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RESEARCH ARTICLE

Parameter Selection Impacting Software Reliability by Utilizing WASPAS Technique Based on Tangent Trigonometric Complex Fuzzy Aggregation Operators

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ABSTRACT Software reliability is a paramount quality attribute that manifests itself in the likelihood of failure-free operation. The development of systems with growing complexity complicates the assessment and enhancement of reliability owing to technical and managerial factors. The vague concept of fuzzy sets and their generalizations have been useful since they can be applied to dealing with imprecise and unreliable information. To overcome the issues of software reliability and the complex, ambiguous nature of information parameters, this paper suggests a new way to use the fuzzy complex set in addition to multi-criteria decision-making to analyze the effect of various software parameters. The investigation starts by explaining the concept of the tangent trigonometric complex fuzzy set and the operations connected with it. Then, it gives the aggregation operators in the complex fuzzy sets, including the tangent trigonometric complex fuzzy weighted averaging and the tangent trigonometric complex fuzzy weighted geometric operators with their basic properties. Consequently, we apply the obtained operators to a multi-criteria decision-making method called the "Weighted Aggregated Sum Product Assessment" in the context of complex fuzzy sets. Next, we go into details of the proposed technique by taking a case study of a "Parameter Selection Impacting Software Reliability". Furthermore, the study analyzes the given approach with other existing theories to show the benefits and superiority of the newly developed approach. This paper provides a detailed and original approach that combines complex fuzzy sets of and multi-criteria decision-making methods in order to overcome the obstacles of software reliability evaluation in the presence of complex and ambiguous information.

INDEX TERMS Software reliability, complex fuzzy set, tangent trigonometric complex fuzzy aggregation operators, WASPAS technique.

I. INTRODUCTION

Software reliability is an important quality attribute that reflects the possibility of failure-free operation of a software system for a specified period. As software systems grow larger and more complex, assessing and improving reliability has become a significant challenge. It includes the system's capacity to work as intended, free from glitches, failures, and unanticipated crashes. Attaining high software dependability is essential for guaranteeing user contentment,

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preserving company continuity, and preserving the software provider's reputation. It has a direct effect on user experience, and operational effectiveness, and may even have a big influence on safety-critical systems like transportation or medical devices. When it comes to software dependability, parameter selection is carefully selecting and configuring the different measurements, thresholds, and criteria that are used to evaluate and quantify a software system's reliability. These characteristics may include things like availability, fault tolerance, failure rates, mean time to failure (MTTF), and mean time between failures (MTBF), among other things. The significance of parameter selection stems from its direct impact on the assessment, forecasting, and, eventually, enhancement of dependability.

A thorough grasp of the software's design, intended usage, operating environment, user expectations, and past failure or error statistics are all necessary for choosing the right parameters. Parameter selection errors can result in deceptive reliability evaluations, which can underestimate or overestimate the system's real performance. For example, if the parameters are set too loosely, they may not appropriately account for probable failures that result from the system's vulnerabilities. On the other hand, too strict specifications may cause the system to perform poorly or incorrectly indicating frequent failures while the system is operating within reasonable bounds. Thus, careful parameter selection is essential to software reliability engineering as it serves as the foundation for test design, enhancement implementation, and software robustness assessment. A well-considered choice of parameters leads to more accurate dependability forecasts, aids in finding weak places in the system, and directs efforts to improve the software's resilience and dependability, all of which contribute to more dependable and trustworthy software output.

MCDM refers to making choices in the presence of multiple, and often competing, decision criteria. MCDM provides a structured framework for considering the trade-offs between criteria like cost, quality, risk, and performance when making complex decisions. The key steps in MCDM involve determining the relevant criteria, weighing their importance, scoring alternatives, and applying decision rules to find the optimal choice. MCDM models, such as the analytical hierarchy process, have become invaluable decision-aiding tools across domains like business, engineering, healthcare, and public policy. By incorporating both quantitative and qualitative factors into the decision process, MCDM allows for more informed, thorough, and justifiable decision-making. It brings rigor and rationality to decisions that must balance multiple competing objectives. With many real-world decisions involving selection among alternatives with multiple attributes, MCDM provides an essential methodology for identifying the optimal choice while considering the nuanced priorities and preferences of stakeholders. The field of operations research gave rise to MCDM. After continuous evolution and development, material science [1], energy [2], geography [3], management science [4], environmental science [5], mathematics [6], information and computer science [7], etc. have all made extensive use of MCDM.

Fuzzy set [8] theory models vagueness and imprecise information using partial membership values between 0 and 1. Elements have a degree of belonging to fuzzy sets, unlike the crisp binary membership of classical sets. Fuzzy sets use membership functions to map elements to grades of membership representing vagueness or ambiguity about class boundaries. Fuzzy MCDM is an approach for handling decisions with multiple criteria under uncertainty. It combines FS theory with traditional MCDM methods to enable decision-making where the goals, constraints, and consequences are vague or imprecise. This allows for capturing the inherent ambiguity and subjectivity in many real-world decision problems. In a decision-making situation involving many information sources, Dursun and Karsak [9] suggested managing information appraised using both linguistic and numerical scales. Wu et al. [10] introduced the fuzzy MCDM approach for assessing banking performance based on Balanced Scorecard. Chu and Lin [11] propose an addition to the fuzzy MCDM model in which the importance weights of each criterion and the ratings of alternatives against criteria are evaluated in terms of linguistic values represented by fuzzy numbers. Airline service quality is a mixture of many different factors, many of which are intangible and challenging to quantify. To address this problem, fuzzy MCDM was introduced into performance evaluation by Tsaur et al. [12]. A different method, the non-additive fuzzy integral, was proposed by Chiou et al. [13] to handle the evaluation of fuzzy MCDM problems, especially when there are dependencies among the criteria taken into consideration. The creation of a fuzzy decision support system using a multi-criteria analysis technique is described by Chen et al. [14] as a means of choosing the best plan options or strategies in an environment watershed.

In various real-life dilemmas, there is always a need for extra fuzzy information, to overcome this, Ramot et al. [15] originated the theory of complex fuzzy set (CFS) in the polar framework. In this notion, the degree of belonging contains amplitude and phase terms which are placed in a unit circle of a complex plane. After that, Tamir et al. [16] deduced another theory of CFS in a cartesian framework, where each degree of belonging contains real and unreal parts which are placed in unit squares of a complex plane. CFS can be applied to model more complicated interactions and uncertainties in various domains, including artificial intelligence, control systems, pattern recognition, and decision-making. It's essential to remember that, in contrast to FS, working with CFS may require more intricate computations, and their use may necessitate a deep understanding of fuzzy logic and associated concepts. Sobhi and Dick's [17] study on CFS for massive learning was presented. Bi et al. [18] developed entropy measurements for CFS, while Luqman et al. [19] identified a hypergraph. CF morphisms were covered by Imtiaz et al. [20]. Rehman [21] devised certain AOs for CFSs.

