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RESEARCH ARTICLE

Imperfect Production–Inventory Models for Deteriorating Items With Carbon Cap-and-Trade Policy and Advance-Cash-Credit Payment

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ABSTRACT Global warming, driven by excessive carbon emissions, harms nature and society. Governments' carbon reduction policies are crucial for sustainable industrial growth, integrating eco-friendly practices into production decisions. In addition, sellers often demand upfront payment for goods as a performance guarantee, receive a portion upon delivery, and finalize the transaction on credit. This advance-cash-credit (ACC) payment scheme is becoming increasingly common. Based on the above, this study proposes a production–inventory model with a carbon cap-and-trade policy and an ACC payment scheme and discusses optimal production and replenishment strategies. We also consider deteriorating raw materials, deteriorating finished products, and an imperfect production system. Mathematical methods and numerical examples are discussed to clarify the solution process, and a sensitivity analysis of parameters is performed. We demonstrate that under the two-stage trade credit and ACC scheme, a larger credit payment component enables the production–inventory system to generate higher total profits, and that a lower charged interest, higher earned interest, or later overall payment (for the ACC scheme) leads to lower carbon emissions. Furthermore, changing carbon emission quota does not affect annual carbon emissions but increasing carbon trading price does, thereby encouraging companies to reduce annual carbon emissions under a carbon cap-and-trade policy.

INDEX TERMS Production–inventory model, advance-cash-credit payment, deterioration, defective item, carbon emissions.

I. INTRODUCTION

In today's highly competitive market environment, companies must apply strategies involving effective production and inventory management to reduce operating costs. Companies must consider how they can establish a better production–inventory management mechanism to reduce internal and external costs and enhance their competitiveness and corporate image. In 1913, Harris proposed the Economic Order Quantity (EOQ) model, which was applied in a quantitative ordering strategy that achieves profit optimization by balancing inventory and ordering costs [1]. The Economic Production Quantity (EPQ) model proposed by Taft is an extension of Harris's EOQ model for simultaneously addressing production and inventory optimization [2]. Traditional EPQ models assume a perfect production process

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in manufacturing; that is, all products are non-defective. However, the production system is imperfect in practice; defective products are produced because of problems such as imperfect production procedures, operator negligence, and aging equipment. Incomplete production systems affect optimal production–inventory decisions. Therefore, when a production batch contains defective products, production and replenishment strategies should be modified to minimize inventory costs and maximize profits.

Moreover, problems such as item corruption, deterioration, and damage may occur during the inventory management process of most items; these problems are collectively referred to as item deterioration. The characteristics of deteriorating items affect the production–inventory management decisions of a company or supply chain system. To address this problem, scholars have proposed the implementation of production–inventory models for deteriorating items. However, most studies have only discussed the effect of finished product deterioration on production–inventory decisions and total cost or profit. A realistic supply chain system implements inventories for multiple items, including raw materials and finished products. Therefore, the deterioration characteristics of raw materials and finished products must be considered when developing production–inventory models.

Traditional EPQ and EOQ models usually assume that the buyer pays cash on delivery; however, fixed cash transaction is not the only form of transaction that is used. To increase orders and consumption, numerous companies now allow credit transactions. However, when the manufacturer's production cost or retailer's purchase volume is high, a full credit transaction arrangement is adopted, exposing the seller to a high level of credit risk. Therefore, in practice, a supplier typically requires a purchaser to pay a deposit as a guarantee for order fulfillment during an order placement and, subsequently, a proportion of the purchase price upon delivery of goods. Payment of the remaining amount can be postponed through a credit period. In contrast to full credit transactions, this scheme of advance-cash-credit (ACC) payment not only increases purchase orders but also reduces default risks.

Global warming and climate change are increasingly affecting the natural world and human life. Carbon emissions caused by production activities are a key contributor to extreme climate events. In 2021, numerous countries are vying to implement carbon reduction measures. The United States has committed to reducing its net carbon emissions by 52% relative to 2005 levels by 2030, and Japan has pledged to reduce its net carbon emissions by 46% relative to 2013 levels by 2030. China has set a carbon emissions peak for 2030 and has pledged to become carbon neutral by 2060. Governments worldwide are actively promoting measures such as the development of renewable energy sources, introduction of energy efficiency directives, promotion of carbon trading markets, and levying of carbon taxes. Among these practices, carbon emission trading allows for the establishment of emission quotas for factories and facilities through market transactions and under a mechanism involving total volume control and trading rules. When an enterprise exceeds its emission quota, it can achieve its production goals by purchasing emission rights from others that still have surplus quotas. The European Union currently has the world's largest carbon trading market. Notably, China has established a carbon trading platform in Shanghai that is projected to become the world's largest. From the corporate perspective, an enterprise must consider not only profit maximization but also the means to reducing the adverse environmental and social effects of their operating activities [3]. In line with global efforts to promote carbon reduction policies, most companies are focusing on measures such as improving physical processes, phasing out less energy-efficient equipment, updating product packaging, and utilizing low-polluting energy sources [4]. However, scholars have discovered that companies can also reduce carbon emissions through operation improvements such as the optimization of production–inventory management, which does not substantially increase costs [5].

On the basis of the aforementioned findings, the present study proposes a production–inventory model for deteriorating items with defective products in which the ACC payment scheme is applied. Moreover, we introduce a carbon cap-and-trade policy and establish a corresponding total profit function. Through mathematical analyses, we determine the adequate and necessary conditions for the implementation of optimal solutions. Numerical examples are provided to clarify the solution procedure and compare payment schemes. In addition, managerial implications and decision-making considerations are presented.

II. LITERATURE REVIEW

Our study mainly relates to four topics covered by the existing literature, namely ACC payment, carbon emissions, deteriorating items and imperfect EPQ systems.

The traditional EOQ model assumes the use of cash-on-delivery transactions. Such transactions are only one of multiple forms of transactions that are relatively used today. In a rapidly changing market, companies often use credit transactions to achieve mutual benefits for both buyers and sellers. Goyal [6] was the first to add credit transaction conditions under which a seller can allow a buyer to postpone payments in the EOQ model. Since then, scholars have conducted more extensive and in-depth research on inventory models of credit transactions. Huang and Chung [7] extended Goyal's [6] model by integrating a cash discount component. Teng et al. [8] established modes in which the manufacturer simultaneously receives the supplier's trade credit and provides the customer's trade credit. Yang et al. [9] proposed that a supplier may offer its retailers either a cash discount or credit period to increase sales and reduce default risks. Moradi et al. [10] constructed an inventory model for imperfect quality items considering learning effects and partial trade credit policy. Lin et al. [11] discussed optimizing order policy and credit term for items with inventory-level-dependent demand under trade credit limit. Akhtar et al. [12]

developed an interval valued inventory model for deterioration, carbon emissions and selling price dependent demand considering buy now and pay later facility.

Li et al. [13] developed a supplier–retailer–customer chain in which the retailer receives an upstream ACC payment from the supplier and, in return, makes a downstream cash-credit payment to customers. Wu et al. [14] formulated an inventory model for ACC payment schemes for perishable products with expiration dates. Li et al. [15] discussed sellers' optimal replenishment policies and payment terms for advance, cash, and credit payments. Li et al. [16] discussed optimal pricing, lot sizing, and backordering decisions that are involved when a seller in a supplier–retailer chain demands an ACC payment. Tsao et al. [17] developed an EPQ model for perishable products with an ACC payment scheme, and they determined the optimal selling price and time cycle for maximizing profit by performing a discounted cash flow analysis. Shi et al. [18] discussed the optimal replenishment strategies for deteriorating items under carbon tax regulation and ACC payment conditions. Tsao et al. [19] discussed optimal production and predictive maintenance decisions for deteriorated products under ACC payments.

