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RESEARCH ARTICLE

Integral Power Theory and Active Filtering Using the α-β Reference Frame

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ABSTRACT The paper describes a new version of the integral power theory in the α-β reference frame, which combines the advantages of the well-known instantaneous and integral power theories, and a shunt active filtering strategy based on it. From *p-q* power theory, it takes the α-β reference frame and separation of instantaneous powers on average and oscillating components. The three-phase current decomposition of integral Currents' Physical Components power theory in the frequency domain was improved by dividing the unbalance power component into two orthogonal components with separate integral unbalance powers. The method is proposed for determining these two integral unbalance power components using double fundamental frequency functions. This method was practically implemented by performing real-time algebraic operations with the instantaneous power oscillating components and α-β voltage coordinates synchronized with a three-phase network. Based on this current decomposition in α-β reference frame, the compensation three-component signal for shunt active filtering is proposed, each component of which is proportional to a separate integral inactive power. However, to save voltage and current sensors and avoid the need for a voltage reference point, the input and output information for the shunt active filter control system is set in the two-wattmeter reference frame. The proposed control system successfully passed the verification by computer simulation with selective compensation of the inactive power components. The suggested compensation algorithm has shown its effectiveness and reliability for a linear and even a non-linear load.

INDEX TERMS CPC power theory, *p-q* power theory, α-β reference frame, power component decomposition, unbalance power, reactive power control, distributed active filtering, selective power compensation, power quality.

I. INTRODUCTION

The increasing number of non-linear powerful consumers asymmetrically connected to the power supply system leads to a deterioration in the power quality and increasing power losses. The most effective hardware for improving power quality in points of common coupling is shunt active filters (SAFs), which improve the harmonic content of the consumed currents and compensate for inactive power components, minimizing power losses in the transmission line [1], [2], [3]. These works also aim to assess the most advanced SAFs by reducing the number of power switches and focusing on reducing grid-connected inverters' cost, size, and weight. SAF control strategies are based on various versions of power theory, which has been evolving for over a century and a half, an overview of which is presented in [4] and [5]. They can be conditionally divided into instantaneous and integral power theories. Instantaneous power theories were initiated by H. Akagi et al. [6], who proposed an algorithm for compensating the instantaneous imaginary

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power of a three-phase, three-wire system in the α-β reference frame by applying the Clarke transformation. Further improvements and modifications of the instantaneous power theories, a comparative analysis of which was carried out in [7], concerned its extension to a three-phase four-wire system by transferring to the α-β-0 [8], p-q-r [9], d-q-0 reference frames [10]. Cross-vector [11] and vectorial [7] instantaneous power theories that do not require coordinate axes have also been proposed. The main advantages of active filtering based on instantaneous power theories are real-time compensation without time delay and reduced capacitance of energy storage elements. However, using integral power theories provides higher power factor values, as it justifies the expression for the active current of the power system according to the Fryze concept [12], [13], which minimizes the power loss in the transmission line. Integral power theories acquired more advanced forms within the framework of Fryze-Buchholz-Depenbrock Power Theory (FBD Theory [14]), Currents' Physical Components Power Theory (CPC Theory by L. Czarnecki [15]), Conservative Power Theory (CPT by Tenti and Mattavelli [16]), a comparative analysis of which was conducted in [17]. These theories use different methods to decompose the load current into components proportional to the separate components of the apparent power. Its implementation in the transmission line using shunt active filtering is not difficult [18], [19], and attempts are also being made to predictively control the SAF in transient operating modes [20], [21], [22] when many different loads and generating units can be simultaneously switched on or off. In such cases, active filters are a means of mitigating transients by partially storing energy in the DC links.

New ideas and realities of electric energy generation, transmission, and distribution such as distributed generation (DG) and reactive power control, generate new requirements for active filtering strategies. SAFs are relied on to generate a given amount of reactive power to stabilize the voltage in transmission lines and increase the overall stability of distributed generation grids [23], [24], [25]. The main idea of distributed active filtering is to compensate for all microgrid current inactive components, mainly with embedded semiconductor converters of DG supplies. As a result, the energy storage capability of input microgrid SAF decreases [26], [27], [28], [29]. The problem arises in establishing the particle of inactive power that falls on each converter and its identification in compensation currents. This requires revising well-known active filtering strategies to obtain unified high-speed SAF control strategies that operate in autonomous mode and as part of DG semiconductor converters, selectively compensating for inactive powers. In particular, the classical p-q theory of instantaneous power justifies the compensation of only one three-phase current component, which is proportional to the instantaneous inactive power and is inferior to integral power theories by the value of the power factor. Also, it gives rise to the problem of the third harmonic under asymmetric load conditions.

The goal of this paper is to substantiate the power theory, which combines the advantages of instantaneous and integral power theories by decomposing three-phase currents of a three-wire power system into four mutually orthogonal components, one of which is an active current, and the other three are proportional to inactive integral powers, that are instantaneously determined and independently compensated by different SAFs. This decomposition was carried out in the α-β reference frame to reduce the dimensionality of the current and voltage vectors that describe the system under study and to simplify the construction of the SAF control system.

In the rest of the paper, Section II locates specific provisions of the classical p-q theory of instantaneous power and CPC power theory, on which the proposed integral power theory is based, and highlights the shortcomings of these theories that are overcome within the framework of the new approach. Section III substantiates the proposed current decomposition within the new power theory into four mutually orthogonal components with integral powers in the α-β reference frame, one of which is an active current, and the other three are proportional to inactive integral powers that can be independently compensated by semiconductor converters DG. Also, this Section proposes the method for determining two unbalance integral powers from instantaneous power oscillating components and α-β voltage coordinates. Section IV describes the practical implementation of the theoretical provisions of the previous section in the form of the shunt active filter scheme with a three-component structure of the compensation signal in the α-β reference frame, each component of which is proportional to a separate inactive power. Section V presents the simulation results, showing the effectiveness of the proposed method of real-time SAF control with selective compensation of integral inactive powers for linear and nonlinear load, ensuring minimal power losses in the transmission line with symmetric quasi-sinusoidal consumed currents.

