IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received 3 April 2024, accepted 18 April 2024, date of publication 25 April 2024, date of current version 2 May 2024.

Digital Object Identifier 10.1109/ACCESS.2024.3393574

RESEARCH ARTICLE

Cooperative Optimization With Globally Coupled Cost Function and Coupled Constraints

TAI-FANG LI, JINGLONG WANG, AND HAOZHENG MENG
College of Control Science and Engineering, Bohai University, Jinzhou 121013, China
Corresponding author: Tai-Fang Li (taifang0416@bhu.edu.cn)

This work was supported by the National Natural Science Foundation of China under Grant 61873041.

ABSTRACT In this paper, we study the cooperative optimization problem of multi-agent systems with globally coupled cost function and coupled constraints, and design a distributed computing framework combining potential game theory underlying geometric projection. This design framework has the advantage of being able to solve the cooperative optimization problem with globally coupled cost function and coupled constraints in distributed way. Firstly, the studied problem with coupled constraints is converted to an unconstrained one by using barrier and penalty methods, respectively, and then the cost function with n variables to be optimized is decoupled by projecting it to n hyperplanes, and n decoupled sub-optimization problems are established. Underlying this design, we exploit an equivalently changing relationship during the optimizing process between each decoupled cost function and the original global function in a fixed communication topology, which forms a potential game, and derive that the optimal solution of the cooperative optimization problem is equivalent with Nash equilibrium of the potential game. The obtained sub-optimization problems can be solved in distributed manner and two improved distributed gradient algorithms are proposed. Finally, the distributed design is applied to the economic dispatch problem in power system to verify the superiority of our proposed algorithms.

INDEX TERMS Cooperative optimization, coupled inequality constraint, potential game, barrier method, penalty method.

I. INTRODUCTION

A multi-agent cooperative optimization problem (COP) means that agents cooperate to find the best solution to minimize the global cost function relying on the information obtaining from their neighbors through a communication network [2], [3], [4], [5], [6], [7], [8], [9]. Many achievements focus on this issue and study COP with coupled constraints. For example, a fixed-step algorithm is raised to solve a smooth COP with coupling constraints in [6]. In [7], a distributed online optimization problem with time-varying coupled constraints is studied. As we all know that constraints in a COP are usually of great practical importance, such as residential energy consumption scheduling problems [10], [11], [12], [13], network resource allocation problems [14], [15], [16] and power-traffic network allocation problems [17], [18], [19], [20]. However, the existing of coupled constraints definitely increases difficulty of seeking optimal solution and complexity of designing algorithms in a great extent.

Many algorithms are proposed to solve COP in which agents learn their associated decision sets (feasible regions) during optimizing process, including but not limited to dual gradient algorithms [21], [22], [23], [24], [25] and primal-dual algorithms [26], [27], [28], [29], [30]. However, not every agent knows its own feasible region in real applications causing the above methods to be infeasible [31], [32], [33], [34]. Reference [31] studies the information exchange and equilibrium computation of distributed algorithms in communication networks for a kind of games, which converge to the equilibrium point both synchronously and asynchronously. Convergences of two distributed algorithms are investigated and analyzed in the communication environment with transmission delays and time-varying network topographies

The associate editor coordinating the review of this manuscript and approving it for publication was Xiwang Dong.

in [33]. Note that in most references mentioned above, the cost function in a COP to be optimized is given in summation form of local cost functions directly, in which each agent possesses an independent cost function. However, not every COP can be modeled by a global cost function in summation form of independent local cost functions, and some existing distributed algorithms would lose efficacy if the cost function of a COP is in a general form. How to design a distributed computation procedure to solve COP with general cost functions and constraints motivates our interests.

In recent years, potential game theory has been introduced into the theoretical analysis of distributed optimization since it is a better way to provide a structural decomposition between COP and specific decision rules for agents [35]. In addition, it is necessary to adopt some optimization techniques to deal with the coupled constraints [34], [36], [37], [38], [39], [40], [41], such as the multiplier method [34], [37] and the penalty method [39], [40], [41]. By combining game theory with barrier and penalty methods, we concentrate on developing a decoupling design framework for the multi-agent COP with globally coupled cost function and constraints. The main difficulties solved by us are in two aspects: The first one is how to obtain the local cost function for each agent by decoupling from the global cost function, under which what kind of distributed algorithms can be used to solve the decoupled COP. The second one is how to guarantee equivalence between the joint optimal solution of the decoupled COP and the optimal solution of the original COP. Based on the above analysis, the main contributions of this paper are listed below.

1) Ideological innovation. A kind of multi-agent COP with globally coupled cost function and constraints is studied, in which the cost function is not necessarily the summation of local utility functions. We design two improved gradient projection algorithms to solve the multi-agent COP in distributed manner. This undoubtedly relaxes the restriction on the form of the global cost function for modeling multi-agent COP and provides effective algorithms to solve this kind of COP in distributed way.

2) Technological innovation. Barrier and penalty methods are used to convert the COP with coupled constraints to an unconstrained one, respectively, and the globally coupling cost function with n variables is decoupled by projecting it to n hyperplanes, then n decoupled sub-optimization problems are established. We exploit an equivalently changing relationship during the optimizing process between each decoupled cost function and the original global cost function, which forms a potential game. Through this design, the final solution of COP is proved to be equivalent with the Nash equilibrium point of the potential game.

The rest organization of this paper is as follows. Section II describes the multi-agent COP and give some related definitions and lemmas. In Section III, we present the detailed process of designing framework, and propose two improved distributed gradient algorithms. In Section IV, two simulation examples are given to verify our proposed algorithms. Section V concludes the paper.

II. PRELIMINARIES

Let $N = \{1, 2, \cdots, n\}$ be the set of agents. Every agent $i \in N$ has a convex decision set $y_i \subseteq \mathbb{R}$. Define $y = (y_1, \cdots, y_n)^T \in Y = \prod_{i\in N} Y_i$ as the global decision vector and $\{y_j\}_i$ is the set of neighbors of agent i. A general multi-agent COP with coupled constraints is in form:

$$\begin{aligned} \min_{y\in Y} \quad & f(y), \\ s.t. \quad & h_u(y) = 0, \quad u = 1, 2, \cdots, m_1, \\ & g_v(y) \leqslant 0, \quad v = 1, 2, \cdots, m_2, \end{aligned} \tag{1}$$

where $f(y) : \mathbb{R}^n \to \mathbb{R}$ is convex and quadratic differentiable, $\{h_u(y) = 0, u = 1, 2, \cdots, m_1\}$ and $\{g_v(y) \leqslant 0, v = 1, 2, \cdots, m_2\}$ are the equality constraint set and the inequality constraint set, respectively. In addition, we assume that the optimal solution y^* of (1) uniquely exists, and $f(y^*) = p^*$ is the optimal value. A connected graph $\mathcal{G} = (N, \mathcal{E})$ represents the communication rule between agents, where $\mathcal{E}$ means the edge set. The neighbor set of i is denoted by $N_i = \{j \in N : (i, j) \in \mathcal{E}\}$ and we define $i \in N_i$ specially. Agent i is only allowed to communicate with its neighbors based on a connected graph. Our goal is to derive the optimal solution y^* of (1) by each agent i calculating its variable value in the light of the communication with its neighbors in the connected graph $\mathcal{G}$.

Before proceeding, we state preliminary knowledge of potential game and two relative lemmas for the analysis process of our main result.

A strategic game is characterized by the player (agent) set $N = \{1, 2, \cdots, n\}$, in which every player $i \in N$ has one action profile $\mathcal{A}$ and cost function $f_i : \mathcal{A} \to \mathbb{R}$ where $\mathcal{A} = \prod_{i\in N} \mathcal{A}_i$ stands for the combination action profile. For an action profile $y = (y_1, \cdots, y_n)^T$, let $y_{-i} = (y_1, \cdots, y_{i-1}, y_{i+1}, \cdots, y_n)^T$ be the actions of players other than i. The following definition is meaningful for our main work.

