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RESEARCH ARTICLE

# Decision Algorithm With Interval-Valued Intuitionistic Fuzzy Hamy Mean Aggregation Operators for Assessment of Agricultural Education Practice

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**ABSTRACT** The assessment of performance in agricultural education practice is an important issue because it concerns the quality and effectiveness of agricultural education. This evaluation requires effective assessment methods. Interval-valued intuitionistic fuzzy set (IVIFS) is a powerful extension of fuzzy sets and intuitionistic fuzzy sets, conceived to handle uncertainty and vagueness more effectively in agricultural education practice assessment performance. A comprehensive exploration of interval-valued intuitionistic fuzzy (IVIF) information offers some flexible methods for assessment of the performance of agricultural education practice. This article identifies the relationship among input arguments with the help of Hamy mean aggregation models. In order to avail smooth approximated results during the aggregation process, some prominent operations of Aczel Alsina aggregation operators have also been adopted in light of an IVIF theory. The robustness of this article is to develop a family of new mathematical strategies based on IVIF information, namely IVIF Hamy mean (IVIFHM), IVIF weighted Hamy mean (IVIFWHM), IVIF Dual Hamy mean (IVIFDHM) and IVIF weighted Dual Hamy mean (IVIFWDHM) operators. Some dominant properties of diagnosed aggregation operators are also discussed to show their validity and effectiveness. Moreover, the decision algorithm of the multi-attribute decision-making (MAMD) problem is also adopted to reveal the versatility and adaptability of IVIF contexts. A case study of agricultural education practice assessment performance is also illustrated to showcase the practicability of derived approaches. The advantages and supremacy of invented research work are verified with a comparative study of prevailing research work that exists in the literature.

**INDEX TERMS** Interval-valued intuitionistic fuzzy values, Aczel Alsina t-norms and t-conorms, agricultural education practice, decision support system.

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## I. INTRODUCTION

The assessment of performance in agricultural education practice is of great significance to educational institutions, students, and society. It helps improve the quality of education, promote student learning and growth, and drive the development and progress of the agricultural field. The methods for assessing the performance of agricultural education practice do have some shortcomings, such as the lack of standardized indicators, insufficient diversity in evaluation tools, incomplete evaluation processes, inadequate feedback mechanisms, and insufficient data collection and analysis. This necessitates continuous improvement and refinement of evaluation methods, including the establishment of unified evaluation standards and indicators, diversified evaluation tools, comprehensive consideration of evaluation factors, the establishment of sound feedback mechanisms, and strengthening of data collection and analysis to enhance the effectiveness and accuracy of the evaluation. Decision Making (DM) is involved in every activity of life. DM is the most crucial part of a job because the consequences depend on the previous decision. Decision makers and DM are the backbone of problems containing complex and uncertain information. Multi-attribute decision-making (MADM) is a method to choose more useful information from a finite set of alternatives collected by different observations about a specific thing in different ways. It is an efficient technique that is used in different fields such as the treatment of diseases, engineering, business purposes, selection of routes to any destiny calculations, etc. In set theory, for an element, there were only two possibilities: either it belongs to a set with membership degree 1 (satisfying the criteria defined for the set) or not with membership degree 0 (does not satisfy the given criteria for the set), there was no third choice. The set theory was unable to deal with the situation in which there is flexibility for an element that is somehow near to fulfilling the given criteria to be an element of the set. In this situation, the decision maker was unable to ensure an element to include in the set or not. In the effectual aspect of decision-making and problem-solving, the incorporation of fuzzy set (FS) theory formulated by Zadeh [1] has undeniably enriched the direction and perfection of choices across a wide range of domains. In his theory, he introduced a new set with elements having membership degrees between 0 and 1. This set was named by FS. FS has initially been instrumental in handling uncertainty and perfection in decision-making. However, as real-world problems grow in complexity, the classical fuzzy sets encounter their limitations in fully capturing the nuances of uncertainty and vagueness. Subsequently, to attain greater accuracy, the notion of FS evolved into Interval Value Fuzzy Sets (IVFS) by Gehrke et al. [2]. This adaptation introduced a heightened level of flexibility for decision-makers by allowing them to assign membership degrees within a specified range, thereby relaxing the constraints on their decision-making process.

To address these shortcomings, the concept of intuitionistic fuzzy sets (IFS) was introduced by Atanassov [3], offering a broader framework for DM under ambiguity. In the concept of IFS, he discussed membership degree as well as the non-membership degree of element. The ordering of IF numbers is crucial in addressing practical challenges related to decision-making and clustering in the realm of intuitionistic fuzzy systems. In the pursuit of further enhancing the efficacy of IFS in practical applications, IVIFS [4], [5], [6], [7] emerged as a valuable extension. In IVIFS, a membership degree (MD), a non-membership degree (NMD), and a hesitancy degree are defined as providing clear information for an element. After the concept of IVIFS its basic properties and operations were found. IVIFS is more useful than classical FS in handling doubtful information. Furthermore, IVIFS offers utilization techniques for unclear properties in the judgment of doubtful information. IVIFS is applicable in different fields, especially when assessing multiple pieces of information about a decision. This paper delves into the intriguing realm of decision-making, bridging the gap between the complexities of IVIFS and the powerful Hamy mean operators [8] while drawing from the rich toolkit of Aczel-Alsina IVFS [9], [10], [11], [12]. The primary objective is to furnish an exhaustiveframework for decision-makers and authorize them with a robust and adaptable toolset to navigate the intricate landscape of uncertain choices. Atanassov extended FS theory by introducing the concept of an IFS with MD and NMD. The sum of MD and NMD is constrained to be between 0 and 1. However, scenarios arose where IFS faced challenges, as exemplified by a case with MD at 0.65 and NMD at 0.55, resulting in a sum exceeding 1. To address such situations, Yager [13], [14] introduced Pythagorean fuzzy sets (PyFS), where the sum of the squares of MD and NMD is limited to 0 and 1. Mahmood and Ali utilized the properties of entropy measures for resolving genuine real-life applications by incorporating the theory of q-rung [14] orthopair FS and also derived some new mathematical approaches for an advanced decision-making approach of the TOPSIS method. Hussain et al. [15] established a decision algorithm for the MADM problem with prioritized aggregation operators. Yager further developed q-rung orthopair fuzzy sets (q-ROFS) by generalizing PyFS [14]. Cuong proposed [16] the concept of picture fuzzy sets (PFS), encompassing characteristic functions like MD, abstinence degree (AD), NMD, and refusal degree (RD). PFS has a structure with the sum of three terms, constraining MD, NMD, and RD. Hussain et al. [17] deliberated Dombi aggregation operators taking into account the t-spherical fuzzy theory. Lu et al. [18] extended PFS concepts to PF rough sets for addressing real-life problems. Researchers have explored limitations and proposed solutions in various fields. Aggregation operators (AOs) serve as mathematical models for investigating fuzzy information. Scholars have presented AOs for intuitionistic fuzzy sets (IFS), PyFSs, and q-ROFSs, utilizing operations like Schweizer [19] and Sklar power operations, Einstein T-norm [20] and T-conorm, and VIKOR method. An innovative mathematical strategy of Heronian mean operators and

Aczel Alsina operators was developed by Hussain et al. [21]. Novel AOs have been introduced for IVIFSs and bipolar valued fuzzy hesitant systems, incorporating operations of Hamacher [22] T-norm and T-conorm, Aczel Alsina [23] T-norm and T-conorm [24]. These AOs address MADM techniques in diverse scenarios, demonstrating the versatility of FS theories in handling complex information and uncertainty.

This research offers an in-depth exploration of the theoretical foundations, practical applications, and algorithmic implementations of DM processes augmented by IVIFSs Hamy mean operators based on Aczel-Alsina tools, paving the way for improved decision quality in settings fraught with uncertainty and imprecision. Mahmood and Ali utilized the properties of entropy measures for resolving genuine real-life applications by incorporating the theory of q-ROFSs and also derived some new mathematical approaches for an advanced DM approach of the TOPSIS method.

Menger [25] introduced the theoretical foundations of T-norms within statistical metric spaces, outlining specific properties. Numerous researchers have delved into t-norms capabilities, employing advanced mathematical tools like algebraic products. Lee [26] provided the probabilistic algebraic sum of triangular norms within fuzzy systems. Schweizer and Sklar [27] expanded on the theoretical aspects of triangular norms in topological metric spaces. Garg [28] proposed novel approaches using Hamacher aggregation tools to address information loss during the aggregation process, particularly in the context of IFSs. Ali et al. [29] developed aggregation tools based on complex T-Spherical FS, while Mahmood [30] introduced new approaches to complex bipolar FS and applied them to solve MADM problems. Hussain et al. [31] presented a MADM algorithm for evaluating unreliable and unpredictable information with an emphasis on IVPyF systems. Li et al. [32] suggested specific approaches to handle unpredictable situations in human opinions based on different attributes within the framework of IFSs. Akram et al. [33] explored the speculative theory of Dombi aggregation tools, offering methodologies for evaluating optimal options based on various criteria within the PyF information system. Ali Khan et al. [34] introduced new approaches using Einstein's prioritized aggregation tools and applied them to a real-life problem within the MADM technique. Hussain et al. [35] presented robust aggregation tools based on the speculative hypothesis of an Aczel Alsina aggregation model in PyF environments. Zhang [36] extended the theory of Frank aggregation operators to IVIFSs. Yahya et al. [37] introduced new aggregation models based on hesitant fuzzy information. Mahmood et al. [38] proposed Frank aggregation operators and advanced decision-making process of analytic hierarchy method under consideration of interval-valued PF information. Wei [39] anticipated cosine similarity measures and mathematical approaches. Hussain et al. [40] employed aggregation operators to address complex decision-making scenarios that arise in everyday life. These situations involve multiple options for a decision, and Hussain utilized aggregation operators to systematically evaluate and combine various criteria, aiding in the selection of the most suitable alternative.

However, the above-discussed mathematical approaches and research work have a lot of advantages. However, decision-makers face complicated challenges during decision analysis due to incomplete and redundant information about any object. In order to address these challenges, we introduced a family of interval-valued intuitionistic fuzzy theory and Hamy mean models with Aczel Alsina operators, namely IVIFAAWHM and IVIFAAWDHM operators. This paper commences with an introduction named section I that elucidates fundamental concepts of fuzzy sets and their various types.

The remaining parts of this research work are organized as follows: Section II explores some basic notions of Aczel Alsina t-norms and t-conorms, IFSs, and a comparison technique for an intuitionistic fuzzy value. In this section, we also revised the basic definition of the Hamy Mean operator. In section III, we formulated the operators IVIFAAWHM and IVIFAAWDHM by building upon the principles of AAHM. In section IV, We have outlined a step-by-step algorithm for the assessment performance of agricultural education practice. This approach aims to provide more effective results and simplify the decision-making process for users in their daily life alternatives. In Section V, we examined a case study, identified various alternatives, and employed our proposed algorithm to apply the derived operators. By utilizing different parameters, we observed the corresponding score values. In Section VI, we conducted a comparative analysis of our derived operators in contrast to existing ones. We ranked the score values to assess the utility of our operators in comparison to those already in existence. Section VII presents the conclusive results of our proposed operators, supported by a comparative analysis that highlights the authenticity of the operators derived in this research paper. Figure 1 explores the characteristics of the proposed work.

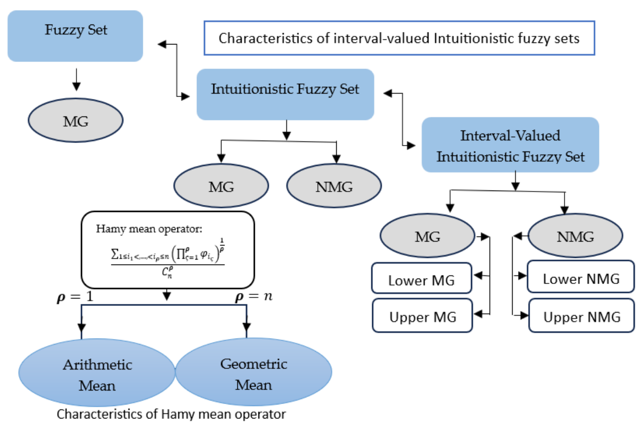


**FIGURE 1.** Diagram for proposed work.

## II. PRELIMINARIES

This section explores basic preliminaries and fundamental notions of Aczel Alsina operations, IVIFSs, and many other prominent principles.

*Definition 1 [41]:* A function $T : [0,1]^2 \rightarrow [0,1]$ is a t-norm, if the consecutive axioms are fulfilled

a. Symmetry: $T(p,s) = T(s,f)$
b. Monotonically: $T(p,s) \leq T(p,l)$ if $s \leq l$
c. Associativity: $T(p, T(s,l)) = T(T(p,s), l)$
d. One identity: $T(p,1) = p$

For all $p, s, l \in [0,1]$.

*Example 1:* Some examples of t-norms are given below:

a. Product t-norm: $T_p(p,s) = p.s$
b. Minimum t-norm: $T_m(p,s) = min(p,s)$
c. Lukasiewicz t-norm $T_l(p,s) = max(p+s-1, 0)$
d. Drastic t-norm

$$T_d(p,s) = \begin{cases} p, & if\ s = 1 \\ s & if\ p = 1 \\ 0 & otherwise \end{cases}$$

For all $p, s \in [0,1]$.

*Definition 2 [34]:* A function $R : [0,1]^2 \rightarrow [0,1]$ is a t-conorm if the following axioms are fulfilled

a. Symmetry: $R(p,s) = R(s,p)$
b. Monotonically: $R(p,s) \leq R(p,l)$ if $s \leq l$
c. Associativity: $R(p, R(s,l)) = R(R(p,s), l)$
d. Zero identity: $R(p,0) = p$

for all $p, s, l \in [0,1]$.

*Example 2:* Some examples of t-conorm are given as:

a. Probabilistic sum: $R_p(p,s) = p + s - p.s$
b. Maximum t-conorm: $R_m(p,s) = max(p,s)$
c. Lukasiewicz t-conorm: $R_l(p,s) = min(p+s, 1)$
d. Drastic t-conorm:

$$R_d(p,s) = \begin{cases} p, & if\ s = 0 \\ s, & if\ p = 0 \\ 1, & otherwise \end{cases}$$

For all $p, s \in [0,1]$.

