normalized matrix.

S_i	C_1	C_2	C_3	C_4
S_1	(0.376,0.557)	(0.242,0.350)	(0.265,0.643)	(0.325,0.478)
S_2	(0.073,0.145)	(0.093,0.457)	(0.186,0.275)	(0.564,0.463)
S_3	(0.368,0.341)	(0.190,0.638)	(0.357,0.546)	(0.185,0.562)
S_4	(0128,0.548)	(0.567,0.470)	(0.574,0.382)	(0.239,0.458)
S_5	(0.435,0.377)	(0.078,0.158)	(0.284,0.383)	(0.390,0.290)

Phase 4. Using Equations (8) and (12) to calculate the weighted normalized assessment matrix $\mathcal{W}$. The results are summarized in Table 10.

Phase 5. Sum up the benefit criteria by using Equation (13), $\mathcal{R}_1^+ = (0.3776, 0.7011)$, $\mathcal{R}_2^+ = \big(0.5291, 0.7953\big)$, $\mathcal{R}_3^+ = (0.2904, 0.3064)$, $\mathcal{R}_4^+ = (0.3910, 0.7144)$.

Phase 6. Sum up the cost criteria by using Eq. (14), $\mathcal{R}_1^- = (0.2365, 0.7595)$, $\mathcal{R}_2^- = (0.2365, 0.8913)$, $\mathcal{R}_3^- = (0.1013, 0.8948)$, $\mathcal{R}_4^- = (0.2567, 0.8821)$.

Phase 7. Calculate the COPRAS index using Equation (15) to evaluate the relative relevance of each supplier based on benefit and cost criteria. We have $\mathcal{Q}_1 = 0.6117$, $\mathcal{Q}_2 = 0.6345$, $\mathcal{Q}_3 = 0.6293$, $\mathcal{Q}_4 = 0.6548$, and $\mathcal{Q}_5 = 0.6892$.

The obtained rank of available green suppliers using the $p, q-$QOF-COPRAS method is $S_5 \succ S_4 \succ S_2 \succ S_3 \succ S_1$.

The decision-makers can consider the membership function as a satisfaction degree and the non-membership function as a dissatisfaction degree. The evaluations can make in the form of satisfaction and dissatisfaction by the decision-makers as $p, q-$QOFSs. The appropriate green supplier is S_5.

A. SENSITIVITY ANALYSIS

To assess the impact of changes in the parameters p and q on the decision outcomes, a sensitivity analysis is conducted. This analysis aids in comprehending the extent to which modifications to these parameters affect the final decisions made. We first set value q to 2 and then gradually changed parameter p between 2 and 6. Table 11 provides a comprehensive summary of the investigations' outcomes.

Analyzing Table 11 demonstrates variations in the relative relevance of each supplier considering benefit and cost criteria when we raise the parameter p while maintaining q constant at 2. Figure 5 shows the results of this sensitivity study graphically.

TABLE 7. $p, q-$QOF weighted geometric normalized matrix.

S_i	C_1	C_2	C_3	C_4
S_1	(0.335,0.501)	(0.203,0.310)	(0.214,0.601)	(0.291,0.430)
S_2	(0.052,0.101)	(0.069,0.419)	(0.139,0.229)	(0.524,0.429)
S_3	(0.320,0.308)	(0.149,0.600)	(0.316,0.508)	(0.115,0.518)
S_4	(0.096,0.494)	(0.517,0.427)	(0.522,0.321)	(0.201,0.410)
S_5	(0.401,0.331)	(0.049,0.119)	(0.242,0.346)	(0.356,0.251)

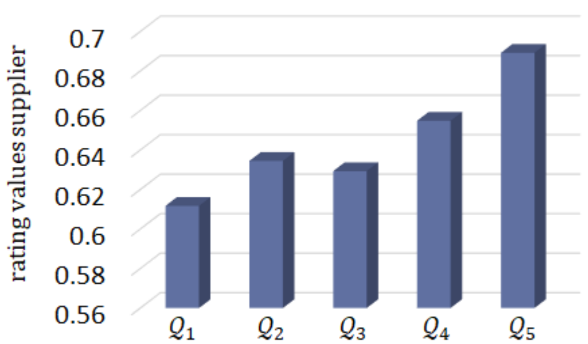

FIGURE 4. The comparative importance or significance of each supplier.