The challenge of carbon reduction is widely discussed in inventory management research. Hua et al. [20] investigated how firms manage carbon footprints in inventory management through a carbon emission trading mechanism. Song and Leng [21] proposed a single-period inventory (Newsvendor) model that incorporates mandatory carbon emission capacity, carbon tax, and cap-and-trade policies. Zhang and Xu [22] discussed the application of a carbon cap-and-trade mechanism to address a multi-item production planning problem, reporting that carbon trading is an effective means to reduce carbon emissions. Battini et al. [23] incorporated the concept of carbon footprint management into a sustainable EOQ (S-EOQ) model and compared it with the traditional EOQ model. Toptal et al. [24] established an EOQ model to determine the optimal carbon reduction investment and order quantity. He et al. [25] employed an EOQ model in examining the production lot sizing problems of a firm subjected to cap-and-trade and carbon tax regulations. Dye and Yang [26] established an inventory model for determining the optimal credit transaction periods and replenishment strategies for various carbon emission policies and credit transaction types. Qi et al. [27] discussed inventory and pricing strategies in a supply chain subjected to a carbon cap policy. Xu et al. [28] discussed governmental subsidy policies and supply chain decisions with carbon emission limit and consumer's environmental awareness. Wee and Daryanto [29] proposed two EOQ models considering carbon emissions and imperfect quality under various out-of-stock conditions. Datta et al. [30] analyzed a production–inventory system with a mixed carbon policy that combines carbon tax and cap-and-trade components. Lu et al. [31] used the Stackelberg game method to explore competitive and cooperative problems associated with a production–inventory model subjected to carbon reduction policies. Mashud et al. [32] proposed the implementation of green technology investment and preservation technology to reduce carbon emission and product deterioration in a sustainable inventory model. Jauhari et al. [33] discussed pricing and green inventory decisions for a supply chain system with green investment and carbon tax regulation. Priyan [34] established a blockchain-based inventory system with lot size-dependent lead times and uncertain carbon footprints.

Numerous inventory models for deteriorating items have been formulated. Ghare and Schrader [35] first integrated deteriorating items into an EOQ model and proposed that deterioration rate exhibits an exponential distribution. Covert and Philip [36] extended this model, establishing an EOQ model in which deterioration rate is a Weibull distribution of two parameters. Philip [37] further constructed an inventory model in which deterioration rate is a Weibull distribution of three parameters. Wu et al. [38] proposed the first generalized inventory model for non-instantaneous deteriorating items. Hsu et al. [39] were the first to explore the effect of preservation technology investment on a deteriorating inventory model to identify the optimal replenishment and investment strategies for maximizing the retailer's total profits. Lee and Dye [40] formulated a deteriorating inventory model with stock-dependent demand by considering preservation technology cost as a decision variable. Dye [41] extended the inventory model by incorporating a general deterioration rate, concluding that investing more in preservation technology leads to higher service standards. Yang et al. [42] examined trade credit conditions and explored inventory models that allow technology investments to alleviate deterioration. Zhang et al. [43] investigated and clarified the optimal dynamic pricing strategy and replenishment cycle for non-instantaneous deteriorating items with respect to situations where consumer demand is dependent on sales price and the quantity of items displayed in stores. Pal et al. [44] studied an inventory model for non-instantaneous deteriorating items that are characterized by constant demand and inventory shortages. Khakzad and Gholamian [45] discussed the effect of inspection time on average deterioration rate during the replenishment period and established an inventory model for deteriorating items. Mishra et al. [46] considered carbon emission reduction and established a sustainable inventory model for deteriorating items.

For imperfect production systems, Porteus [47] and Rosenblatt and Lee [48] were the first to consider the effect of imperfect production processes on EPQ models. Lee and Rosenblatt [49] and Groenevelt et al. [50] added maintenance and repair factors (relating to production equipment) to an EPQ model with incomplete production procedures. Salameh and Jaber [51] proposed an inventory model based on EPQ and EOQ models to analyze item quality. Chan et al. [52] established an EPQ model for comprehensive item inspection and classified defective products as low priced, reworked, or discarded. Huang [53] examined an inventory model in which defective products are inspected by the buyer. Chiu et al. [54] determined the optimal run time for an imperfect finite production rate model with scrap, rework, and stochastic machine breakdown. Wee and Widyadana [55] developed EPQ models for deteriorating items that incorporated rework and stochastic preventive maintenance time. Taleizadeh et al. [56] considered an EPQ model that allows for defective products to be reprocessed and reshipped to determine the optimal price and replenishment strategies. Hsu and Hsu [57] formulated EPQ models to determine the optimal production lot size and backorder quantity for a manufacturer with an imperfect production process. Tsao et al. [58] discussed imperfect EPQ models that consider the predictive maintenance and reworking of defective products to obtain the optimal production runtime and minimize total projected cost. Khanna et al. [59] discussed integrated vendor–buyer strategies for imperfect production systems with maintenance and warranty policy.

The main differences between the present study and the studies discussed thus far are outlined in Table 1. In contrast to previous studies, the present study incorporates the two characteristics of imperfect production process and inventory deterioration, which can make a model more realistic. The model assumption of payment type has gradually developed from the assumption of early cash payment to that of a comparative decision involving scenarios such as payment in advance, cash on delivery, permissible delayed payment, and trade credit. ACC payment, the most recently proposed scheme of payment, can be regarded as a general setting for the aforementioned forms of payment under dynamic adjustments. Furthermore, ACC payment is a relatively flexible and comprehensive scheme. Although studies have discussed the production–inventory problem in relation to deteriorating items and defective products in the presence of carbon emission policies, no study has discussed the effect of ACC payment on carbon emissions and carbon reduction policies. With countries continually working to improve their carbon trading markets, cap-and-trade policies have become the most common industrial carbon policies; therefore, we selected the cap-and-trade policy as the carbon reduction policy constraint of our proposed model. The present study fills a research gap relating to the production–inventory model and discusses the effect of ACC payment on carbon emissions and carbon cap-and-trade policy.

III. NOTATION AND ASSUMPTIONS

To establish the production–inventory model, the following notation and assumptions are used.

A. NOTATION

α	Percentage of payment in advance
β	Percentage of cash on delivery
γ	Percentage of credit payment ($\alpha + \beta + \gamma = 1$)
D	Demand for finished products per unit time
P	Production quantity per unit time
λ	Defective rate of finished products
S	Setup cost per production cycle
$\hat{S}$	Fixed carbon emissions per production setup
A	Ordering cost of raw materials per production cycle
$\hat{A}$	Fixed carbon emissions from raw material orders per production cycle
c_m	Unit purchase cost of raw materials
$\hat{c}_m$	Carbon emissions from procurement of raw materials per unit
c	Unit production cost
$\hat{c}$	Carbon emissions from production of one unit of finished product
s	Unit selling price of finished products ($s > c$)
h_m	Unit holding cost of materials per unit time (excluding interest charged)
h_i	Unit holding cost for i product per unit time (excluding interest charged), with an i of 1 and 2 denoting non-defective and defective products, respectively
$\hat{h}_m$	Carbon emissions from storage of raw materials per unit per unit time
$\hat{h}_i$	Carbon emissions from storage of i product per unit per unit time, with an i of 1 and 2 denoting non-defective and defective products, respectively
k	Unit disposal cost of defective products
p_c	Unit price of carbon emissions trading
θ_m	Deterioration rate of raw materials
θ_f	Deterioration rate of finished products
r	Quantity of raw materials required to produce one unit of finished product
ϖ	Carbon emission quota
N	Trade credit period provided to retailer ($N \geq 0$)
M	Trade credit period offered by supplier ($M \geq N \geq 0$)
l	Length of period during which manufacturer pays in advance ($l \geq 0$)
I_c	Interest charged per dollar per unit time
I_e	Interest earned per dollar per unit time
t_1	Length of production period (a decision variable)
T	Length of replenishment cycle (a decision variable)
$I_m(t)$	Inventory level of raw materials at time $t \in [0, t_1]$
$I_1(t)$	Inventory level of finished products at time $t \in [0, t_1]$
$I_2(t)$	Inventory level of finished products at time $t \in [t_1, T]$
$TP_W(t_1, T)$	Total profits per production cycle (without charged and earned interest) as a function of t_1 and T

TABLE 1. Principal characteristics of related studies and the present study.