*Definition 3 [42]:* The mathematical expressions of Aczel Alsina aggregation tools are explored as follows:

$$T^{\daleth}(p,s) = \begin{cases} T(p,s) & \daleth = 0 \\ min(p,s) & \daleth = \infty \\ e^{-((-logp)^{\daleth} + (-logs)^{\daleth})^{\frac{1}{\daleth}}} & Otherwise \end{cases}$$

and

$$R^{\daleth}(p,s) = \begin{cases} R(p,s) & \daleth = 0 \\ max(p,s) & \daleth = \infty \\ 1 - e^{-\left((-log(1-p))^{\daleth}\right)^{\frac{1}{\daleth}}} & Otherwise \end{cases}$$

where $\daleth > 1$, $T^{\daleth}$ and $R^{\daleth}$ are represented by the discrete t-norm, such as

$$T = \begin{cases} p & s = 1 \\ s & p = 1 \\ 0 & otherwise \end{cases}$$

$$R = \begin{cases} p & s = 0 \\ s & p = 0 \\ 1 & otherwise \end{cases}$$

For all $p, s \in [0,1]$.

*Definition 4 [3]:* Consider a non-empty set $Y$ and IFS Z is expressed as follows:

$$Z = \{(t, \varphi(t), \upsilon(t)) | t \in Y\}$$

where $\varphi : Y \rightarrow [0,1]$ be the MG and $\upsilon : Y \rightarrow [0,1]$ be the NMG respectively with the given condition

$$0 \leq \varphi(t) + \upsilon(t) \leq 1$$

Moreover, the hesitancy degree is denoted by $Ħ(t) = 1 - (\varphi(t) + \upsilon(t))$, $Ħ(t) \in [0,1]$ and a pair $(\varphi(t), \upsilon(t))$ known as an intuitionistic fuzzy value (IFV).

*Definition 5 [4]:* Consider an IVIFS $G$ on $Y$ is particularized as follows:

$$G = \{(t, (\varphi(t), \upsilon(t))) | t \in Y\}$$

where $\varphi(t) : Y \rightarrow [0,1]$, $\upsilon(t) : Y \rightarrow [0,1]$. The intervals $\varphi(t)$ and $\upsilon(t)$ indicate the intervals of MG and NMG of the element $t$ in the set $G$, where $\varphi(t) = [\varphi^L(t), \varphi^U(t)]$ and $\upsilon(t) = [\upsilon^L(t), \upsilon^U(t)]$, for all $t \in Y$, including the condition $0 \leq \varphi^U(t) + \upsilon^U(t) \leq 1$. $\pi(t) = [\pi^L(t), \pi^U(t)]$ denotes the indeterminacy degree of element $t$ that belongs to $\hat{I}$, where $\pi^L(t) = 1 - (\varphi(t) + \upsilon(t))$ and $\pi^U(t) = 1 - \varphi^L(t) - \upsilon^L(t)$. Furthermore, an IVIFV is denoted by $G = ([\varphi^L(t), \varphi^U(t)], [\upsilon^L(t), \upsilon^U(t)])$.

*Definition 6 [43]:* For an IVIFV $\zeta = ([\varphi^L(t), \varphi^U(t)], [\upsilon^L(t), \upsilon^U(t)])$. The score function $Sc(\zeta)$ and accuracy function $Acc(\zeta)$ are defined as follows:

$$Sc(\zeta) = \frac{1}{2}\left(\varphi^L(t) + \varphi^U(t) - \upsilon^L(t) - \upsilon^U(t)\right)$$

$$Acc(\zeta) = \frac{1}{2}\left(\varphi^L(t) + \varphi^U(t) + \upsilon^L(t) + \upsilon^U(t)\right)$$

where $Sc(\zeta) \in [-1, 1]$ and $Acc(\zeta) \in [0,1]$.

*Definition 7* Let $\zeta_1 = ([\varphi_1^L(t), \varphi_1^U(t)], [\upsilon_1^L(t), \upsilon_1^U(t)])$ and $\zeta_2 = ([\varphi_2^L(t), \varphi_2^U(t)], [\upsilon_2^L(t), \upsilon_2^U(t)])$ are two IVIFVs. Then

a. If $Sc(\zeta_1) < Sc(\zeta_2)$, then $\zeta_1 < \zeta_2$
b. If $Sc(\zeta_1) = Sc(\zeta_2)$, then
   i. If $Acc(\zeta_1) < Acc(\zeta_2)$, then $\zeta_1 < \zeta_2$
   ii. If $Acc(\zeta_1) = Acc(\zeta_2)$, then $\zeta_1 = \zeta_2$

*Definition 8 [44]:* Let $\zeta_i = ([\varphi^L(t), \varphi^U(t)], [\upsilon^L(t), \upsilon^U(t)])(i = 1, 2, \ldots, n)$ be an accumulation of IVIFSs and $\omega = (\omega_1, \omega_2, \omega_3, \ldots, \omega_n)$ represents weight vector of

$\zeta_i (i = 1, 2, \ldots, n)$ in a manner allowing $\omega \in [0, 1]$, $i = 1, 2, \ldots, n$ and $\sum_{i=1}^{n} \omega = 1$.

$$IVIFWA\left(\zeta_1, \zeta_2, \ldots, \zeta_i\right) = \overset{n}{\underset{i=1}{\oplus}} \left(\omega \zeta_i\right) = \left( \begin{array}{c} \left[1 - \prod_{i=1}^{n} \left(1 - \varphi_{\zeta_i}^{L}(t)\right)^{\omega}, 1 - \prod_{i=1}^{n} \left(1 - \varphi_{\zeta_i}^{U}(t)\right)^{\omega}\right], \\ \left[\prod_{i=1}^{n} \left(\upsilon_{\zeta_i}^{L}(t)\right)^{\omega}, \prod_{i=1}^{n} \left(\upsilon_{\zeta_i}^{U}(t)\right)^{\omega}\right] \end{array} \right)$$

*Definition 9 [43]:* Let $\zeta_1 = ([\varphi_1^L(t), \varphi_1^U(t)], [\upsilon_1^L(t), \upsilon_1^U(t)])$, $\zeta_2 = ([\varphi_2^L(t), \varphi_2^U(t)], [\upsilon_2^L(t), \upsilon_2^U(t)])$ and $\zeta_3 = ([\varphi_3^L(t), \varphi_3^U(t)], [\upsilon_3^L, \upsilon_3^U])$ be three IVIFSs, $\beth \geq 1$ and $\beth > 0$. Then:

a)

$$\zeta_1 \oplus \zeta_2 = \left( \begin{array}{c} \left[ \begin{array}{c} 1 - e^{-\left(\left(-log\left(1-\varphi_1^L(t)\right)\right)^{\beth} + \left(-log\left(1-\varphi_2^L(t)\right)\right)^{\beth}\right)^{\frac{1}{\beth}}}, \\ 1 - e^{-\left(\left(-log\left(1-\varphi_1^U(t)\right)\right)^{\beth} + \left(-log\left(1-\varphi_2^U(t)\right)\right)^{\beth}\right)^{\frac{1}{\beth}}} \end{array} \right], \\ \left[ \begin{array}{c} e^{-\left(\left(-log\left(1-\upsilon_1^L(t)\right)\right)^{\beth} + \left(-log\left(1-\upsilon_2^L(t)\right)\right)^{\beth}\right)^{\frac{1}{\beth}}}, \\ e^{-\left(\left(-log\left(1-\upsilon_1^U(t)\right)\right)^{\beth} + \left(-log\left(1-\upsilon_2^U(t)\right)\right)^{\beth}\right)^{\frac{1}{\beth}}} \end{array} \right] \end{array} \right)$$

b)

$$\zeta_1 \otimes \zeta_2 = \left( \begin{array}{c} \left[ \begin{array}{c} e^{-\left(\left(-log\left(1-\varphi_1^L(t)\right)\right)^{\beth} + \left(-log\left(1-\varphi_2^L(t)\right)\right)^{\beth}\right)^{\frac{1}{\beth}}}, \\ e^{-\left(\left(-log\left(1-\varphi_1^U(t)\right)\right)^{\beth} + \left(-log\left(1-\varphi_2^U(t)\right)\right)^{\beth}\right)^{\frac{1}{\beth}}} \end{array} \right], \\ \left[ \begin{array}{c} 1 - e^{-\left(\left(-log\left(1-\upsilon_1^L(t)\right)\right)^{\beth} + \left(-log\left(1-\upsilon_2^L(t)\right)\right)^{\beth}\right)^{\frac{1}{\beth}}}, \\ 1 - e^{-\left(\left(-log\left(1-\upsilon_1^U(t)\right)\right)^{\beth} + \left(-log\left(1-\upsilon_2^U(t)\right)\right)^{\beth}\right)^{\frac{1}{\beth}}} \end{array} \right] \end{array} \right).$$

c)

$$\beth^{\zeta_3} = \left( \begin{array}{c} \left[ \begin{array}{c} 1 - e^{-\left(\beth\left(-log\left(1-\varphi_3^L(t)\right)\right)^{\beth}\right)^{\frac{1}{\beth}}}, \\ 1 - e^{-\left(\beth\left(-log\left(1-\varphi_3^U(t)\right)\right)^{\beth}\right)^{\frac{1}{\beth}}} \end{array} \right], \\ \left[ -e^{-\left(\beth\left(-log\left(\upsilon_3^L(t)\right)\right)^{\beth}\right)^{\frac{1}{\beth}}}, -e^{-\left(\beth\left(-log\left(\upsilon_3^U(t)\right)\right)^{\beth}\right)^{\frac{1}{\beth}}} \right] \end{array} \right)$$

d)

$$\zeta_3^{\beth} = \left( \begin{array}{c} \left[ e^{-\left(\beth\left(-log\left(\varphi_3^L(t)\right)\right)^{\beth}\right)^{\frac{1}{\beth}}}, e^{-\left(\beth\left(-log\left(\varphi_3^U(t)\right)\right)^{\beth}\right)^{\frac{1}{\beth}}} \right], \\ \left[ \begin{array}{c} 1 - e^{-\left(\beth\left(-log\left(1-\upsilon_3^L(t)\right)\right)^{\beth}\right)^{\frac{1}{\beth}}}, \\ 1 - e^{-\left(\beth\left(-log\left(1-\upsilon_3^U(t)\right)\right)^{\beth}\right)^{\frac{1}{\beth}}} \end{array} \right] \end{array} \right)$$

*Definition 10 [45]:* Let $\varphi_i$ be a family of non-nagative real numbers and the HM operator is expressed as follows:

$$HM^{(\rho)}\left(\varphi_1, \varphi_2, \varphi_3, \ldots, \varphi_n\right) = \frac{\sum_{1 \leq i_1 <, \ldots, < i_\rho \leq n} \left(\prod_{\varsigma=1}^{\rho} \varphi_{i_\varsigma}\right)^{\frac{1}{\rho}}}{C_n^{\rho}}$$

where $\rho$ is such that $1 \leq \rho \leq n$ and $C_n^{\rho}$ represents the Binomial coefficient, i.e $C_n^{\rho} = \frac{n!}{\rho!(n-\rho)!}$.

*Definition 11 [45]:* Let $\varphi_i$ be a family of non-nagative real numbers and the Dual HM (DHM) operator is expressed as follows:

$$DHM^{(\rho)}\left(\varphi_1, \varphi_2, \varphi_3, \ldots, \varphi_n\right) = \left( \prod_{1 \leq i_1 <, \ldots, < i_\rho \leq n} \frac{\sum_{\varsigma=1}^{n} \varphi_{i_\varsigma}}{\rho} \right)^{\frac{1}{C_n^{\rho}}}$$

## III. INTERVAL-VALUED INTUITIONISTIC FUZZY ACZEL ALSINA HAMY MEAN OPERATORS

This section shows the robustness of Hamy mean models and Aczel Alsina operations by developing new aggregation operators of the IVIFAAHM and IVIFAAWHM operators.

*Definition 12:* Consider a list of IVIFVs $\zeta_i = ([\varphi_i^L(t), \varphi_i^U(t)], [\upsilon_i^L(t), \upsilon_i^U(t)])$, $i = 1, 2, \ldots, n$. The IVIFAAHM operator is given by:

$$\text{IVIFAAHM}^{(\rho)}\left(\zeta_1, \zeta_2, \ldots, \zeta_n\right) = \frac{\underset{1 \leq i_1 <, \ldots, < i_\rho \leq n}{\oplus} \left( \overset{\rho}{\underset{\varsigma=1}{\otimes}} \zeta_{i_\varsigma} \right)^{\frac{1}{\rho}}}{C_n^{\rho}} \quad (1)$$

*Theorem 1:* Consider a list of IVIFVs $\zeta_i = ([\varphi_i^L(t), \varphi_i^U(t)], [\upsilon_i^L(t), \upsilon_i^U(t)])$, $i = 1, 2, \ldots, n$. The aggregated outcome by the IVIFAAHM operator is still an IVIFV and we have (2), as shown at the bottom of the next page.

*Proof:* By using the operational laws of Aczel Alsina operators, we prove the above expression as follows:

Firstly, we use the multiplication rule that is discussed in definition 9 and we have:

$$\overset{\rho}{\underset{\varsigma=1}{\otimes}} \zeta_{i_\varsigma} = \left( \begin{array}{c} \left[ \begin{array}{c} e^{-\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\varphi_{i_\varsigma}^L(t)\right)\right)^{\beth}\right)^{\frac{1}{\beth}}}, \\ e^{-\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\varphi_{i_\varsigma}^U(t)\right)\right)^{\beth}\right)^{\frac{1}{\beth}}} \end{array} \right], \\ \left[ \begin{array}{c} 1 - e^{-\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(1-\upsilon_{i_\varsigma}^L(t)\right)\right)^{\beth}\right)^{\frac{1}{\beth}}}, \\ 1 - e^{-\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(1-\upsilon_{i_\varsigma}^U(t)\right)\right)^{\beth}\right)^{\frac{1}{\beth}}} \end{array} \right] \end{array} \right)$$

Secondly, we have applied the power rule by definition 9 and we have:

$$\left(\overset{\rho}{\underset{i=1}{\otimes}} \zeta_{i_\varsigma}\right)^{\frac{1}{\rho}} = \left( \begin{bmatrix} e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\varphi_{i_\varsigma}^{L}(t)\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}, \\ e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\varphi_{i_\varsigma}^{U}(t)\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}} \end{bmatrix}, \begin{bmatrix} 1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(1-\upsilon_{i_\varsigma}^{L}(t)\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}, \\ 1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(1-\upsilon_{i_\varsigma}^{U}(t)\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}} \end{bmatrix} \right)$$

Then, we have a new expression by employing the sum rule on the previous result, as shown in the equation at the bottom of the next page.