II. THEORETICAL PREMISES OF THE INTEGRAL POWER THEORY CONSTRUCTION IN THE α-β REFERENCE FRAME

A. ELEMENTS OF INTEGRAL POWER THEORY IN THE INSTANTANEOUS REACTIVE POWER THEORY

The electrical variables of a three-phase, three-wire power supply system in the ABC reference frame are linearly dependent. Namely, for supply currents

$$i_A(t) + i_B(t) + i_C(t) = 0 \tag{1}$$

according to Kirchhoff's current law, a similar equation is valid for the phase voltages referenced to an artificial zero (from now on, we omit the notation of the time dependence for vector coordinates):

$$u_A + u_B + u_C = 0. \tag{2}$$

Therefore, the transformation to the α-β reference frame using the reduced Clarke matrix [30].

$$\mathbf{C} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix},$$

namely

$$\mathbf{i}(t) = \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \mathbf{C} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix}; \ \mathbf{u}(t) = \begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = \mathbf{C} \begin{bmatrix} u_A \\ u_B \\ u_C \end{bmatrix}$$

is a natural way to operate with two-coordinate vectors of linearly independent variables. Then, the real power

$$p(t) = u_\alpha i_\alpha + u_\beta i_\beta = u_A i_A + u_B i_B + u_C i_C \tag{3}$$

is the instantaneous active power and instantaneous imaginary power

$$q(t) = u_\beta i_\alpha - u_\alpha i_\beta \tag{4}$$

is a measure of the disproportionality of the instantaneous vectors $\mathbf{i}(t)$ and $\mathbf{u}(t)$. Equation (4) for imaginary power is presented so that its value, averaged over the period of symmetrical network voltages, corresponds to the reactive power [30]. Transformation to the matrix notation of the equation system (3), (4)

$$\begin{bmatrix} p(t) \\ q(t) \end{bmatrix} = \begin{bmatrix} u_\alpha & u_\beta \\ u_\beta & -u_\alpha \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}$$

and the reverse transformation to the current vector

$$\begin{aligned} \mathbf{i}(t) &= \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} u_\alpha & u_\beta \\ u_\beta & -u_\alpha \end{bmatrix}^{-1} \begin{bmatrix} p(t) \\ q(t) \end{bmatrix} = \\ &= \frac{p(t)}{u_\alpha^2 + u_\beta^2} \begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} + \frac{q(t)}{u_\alpha^2 + u_\beta^2} \begin{bmatrix} -u_\beta \\ u_\alpha \end{bmatrix} = \\ &= \frac{p(t)}{u^2(t)} \mathbf{u}(t) + \frac{q(t)}{u^2(t)} \mathbf{u}_q(t) \end{aligned} \tag{5}$$

allows its decomposition into components proportional to instantaneous powers. The reference voltage vectors $\mathbf{u}(t)$, $\mathbf{u}_q(t)$ have the same norms

$$u = \sqrt{\mathbf{u}(t)^\wedge \mathbf{u}(t)} = \sqrt{\mathbf{u}(t)_q^\wedge \mathbf{u}_q(t)} = \sqrt{u_\alpha^2 + u_\beta^2},$$

where ^ is the transposition sign. These vectors are mutually orthogonal because $\mathbf{u}(t)^\wedge \mathbf{u}_q(t) = 0$, so the equation for the currents' norm is valid:

$$\begin{aligned} i^2(t) = \mathbf{i}^\wedge(t)\,\mathbf{i}(t) &= \frac{p^2(t)}{u^4(t)} \mathbf{u}^\wedge(t)\,\mathbf{u}(t) + \\ &+ \frac{q^2(t)}{u^4(t)} \mathbf{u}_q^\wedge(t)\,\mathbf{u}_q(t) = \frac{p^2(t) + q^2(t)}{u^2(t)}. \end{aligned} \tag{6}$$

This leads to the expressions [31] for the instantaneous apparent power $s(t) = u(t)i(t)$ and the decomposition of its square:

$$s^2(t) = p^2(t) + q^2(t). \tag{7}$$

The first term of the current decomposition (5)

$$\mathbf{i}_p(t) = \frac{p(t)}{u^2(t)} \mathbf{u}(t) \tag{8}$$

is the instantaneous active current [11], it provides the same instantaneous active power as the whole current (5), minimal instantaneous power losses in the transmission line, and a unity value of the instantaneous power factor $\lambda(t) = p(t)/s(t)$.

Further reduction of power losses in the transmission line is associated with compensation of instantaneous powers oscillating components [30].

$$\tilde{p}(t) = p(t) - P; \tilde{q}(t) = q(t) - Q, \tag{9}$$

where P and Q are integral active and reactive powers. Substitution of (9) into (5) allows us to identify four current components

$$\begin{aligned} \mathbf{i}(t) &= \frac{P + \tilde{p}(t)}{u^2(t)} \mathbf{u}(t) + \frac{Q + \tilde{q}(t)}{u^2(t)} \mathbf{u}_q(t) = \\ &= \frac{P}{u^2(t)} \mathbf{u}(t) + \frac{\tilde{p}(t)}{u^2(t)} \mathbf{u}(t) + \\ &+ \frac{Q}{u^2(t)} \mathbf{u}_q(t) + \frac{\tilde{q}(t)}{u^2(t)} \mathbf{u}_q(t) \end{aligned} \tag{10}$$

Assuming a symmetrical sinusoidal three-phase source

$$u^2(t) = u_\alpha^2 + u_\beta^2 = u_A^2 + u_B^2 + u_C^2 = 3U^2 = V^2,$$

where U is the RMS value of the source phase voltages, $V = \sqrt{3}U$ is the RMS value of the source line voltages. Then, all four current components (10) are mutually orthogonal concerning the integral scalar product. Hence, the square of the current integral norm will be as follows

$$\begin{aligned} I^2 &= T^{-1} \int_T \mathbf{i}^\wedge(t)\,\mathbf{i}(t)\,dt = \\ &= \frac{P^2 + T^{-1}\int_T \tilde{p}^2(t)\,dt}{TV^4} \int_T \mathbf{u}^\wedge(t)^\wedge \mathbf{u}(t)\,dt + \\ &+ \frac{Q^2 + T^{-1}\int_T \tilde{q}^2(t)\,dt}{TV^4} \int_T \mathbf{u}_q^\wedge(t) \mathbf{u}_q(t)\,dt = \\ &= \frac{P^2 + \tilde{P}^2 + Q^2 + \tilde{Q}^2}{V^2}, \end{aligned} \tag{11}$$

where the effect of power losses from oscillating power components caused by the unbalanced load is represented by the sum of the power squares

$$\tilde{P}^2 + \tilde{Q}^2 = T^{-1} \int_T \tilde{p}^2(t)\,dt + T^{-1} \int_T \tilde{q}^2(t)\,dt. \tag{12}$$

From (11) follows the equation for the decomposition of the integral apparent power

$$S^2 = V^2 I^2 = P^2 + \tilde{P}^2 + Q^2 + \tilde{Q}^2. \tag{13}$$

However, the practical implementation of selective oscillating powers compensation within the p-q instantaneous power theory framework encounters the problem of the third harmonic [31], [32]. Really, the compensation current, e.g.,