Software reliability is critical for quality and dependability. However, it is impacted by many technical and managerial parameters related to software development, making it difficult to analyze and optimize. Subjective and qualitative factors also affect reliability measurement, adding uncertainty. Existing reliability models using statistical distributions have limitations in handling real-world complexity. Fuzzy set techniques can address uncertainty but lack multi-dimensional modeling capabilities. This research is motivated by the need for a comprehensive methodology to assess the influence of different software parameters on reliability under uncertainty. CFS can capture nonlinear interactions between decision variables better than traditional fuzzy sets. Integrating complex fuzzy numbers with MCDM provides a promising approach to evaluating alternatives and making optimal choices for maximizing reliability. The proposed technique will enhance decision support during software design and development by consolidating the multiple factors affecting reliability into a single analysis framework. This can guide effective parameter selection to improve software reliability based on quantitative data as well as qualitative insights. The flowchart of the research methodology is shown in Figure 1.

The rest of the manuscript is organized as: In Section II, we review basic notions such as FS, CFS, and linked properties of CFSs. In Section III, we first develop the notion of TTCFN and then devise some elementary operational laws. In Section IV, we propose aggregation operators in the setting of CFS such as TTCFWA and TTCFWG, and indicate their properties. In Section V, we devise the concept of the WASPAS technique in the setting of CFS. Section VI, contains the results and discussion, and Section VII contains the comparative analysis. The concluding remarks are devised in Section 8. The flow chart of the section-wise study is given in Figure 2.

II. PRELIMINARIES

To make the manuscript self-contained this section is dedicated to the basic view of FS, CFS, and linked properties of CFS.

Definition 1 [8]: A FS is a mathematical structure $U_{FS} = \left\{x,\ \delta_{U_{FS}}(x) : x \in X\right\}$, where, the degree of belonging is placed in [0, 1] and identified by $\delta_{U_{FS}}(x)$.

Definition 2 [16]: A CFS is a mathematical structure $U_{CFS} = \left\{x,\ \delta_{U_{CFS}}(x) : x \in X\right\} = \left\{x,\ \delta^{R}_{U_{CFS}}(x) + \iota\, \delta^{I}_{U_{CFS}}(x) : x \in X\right\}$, where degree of belonging $\delta_{U_{CFS}}(x)$ has real part $\delta^{R}_{U_{CFS}}(x)$ and unreal part $\delta^{I}_{U_{CFS}}(x)$ and placed in complex plane's unit square. The CF number (CFN) would be disclosed as

$$U_{CFS} = \left(\delta_{U_{CFS}}\right) = \left(\delta^{R}_{U_{CFS}}(x) + \iota\, \delta^{I}_{U_{CFS}}\right)$$

Definition 3 [21]: Let two CFNs $U_{CFS-1} = \left(\delta_{U_{CFS-1}}\right) = \left(\delta^{R}_{U_{CFS-1}} + \iota\, \delta^{I}_{U_{CFS-1}}\right)$ and $U_{CFS-2} = \left(\delta_{U_{CFS-2}}\right) = \left(\delta^{R}_{U_{CFS-2}} + \iota\, \delta^{I}_{U_{CFS-2}}\right)$ and $w \geq 0$, then

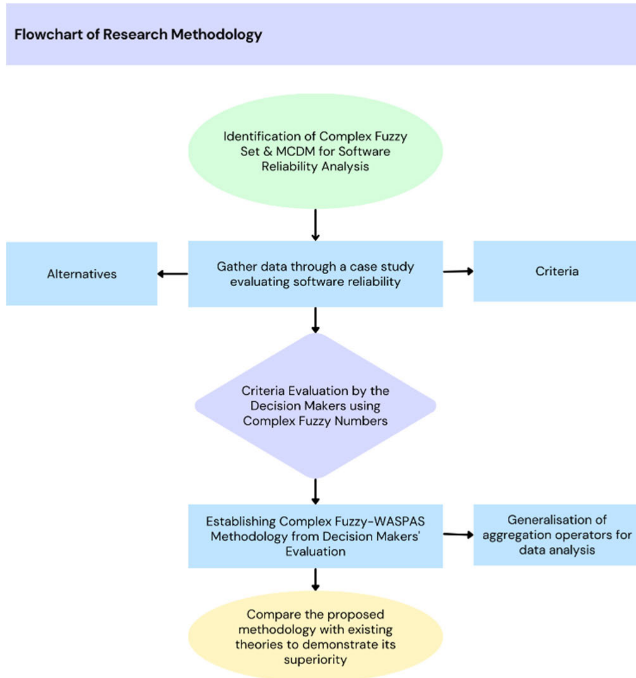

FIGURE 1. **The flowchart of research methodology.**

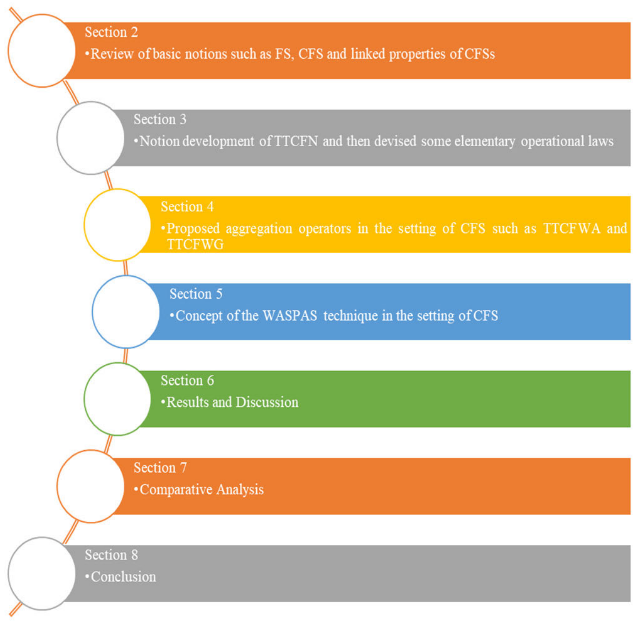

FIGURE 2. **The flowchart of the section-wise study.**

1. $U_{CFS-1} \oplus U_{CFS-2}$
$$= \begin{pmatrix} \left(\delta^{R}_{U_{CFS-1}} + \delta^{R}_{U_{CFS-2}} - \left(\delta^{R}_{U_{CFS-1}} \delta^{R}_{U_{CFS-2}}\right)\right) \\ +\iota(\delta^{I}_{U_{CFS-1}} + \delta^{I}_{U_{CFS-2}} - \left(\delta^{I}_{U_{CFS-1}} \delta^{I}_{U_{CFS-2}}\right) \end{pmatrix}$$

2. $U_{CFS-1} \otimes U_{CFS-2} = \begin{pmatrix} \delta^{R}_{U_{CFS-1}} \delta^{R}_{U_{CFS-2}} \\ +\iota\, \delta^{I}_{U_{CFS-1}} \delta^{I}_{U_{CFS-2}} \end{pmatrix}$

3. $wU_{CFS-1} = \begin{pmatrix} \left(1 - \left(1 - \delta^{R}_{U_{CFS-1}}\right)\right)^{w} \\ +\iota\left(1 - \left(1 - \delta^{I}_{U_{CFS-1}}\right)\right)^{w} \end{pmatrix}$

4. $U^{w}_{CFS-1} = \left(\left(\delta^{R}_{U^{w}_{CFS-1}}\right) + \iota\left(\delta^{R}_{U^{w}_{CFS-1}}\right)\right)$