Definition 1 [35]: For a potential game $G = \{N, \{\mathcal{A}_i\}, \{f_i\}\}$, there is a function $F : \mathcal{A} \to \mathbb{R}$ satisfying

$$f_i(y_i', y_{-i}) - f_i(y_i'', y_{-i}) = F(y_i', y_{-i}) - F(y_i'', y_{-i})$$

for every player i, where $y_{-i} \in \mathcal{A}_{-i}$ and $y_i', y_i'' \in \mathcal{A}_i$. The game G must have a pure Nash equilibrium point $y^* \in \mathcal{A}$, if the action profile $y^* \in \mathcal{A}$ satisfies

$$f_i(y_i^*, y_{-i}^*) = \min_{y_i\in\mathcal{A}_i} f_i(y_i, y_{-i}^*).$$

Lemma 1 [42]: Define a convex set $Z \subseteq \mathbb{R}^n$, and it is closed and nonempty.

i) For each $y \in \mathbb{R}^n$, there is only one $z^* \in Z$ that minimizes $\|y - z\|$ overall $z \in Z$, and z^* is called the projection of y on Z and is recorded as $[y]^+$.

ii) The projection $[y]^+$ and $z^* \in Z$ are consistent if and only if

$$(y - z^*)^T (z - z^*) \leqslant 0, \quad \forall z \in Z$$

holds for some $y \in \mathbb{R}^n$.

Lemma 2 [42]: Let $f : \mathbb{R}^n \rightarrow \mathbb{R}$ be a continuously differentiable vector function and $y, x \in \mathbb{R}^n$ be two real vectors. If

$$\|\nabla f(y + rx) - \nabla f(y)\| \leqslant Lr\|x\|, \quad \forall r \in [0, 1],$$

holds, where L is a scalar, then

$$f(y + x) \leqslant f(y) + x^T \nabla f(y) + \frac{L}{2}\|x\|^2.$$

III. BARRIER FUNCTION METHOD

In this section, we solve the optimization problem (1) relying on logarithmic barrier function method.

It is well known that an equality constraint can be equivalently replaced by two inequality constraints, we hence only need to consider problem (1) with inequality constraints:

$$\begin{aligned} &\min_{y \in Y} \quad f(y), \\ &s.t. \quad g_v(y) \leqslant 0, \quad v = 1, 2, \cdots, m. \end{aligned} \tag{2}$$

The problem (2) with inequality constraints can be converted to the equivalent unconstrained optimization problem:

$$\min_{y,t} \ tf(y, t) + \phi(y, t), \tag{3}$$

where $\phi(y, t)$ is a barrier function, $t > 0$ is a barrier parameter with its variation law $t^{s+1} = \mu t^s$ to determine the approximate precision, where $\mu > 1$ is constant. It needs to point out that y changes once t changes. But we can analyze the updating process of each component of vector y while remains t unchanged. To facilitate subsequent exposition, we adopt y^{s*} to denote the optimal solution of (3) at $t = t^s$. We take the barrier function $\phi(y, t)$ in (3) as

$$\phi(y, t) = -\sum_{v=1}^{m} \log(-g_v(y, t)), \tag{4}$$

where the variable y in $\phi(y, t)$ strictly satisfies inequality constraints of (1) and $\phi(y, t)$ is convex and quadratic differentiable with y. It is obvious from (4) that $\phi(y, t) \rightarrow \infty$ for $\forall t > 0$, when $g_v(y, t)$ goes to zero. Replace $\phi(y, t)$ in (3) with (4), we obtain an equivalent optimization problem of (2):

$$\min_{y,t} \ F(y, t) = tf(y, t) - \sum_{v=1}^{m} \log(-g_v(y, t)). \tag{5}$$

From the convexity operation, $F(y, t)$ in (5) is convex and quadratic differentiable with respect to y. Now, the problem (1) has been transformed to its equivalent optimization problem (5). In the following, we exploit a decoupling design method and develop a distributed algorithm for solving the optimization problem (5). We will state this design process in two aspects.

A. DECOUPLING DESIGN UNDER BARRIER METHOD

We first present the main result of decoupling design for the optimization problem (5) in following theorem.

Theorem 1: Consider the optimization problem (5), if there exists a local cost function $f_i(y, t)$ for every $i \in N$ and for $\forall y_i', y_i'' \in Y_i, y_{-i} \in Y_{-i}$ such that

$$f_i(y_i', y_{-i}, t) - f_i(y_i'', y_{-i}, t) = F(y_i', y_{-i}, t) - F(y_i'', y_{-i}, t), \tag{6}$$

then $G = \{N, \{Y_i\}, \{f_i\}\}$ is a potential game, and the optimization problem (5) could be decoupled into n sub-optimization problems

$$\min_{y_i \in y_i} \ f_i(y_i, y_{-i}^s, t^s), \quad s = 1, 2, \ldots \tag{7}$$

and (y_i^{s*}, t^s) is one stage Nash equilibrium of $G = \{N, \{Y_i\}, \{f_i\}\}$ at $t = t^s > 0$ if

$$f_i(y_i^*, y_{-i}^{s*}, t^s) = \min_{y_i \in Y_i} \ f_i(y_i, y_{-i}^{s*}, t^s) \tag{8}$$

for every agent i. The solution y^{s*} of (7) is a stage optimal solution of (5) at $t = t^s$, and y^{s*} will gradually approach the optimal solution y^* for (5) with $t^s \rightarrow \infty$.

Proof. Combining (6), (7) and (8), the equivalence of solutions between optimization problems (5) and (7) will be proved by a spatial projection segmentation technique.

Consider the problem (5) with n agents in $n+1$ dimensional Cartesian coordinate system. For a given initial state (y^0, t^0), we establish the hyperplane $h_i^0 = \{y | y_i \in Y_i, y_{-i} = y_{-i}^0\}$ for each $i \in N$. The optimizing process of the variable vector y is as follows:

(i) Underlying the $n + 1$ dimensional Cartesian coordinate system, a cross-sectional function $f_i(y_i, y_{-i}^0, t^0)$ would yield on the intersecting surface between the hyperplane h_i^0 and the cost function $F(y, t)$. Variable y_i can be optimized to get the local optimal value $f_i(y_i^1, y_{-i}^0, t^0)$ along y_i direction, where y_i^1 is the optimal variable response to y_{-i}^0 under $t = t^0$. Underlying the point (y_i^1, y_{-i}^0, t^0), one neighbor of agent i, y_j, is chosen. Hyperplane $h_j = \{y | y_j \in Y_j, y_i = y_i^1, y_{-\{i,j\}} = y_{-\{i,j\}}^0\}$ is set up corresponding to y_j which is used to cut the image of function $F(y, t)$ to obtain $f_j(y_i^1, y_j^1, y_{-\{i,j\}}^0, t^0)$ on the cutting surface. y_j can be optimized to obtain the local optimal value $f_j(y_i^1, y_j^1, y_{-\{i,j\}}^0, t^0)$ at y_j direction. Repeat the above process until we obtain the processing solution (y^1, t^0) for updating all variables once under $t = t^0$. We present a simplified decomposition process in 3D space as an example in FIGURE 1.

(ii) Repeat the updating process of (i) by replacing y^0 with y^1 and optimize each component of vector y to obtain the processing solution (y^2, t^0) for updating all variables once again under $t = t^0$.

(iii) Repeat processes (i) and (ii) until we get the stage optimal solution (y^{0*}, t^0), where each y_i^{0*}, $i \in N$, is the optimal response to y_{-i}^{0*} under $t = t^0$.

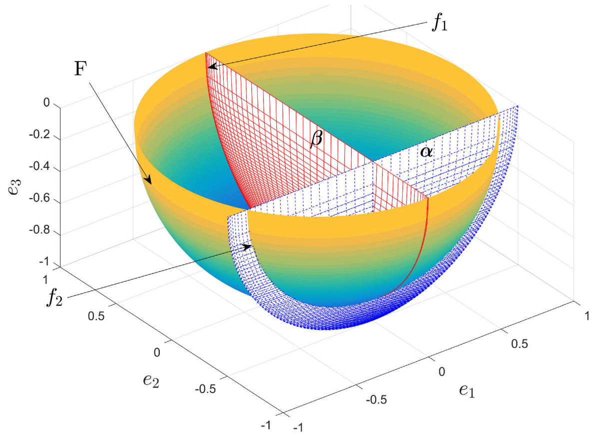


FIGURE 1. Illustration of decoupled design in 3D space, where α and β denote two different hyperplanes, f_1 and f_2 denote two different cross-sectional functions.