Finally, we have obtained a final expression by using the scalar multiplication rule of the definition 9, as shown in the equation at the bottom of the next page.

*Theorem 2:* Consider a list of IVIFVs $\zeta_i = ([\varphi_i^L(t), \varphi_i^U(t)], [\upsilon_i^L(t), \upsilon_i^U(t)]), i = 1, 2, \ldots, n$ such that $\zeta_i = \zeta$. Then:

$$\text{IVIFAAHM}^{(\rho)}(\zeta_1, \zeta_2, \ldots, \zeta_n) = \zeta$$

*Proof:* Since a list of identical IVIFVs $\zeta_i = ([\varphi_i^L(t), \varphi_i^U(t)], [\upsilon_i^L(t), \upsilon_i^U(t)]), i = 1, 2, \ldots, n$. We can prove the above expression, as shown in the equation at the page 8.

*Theorem 3:* Consider two sets of IVIFVs $\zeta_i = ([\varphi_i^L(t), \varphi_i^U(t)], [\upsilon_i^L(t), \upsilon_i^U(t)])$ and $\xi_i = ([\gamma_i^L(t), \gamma_i^U(t)], [\delta_i^L(t), \delta_i^U(t)]),\ i = 1, 2, \ldots, n$. If $\varphi_i(t) \leq \gamma_i(t), \upsilon_i(t) \geq \delta_i(t)$, $\forall\, i$, Then:

$$\text{IVIFAAHM}^{(\rho)}(\zeta_1, \zeta_2, \ldots, \zeta_n) \leq \text{IVIFAAHM}^{(\rho)}(\xi_1, \xi_2, \ldots, \xi_n)$$

*Proof:* Since two sets of IVIFVs $\zeta_i = ([\varphi_i^L(t), \varphi_i^U(t)], [\upsilon_i^L(t), \upsilon_i^U(t)])$ and $\xi_i = ([\gamma_i^L(t), \gamma_i^U(t)], [\delta_i^L(t), \delta_i^U(t)]),$ $i = 1, 2, \ldots, n$. If $\varphi_i(t) \leq \gamma_i(t), \upsilon_i(t) \geq \delta_i(t),\ \forall\, i$, Then, as shown in the equation at the page 9.

This means that $\varphi_{i_\varsigma}(t) \leq \gamma_{i_\varsigma}(t)$.

Similarly, we can show $\upsilon_{i_\varsigma}(t) \geq \delta_{i_\varsigma}(t)$.

If $\varphi_{i_\varsigma}(t) < \gamma_{i_\varsigma}(t), \upsilon_{i_\varsigma}(t) > \delta_{i_\varsigma}(V)$, then:

$$\text{IVIFAAHM}^{(\rho)}(\zeta_1, \zeta_2, \ldots, \zeta_n) < \text{IVIFAAHM}^{(\rho)}(\xi_1, \xi_2, \ldots, \xi_n)$$

If $\varphi_{i_\varsigma}(t) = \gamma_{i_\varsigma}(t), \upsilon_{i_\varsigma}(t) = \delta_{i_\varsigma}(t)$, then:

$$\text{IVIFAAHM}^{(\rho)}(\zeta_1, \zeta_2, \ldots, \zeta_n) = \text{IVIFAAHM}^{(\rho)}(\xi_1, \xi_2, \ldots, \xi_n)$$

*Theorem 4:* Consider a list of IVIFVs $\zeta_i = ([\varphi_i^L(t), \varphi_i^U(t)], [\upsilon_i^L(t), \upsilon_i^U(t)])i = 1, 2, \ldots, n$. If $\zeta_i^- = min(\zeta_1, \zeta_2, \zeta_3, \ldots, \zeta_n)$ and $\zeta_i^+ = max(\zeta_1, \zeta_2, \zeta_3, \ldots, \zeta_n)$. Then:

$$\zeta^- \leq \text{IVIFAAHM}(\zeta_1, \zeta_2, \ldots, \zeta_n) \leq \zeta^+$$

*Proof:* By using theorem 1 and 2. We have:

$$\text{IVIFAAHM}(\zeta_1, \zeta_2, \ldots, \zeta_n) \geq \text{IVIFAAHM}\left(\zeta_1^-, \zeta_2^-, \ldots, \zeta_n^-\right) = \zeta^-$$

$$\text{IVIFAAHM}(\zeta_1, \zeta_2, \ldots, \zeta_n) \leq \text{IVIFAAHM}\left(\zeta_1^+, \zeta_2^+, \ldots, \zeta_n^+\right) = \zeta^+$$

Therefore

$$\zeta^- \leq \text{IVIFAAHM}(\zeta_1, \zeta_2, \ldots, \zeta_n) \leq \zeta^+$$

$$\text{IVIFAAHM}^{(\rho)}(\zeta_1, \zeta_2, \ldots, \zeta_n) = \left( \begin{bmatrix} 1-e^{-\left(\left(\frac{1}{C_n^\rho}\right)\left(\left(\sum_{1\leq i_1<,\ \ldots,\ <i_\rho\leq n}\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\varphi_{i_\varsigma}^{L}(t)\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}\right)\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}, \\ 1-e^{-\left(\left(\frac{1}{C_n^\rho}\right)\left(\left(\sum_{1\leq i_1<,\ \ldots,\ <i_\rho\leq n}\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\varphi_{i_\varsigma}^{U}(t)\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}\right)\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}} \end{bmatrix}, \begin{bmatrix} e^{-\left(\left(\frac{1}{C_n^\rho}\right)\left(\sum_{1\leq i_1<,\ \ldots,\ <i_\rho\leq n}\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(1-\upsilon_{i_\varsigma}^{L}(t)\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}, \\ e^{-\left(\left(\frac{1}{C_n^\rho}\right)\left(\sum_{1\leq i_1<,\ \ldots,\ <i_\rho\leq n}\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(1-\upsilon_{i_\varsigma}^{U}(t)\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}} \end{bmatrix} \right) \quad (2)$$

*Definition 13:* Consider a list of IVIFVs $\zeta_i = ([\varphi_i^L(\mathfrak{t}), \varphi_i^U(\mathfrak{t})], [\upsilon_i^L(\mathfrak{t}), \upsilon_i^U(\mathfrak{t})]), i = 1, 2, \ldots, n$ and a family of weight vector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$, $\omega_i \in [0, 1]$ and $\sum_{i=1}^{n} \omega_i = 1$. The IVIFAAWHM operator is given by:

$$\mathrm{IVIFAAWHM}^{(\rho)}(\zeta_1, \zeta_2, \ldots, \zeta_n) = \begin{cases} \dfrac{\underset{1\le i_1<,\ldots,<i_\rho\le n}{\oplus}\left(1-\sum_{\varsigma=1}^{\rho}\omega_\varsigma\right)\left(\overset{\rho}{\underset{\varsigma=1}{\otimes}}\zeta_{i_\varsigma}\right)^{\frac{1}{\rho}}}{C_{n-1}^{\rho}} & 1\le\rho<n \\ \overset{\rho}{\underset{\varsigma=1}{\otimes}}\zeta_{i_\varsigma}^{\frac{1-\omega_\varsigma}{n-1}} & \rho=n \end{cases} \tag{3}$$

*Theorem 5:* Consider a list of IVIFVs $\zeta_i = ([\varphi_i^L(\mathfrak{t}), \varphi_i^U(\mathfrak{t})], [\upsilon_i^L(\mathfrak{t}), \upsilon_i^U(\mathfrak{t})]), i = 1, 2, \ldots, n$. The aggregated outcome by the IVIFAAWHM operator is still an IVIFV and we have (4), as shown at the bottom of page 10.

*Proof:* Let $\zeta_i = ([\varphi_i^L(\mathfrak{t}), \varphi_i^U(\mathfrak{t})], [\upsilon_i^L(\mathfrak{t}), \upsilon_i^U(\mathfrak{t})]), i = 1, 2, \ldots, n$ be the family of IVIFVs.

**Case 1:** We can prove Eq. 8 for $1\le\rho<n$, as shown in the equation at the pages 11 and 12.

**Case 2:** We can prove Eq. 8 for $\rho = n$ we have:

$$\zeta_{i_\varsigma}^{\frac{1-\omega_\varsigma}{n-1}} = \left( \begin{bmatrix} e^{-\left(\left(\frac{1-\omega_\varsigma}{n-1}\right)\left(-log\left(\varphi_{i_\varsigma}^L(\mathfrak{t})\right)\right)^{\daleth}\right)^{\frac{1}{\daleth}}}, \\ e^{-\left(\left(\frac{1-\omega_\varsigma}{n-1}\right)\left(-log\left(\varphi_{i_\varsigma}^U(\mathfrak{t})\right)\right)^{\daleth}\right)^{\frac{1}{\daleth}}} \end{bmatrix}, \begin{bmatrix} 1-e^{-\left(\left(\frac{1-\omega_\varsigma}{n-1}\right)\left(\left(-log\left(1-\upsilon_{i_\varsigma}^L(\mathfrak{t})\right)\right)^{\daleth}\right)\right)^{\frac{1}{\daleth}}}, \\ 1-e^{-\left(\left(\frac{1-\omega_{i_\varsigma}}{n-1}\right)\left(\left(-log\left(1-\upsilon_{i_\varsigma}^U(\mathfrak{t})\right)\right)^{\daleth}\right)\right)^{\frac{1}{\daleth}}} \end{bmatrix} \right)$$

$$\oplus_{1\le i_1<,\ldots,<i_\rho\le n}\left(\overset{\rho}{\underset{\varsigma=1}{\otimes}}\zeta_{i_\varsigma}\right)^{\frac{1}{\rho}} = \left( \begin{bmatrix} 1-e^{-\left(\sum_{1\le i_1<,\ldots,<i_\rho\le n}\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\varphi_{i_\varsigma}^L(\mathfrak{t})\right)\right)^{\daleth}\right)\right)^{\frac{1}{\daleth}}}\right)\right)^{\daleth}\right)^{\frac{1}{\daleth}}}, \\ 1-e^{-\left(\sum_{1\le i_1<,\ldots,<i_\rho\le n}\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\varphi_{i_\varsigma}^U(\mathfrak{t})\right)\right)^{\daleth}\right)\right)^{\frac{1}{\daleth}}}\right)\right)^{\daleth}\right)^{\frac{1}{\daleth}}} \end{bmatrix}, \begin{bmatrix} e^{-\left(\sum_{1\le i_1<,\ldots,<i_\rho\le n}\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(1-\upsilon_{i_\varsigma}^L(\mathfrak{t})\right)\right)^{\daleth}\right)\right)^{\frac{1}{\daleth}}}\right)\right)^{\daleth}\right)^{\frac{1}{\daleth}}}, \\ e^{-\left(\sum_{1\le i_1<,\ldots,<i_\rho\le n}\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(1-\upsilon_{i_\varsigma}^U(\mathfrak{t})\right)\right)^{\daleth}\right)\right)^{\frac{1}{\daleth}}}\right)\right)^{\daleth}\right)^{\frac{1}{\daleth}}} \end{bmatrix} \right)$$

$$\frac{\oplus_{1\le i_1<,\ldots,<i_\rho\le n}\left(\overset{\rho}{\underset{\varsigma=1}{\otimes}}\zeta_\varsigma\right)^{\frac{1}{\rho}}}{C_n^{\rho}} = \left( \begin{bmatrix} 1-e^{-\left(\left(\frac{1}{C_n^{\rho}}\right)\left(\left(\sum_{1\le i_1<,\ldots,<i_\rho\le n}\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\varphi_{i_\varsigma}^L(\mathfrak{t})\right)\right)^{\daleth}\right)\right)^{\frac{1}{\daleth}}}\right)\right)^{\daleth}\right)\right)\right)^{\frac{1}{\daleth}}}, \\ 1-e^{-\left(\left(\frac{1}{C_n^{\rho}}\right)\left(\left(\sum_{1\le i_1<,\ldots,<i_\rho\le n}\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\varphi_{i_\varsigma}^U(\mathfrak{t})\right)\right)^{\daleth}\right)\right)^{\frac{1}{\daleth}}}\right)\right)^{\daleth}\right)\right)\right)^{\frac{1}{\daleth}}} \end{bmatrix}, \begin{bmatrix} e^{-\left(\left(\frac{1}{C_n^{\rho}}\right)\left(\sum_{1\le i_1<,\ldots,<i_\rho\le n}\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(1-\upsilon_{i_\varsigma}^L(\mathfrak{t})\right)\right)^{\daleth}\right)\right)^{\frac{1}{\daleth}}}\right)\right)^{\daleth}\right)\right)^{\frac{1}{\daleth}}}, \\ e^{-\left(\left(\frac{1}{C_n^{\rho}}\right)\left(\sum_{1\le i_1<,\ldots,<i_\rho\le n}\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(1-\upsilon_{i_\varsigma}^U(\mathfrak{t})\right)\right)^{\daleth}\right)\right)^{\frac{1}{\daleth}}}\right)\right)^{\daleth}\right)\right)^{\frac{1}{\daleth}}} \end{bmatrix} \right)$$