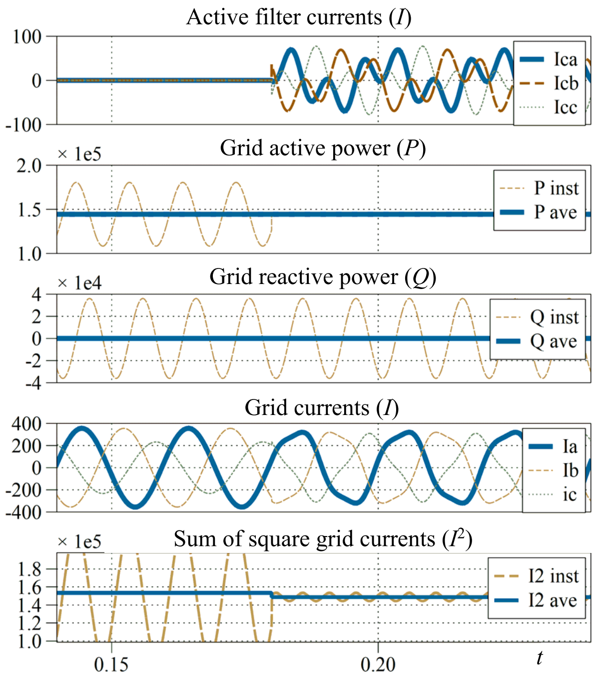

FIGURE 1. **Only active powers oscillating component compensation.**

$\tilde{p}(t)\mathbf{u}(t)/V^2$ contains the third harmonic of the fundamental frequency due to multiplying the harmonic functions $\mathbf{u}(t)$ of the fundamental frequency by the oscillating power $\tilde{p}(t)$ harmonics of the double frequency. Fig. 1 shows the result of such compensation for a standard three-phase, three-wire 380 V 50 Hz network with an unbalanced load $R_{ab} = 2\ Ohm$; $R_{bc} = R_{ca} = 4\ Ohm$, where the specified compensation current is turned on from time 0.18 s.

It can be seen that the oscillations of the instantaneous active power are completely eliminated, and the total RMS values of the grid currents are reduced accordingly. Still, the waveform of the grid currents differs significantly from the sinusoidal one and has distortions characteristic of the third harmonic presence. Thus, selective compensation of inactive powers requires integral methods of active filtering to overcome the problem of supply current harmonic distortion, which forms average values of inactive powers to overcome the problem of supply current harmonic distortion.

B. OUTLINE OF CPC THEORY FOR THREE–PHASE THREE–WIRE SYSTEMS UNDER SINUSOIDAL CONDITIONS

This theory, specified in [31] and [33] for a symmetric three-phase source and a linear unbalanced load identified by admittances $\bar{Y}_{AB}, \bar{Y}_{BC}, \bar{Y}_{CA}$, operates with equivalent admittance $\bar{Y}_e = \bar{Y}_{AB} + \bar{Y}_{BC} + \bar{Y}_{CA} = G_e + jB_e$ and unbalanced admittance

$$\bar{Y}_u = -(\bar{Y}_{BC} + \dot{a}\bar{Y}_{CA} + \tilde{a}\bar{Y}_{AB}) = Y_u e^{j\psi};\ \dot{a} = e^{j120^\circ};\ \tilde{a} = e^{-j120^\circ}.$$

For a symmetrical three-phase voltage of the positive sequence with an instantaneous phase voltage $u_A = \sqrt{2}U\cos(\omega t)$, the decomposition of the three-phase currents in the frequency domain is as follows

$$\begin{aligned}\bar{\boldsymbol{I}} &= \begin{bmatrix}\dot{I}_A\\ \dot{I}_B\\ \dot{I}_C\end{bmatrix} = \bar{Y}_e\begin{bmatrix}\dot{U}_A\\ \dot{U}_B\\ \dot{U}_C\end{bmatrix} + \bar{Y}_u\begin{bmatrix}\dot{U}_A\\ \dot{U}_C\\ \dot{U}_B\end{bmatrix} = \\ &= U\left(G_e\begin{bmatrix}1\\ \tilde{a}\\ \dot{a}\end{bmatrix} + B_e j\begin{bmatrix}1\\ \tilde{a}\\ \dot{a}\end{bmatrix} + Y_u e^{j\psi}\begin{bmatrix}1\\ \dot{a}\\ \tilde{a}\end{bmatrix}\right) = \\ &= \bar{\boldsymbol{I}}_G + \bar{\boldsymbol{I}}_B + \bar{\boldsymbol{I}}_Y.\end{aligned} \tag{14}$$

Due to the mutual integral orthogonality of the reference voltage vectors (14), the quadratic relationship for the RMS values of current components is given by

$$I^2 = \bar{\boldsymbol{I}}^{\wedge}\bar{\boldsymbol{I}}^* = I_G^2 + I_B^2 + I_Y^2, \tag{15}$$

where * is the complex conjugation symbol;

$$I_G = \sqrt{3}G_e U;\ I_B = \sqrt{3}\,|B_e|\,U;\ I_Y = \sqrt{3}Y_u U.$$

Multiplying (15) by the square of the supply voltage RMS value $3U^2 = V^2$, the power equation was obtained in [35].

$$S^2 = V^2 I^2 = P^2 + Q^2 + D^2, \tag{16}$$

where $P = G_e V^2$; $Q = -B_e V^2$; $D = Y_u V^2$ is the unbalanced power of the load.

Also, instantaneous active and reactive powers in the p-q theory were expressed in [31] as

$$p(t) = P + D\cos(2\omega t + \psi);\ \ q(t) = Q + D\sin(2\omega t + \psi). \tag{17}$$

From (17), considering (9), it follows that for a symmetrical three-phase source and a linear load

$$\tilde{P}^2 + \tilde{Q}^2 = D^2. \tag{18}$$

The use of CPC theory for the practical implementation of shunt active filtering encounters several difficulties.

The initial information for this theory is the load admittances $\bar{Y}_{AB}, \bar{Y}_{BC}, \bar{Y}_{CA}$. Measuring their parameters requires current and voltage sensors in each of the three phases of the linear load.

Each quadratic component of the apparent power (16) and the parameter ψ of the instantaneous power (17) are calculated using complex numbers, which require a complicated hardware implementation.

Distributed shunt active filtering requires extracting as many orthogonal current components as possible [29], corresponding to the inactive power components. In contrast, in CPC theory, the unbalance power is represented by only one component. In addition, in the description (16), it is defined as a modular quantity, which, unlike reactive power Q, cannot take negative values, complicating its measurement and compensation.

CPC theory operates with three-coordinate current and voltage vectors; this information is redundant due to the linear dependence of these vector coordinates.