According to the above analysis, the decoupled optimization problem for agent i can be derived from (7) as

$$\min_{y_i} \; f_i \left(y_i | \{y_j\}_i, t^0 \right), \tag{9}$$

where f_i is the cost function for agent i with y_i and $\{y_j\}_i$ is the decision variable for neighbor j of i. It is obvious that the cross-sectional function f_i we obtained by using the spatial projection segmentation technique from $F(y, t)$ satisfies (6). Therefore, $\{N, \{y_i\}, \{f_i\}\}$ is a potential game where $F(y, t)$ is the potential function.

We describe the optimizing process of variable y and derive the stage optimal solution y^{0*} of (5) when $t = t^0$. In what following, we analyze the optimizing process of the barrier parameter t.

It is easy to conclude from the definition of potential game that the stage optimal solution y^{s*} of (5) is also the optimal solution of (7) at $t = t^s$. We only need to prove that $y^{s*} \to y^*$ when $t^s \to \infty$. We refer to the solution (y^{s*}, t^s) of (5) as the central point. The set $\{(y^{s*}, t^s) | t^s > 0, s = 1, 2, \cdots\}$ formed by central points is regarded as the central path of (2), and for (y^{s*}, t^s), we have

$$\begin{aligned} &gv(y^{s*}) < 0, \quad v \in [1, m], \\ &\quad 0 = t\nabla f(y^{s*}, t^s) + \nabla\phi(y^{s*}, t^s) \\ &\quad\;\; = t\nabla f(y^{s*}, t^s) + \sum_{v=1}^{m} \frac{1}{-g_v(y^{s*}, t^s)} \nabla g_v(y^{s*}, t^s). \end{aligned} \tag{10}$$

Define λ_v^{s*} corresponding to the central point (y^{s*}, t^s) as

$$\lambda_v^{s*} = -\frac{1}{t g_v(y^{s*}, t^s)}, \quad v \in [1, m], \tag{11}$$

where $g_v < 0$ and $\lambda_v^{s*} > 0$. Instead of λ_v^{s*} in (10) with (11), we obtain

$$\nabla f(y^{s*}, t^s) + \sum_{v=1}^{m} \lambda_v^{s*} \nabla g_v(y^{s*}, t^s) = 0. \tag{12}$$

The Lagrange function of (2) is

$$\mathcal{L}^b(y, \lambda, t) = f(y, t) + \sum_{v=1}^{m} \lambda_v g_v(y, t). \tag{13}$$

With the optimal dual solution λ^{s*} and the central point (y^{s*}, t^s), it can be concluded that $\lambda = \lambda^{s*}$ when $y = y^{s*}$ and the Lagrange function (13) reaches the minimum, which means that λ^{s*} is a dual feasible solution at $t = t^s$, the dual function is finite. We denote the dual function by $d(\lambda^{s*}, t^s)$, then

$$d(\lambda^{s*}, t^s) = f(y^{s*}, t^s) + \sum_{v=1}^{m} \lambda_v^{s*} g_v(y^{s*}, t^s) = f(y^{s*}, t^s) - \frac{m}{t^s}. \tag{14}$$

It is not difficult to see from (14) that m/t^s is the duality gap between the dual feasible solution λ^* and y^*. y^* represents the optimal solution of (2). We thus have

$$f(y^{s*}, t^s) - p^* \leqslant \frac{m}{t^s}. \tag{15}$$

From (15), it is obvious that $y^{s*} \to y^*$ when $t^s \to \infty$. $\square$

Remark 1: For the COP with globally coupled cost function, a novel design framework based on potential game is proposed in Theorem 1. Underlying this framework, we can obtain the decoupled local cost function for each agent from the global cost function. For the coupled inequality constraints, the barrier method is used to incorporate constraints into the proposed design framework. The complexity of the proposed design framework is how to find an effective way to prove equivalence between the joint optimal solution

---

Algorithm 1 Gradient Projection Algorithm

Input: An initial value $(y^0, t^0) = (y_1^0, y_2^0, \cdots, y_n^0, t^0)$, where y^0 satisfies $g_v(y^0) < 0$

Output: The optimal solution $t = t^s, y^* = y^{s*}$ of the problem (1)

$c = 0, \;\; s = 0$
while $\|y^{(s+1)*} - y^{s*}\| > \varepsilon_2$ **do**
  while $\|y^{c+1} - y^c\| > \varepsilon_1$ **do**
    Calculate $\nabla f_i(y_i, y_{-i}, t^s)$ by

$$\begin{aligned} \nabla f_i(y_i, y_{-i}, t^s) &= \frac{\partial f_i(y_i, y_{-i}, t^s)}{\partial y_i} \\ &= \frac{t \partial f(y_i, y_{-i}, t^s)}{\partial y_i} - \frac{\partial \phi_i(y_i, y_{-i}, t^s)}{\partial y_i} \end{aligned} \tag{16}$$

    $y_i^{c+1} = y_i^c - \omega \nabla f_i(y_i^c, y_{-i}^c, t^s)$
    $c = c + 1$
  end while
  Get $y^{s*} = y^c = (y_1^c, \cdots, y_n^c)$
  $s = s + 1$
  $t^s = \mu t^{s-1}$
end while

---

obtained from local optimization problem (7) and the optimal solution of original optimization problem (2).

B. ALGORITHM DESIGN UNDERLYING BARRIER METHOD

A gradient algorithm is proposed and its convergence is proved under the above designed framework. The detailed procedure of the proposed algorithm is shown in Algorithm 1 and its convergence will be proved followed from [43] in Theorem 2.

Theorem 2: Algorithm 1 converges and the variable y will gradually approach the equilibrium point y^* if and only if the step $\omega_i < 2/L_i$.

Proof. Consider the local optimization problem for agent i with cost function $f_i(y_i, y_{-i}, t)$, an initial value (y_i^0, y_{-i}^0, t^0), and the ensuing state $(y_i^{c+1}, y_{-i}^{c+1}, t^0)$. From Theorem 1 and algorithm 1, we obtain

$$\begin{aligned} F(y_i^{c+1}, y_{-i}^0, t^0) &= f_i(y_i^{c+1}, y_{-i}^0, t^0) \\ &\leqslant f_i(y_i^0, y_{-i}^0, t^0) = F(y_i^0, y_{-i}^0, t^0), \end{aligned} \tag{17}$$

which illustrates that $F(y_i, y_{-i}, t)$ is monotonically decreasing with y_i. Now we prove that $F(y_i, y_{-i}, t)$ is monotonically decreasing with y_{-i}, i.e., $F(y_i^{c+1}, y_{-i}^{c+1}, t^0) \leqslant F(y_i^{c+1}, y_{-i}^0, t^0)$. According to Lemma 1

$$\begin{aligned} &F(\tilde{y}_i, \tilde{y}_{-i}, t) - F(\tilde{y}_i, y_{-i}^0, t) \\ &= F(\tilde{y}_i, y_{-i}^0 + \Delta y_{-i}, t) - F(\tilde{y}_i, y_{-i}^0, t) \\ &= F(y + \Delta y_{-i}) - F(y) \\ &\leqslant \Delta y_{-i}^T \nabla F(y) + \frac{L_i}{2} \|\Delta y_{-i}\|^2 \\ &= \Delta y_{-i}^T \frac{\partial F(\tilde{y}_i, y_{-i}, t)}{\partial y_{-i}} + \frac{L_i}{2} \|\Delta y_{-i}\|^2, \end{aligned} \tag{18}$$

where $\Delta Y_{-i} = (\Delta y_1, \cdots, \Delta y_{i-1}, \Delta y_{i+1}, \cdots, \Delta y_n)$, Δy_{-i} is a variable of y_{-i}. By Theorem 1, we have

$$\left(-\varepsilon_{-ik} \cdot \frac{\partial F(y_i^{c+1}, y_{-i}, t^0)}{\partial y_{-i}} - \Delta y_{-i}\right)^T \cdot (-\Delta y_{-i}) \leqslant 0. \tag{19}$$