$$
\begin{aligned}
\mathrm{IVIFAAHM}^{(\rho)}\left(\zeta_1,\zeta_2,\ldots,\zeta_n\right) &= \left(\begin{array}{l}
\left[\begin{array}{l}
1-e^{-\left(\left(\frac{1}{C_n^{\rho}}\right)\left(\left(\sum_{1\le i_1<,\ \ldots,\ <i_\rho\le n}\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\varphi_{i_\varsigma}^{L}\left(\mathfrak{t}\right)\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}\right)\right)^{\beth}\right)\right)\right)^{\frac{1}{\beth}}},\\
1-e^{-\left(\left(\frac{1}{C_n^{\rho}}\right)\left(\left(\sum_{1\le i_1<,\ \ldots,\ <i_\rho\le n}\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\varphi_{i_\varsigma}^{U}\left(\mathfrak{t}\right)\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}\right)\right)^{\beth}\right)\right)\right)^{\frac{1}{\beth}}}
\end{array}\right],\\
\left[\begin{array}{l}
e^{-\left(\left(\frac{1}{C_n^{\rho}}\right)\left(\sum_{1\le i_1<,\ \ldots,\ <i_\rho\le n}\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(1-\upsilon_{i_\varsigma}^{L}\left(\mathfrak{t}\right)\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}},\\
e^{-\left(\left(\frac{1}{C_n^{\rho}}\right)\left(\sum_{1\le i_1<,\ \ldots,\ <i_\rho\le n}\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(1-\upsilon_{i_\varsigma}^{U}\left(\mathfrak{t}\right)\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}
\end{array}\right]
\end{array}\right)\\
&= \left(\begin{array}{l}
\left[\begin{array}{l}
1-e^{-\left(\left(\frac{1}{C_n^{\rho}}\right)\left(\left(\sum_{1\le i_1<,\ \ldots,\ <i_\rho\le n}\left(-log\left(1-e^{-\left(\left(-log\left(\varphi_{i}^{L}\left(\mathfrak{t}\right)\right)\right)^{\beth}\right)^{\frac{1}{\beth}}}\right)\right)^{\beth}\right)\right)\right)^{\frac{1}{\beth}}},\\
1-e^{-\left(\left(\frac{1}{C_n^{\rho}}\right)\left(\left(\sum_{1\le i_1<,\ \ldots,\ <i_\rho\le n}\left(-log\left(1-e^{-\left(\left(-log\left(\varphi_{i}^{U}\left(\mathfrak{t}\right)\right)\right)^{\beth}\right)^{\frac{1}{\beth}}}\right)\right)^{\beth}\right)\right)\right)^{\frac{1}{\beth}}}
\end{array}\right],\\
\left[\begin{array}{l}
e^{-\left(\left(\frac{1}{C_n^{\rho}}\right)\left(\sum_{1\le i_1<,\ \ldots,\ <i_\rho\le n}\left(-log\left(1-e^{-\left(\left(-log\left(1-\upsilon_{i}^{L}\left(\mathfrak{t}\right)\right)\right)^{\beth}\right)^{\frac{1}{\beth}}}\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}},\\
e^{-\left(\left(\frac{1}{C_n^{\rho}}\right)\left(\sum_{1\le i_1<,\ \ldots,\ <i_\rho\le n}\left(-log\left(1-e^{-\left(\left(-log\left(1-\upsilon_{i}^{U}\left(\mathfrak{t}\right)\right)\right)^{\beth}\right)^{\frac{1}{\beth}}}\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}
\end{array}\right]
\end{array}\right)\\
&= \left(\begin{array}{l}
\left[\begin{array}{l}
1-e^{-\left(\left(-log\left(1-e^{-\left(\left(-log\left(\varphi_{i}^{L}\left(\mathfrak{t}\right)\right)\right)^{\beth}\right)^{\frac{1}{\beth}}}\right)\right)^{\beth}\right)^{\frac{1}{\beth}}},\\
1-e^{-\left(\left(-log\left(1-e^{-\left(\left(-log\left(\varphi_{i}^{U}\left(\mathfrak{t}\right)\right)\right)^{\beth}\right)^{\frac{1}{\beth}}}\right)\right)^{\beth}\right)^{\frac{1}{\beth}}}
\end{array}\right],\\
\left[\begin{array}{l}
e^{-\left(\left(-log\left(1-e^{-\left(\left(-log\left(1-\upsilon_{i}^{L}\left(\mathfrak{t}\right)\right)\right)^{\beth}\right)^{\frac{1}{\beth}}}\right)\right)^{\beth}\right)^{\frac{1}{\beth}}},\\
e^{-\left(\left(-log\left(1-e^{-\left(\left(-log\left(1-\upsilon_{i}^{U}\left(\mathfrak{t}\right)\right)\right)^{\beth}\right)^{\frac{1}{\beth}}}\right)\right)^{\beth}\right)^{\frac{1}{\beth}}}
\end{array}\right]
\end{array}\right) = \left(\varphi\left(\mathfrak{t}\right), \upsilon\left(\mathfrak{t}\right)\right) = \zeta
\end{aligned}
$$

$$\left[\begin{array}{l} e^{-\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\varphi_{i_\varsigma}^{L}(\mathfrak{t})\right)\right)^{\sqsupset}\right)^{\frac{1}{\sqsupset}}}, \\ e^{-\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\varphi_{i_\varsigma}^{U}(\mathfrak{t})\right)\right)^{\sqsupset}\right)^{\frac{1}{\sqsupset}}} \end{array}\right] \leq \left[\begin{array}{l} e^{-\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\gamma_{i_\varsigma}^{L}(\mathfrak{t})\right)\right)^{\sqsupset}\right)^{\frac{1}{\sqsupset}}}, \\ e^{-\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\gamma_{i_\varsigma}^{U}(\mathfrak{t})\right)\right)^{\sqsupset}\right)^{\frac{1}{\sqsupset}}} \end{array}\right]$$

$$\left[\begin{array}{l} e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\varphi_{i_\varsigma}^{L}(\mathfrak{t})\right)\right)^{\sqsupset}\right)\right)^{\frac{1}{\sqsupset}}}, \\ e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\varphi_{i_\varsigma}^{U}(\mathfrak{t})\right)\right)^{\sqsupset}\right)\right)^{\frac{1}{\sqsupset}}} \end{array}\right] \geq \left[\begin{array}{l} e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\gamma_{i_\varsigma}^{L}(\mathfrak{t})\right)\right)^{\sqsupset}\right)\right)^{\frac{1}{\sqsupset}}}, \\ e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\gamma_{i_\varsigma}^{U}(\mathfrak{t})\right)\right)^{\sqsupset}\right)\right)^{\frac{1}{\sqsupset}}} \end{array}\right]$$

$$\left[\begin{array}{l} 1-e^{-\left(\sum_{1\leq i_1<,\ \ldots,\ <i_\rho\leq n}\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\varphi_{i_\varsigma}^{L}(\mathfrak{t})\right)\right)^{\sqsupset}\right)\right)^{\frac{1}{\sqsupset}}}\right)\right)^{\sqsupset}\right)^{\frac{1}{\sqsupset}}}, \\ 1-e^{-\left(\sum_{1\leq i_1<,\ \ldots,\ <i_\rho\leq n}\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\varphi_{i_\varsigma}^{U}(\mathfrak{t})\right)\right)^{\sqsupset}\right)\right)^{\frac{1}{\sqsupset}}}\right)\right)^{\sqsupset}\right)^{\frac{1}{\sqsupset}}} \end{array}\right]$$

$$\leq \left[\begin{array}{l} 1-e^{-\left(\sum_{1\leq i_1<,\ \ldots,\ <i_\rho\leq n}\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\gamma_{i_\varsigma}^{L}(\mathfrak{t})\right)\right)^{\sqsupset}\right)\right)^{\frac{1}{\sqsupset}}}\right)\right)^{\sqsupset}\right)^{\frac{1}{\sqsupset}}}, \\ 1-e^{-\left(\sum_{1\leq i_1<,\ \ldots,\ <i_\rho\leq n}\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\gamma_{i_\varsigma}^{U}(\mathfrak{t})\right)\right)^{\sqsupset}\right)\right)^{\frac{1}{\sqsupset}}}\right)\right)^{\sqsupset}\right)^{\frac{1}{\sqsupset}}} \end{array}\right]$$

$$\left[\begin{array}{l} 1-e^{-\left(\left(\frac{1}{C_n^{\rho}}\right)\left(\left(\sum_{1\leq i_1<,\ \ldots,\ <i_\rho\leq n}\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\varphi_{i_\varsigma}^{L}(\mathfrak{t})\right)\right)^{\sqsupset}\right)\right)^{\frac{1}{\sqsupset}}}\right)\right)^{\sqsupset}\right)\right)\right)^{\frac{1}{\sqsupset}}}, \\ 1-e^{-\left(\left(\frac{1}{C_n^{\rho}}\right)\left(\left(\sum_{1\leq i_1<,\ \ldots,\ <i_\rho\leq n}\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\varphi_{i_\varsigma}^{U}(\mathfrak{t})\right)\right)^{\sqsupset}\right)\right)^{\frac{1}{\sqsupset}}}\right)\right)^{\sqsupset}\right)\right)\right)^{\frac{1}{\sqsupset}}} \end{array}\right]$$

$$\leq \left[\begin{array}{l} 1-e^{-\left(\left(\frac{1}{C_n^{\rho}}\right)\left(\left(\sum_{1\leq i_1<,\ \ldots,\ <i_\rho\leq n}\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\gamma_{i_\varsigma}^{L}(\mathfrak{t})\right)\right)^{\sqsupset}\right)\right)^{\frac{1}{\sqsupset}}}\right)\right)^{\sqsupset}\right)\right)\right)^{\frac{1}{\sqsupset}}}, \\ 1-e^{-\left(\left(\frac{1}{C_n^{\rho}}\right)\left(\left(\sum_{1\leq i_1<,\ \ldots,\ <i_\rho\leq n}\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\gamma_{i_\varsigma}^{U}(\mathfrak{t})\right)\right)^{\sqsupset}\right)\right)^{\frac{1}{\sqsupset}}}\right)\right)^{\sqsupset}\right)\right)\right)^{\frac{1}{\sqsupset}}} \end{array}\right]$$

$$\bigotimes_{\varsigma=1}^{\rho} \zeta_{i_\varsigma}^{\frac{1-\omega_\varsigma}{n-1}} = \left( \begin{array}{c} \left[ e^{-\left(\sum_{\varsigma=1}^{\rho}\left(\left(\frac{1-\omega_\varsigma}{n-1}\right)\left(-log\left(\varphi_{i_\varsigma}^{L}(t)\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}, e^{-\left(\sum_{\varsigma=1}^{\rho}\left(\left(\frac{1-\omega_\varsigma}{n-1}\right)\left(-log\left(\varphi_{i_\varsigma}^{U}(t)\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}} \right], \\ \left[ 1-e^{-\left(\sum_{\varsigma=1}^{\rho}\left(\frac{1-\omega_\varsigma}{n-1}\right)\left(\left(-log\left(1-\upsilon_{i_\varsigma}^{L}(t)\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}, 1-e^{-\left(\sum_{\varsigma=1}^{\rho}\left(\frac{1-\omega_\varsigma}{n-1}\right)\left(\left(-log\left(1-\upsilon_{i_\varsigma}^{U}(t)\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}} \right] \end{array} \right)$$

*Theorem 6:* Consider a list of IVIFVs $\zeta_i = ([\varphi_i^L(t), \varphi_i^U(t)], [\upsilon_i^L(t), \upsilon_i^U(t)])$, $i = 1, 2, \ldots, n$ such that $\zeta_i = \zeta$. Then:

$$\text{IVIFAAWHM}^{(\rho)}(\zeta_1, \zeta_2, \ldots, \zeta_n) = \zeta$$

*Theorem 7:* Consider two sets of IVIFVs $\zeta_i = ([\varphi_i^L(t), \varphi_i^U(t)], [\upsilon_i^L(t), \upsilon_i^U(t)])$ and $\xi_i = ([\gamma_i^L(t), \gamma_i^U(t)], [\delta_i^L(t), \delta_i^U(t)])$, $i = 1, 2, \ldots, n$. If $\varphi_i(t) \leq \gamma_i(t)$, $\upsilon_i(t) \geq \delta_i(t)$, $\forall\, i$, Then:

$$\text{IVIFAAWHM}^{(\rho)}(\zeta_1, \zeta_2, \ldots, \zeta_n) \leq \text{IVIFAAWHM}^{(\rho)}(\xi_1, \xi_2, \ldots, \xi_n)$$

*Theorem 8:* Consider a list of IVIFVs $\zeta_i = ([\varphi_i^L(t), \varphi_i^U(t)], [\upsilon_i^L(t), \upsilon_i^U(t)])i = 1, 2, \ldots, n$. If $\zeta_i^- = min(\zeta_1, \zeta_2, \zeta_3, \ldots, \zeta_n)$ and $\zeta_i^+ = max(\zeta_1, \zeta_2, \zeta_3, \ldots, \zeta_n)$. Then:

$$\zeta^- \leq \textbf{IVIFAAWHM}\left(\zeta_1, \zeta_2, \ldots, \zeta_n\right) \leq \zeta^+$$

## IV. INTERVAL-VALUED INTUITIONISTIC FUZZY ACZEL ALSINA DUAL HAMY MEAN OPERATORS

In this section, we derive a family of new mathematical strategies for Dual Hamy mean operators, including IVIFAADHM and IVIFAAWDHM operators.

*Definition 14:* Consider a list of IVIFVs $\zeta_i = ([\varphi_i^L(t), \varphi_i^U(t)], [\upsilon_i^L(t), \upsilon_i^U(t)])$, $i = 1, 2, \ldots, n$. The IVIFAADHM operator is given by:

$$\text{IVIFAADHM}^{(\rho)}(\zeta_1, \zeta_2, \ldots, \zeta_n) = \left( \bigotimes_{1 \leq i_1 <, \ldots, < i_\rho \leq n} \frac{\left(\bigoplus_{\varsigma=1}^{\rho} \zeta_{i_\varsigma}\right)}{\rho} \right)^{\frac{1}{C_n^\rho}} \tag{5}$$