Literature	Model	Carbon emissions	Deterioration		Imperfect production system	Trade credit	
			Finished products	Raw materials		Two-stage	Payment terms
[35], [36], [38], [39], [40], [41], [43], [44]	EOQ		✓				
[4], [23], [24], [25]	EOQ	✓					
[33], [34], [46]	EOQ	✓	✓				
[6], [7]	EOQ						Delayed payment
[42]	EOQ		✓				Delayed payment
[8], [9], [10], [11]	EOQ					✓	Delayed payment
[12], [26]	EOQ	✓					Delayed payment
[13], [45]	EOQ		✓				Advance payment
[32]	EOQ	✓	✓				Advance payment
[15], [16]	EOQ						ACC
[13], [14]	EOQ		✓				ACC
[18], [19]	EOQ	✓	✓				ACC
[47], [49], [50], [51], [52], [54], [56], [57]	EPQ				✓		
[55]	EPQ		✓		✓		
[17]	EPQ		✓				ACC
[53]	Production-inventory model				✓		
[27], [30]	Production-inventory model	✓					
[29]	Production-inventory model	✓			✓		
[31]	Production-inventory model	✓	✓				
This study	**Production-inventory model**	✓	✓	✓	✓	✓	**ACC**

EOQ: economic order quantity; EPQ: economic production quantity; ACC: advance-cash-credit

$TP(t_1, T)$ Total profits per unit time as a function of t_1 and T

$*$ Representation of optimal value.

B. ASSUMPTIONS

1) The production–inventory model considers raw materials and finished products simultaneously.
2) When the purchase price is high, the upstream company usually requires the downstream company to pay a partial deposit (assuming a ratio of α) for an order placement and then a part of the price (assuming a ratio of β) upon delivery of goods. Payment of the remaining amount (assuming a ratio of γ) can be postponed, with $0 \leq \alpha, \beta, \gamma \leq 1$ and $\alpha + \beta + \gamma = 1$ (see Li et al. [13]; Wu et al. [14]; Li et al. [16]; Tsao et al. [17]).
3) The trade credit period offered by the supplier is longer than or equal to that offered by the manufacturer; that is, $M \geq N$.
4) The replenishment rate of raw materials is unlimited, whereas the production rate of finished products is limited and greater than the market demand rate.
5) The lead time for material replenishment is 0.
6) A shortage of finished products is not allowed, and the production of finished products is immediately terminated upon the exhaustion of the raw materials.

IV. MODEL FORMULATION AND SOLUTION

A. MODEL FORMULATION

On the basis of the aforementioned notation and assumptions, the production-inventory system, encompassing raw materials and finished goods, is described as follows. At the beginning of each cycle, the manufacturer purchases and receives q_m units of raw materials and then starts production. The raw materials are continuously used in production, simultaneously deteriorate gradually, and are exhausted by time t_1. Regarding finished goods, the manufacturer begins producing and selling production batches that contain defective

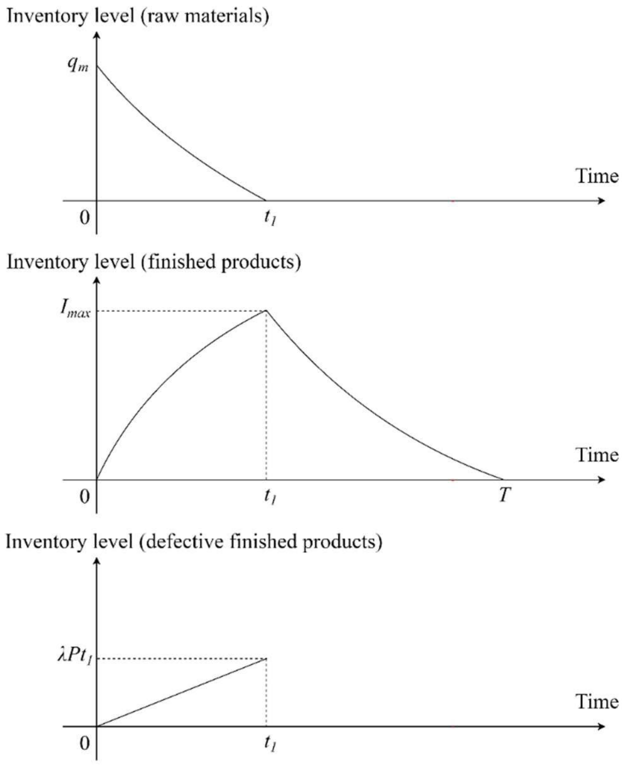

FIGURE 1. Inventory levels of raw materials and finished products in a production cycle.

products. Because the production rate of non-defective products is greater than the demand rate, the inventory of finished goods gradually accumulates until the materials are exhausted. At this time, the inventory level peaks. Thereafter, at the end of the cycle, the inventory level is reduced to zero because of the demand and deterioration. In addition, defective products are detected immediately and gradually accumulate until the end of production, at which point the entire batch is processed. This entire process is then repeated. The relationship between inventory level and timeline for raw materials and finished goods is presented in Figure 1.

Figure 1 indicates that the change in inventory level for raw materials during $[0, t_1]$ is related to production input and deterioration. Therefore, $I_m(t)$ at time t can be expressed by the following differential equation:

$$\frac{dI_m(t)}{dt} + \theta_m I_m(t) = -rP, 0 < t < t_1. \tag{1}$$

With the application of the boundary condition $I_m(t_1) = 0$, Eq. (1) is solved to obtain the inventory level at this stage:

$$I_m(t) = \frac{rP}{\theta_m}\left[e^{\theta_m(t_1-t)} - 1\right], 0 < t < t_1. \tag{2}$$

Subsequently, the manufacturer's order quantity of materials per production cycle can be obtained:

$$q_m = I_m(0) = \frac{rP}{\theta_m}(e^{\theta_m t_1} - 1). \tag{3}$$

For finished goods, during $[0, t_1]$, the inventory level changes because of the production of non-defective goods, demand, and deterioration. Therefore, $I_1(t)$ at time t can be expressed as the following differential equation:

$$\frac{dI_1(t)}{dt} + \theta_f I_1(t) = (1-\lambda)P - D, 0 < t < t_1. \tag{4}$$

With the application of the boundary condition $I_1(0) = 0$, Eq. (4) is solved to obtain the inventory level at this stage:

$$I_1(t) = \frac{(1-\lambda)P - D}{\theta_f}(1 - e^{-\theta_f t}), 0 \le t \le t_1. \tag{5}$$

Similarly, during $[t_1, T]$, the change in inventory level is affected by demand and deterioration. Therefore, $I_2(t)$ at time t can be expressed as the following differential equation:

$$\frac{dI_2(t)}{dt} + \theta_f I_2(t) = -D, t_1 \le t \le T. \tag{6}$$

With the application of the boundary condition $I_2(T) = 0$, Eq. (6) is solved to obtain the inventory level at this stage:

$$I_2(t) = \frac{D}{\theta_f}[e^{\theta_f(T-t)} - 1], t_1 \le t \le T. \tag{7}$$

Next, t_1 is substituted into Eqs. (5) and (7), and because $I_1(t_1) = I_2(t_1)$, the length of the production period t_1 is obtained using the following equation:

$$t_1 = \frac{1}{\theta_f}\ln\left[1 + \frac{D}{(1-\lambda)P}(e^{\theta_f T} - 1)\right]. \tag{8}$$