TABLE 8. Averaging weighted $p, q-$QOF decision matrix.

S_i	C_1	C_2	C_3	C_4
S_1	(0.705,0.221)	(0.837,0.310)	(0.883,0.237)	(0.912,0.217)
S_2	(0.832,0.059)	(0.503,0.175)	(0.814,0.117)	(0.920,0.192)
S_3	(0.852,0.125)	(0.801,0.190)	(0.671,0.194)	(0.704,0.104)
S_4	(0.874,0.216)	(0.604,0.176)	(0.876,0.159)	(0.803,0.202)
S_5	(0.932,0.129)	(0.546,0.675)	(0.836,0.157)	(0.829,0.104)

TABLE 9. Individual criteria weights.

S_i	C_1	C_2	C_3	C_4
S_1	$q_1 = 0.1462$	$q_2 = 0.2509$	$q_3 = 0.3051$	$q_4 = 0.2978$
S_2	$q_1 = 0.2343$	$q_2 = 0.1757$	$q_3 = 0.2145$	$q_4 = 0.3755$
S_3	$q_1 = 0.2511$	$q_2 = 0.1942$	$q_3 = 0.3128$	$q_4 = 0.2419$
S_4	$q_1 = 0.2645$	$q_2 = 0.1889$	$q_3 = 0.3217$	$q_4 = 0.2249$
S_5	$q_1 = 0.1976$	$q_2 = 0.2705$	$q_3 = 0.2922$	$q_4 = 0.2397$

TABLE 10. Weighted normalized assessment matrix.

S_i	C_1	C_2	C_3	C_4
S_1	(0.923,0.012)	(0.914,0.017)	(0.916,0.002)	(0.921,0.001)
S_2	(0.905,0.003)	(0.889,0.010)	(0.965,0.004)	(0.920,0.004)
S_3	(0.938,0.004)	(0.951,0.014)	(0.865,0.012)	(0.810,0.007)
S_4	(0.951,0.005)	(0.876,0.010)	(0.961,0.009)	(0.890,0.013)
S_5	(0.943,0.001)	(0.942,0.012)	(0.902,0.001)	(0.882,0.003)

TABLE 11. Relative relevance of alternatives for $p = 2, 3, 4, 5, 6 (q = 2)$.

p	relative relevance of suppliers				
	Q_1	Q_2	Q_3	Q_4	Q_5
2	0.611	0.634	0.629	0.654	0.689
3	0.624	0.636	0.631	0.655	0.692
4	0.627	0.638	0.632	0.657	0.694
5	0.632	0.641	0.633	0.659	0.695
6	0.635	0.642	0.634	0.660	0.696

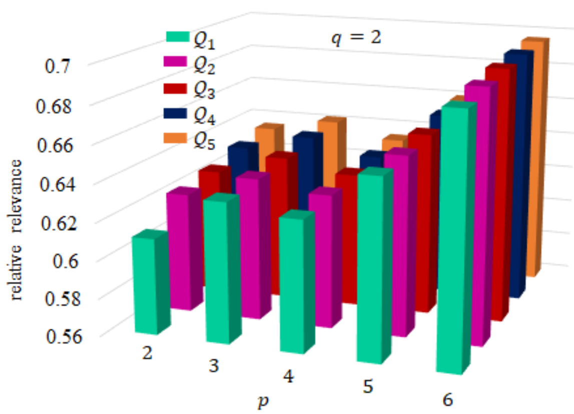

FIGURE 5. Relative relevance of suppliers for $p = 2, 3, 4, 5, 6$.