III. CURRENT DECOMPOSITION WITH INTEGRAL POWERS IN THE α- β REFERENCE FRAME

Let us define a unit vector of positive sequence symmetric components $\bar{\mathbf{e}}_+ = \left[1\ \tilde{a}\ \dot{a}\right]^\wedge/\sqrt{3}$; then, the grid voltage vector may be performed as:

$$\bar{\boldsymbol{U}} = \begin{bmatrix} \dot{U}_A \\ \dot{U}_B \\ \dot{U}_C \end{bmatrix} = \begin{bmatrix} U \\ U\tilde{a} \\ U\dot{a} \end{bmatrix} = \frac{V}{\sqrt{3}} \begin{bmatrix} 1 \\ \tilde{a} \\ \dot{a} \end{bmatrix} = V\,\bar{\mathbf{e}}_+. \tag{19}$$

Then decompose the current vector of complex RMS values by unit vectors of symmetric components [34], [35].

$$\bar{\boldsymbol{I}} = \dot{I}_+\bar{\mathbf{e}}_+ + \dot{I}_-\bar{\mathbf{e}}_- = \frac{\dot{I}_+}{\sqrt{3}} \begin{bmatrix} 1 \\ \tilde{a} \\ \dot{a} \end{bmatrix} + \frac{\dot{I}_-}{\sqrt{3}} \begin{bmatrix} 1 \\ \dot{a} \\ \tilde{a} \end{bmatrix}, \tag{20}$$

where $\bar{\mathbf{e}}_- = \left\| 1\ \dot{a}\ \tilde{a} \right\|^\wedge/\sqrt{3} = \bar{\mathbf{e}}_+^*$ is the unit vector of negative sequence symmetric components; $\dot{I}_+ = I_{R+} + jI_{I+}$; $\dot{I}_- = I_{R-} + jI_{I-}$ are complex coefficients depending on a particular type of linear load.

Let us apply an α-β transformation to the voltage vector from (19)

$$\begin{aligned} \underline{\boldsymbol{U}} &= \begin{bmatrix} \dot{U}_\alpha \\ \dot{U}_\beta \end{bmatrix} = \mathbf{C}\bar{\boldsymbol{U}} = V\,\mathbf{C}\,\bar{\mathbf{e}}_+ = \\ &= \frac{V}{\sqrt{3}} \times \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} 1 \\ \tilde{a} \\ \dot{a} \end{bmatrix} = \\ &= \frac{V}{\sqrt{2}} \begin{bmatrix} 1 \\ -j \end{bmatrix} = V\,\underline{\mathbf{e}}_+. \end{aligned} \tag{21}$$

Thus, the three-coordinate unit vector $\bar{\mathbf{e}}_+$ is transformed in the α-β reference frame to a two-coordinate unit vector $\underline{\mathbf{e}}_+ = \left\| 1\ -j \right\|^\wedge/\sqrt{2} = \left\| \dot{U}_\alpha\ \dot{U}_\beta \right\|^\wedge/V = \underline{\boldsymbol{U}}/V$. Similarly, the three-coordinate current vector (20) is performed in the α-β reference frame as follows

$$\begin{aligned} \underline{\boldsymbol{I}} &= \begin{bmatrix} \dot{I}_\alpha \\ \dot{I}_\beta \end{bmatrix} = \mathbf{C}\bar{\boldsymbol{I}} = \dot{I}_+\mathbf{C}\bar{\mathbf{e}}_+ + \dot{I}_-(\mathbf{C}\bar{\mathbf{e}}_+)^* \\ &= \dot{I}_+\underline{\mathbf{e}}_+ + \dot{I}_-(\underline{\mathbf{e}}_+)^* = \\ &= \frac{I_{R+} + jI_{I+}}{\sqrt{2}} \begin{bmatrix} 1 \\ -j \end{bmatrix} + \frac{I_{R-} + jI_{I-}}{\sqrt{2}} \begin{bmatrix} 1 \\ j \end{bmatrix} = \\ &= \frac{I_{R+}}{\sqrt{2}} \begin{bmatrix} 1 \\ -j \end{bmatrix} + \frac{I_{I+}}{\sqrt{2}} \begin{bmatrix} j \\ 1 \end{bmatrix} + \frac{I_{R-}}{\sqrt{2}} \begin{bmatrix} 1 \\ j \end{bmatrix} + \frac{I_{I-}}{\sqrt{2}} \begin{bmatrix} j \\ -1 \end{bmatrix} = \\ &= \frac{1}{V}\left(I_{R+} \begin{bmatrix} \dot{U}_\alpha \\ \dot{U}_\beta \end{bmatrix} + I_{I+} \begin{bmatrix} -\dot{U}_\beta \\ \dot{U}_\alpha \end{bmatrix} + I_{R-} \begin{bmatrix} \dot{U}_\alpha \\ -\dot{U}_\beta \end{bmatrix} \right. \\ &\quad \left. + I_{I-} \begin{bmatrix} -\dot{U}_\beta \\ -\dot{U}_\alpha \end{bmatrix} \right). \end{aligned} \tag{22}$$

It is easy to see that all four vectors with real coefficients $I_{R+}, I_{I+}, I_{R-}, I_{I-}$ are mutually orthogonal with unit norms, so the first pair of coefficients is calculated as follows

$$I_{R+} = \frac{1}{V}\mathrm{Re}\left(\begin{bmatrix} \dot{I}_\alpha \\ \dot{I}_\beta \end{bmatrix}^\wedge \begin{bmatrix} \dot{U}_\alpha \\ \dot{U}_\beta \end{bmatrix}^* \right) = \frac{\mathrm{Re}\left(\dot{I}_\alpha\dot{U}_\alpha^* + \dot{I}_\beta\dot{U}_\beta^* \right)}{V} = \frac{\int_T \left(i_\alpha u_\alpha + i_\beta u_\beta\right) dt}{TV} = \frac{\int_T p\left(t\right) dt}{TV} = \frac{P}{V};$$

$$I_{I+} = \frac{1}{V}\mathrm{Re}\left(\begin{bmatrix} \dot{I}_\alpha \\ \dot{I}_\beta \end{bmatrix}^\wedge \begin{bmatrix} -\dot{U}_\beta \\ \dot{U}_\alpha \end{bmatrix}^* \right) = \frac{\mathrm{Re}\left(\dot{I}_\beta\dot{U}_\alpha^* - \dot{I}_\alpha\dot{U}_\beta^* \right)}{V} = = -\frac{\int_T \left(i_\alpha u_\beta - i_\beta u_\alpha\right) dt}{TV} = -\frac{\int_T q\left(t\right) dt}{TV} = -\frac{Q}{V}.$$