The total profits per production cycle of the production–inventory system comprises sales revenue, setup cost, material ordering and purchase costs, production cost, holding costs for materials and finished products, and disposal cost of defective products. $TP_W(t_1, T)$, the total profits excluding interests charged and earned per production cycle can be calculated as follows:

$$\begin{aligned} TP_W(t_1, T) &= sDT - S - A - c_m q_m - cPt_1 \\ &\quad - h_1\left[\int_0^{t_1} I_1(t)dt + \int_{t_1}^{T} I_2(t)dt\right] \\ &\quad - \frac{h_2\lambda Pt_1^2}{2} - h_m\int_0^{t_1} I_m(t)dt - k\lambda Pt_1 \\ &= sDT - S - A - c_m q_m - (c + k\lambda)Pt_1 \\ &\quad - h_1\left\{\frac{(1-\lambda)P - D}{\theta_f^2}(e^{-\theta_f t_1} + \theta_f t_1 - 1)\right. \\ &\quad \left. + \frac{D}{\theta_f^2}[e^{\theta_f(T-t_1)} - \theta_f(T - t_1) - 1]\right\} \\ &\quad - \frac{h_2\lambda Pt_1^2}{2} - \frac{h_m rP}{\theta_m^2}(e^{\theta_m t_1} - \theta_m t_1 - 1). \end{aligned} \tag{9}$$

Interest charged for raw material (represented by IC_m)

The downstream company pays a deposit of α percent of the purchase price first when placing an order and then cash-on-delivery payment of β percent of the purchase price after a period of l. Payment of the remaining γ percent of the

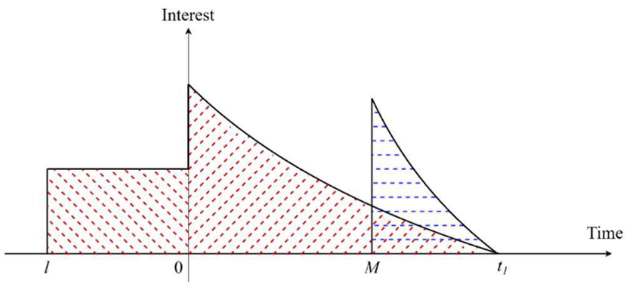

FIGURE 2. Interest charged for raw materials when $M \leq t_1$.

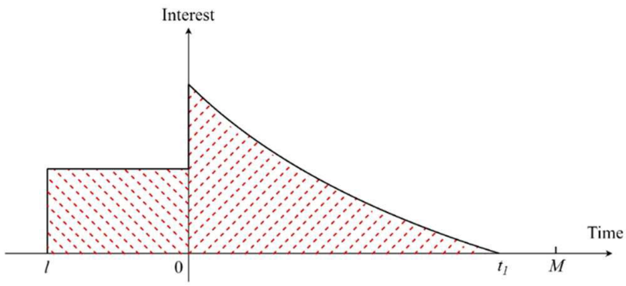

FIGURE 3. Interest charged for raw materials when $M \geq t_1$.

purchase price can be postponed for a period of M. Regarding the interest charged, the trade credit period M offered by the upstream supplier differs from the production period t_1; thus, two possible scenarios apply, namely $M \leq t_1$ and $M \geq t_1$ (Figs. 2 and 3, respectively). Denoted as IC_m, the interest charged per replenishment cycle is calculated as follows:

$$IC_m = \begin{cases} IC_{1m} \; if M \leq t_1, \\ IC_{2m} \; if M \geq t_1, \end{cases}$$

where

$$\begin{aligned} IC_{1m} &= c_m I_c \left\{ +(\alpha+\beta)\int_0^{t_1} I_m(t)dt + \gamma \int_M^{t_1} I_m(t)dt \right\} \\ &= c_m I_c r P \left\{ \frac{[\alpha(M+l)+\beta M]}{\theta_m}(e^{\theta_m t_1}-1) \right. \\ &\quad + \frac{(\alpha+\beta)}{\theta_m^2}(e^{\theta_m t_1}-\theta_m t_1 - 1) \\ &\quad \left. + \frac{\gamma}{\theta_m^2}\left[e^{\theta_m(t_1-M)} - \theta_m(t_1-M) - 1\right] \right\}, \qquad (10) \end{aligned}$$

and

$$\begin{aligned} IC_{2m} = c_m I_c r P &\left\{ \frac{[\alpha(M+l)+\beta M]}{\theta_m}(e^{\theta_m t_1}-1) \right. \\ &\left. + \frac{(\alpha+\beta)}{\theta_m^2}\left(e^{\theta_m t_1}-\theta_m t_1 - 1\right) \right\}. \qquad (11) \end{aligned}$$

Interest charged for finished products (represented by IC_g)

The credit transaction period provided by the manufacturer to the downstream retailer is N, and interest is charged due to prior inventory cost. As N, the production period t_1 and replenishment cycle T are different, three scenarios are

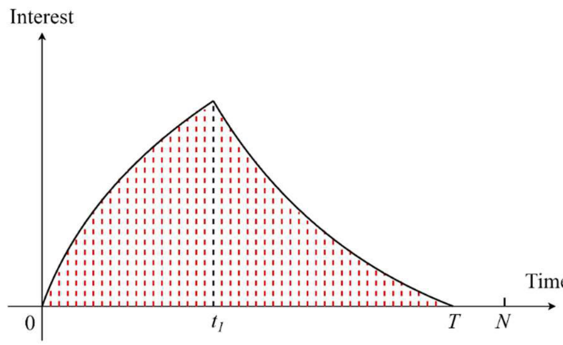

FIGURE 4. Interest charged for finished products when $N \leq t_1$.

possible, namely $N \leq t_1$, $t_1 \leq N \leq T$, and $N \geq T$ (Figs. 4, 5, and 6, respectively). The interest charged per replenishment cycle is expressed as follows:

$$IC_g = \begin{cases} IC_{1g}, & if N \leq t_1, \\ IC_{2g}, & if t_1 \leq N \leq T, \\ IC_{3g}, & if N \geq T, \end{cases}$$

where

$$\begin{aligned} IC_{1g} &= cI_c \int_0^N I_1(t)dt \\ &= \frac{cI_c[(1-\lambda)P - D]}{\theta_f^2}(e^{-\theta_f N} + \theta_f N - 1), \qquad (12) \end{aligned}$$

$$\begin{aligned} IC_{2g} &= cI_c \left[\int_0^{t_1} I_1(t)dt + \int_{t_1}^N I_2(t)dt \right] \\ &= cI_c \left\{ \frac{(1-\lambda)P - D}{\theta_f^2}(e^{-\theta_f t_1} + \theta_f t_1 - 1) \right. \\ &\quad \left. + \frac{D}{\theta_f^2}\left[e^{\theta_f(T-t_1)} - e^{\theta_f(T-N)} - \theta_f(N-t_1)\right] \right\}, \qquad (13) \end{aligned}$$

and

$$\begin{aligned} IC_{3g} &= cI_c \left[\int_0^{t_1} I_1(t)dt + \int_{t_1}^T I_2(t)dt \right] \\ &= cI_c \left\{ \frac{(1-\lambda)P - D}{\theta_f^2}(e^{-\theta_f t_1} + \theta_f t_1 - 1) \right. \\ &\quad \left. + \frac{D}{\theta_f^2}\left[e^{\theta_f(T-t_1)} - \theta_f(T-t_1) - 1\right] \right\}. \qquad (14) \end{aligned}$$

Interest earned (represented by IE)

The upstream supplier provides a trade credit period M, and the manufacturer can earn interest through sales income before payment. As M, the credit transaction period N provided to the retailer, and replenishment cycle T are different, two scenarios are possible, namely $M \leq T + N$ and $M \geq T + N$ (Figs. 7 and 8, respectively). Therefore, the interest earned per replenishment cycle is:

$$IE = \begin{cases} IE_1 & if \; M \leq T + N, \\ IE_2 & if \; M \geq T + N, \end{cases}$$

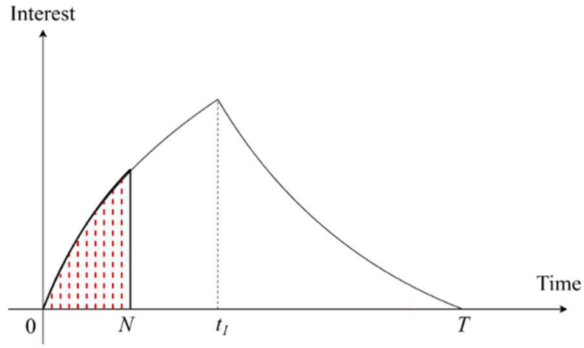

FIGURE 5. Interest charged for finished products when $t_1 \leq N \leq T$.