Definition 4 [21]: For $U_{CFS} = \delta_{U_{CFS}} = (\delta^{R}_{U_{CFS}}(x) + \iota\,\delta^{I}_{U_{CFS}})$ the score and accuracy functions are defined as follows

$$\tilde{Y}(U_{CFS}) = \delta^{R}_{U_{CFS}} - \delta^{I}_{U_{CFS}}, \ \tilde{Y}(U_{CFS}) \in [-1,\ 1] \quad (1)$$

$$\tilde{H}(U_{CFS}) = \frac{\delta^{R}_{U_{CFS}} + \delta^{I}_{U_{CFS}}}{2}, \ \tilde{H}(U_{CFS}) \in [0,\ 1] \quad (2)$$

From Eq. (1) and (2), we have

1. If $\tilde{Y}(U_{CFS-1}) < \tilde{Y}(U_{CFS-2})$ then $U_{CFS-1} < U_{CFS-2}$
2. If $\tilde{Y}(U_{CFS-1}) > \tilde{Y}(U_{CFS-2})$ then $U_{CFS-1} > U_{CFS-2}$
3. If $\tilde{Y}(U_{CFS-1}) = \tilde{Y}(U_{CFS-2})$ then we have $U_{CFS-1} = U_{CFS-2}$
   i. *If* $\tilde{H}(U_{CFS-1}) < \tilde{H}(U_{CFS-2})$ then $U_{CFS-1} < U_{CFS-2}$
   ii. *If* $\tilde{H}(U_{CFS-1}) > \tilde{H}(U_{CFS-2})$ then $U_{CFS-1} > U_{CFS-2}$
   iii. *If* $\tilde{H}(U_{CFS-1}) = \tilde{H}(U_{CFS-2})$ then $U_{CFS-1} = U_{CFS-2}$

III. TTCFN OPERATIONAL LAWS

In this section, we first develop the notion of TTCFN and then devise some elementary operational laws.

Definition 5: Set CFN as $U_{CFS} = (\delta_{U_{CFS}}) = (\delta^{R}_{U_{CFS}}(x) + \iota\,\delta^{I}_{U_{CFS}})$ then TTCFN is defined as follows:

$$tan\,(U_{CFS}) = \left(\tan\left(\frac{\pi}{4}\delta^{R}_{U_{CFS}}\right) + \iota\tan\left(\frac{\pi}{4}\delta^{I}_{U_{CFS}}\right)\right)$$

Observed that the real and unreal parts of the degree of belonging are given as

$$\tan\left(\frac{\pi}{4}\delta^{R}_{U_{CFS}}\right) : Y \to [0,1],\ \tan\left(\frac{\pi}{4}\delta^{I}_{U_{CFS}}\right) : Y \to [0,\ 1]$$

Then, $tan\,(U_{CFS})$ is named the tangent trigonometric operator and its value is TTCFN.

Definition 6: Set two TTCFNs as $tan\,(U_{CFS-1}) = \left(\tan\left(\frac{\pi}{4}\delta^{R}_{U_{CFS-1}}\right) + \iota\tan\left(\frac{\pi}{4}\delta^{I}_{U_{CFS-1}}\right)\right)$ and $tan\,(U_{CFS-2}) = \left(\tan\left(\frac{\pi}{4}\delta^{R}_{U_{CFS-2}}\right) + \iota\tan\left(\frac{\pi}{4}\delta^{I}_{U_{CFS-2}}\right)\right)$ then their operational law is defined as:

1. $tan\,(U_{CFS-1}) \oplus tan\,(U_{CFS-2})$
$$= \begin{pmatrix} \left(1-\tan\left(\frac{\pi}{4}\delta^{R}_{U_{CFS-1}}\right)\right)\left(1-\tan\left(\frac{\pi}{4}\delta^{R}_{U_{CFS-2}}\right)\right) \\ +\iota\left(1-\tan\left(\frac{\pi}{4}\delta^{I}_{U_{CFS-1}}\right)\right)\left(1-\tan\left(\frac{\pi}{4}\delta^{I}_{U_{CFS-2}}\right)\right) \end{pmatrix}$$
2. $tan\,(U_{CFS-1}) \otimes tan\,(U_{CFS-2})$
$$= \begin{pmatrix} \left(\tan\left(\frac{\pi}{4}\delta^{R}_{U_{CFS-1}}\right)\right)\left(\tan\left(\frac{\pi}{4}\delta^{R}_{U_{CFS-2}}\right)\right) \\ +\iota\left(\tan\left(\frac{\pi}{4}\delta^{I}_{U_{CFS-1}}\right)\right)\left(\tan\left(\frac{\pi}{4}\delta^{I}_{U_{CFS-2}}\right)\right) \end{pmatrix}$$
3. $$w.tan\,(U_{CFS-1}) = \begin{pmatrix} 1-\left(1-\tan\left(\frac{\pi}{4}\delta^{R}_{U_{CFS-1}}\right)\right)^{w} \\ +\iota\left(1-\left(1-\tan\left(\frac{\pi}{4}\delta^{I}_{U_{CFS-1}}\right)\right)^{w}\right) \end{pmatrix}$$
4. $$(tan\,(U_{CFS-1}))^{w} = \begin{pmatrix} \left(\tan\left(\frac{\pi}{4}\delta^{R}_{U_{CFS-1}}\right)\right)^{w} \\ +\iota\left(\tan\left(\frac{\pi}{4}\delta^{I}_{U_{CFS-1}}\right)\right)^{w} \end{pmatrix}$$

IV. TANGENT TRIGONOMETRIC COMPLEX FUZZY AGGREGATION OPERATORS

This section proposes aggregation operators in the setting of CFS such as TTCFWA and TTCFWGand indicates their properties.

Definition 7: Let $U_{CFS-g} = \left(\delta^{R}_{U_{CFS-g}} + \iota\,\delta^{I}_{U_{CFS-g}}\right)$ where $(g = 1, 2, \ldots, n)$ be a collection of CFNs and $W = (w_1, w_2, \ldots, w_n)$ be a weight vector with $w_g \in [0, 1]$ and $\sum_{g=1}^{n} w_g = 1$. A TTCFWA operator is developed as

$$TTCFWA\,(U_{CFS-1}, U_{CFS-2}, \ \ldots, \ U_{CFS-n}) = \bigoplus_{g=1}^{n} w_g\left(\tan\left(U_{CFS-g}\right)\right)$$

Theorem 1: Set $U_{CFS-g} = \left(\delta^{R}_{U_{CFS-g}} + \iota\,\delta^{I}_{U_{CFS-g}}\right)$ where $(g = 1, 2, \ldots, n)$ as a collection of CFNs, then the value after using TTCFWA is again a CFN i.e.,

$$TTCFWA\left(U_{CFS-1}, U_{CFS-2}\ldots\ldots U_{CFS-g}\right) = \begin{pmatrix} 1-\prod_{g=1}^{n}\left(1-\tan\left(\frac{\pi}{4}\left(\delta^{R}_{U_{CFS-g}}\right)\right)\right)^{w_g} \\ +\iota\left(1-\prod_{g=1}^{n}\left(1-\tan\left(\frac{\pi}{4}\left(\delta^{I}_{U_{CFS-g}}\right)\right)\right)^{w_g}\right) \end{pmatrix} \quad (3)$$