Thus,

$$\dot{I}_+ = I_{R+} + jI_{I+} = (P - jQ)/V. \tag{23}$$

By analogy, the second complex coefficient (20), which determines the unbalance power, can be expressed in terms of two scalar powers [35]:

$$\dot{I}_- = I_{R-} + jI_{I-} = (D_R - jD_I)/V, \tag{24}$$

where

$$D_R = \mathrm{Re}\left(\dot{I}_\alpha\dot{U}_\alpha^* - \dot{I}_\beta\dot{U}_\beta^* \right) = T^{-1}\int_T \left(i_\alpha u_\alpha - i_\beta u_\beta\right) dt;$$

$$D_I = \mathrm{Re}\left(\dot{I}_\alpha\dot{U}_\beta^* + \dot{I}_\beta\dot{U}_\alpha^* \right) = T^{-1}\int_T \left(i_\alpha u_\beta + i_\beta u_\alpha\right) dt.$$

Substitution of (23) and (24) into (22) gives a decomposition of the current vector into four components proportional to the corresponding scalar powers

$$\begin{aligned} \underline{\boldsymbol{I}} &= \frac{1}{V^2}\left(P \begin{bmatrix} \dot{U}_\alpha \\ \dot{U}_\beta \end{bmatrix} + Q \begin{bmatrix} \dot{U}_\beta \\ -\dot{U}_\alpha \end{bmatrix} \right. \\ &\quad \left. + D_R \begin{bmatrix} \dot{U}_\alpha \\ -\dot{U}_\beta \end{bmatrix} + D_I \begin{bmatrix} \dot{U}_\beta \\ \dot{U}_\alpha \end{bmatrix} \right) = \\ &= \left(P\,\underline{\mathbf{u}}_P + Q\,\underline{\mathbf{u}}_Q + D_R\underline{\mathbf{u}}_{DR} + D_I\underline{\mathbf{u}}_{DI} \right)/V^2. \end{aligned} \tag{25}$$

From (25) follows the power equation for sinusoidal voltages and currents in the α-β reference frame

$$\begin{aligned} S^2 &= \left(\underline{\boldsymbol{U}}^\wedge\underline{\boldsymbol{U}}^*\right) \times \left(\underline{\boldsymbol{I}}^\wedge\underline{\boldsymbol{I}}^*\right) = \\ &= T^{-1}\int_T \left(u_\alpha^2 + u_\beta^2\right) dt \times T^{-1}\int_T \left(i_\alpha^2 + i_\beta^2\right) dt = \\ &= P^2 + Q^2 + D_R^2 + D_I^2. \end{aligned} \tag{26}$$

The first term of the current decomposition (25) in the time domain is the α-β transformation of the integral active current according to the Fryze concept [13].

$$\mathbf{i}_P\left(t\right) = \frac{P}{V^2}\mathbf{u}\left(t\right),$$

that provides the same active power as the whole current (25), minimal power losses in the transmission line, and a unity value of the power factor $\Lambda = P/S$.

Each of the four integral powers can be measured using the following equations by algebraically adding four wattmeter readings [35]. Consider another way to measure two separate unbalance power components. To do this, using (21) and (22), we find the oscillating component of the instantaneous active power

$$\begin{aligned} &\tilde{p}\left(t\right) \\ &= \mathrm{Re}\left(\underline{\boldsymbol{U}}^\wedge\underline{\boldsymbol{I}}e^{j2\omega t} \right) = \\ &= V\mathrm{Re}\left\{ \underline{\mathbf{e}}_+^\wedge \left[\dot{I}_+\underline{\mathbf{e}}_+ + \dot{I}_-\left(\mathbf{e}_+\right)^* \right] e^{j2\omega t} \right\} = V\mathrm{Re}\left(\dot{I}_-e^{j2\omega t} \right) = \end{aligned}$$

$$= \text{Re}\left\{(D_R - jD_I)\left[\cos(2\omega t) + j\sin(2\omega t)\right]\right\} =$$
$$= D_R\cos(2\omega t) + D_I\sin(2\omega t) \tag{27}$$

Similarly, the oscillating component of instantaneous reactive power

$$\tilde{q}(t) = \text{Re}\left(\left[\,\dot{U}_\beta \quad -\dot{U}_\alpha\,\right]\underline{\boldsymbol{I}}e^{j2\omega t}\right) = \text{Re}\left(-j\underline{U}^\wedge\underline{\boldsymbol{I}}e^{j2\omega t}\right) =$$
$$= \text{Re}\left\{(D_R - jD_I)\left[\sin(2\omega t) - j\cos(2\omega t)\right]\right\} =$$
$$= D_R\sin(2\omega t) - D_I\cos(2\omega t)\,. \tag{28}$$

A matrix system of equations can be formed from (27) and (28) for the unbalance power components

$$\begin{bmatrix}\cos(2\omega t) & \sin(2\omega t)\\ \sin(2\omega t) & -\cos(2\omega t)\end{bmatrix}\begin{bmatrix}D_R\\ D_I\end{bmatrix} = \begin{bmatrix}\tilde{p}(t)\\ \tilde{q}(t)\end{bmatrix},$$

the solution of which is

$$\begin{bmatrix}D_R\\ D_I\end{bmatrix} = \begin{bmatrix}\cos(2\omega t) & \sin(2\omega t)\\ \sin(2\omega t) & -\cos(2\omega t)\end{bmatrix}\begin{bmatrix}\tilde{p}(t)\\ \tilde{q}(t)\end{bmatrix}. \tag{29}$$

Double-frequency harmonic functions from (29) can be synchronized with instantaneous values of the grid voltage vector

$$\mathbf{u}^\wedge(t) = \left[\,u_\alpha \quad u_\beta\,\right] = V\left[\,\cos(\omega t) \quad \sin(\omega t)\,\right]:$$
$$\cos(2\omega t) = \cos^2(\omega t) - \sin^2(\omega t) = \left(u_\alpha^2 - u_\beta^2\right)\Big/V^2;$$
$$\sin(2\omega t) = 2\sin(\omega t)\cos(\omega t) = 2\,u_\alpha u_\beta\Big/V^2.$$

Then, the unbalance power components will be determined from the measured instantaneous values as follows

$$\begin{bmatrix}D_R\\ D_I\end{bmatrix} = \frac{1}{u_\alpha^2 + u_\beta^2}\begin{bmatrix}\left(u_\alpha^2 - u_\beta^2\right)\tilde{p} + 2\,u_\alpha u_\beta\,\tilde{q}\\ 2u_\alpha u_\beta\,\tilde{p} - \left(u_\alpha^2 - u_\beta^2\right)\tilde{q}\end{bmatrix}. \tag{30}$$

Thus, the method is proposed for the instantaneous determination of two unbalance integral powers D_R, D_I from two instantaneous power oscillating components and α-β voltage coordinates. This approach increases the speed of obtaining the values of unbalance integral powers for a variable load, although it worsens the noise immunity compared to integral methods [35].