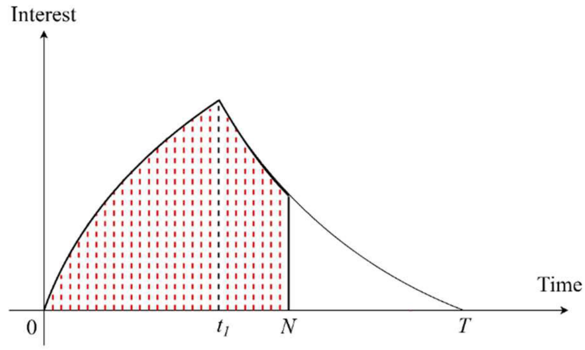

FIGURE 6. Interest charged for finished products when $N \geq T$.

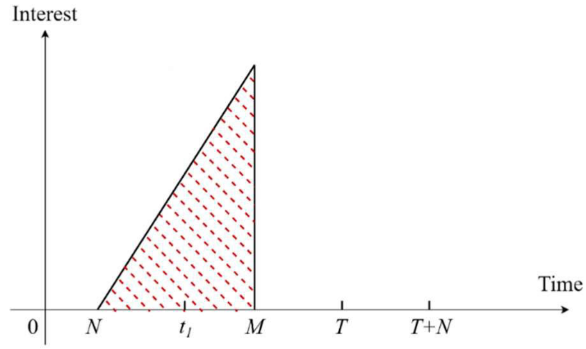

FIGURE 7. Interest earned when $M \leq T + N$.

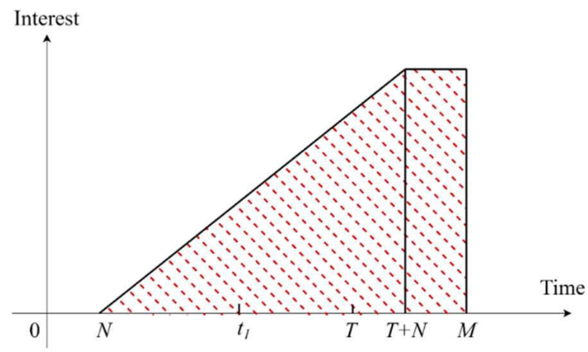

FIGURE 8. Interest earned when $M \geq T + N$.

where

$$IE_1 = \frac{\gamma s I_e D(M-N)^2}{2}, \tag{15}$$

and

$$IE_2 = \gamma s I_e \left[\frac{DT^2}{2} + DT(M-T-N) \right]. \tag{16}$$

Therefore, we can obtain $TP(t_1, T)$, the total profits of the manufacturer per unit time, as follows:

$$TP(t_1,T) = \begin{cases} TP_1(t_1,T), & \text{if } M \leq t_1, \\ TP_2(t_1,T), & \text{if } N \leq t_1 \leq M \leq T+N, \\ TP_3(t_1,T), & \text{if } N \leq t_1 \leq T+N \leq M, \\ TP_4(t_1,T), & \text{if } t_1 \leq N \leq M \leq T+N, \\ TP_5(t_1,T), & \text{if } t_1 \leq N \leq T+N \leq M, \\ TP_6(t_1,T), & \text{if } T \leq N \leq M \leq T+N, \\ TP_7(t_1,T), & \text{if } N \leq T+N \leq M, \end{cases} \tag{17}$$

where

$$\begin{aligned} &TP_1(t_1,T) \\ &= [TP_W(t_1,T) - IC_{1m} - IC_{1g} + IE_1]/T \\ &= sD - \frac{1}{T}\left\{ S + A + \frac{c_m r P}{\theta_m}(e^{\theta_m t_1} - 1) + cPt_1 \right. \\ &\quad + h_1 \frac{(1-\lambda)P - D}{\theta_f^2}\left(e^{-\theta_f t_1} + \theta_f t_1 - 1\right) \\ &\quad + \frac{D}{\theta_f^2}[e^{\theta_f (T-t_1)} - \theta_f (T-t_1) - 1]\} \\ &\quad + \frac{h_2 \lambda P t_1^2}{2} + \frac{h_m r P}{\theta_m^2}(e^{\theta_m t_1} - \theta_m t_1 - 1) \\ &\quad + k\lambda P t_1 + c_m I_c r P \left\{ \frac{[\alpha(M+l) + \beta M]}{\theta_m} \right. \\ &\quad \left. \times (e^{\theta_m t_1} - 1) + \frac{(\alpha+\beta)(e^{\theta_m t_1} - \theta_m t_1 - 1)}{\theta_m^2} \right\} \\ &\quad \left. + \frac{\gamma}{\theta_m^2}\left[e^{\theta_m (t_1 - M)} - \theta_m (t_1 - M) - 1 \right] \right\} \\ &\quad + \frac{c I_c [(1-\lambda)P - D]}{\theta_f^2}(e^{-\theta_f N} + \theta_f N - 1) \\ &\quad \left. - \frac{\gamma s I_e D(M-N)^2}{2} \right\}, \end{aligned} \tag{18}$$

$$\begin{aligned} &TP_2(t_1,T) \\ &= [TP_W(t_1,T) - IC_{2m} - IC_{1g} + IE_1]/T \\ &= sD - \frac{1}{T}\left\{ S + A + \frac{c_m r P}{\theta_m}(e^{\theta_m t_1} - 1) + cPt_1 \right. \\ &\quad + h_1 \left\{ \frac{(1-\lambda)P - D}{\theta^2}(e^{-\theta_f t_1} + \theta_f t_1 - 1) \right. \\ &\quad \left. + \frac{D}{\theta_f^2}[e^{\theta_f (T-t_1)} - \theta_f (T-t_1) - 1] \right\} + \frac{h_2 \lambda P t_1^2}{2} \end{aligned}$$

$$+\frac{h_m rP}{\theta_m^2}(e^{\theta_m t_1}-\theta_m t_1-1)+k\lambda Pt_1$$
$$+c_m I_c rP\left\{\frac{[\alpha(M+l)+\beta M]}{\theta_m}(e^{\theta_m t_1}-1)\right.$$
$$\left.+\frac{(\alpha+\beta)}{\theta_m^2}(e^{\theta_m t_1}-\theta_m t_1-1)\right\}$$
$$+\frac{cI_c[(1-\lambda)P-D]}{\theta_f^2}(e^{-\theta_f N}+\theta_f N-1)$$
$$\left.-\frac{\gamma sI_e D(M-N)^2}{2}\right\}, \qquad (19)$$

$$TP_3(t_1,T)$$
$$=[TP_W(t_1,T)-IC_{2m}-IC_{1g}+IE_2]/T$$
$$=sD-\frac{1}{T}\left\{S+A+\frac{c_m rP}{\theta_m}(e^{\theta_m t_1}-1)+cPt_1\right.$$
$$+h_1\left\{\frac{(1-\lambda)P-D}{\theta_f^2}(e^{-\theta_f t_1}+\theta_f t_1-1)\right.$$
$$\left.+\frac{D}{\theta_f^2}\left[e^{\theta_f(T-t_1)}-\theta_f(T-t_1)-1\right]\right\}$$
$$+\frac{h_2\lambda Pt_1^2}{2}+\frac{h_m rP}{\theta_m^2}(e^{\theta_m t_1}-\theta_m t_1-1)+k\lambda Pt_1$$
$$+c_m I_c rP\left\{\frac{[\alpha(M+l)+\beta M]}{\theta_m}(e^{\theta_m t_1}-1)\right.$$
$$\left.+\frac{(\alpha+\beta)}{\theta_m^2}(e^{\theta_m t_1}-\theta_m t_1-1)\right\}$$
$$+\frac{cI_c[(1-\lambda)P-D]}{\theta_f^2}(e^{-\theta_f N}+\theta_f N-1)$$
$$\left.-\gamma sI_e\left[\frac{DT^2}{2}+DT(M-T-N)\right]\right\}, \qquad (20)$$