From (19), we have

$$\Delta y_{-i}^T \cdot \frac{\partial F(y_i^{c+1}, y_{-i}, t^0)}{\partial y_{-i}} \leqslant -\frac{1}{\varepsilon_{-ik}} \|\Delta y_{-i}\|^2. \tag{20}$$

Combining (18) and (20), we have

$$\begin{aligned} &F(y_i^{c+1}, y_{-i}^{c+1}, t^0) - F(y_i^{c+1}, y_{-i}^0, t^0) \\ &\leqslant \sum \left(\frac{L_i}{2} - \frac{1}{\varepsilon_{-ik}}\right) \|\Delta y_{-i}\|^2 \leqslant 0. \end{aligned} \tag{21}$$

Therefore,

$$\begin{aligned} F(y_i^{c+1}, y_{-i}^{c+1}, t^0) &\leqslant F(y_i^{c+1}, y_{-i}^0, t^0) = f_i(y_i^{c+1}, y_{-i}^0, t^0) \\ &\leqslant f_i(y_i^0, y_{-i}^0, t^0) = F(y_i^0, y_{-i}^0, t^0), \end{aligned} \tag{22}$$

from which $F(y, t)$ is monotonically decreasing along y_i direction as $f_i(y_i, y_{-i})$ until it arrives at a fixed point, at which point there is $\Delta y_i = 0$ for $i = 1, \cdots, n$, and this fixed point is the optimal solution of (2).

Remark 2: The iteration process of Algorithm 1 has been proved convergent to the optimal solution. It realizes distributed computation in which each agent updates only utilizing its neighbors' information, although the complexity of Algorithm 1 increases compared with centralized algorithm.

IV. PENALTY FUNCTION METHOD

In this section, we solve the optimization problem (1) by using penalty function method.

We introduce slack variables $\theta_v (v = 1, 2, \cdots, m_2)$ and convert problem (1) to its equivalent problem:

$$\begin{aligned} \min_y \quad & f(y), \\ s.t. \quad & h_u(y) = 0, \qquad u = 1, 2, \ldots, m_1, \\ & g_v(y) + \theta_v^2 = 0, \quad v = 1, 2, \ldots, m_2. \end{aligned} \tag{23}$$

Let $\frac{\sigma}{2} \sum_{u=1}^{m_1} h_u^2(y)$, $\frac{\sigma}{2} \sum_{v=1}^{m_2} \left(g_v(y) + \theta_v^2\right)^2$ be penalty functions. The augmented Lagrange function of (23) is

$$\begin{aligned} \mathcal{L}^p(y, \alpha_u, \beta_v, \theta_v, \sigma) = f(y) &+ \sum_{u=1}^{m_1} \alpha_u h_u(y) \\ &+ \frac{\sigma}{2} \sum_{u=1}^{m_1} h_u^2(y) + \sum_{v=1}^{m_2} \beta_v \left(g_v(y) + \theta_v^2\right) \\ &+ \frac{\sigma}{2} \sum_{v=1}^{m_2} \left(g_v(y) + \theta_v^2\right)^2, \end{aligned} \tag{24}$$

where $\alpha_u > 0$ and $\beta_v > 0$ are Lagrange multipliers and $\sigma > 0$ is the penalty factor. Then we have the equivalent problem of (24) as

$$\min_y \mathcal{L}^p(y, \alpha_u, \beta_v, \theta_v, \sigma). \tag{25}$$

To eliminate the slack variable θ_v, we present the first-order necessary conditions for (25)

$$\frac{\partial \mathcal{L}^p(y, \alpha_u, \beta_v, \theta_v, \sigma)}{\partial \theta_v} = \theta_v \left(\beta_v + \sigma(g_v(y) + \theta_v^2)\right) = 0. \tag{26}$$

Therefore

$$\theta_v^2 = \begin{cases} -\dfrac{\beta_v}{\sigma} - g_v(y), & g_v(y) + \theta_v^2 < 0, \\ 0, & g_v(y) + \theta_v^2 \geqslant 0. \end{cases} \tag{27}$$

We give second-order sufficient conditions of (25)

$$\frac{\partial^2 \mathcal{L}^p(y, \alpha_u, \beta_v, \theta_v, \sigma)}{\partial \theta_v^2} = 2(3\sigma\theta_v^2 + \sigma g_v(y) + \beta_v) \geqslant 0. \tag{28}$$

Replacing θ_v^2 in (27) with (28), we can verify that (27) is minimum point of (25). Replacing θ_v^2 in (25) with (27), we obtain

$$\min_y \mathcal{L}^p(y, \alpha_u, \beta_v, \sigma) = f(y) + H(y, \alpha_u, \beta_v, \sigma) + G(y, \alpha_u, \beta_v, \sigma) \tag{29}$$

where

$$H(y, \alpha_u, \beta_v, \sigma) = \sum_{u=1}^{m_1} \alpha_u h_u(y) + \frac{\sigma}{2} \sum_{u=1}^{m_1} h_u^2(y),$$
$$G(y, \alpha_u, \beta_v, \sigma) = \frac{1}{2\sigma} \sum_{v=1}^{m_2} \left\{ [\max(0, \beta_v + \sigma g_v(y))]^2 - \beta_v^2 \right\}.$$

Let y^{c*} be the optimal solution of (29). If $\sigma = \sigma^*$ is sufficiently large and $\alpha = \alpha^*$, $\beta = \beta^*$, then the optimal solution y^{c*} of (25) converges to the equilibrium point y^* of the original problem (1) followed from [44].

A. DECOUPLING DESIGN UNDER PENALTY METHOD

Similarly to Theorem 1, we give the decoupling decomposition of (29) in following theorem.

Theorem 3: For the optimization problem (29), if there is a local cost function $J_i(y, \alpha_u, \beta_v, \sigma)$ for every $i \in N$ and for $\forall y_i', y_i'' \in Y_i, y_{-i} \in Y_{-i}$ such that

$$\begin{aligned} &J_i(y_i', y_{-i}, \alpha_u, \beta_v, \sigma) - J_i(y_i'', y_{-i}, \alpha_u, \beta_v, \sigma) \\ &\quad = \mathcal{L}^p(y_i', y_{-i}, \alpha_u, \beta_v, \sigma) - \mathcal{L}^p(y_i'', y_{-i}, \alpha_u, \beta_v, \sigma), \end{aligned} \tag{30}$$

then $G = \{N, \{Y_i\}, \{J_i\}\}$ is a potential game, and (29) can be decoupled into n sub-optimization problems

$$\min_{y_i \in Y_i} J_i(y_i, y_{-i}, \alpha_u, \beta_v, \sigma), \quad u = 1, \ldots, m_1, v = 1, \ldots, m_2 \tag{31}$$

and $(y^{c*}, \alpha^*, \beta^*, \sigma^*)$ is a stagewise Nash equilibrium point of $G = \{N, \{Y_i\}, \{J_i\}\}$ if

$$J_i(y_i^*, y_{-i}^{c*}, \alpha_u^*, \beta_v^*, \sigma^*) = \min_{y_i \in Y_i} J_i(y_i, y_{-i}^{c*}, \alpha_u^*, \beta_v^*, \sigma^*) \tag{32}$$

where $i \in N$. The solution y^{c*} of (31) is a stage optimal solution of (25) with positive parameters $\alpha_u, \beta_v, \sigma$ and y^{c*} converges to the optimal solution y^* of (1) when $\sigma = \sigma^*$ is sufficiently large and $\alpha = \alpha^*, \beta = \beta^*$.

Proof. The proof of Theorem 3 can be derived by replacing the cost function $F(y, t)$ in the proof of Theorem 1 with $\mathcal{L}^p(y, \alpha_u, \beta_v, \sigma)$. Finally, we obtain the decoupled optimization problem for every agent i from (31) as

$$\min_{y_i} J_i(y_i | \{y_j\}_i, \alpha_u, \beta_v, \sigma). \tag{33}$$

B. ALGORITHM DESIGN UNDERLYING PENALTY METHOD

A brief calculation process and related parameters of the distributed gradient algorithm are presented and is executed in Algorithm 2.