With q set at 2, and p values ranging from 2 to 5, the suppliers are ranked as follows: $S_5 \succ S_4 \succ S_2 \succ S_3 \succ S_1$. But if p is set to 6, then $S_5 \succ S_4 \succ S_3 \succ S_2 \succ S_1$ is the new ranking order. This discovery suggests that adjustments in the parameter p may cause the ranking order of suppliers to alter. Similarly,

TABLE 12. Relative relevance of alternatives for $q = 2, 3, 4, 5, 6 (p = 2)$.

q	Relative relevance of suppliers				
	Q_1	Q_2	Q_3	Q_4	Q_5
2	0.611	0.634	0.629	0.654	0.689
3	0.615	0.635	0.630	0.656	0.690
4	0.617	0.637	0.631	0.658	0.691
5	0.619	0.638	0.633	0.659	0.693
6	0.620	0.639	0.633	0.661	0.697

when we set p to 2 and systematically adjust the parameter q within the range of 1 to 5, the outcomes are consolidated and presented in Table 12.

The relative relevance of the alternative's changes across different combinations of parameters p and q, according to an analysis of Tables 11 and 12. It is important to note, nevertheless, that the best option stays the same regardless of the precise combinations of p and q.

B. COMPARATIVE STUDY

We compare the results of the suggested strategy against a number of other strategies [56], [57], [58], [59], [60], [61], and [62] in order to verify its findings. Table 13 provides a concise summary of the results obtained from this comparative investigation. The results show that, for the most part, the supplier rating order is stable across the various techniques.

TABLE 13. Existing approaches and ranking order of suppliers.

Approaches	S_1	S_2	S_3	S_4	S_5
Kumar and Mishra **[56]**	0.1287	0.1456	0.1401	0.2395	0.2974
Thakur et al. **[57]**	0.1034	0.1514	0.1302	0.2041	0..2469
Dorfeshan and Mousavi **[58]**	0.4511	0.5132	0.4853	0.5344	0.5560
Krishankumar and Ecer **[59]**	0.3701	0.4552	0.4394	0.4958	0.5032
Rani et al. **[60]**	0.4193	0.4461	0.4352	0.4877	0.5409
Bolturk **[61]**	0.6475	0.6765	0.6881	0.7194	0.7463
Naeem et al. **[62]**	0.3117	0.3743	0.3901	0.4381	0.4861

The COPRAS techniques described in Table 13 do not provide a parameter for modifying the membership degree based on specific situations. In contrast, the suggested

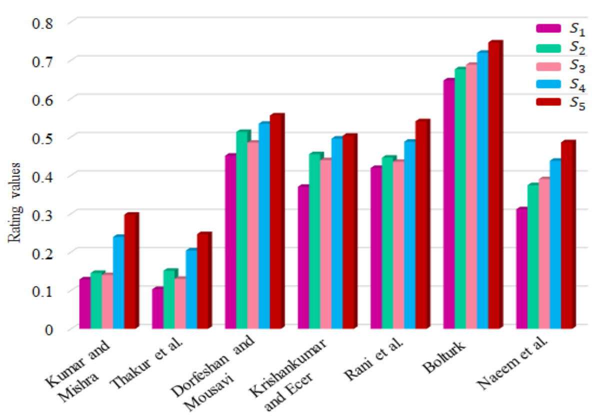

FIGURE 6. Comparison of suppliers.

COPRAS includes two parameters, p and q, with p regulating the effect of the membership degree and q governing the non-membership degree throughout the decision-making process. This addition makes the suggested technique more flexible and realistic than previous approaches, allowing for finer and context-dependent adjustments to membership degrees during decision-making. Figure 6 depicts the supplier ranking order determined by existing techniques.

Another strength of the proposed COPRAS method lies in its ability to compute both maximizing and minimizing criteria. This flexibility allows for a more balanced assessment, considering both positive and negative aspects of supplier performance. Moreover, our approach can handle both qualitative and quantitative criteria, further improving its versatility and applicability in real-world decision-making scenarios. The characteristics of the proposed and existing approaches are shown in Table 14.