Proof: We can verify this Theorem 1 in view of the mathematical induction for $n = 2$ we obtain the operational result:

$$TTCFWA\left(\delta_{U_{CFS-1}},\ \delta_{U_{CFS-2}}\right) = \bigoplus_{g=1}^{2} w_g\left(\tan\left(U_{CFS-g}\right)\right)$$
$$= w_1\left(\tan\left(U_{CFS-1}\right)\right) \oplus w_2\left(\tan\left(U_{CFS-2}\right)\right)$$
$$= \begin{pmatrix} \begin{pmatrix} 1-\left(1-\tan\left(\frac{\pi}{4}\left(\delta^{R}_{U_{CFS-1}}\right)\right)\right)^{w_1} \\ +\iota\left(1-\left(1-\tan\left(\frac{\pi}{4}\left(\delta^{I}_{U_{CFS-1}}\right)\right)\right)^{w_1}\right) \end{pmatrix} \\ \oplus\begin{pmatrix} 1-\left(1-\tan\left(\frac{\pi}{4}\left(\delta^{R}_{U_{CFS-2}}\right)\right)\right)^{w_2} \\ +\iota\left(1-\left(1-\tan\left(\frac{\pi}{4}\left(\delta^{I}_{U_{CFS-2}}\right)\right)\right)^{w_2}\right) \end{pmatrix} \end{pmatrix}$$
$$= \begin{pmatrix} 1-\prod_{g=1}^{2}\left(1-\tan\left(\frac{\pi}{4}\left(\delta^{R}_{U_{CFS-g}}\right)\right)\right)^{w_g} \\ +\iota\left(1-\prod_{g=1}^{2}\left(1-\tan\left(\frac{\pi}{4}\left(\delta^{I}_{U_{CFS-g}}\right)\right)\right)^{w_g}\right) \end{pmatrix}$$

Assume that Eq. (3) holds for $n = p$ as follows:

$$TTCFWA\left(U_{CFS-1}, U_{CFS-2}\ldots\ldots U_{CFS-p}\right) = \bigoplus_{g=1}^{p} w_g\left(\tan\left(U_{CFS-g}\right)\right)$$

$$= \begin{pmatrix} 1-\prod_{g=1}^{p}\left(1-\tan\left(\frac{\pi}{4}\left(\delta^{R}_{U_{CFS-g}}\right)\right)\right)^{w_g} \\ +\iota\left(1-\prod_{g=1}^{p}\left(1-\tan\left(\frac{\pi}{4}\left(\delta^{I}_{U_{CFS-g}}\right)\right)\right)^{w_g}\right) \end{pmatrix}$$

For $n = p+1$ we have the following results:

$$TTCFWA\left(U_{CFS-1}, U_{CFS-2} \ldots\ldots U_{CFS-p}, U_{CFS-p+1}\right)$$
$$= \underset{g=1}{\overset{p}{\oplus}} w_g\left(\tan\left(U_{CFS-g}\right)\right) \oplus w_{p+1}\left(\tan\left(U_{CFS-p+1}\right)\right)$$
$$= \left(\begin{pmatrix} 1-\prod_{g=1}^{p}\left(1-\tan\left(\frac{\pi}{4}\left(\delta^{R}_{U_{CFS-g}}\right)\right)\right)^{w_g} \\ +\iota\left(1-\prod_{g=1}^{p}\left(1-\tan\left(\frac{\pi}{4}\left(\delta^{I}_{U_{CFS-g}}\right)\right)\right)^{w_g}\right) \end{pmatrix} \oplus \begin{pmatrix} 1-\left(1-\tan\left(\frac{\pi}{4}\left(\delta^{R}_{U_{CFS-p+1}}\right)\right)\right)^{w_1} \\ +\iota\left(1-\left(1-\tan\left(\frac{\pi}{4}\left(\delta^{I}_{U_{CFS-p+1}}\right)\right)\right)^{w_1}\right) \end{pmatrix}\right)$$
$$= \begin{pmatrix} 1-\prod_{g=1}^{p+1}\left(1-\tan\left(\frac{\pi}{4}\left(\delta^{R}_{U_{CFS-g}}\right)\right)\right)^{w_g} \\ +\iota\left(1-\prod_{g=1}^{p+1}\left(1-\tan\left(\frac{\pi}{4}\left(\delta^{I}_{U_{CFS-g}}\right)\right)\right)^{w_g}\right) \end{pmatrix}$$

Thus Eq. (03) can hold for $n = p+1$ and it can be held for any value of n. Hence, the proof is finished.

Theorem 2: The proposed TTCFWA operator contains the underneath properties based on the tangent trigonometric function

[1] Idempotency: set $U_{CFS-g} = \left(\delta^{R}_{U_{CFS-g}} + \iota\delta^{I}_{U_{CFS-g}}\right) = \left(\delta^{R}_{U_{CFS}} + \iota\,\delta^{I}_{U_{CFS}}\right) = U_{CFS}$ where $(g = 1, 2, \ldots, n)$ there is

$$TTCFWA\left(U_{CFS-1}, U_{CFS-2}, \ldots U_{CFS-g}\right) = \tan(U_{CFS})$$

[2] Boundedness: Set $U^{+}_{CFS} = \max_g\left(\delta^{R}_{U_{CFS-g}}\right) + \iota\max_g\left(\delta^{I}_{U_{CFS-g}}\right)$ and $U^{-}_{CFS} = \min_g\left(\delta^{R}_{U_{CFS-g}}\right) + \iota\min_g\left(\delta^{I}_{U_{CFS-g}}\right)$, then, there is

$$\tan(U^{-}_{CFS}) \leq TTCFWA\left(U_{CFS-g}\right) \leq \tan(U^{+}_{CFS})$$

[3] Monotonicity: Set $U_{CFS} = (\delta^{R}_{U_{CFS}} + \iota\,\delta^{I}_{U_{CFS}})$ and $U^{*}_{CFS} = \left(\delta^{R}_{U^{*}_{CFS}} + \iota\,\delta^{I}_{U^{*}_{CFS}}\right)$ where $(g = 1, 2, \ldots, n)$ are two groups of CFS. Then

$$TTCFWA\left(U_{CFS-1}, U_{CFS-2}, \ldots U_{CFS-g}\right) \leq TTCFWA\left(U^{*}_{CFS-1}, U^{*}_{CFS-2}, \ldots U^{*}_{CFS-n}\right)$$

exist, when

$$U_{CFS-g} \leq U^{*}_{CFS-g}$$

Proof:

[1] For $U_{CFS-g} = U_{CFS}$, we obtain

$$TTCFWA\left(U_{CFS-1}, U_{CFS-2}, \ldots U_{CFS-g}\right)$$
$$= \underset{g=1}{\overset{n}{\oplus}} w_g\left(\tan\left(U_{CFS}\right)\right)$$
$$= \begin{pmatrix} 1-\prod_{g=1}^{n}\left(1-\tan\left(\frac{\pi}{4}\left(\delta^{R}_{U_{CFS-g}}\right)\right)\right)^{w_g} \\ +\iota\left(1-\prod_{g=1}^{n}\left(1-\tan\left(\frac{\pi}{4}\left(\delta^{I}_{U_{CFS-g}}\right)\right)\right)^{w_g}\right) \end{pmatrix}$$
$$= \begin{pmatrix} 1-\left(1-\tan\left(\frac{\pi}{4}\left(\delta^{R}_{U_{CFS-g}}\right)\right)\right)^{\sum_{g=1}^{n} w_g} \\ +\iota\left(1-\left(1-\tan\left(\frac{\pi}{4}\left(\delta^{I}_{U_{CFS-g}}\right)\right)\right)^{\sum_{g=1}^{n} w_g}\right) \end{pmatrix}$$
$$= \begin{pmatrix} \tan\left(\frac{\pi}{4}\left(\delta^{R}_{U_{CFS-g}}\right)\right) \\ +\iota\tan\left(\frac{\pi}{4}\left(\delta^{I}_{U_{CFS-g}}\right)\right) \end{pmatrix}$$
$$= \tan U_{CFS}$$

[2] When $U^{-}_{CFS} \leq U_{CFS-g} \leq U^{+}_{CFS}$, $\tan\left(U^{-}_{CFS}\right) \leq \tan\left(U_{CFS-g}\right) \leq \tan\left(U^{+}_{CFS}\right)$ exist since $\tan(x)$ for $0 \leq x \leq \pi/4$ is an increasing function. Then, there is also

$$\underset{g=1}{\overset{n}{\oplus}} w_g \tan\left(U^{-}_{CFS}\right)$$
$$\leq \underset{g=1}{\overset{n}{\oplus}} w_g \tan\left(U_{CFS-g}\right) \leq \underset{g=1}{\overset{n}{\oplus}} w_g\left(\tan\left(U^{+}_{CFS}\right)\right)$$

Therefore, based on the property [1], there is

$$\tan\left(U^{-}_{CFS}\right) \leq TCFWA\left(U_{CFS-n}\right) \leq \tan\left(U^{+}_{CFS}\right)$$

[3] When $U_{CFS} \leq U^{*}_{CFS}$, there is a $\tan\left(U_{CFS}\right) \leq \tan\left(U^{*}_{CFS}\right)$ since $\tan(x)$ for $0 \leq x \leq \pi/4$ is an increasing function. Then, there is also

$$\underset{g=1}{\overset{n}{\oplus}} w_g\left(\tan\left(U_{CFS-g}\right)\right) \leq \underset{g=1}{\overset{n}{\oplus}} w_g\left(\tan(U^{*}_{CFS-g})\right)$$

can hold in view of the property [2]. Thus, the following attributes exist

$$TCFNWA\left(U_{CFS-1}, U_{CFS-2}, \ldots U_{CFS-g}\right) \leq TCFNWA\left(U^{*}_{CFS-1}, U^{*}_{CFS-2} \ldots U^{*}_{CFS-g}\right)$$

Definition 8: Let $U_{CFS-g} = \left(\delta^{R}_{U_{CFS-g}} + \iota\,\delta^{I}_{U_{CFS-g}}\right)$ where $(g = 1, 2, \ldots, n)$ be a collection of CFNs and $W = (w_1, w_2, \ldots, w_n)$ be a weight vector with $w_g \in [0, 1]$ and $\sum_{g=1}^{n} w_g = 1$. A TTCFWG operator is developed as

$$TTCFWG\left(U_{CFS-1}, U_{CFS-2}, \ldots, U_{CFS-n}\right)$$
$$= \underset{g=1}{\overset{n}{\otimes}} \left(\tan\left(U_{CFS-g}\right)\right)^{w_g}$$

Theorem 3: Set $U_{CFS-g} = \left(\delta^R_{U_{CFS-g}} + \iota\, \delta^I_{U_{CFS-g}}\right)$ where $(g = 1, 2, \ldots, n)$ as a collection of CFNs, then the value after using TTCFWG is again a CFN i.e.,

$$TTCFWG\left(U_{CFS-1}, U_{CFS-2}, \ldots, U_{CFS-n}\right) = \left(\begin{array}{c} \prod_{g=1}^{n} \left(\tan\left(\frac{\pi}{4}\delta^R_{U_{CFS-g}}\right)\right)^{w_g} \\ +\iota \left(\prod_{g=1}^{n} \left(\tan\left(\frac{\pi}{4}\delta^I_{U_{CFS-g}}\right)\right)^{w_g}\right) \end{array} \right)$$

In view of the similar proof process of theorem 1, we can easily verify the Theorem 3, which is omitted intentionally.

Theorem 4: The proposed TTCFWG operator contains the underneath properties based on the tangent trigonometric function

[1] Idempotency: set $U_{CFS} = \left(\delta^R_{U_{CFS-g}} + \delta^I_{U_{CFS-g}}\right)$ where $(g = 1, 2, \ldots, n)$ there is

$$TTCFWG\left(\delta_{U_{CFS-1}}, \delta_{U_{CFS-2}} \ldots\ldots \delta_{U_{CFS-g}}\right) = \tan\left(\delta^R_{U_{CFS-g}} + \delta^I_{U_{CFS-g}}\right)$$

[2] Boundedness: Set $U^+_{CFS} = \max_g \left(\delta^R_{U_{CFS-g}} + \delta^I_{U_{CFS-g}}\right)$ and $U^-_{CFS} = \min_g \left(\delta^R_{U_{CFS-g}} + \delta^I_{U_{CFS-g}}\right)$

[3] Monotonicity: Set $U_{CFS} = \left(\delta^R_{U_{CFS-g}} + \delta^I_{U_{CFS-g}}\right)$ and $U^*_{CFS} = \left(\delta^R_{U^*_{CFS-g}} + \delta^I_{U^*_{CFS-g}}\right)$ are two groups of CFS. Then $TTCFWG\left(\delta_{U_{CFS-1}}, \delta_{U_{CFS-2}} \ldots\ldots \delta_{U_{CFS-n}}\right) \leq TTCFWA\left(\delta_{U^*_{CFS-1}}, \delta_{U^*_{CFS-2}} \ldots\ldots \delta_{U^*_{CFS-g}}\right)$ exist when $U_{CFS} \leq U^*_{CFS}$.

The proof process of Theorem 4 is similar to that of Theorem 2. Therefore, it has been omitted intentionally.

V. WASPAS APPROACH WITHIN COMPLEX FUZZY INFORMATION

This section demonstrates the WASPAS method in the environment of complex fuzzy by employing the invented operators.

Let's assume a set comprises over n alternatives $K = \{K_1, K_2, \ldots, K_n\}$ and a set having several criteria $C = \{C_1, C_2, \ldots, C_m\}$ related to a decision-making problem where the decision-maker assesses each option based on the considered criteria. Because of the significance of each criterion, the decision-maker provides weight $W = (w_1, w_2, \ldots, w_m)$ $w_f \in [0, 1]$ and $\sum_{f=1}^{m} w_f = 1$ to the criteria according to his/her preference. The assessment values of the alternatives would be in the model of CFNs that is $U_{CFS} = \left(\delta^R_{U_{CFS-gf}} + \delta^I_{U_{CFS-gf}}\right)_{n\times m}$ which would construct a complex fuzzy decision matrix D. To cope with this decision-matrix we have the underneath WASPAS technique.