1) UPDATE RULES

For the optimization problem (33), the iterative process of y_i is given by

$$y_i^{c+1} = y_i^c - \alpha_{step} \cdot \frac{\partial J_i(y_i, y_{-i})}{\partial y_i}, \tag{34}$$

where α_{step} and c stands for the step size and the iteration number, respectively. Let s denote the iteration number for parameters $\alpha = (\alpha_1, \cdots, \alpha_{m_1})$, $\beta = (\beta_1, \cdots, \beta_{m_2})$ and σ. The updating rules of α, β and σ are

$$\begin{aligned} \alpha^s &= \alpha^{s-1} + \sigma^{s-1} h(y^{s-1}), \\ \beta^s &= \max(\mathbf{0}_{m_2}, \beta^{s-1} + \sigma^{s-1} g(y^{s-1})), \\ \sigma^s &= \eta \sigma^{s-1}, \end{aligned} \tag{35}$$

where $\mathbf{0}_{m_2}$ denotes a zero vector with $m_2 \cdot 1$ dimension and the factor $\eta > 1$ regulates the rate at which σ increases.

2) TERMINATION CONDITION

Repeating (34) until

$$\begin{aligned} &\|y_i^{c+1} - y_i^c\| \leqslant \varepsilon_3, \\ &\left\{ \sum_{u=1}^{m_1} h_u^2(y^{s*}) + \sum_{v=1}^{m_2} n \left[\max\left(g_v(y^{s*}), -\frac{\beta_v}{\sigma} \right) \right]^2 \right\}^{\frac{1}{2}} \leqslant \varepsilon_4, \end{aligned} \tag{36}$$

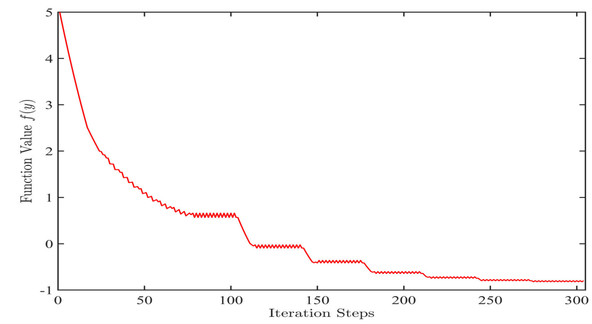
where ε_3 and ε_4 are the preset termination errors.

Algorithm 2 Gradient Algorithm Based on Penalty Function

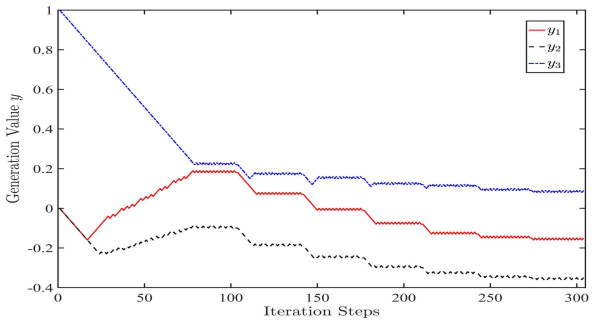
Input: Initial value y^0 and parameters $\sigma^0, \alpha^0, \beta^0, s, c, \eta$
Output: The optimal solution $y^* = y^{s*}, \alpha = \alpha^*, \beta = \beta^*, \sigma = \sigma^*$

1: **while** $\left\{ \sum_{u=1}^{m_1} h_u^2(y^{s-1}) + \sum_{v=1}^{m_2} \left[\max\left(g_v(y^{s-1}), -\frac{\beta_v^{s-1}}{\sigma} \right) \right]^2 \right\}^{\frac{1}{2}} \geqslant \varepsilon_4$ **do**
2: $c = 0$
3: **while** $\|y_i^c - y_i^{c-1}\| \geqslant \varepsilon_3$ **do**
4: $y_i^{c+1} = y_i^c - \alpha_{step} \cdot \frac{\partial J_i(y_i^c, y_{-i}^c)}{\partial y_i}$
5: $y_i^s = y_i^{c+1}$
6: $c = c + 1$
7: **end while**
8: $s = s + 1$
9: Get $y^{s*} = (y_1^s, \ldots, y_n^s)^T$
10: $\alpha^s = \alpha^{s-1} + \sigma^{s-1} h(y^{s-1})$
11: $\beta^s = \max(\mathbf{0}_{m_2}, \beta^{s-1} + \sigma^{s-1} g(y^{s-1}))$
12: $\sigma^s = \eta \sigma^{s-1}$
13: **end while**

Remark 3: The gradient projection algorithm underlying penalty function relaxes the restriction on initial points, that is, the initial point does not need to satisfy the coupling constraints, including coupling equality and inequality constraints, but not be restricted in the feasible region. In Algorithm 2, the advantage of the penalty method with Lagrange multipliers instead of the barrier function method

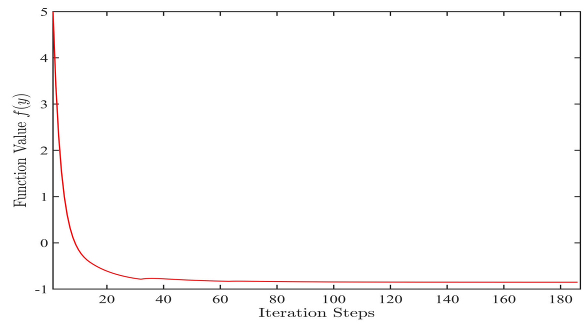


(a) Trajectory of the cost function value $f(y)$.

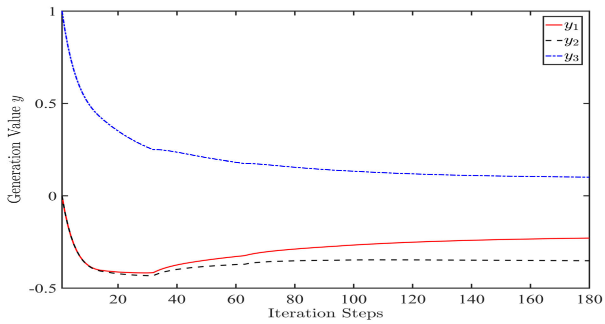


(b) Trajectories of each agent's generation value y_i.

FIGURE 2. **Simulation results of algorithm 1 in example 1.**



(a) Trajectory of the cost function value $f(y)$.



(b) Trajectories of each agent's generation value y_i.

FIGURE 3. **Simulation results of algorithm 2 in example 1.**

is that we can choose the initial iteration point y_i^0 outside the feasible domain and ensure that there is no ill-conditioned property in iteration processes.

V. EXAMPLES

In this section, we verify advantages of our proposed design methods and optimization algorithms through two simulation examples. Furthermore, we highlight the reliability of our proposed gradient algorithms by comparing with the centralized quadratic programming algorithm

$$y_i^{k+1} = y_i^k - \omega \frac{\partial F}{\partial y_i}, \quad \frac{\partial F}{\partial y_i} = t\frac{\partial f}{\partial y_i} + \frac{\partial \phi}{\partial y_i}. \tag{37}$$

Example 1: Consider a cooperative optimization problem of multi-agent system composed of three agents who exchange information through an undirected connected graph which is modeled as

$$\begin{aligned} \min f(y) = &\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}^T \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 4 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \\ &+ \begin{bmatrix} 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}, \\ s.t. \quad & y_1 + y_2 + y_3 = -0.43, \\ & 5y_1^2 + 4y_2^2 + 2y_3 \leqslant 12. \end{aligned} \tag{38}$$

i) Barrier method (Algorithm 1). FIGURE 2 shows the simulation results of Algorithm 1, where $\mu = 5$, the termination error $\varepsilon_1 = \varepsilon_2 = 0.01$ and the step size $\omega = 0.01$. The convergence trajectory of $f(y)$ is shown in FIGURE 2(a). FIGURE 2(b) is the convergence trajectories of the decision variables y_i, and the results indicate that the optimal solution converges to $y^* = (-0.16, -0.35, 0.09)^T$ with the initial value of $y(0) = (0, 0, 1)^T$ and the initial barrier factor $t = t^0 = 0.5$. In addition, it is not difficult to find from FIGURE 2(b) that the variables y_i meet the requirements of the algorithm during the convergence process. In order to verify the results of Algorithm 1, the centralized quadratic programming algorithm (37) is used for simulation verification, and the solution is converged to $y^* = (-0.17, -0.35, 0.10)^T$. The comparison shows that the results of Algorithm 1 and the algorithm (37) are almost the same.

ii) Penalty method (Algorithm 2). With the parameter $\eta = 1.1$, the error $\varepsilon_1 = \varepsilon_2 = 0.005$ and the step size $\omega = 0.005$, the simulation results of Algorithm 2 are presented in FIGURE 3. The value change trajectory of the $f(y)$ is shown in FIGURE 3(a), and FIGURE 3(b) is the convergence trajectories of y_i, where the result converges to $y^* = (-0.18, -0.35, 0.10)^T$. The initial value is set as $y(0) = (0, 0, 1)^T$ and the initial value of the auxiliary variables are $\alpha^0 = -1$, $\beta^0 = -1$, $\sigma^0 = 10$. Clearly, the optimal solution y^* satisfies the requirements and constraints of Algorithm 2.