*Theorem 9:* Consider a list of IVIFVs $\zeta_i = ([\varphi_i^L(t), \varphi_i^U(t)], [\upsilon_i^L(t), \upsilon_i^U(t)])$, $i = 1, 2, \ldots, n$. The aggregated

$$\text{IVIFAAWHM}^{(\rho)}(\zeta_1, \zeta_2, \ldots, \zeta_n) = \left( \begin{array}{c} \left[ 1-e^{-\left(\left(\frac{1}{C_{n-1}^{\rho}}\right)\left(\sum_{1\leq i_1<,\ldots,<i_\rho\leq n}\left(1-\sum_{\varsigma=1}^{\rho}\omega_\varsigma\right)\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\varphi_{i_\varsigma}^{L}(t)\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}, 1-e^{-\left(\left(\frac{1}{C_{n-1}^{\rho}}\right)\left(\sum_{1\leq i_1<,\ldots,<i_\rho\leq n}\left(1-\sum_{\varsigma=1}^{\rho}\omega_\varsigma\right)\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\varphi_{i_\varsigma}^{U}(t)\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}} \right], \\ \left[ e^{-\left(\left(\frac{1}{C_{n-1}^{\rho}}\right)\left(\sum_{1\leq i_1<,\ldots,<i_\rho\leq n}\left(1-\sum_{\varsigma=1}^{\rho}\omega_\varsigma\right)\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(1-\upsilon_{i_\varsigma}^{L}(t)\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}, e^{-\left(\left(\frac{1}{C_{n-1}^{\rho}}\right)\left(\sum_{1\leq i_1<,\ldots,<i_\rho\leq n}\left(1-\sum_{\varsigma=1}^{\rho}\omega_\varsigma\right)\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(1-\upsilon_{i_\varsigma}^{U}(t)\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}} \right] \end{array} \right), 1 \leq \rho < n$$

$$\bigotimes_{\varsigma=1}^{\rho} \zeta_{i_\varsigma}^{\frac{1-\omega_\varsigma}{n-1}} = \left( \begin{array}{c} \left[ e^{-\left(\sum_{\varsigma=1}^{\rho}\left(\left(\frac{1-\omega_\varsigma}{n-1}\right)\left(-log\left(\varphi_{i_\varsigma}^{L}(t)\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}, e^{-\left(\sum_{\varsigma=1}^{\rho}\left(\left(\frac{1-\omega_\varsigma}{n-1}\right)\left(-log\left(\varphi_{i_\varsigma}^{U}(t)\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}} \right], \\ \left[ 1-e^{-\left(\sum_{\varsigma=1}^{\rho}\left(\frac{1-\omega_\varsigma}{n-1}\right)\left(\left(-log\left(1-\upsilon_{i_\varsigma}^{L}(t)\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}, 1-e^{-\left(\sum_{\varsigma=1}^{\rho}\left(\frac{1-\omega_\varsigma}{n-1}\right)\left(\left(-log\left(1-\upsilon_{i_\varsigma}^{U}(t)\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}} \right] \end{array} \right), \rho = n \tag{4}$$

$$\overset{\rho}{\underset{\varsigma=1}{\otimes}} \zeta_{i_\varsigma} = \left( \begin{array}{c} \left[ \begin{array}{c} e^{-\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\varphi_{i_\varsigma}^{L}(\mathfrak{t})\right)\right)^{\sqsupset}\right)^{\frac{1}{\sqsupset}}}, \\ e^{-\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\varphi_{i_\varsigma}^{U}(\mathfrak{t})\right)\right)^{\sqsupset}\right)^{\frac{1}{\sqsupset}}} \end{array} \right], \\ \left[ \begin{array}{c} 1-e^{-\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(1-\upsilon_{i_\varsigma}^{L}(\mathfrak{t})\right)\right)^{\sqsupset}\right)^{\frac{1}{\sqsupset}}}, \\ 1-e^{-\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(1-\upsilon_{i_\varsigma}^{U}(\mathfrak{t})\right)\right)^{\sqsupset}\right)^{\frac{1}{\sqsupset}}} \end{array} \right] \end{array} \right) \left(\overset{\rho}{\underset{\varsigma=1}{\otimes}} \zeta_{i_\varsigma}\right)^{\frac{1}{\rho}} = \left( \begin{array}{c} \left[ \begin{array}{c} e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\varphi_{i_\varsigma}^{L}(\mathfrak{t})\right)\right)^{\sqsupset}\right)\right)^{\frac{1}{\sqsupset}}}, \\ e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\varphi_{i_\varsigma}^{U}(\mathfrak{t})\right)\right)^{\sqsupset}\right)\right)^{\frac{1}{\sqsupset}}} \end{array} \right], \\ \left[ \begin{array}{c} 1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(1-\upsilon_{i_\varsigma}^{L}(\mathfrak{t})\right)\right)^{\sqsupset}\right)\right)^{\frac{1}{\sqsupset}}}, \\ 1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(1-\upsilon_{i_\varsigma}^{U}(\mathfrak{t})\right)\right)^{\sqsupset}\right)\right)^{\frac{1}{\sqsupset}}} \end{array} \right] \end{array} \right)$$

$$\left(1-\sum_{\varsigma=1}^{\rho}\omega_\varsigma\right)\left(\overset{\rho}{\underset{\varsigma=1}{\otimes}} \zeta_{i_\varsigma}\right)^{\frac{1}{\rho}} = \left( \begin{array}{c} \left[ \begin{array}{c} 1-e^{-\left(\left(1-\sum_{\varsigma=1}^{\rho}\omega_\varsigma\right)\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\varphi_{i_\varsigma}^{L}(\mathfrak{t})\right)\right)^{\sqsupset}\right)\right)^{\frac{1}{\sqsupset}}}\right)\right)^{\sqsupset}\right)^{\frac{1}{\sqsupset}}}, \\ 1-e^{-\left(\left(1-\sum_{\varsigma=1}^{\rho}\omega_\varsigma\right)\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\varphi_{i_\varsigma}^{U}(\mathfrak{t})\right)\right)^{\sqsupset}\right)\right)^{\frac{1}{\sqsupset}}}\right)\right)^{\sqsupset}\right)^{\frac{1}{\sqsupset}}} \end{array} \right], \\ \left[ \begin{array}{c} e^{-\left(\left(1-\sum_{\varsigma=1}^{\rho}\omega_\varsigma\right)\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(1-\upsilon_{i_\varsigma}^{L}(\mathfrak{t})\right)\right)^{\sqsupset}\right)\right)^{\frac{1}{\sqsupset}}}\right)\right)^{\sqsupset}\right)^{\frac{1}{\sqsupset}}}, \\ e^{-\left(\left(1-\sum_{\varsigma=1}^{\rho}\omega_\varsigma\right)\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(1-\upsilon_{i_\varsigma}^{U}(\mathfrak{t})\right)\right)^{\sqsupset}\right)\right)^{\frac{1}{\sqsupset}}}\right)\right)^{\sqsupset}\right)^{\frac{1}{\sqsupset}}} \end{array} \right] \end{array} \right)$$

$$\underset{1\leq i_1<,\ \ldots,\ <i_\rho\leq n}{\oplus}\left(1-\sum_{\varsigma=1}^{\rho}\omega_\varsigma\right)\left(\overset{\rho}{\underset{\varsigma=1}{\otimes}} \zeta_{i_\varsigma}\right)^{\frac{1}{\rho}}$$

$$= \left( \begin{array}{c} \left[ \begin{array}{c} 1-e^{-\left(\sum_{1\leq i_1<,\ \ldots,\ <i_\rho\leq n}\left(\left(1-\sum_{\varsigma=1}^{\rho}\omega_\varsigma\right)\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\varphi_{i_\varsigma}^{L}(\mathfrak{t})\right)\right)^{\sqsupset}\right)\right)^{\frac{1}{\sqsupset}}}\right)\right)\right)^{\sqsupset}\right)^{\frac{1}{\sqsupset}}}, \\ 1-e^{-\left(\sum_{1\leq i_1<,\ \ldots,\ <i_\rho\leq n}\left(\left(1-\sum_{\varsigma=1}^{\rho}\omega_\varsigma\right)\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\varphi_{i_\varsigma}^{U}(\mathfrak{t})\right)\right)^{\sqsupset}\right)\right)^{\frac{1}{\sqsupset}}}\right)\right)\right)^{\sqsupset}\right)^{\frac{1}{\sqsupset}}} \end{array} \right], \\ \left[ \begin{array}{c} e^{-\left(\sum_{1\leq i_1<,\ \ldots,\ <i_\rho\leq n}\left(\left(1-\sum_{\varsigma=1}^{\rho}\omega_\varsigma\right)\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(1-\upsilon_{i_\varsigma}^{L}(\mathfrak{t})\right)\right)^{\sqsupset}\right)\right)^{\frac{1}{\sqsupset}}}\right)\right)\right)^{\sqsupset}\right)^{\frac{1}{\sqsupset}}}, \\ e^{-\left(\sum_{1\leq i_1<,\ \ldots,\ <i_\rho\leq n}\left(\left(1-\sum_{\varsigma=1}^{\rho}\omega_\varsigma\right)\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(1-\upsilon_{i_\varsigma}^{U}(\mathfrak{t})\right)\right)^{\sqsupset}\right)\right)^{\frac{1}{\sqsupset}}}\right)\right)\right)^{\sqsupset}\right)^{\frac{1}{\sqsupset}}} \end{array} \right] \end{array} \right)$$

$$\frac{\underset{1\le i_1<,\ \ldots,\ <i_\rho\le n}{\oplus}\left(1-\sum_{\varsigma=1}^{\rho}\omega_\varsigma\right)\left(\overset{\rho}{\underset{\varsigma=1}{\otimes}}\zeta_{i_\varsigma}\right)^{\frac{1}{\rho}}}{C_{n-1}^{\rho}}$$

$$=\left(\begin{array}{l}\left[\begin{array}{l}1-e^{-\left(\left(\frac{1}{C_{n-1}^{\rho}}\right)\left(\sum_{1\le i_1<,\ \ldots,\ <i_\rho\le n}\left(\left(1-\sum_{\varsigma=1}^{\rho}\omega_\varsigma\right)\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\varphi_{i_\varsigma}^{L}(t)\right)\right)^{\sqsupset}\right)\right)^{\frac{1}{\sqsupset}}}\right)\right)^{\sqsupset}\right)\right)\right)^{\frac{1}{\sqsupset}}},\\ 1-e^{-\left(\left(\frac{1}{C_{n-1}^{\rho}}\right)\left(\sum_{1\le i_1<,\ \ldots,\ <i_\rho\le n}\left(\left(1-\sum_{\varsigma=1}^{\rho}\omega_\varsigma\right)\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\varphi_{i_\varsigma}^{U}(t)\right)\right)^{\sqsupset}\right)\right)^{\frac{1}{\sqsupset}}}\right)\right)^{\sqsupset}\right)\right)\right)^{\frac{1}{\sqsupset}}}\end{array}\right],\\ \left[\begin{array}{l}e^{-\left(\left(\frac{1}{C_{n-1}^{\rho}}\right)\left(\sum_{1\le i_1<,\ \ldots,\ <i_\rho\le n}\left(\left(1-\sum_{\varsigma=1}^{\rho}\omega_\varsigma\right)\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(1-\upsilon_{i_\varsigma}^{L}(t)\right)\right)^{\sqsupset}\right)\right)^{\frac{1}{\sqsupset}}}\right)\right)^{\sqsupset}\right)\right)\right)^{\frac{1}{\sqsupset}}},\\ e^{-\left(\left(\frac{1}{C_{n-1}^{\rho}}\right)\left(\sum_{1\le i_1<,\ \ldots,\ <i_\rho\le n}\left(\left(1-\sum_{\varsigma=1}^{\rho}\omega_\varsigma\right)\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(1-\upsilon_{i_\varsigma}^{U}(t)\right)\right)^{\sqsupset}\right)\right)^{\frac{1}{\sqsupset}}}\right)\right)^{\sqsupset}\right)\right)\right)^{\frac{1}{\sqsupset}}}\end{array}\right]\end{array}\right)$$

outcome by the IVIFAADHM operator is still an IVIFV and we have (6), as shown at the bottom of the next page.

*Theorem 10:* Consider a list of IVIFVs $\zeta_i = ([\varphi_i^L(t), \varphi_i^U(t)], [\upsilon_i^L(t), \upsilon_i^U(t)]), i = 1, 2, \ldots, n$ such that $\zeta_i = \zeta$. Then:

$$\text{IVIFAADHM}^{(\rho)}(\zeta_1, \zeta_2, \ldots, \zeta_n) = \zeta$$

*Theorem 11:* Consider two sets of IVIFVs $\zeta_i = ([\varphi_i^L(t), \varphi_i^U(t)], [\upsilon_i^L(t), \upsilon_i^U(t)])$ and $\xi_i = ([\gamma_i^L(t), \gamma_i^U(t)], [\delta_i^L(t), \delta_i^U(t)]),\ i = 1, 2, \ldots, n$. If $\varphi_i(t)\le\gamma_i(t),\ \upsilon_i(t)\ge\delta_i(t)$, $\forall\ i$, Then:

$$\begin{aligned}&\text{IVIFAADHM}^{(\rho)}(\zeta_1, \zeta_2, \ldots, \zeta_n)\\ &\quad\le\text{IVIFAADHM}^{(\rho)}(\xi_1, \xi_2, \ldots, \xi_n)\end{aligned}$$

*Theorem 12:* Consider a list of IVIFVs $\zeta_i = ([\varphi_i^L(t), \varphi_i^U(t)], [\upsilon_i^L(t), \upsilon_i^U(t)])i = 1, 2, \ldots, n$. If $\zeta_i^- = min(\zeta_1, \zeta_2, \zeta_3, \ldots, \zeta_n)$ and $\zeta_i^+ = max(\zeta_1, \zeta_2, \zeta_3, \ldots, \zeta_n)$. Then:

$$\zeta^-\le\text{IVIFAADHM}(\zeta_1, \zeta_2, \ldots, \zeta_n)\le\zeta^+$$

*Definition 15:* Consider a list of IVIFVs $\zeta_i = ([\varphi_i^L(t), \varphi_i^U(t)], [\upsilon_i^L(t), \upsilon_i^U(t)]), i = 1, 2, \ldots, n$ and a family of weight vector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$, $\omega_i\in[0, 1]$ and $\sum_{i=1}^n\omega_i = 1$. The IVIFAAWDHM operator is given by:

$$\text{IVIFAAWDHM}^{(\rho)}(\zeta_1, \zeta_2, \ldots, \zeta_n)$$

$$=\begin{cases}\left(\underset{1\le i_1<,\ldots,<i_\rho\le n}{\otimes}\left(1-\sum_{\varsigma=1}^{\rho}\omega_\varsigma\right)\left(\frac{\overset{\rho}{\underset{\varsigma=1}{\oplus}}\zeta_{i_\varsigma}}{\rho}\right)\right)^{\frac{1}{C_{n-1}^{\rho}}} & 1\le\rho<n\\ \overset{\rho}{\underset{\varsigma=1}{\oplus}}\zeta_{i_\varsigma}^{\frac{1-\omega_\varsigma}{n-1}} & \rho=n\end{cases}\tag{7}$$

*Theorem 13:* Consider a list of IVIFVs $\zeta_i = ([\varphi_i^L(t), \varphi_i^U(t)], [\upsilon_i^L(t), \upsilon_i^U(t)]), i = 1, 2, \ldots, n$. The aggregated outcome by the IVIFAAWDHM operator is still an IVIFV and we have (8), as shown at the bottom of page 14.