TABLE 14. Features comparison.

Approaches	MD	NMD	p	q
IFS-COPRAS	yes	yes	no	no
PFS-COPRAS	yes	yes	no	no
q −ROF-COPRAS	yes	yes	no	yes
p, q −COPRAS	yes	yes	yes	yes

C. ADVANTAGES

1. The proposed method is made to handle $p, q-$ QOFinformation, enabling a more thorough portrayal of uncertainty and ambiguity in scenarios involving decision-making. This approach may collect a wider range of evaluation data by incorporating cubic information, resulting in more precise and reliable supplier ratings.
2. The process successfully combines a variety of factors and qualities important for choosing green suppliers. To provide a comprehensive evaluation of suppliers based on many aspects, it considers both qualitative and quantitative variables, such as environmental effect, service level, waste disposal, and team capabilities.
3. The CRITIC method is used in the procedure to include inter-criteria correlations. By ensuring that the most important factors are weighted, this improves the objectivity of the MD process while lowering subjectivity.
4. One of the suggested method's features is its capacity to analyze both maximizing and minimizing criteria. This feature enables a balanced assessment, in which both positive and negative elements of supplier performance are considered, resulting in more well-rounded conclusions.
5. The suggested technique seeks to provide more reasonable and trustworthy findings by considering a bigger variety of data and utilizing robust mathematical concepts. This promotes decision-making that is more closely aligned with real-world events and increases the trustworthiness of the results.

6. Its capacity to manage a wide range of criteria and uncertainty makes it an invaluable tool in such complex decision-making circumstances.

D. LIMITATIONS

1. The efficacy of the proposed approach, like that of many other decision-making approaches, is strongly dependent on the availability and quality of data. Obtaining accurate and complete data on supplier performance and environmental factors may offer issues in real-world applications. Inadequate or untrustworthy data might have an influence on the method's reliability and validity.
2. The proposed approach may include computing demands that might be time-consuming for large-scale applications, depending on the size and complexity of the decision issue. It may need a lot of computing power to ensure efficient processing of cubic sets and inter-criteria correlations.
3. The procedure of allocating criteria weights still requires considerable subjectivity despite the use of the CRITIC technique to address inter-criteria correlations. The ultimate outcomes might differ depending on the decision-maker's preferences, which can compromise the method's overall impartiality.

V. CONCLUSION AND FUTURE WORK

In this study, we provide a new extension of the COPRAS method that smoothly integrates the conventional COPRAS approach with the idea of p, q-QOFSs. Because of its parametric character, this innovative technology provides greater flexibility and realism. The addition of two factors, p and q, is critical in managing the effect of membership and non-membership degrees, resulting in a complex and adaptive decision-making model. The proposed technique is shown in the context of green supplier selection. Furthermore, we provide a thorough description of how these characteristics affect decision results. Finally, the study discusses both the benefits and drawbacks of the suggested technique.

In terms of future directions for research, it would be beneficial to investigate at different approaches to decision-making and expand the study by including more participants—especially a broader spectrum of professionals. Moreover, future investigation may focus on how outside variables, including the state of the economy or political unrest, affect supplier selection alternatives. It is important to take into account the enduring consequences of supplier selection choices, encompassing their impacts on the overall efficacy and durability of the supply chain. The study provides insightful information that may be used by researchers and practitioners alike, highlighting the significance of relying supplier selection decisions on careful investigation and analysis.

REFERENCES

[1] S. K. Srivastava, "Green supply-chain management: A state-of-the-art literature review," *Int. J. Manage. Rev.*, vol. 9, no. 1, pp. 53–80, Mar. 2007.

[2] R.-J. Lin, R.-H. Chen, and T.-H. Nguyen, "Green supply chain management performance in automobile manufacturing industry under uncertainty," *Proc. Social Behav. Sci.*, vol. 25, pp. 233–245, 2011.