The three-component structure of the inactive powers compensation signal in the time domain follows from (25)

$$\mathbf{i}^C(t) = \begin{bmatrix}i_\alpha^C\\ i_\beta^C\end{bmatrix} = \frac{Q}{V^2}\begin{bmatrix}u_\beta\\ -u_\alpha\end{bmatrix} + \frac{D_R}{V^2}\begin{bmatrix}u_\alpha\\ -u_\beta\end{bmatrix} + \frac{D_I}{V^2}\begin{bmatrix}u_\beta\\ u_\alpha\end{bmatrix}. \tag{31}$$

Its implementation requires only the values of instantaneous powers (3) and (4) in the α-β reference frame and the RMS value of the grid voltage.

IV. PRACTICAL IMPLEMENTATION OF THE SHUNT ACTIVE FILTER SCHEME

To reduce the number of voltage and current sensors and avoid the need to provide an artificial grounding point, the input information about the power state of the system will be set in the two-wattmeter reference frame [36] that corresponds to a known method of three-phase active power measurement using two wattmeters according to Aron's scheme [37]. This reference frame is formed by a pair of line voltages measured concerning a common point, for example, u_{AC}, u_{BC}, and two corresponding line currents i_A, i_B.

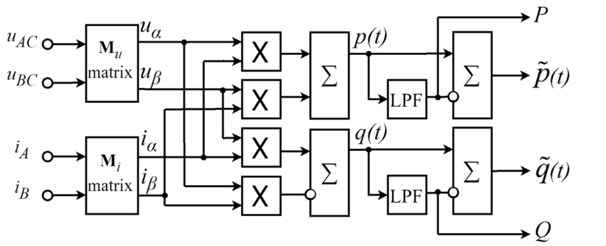

FIGURE 2. A diagram of the instantaneous power components extraction.

Then, the current and voltage vector in the α-β reference frame are obtained as follows

$$\begin{bmatrix}i_\alpha\\ i_\beta\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}\sqrt{3} & 0\\ 1 & 2\end{bmatrix}\begin{bmatrix}i_A\\ i_B\end{bmatrix} = \mathbf{M}_i\begin{bmatrix}i_A\\ i_B\end{bmatrix};$$
$$\begin{bmatrix}u_\alpha\\ u_\beta\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}2/\sqrt{3} & -1/\sqrt{3}\\ 0 & 1\end{bmatrix}\begin{bmatrix}u_{AC}\\ u_{BC}\end{bmatrix} = \mathbf{M}_u\begin{bmatrix}u_{AC}\\ u_{BC}\end{bmatrix}.$$

These matrices perform input coordinate transformations, then current and voltage α-β coordinates are used to generate instantaneous powers, and their components are separated with the LPF following the block diagram (Fig. 2).

Then, according to (30), double-frequency harmonic functions are formed (Fig. 3) and used to calculate the values of the unbalance power components D_R and D_I (Fig. 4).

The output stage of the control system (Fig. 5) synthesizes a compensation signal according to (31). The values of inactive powers Q, D_R, and D_I can be calculated according to the previously mentioned diagrams in the autonomous mode of the SAF operation or provided by a centralized control system when operating as part of a microgrid [29], [38].

Using a transposed Clarke matrix, the output unit performs a standard inverse current transformation to the ABC reference frame. All three coordinates are used for the standard three-phase SAF inverter circuit configuration, with two current coordinates being sufficient to control the half-bridge two-arm inverter circuit [36].

V. VERIFICATION OF SELECTIVE COMPENSATION OF INTEGRAL INACTIVE POWERS IN REAL-TIME

With known load admittances connected by the triangle, all four powers can be expressed from admittance parameters. Indeed, in this case (20) takes the form

$$\bar{\boldsymbol{I}} = \begin{bmatrix}\dot{U}_{AB}\bar{Y}_{AB} - \dot{U}_{CA}\bar{Y}_{CA}\\ \dot{U}_{BC}\bar{Y}_{BC} - \dot{U}_{AB}\bar{Y}_{AB}\\ \dot{U}_{CA}\bar{Y}_{CA} - \dot{U}_{BC}\bar{Y}_{BC}\end{bmatrix} = \frac{P - jQ}{V}\bar{\mathbf{e}}_+ + \frac{D_R - jD_I}{V}\bar{\mathbf{e}}_-,$$

from which we find expressions [34] for numerators of fractions

$$P - jQ = V\,\bar{\mathbf{e}}_-^\wedge\bar{\boldsymbol{I}} = V^2\left(\bar{Y}_{AB} + \bar{Y}_{BC} + \bar{Y}_{CA}\right);$$
$$D_R - jD_I = V\,\bar{\mathbf{e}}_+^\wedge\bar{\boldsymbol{I}} = V^2\left(e^{j\pi/3}\bar{Y}_{AB} - \bar{Y}_{BC} + e^{-j\pi/3}\bar{Y}_{CA}\right).$$

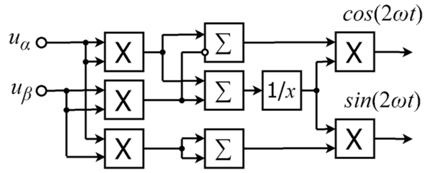

FIGURE 3. A diagram of the double-frequency harmonic functions formation.

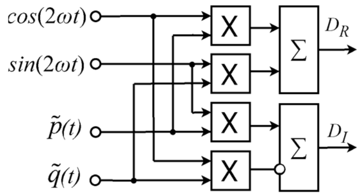

FIGURE 4. A diagram of the unbalance power components extraction.

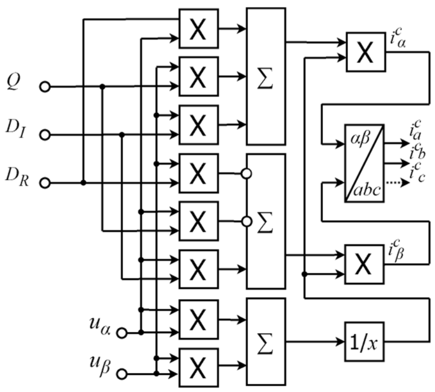

FIGURE 5. A diagram of the final compensation currents formation.