*Theorem 14:* Consider a list of IVIFVs $\zeta_i = ([\varphi_i^L(t), \varphi_i^U(t)], [\upsilon_i^L(t), \upsilon_i^U(t)]), i = 1, 2, \ldots, n$ such that $\zeta_i = \zeta$. Then:

$$\text{IVIFAAWDHM}^{(\rho)}(\zeta_1, \zeta_2, \ldots, \zeta_n) = \zeta$$

*Theorem 15:* Consider two sets of IVIFVs $\zeta_i = ([\varphi_i^L(t), \varphi_i^U(t)], [\upsilon_i^L(t), \upsilon_i^U(t)])$ and $\xi_i = ([\gamma_i^L(t), \gamma_i^U(t)], [\delta_i^L(t), \delta_i^U(t)]),\ i = 1, 2, \ldots, n$. If $\varphi_i(t)\le\gamma_i(t)$, $\upsilon_i(t)\ge\delta_i(t)$, $\forall\ i$, Then:

$$\begin{aligned}&\text{IVIFAAWDHM}^{(\rho)}(\zeta_1, \zeta_2, \ldots, \zeta_n)\\ &\quad\le\text{IVIFAAWDHM}^{(\rho)}(\xi_1, \xi_2, \ldots, \xi_n)\end{aligned}$$

*Theorem 16:* Consider a list of IVIFVs $\zeta_i = ([\varphi_i^L(t), \varphi_i^U(t)], [\upsilon_i^L(t), \upsilon_i^U(t)])i = 1, 2, \ldots, n$. If $\zeta_i^- = min(\zeta_1, \zeta_2, \zeta_3, \ldots, \zeta_n)$ and $\zeta_i^+ = max(\zeta_1, \zeta_2, \zeta_3, \ldots, \zeta_n)$. Then:

$$\zeta^-\le\textbf{IVIFAAWDHM}(\zeta_1,\zeta_2,\ldots,\zeta_n)\le\zeta^+$$

## V. SOLUTION OF THE MADM PROBLEM BASED ON DERIVED MATHEMATICAL APPROACHES

Multi-attribute decision-making (MADM) is a method to choose more useful information from a finite set of alternatives collected by different observations about a specific thing in different ways. It is used when a decision maker or expert needs the best information among all collected qualitative or quantitative information. It is an efficient technique that is used in different fields such as the treatment of diseases, engineering, business purposes, selection of routes to any destiny calculations, etc. MADM enables us to make better decisions about anything by comparing different observations and calculations based on certain criteria. We propose a method for multi-attribute decision-making with weights.

To serve this purpose, suppose that a set of finite alternative $\{\Xi_1, \Xi_2, \ldots, \Xi_n\}$ and a family of attributes $\{\mathfrak{D}_1, \mathfrak{D}_2, \ldots, \mathfrak{D}_m\}$. Let $\omega = (\omega_1, \omega_2, \omega_3, \ldots, \omega_m)$ be a set of weight vectors that assign a specific value to each attribute such that $\omega_{\jmath} \in [0, 1]$ and $\sum_{\jmath=1}^{m} \omega_{\jmath} = 1$. Each alternative carries different types of information in the form of IVIFVs $\zeta_{\imath\jmath} = \left(\left[\varphi^L_{\imath\jmath}(t), \varphi^U_{\imath\jmath}(t)\right], \left[\upsilon^L_{\imath\jmath}(t), \upsilon^U_{\imath\jmath}(t)\right]\right), t = 1, 2, \ldots, n \& \imath = 1, 2, \ldots, m$. The decision maker arranges information and lists it in the following decision matrix $\Omega = \left(\zeta_{\imath\jmath}\right)_{m\times n}, \imath = 1, 2, \ldots, n \& \jmath = 1, 2, \ldots, m$.

$$\Omega = \begin{pmatrix} \zeta_{11} & \zeta_{12} & \cdots & \zeta_{1n} \\ \zeta_{21} & \zeta_{22} & \cdots & \zeta_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \zeta_{m1} & \zeta_{m2} & \cdots & \zeta_{mn} \end{pmatrix}$$

In order to evaluate the MADM problem, the decision maker illustrates some particular steps of an algorithm of the MADM problem. Figure 2 also depicts various steps of the MADM problem.

**Step 1:**Construct a decision matrix $\Omega = \left(\zeta_{\imath\jmath}\right)_{m\times n}, \imath = 1, 2, \ldots, n \& \jmath = 1, 2, \ldots, m$ of an IVIF information.

**Step 2:**Generally, a decision matrix contains two types of information such as beneficial and non-beneficial. Before the aggregation process, all types of information should be the same types.

**Step 3.** Utilize our suggested operators to assess alternatives progressively.

**Step 4:**To assess an optimal option during decision analysis, compute the score values of each alternative or individual.

**Step 5:**Re-arrange all alternatives or individuals based on computed score values to choose the most appropriate optimal option.

### A. EXPERIMENTAL CASE STUDY

In this experimental case study, we expressed some methods for assessing the performance of agricultural education practice. The important role of performance evaluation in agricultural education practice:

1) Enhancing Education Quality: Evaluation helps educational institutions and teachers understand the effectiveness of practical teaching, thereby adjusting teaching methods and content in a timely manner to improve education quality.
2) Promoting Student Learning: Through evaluation, students can clearly understand their performance and growth in practice, stimulating learning motivation and increasing learning enthusiasm.
3) Developing Practical Skills: Performance evaluation emphasizes the skills and experiences students gain in

$$\mathrm{IVIFAADHM}^{(\rho)}(\zeta_1, \zeta_2, \ldots, \zeta_n) = \left(\begin{array}{c} \left[ e^{-\left(\left(\frac{1}{C_n^{\rho}}\right)\left(\sum_{1\le i_1<,\ldots,<i_\rho\le n}\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(1-\varphi^L_{i_\varsigma}(t)\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}, e^{-\left(\left(\frac{1}{C_n^{\rho}}\right)\left(\sum_{1\le i_1<,\ldots,<i_\rho\le n}\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(1-\varphi^U_{i_\varsigma}(t)\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}} \right], \\ \left[ 1-e^{-\left(\left(\frac{1}{C_n^{\rho}}\right)\left(\sum_{1\le i_1<,\ldots,<i_\rho\le n}\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\upsilon^L_{i_\varsigma}(t)\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}, 1-e^{-\left(\left(\frac{1}{C_n^{\rho}}\right)\left(\sum_{1\le i_1<,\ldots,<i_\rho\le n}\left(-log\left(1-e^{-\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\upsilon^U_{i_\varsigma}(t)\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}} \right] \end{array}\right) \quad (6)$$

practical operations, helping them better develop and enhance practical abilities.

4) Broadening Career Perspectives: Evaluation helps students understand actual work environments and job requirements, broadening their career perspectives and preparing them for future career development.
5) Facilitating Industry-Academia Collaboration: Through evaluation, educational institutions and businesses can better understand the needs and effects of practical teaching, promoting collaboration between academia and industry and driving industrial development.
6) Enhancing Social Impact: Evaluation of the performance of agricultural education practice can promote the dissemination and application of agricultural technology and knowledge, enhance agricultural productivity, promote sustainable agricultural development, and have a positive impact on society.

Overall, performance evaluation in agricultural education practice plays an important role at the levels of education, students, industry, and society, helping to improve education quality, promote student learning and growth, and drive development and progress in the agricultural field. In this numerical example, we also explore some of the assessment performance of agricultural education practice as follows:

**Vertical Farming Education $\Xi_1$:** Vertical farming education is a method of growing crops in vertically stacked layers, often in controlled indoor environments. This approach maximizes space utilization and allows for year-round cultivation, independent of external weather conditions. Vertical farming

$$\mathrm{IVIFAAWDHM}^{(\rho)}(\zeta_1,\zeta_2,\ldots,\zeta_n) = \left(\begin{array}{l}\left[\begin{array}{l} e^{-\left(\left(\frac{1}{C_{n-1}^{\rho}}\right)\left(\sum_{1\le i_1<,\ldots,<i_\rho\le n}\left(-log\left(1-e^{-\left(\left(1-\sum_{\varsigma=1}^{\rho}\omega_\varsigma\right)\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(1-\varphi_{i_\varsigma}^{L}(\mathfrak{t})\right)\right)^{\beth}\right)\right)\right)^{\frac{1}{\beth}}}\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}},\\ e^{-\left(\left(\frac{1}{C_{n-1}^{\rho}}\right)\left(\sum_{1\le i_1<,\ldots,<i_\rho\le n}\left(-log\left(1-e^{-\left(\left(1-\sum_{\varsigma=1}^{\rho}\omega_\varsigma\right)\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(1-\varphi_{i_\varsigma}^{U}(\mathfrak{t})\right)\right)^{\beth}\right)\right)\right)^{\frac{1}{\beth}}}\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}\end{array}\right],\\ \left[\begin{array}{l} 1-e^{-\left(\left(\frac{1}{C_{n-1}^{\rho}}\right)\left(\sum_{1\le i_1<,\ldots,<i_\rho\le n}\left(-log\left(1-e^{-\left(\left(1-\sum_{\varsigma=1}^{\rho}\omega_\varsigma\right)\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\upsilon_{i_\varsigma}^{L}(\mathfrak{t})\right)\right)^{\beth}\right)\right)\right)^{\frac{1}{\beth}}}\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}},\\ 1-e^{-\left(\left(\frac{1}{C_{n-1}^{\rho}}\right)\left(\sum_{1\le i_1<,\ldots,<i_\rho\le n}\left(-log\left(1-e^{-\left(\left(1-\sum_{\varsigma=1}^{\rho}\omega_\varsigma\right)\left(\left(\frac{1}{\rho}\right)\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(\upsilon_{i_\varsigma}^{U}(\mathfrak{t})\right)\right)^{\beth}\right)\right)\right)^{\frac{1}{\beth}}}\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}\end{array}\right]\end{array}\right), 1\le\rho<n$$

$$\mathrm{IVIFAAWDHM}^{(\rho)}(\zeta_1,\zeta_2,\ldots,\zeta_n) = \left(\begin{array}{l}\left[\begin{array}{l} 1-e^{-\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(1-e^{-\left(\left(\frac{1-\omega_\varsigma}{n-1}\right)\left(-log\left(\varphi_{i_\varsigma}^{L}(\mathfrak{t})\right)\right)^{\beth}\right)^{\frac{1}{\beth}}}\right)\right)^{\beth}\right)^{\frac{1}{\beth}}},\\ 1-e^{-\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(1-e^{-\left(\left(\frac{1-\omega_\varsigma}{n-1}\right)\left(-log\left(\varphi_{i_\varsigma}^{U}(\mathfrak{t})\right)\right)^{\beth}\right)^{\frac{1}{\beth}}}\right)\right)^{\beth}\right)^{\frac{1}{\beth}}}\end{array}\right],\\ \left[\begin{array}{l} e^{-\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(1-e^{-\left(\left(\frac{1-\omega_\varsigma}{n-1}\right)\left(\left(-log\left(1-\upsilon_{i_\varsigma}^{L}(\mathfrak{t})\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}\right)\right)^{\beth}\right)^{\frac{1}{\beth}}},\\ e^{-\left(\sum_{\varsigma=1}^{\rho}\left(-log\left(1-e^{-\left(\left(\frac{1-\omega_\varsigma}{n-1}\right)\left(\left(-log\left(1-\upsilon_{i_\varsigma}^{U}(\mathfrak{t})\right)\right)^{\beth}\right)\right)^{\frac{1}{\beth}}}\right)\right)^{\beth}\right)^{\frac{1}{\beth}}}\end{array}\right]\end{array}\right), \rho=n \quad (8)$$

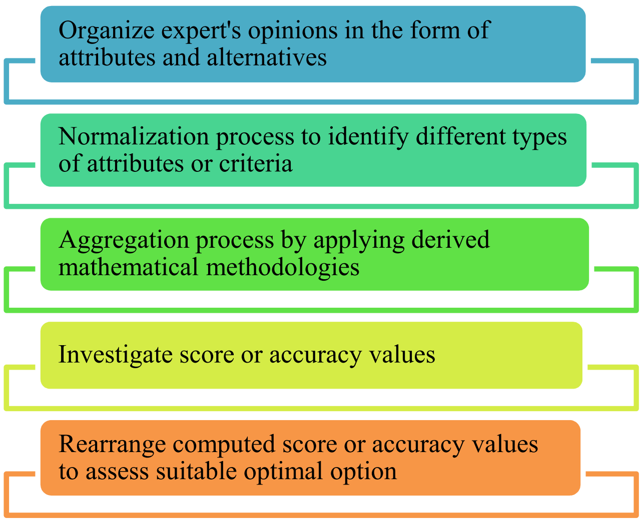


**FIGURE 2.** Steps of the MADM problem.

education can be particularly beneficial in urban areas where arable land is limited.

**Aquaponics and Hydroponics Education $\Xi_2$:** Aquaponics and hydroponics education are soilless farming techniques that involve growing plants in nutrient-rich water. Aquaponics combines plant cultivation with aquaculture (fish farming), creating a symbiotic relationship where fish waste provides nutrients for plants, and plants filter and purify the water for the fish. Hydroponics relies on nutrient solutions for plant growth.

**Blockchain Education in Agriculture $\Xi_3$:** Blockchain technology education is being employed to enhance transparency and traceability in the agricultural supply chain. It ensures the secure and transparent recording of educational information related to agricultural product production, processing, and distribution, reducing fraud and improving accountability.

**Agroforestry Education $\Xi_4$:** Agroforestry education integrates trees and shrubs into traditional agricultural practices. This education approach enhances biodiversity, improves soil health, conserves water, and provides additional revenue streams for farmers through the sustainable harvesting of timber, fruits, or nuts.

**Drone Technology Education $\Xi_5$:**Drone education equipped with sensors and cameras is increasingly used in agriculture for crop monitoring, pest control, and precision farming. They can provide high-resolution images of fields, helping farmers identify issues such as disease outbreaks or nutrient deficiencies.

The above-discussed innovative agricultural methods are evaluated using the following particular characteristics or attributes. A detailed discussion about these attributes is expressed as follows:

**Economic Viability Education $C_1$:** Economic viability education refers to educational programs or initiatives focused on teaching individuals about the economic aspects of various endeavours, industries, or sectors to enhance their understanding of financial principles, economic decision-making, and strategies for sustainable economic development.

**Sustainability Education $C_2$:** Sustainability education often covers climate change, biodiversity conservation, renewable energy, sustainable agriculture, green technology, social equity, ethical business practices, and global citizenship. It seeks to empower individuals and communities to address pressing sustainability issues, adopt more sustainable lifestyles, and contribute to the transition towards a more sustainable and resilient society.

**Adaptability Education $C_3$:** Adaptability education may include teaching strategies such as scenario planning, critical thinking, creativity, communication skills, and stress management techniques. It often emphasizes the importance of lifelong learning and encourages individuals to embrace change as an opportunity for growth and innovation. By fostering adaptability, this type of education equips individuals with the skills and mindset needed to thrive in dynamic and uncertain environments, whether in the workplace, community, or broader society.

**Resource Efficiency Education $C_4$:** Resource efficiency education aims to empower individuals, communities, businesses, and governments to make informed decisions and take actions that promote the efficient use of resources, reduce environmental impact, and build a more sustainable future.