[3] Z. Ali, T. Mahmood, and M.-S. Yang, "Weighted Bonferroni aggregation operators on complex q-rung orthopair 2-tuple linguistic variables with application to green supply chain management," *IEEE Access*, vol. 11, pp. 139557–139574, 2023.

[4] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets Syst.*, vol. 20, no. 1, pp. 87–96, Aug. 1986.

[5] R. R. Yager, "Pythagorean fuzzy subsets," in *Proc. Joint IFSA World Congr. NAFIPS Annu. Meeting (IFSA/NAFIPS)*, Jun. 2013, pp. 57–61.

[6] R. R. Yager, "Generalized orthopair fuzzy sets," *IEEE Trans. Fuzzy Syst.*, vol. 25, no. 5, pp. 1222–1230, Oct. 2017.

[7] M. R. Seikh and U. Mandal, "Multiple attribute group decision making based on quasirung orthopair fuzzy sets: Application to electric vehicle charging station site selection problem," *Eng. Appl. Artif. Intell.*, vol. 115, Oct. 2022, Art. no. 105299.

[8] E. Zavadskas and A. Kaklauskas, *Multiple Criteria Analysis of Buildings*. Technika Vilnius, 1996.

[9] P. Chatterjee, V. M. Athawale, and S. Chakraborty, "Materials selection using complex proportional assessment and evaluation of mixed data methods," *Mater. Design*, vol. 32, no. 2, pp. 851–860, Feb. 2011.

[10] E. T. Bekar, M. Cakmakci, and C. Kahraman, "Fuzzy COPRAS method for performance measurement in total productive maintenance: A comparative analysis," *J. Bus. Econ. Manage.*, vol. 17, no. 5, pp. 663–684, Oct. 2016.

[11] A. Kaklauskas, E. K. Zavadskas, S. Raslanas, R. Ginevicius, A. Komka, and P. Malinauskas, "Selection of low-e windows in retrofit of public buildings by applying multiple criteria method COPRAS: A Lithuanian case," *Energy Buildings*, vol. 38, no. 5, pp. 454–462, May 2006.

[12] S. Datta, G. Beriha, B. Patnaik, and S. Mahapatra, "Use of compromise ranking method for supervisor selection: A multi-criteria decision making (MCDM) approach," *Int. J. Vocational Tech. Educ.*, vol. 1, no. 1, pp. 7–13, 2009.

[13] A. Soni, S. Chakraborty, P. K. Das, and A. K. Saha, "Material selection of sustainable composites by recycling of waste plastics and agro-industrial waste for structural applications: A fuzzy group decision-making approach," *J. Building Eng.*, vol. 73, Aug. 2023, Art. no. 106787.

[14] N. Agarwal and D. K. Tayal, "A new model based on the extended COPRAS method for improving performance during the accreditation process of Indian higher educational institutions," *Comput. Appl. Eng. Educ.*, vol. 31, no. 3, pp. 728–754, May 2023.

[15] Z. Li, W. Liang, and G. Zhao, "Rockburst hazard evaluation using an extended COPRAS method with interval-valued fuzzy information," *Appl. Sci.*, vol. 13, no. 17, p. 9941, Sep. 2023.

[16] B. Erdebilli, İ. Yilmaz, T. Aksoy, U. Hacıoglu, S. Yüksel, and H. Dinçer, "An interval-valued Pythagorean fuzzy AHP and COPRAS hybrid methods for the supplier selection problem," *Int. J. Comput. Intell. Syst.*, vol. 16, no. 1, Jul. 2023.

[17] L. A. Zadeh, "Fuzzy sets," *Inf. Control*, vol. 8, no. 3, pp. 338–353, 1965.

[18] K.-P. Lu, S.-T. Chang, and M.-S. Yang, "Change-point detection for shifts in control charts using fuzzy shift change-point algorithms," *Comput. Ind. Eng.*, vol. 93, pp. 12–27, Mar. 2016.