A. COMPLEX FUZZY WASPAS (CF-WAPAS) TECHNIQUE

This technique has the following steps.

Step 1: Because of the cost and benefit types of attributes, the first step is to standardize the complex fuzzy decision matrix D. This would be done by following the formulas

$$\mathrm{U}_{\mathrm{CFS}-\mathrm{g}} = \frac{\mathrm{U}_{\mathrm{CFS-gf}}}{\max_{\mathrm{g}} \mathrm{U}_{\mathrm{CFS-gf}}} = \left(\frac{\delta^{\mathrm{R}}_{\mathrm{U_{CFS-gf}}} + \iota\, \delta^{\mathrm{I}}_{\mathrm{U_{CFS-gf}}}}{\max_{\mathrm{g}}\left(\delta^{\mathrm{R}}_{\mathrm{U_{CFS-gf}}} + \iota\, \delta^{\mathrm{I}}_{\mathrm{U_{CFS-gf}}}\right)}\right) \tag{4}$$

where,

$$\begin{aligned} &\max_{\mathrm{g}}\left(\delta^{\mathrm{R}}_{\mathrm{U_{CFS-gf}}} + \iota\, \delta^{\mathrm{I}}_{\mathrm{U_{CFS-gf}}}\right) \\ &\quad = \max_{\mathrm{g}} \delta^{\mathrm{R}}_{\mathrm{U_{CFS-gf}}} + \iota \max_{\mathrm{g}} \delta^{\mathrm{I}}_{\mathrm{U_{CFS-gf}}} \mathrm{U}_{\mathrm{CFS-gf}} \\ &\quad = \frac{\min_{\mathrm{g}} \mathrm{U}_{\mathrm{CFS-gf}}}{\mathrm{U}_{\mathrm{CFS-gf}}} = \left(\frac{\min_{\mathrm{g}}\left(\delta^{\mathrm{R}}_{\mathrm{U_{CFS-gf}}} + \iota\, \delta^{\mathrm{I}}_{\mathrm{U_{CFS-gf}}}\right)}{\delta^{\mathrm{R}}_{\mathrm{U_{CFS-gf}}} + \iota\, \delta^{\mathrm{I}}_{\mathrm{U_{CFS-gf}}}}\right) \end{aligned} \tag{5}$$

The division of Eq. (4) and Eq. (5) would be performed as follows

$$\frac{\mathrm{U}_{\mathrm{CFS}-1}}{\mathrm{U}_{\mathrm{CFS}-2}} = \left(\begin{array}{c} \dfrac{\delta^{\mathrm{R}}_{\mathrm{U_{CFS-1}}}\delta^{\mathrm{R}}_{\mathrm{U_{CFS-2}}}}{\delta^{\mathrm{R}}_{\mathrm{U_{CFS-1}}} + \delta^{\mathrm{R}}_{\mathrm{U_{CFS-2}}} - \delta^{\mathrm{R}}_{\mathrm{U_{CFS-1}}}\delta^{\mathrm{R}}_{\mathrm{U_{CFS-2}}}} \\ +\iota \dfrac{\delta^{\mathrm{I}}_{\mathrm{U_{CFS-1}}}\delta^{\mathrm{I}}_{\mathrm{U_{CFS-2}}}}{\delta^{\mathrm{I}}_{\mathrm{U_{CFS-1}}} + \delta^{\mathrm{I}}_{\mathrm{U_{CFS-2}}} - \delta^{\mathrm{I}}_{\mathrm{U_{CFS-1}}}\delta^{\mathrm{I}}_{\mathrm{U_{CFS-2}}}}, \end{array}\right) \tag{6}$$

where $\mathrm{U}_{\mathrm{CFS}-1}$ and $\mathrm{U}_{\mathrm{CFS}-2}$ are CFNs.

Step 2: Employing TTCFWA operator to determine the importance of each alternative based on the WSM

Step 3: Employing TTCFWG operator to determine the importance of each alternative based on the WPM

Step 4: After applying both the sum and the product operations. This method combines the advantages of both operations to obtain the final importance of each alternative by the following align:

$$\tilde{Y}(K_g) = 0.5\mathrm{WSM}\left(K_g\right) + 0.5\mathrm{WPM}\left(K_g\right) \tag{7}$$

Step 5: Rank the alternatives based on their score values. The alternative with the highest score is considered the best choice.

Step 6: Determining the stability of the acquired results is crucial for demonstrating the efficacy of an MCDM-based framework. The weights assigned to the selection criteria have a significant impact on how the alternatives rank. Small adjustments to the weights of the selection criteria may have an impact on the stability of the ranks. We used the technique outlined in [22] to conduct a sensitivity analysis in order to verify the consistency of the findings. The weights of the selection criteria are changed independently by 10% to 30% in order to examine the performance sensitivity of the alternatives.

B. RESULTS AND DISCUSSION OF CASE STUDY

In this case study, data was collected to evaluate software reliability across four alternatives - software modules, operational profiles, testing techniques, and development teams. Four criteria were selected - failure intensity, accuracy, failure rate, and fault tolerance. To capture the multidimensional aspects, a CFS was used. The real part quantified the probability of failure-free operation i.e. reliability. The imaginary part measured defect density which indicates quality. i.e. $U_{CFS-gf} = \left(\delta^R_{U_{CFS-gf}} + \iota\, \delta^I_{U_{CFS-gf}}\right)$. Data was synthesized through rigorous testing and defect tracking across the execution cycles. Failures were mapped to modules, operational profiles, test techniques, etc. Defect density was calculated per line of code. The probability of failure-free operation was computed from reliability models like execution time, failures observed, etc. The proposed WASPAS approach within CFS is applied to evaluate software parameters for maximizing reliability.

TABLE 1 shows the software reliability matrix of alternatives over criteria. The data is expressed in terms of CFNs where the real part (a) represents reliability measured by the probability of failure-free operation and the imaginary part (b) represents reliability measured by defect density. In the data expressed in TABLE 2. it can be observed that the development teams have the highest failure intensity, testing techniques have the lowest failure rate, and the operational profiles have the lowest fault tolerance. The following section discusses and analyzes these data in detail.

TABLE 2 shows the standardized data matrix, here standardization and weighting allow transforming raw criteria data into comparable scales and integrating decision-maker preferences into the analysis.

Weights are assigned to each criterion based on its relative importance. The weights sum to 1. Criteria like failure rate and failure intensity are given higher weights, indicating their higher importance for software reliability. Criteria like accuracy and fault tolerance have lower weights, suggesting they are less critical factors. The assigned weights reflect the decision maker's preferences and priorities regarding the criteria. Multiplying the standardized values with criteria weights incorporates the relative importance of each criterion in the analysis.

The approach provides a robust analysis to capture software reliability. WSM stands for "Weighted Sum Model". Table 4, shows the importance of each alternative based on the WSM and WPM.

The overall importance of each alternative is devised in Table 5, where (a) signifies the real part and (b) signifies the unreal part of a CFN. They provide a more realistic representation of each alternative's performance with incorporated uncertainty. The alternatives are ranked based on these scores to identify the most reliable software module.