The decision-maker assigns some particular value to each attribute (0.30, 0.35, 0.15, 0.20) corresponding to each alternative or individual as a hypothetically. To evaluate the performance of agricultural education practice, the aggregate information of an experimental case study is given using the following procedure of an algorithm for the MADM problem.

**Step 1:** The decision maker constructs a decision matrix of IVIF information in the form of alternatives and attributes about the discussed experimental case study and given information listed in Table 1.

**Step 2:** We noticed that all given attributes are the same types. Therefore, we can avoid doing the normalization process.

**Step 3:** Utilized derived mathematical approaches of the IVIFAAWHM and IVIFAWDHM operators. Table 2 demonstrates the aggregated outcomes by the invented aggregation operators.

**Step 4:** Investigate score values for all alternatives or individuals by using Definition 6. Table 3 illustrates aggregated outcomes of the score values corresponding to each alternative.

**Step 5:** By ranking all individuals, we investigate an ideal optimal option by considering some particular criteria or attributes. From Table 3, we see $\Xi_3$ and $\Xi_5$ are the most appropriate optimal options for the IVIFAAWHM and IVIFAAWDHM operators, respectively. A bar chart in Figure 3 also explores the computed score values of all alternatives or individuals. To facilitate a better understanding of aggregated outcomes, graphical representations play an essential

**TABLE 1.** Interval-valued intuitionistic fuzzy information.

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $\Xi_1$ | $\begin{pmatrix}[0.24, 0.31],\\ [0.62,0.66]\end{pmatrix}$ | $\begin{pmatrix}[0.22, 0.34],\\ [0.36,0.45]\end{pmatrix}$ | $\begin{pmatrix}[0.25, 0.43],\\ [0.47,0.51]\end{pmatrix}$ | $\begin{pmatrix}[0.33, 0.39],\\ [0.36,0.76]\end{pmatrix}$ |
| $\Xi_2$ | $\begin{pmatrix}[0.19, 0.71],\\ [0.45,0.54]\end{pmatrix}$ | $\begin{pmatrix}[0.31, 0.54],\\ [0.32,0.57]\end{pmatrix}$ | $\begin{pmatrix}[0.26, 0.69],\\ [0.18,0.34]\end{pmatrix}$ | $\begin{pmatrix}[0.43, 0.45],\\ [0.32,0.64]\end{pmatrix}$ |
| $\Xi_3$ | $\begin{pmatrix}[0.53, 0.61],\\ [0.46,0.57]\end{pmatrix}$ | $\begin{pmatrix}[0.53, 0.65],\\ [0.38,0.59]\end{pmatrix}$ | $\begin{pmatrix}[0.44, 0.49],\\ [0.38,0.56]\end{pmatrix}$ | $\begin{pmatrix}[0.39, 0.56],\\ [0.38,0.59]\end{pmatrix}$ |
| $\Xi_4$ | $\begin{pmatrix}[0.27, 0.7],\\ [0.38,0.43]\end{pmatrix}$ | $\begin{pmatrix}[0.27, 0.43],\\ [0.44,0.48]\end{pmatrix}$ | $\begin{pmatrix}[0.21, 0.51],\\ [0.28,0.69]\end{pmatrix}$ | $\begin{pmatrix}[0.46, 0.53],\\ [0.34,0.47]\end{pmatrix}$ |
| $\Xi_5$ | $\begin{pmatrix}[0.29, 0.45],\\ [0.18,0.34]\end{pmatrix}$ | $\begin{pmatrix}[0.49, 0.5],\\ [0.18,0.69]\end{pmatrix}$ | $\begin{pmatrix}[0.22, 0.53],\\ [0.31,0.61]\end{pmatrix}$ | $\begin{pmatrix}[0.45, 0.65],\\ [0.26,0.65]\end{pmatrix}$ |

**TABLE 2.** Integrated results by the derived mathematical strategies.

| | IVIFAAWHM | IVIFAAWDHM |
|---|---|---|
| $\Xi_1$ | $\begin{pmatrix}[0.1402, 0.2065],\\ [0.7281,0.7888]\end{pmatrix}$ | $\begin{pmatrix}[0.3340, 0.3988],\\ [0.4002,0.4502]\end{pmatrix}$ |
| $\Xi_2$ | $\begin{pmatrix}[0.1591, 0.3648],\\ [0.6619,0.7558]\end{pmatrix}$ | $\begin{pmatrix}[0.3612, 0.5174],\\ [0.3187,0.3976]\end{pmatrix}$ |
| $\Xi_3$ | $\begin{pmatrix}[0.2690, 0.3447],\\ [0.7038,0.7764]\end{pmatrix}$ | $\begin{pmatrix}[0.4486, 0.5060],\\ [0.3686,0.4440]\end{pmatrix}$ |
| $\Xi_4$ | $\begin{pmatrix}[0.1606, 0.3222],\\ [0.6833,0.7589]\end{pmatrix}$ | $\begin{pmatrix}[0.3595, 0.4858],\\ [0.3418,0.4264]\end{pmatrix}$ |
| $\Xi_5$ | $\begin{pmatrix}[0.1921, 0.3183],\\ [0.6206,0.7793]\end{pmatrix}$ | $\begin{pmatrix}[0.3944, 0.4900],\\ [0.2891,0.4219]\end{pmatrix}$ |

role in the assessment performance of agricultural education practice.

### B. INFLUENCE OF DIFFERENT PARAMETRIC VALUES ON THE RESULTS OF THE MADM PROBLEM

In this section, we examine the reliability and validity of derived mathematical approaches by considering an algorithm for the MADM problem. Sometimes, decision-makers face a lot of challenges during decision analysis in finding a desirable optimal option based on certain criteria or characteristics.

By setting different parametric values of Aczel Alsina operations $\daleth$, we reveal the validity of proposed mathematical approaches under considering derived approaches and the MADM problem. We also analyze aggregated outcomes by the changing of different variable values $\rho$ in the decision-making process. Table 4 considers the ranking of all alternatives or individuals that the IVIFAAWHM operator investigates.

Similarly, we also examined the reliability of the derived mathematical expression IVIFAAWDHM operator

**TABLE 3.** Ranking of alternatives based on corresponding score values.

| Aggregation operators | $Sc(\Xi_1)$ | $Sc(\Xi_2)$ | $Sc(\Xi_3)$ | $Sc(\Xi_4)$ | $Sc(\Xi_5)$ |
|---|---|---|---|---|---|
| IVIFAAWHM | −0.5851 | −0.4469 | −0.4332 | −0.4797 | −0.4448 |
| IVIFAAWDHM | −0.0588 | 0.0811 | 0.0710 | 0.0386 | 0.0868 |
| **Aggregation operators** | **Ranking of score values** | | | | |
| IVIFAAWHM | $\Xi_3 \succ \Xi_5 \succ \Xi_2 \succ \Xi_4 \succ \Xi_1$ | | | | |
| IVIFAAWDHM | $\Xi_5 \succ \Xi_2 \succ \Xi_3 \succ \Xi_4 \succ \Xi_1$ | | | | |

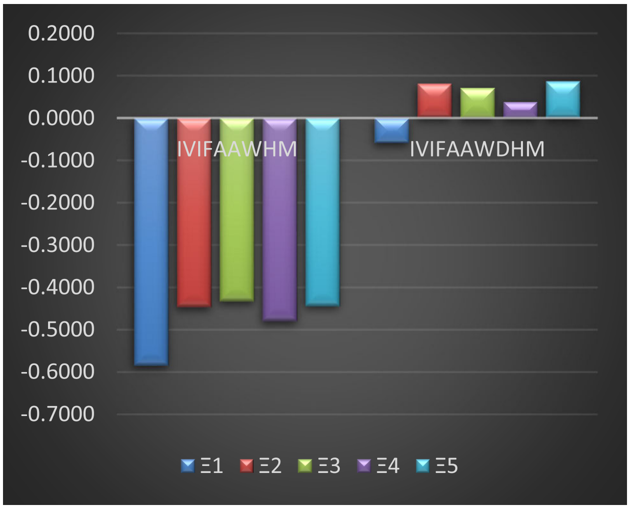


**FIGURE 3.** Explores the results of derived mathematical approaches.

at different parametric values of Aczel Alsina operations $\daleth$ and $\rho = 1, 2, 3, 4$. Table 5 also investigated the ranking of all individuals by the IVIFAAWDHM operator. The ranking of alternatives becomes unchanged after some particular values of Aczel Asina operations. Furthermore, we also understand the structural behaviour of alternatives or individuals under considering the investigated outcomes of the derived approaches.

## VI. COMPARATIVE STUDY

This section demonstrates the advantages and reliability of deduced research work by incorporating the process of the MADM technique. To serve this purpose, we applied different existing mathematical approaches [43], [44], [46], [47], [48], [49], [50], [51] to the experimental case study based on a discussed algorithm of the MADM technique.

**TABLE 4.** Aggregated outcomes by applying the IVIFAAWHM operator for different values of $\rho = 1, 2, 3, 4$ and $\beth$.

| Parameters | $\rho = 1$ | $\rho = 2$ | $\rho = 3$ | $\rho = 4$ |
|---|---|---|---|---|
| $\beth = 1$ | $\Xi_5 \succ \Xi_2 \succ \Xi_3 \succ \Xi_4 \succ \Xi_1$ | $\Xi_3 \succ \Xi_5 \succ \Xi_2 \succ \Xi_4 \succ \Xi_1$ | $\Xi_5 \succ \Xi_2 \succ \Xi_3 \succ \Xi_4 \succ \Xi_1$ | $\Xi_3 \succ \Xi_5 \succ \Xi_2 \succ \Xi_4 \succ \Xi_1$ |
| $\beth = 10$ | $\Xi_2 \succ \Xi_5 \succ \Xi_4 \succ \Xi_3 \succ \Xi_1$ | $\Xi_3 \succ \Xi_5 \succ \Xi_2 \succ \Xi_4 \succ \Xi_1$ | $\Xi_3 \succ \Xi_5 \succ \Xi_2 \succ \Xi_4 \succ \Xi_1$ | $\Xi_3 \succ \Xi_5 \succ \Xi_4 \succ \Xi_2 \succ \Xi_1$ |
| $\beth = 25$ | $\Xi_3 \succ \Xi_5 \succ \Xi_2 \succ \Xi_4 \succ \Xi_1$ | $\Xi_3 \succ \Xi_5 \succ \Xi_2 \succ \Xi_4 \succ \Xi_1$ | $\Xi_3 \succ \Xi_5 \succ \Xi_4 \succ \Xi_2 \succ \Xi_1$ | $\Xi_3 \succ \Xi_5 \succ \Xi_4 \succ \Xi_2 \succ \Xi_1$ |
| $\beth = 35$ | $\Xi_4 \succ \Xi_5 \succ \Xi_2 \succ \Xi_3 \succ \Xi_1$ | $\Xi_3 \succ \Xi_5 \succ \Xi_2 \succ \Xi_4 \succ \Xi_1$ | $\Xi_3 \succ \Xi_4 \succ \Xi_5 \succ \Xi_2 \succ \Xi_1$ | $\Xi_3 \succ \Xi_5 \succ \Xi_4 \succ \Xi_2 \succ \Xi_1$ |
| $\beth = 50$ | $\Xi_2 \succ \Xi_5 \succ \Xi_4 \succ \Xi_3 \succ \Xi_1$ | $\Xi_3 \succ \Xi_5 \succ \Xi_2 \succ \Xi_4 \succ \Xi_1$ | $\Xi_3 \succ \Xi_4 \succ \Xi_5 \succ \Xi_2 \succ \Xi_1$ | $\Xi_3 \succ \Xi_5 \succ \Xi_4 \succ \Xi_2 \succ \Xi_1$ |
| $\beth = 65$ | $\Xi_2 \succ \Xi_5 \succ \Xi_4 \succ \Xi_3 \succ \Xi_1$ | $\Xi_3 \succ \Xi_5 \succ \Xi_2 \succ \Xi_4 \succ \Xi_1$ | $\Xi_3 \succ \Xi_4 \succ \Xi_5 \succ \Xi_2 \succ \Xi_1$ | $\Xi_3 \succ \Xi_5 \succ \Xi_4 \succ \Xi_2 \succ \Xi_1$ |
| $\beth = 85$ | $\Xi_2 \succ \Xi_5 \succ \Xi_4 \succ \Xi_3 \succ \Xi_1$ | $\Xi_3 \succ \Xi_5 \succ \Xi_2 \succ \Xi_4 \succ \Xi_1$ | $\Xi_3 \succ \Xi_4 \succ \Xi_5 \succ \Xi_2 \succ \Xi_1$ | $\Xi_3 \succ \Xi_5 \succ \Xi_4 \succ \Xi_2 \succ \Xi_1$ |
| $\beth = 100$ | $\Xi_2 \succ \Xi_5 \succ \Xi_4 \succ \Xi_3 \succ \Xi_1$ | $\Xi_3 \succ \Xi_5 \succ \Xi_2 \succ \Xi_4 \succ \Xi_1$ | $\Xi_3 \succ \Xi_4 \succ \Xi_5 \succ \Xi_2 \succ \Xi_1$ | $\Xi_3 \succ \Xi_5 \succ \Xi_4 \succ \Xi_2 \succ \Xi_1$ |
| $\beth = 125$ | $\Xi_2 \succ \Xi_5 \succ \Xi_4 \succ \Xi_3 \succ \Xi_1$ | $\Xi_3 \succ \Xi_5 \succ \Xi_2 \succ \Xi_4 \succ \Xi_1$ | $\Xi_3 \succ \Xi_4 \succ \Xi_5 \succ \Xi_2 \succ \Xi_1$ | $\Xi_3 \succ \Xi_5 \succ \Xi_4 \succ \Xi_2 \succ \Xi_1$ |
| $\beth = 150$ | $\Xi_2 \succ \Xi_5 \succ \Xi_4 \succ \Xi_3 \succ \Xi_1$ | $\Xi_3 \succ \Xi_5 \succ \Xi_2 \succ \Xi_4 \succ \Xi_1$ | $\Xi_3 \succ \Xi_4 \succ \Xi_5 \succ \Xi_2 \succ \Xi_1$ | $\Xi_3 \succ \Xi_5 \succ \Xi_4 \succ \Xi_2 \succ \Xi_1$ |

**TABLE 5.** Aggregated outcomes by applying the IVIFAAWDHM operator for different values of $\rho = 1, 2, 3, 4$ and $\beth$.