[19] S. Poulik, G. Ghorai, and Q. Xin, "Explication of crossroads order based on Randic index of graph with fuzzy information," *Soft Comput.*, vol. 28, no. 3, pp. 1851–1864, Feb. 2024.

[20] S. Das, S. Poulik, and G. Ghorai, "Picture fuzzy Φ-tolerance competition graphs with its application," *J. Ambient Intell. Humanized Comput.*, vol. 15, no. 1, pp. 547–559, Jan. 2024.

[21] S. Poulik and G. Ghorai, "Connectivity concepts in bipolar fuzzy incidence graphs," *Thai J. Math.*, vol. 20, no. 4, pp. 1609–1619, 2022.

[22] C. M. Hwang and M. S. Yang, "New construction for similarity measures between intuitionistic fuzzy sets based on lower, upper and middle fuzzy sets," *Int. J. Fuzzy Syst.*, vol. 15, pp. 359–366, Sep. 2013.

[23] M.-S. Yang, Z. Hussain, and M. Ali, "Belief and plausibility measures on intuitionistic fuzzy sets with construction of belief-plausibility TOPSIS," *Complexity*, vol. 2020, pp. 1–12, Aug. 2020.

[24] J. Mahanta and S. Panda, "A novel distance measure for intuitionistic fuzzy sets with diverse applications," *Int. J. Intell. Syst.*, vol. 36, no. 2, pp. 615–627, Feb. 2021.

[25] J. Ding, C. Zhang, D. Li, J. Zhan, W. Li, and Y. Yao, "Three-way decisions in generalized intuitionistic fuzzy environments: Survey and challenges," *Artif. Intell. Rev.*, vol. 57, no. 2, pp. 1–45, Feb. 2024.

[26] X. Zhang, "Multicriteria Pythagorean fuzzy decision analysis: A hierarchical QUALIFLEX approach with the closeness index-based ranking methods," *Inf. Sci.*, vol. 330, pp. 104–124, Feb. 2016.

[27] M. Rahim, F. Amin, A. Ali, and K. Shah, "An extension of Bonferroni mean under cubic Pythagorean fuzzy environment and its applications in selection-based problems," *Math. Problems Eng.*, vol. 2022, pp. 1–28, Nov. 2022.

[28] G. Wei, H. Gao, and Y. Wei, "Some q-rung orthopair fuzzy heronian mean operators in multiple attribute decision making," *Int. J. Intell. Syst.*, vol. 33, no. 7, pp. 1426–1458, Jul. 2018.

[29] M.-S. Yang, Z. Ali, and T. Mahmood, "Three-way decisions based on Q-rung orthopair fuzzy 2-tuple linguistic sets with generalized Maclaurin symmetric mean operators," *Mathematics*, vol. 9, no. 12, p. 1387, Jun. 2021.

[30] C. Zhang, J. Ding, D. Li, and J. Zhan, "A novel multi-granularity three-way decision making approach in q-rung orthopair fuzzy information systems," *Int. J. Approx. Reasoning*, vol. 138, pp. 161–187, Nov. 2021.

[31] R. M. Zulqarnain, R. Ali, J. Awrejcewicz, I. Siddique, F. Jarad, and A. Iampan, "Some Einstein geometric aggregation operators for q-rung orthopair fuzzy soft set with their application in MCDM," *IEEE Access*, vol. 10, pp. 88469–88494, 2022.

[32] M. R. Seikh and U. Mandal, "Q-rung orthopair fuzzy Frank aggregation operators and its application in multiple attribute decision-making with unknown attribute weights," *Granular Comput.*, vol. 7, no. 3, pp. 709–730, Jul. 2022.

[33] C. Zhang, W. Bai, D. Li, and J. Zhan, "Multiple attribute group decision making based on multigranulation probabilistic models, MULTIMOORA and TPOP in incomplete q-rung orthopair fuzzy information systems," *Int. J. Approx. Reasoning*, vol. 143, pp. 102–120, Apr. 2022.