The top-ranked alternative has the best tradeoff between the deterministic score and the uncertainty, based on the computations the score values against the alternatives are mentioned in TABLE 6 and Figure 3. The score values obtained indicate that software modules received the highest positive score (0.00996), followed by operational profiles (−0.00144), testing techniques (−0.00243), and development teams (−0.00146). The positive score for software modules highlights their importance as a key parameter for enhancing software reliability. This aligns with previous research showing modular software design can contain failures and support fault isolation. The modular structure also allows easier testing, maintenance and troubleshooting. However, while important, relying solely on software modules is insufficient. Operational profiles received the second-highest score, indicating they also play a role in software reliability. Well-defined operational profiles allow testing to better reflect real-world usage scenarios. However, challenges in accurately capturing dynamic production environments mean operational profiles alone are inadequate. Comparatively lower scores were obtained for testing techniques and development teams. While improved testing methodologies can reveal failures earlier, exhaustively testing all paths and inputs is infeasible for complex systems. Development teams follow established processes, but human errors still occur. Comprehensive testing and best practices are beneficial but cannot fully prevent reliability issues.

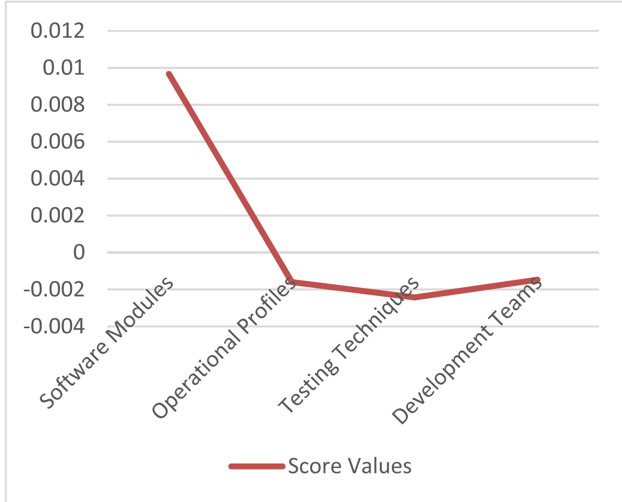

FIGURE 3. **The determined score values of alternative.**

Therefore, the results suggest that focusing on software modules provides the most impact, but needs to be complemented by the other alternatives to holistically maximize software reliability. A balanced approach across all four parameters is recommended, rather than over-emphasizing any single factor. Further studies could explore relative weightings to quantify the contribution of each alternative. Overall, the complex fuzzy-WASPAS method has demonstrated its suitability for handling the imprecision and subjectivity inherent in this multi-criteria decision problem.

The complex fuzzy formulation allowed the consolidation of these quantitative and qualitative metrics into a single model. The data collection focused on failure monitoring as well as defect analysis. Trends were observed on how

TABLE 1. Software reliability matrix of alternatives over criteria's.

	Failure intensity		Accuracy		Failure rate		Fault tolerance	
	(a)	**(b)**	**(a)**	**(b)**	**(a)**	**(b)**	**(a)**	**(b)**
Software Modules	0.174	0.771	0.432	0.669	0.803	0.508	0.147	0.61
Operational Profiles	0.187	0.641	0.456	0.618	0.982	0.492	0.342	0.242
Testing Techniques	0.288	0.412	0.459	0.599	0.99	0.304	0.374	0.554
Development Teams	0.82	0.311	0.562	0.505	0.998	0.433	0.511	0.464

TABLE 2. Standardized data matrix.

	Failure Intensity		Accuracy		Failure rate		Fault tolerance	
Software Modules	0.168	0.627	0.323	0.503	0.802	0.34	0.129	0.439
Operational Profiles	0.18	0.538	0.336	0.473	0.98	0.333	0.258	0.21
Testing Techniques	0.271	0.367	0.338	0.462	0.988	0.235	0.275	0.409
Development Teams	0.695	0.285	0.391	0.404	0.996	0.305	0.343	0.358

TABLE 3. Computed weights.

	Failure intensity	Accuracy	Failure rate	Fault tolerance
Software Modules	0.252141593	0.232037735	0.338218394	0.177602277
Operational Profiles	0.225648729	0.230827728	0.411494116	0.132029427
Testing Techniques	0.179786337	0.225629144	0.400228062	0.194356456
Development Teams	0.263444523	0.197198447	0.365425277	0.173931754

TABLE 4. Computed WSM & WPM.

	WSM		WPM	
Software Modules	0.996360	0.998860	0.262847	0.375377
Operational Profiles	0.999776	0.997467	0.381302	0.308305
Testing Techniques	0.999883	0.996973	0.418394	0.266395
Development Teams	0.999989	0.996099	0.557668	0.261624

TABLE 5. Operational values of alternatives.

	(a)	(b)
Software Modules	0.109363	0.067373
Operational Profiles	0.04558	0.092802
Testing Techniques	0.041485	0.101338
Development Teams	0.034963	0.112584

TABLE 6. Score values of alternatives.

	Score Values
Software Modules	0.00996
Operational Profiles	-0.00144
Testing Techniques	-0.00243
Development Teams	-0.00146

failure intensity and defect density changed over time for the alternatives. This provided insights into the reliability behaviors. The case study demonstrated the feasibility of gathering multifaceted data for software reliability measurement using complex fuzzy numbers. However, more streamlined processes may be needed for large projects. Automated data

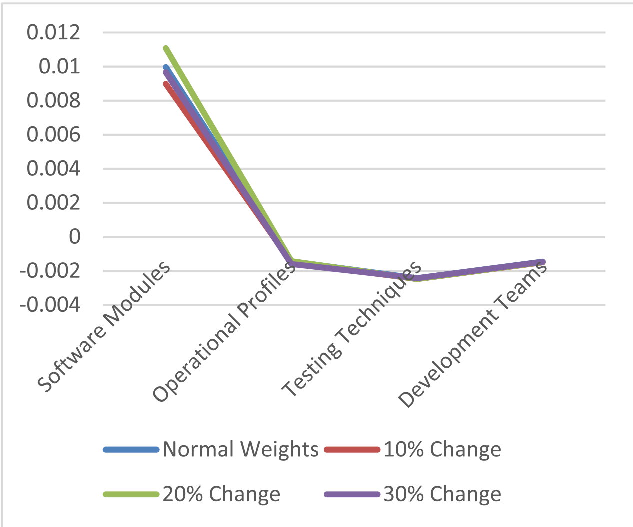

FIGURE 4. The sensitivity analysis of determined score values.

collection tools could help with managing the scale while retaining the integrated metrics. This study demonstrated the application of complex fuzzy numbers for MCDM in software reliability assessment. The key criteria considered were failure intensity, accuracy, failure rate, and fault tolerance. These encompass both quantitative metrics and qualitative aspects relevant to reliability measurement. The alternatives evaluated were software modules, operational profiles, testing techniques, and development teams. The data showed that testing techniques had the highest failure intensity while operational profiles had the lowest fault tolerance. This indicates testing techniques are most impacting in revealing failures early which is positive. However, operational profiles need to be enhanced to improve fault tolerance. Development teams had a tradeoff between high failure intensity and also highest accuracy.