| Parameters | $\rho = 1$ | $\rho = 2$ | $\rho = 3$ | $\rho = 4$ |
|---|---|---|---|---|
| $\beth = 1$ | $\Xi_5 \succ \Xi_3 \succ \Xi_2 \succ \Xi_4 \succ \Xi_1$ | $\Xi_5 \succ \Xi_2 \succ \Xi_3 \succ \Xi_4 \succ \Xi_1$ | $\Xi_3 \succ \Xi_1 \succ \Xi_5 \succ \Xi_4 \succ \Xi_2$ | $\Xi_3 \succ \Xi_5 \succ \Xi_2 \succ \Xi_4 \succ \Xi_1$ |
| $\beth = 10$ | $\Xi_3 \succ \Xi_5 \succ \Xi_2 \succ \Xi_4 \succ \Xi_1$ | $\Xi_2 \succ \Xi_5 \succ \Xi_3 \succ \Xi_4 \succ \Xi_1$ | $\Xi_3 \succ \Xi_5 \succ \Xi_1 \succ \Xi_4 \succ \Xi_1$ | $\Xi_5 \succ \Xi_2 \succ \Xi_3 \succ \Xi_4 \succ \Xi_1$ |
| $\beth = 25$ | $\Xi_5 \succ \Xi_2 \succ \Xi_4 \succ \Xi_3 \succ \Xi_1$ | $\Xi_5 \succ \Xi_2 \succ \Xi_4 \succ \Xi_3 \succ \Xi_1$ | $\Xi_5 \succ \Xi_3 \succ \Xi_1 \succ \Xi_4 \succ \Xi_2$ | $\Xi_5 \succ \Xi_2 \succ \Xi_3 \succ \Xi_4 \succ \Xi_1$ |
| $\beth = 35$ | $\Xi_3 \succ \Xi_5 \succ \Xi_2 \succ \Xi_4 \succ \Xi_1$ | $\Xi_5 \succ \Xi_2 \succ \Xi_3 \succ \Xi_4 \succ \Xi_1$ | $\Xi_5 \succ \Xi_3 \succ \Xi_1 \succ \Xi_4 \succ \Xi_2$ | $\Xi_5 \succ \Xi_2 \succ \Xi_4 \succ \Xi_3 \succ \Xi_1$ |
| $\beth = 50$ | $\Xi_3 \succ \Xi_5 \succ \Xi_2 \succ \Xi_4 \succ \Xi_1$ | $\Xi_2 \succ \Xi_5 \succ \Xi_3 \succ \Xi_4 \succ \Xi_1$ | $\Xi_5 \succ \Xi_3 \succ \Xi_1 \succ \Xi_4 \succ \Xi_2$ | $\Xi_5 \succ \Xi_2 \succ \Xi_4 \succ \Xi_3 \succ \Xi_1$ |
| $\beth = 65$ | $\Xi_3 \succ \Xi_5 \succ \Xi_2 \succ \Xi_4 \succ \Xi_1$ | $\Xi_5 \succ \Xi_2 \succ \Xi_3 \succ \Xi_4 \succ \Xi_1$ | $\Xi_5 \succ \Xi_3 \succ \Xi_1 \succ \Xi_4 \succ \Xi_2$ | $\Xi_5 \succ \Xi_2 \succ \Xi_4 \succ \Xi_3 \succ \Xi_1$ |
| $\beth = 85$ | $\Xi_3 \succ \Xi_5 \succ \Xi_2 \succ \Xi_4 \succ \Xi_1$ | $\Xi_5 \succ \Xi_3 \succ \Xi_2 \succ \Xi_4 \succ \Xi_1$ | $\Xi_5 \succ \Xi_3 \succ \Xi_1 \succ \Xi_4 \succ \Xi_2$ | $\Xi_5 \succ \Xi_2 \succ \Xi_4 \succ \Xi_3 \succ \Xi_1$ |
| $\beth = 100$ | $\Xi_3 \succ \Xi_5 \succ \Xi_2 \succ \Xi_4 \succ \Xi_1$ | $\Xi_2 \succ \Xi_5 \succ \Xi_3 \succ \Xi_4 \succ \Xi_1$ | $\Xi_5 \succ \Xi_3 \succ \Xi_1 \succ \Xi_4 \succ \Xi_2$ | $\Xi_5 \succ \Xi_2 \succ \Xi_4 \succ \Xi_3 \succ \Xi_1$ |
| $\beth = 125$ | $\Xi_3 \succ \Xi_5 \succ \Xi_2 \succ \Xi_4 \succ \Xi_1$ | $\Xi_5 \succ \Xi_3 \succ \Xi_2 \succ \Xi_4 \succ \Xi_1$ | $\Xi_5 \succ \Xi_3 \succ \Xi_1 \succ \Xi_4 \succ \Xi_2$ | $\Xi_5 \succ \Xi_2 \succ \Xi_4 \succ \Xi_3 \succ \Xi_1$ |
| $\beth = 150$ | $\Xi_3 \succ \Xi_5 \succ \Xi_2 \succ \Xi_4 \succ \Xi_1$ | $\Xi_2 \succ \Xi_5 \succ \Xi_3 \succ \Xi_4 \succ \Xi_1$ | $\Xi_5 \succ \Xi_3 \succ \Xi_1 \succ \Xi_4 \succ \Xi_2$ | $\Xi_5 \succ \Xi_2 \succ \Xi_4 \succ \Xi_3 \succ \Xi_1$ |

A detailed discussion of existing research work and mathematical approaches is expressed as follows:

a) Senapati et al. [43] utilized the operational laws of Aczel Alsina operations to develop weighted averaging operators under considering IVIF theory.
b) Senapati et al. [46] also deduced weighted geometric operators with Aczel Alsina operations and IVIF information.
c) Wang et al. [44] diagnosed aggregation operators of weighted average and weighted geometric operators.
d) Wu et al. [47] proposed the theory of Hamy mean operators and Dombi operational laws to derive new approaches for handling real-life applications.
e) Zhang [48] anticipated a family of Frank aggregation operators, namely IVIF Frank weighted average and IVIF Frank weighted geometric operators.

**TABLE 6.** Explored aggregated results by the existing mathematical methodologies.

| Aggregation Operators | Score values | Ranking of alternatives |
|---|---|---|
| IVIFAAWHM | $Sc(\Xi_1)$ = -0.5851, $Sc(\Xi_2) = -0.4469$, $Sc(\Xi_3)$ =-0.4332 $Sc(\Xi_4) = -0.4797$, $Sc(\Xi_5) = -0.4448$ | $\Xi_3 \succ \Xi_5 \succ \Xi_2 \succ \Xi_4 \succ \Xi_1$ |
| IVIFAAWDHM | $Sc(\Xi_1)$ = -0.0588, $Sc(\Xi_2) = -0.0811$, $Sc(\Xi_3) = -0.0710$ $Sc(\Xi_4) = -0.0386$, $Sc(\Xi_5) = -0.0868$, | $\Xi_4 \succ \Xi_1 \succ \Xi_3 \succ \Xi_2 \succ \Xi_5$ |
| IVIFAAWA **[43]** | $Sc(\Xi_1)$ = -0.4799, $Sc(\Xi_2) = -0.4396$, $Sc(\Xi_3) = -0.4656$ $Sc(\Xi_4) = -0.3941$, $Sc(\Xi_5) = -0.3144$, | $\Xi_5 \succ \Xi_4 \succ \Xi_2 \succ \Xi_3 \succ \Xi_1$ |
| IVIFAAWG **[46]** | $Sc(\Xi_1) = 0.3623$, $Sc(\Xi_2) = 0.4568$, $Sc(\Xi_3) = 0.4307$ $Sc(\Xi_4) = 0.4654$, $Sc(\Xi_5) = 0.4130$, | $\Xi_4 \succ \Xi_2 \succ \Xi_3 \succ \Xi_5 \succ \Xi_1$ |
| IVIFWA **[44]** | $Sc(\Xi_1)$ = -0.0602, $Sc(\Xi_2) = 0.0326$, $Sc(\Xi_3) = -0.0075$ $Sc(\Xi_4) = 0.0730$, $Sc(\Xi_5) = 0.1883$, | $\Xi_5 \succ \Xi_4 \succ \Xi_2 \succ \Xi_3 \succ \Xi_1$ |
| IVIFWG **[44]** | $Sc(\Xi_1)$ = -0.1222, $Sc(\Xi_2) = 0.0022$, $Sc(\Xi_3) = -0.0422$ $Sc(\Xi_4) = 0.0299$, $Sc(\Xi_5) = -0.0629$, | $\Xi_4 \succ \Xi_2 \succ \Xi_3 \succ \Xi_5 \succ \Xi_1$ |
| IVIFDWA **[47]** | $Sc(\Xi_1) = 0.0310$, $Sc(\Xi_2) = 0.0645$, $Sc(\Xi_3) = 0.0378$ $Sc(\Xi_4) = 0.1336$, $Sc(\Xi_5) = 0.2383$, | $\Xi_5 \succ \Xi_4 \succ \Xi_2 \succ \Xi_3 \succ \Xi_1$ |
| IVIFDWG **[47]** | $Sc(\Xi_1)$ = -0.1911, $Sc(\Xi_2) = -0.0602$, $Sc(\Xi_3) = -0.1083$ $Sc(\Xi_4) = -0.1001$, $Sc(\Xi_5) = -0.0686$, | $\Xi_2 \succ \Xi_5 \succ \Xi_4 \succ \Xi_3 \succ \Xi_1$ |
| IVIFFWA **[48]** | $Sc(\Xi_1)$ = -0.0650, $Sc(\Xi_2) = 0.0306$, $Sc(\Xi_3) = -0.0101$ $Sc(\Xi_4) = 0.0697$, $Sc(\Xi_5) = 0.1902$, | $\Xi_5 \succ \Xi_4 \succ \Xi_2 \succ \Xi_3 \succ \Xi_1$ |
| IVIFFWG **[48]** | $Sc(\Xi_1)$ = -0.1175, $Sc(\Xi_2) = 0.0046$, $Sc(\Xi_3) = -0.0395$ $Sc(\Xi_4) = 0.0330$, $Sc(\Xi_5) = -0.0648$, | $\Xi_4 \succ \Xi_2 \succ \Xi_3 \succ \Xi_5 \succ \Xi_1$ |
| Ali et al. **[49]** | ×#×#×#×#×#×#×#×#×#× | ×#×#×#×#×#×#×#×#×#×#× |
| Mahmood and Ur Rehman **[50]** | ×#×#×#×#×#×#×#×#×#× | ×#×#×#×#×#×#×#×#×#×#× |
| Mahmood et al. **[51]** | ×#×#×#×#×#×#×#×#×#× | ×#×#×#×#×#×#×#×#×#×#× |

f) Ali et al. [49] developed Bonferroni Mean Operators based on complex spherical fuzzy theory.
g) Mahmood and Ur Rehman [50] generalized bipolar complex fuzzy and derived a family of weighted average and weighted geometric operators.
h) Mahmood et al. [51] presented averaging aggregation operators by incorporating the system of an interval-valued T-spherical fuzzy Soft theory.

Table 6 considers aggregated outcomes based on the existing mathematical approaches and research work that

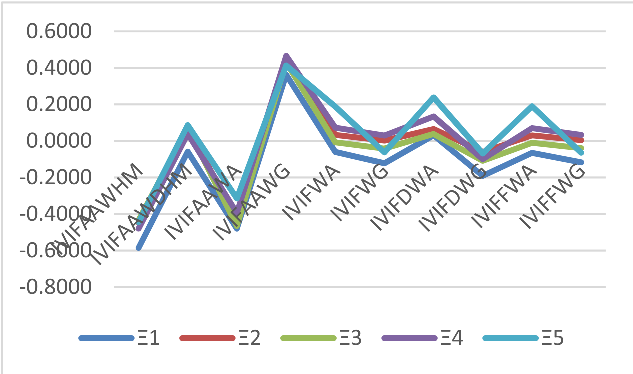


**FIGURE 4.** Explores the results of existing approaches.

exist in the literature. To seek a comparison between existing approaches and pioneered mathematical approaches, we employed the above-discussed aggregation operators. After analyzing the aggregated outcomes of the existing approaches, we concluded that currently developed approaches are more convenient and efficient. The uniqueness of designed mathematical approaches is that they express relationships among different attributes of information during the decision-analysis process. To demonstrate the results of existing approaches, we displayed all computed aggregated outcomes in a graphical representation of Figure 4.

### A. ADVANTAGES OF DEDUCED MATHEMATICAL APPROACHES

1) Diagnosed mathematical approaches enable the exposure of ambiguous and uncertain information about human opinion.
2) To compare experts' opinions using score and accuracy values based on interval-valued intuitionistic fuzzy context.
3) Utilized robust mathematical methodologies to aggregate complicated genuine real-life applications with the help of the algorithm discussed in the MADM problem.
4) Deduced theories and mathematical approaches are capable of demonstrating the drawbacks of existing theories and models.

## VII. CONCLUSION

In this paper, we utilized the theory of HM operators to resolve the assessment performance of agricultural education practice with the help of the MADM technique and mathematical approaches. Mathematical approaches are more helpful for handling unpredictable and complex information about any object. The primary objects of this presented research work are given below:

a) Despite the theory of IVIF information and Hamy mean operators, some dominant operational laws of Aczel Alsina operations are also presented.
b) We derived a family of new mathematical strategies of IVIF information, namely, IVIFAAHM, IVIFAAWHM, IVIFAADHM, and IVIFAAWDHM operators. Some viable properties and characteristics are also described to reveal the validity and compatibility of diagnosed approaches.
c) A robust technique for solving the MADM problem is established to resolve complications in the assessment performance of agricultural education practice with the help of pioneered mathematical methodologies.
d) The comparative study verifies the advantages and feasibility of developed research work. The aggregated outcomes of pioneered approaches contrast with existing research work in the literature.

In the future, we will explore developed research work in different fuzzy domains. We noticed that developed methodologies cannot deal with the unknown degree of weight of the attributes or criteria. We can derive new approaches for power operators and prioritized aggregation operators to handle such a situation. Moreover, we can try to resolve the assessment performance of agricultural education practice and numerical examples with the help of advanced decision-making processes like the TOPSIS method [52], WASPAS [53], EDAS method [54], and COCOSO method [56]. Further, we can also apply deduced theories and mathematical approaches to resolving the assessment performance of agricultural education practice of decision-making based on blockchain [57], medical problems [55], digital innovation [58], [59] and machine learning [60].

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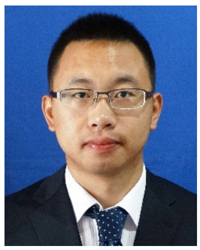

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