$$TP_4(t_1,T)$$
$$=[TP_W(t_1,T)-IC_{2m}-IC_{2g}+IE_1]/T$$
$$=sD-\frac{1}{T}\left\{S+A+\frac{c_m rP}{\theta_m}(e^{\theta_m t_1}-1)+cPt_1\right.$$
$$+h_1\left\{\frac{(1-\lambda)P-D}{\theta_f^2}(e^{-\theta_f t_1}+\theta_f t_1-1)\right.$$
$$\left.+\frac{D}{\theta_f^2}\left[e^{\theta_f(T-t_1)}-\theta_f(T-t_1)-1\right]\right\}$$
$$+\frac{h_2\lambda Pt_1^2}{2}+\frac{h_m rP}{\theta_m^2}(e^{\theta_m t_1}-\theta_m t_1-1)$$
$$+k\lambda Pt_1+c_m I_c rP\left\{\frac{[\alpha(M+l)+\beta M]}{\theta_m}(e^{\theta_m t_1}-1)\right.$$
$$\left.+\frac{(\alpha+\beta)}{\theta_m^2}(e^{\theta_m t_1}-\theta_m t_1-1)\right\}$$
$$+cI_c\left\{\frac{(1-\lambda)P-D}{\theta_f^2}(e^{-\theta_f t_1}+\theta_f t_1-1)\right.$$
$$\left.+\frac{D}{\theta^2}\left[e^{\theta_f(T-t_1)}-e^{\theta_f(T-N)}-\theta_f(N-t_1)\right]\right\}$$
$$\left.-\frac{\gamma sI_e D(M-N)^2}{2}\right\}, \qquad (21)$$

$$TP_5(t_1,T)$$
$$=[TP_W(t_1,T)-IC_{2m}-IC_{2g}+IE_2]/T$$
$$=sD-\frac{1}{T}\left\{S+A+\frac{c_m rP}{\theta_m}(e^{\theta_m t_1}-1)+cPt_1\right.$$
$$+h_1\left\{\frac{(1-\lambda)P-D}{\theta_f^2}(e^{-\theta_f t_1}+\theta_f t_1-1)\right.$$
$$\left.+\frac{D}{\theta_f^2}[e^{\theta_f(T-t_1)}-\theta_f(T-t_1)-1]\right\}+\frac{h_2\lambda Pt_1^2}{2}$$
$$+\frac{h_m rP}{\theta_m^2}(e^{\theta_m t_1}-\theta_m t_1-1)+k\lambda Pt_1$$
$$+c_m I_c rP\left\{\frac{[\alpha(M+l)+\beta M]}{\theta_m}(e^{\theta_m t_1}-1)\right.$$
$$\left.+\frac{(\alpha+\beta)}{\theta_m^2}(e^{\theta_m t_1}-\theta_m t_1-1)\right\}$$
$$+cI_c\left\{\frac{(1-\lambda)P-D}{\theta_f^2}(e^{-\theta_f t_1}+\theta_f t_1-1)\right.$$
$$\left.+\frac{D}{\theta_f^2}\left[e^{\theta_f(T-t_1)}-e^{\theta_f(T-N)}-\theta_f(N-t_1)\right]\right\}$$
$$\left.-\gamma sI_e\left[\frac{DT^2}{2}+DT(M-T-N)\right]\right\}, \qquad (22)$$

$$TP_6(t_1,T)$$
$$=[TP_W(t_1,T)-IC_{2m}-IC_{3g}+IE_1]/T$$
$$=sD-\frac{1}{T}\left\{S+A+\frac{c_m rP}{\theta_m}(e^{\theta_m t_1}-1)+cPt_1\right.$$
$$+h_1\left\{\frac{(1-\lambda)P-D}{\theta_f^2}(e^{-\theta_f t_1}+\theta_f t_1-1)\right.$$
$$\left.+\frac{D}{\theta_f^2}[e^{\theta_f(T-t_1)}-\theta_f(T-t_1)-1]\right\}$$
$$+\frac{h_2\lambda Pt_1^2}{2}+\frac{h_m rP}{\theta_m^2}(e^{\theta_m t_1}-\theta_m t_1-1)+k\lambda Pt_1$$
$$+c_m I_c rP\left\{\frac{[\alpha(M+l)+\beta M]}{\theta_m}(e^{\theta_m t_1}-1)\right.$$
$$\left.+\frac{(\alpha+\beta)}{\theta_m^2}(e^{\theta_m t_1}-\theta_m t_1-1)\right\}$$
$$+cI_c\left\{\frac{(1-\lambda)P-D}{\theta_f^2}(e^{-\theta_f t_1}+\theta_f t_1-1)\right.$$
$$\left.+\frac{D}{\theta_f^2}\left[e^{\theta_f(T-t_1)}-\theta_f(T-t_1)-1\right]\right\}$$
$$\left.-\frac{\gamma sI_e D(M-N)^2}{2}\right\}, \qquad (23)$$

and

$$\begin{aligned} TP_7(t_1,T) &= [TP_W(t_1,T) - IC_{2m} - IC_{3g} + IE_2]/T \\ &= sD - \frac{1}{T}\left\{S + A + \frac{c_m rP}{\theta_m}(e^{\theta_m t_1}-1) + cPt_1 \right. \\ &\quad + h_1\left\{\frac{(1-\lambda)P-D}{\theta_f^2}(e^{-\theta_f t_1}+\theta_f t_1 - 1)\right. \\ &\quad \left. + \frac{D}{\theta_f^2}[e^{\theta_f(T-t_1)} - \theta_f(T-t_1) - 1]\right\} + \frac{h_2\lambda P t_1^2}{2} \\ &\quad + \frac{h_m rP}{\theta_m^2}(e^{\theta_m t_1} - \theta_m t_1 - 1) + k\lambda P t_1 \\ &\quad + c_m I_c rP\left\{\frac{[\alpha(M+l)+\beta M]}{\theta_m}(e^{\theta_m t_1}-1)\right. \\ &\quad \left. + \frac{(\alpha+\beta)}{\theta_m^2}(e^{\theta_m t_1} - \theta_m t_1 - 1)\right\} \\ &\quad + cI_c\left\{\frac{(1-\lambda)P-D}{\theta_f^2}(e^{-\theta_f t_1}+\theta_f t_1 - 1)\right. \\ &\quad \left. + \frac{D}{\theta_f^2}\left[e^{\theta_f(T-t_1)} - \theta_f(T-t_1) - 1\right]\right\} \\ &\quad \left. - \gamma s I_e\left[\frac{DT^2}{2} + DT(M-T-N)\right]\right\}. \end{aligned} \tag{24}$$

The manufacturer's carbon emissions per production cycle are related to production, setup, material ordering and purchase, disposal of defective products, and inventory levels. Thus, the carbon emission per unit time, denoted as $E(t_1, T)$, can be computed as follows:

$$\begin{aligned} E(t_1,T) &= \frac{1}{T}\left\{\hat{S} + \hat{A} + \frac{\hat{c}_m rP}{\theta_m}(e^{\theta_m t_1}-1) - \hat{c}Pt_1 \right. \\ &\quad - \hat{h}_1\left\{\frac{(1-\lambda)P-D}{\theta_f^2}(e^{-\theta_f t_1}+\theta_f t_1 - 1)\right. \\ &\quad \left. + \frac{D}{\theta_f^2}[e^{\theta_f(T-t_1)} - \theta_f(T-t_1) - 1]\right\} \\ &\quad + \frac{\hat{h}_2\lambda P t_1^2}{2} + \frac{\hat{h}_m rP}{\theta_m^2}(e^{\theta_m t_1} - \theta_m t_1 - 1) + \hat{k}\lambda P t_1. \end{aligned} \tag{25}$$