TABLE 1 shows the specific cost function values with the steps for Algorithm 1, Algorithm 2 and Algorithm (37). By comparing the final solutions of the three algorithms,

TABLE 1. Comparison of the cost function values of example 1.

Algorithms \ Steps	1	36	66	96	126	156	186	...	276	306
Algorithm (37)	4.89	1.86	0.70	0.66	0.13	-0.01	-0.40	...	-0.78	-0.82
Algorithm 1	4.82	1.54	0.76	0.66	-0.03	-0.42	-0.64	...	-0.80	-0.80
Algorithm 2	3.40	-0.79	-0.84	-0.85	-0.85	-0.85	-0.85	...	...	...

we can see that the results are basically consistent, which verifies the effectiveness of our proposed Algorithms. Furthermore, comparing the convergence process of the three algorithms, it can be concluded that the final result of Algorithm 1 is closer to the optimal solution of the centralized algorithm than Algorithm 2, and Algorithm 2 converges faster than Algorithm 1 and Algorithm (37).

Remark 4: Example 1 is a numerical simulation characterized by an inequality constraint function that is a nonlinear function. This example is intended to show that our proposed design framework and algorithm are inclusive to constraint functions, and can solve more general forms of multi-agent cooperative optimization problem.

Example 2: Consider a microgrid economic dispatch problem (EDP) in a power system with two dispatchable generators (DG1 and DG2), an energy storage system (ESS), and advanced control technologies. Each generating unit in the grid can be regarded as an agent, and they communicate directly with their neighbors by using the communication network. The primary mission of microgrid EDP is to minimize the generation cost function while keeping power balance and meeting the capacity constraints of agents. The three agents communicate according to the undirected connected graph which is shown in FIGURE 4. Suppose y_i represents the power output of the ith agent, and $D \geqslant 0$ represents the total generated power demand of the three dispatchable agents. $f(y)$ represents the total cost of three dispatchable agents. y_i^{min} and y_i^{max} denote the minimum and maximum generated power of the ith generator unit. All the agents operate at rated power after the microgrid system is stabilized. The EDP can be modeled as COP of a multi-agent system as

$$\begin{aligned} \min f(y) &= -2y_1y_2 + \sum_{i=1}^{3} a_i y_i^2 + b_i y_i + c_i, \\ s.t. \quad & y_1 + y_2 + y_3 = 1.6, \\ & -9y_1 - 7y_2 - 16y_3 \leqslant 100, \end{aligned} \tag{39}$$

where a_i, b_i and c_i are cost coefficients of the three schedulable agents which values are shown in TABLE 3 along with the range of generated power.

In the ideal case, the initial generation power of the agents is set to $y(0) = (1, 1, 1)^T$, The generation demand is met by the equality constraint in (39), and the total demand of the three agents is $D = 1.6\ MW$. Algorithm 1 and Algorithm 2 are used, respectively, to deal with the

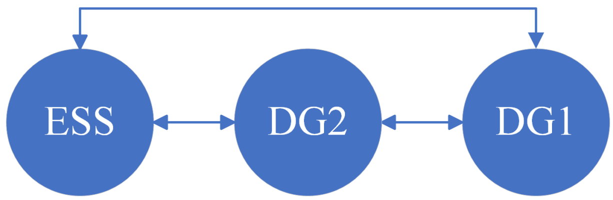
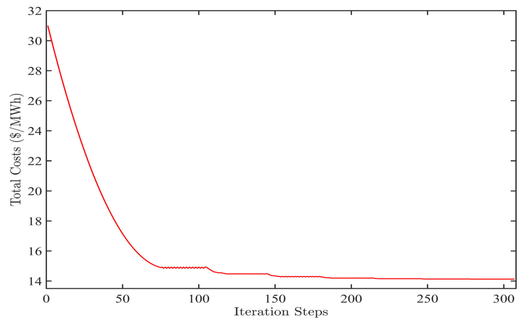
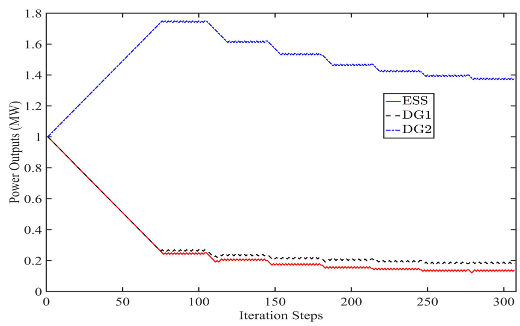


FIGURE 4. The undirected network graph of the three dispatchable agents.



(a) Trajectory of the total cost $f(y)$.



(b) Trajectories of each agent's power outputs y_i.

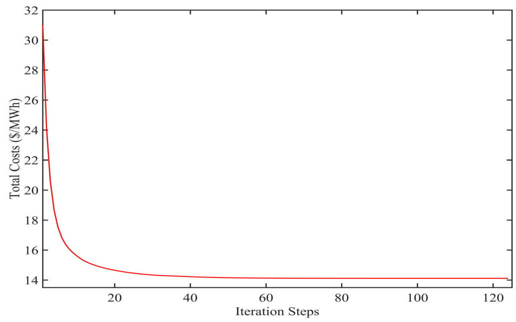
FIGURE 5. Simulation results of algorithm 1 in example 2.

optimization problem (39). The simulation results and statistical analysis are shown in FIGUREs 5-6 and TABLE 2.

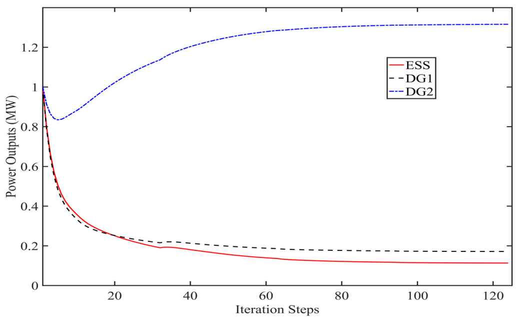
Simulation results of Algorithm 1 are presented in FIGURE 5 with parameters $t = t^0 = 0.5$, $\mu = 5$, the error $\varepsilon_1 = \varepsilon_2 = 0.01$ and the step size $\omega = 0.01$. The generation cost optimization trajectory for agents is shown in FIGURE 5(a). FIGURE 5(b) shows the output power trajectories of agents. Simulation results show that the output power of agents rapidly approaches $y^* = (0.11\ MW, 0.17\ MW, 1.29\ MW)$ which is close to the optimal

TABLE 2. Comparison of total generation costs of example 2.

Algorithms \ Steps	1	20	40	60	80	100	120	...	306	376
Algorithm (37)	30.72	25.86	21.75	18.57	16.35	15.15	14.76	...	14.16	14.12
Algorithm 1	30.59	23.84	18.76	15.76	14.85	14.83	14.46	...	14.12	...
Algorithm 2	24.15	14.61	14.21	14.13	14.12	14.11	14.11	...	...	...



(a) Trajectory of the total cost $f(y)$.



(b) Trajectories of each agent's power outputs y_i.

FIGURE 6. Simulation results of algorithm 2 in example 2.

TABLE 3. Parameters of three dispatchable agents.