Overall, the complex fuzzy number approach allowed the consolidation of the multidimensional criteria into a single model for comparison. The subjective and conflicting nature of reliability measurement was handled through the real and imaginary components. This provides a more nuanced evaluation than using single-dimension criteria.

Future research can validate this approach on real software projects and with larger criteria and alternative sets. Comparative analysis with other multi-criteria methods like AHP, TOPSIS, etc. will also be relevant.

VI. COMPARATIVE ANALYSIS

The proposed approach integrates complex fuzzy sets with the weighted aggregated sum product assessment (WASPAS) method to evaluate software reliability parameters. Tangent Trigonometric complex fuzzy aggregation operators provide a way to aggregate fuzzy information and deal with uncertainties in data. This is useful for parameter estimation problems where the data contains ambiguities or imprecision. In our case study there are four criteria's and all such criteria

Algorithm 1 Implementation Algorithm for Python Platform

1. Import necessary modules: cmath, math, itertools
2. Define input data as a 2D list of complex numbers
3. Extract real and imaginary parts into separate 2D lists
4. For each column:
  - Extract column from real and imaginary parts
  - Find min ans for real and imaginary maxima using column maximad max of each column
  - Store in separate lists for real and imaginary parts
5. Create 2D list
6. Combine real and imaginary maxima lists into 2D list of complex numbers (complex max matrix)
7. Repeat steps 4-6 for minima instead of maxima (complex min matrix)
8. Print complex max and min matrices
9. Normalize input data:
  - For each element:
    - Divide real part by corresponding real max
    - Divide imaginary part by corresponding imaginary max
  - Store normalized values in 2D list
10. Print normalized matrix
11. Compute row weights:
    - Sum absolute values in each row
    - Divide each element by row sum to get weights
    - Store weights in 2D list
12. Print weights
13. Compute significance scores:
  - For each row:
    - Extract real and imaginary parts
    - Compute product of tangent raised to weight power for each
    - Subtract from 1 to get score
  - Store scores as complex numbers
14. Print significance scores

reflecting both probability of failure-free operation and defect density.

These operators utilize trigonometric functions like sine, cosine, tangent etc. to map the fuzzy inputs into a range of $[-1, 1]$. This normalized range allows easier mathematical operations and aggregation of fuzzy sets. Here, tangent function used in these operators also captures both positive and negative correlations in the data, making the aggregated results more reasonable. This is important for parameter estimation where both positive and negative relationships exist. Furthermore, compared to simpler weighted averaging operators, tangent trigonometric complex fuzzy aggregation operators consider interaction and correlation between input variables. This avoids loss of information and improves accuracy of estimated parameters. Keeping in view, the proposed methodology has several enhancements that is expressed in TABLE 7:

The mean absolute percentage error (MAPE) between the actual and estimated fuzzy data sets was calculated to be 19.51% using the Algo (2) implemented in Python.

Algorithm 2 Compute MAPE forActual and Normalized Fuzzy Data

INPUT:

data1[][]: 2D array containing actual fuzzy data
data_norm[][]: 2D array containing estimated/normalized fuzzy data

STEPS:

Initialize empty list errors to store absolute percentage errors
Loop through each row i in data1:
2.1 Loop through each element j in row i:
2.1.1 Get actual value a = data1[i][j]
2.1.2 Get estimated value e = data_norm[i][j]
*2.1.3 Compute absolute percentage error = (|a - e|)/ |a| * 100*
2.1.4 Append computed error to list errors

Calculate mean of all errors in list:
mape = sum(errors) / length(errors)

Print computed MAPE value

TABLE 7. Comparison of standard and proposed complex fuzzy WASPAS method.

Basis for Comparison	Standard fuzzy WASPAS	Complex fuzzy WASPAS
Methodology	Uses fuzzy set theory to handle uncertainty in criteria weights and performance ratings	Uses complex fuzzy set theory to handle uncertainty in criteria weights and performance ratings
Nature of data	Real-valued, fuzzy/uncertain data	Real-valued, fuzzy data and Complex- Fuzzy data
Criteria weights	Determined using fuzzy numbers	Determined using complex fuzzy numbers
Information representation	Less informative, handles real-valued uncertainty	More informative, handles complex uncertainty involving real and imaginary terms
Computation	Simpler, uses standard fuzzy aggregation operations	More complex, uses complex fuzzy aggregation operations
Result interpretation	Easier to interpret crisp ranking order	Harder to interpret crisp ranking order due to complex operations
Suitability	Well-suited for problems with fuzzy/real-valued data	Better suited for problems with oscillating, periodic or wave-like complex data

This provides a quantitative measure of the error or accuracy of the parameter estimation method used in the study. The relatively low MAPE value indicates that the parameter estimation technique based on tangent trigonometric complex fuzzy aggregation operators and WASPAS had a reasonably good performance. It was able to estimate the complex fuzzy parameter within about 20% of the actual observed values on average. While there is still scope to further reduce the errors, the obtained MAPE is encouraging and demonstrates the applicability of the proposed approach for parameter estimation tasks involving ambiguous, vague data. Overall, the low MAPE highlights the benefits of using tangent trigonometric complex fuzzy aggregation operators over traditional methods. In future work, we can explore fine-tuning the operator parameters or trying different fuzzy set representations to further minimize the estimation errors. However, the current results provide a proof of concept and confirm the superiority of the proposed approach.

VII. CONCLUSION

This work presented an innovative method of MCDM for software reliability analysis involving complex fuzzy sets combined with the WASPAS technique. In particular, the tangent trigonometric complex fuzzy aggregation operators were proposed as an efficient way to take into account the complexities and uncertainties involved in the relationships between the software reliability parameters. The operators were used to determine four main alternatives such as software modules, operational profiles, testing methods, and development teams that were evaluated based on failure intensity, correctness, failure rate, and fault tolerance. The software modules drew the most positive responses from the participants, indicating their high importance in the context of reliability. On the other hand, all parameters must be given equal weight rather than the modules being the main focal point. The complex fuzzy formulation was used to tie the multidimensional quantitative and qualitative criteria into a single integrated model. It facilitated a more detailed and complex analysis of the uncertainty alternatives than the standard methods. The suggested technique expands the power of standard fuzzy WASPAS via a hybrid model that can take into account the weighted sum and product models, use trigonometric computing, and handle higher dimensionality. It offers an advanced decision support methodology that can deal with the imprecise data and complexity present in the software reliability assessment.

A. LIMITATIONS AND FUTURE DIRECTION

The research can't cope with information that is in the generalized form of CFS such as bipolar complex fuzzy set [23], and hesitant bipolar complex fuzzy sets [24]. This work also can't cope with the negative aspect that is the concept of bipolar fuzzy set [25]. That's why, in the future, we aim to expand this work in these domains.

Further, future research can be validated through real-world case studies across different domains. Comparative analysis with other MCDM techniques like AHP, TOPSIS, etc. will also be relevant. The complex fuzzy aggregation operators can be further enhanced by exploring different membership functions and decision weight models. Opportunities exist to apply the developed approach for software quality factors beyond reliability. As software systems continue to grow more complex, the proposed methodology can provide an effective solution for MCDM under uncertainty.

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