Under the carbon cap-and-trade policy, the manufacturer's total carbon emissions are subjected to a quota limit of ϖ. When the manufacturer's carbon emissions exceed this limit, the excess portion must be purchased at a market price of p_c. By contrast, when the manufacturer does not reach its carbon emission limit, the unused portion of the quota can be sold at a price p_c. In this context, we assume that emission allowances can be sold and purchased in the market. Under the cap-and-trade policy, the total profits of the manufacturer per unit time, denoted as $TP_{CC}(t_1, T)$, is calculated as follows:

$$TP_{CC}(t_1,T) = TP(t_1,T) - p_c[E(t_1,T) - \varpi], \tag{26}$$

where $TP(t_1, T)$ and $E(t_1, T)$ are expressed using Eq. (17) and Eq. (25), respectively.

B. MODEL SOLUTION

The present study aims to determine the optimal production and replenishment cycle lengths for maximizing the total profits per unit time under a carbon cap-and-trade policy. Eq. (8) indicates that T is a function of t_1, suggesting that $TP_{CC_i}(t_1, T)$ can be reduced to $TP_{CC_i}(t_1)$, $i = 1, 2,\ldots,7$.

To verify *the* concavity of $TP_{CC_i}(t_1, T)$ with respect to t_1, the following lemma is required.

Lemma 1: T is increasing and concave in t_1.

Proof: Based on Eq. (8), the first and second derivatives of T with respect to t_1 are used to obtain the following equations:

$$\frac{dT}{dt_1} = \frac{(1-\lambda)Pe^{\theta_f t_1}}{(1-\lambda)P(e^{\theta_f t_1}-1)+D} > 0,$$

and

$$\frac{d^2T}{dt_1^2} = \frac{-(1-\lambda)P\theta_f e^{\theta_f t_1}[(1-\lambda)P-D]}{[(1-\lambda)P(e^{\theta_f t_1}-1)+D]^2} < 0.$$

Therefore, T is increasing and concave in t_1. Thus, the proof is complete.

Next, the concavity of $TP_{CC_i}(t_1)$, $i = 1, 2, 3, 4, 5, 6, 7$ with respect to t_1, and the solution process for multiple scenarios are described.

Situation 1. $M \le t_1$.

The necessary condition for maximizing the total profits per unit time $TP_{CC_1}(t_1)$ is $\frac{dTP_{CC_1}(t_1)}{dt_1} = 0$, from which the following equation can be derived:

$$\begin{aligned} &\frac{dTP_{CC_1}(t_1)}{dt_1} \\ &= \frac{1}{[T(t_1)]^2}\left(\frac{dT}{dt_1}\right)\left\{(S+A) + p_c(\hat{S}+\hat{A})\right. \\ &\quad + \frac{(c_m + p_c\hat{c}_m)rP}{\theta_m}(e^{\theta_m t_1}-1) \\ &\quad + [(c+\lambda k) + p_c(\hat{c}+\lambda\hat{k})]Pt_1 \\ &\quad + \frac{\left(h_1 + p_c\hat{h}_1\right)[(1-\lambda)P-D]\,t_1}{\theta_f^2} \\ &\quad + \frac{\left(h_2+p_c\hat{h}_2\right)\lambda P t_1^2}{2} + \frac{\left(h_m + p_c\hat{h}_m\right)rP}{\theta_m^2}(e^{\theta_m t_1} \\ &\quad - \theta_m t_1 - 1) + c_m I_c rP\left\{\frac{[\alpha(M+l)+\beta M]}{\theta_m}\left(e^{\theta_m t_1}-1\right)\right. \\ &\quad + \frac{(\alpha+\beta)}{\theta_m^2}(e^{\theta_m t_1} - \theta_m t_1 - 1) + \frac{cI_c[(1-\lambda)P-D]}{\theta_f^2} \\ &\quad \times(e^{-\theta_f N} + \theta_f N - 1) \\ &\quad \left. + \frac{\gamma}{\theta_m^2}\left[e^{\theta_m(t_1-M)} - \theta_m(t_1-M) - 1\right]\right\} \\ &\quad \left. - \frac{\gamma s I_e D(M-N)^2}{2}\right\} - \frac{1}{T(t_1)}\{\} \\ &\quad + p_c(\hat{c}+\lambda\hat{k})]P + \frac{\left(h_1+p_c\hat{h}_1\right)[(1-\lambda)P-D]}{\theta_f} \end{aligned}$$

$$+(h_2+p_c\hat{h}_2)\lambda Pt_1+\frac{\left(h_m+p_c\hat{h}_m\right)rP}{\theta_m}(e^{\theta_m t_1}-1)$$
$$+c_mI_crP[\alpha(M+l)+\beta M]e^{\theta_m t_1}$$
$$+\frac{(\alpha+\beta)}{\theta_m}\left(e^{\theta_m t_1}-1\right)$$
$$+\frac{\gamma}{\theta_m}\left[e^{\theta_m(t_1-M)}-1\right]\Big\}\Big\}=0. \quad (27)$$

Based on Lemma 1, the following theorem is obtained:

Theorem 1: The total profits per unit time $TP_{CC_1}(t_1)$ is concave and reaches its local maximum at point t_{11}; that is, the point that satisfies Eq.(27) and $M\le t_{11}$.

Proof: Please see Appendix A.

Situation 2. $N\le t_1\le M\le T+N$.

The necessary condition for maximizing total profits per unit time $TP_{CC_2}(t_1)$ is $\frac{dTP_{CC_2}(t_1)}{dt_1}=0$, from which the following equation can be derived:

$$\frac{dTP_{CC_2}(t_1)}{dt_1}=\frac{1}{[T(t_1)]^2}\left(\frac{dT}{dt_1}\right)\Big\{(S+A)+p_c(\hat{S}+\hat{A})$$
$$+\frac{(c_m+p_c\hat{c}_m)rP}{\theta_m}(e^{\theta_m t_1}-1$$
$$+[(c+\lambda k)+p_c(\hat{c}+\lambda\hat{k})]Pt_1$$
$$+\frac{\left(h_1+p_c\hat{h}_1\right)[(1-\lambda)P-D]t_1}{\theta_f^2}$$
$$+\frac{\left(h_2+p_c\hat{h}_2\right)\lambda Pt_1^2}{2}$$
$$+\frac{\left(h_m+p_c\hat{h}_m\right)rP}{\theta_m^2}(e^{\theta_m t_1}-\theta_m t_1-1)$$
$$+c_mI_crP\left\{\frac{[\alpha(M+l)+\beta M]}{\theta_m}\left(e^{\theta_m t_1}-1\right)\right.$$
$$\left.+\frac{(\alpha+\beta)}{\theta_m^2}\left(e^{\theta_m t_1}-\theta_m t_1-1\right)\right\}$$
$$+\frac{cI_c[(1-\lambda)P-D]}{\theta_f^2}(e^{-\theta_f N}+\theta_f N-1)$$
$$\left.-\frac{\gamma sI_eD(M-N)^2}{2}\right\}$$
$$-\frac{1}{T(t_1)}\{$$
$$+p_c(\hat{c}+\lambda\hat{k})]P+\frac{(h_1+p_c\hat{h}_1)[(1-\lambda)P-D]}{\theta_f}$$
$$+(h_2+p_c\hat{h}_2)\lambda Pt_1+\frac{(h_m+p_c\hat{h}_m)rP}{\theta_m}$$
$$\times(e^{\theta_m t_1}-1)+c_mI_crP\left\{[\alpha(M+l)+\beta M]e^{\theta_m t_1}\right.$$
$$\left.+\frac{(\alpha+\beta)}{\theta_m}(e^{\theta_m t_1}-1)\right\}\Big\}=0. \quad (28)$$

Subsequently, the following theorem is obtained:

Theorem 2: The total profits per unit time $TP_{CC_2}(t_1)$ is concave and reaches its local maximum at point t_{12}; that is, the point that satisfies Eq.(28) and $N\le t_{12}\le M\le T(t_{12})+N$.