Agents	DG1	DG2	ESS
a_i	10	15	3
b_i	-2	-5	-8
c_i	5	5	10
$y^{\min}$ (MW)	0	0	-5
$y^{\max}$ (MW)	10	10	5

result $y^* = (0.10\ MW, 0.16\ MW, 1.28\ MW)$ obtained from algorithm (37). However, the total demand d of the agent is not strictly satisfied, this is because the equality constraints are transformed into two inequality constraints in Algorithm 1, and then errors are generated.

FIGURE 6 shows the simulation results of Algorithm 2, where the algorithm parameters $\alpha^0 = -1$, $\beta^0 = -1$, $\sigma^0 = 10$, $\eta = 1.1$, the error $\varepsilon_1 = \varepsilon_2 = 0.001$ and the step size $\omega = 0.001$. FIGURE 6(a) and FIGURE 6(b) show the total generation cost trajectory of the agents and the power output trajectories of the agents, respectively. The simulation results show that the power outputs of the agents quickly converges to $y^* = (0.11\ MW, 0.17\ MW, 1.32\ MW)$, and this results strictly meets the preset generation demand D.

To further analyze the simulation results, the total costs of the three algorithms are listed in TABLE 2. The optimal values of our proposed algorithms are basically the same as the optimal values of the centralized algorithm (37), but Algorithm 1 and Algorithm 2 ensure the privacy of the agent information during the optimization process. Moreover, the convergence rate of Algorithm 2 is faster than that of Algorithm 1 due to the optimization method itself which speculation was verified in the study [40].

VI. CONCLUSION

The focus of this paper is how to design local cost functions for cooperative optimization problem with a global coupled cost function and constraints. A distributed design framework based on potential game theory has been developed to achieve this goal. Specifically, we have transformed the COP with coupled constraints to an unconstrained one by using barrier function and penalty function, respectively, and then decoupled the global cost function in Cartesian coordinate system to obtain each agent's private cost function. Through the design framework, it has been turn out that the optimal solution of the original COP and Nash equilibrium of the potential game formed by multi-agent with decoupled cost function are equivalent. Furthermore, two distributed gradient algorithms have been proposed and verified through two examples.

The idea of our design method aims to solve the cooperative optimization problem with general global cost function and constraints in distributed computation way, which provides a general calculation framwork for cooperative optimization problems. In this paper, we only consider the cooperative optimization problem with time-invariant cost function under undirected communication topology, cooperative optimization problems in time-varying communication graphs and the finite signal transmission situation are interesting topics will be left as our future work.

ACKNOWLEDGMENT

An earlier version of this paper was presented in part at the 35th Chinese Control and Decision Conference [DOI: 10.1109/ACCESS.2024.3393574].

REFERENCES

[1] J. Wang, T.-F. Li, and S. Wang, "Cooperative optimization for inseparable objective functions with coupled inequality constraints," in *Proc. 35th Chin. Control Decis. Conf. (CCDC)*, May 2023, pp. 1843–1848, doi: 10.1109/CCDC58219.2023.10326521.

[2] Q. Yang and G. Chen, "Primal-dual subgradient algorithm for distributed constraint optimization over unbalanced digraphs," *IEEE Access*, vol. 7, pp. 85190–85202, 2019, doi: 10.1109/ACCESS.2019.2925053.

[3] A. Nedic, A. Ozdaglar, and P. A. Parrilo, "Constrained consensus and optimization in multi-agent networks," *IEEE Trans. Autom. Control*, vol. 55, no. 4, pp. 922–938, Apr. 2010, doi: 10.1109/TAC.2010.2041686.

[4] Y. Wang, J. Wang, X. Song, and Y. Hu, "Distributed interval optimization over time-varying networks: A numerical programming perspective," *IEEE Access*, vol. 11, pp. 122541–122553, 2023, doi: 10.1109/ACCESS.2023.3327199.

[5] F. Chen and J. Chen, "Minimum-energy distributed consensus control of multiagent systems: A network approximation approach," *IEEE Trans. Autom. Control*, vol. 65, no. 3, pp. 1144–1159, Mar. 2020, doi: 10.1109/TAC.2019.2917279.

[6] S. Liang, L. Y. Wang, and G. Yin, "Distributed smooth convex optimization with coupled constraints," *IEEE Trans. Autom. Control*, vol. 65, no. 1, pp. 347–353, Jan. 2020, doi: 10.1109/TAC.2019.2912494.

[7] X. Yi, X. Li, L. Xie, and K. H. Johansson, "Distributed online convex optimization with time-varying coupled inequality constraints," *IEEE Trans. Signal Process.*, vol. 68, pp. 731–746, 2020, doi: 10.1109/TSP.2020.2964200.

[8] X. Zeng, J. Chen, S. Liang, and Y. Hong, "Generalized Nash equilibrium seeking strategy for distributed nonsmooth multi-cluster game," *Automatica*, vol. 103, pp. 20–26, May 2019, doi: 10.1016/j.automatica.2019.01.025.

[9] A. Olshevsky and J. N. Tsitsiklis, "Convergence speed in distributed consensus and averaging," *SIAM J. Control Optim.*, vol. 48, no. 1, pp. 33–55, Jan. 2009, doi: 10.1137/060678324.

[10] R. Deng, Z. Yang, J. Chen, N. R. Asr, and M.-Y. Chow, "Residential energy consumption scheduling: A coupled-constraint game approach," *IEEE Trans. Smart Grid*, vol. 5, no. 3, pp. 1340–1350, May 2014, doi: 10.1109/TSG.2013.2287494.

[11] A.-H. Mohsenian-Rad and A. Leon-Garcia, "Optimal residential load control with price prediction in real-time electricity pricing environments," *IEEE Trans. Smart Grid*, vol. 1, no. 2, pp. 120–133, Sep. 2010, doi: 10.1109/TSG.2010.2055903.

[12] P. Samadi, H. Mohsenian-Rad, R. Schober, and V. W. S. Wong, "Advanced demand side management for the future smart grid using mechanism design," *IEEE Trans. Smart Grid*, vol. 3, no. 3, pp. 1170–1180, Sep. 2012, doi: 10.1109/TSG.2012.2203341.

[13] N. Li, L. Chen, and S. H. Low, "Optimal demand response based on utility maximization in power networks," in *Proc. IEEE Power Energy Soc. Gen. Meeting*, Jul. 2011, pp. 1–8, doi: 10.1109/PES.2011.6039082.

[14] I.-Y. Joo and D.-H. Choi, "Distributed optimization framework for energy management of multiple smart homes with distributed energy resources," *IEEE Access*, vol. 5, pp. 15551–15560, 2017, doi: 10.1109/ACCESS.2017.2734911.

[15] C. U. Saraydar, N. B. Mandayam, and D. J. Goodman, "Efficient power control via pricing in wireless data networks," *IEEE Trans. Commun.*, vol. 50, no. 2, pp. 291–303, Feb. 2002, doi: 10.1109/26.983324.

[16] T. Chen, Q. Ling, and G. B. Giannakis, "An online convex optimization approach to proactive network resource allocation," *IEEE Trans. Signal Process.*, vol. 65, no. 24, pp. 6350–6364, Dec. 2017, doi: 10.1109/TSP.2017.2750109.

[17] Y. Sun, Z. Chen, Z. Li, W. Tian, and M. Shahidehpour, "EV charging schedule in coupled constrained networks of transportation and power system," *IEEE Trans. Smart Grid*, vol. 10, no. 5, pp. 4706–4716, Sep. 2019, doi: 10.1109/TSG.2018.2864258.

[18] A. Y. S. Lam, K.-C. Leung, and V. O. K. Li, "Vehicular energy network," *IEEE Trans. Transport. Electrific.*, vol. 3, no. 2, pp. 392–404, Jun. 2017, doi: 10.1109/TTE.2017.2649887.

[19] J. J. Q. Yu, A. Y. S. Lam, and S.-C. Tan, "Energy exchange coordination of off-grid charging stations with vehicular energy network," in *Proc. IEEE Int. Conf. Smart Grid Commun. (SmartGridComm)*, Oct. 2017, pp. 375–380, doi: 10.1109/SMARTGRIDCOMM.2017.8340687.