[34] M. Rahim, K. Shah, T. Abdeljawad, M. Aphane, A. Alburaikan, and H. A. E.-W. Khalifa, "Confidence levels-based p, q-quasirung orthopair fuzzy operators and its applications to criteria group decision making problems," *IEEE Access*, vol. 11, pp. 109983–109996, 2023.

[35] J. Ali and M. Naeem, "Analysis and application of p, q-quasirung orthopair fuzzy Aczel–Alsina aggregation operators in multiple criteria decision-making," *IEEE Access*, vol. 11, pp. 49081–49101, 2023.

[36] M. Rahim, H. Garg, S. Khan, H. Alqahtani, and H. A. E.-W. Khalifa, "Group decision-making algorithm with sine trigonometric P, Q-quasirung orthopair aggregation operators and their applications," *Alexandria Eng. J.*, vol. 78, pp. 530–542, Sep. 2023.

[37] Z. Chen, X. Ming, T. Zhou, and Y. Chang, "Sustainable supplier selection for smart supply chain considering internal and external uncertainty: An integrated rough-fuzzy approach," *Appl. Soft Comput.*, vol. 87, Feb. 2020, Art. no. 106004.

[38] C. A. Weber, J. R. Current, and W. C. Benton, "Vendor selection criteria and methods," *Eur. J. Oper. Res.*, vol. 50, no. 1, pp. 2–18, Jan. 1991.

[39] H. Min and I. Kim, "Green supply chain research: Past, present, and future," *Logistics Res.*, vol. 4, nos. 1–2, pp. 39–47, Mar. 2012.

[40] R. P. Mohanty and A. Prakash, "Green supply chain management practices in india: An empirical study," *Prod. Planning Control*, vol. 25, no. 16, pp. 1322–1337, Dec. 2014.

[41] S. Akcan and M. A. Taş, "Green supplier evaluation with SWARA-TOPSIS integrated method to reduce ecological risk factors," *Environ. Monitor. Assessment*, vol. 191, no. 12, pp. 1–22, Dec. 2019.

[42] H. S. Kilic and A. S. Yalcin, "Modified two-phase fuzzy goal programming integrated with IF-TOPSIS for green supplier selection," *Appl. Soft Comput.*, vol. 93, Aug. 2020, Art. no. 106371.

[43] G. Pishchulov, A. Trautrims, T. Chesney, S. Gold, and L. Schwab, "The voting analytic hierarchy process revisited: A revised method with application to sustainable supplier selection," *Int. J. Prod. Econ.*, vol. 211, pp. 166–179, May 2019.

[44] G. Wei, C. Wei, J. Wu, and H. Wang, "Supplier selection of medical consumption products with a probabilistic linguistic MABAC method," *Int. J. Environ. Res. Public Health*, vol. 16, no. 24, p. 5082, Dec. 2019.

[45] Z. Xu, J. Qin, J. Liu, and L. Martínez, "Sustainable supplier selection based on AHPSort II in interval type-2 fuzzy environment," *Inf. Sci.*, vol. 483, pp. 273–293, May 2019.

[46] L. Fei, J. Xia, Y. Feng, and L. Liu, "An ELECTRE-based multiple criteria decision making method for supplier selection using dempster-shafer theory," *IEEE Access*, vol. 7, pp. 84701–84716, 2019.

[47] R. Kaya and B. Yet, "Building Bayesian networks based on DEMATEL for multiple criteria decision problems: A supplier selection case study," *Expert Syst. Appl.*, vol. 134, pp. 234–248, Nov. 2019.

[48] H. Liao, Y. Long, M. Tang, A. Mardani, and J. Xu, "Low carbon supplier selection using a hesitant fuzzy linguistic SPAN method integrating the analytic network process," *Transformations Bus. Econ.*, vol. 18, no. 2, pp. 67–78, 2019.