Active and reactive powers are determined as follows

$$P = V^2 (G_{AB} + G_{BC} + G_{CA}) = V^2 G_e;$$
$$Q = -V^2 (B_{AB} + B_{BC} + B_{CA}) = -V^2 B_e, \tag{32}$$

that coincide with (16). The unbalance powers are determined as follows [35]

$$D_R = V^2 \mathrm{Re}\left(e^{j\pi/3}\bar{Y}_{AB} - \bar{Y}_{BC} + e^{-j\pi/3}\bar{Y}_{CA}\right);$$
$$D_I = -V^2 \mathrm{Im}\left(e^{j\pi/3}\bar{Y}_{AB} - \bar{Y}_{BC} + e^{-j\pi/3}\bar{Y}_{CA}\right). \tag{33}$$

The module of unbalance power

$$D = \sqrt{D_R^2 + D_I^2} = V^2 \left|e^{j\pi/3}\bar{Y}_{AB} - \bar{Y}_{BC} + e^{-j\pi/3}\bar{Y}_{CA}\right| =$$
$$= V^2 \left|\bar{Y}_{AB} + \dot{a}\bar{Y}_{BC} + \tilde{a}\bar{Y}_{CA}\right| = V^2 Y_u$$

coincides with what is defined in [31] and (16) via unbalanced admittance Y_u. The advantage of unbalance power separation on two orthogonal components is the ability to control each of them in the process of distributed active filtration.

The simulation was carried out using a symmetric mains voltage with parameters $U = 380\,V, f_{mains} = 50\,Hz$ connected

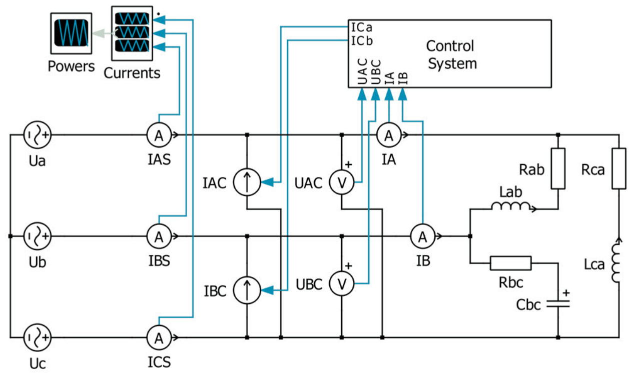

FIGURE 6. A model circuit diagram, linear model.

to a linear unbalanced load with parameters

$$R_{AB} = 1\,\Omega, X_{AB} = 7\,\Omega \rightarrow L_{AB} = 22.282mH,$$
$$R_{BC} = 2\,\Omega,\ X_{BC} = -5\,\Omega \rightarrow C_{BC} = 636.6\,\mu F;$$
$$R_{CA} = 1\Omega,\ X_{CA} = 5\,\Omega \rightarrow L_{CA} = 15.915\,mH,$$

First, admittances are calculated

$$\bar{Y}_0 = 1/\bar{Z}_{AB} + 1/\bar{Z}_{BC} + 1/\bar{Z}_{CA} = Y_0^R + jY_0^I;$$
$$\bar{Y}_- = e^{j\pi/3}/\bar{Z}_{AB} - 1/\bar{Z}_{BC} + e^{-j\pi/3}/\bar{Z}_{CA} = Y_-^R + jY_-^I,$$

and then the powers are determined by (32), (33) as follows

$$P = V^2 Y_0^R;\ Q = -V^2 Y_0^I;\ D_R = V^2 Y_-^R;\ D_I = -V^2 Y_-^I.$$

By substituting the known load parameters, we obtain the power values:

$$P = 18400.5;\ Q = 23088,7;\ D_R = -12279;\ \ D_I = 51198.$$

PLECS Circuit Simulator was used to simulate the selective compensation of inactive powers in the autonomous mode of operation of a two-phase SAF; the model diagram for a linear load is shown in Fig. 6.

The effect of shunt active filtering is represented by the dependent sources of compensation currents I_{AC} and I_{BC}, the values of which are calculated by the Control System according to the circuit diagrams of Section IV based on the input information in the two-wattmeter reference frame.

Fig. 7 shows the simulation results for the operating intervals with compensation of various inactive power components. Interval "1" is without compensation. Starting from time 0.1 s, the SAF is switched on (interval "2") with compensation of reactive power Q only.

In the following intervals "3" and "4", the unbalance power components D_r, D_i are selectively compensated with Q, and in interval "5" they are compensated together without the Q component, which ensures complete elimination of instantaneous power fluctuations inherent in a symmetrical load. In the interval "6" the reactive component Q is additionally compensated; thus, the maximum gain W in terms of power losses in the transmission line is obtained.

TABLE 1. The correspondence between the time intervals and the achieved gains *W* under linear load.

Time interval	Compensated powers	*W*
1	-	1
2	*Q*	1.171
3	*Q, Dr*	1.231
4	*Q, Di*	7.446
5	*Dr, Di*	4.180
6	*Q, Dr, Di*	10.761

TABLE 2. The truth table of inverter switches.

Switch	$T_A=1$	$T_A=-1$	$T_B=1$	$T_B=-1$	$T_{A,B}=0$
K_1	1	0	-	-	0
K_2	0	1	-	-	0
K_3	-	-	1	0	0
K_4	-	-	0	1	0

The gain W is estimated as the ratio of the three-phase current norm squares in the absence and presence of a SAF. The predicted interval values of these quantities, depending on the compensated inactive power components, can be calculated as follows

$$W(D_R)=\frac{S^2}{P^2+Q^2+D_I^2};\quad W(D_I)=\frac{S^2}{P^2+Q^2+D_R^2};$$
$$W(D_R,D_I)=\frac{S^2}{P^2+Q^2};\ W(D_R,D_I,Q)=\frac{S^2}{P^2}=\frac{1}{\Lambda^2}.$$

Table 1 shows the correspondence between the time intervals, the compensated power components, and the achieved gains W in terms of power losses.

The developed switching model of the selective compensator was tested on a widely used nonlinear load formed by a rectifier with a bulky capacitor filtering on the DC side (Fig. 8).

For compensation currents forming, the half-bridge two-arm inverter topology is used [36].

Hysteresis law was realized by using reference I_{AB}^{ref} and measured I_{AB}^{Lcomp} compensator currents. The hysteresis gap width is set by I_{GAP} parameter where currents can change for individual stationary states of the inverter. Switch patterns on each time point are set with signum function [39]

$$T_{AB}^{i+1}=sgn\left(I_{AB}^{ref}-I_{AB}^{Lcomp}+I_{GAP}\cdot T_{AB}^{i}\Big/2\right),$$

where T_{AB}^{i} is the current inverter leg state, T_{AB}^{i+1} is the next inverter leg state. The truth table for individual switches K_1-K_4 are shown in the table below.