[20] S. D. Manshadi, M. E. Khodayar, K. Abdelghany, and H. Üster, "Wireless charging of electric vehicles in electricity and transportation networks," *IEEE Trans. Smart Grid*, vol. 9, no. 5, pp. 4503–4512, Sep. 2018, doi: 10.1109/TSG.2017.2661826.

[21] I. Necoara and V. Nedelcu, "On linear convergence of a distributed dual gradient algorithm for linearly constrained separable convex problems," *Automatica*, vol. 55, pp. 209–216, May 2015, doi: 10.1016/j.automatica.2015.02.038.

[22] J. Wang and G. Hu, "Composite optimization with coupling constraints via dual proximal gradient method with applications to asynchronous networks," *Int. J. Robust Nonlinear Control*, vol. 32, no. 8, pp. 4752–4770, May 2022, doi: 10.1002/rnc.6048.

[23] A. Falsone, K. Margellos, S. Garatti, and M. Prandini, "Dual decomposition for multi-agent distributed optimization with coupling constraints," *Automatica*, vol. 84, pp. 149–158, Oct. 2017, doi: 10.1016/j.automatica.2017.07.003.

[24] J. Li, G. Chen, Z. Dong, and Z. Wu, "A fast dual proximal-gradient method for separable convex optimization with linear coupled constraints," *Comput. Optim. Appl.*, vol. 64, no. 3, pp. 671–697, Jul. 2016, doi: 10.1007/s10589-016-9826-0.

[25] H. Xia and K. Xia, "Improved dual fast gradient method for model predictive control," in *Proc. 37th Chin. Control Conf. (CCC)*, Jul. 2018, pp. 3640–3645, doi: 10.23919/CHICC.2018.8482643.

[26] J. Lei, H.-F. Chen, and H.-T. Fang, "Primal–dual algorithm for distributed constrained optimization," *Syst. Control Lett.*, vol. 96, pp. 110–117, Oct. 2016, doi: 10.1016/j.sysconle.2016.07.009.

[27] A. Nedić and A. Ozdaglar, "Subgradient methods for saddle-point problems," *J. Optim. Theory Appl.*, vol. 142, no. 1, pp. 205–228, Jul. 2009, doi: 10.1007/s10957-009-9522-7.

[28] D. Feijer and F. Paganini, "Stability of primal–dual gradient dynamics and applications to network optimization," *Automatica*, vol. 46, no. 12, pp. 1974–1981, Dec. 2010, doi: 10.1016/j.automatica.2010.08.011.

[29] H. Sakuma, N. Hayashi, and S. Takai, "Distributed primal-dual perturbation algorithm over unbalanced directed networks," *IEEE Access*, vol. 9, pp. 75324–75335, 2021, doi: 10.1109/ACCESS.2021.3082537.

[30] Y. Su, Q. Wang, and C. Sun, "Distributed primal-dual method for convex optimization with coupled constraints," *IEEE Trans. Signal Process.*, vol. 70, pp. 523–535, 2022, doi: 10.1109/TSP.2021.3123888.

[31] J. Koshal, A. Nedić, and U. V. Shanbhag, "Distributed algorithms for aggregative games on graphs," *Oper. Res.*, vol. 64, no. 3, pp. 680–704, Jun. 2016, doi: 10.1287/opre.2016.1501.

[32] D. Paccagnan, B. Gentile, F. Parise, M. Kamgarpour, and J. Lygeros, "Distributed computation of generalized Nash equilibria in quadratic aggregative games with affine coupling constraints," in *Proc. IEEE 55th Conf. Decis. Control (CDC)*, Dec. 2016, pp. 6123–6128, doi: 10.1109/CDC.2016.7799210.

[33] M. Zhu and E. Frazzoli, "Distributed robust adaptive equilibrium computation for generalized convex games," *Automatica*, vol. 63, pp. 82–91, Jan. 2016, doi: 10.1016/j.automatica.2015.10.012.

[34] S. Liang, X. Zeng, and Y. Hong, "Distributed nonsmooth optimization with coupled inequality constraints via modified Lagrangian function," *IEEE Trans. Autom. Control*, vol. 63, no. 6, pp. 1753–1759, Jun. 2018, doi: 10.1109/TAC.2017.2752001.

[35] N. Li and J. R. Marden, "Designing games for distributed optimization," *IEEE J. Sel. Topics Signal Process.*, vol. 7, no. 2, pp. 230–242, Apr. 2013, doi: 10.1109/JSTSP.2013.2246511.

[36] E. Ghadimi, A. Teixeira, I. Shames, and M. Johansson, "Optimal parameter selection for the alternating direction method of multipliers (ADMM): Quadratic problems," *IEEE Trans. Autom. Control*, vol. 60, no. 3, pp. 644–658, Mar. 2015, doi: 10.1109/TAC.2014.2354892.

[37] Z. Huang, R. Hu, Y. Guo, E. Chan-Tin, and Y. Gong, "DP-ADMM: ADMM-based distributed learning with differential privacy," *IEEE Trans. Inf. Forensics Security*, vol. 15, pp. 1002–1012, 2020, doi: 10.1109/TIFS.2019.2931068.

[38] J. B. Krawczyk, "Coupled constraint Nash equilibria in environmental games," *Resource Energy Econ.*, vol. 27, no. 2, pp. 157–181, Jun. 2005, doi: 10.1016/j.reseneeco.2004.08.001.

[39] N. Li and J. R. Marden, "Decoupling coupled constraints through utility design," *IEEE Trans. Autom. Control*, vol. 59, no. 8, pp. 2289–2294, Aug. 2014, doi: 10.1109/TAC.2014.2304373.

[40] N. Li and J. R. Marden, "Designing games to handle coupled constraints," in *Proc. 49th IEEE Conf. Decis. Control (CDC)*, Dec. 2010, pp. 250–255, doi: 10.1109/CDC.2010.5718136.

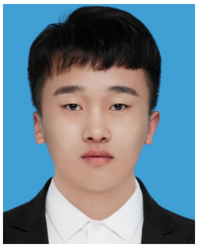
[41] C.-K. Yu, M. van der Schaar, and A. H. Sayed, "Distributed learning for stochastic generalized Nash equilibrium problems," *IEEE Trans. Signal Process.*, vol. 65, no. 15, pp. 3893–3908, Aug. 2017, doi: 10.1109/TSP.2017.2695451.

[42] A. Nedic and A. Ozdaglar, "Distributed subgradient methods for multi-agent optimization," *IEEE Trans. Autom. Control*, vol. 54, no. 1, pp. 48–61, Jan. 2009, doi: 10.1109/TAC.2008.2009515.

[43] T. Li, H. Lin, and J. Zhao, "Cooperative optimisation with inseparable cost functions," *IET Control Theory Appl.*, vol. 9, no. 16, pp. 2430–2437, Oct. 2015, doi: 10.1049/iet-cta.2014.1161.

[44] M. R. Hestenes, "Multiplier and gradient methods," *J. Optim. Theory Appl.*, vol. 4, no. 5, pp. 303–320, Nov. 1969, doi: 10.1007/bf00927673.

TAI-FANG LI received the Ph.D. degree in control theory and control engineering from Northeastern University, Shenyang, China, in 2015. One academic year of her Ph.D. study for a Joint Ph.D. Program, from 2012 to 2013, and one academic year of her Visiting Scholar, from 2018 to 2019, were spent in the College of Engineering, University of Notre Dame, Notre Dame, IN, USA. She is currently an Associate Professor with the College of Control Science and Engineering, Bohai University, Jinzhou, China. Her current research interests include switched systems, hybrid control, optimal control, distributed optimization, and numerical computation.



JINGLONG WANG received the B.S. degree in electrical engineering and automation from Linyi University, China, in 2021. He is currently pursuing the M.S. degree in control theory and control engineering with the College of Control Science and Engineering, Bohai University, Jinzhou, China. His current research interests include distributed optimization and multi-agent systems.

HAOZHENG MENG received the B.S. degree in measurement and control technology and instrumentation from the Qilu University of Technology (Shandong Academy of Sciences), China, in 2020. He is currently pursuing the M.S. degree in control theory and control engineering with the College of Control Science and Engineering, Bohai University, Jinzhou, China. His current research interests include distributed optimization and multi-agent systems.