[49] R. M. X. Wu, Z. Zhang, W. Yan, J. Fan, J. Gou, B. Liu, E. Gide, J. Soar, B. Shen, S. Fazal-e-Hasan, Z. Liu, P. Zhang, P. Wang, X. Cui, Z. Peng, and Y. Wang, "A comparative analysis of the principal component analysis and entropy weight methods to establish the indexing measurement," *PLoS ONE*, vol. 17, no. 1, Jan. 2022, Art. no. e0262261.

[50] N. Sharkasi and S. Rezakhah, "A modified CRITIC with a reference point based on fuzzy logic and Hamming distance," *Knowledge-Based Syst.*, vol. 255, Nov. 2022, Art. no. 109768.

[51] H. Danai, S. Hashemnia, R. Ahmadi, and S. H. Bazazzadeh, "Application of fuzzy ANP method to select the best supplier in the supply chain," *Int. J. Oper. Res.*, vol. 35, no. 1, pp. 1–19, 2019.

[52] R. Rostamzadeh, M. K. Ghorabaee, K. Govindan, A. Esmaeili, and H. B. K. Nobar, "Evaluation of sustainable supply chain risk management using an integrated fuzzy TOPSIS-CRITIC approach," *J. Cleaner Prod.*, vol. 175, pp. 651–669, Feb. 2018.

[53] D. Pamucar, M. Žižović, and D. Duričić, "Modification of the CRITIC method using fuzzy rough numbers," *Decis. Making, Appl. Manage. Eng.*, vol. 5, no. 2, pp. 362–371, Oct. 2022.

[54] D. Diakoulaki, G. Mavrotas, and L. Papayannakis, "Determining objective weights in multiple criteria problems: The critic method," *Comput. Operations Res.*, vol. 22, no. 7, pp. 763–770, Aug. 1995.

[55] A. Konys, "Green supplier selection criteria: From a literature review to a comprehensive knowledge base," *Sustainability*, vol. 11, no. 15, p. 4208, Aug. 2019.

[56] R. Kumari and A. R. Mishra, "Multi-criteria COPRAS method based on parametric measures for intuitionistic fuzzy sets: Application of green supplier selection," *Iranian J. Sci. Technol., Trans. Electr. Eng.*, vol. 44, no. 4, pp. 1645–1662, Dec. 2020.

[57] P. Thakur, B. Kizielewicz, N. Gandotra, A. Shekhovtsov, N. Saini, A. B. Saeid, and W. Sałabun, "A new entropy measurement for the analysis of uncertain data in MCDA problems using intuitionistic fuzzy sets and COPRAS method," *Axioms*, vol. 10, no. 4, p. 335, Dec. 2021.

[58] Y. Dorfeshan and S. M. Mousavi, "A group TOPSIS-COPRAS methodology with Pythagorean fuzzy sets considering weights of experts for project critical path problem," *J. Intell. Fuzzy Syst.*, vol. 36, no. 2, pp. 1375–1387, Mar. 2019.

[59] R. Krishankumar and F. Ecer, "Selection of IoT service provider for sustainable transport using q-rung orthopair fuzzy CRADIS and unknown weights," *Appl. Soft Comput.*, vol. 132, Jan. 2023, Art. no. 109870.

[60] P. Rani, A. R. Mishra, and A. Mardani, "An extended Pythagorean fuzzy complex proportional assessment approach with new entropy and score function: Application in pharmacological therapy selection for type 2 diabetes," *Appl. Soft Comput.*, vol. 94, Sep. 2020, Art. no. 106441.

[61] E. Bolturk, "Pythagorean fuzzy CODAS and its application to supplier selection in a manufacturing firm," *J. Enterprise Inf. Manage.*, vol. 31, no. 4, pp. 550–564, Jul. 2018.

[62] K. Naeem, M. Riaz, X. Peng, and D. Afzal, "Pythagorean fuzzy soft MCGDM methods based on TOPSIS, VIKOR and aggregation operators," *J. Intell. Fuzzy Syst.*, vol. 37, no. 5, pp. 6937–6957, Nov. 2019.

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