The diagrams in Fig. 9 show the compensation results of all power components with a two-period ramp increase of compensation currents at time 0.22 s starting. The following parameters are selected for this model: $I_{GAP}=20$ A,

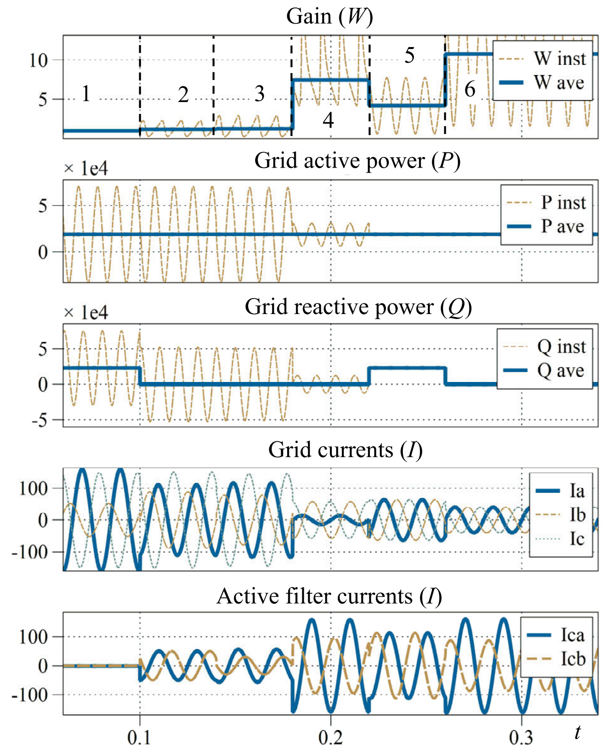

FIGURE 7. Simulation results under linear load.

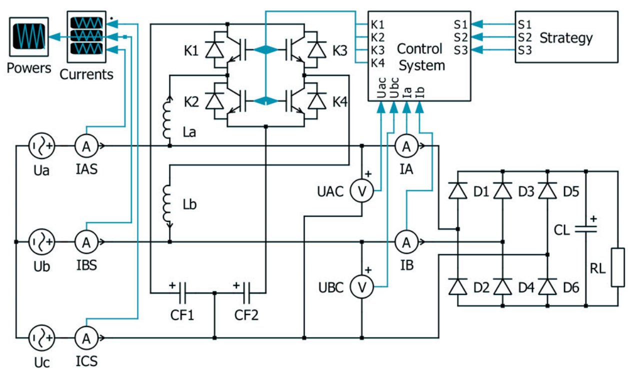

FIGURE 8. The model circuit diagram, non-linear load.

$L_f=0.001$ H, and $U_{DC}=1600$ V. In this way, the approximate maximum switching frequency will not be higher than 20.0 kHz, which is the normal mode of operation of such filters.

An enlarged fragment of current changes in Fig. 9, reproduced in Fig. 10, shows how the hysteresis control algorithm works. Furthermore, the modeling results indicate that the speed at which the compensator current changes is limited by its inductance and the voltages it operates with. As a result, inaccuracies in reference tracking can be observed as spikes in grid current in Fig. 10.

The obtained profit W for a non-linear load with a rectifier is only 1.131, primarily due to decreased active power fluctuations that have shifted to a higher frequency range (harmonics in current).

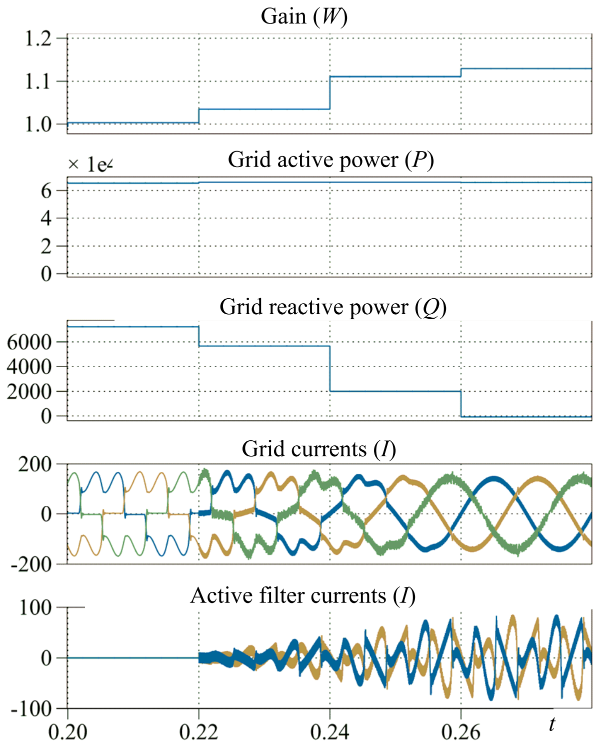

FIGURE 9. Simulation results under non-linear load.

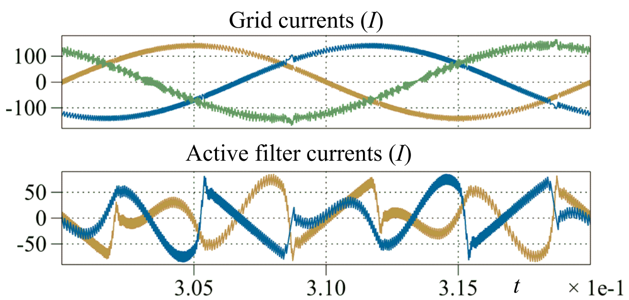

FIGURE 10. Hysteresis control and limitations of current speed change.

Thus, the simulation results showed the effectiveness of the proposed method of real-time SAF control with selective compensation of integral inactive powers for a linear and non-linear load.

VI. CONCLUSION

It was suggested to highlight two orthogonal components in α-β current decomposition of the three-phase three-wire power supply system, each proportional to a separate component of unbalance power. They can be used to simplify the control process in distributed active filtering.

The method is proposed for determining two integral components of the unbalance power by performing real-time algebraic operations on the instantaneous power oscillating components and α-β voltage coordinates synchronized with a three-phase network. This improves the performance of the SAF control system compared to integral methods for determining unbalance powers.

To simplify the construction of a SAF control system with selective real-time compensation of integral inactive powers, a three-component structure of the compensation signal in the α-β reference frame is proposed, each component of which is proportional to a separate inactive power. However, to save voltage and current sensors and avoid the need for an artificial grounding point, the input and output information of the control system is set in the two-wattmeter reference frame.

The computer verification of the proposed SAF control system with selective compensation of the inactive power components has shown its effectiveness for a linear load. In the case of a non-linear load, the balance of the grid currents is achieved by simultaneously compensating for both components of the unbalance powers.

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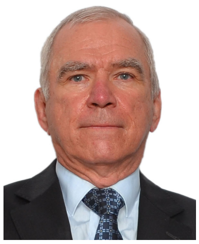

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