Proof: Please see Appendix B.

Situation 3. $N\le t_1\le T+N\le M$.

The necessary condition for maximizing total profits per unit time $TP_{CC_3}(t_1)$ is $\frac{dTP_{CC_3}(t_1)}{dt_1}=0$, from which the following equation can be derived:

$$\frac{dTP_{CC_3}(t_1)}{dt_1}$$
$$=-\frac{\gamma sI_e}{2}\left(\frac{dT}{dt_1}\right)+\frac{1}{[T(t_1)]^2}\left(\frac{dT}{dt_1}\right)\{(S+A)$$
$$+p_c(\hat{S}+\hat{A})+\frac{(c_m+p_c\hat{c}_m)rP}{\theta_m}(e^{\theta_m t_1}-1)$$
$$+[(c+\lambda k)+p_c(\hat{c}+\lambda\hat{k})]Pt_1$$
$$+\frac{(h_1+p_c\hat{h}_1)[(1-\lambda)P-D]t_1}{\theta_f^2}$$
$$+\frac{(h_2+p_c\hat{h}_2)\lambda Pt_1^2}{2}$$
$$+\frac{(h_m+p_c\hat{h}_m)rP}{\theta_m^2}(e^{\theta_m t_1}-\theta_m t_1-1)$$
$$+c_mI_crP\left\{\frac{[\alpha(M+l)+\beta M]}{\theta_m}(e^{\theta_m t_1}-1)\right.$$
$$\left.+\frac{(\alpha+\beta)}{\theta_m^2}(e^{\theta_m t_1}-\theta_m t_1-1)\right\}$$
$$+\frac{cI_c[(1-\lambda)P-D]}{\theta_f^2}(e^{-\theta_f N}+\theta_f N-1)-\frac{1}{T(t_1)}\{$$
$$+p_c(\hat{c}+\lambda\hat{k})]P+\frac{(h_1+p_c\hat{h}_1)[(1-\lambda)P-D]}{\theta_f}$$
$$+(h_2+p_c\hat{h}_2)\lambda Pt_1+\frac{(h_m+p_c\hat{h}_m)rP(e^{\theta_m t_1}-1)}{\theta_m})$$
$$+c_mI_crP\left\{[\alpha(M+l)+\beta M]e^{\theta_m t_1}\right.$$
$$\left.+\frac{(\alpha+\beta)}{\theta_m}(e^{\theta_m t_1}-1)\right\}\Big\}=0. \quad (29)$$

Subsequently, the following theorem is obtained:

Theorem 3: The total profits per unit time $TP_{CC_3}(t_1)$ is concave and reaches its local maximum at point t_{13}; that is, the point that satisfies Eq.(29) and $N\le t_{13}\le T(t_{13})+N\le M$.

Proof: Please see Appendix C.

Situation 4. $t_1\le N\le M\le T+N$.

The necessary condition for maximizing total profits per unit time $TP_{CC_4}(t_1)$ is $\frac{dTP_{CC_4}(t_1)}{dt_1}=0$, from which the following equation can be derived:

$$\frac{dTP_{CC_4}(t_1)}{dt_1}$$
$$=\frac{1}{[T(t_1)]^2}\left(\frac{dT}{dt_1}\right)\Big\{(S+A)+p_c(\hat{S}+\hat{A})$$
$$+\frac{(c_m+p_c\hat{c}_m)rP}{\theta_m}(e^{\theta_m t_1}-1)$$

$$
\begin{aligned}
&+[(c+\lambda k)+p_c(\hat{c}+\lambda\hat{k})]Pt_1\\
&+\frac{\left(h_1+p_c\hat{h}_1\right)[(1-\lambda)P-D]t_1}{\theta_f^2}\\
&+\frac{\left(h_2+p_c\hat{h}_2\right)\lambda Pt_1^2}{2}\\
&+\frac{\left(h_m+p_c\hat{h}_m\right)rP}{\theta_m^2}(e^{\theta_m t_1}-\theta_m t_1-1)\\
&+c_mI_crP\left\{\frac{[\alpha(M+l)+\beta M]}{\theta_m}\left(e^{\theta_m t_1}-1\right)\right.\\
&\left.+\frac{(\alpha+\beta)}{\theta_m^2}\left(e^{\theta_m t_1}-\theta_m t_1-1\right)\right\}\\
&+cI_c\left\{\frac{(1-\lambda)P-D}{\theta_f^2}\left(e^{-\theta_f t_1}+\theta_f t_1-1\right)\right.\\
&+\frac{\left(e^{-\theta_f t_1}-e^{-\theta_f N}\right)}{\theta_f^2}\left\{(1-\lambda)Pe^{\theta_f t_1}-[(1-\lambda)P-D]\right\}\\
&\left.-\frac{D}{\theta_f}(N-t_1)\right\}\\
&\left.-\frac{\gamma sI_eD(M-N)^2}{2}\right\}-\frac{1}{T(t_1)}\{\\
&+[(c+\lambda k)+p_c(\hat{c}+\lambda\hat{k})]P\\
&+\frac{\left(h_1+p_c\hat{h}_1\right)[(1-\lambda)P-D]}{\theta_f}\\
&+(h_2+p_c\hat{h}_2)\lambda Pt_1+\frac{\left(h_m+p_c\hat{h}_m\right)rP}{\theta_m}(e^{\theta_m t_1}-1)\\
&+c_mI_crP[\alpha(M+l)+\beta M]e^{\theta_m t_1}\\
&+\frac{(\alpha+\beta)}{\theta_m}(e^{\theta_m t_1}-1)\}\\
&\left.+\frac{cI_c(1-\lambda)P}{\theta_f}[1-e^{-\theta_f(N-t_1)}]\right\}=0. \qquad (30)
\end{aligned}
$$

Subsequently, the following theorem is obtained:

Theorem 4: The total profits per unit time $TP_{CC_4}(t_1)$ is concave and reaches its local maximum at point t_{14}; that is, the point that satisfies Eq.(30) and $t_{14}\leq N\leq M\leq T(t_{14})+N$.

Proof: Please see Appendix D.

Situation 5. $t_1\leq N\leq T+N\leq M$.

The necessary condition for maximizing total profits per unit time $TP_{CC_5}(t_1)$ is $\frac{dTP_{CC_5}(t_1)}{dt_1}=0$, from which the following equation can be derived:

$$
\begin{aligned}
\frac{dTP_{CC_5}(t_1)}{dt_1}=&-\frac{\gamma sI_e}{2}\left(\frac{dT}{dt_1}\right)+\frac{1}{[T(t_1)]^2}\left(\frac{dT}{dt_1}\right)\{(S+A)\\
&+p_c(\hat{S}+\hat{A})+\frac{\left(c_m+p_c\hat{c}_m\right)rP}{\theta_m}(e^{\theta_m t_1}-1)\\
&+[(c+\lambda k)+p_c(\hat{c}+\lambda\hat{k})]Pt_1\\
&+\frac{\left(h_1+p_c\hat{h}_1\right)[(1-\lambda)P-D]t_1}{\theta_f^2}\\
&+\frac{\left(h_2+p_c\hat{h}_2\right)\lambda Pt_1^2}{2}\\
&+\frac{\left(h_m+p_c\hat{h}_m\right)rP}{\theta_m^2}(e^{\theta_m t_1}-\theta_m t_1-1)\\
&+c_mI_crP\left\{\frac{[\alpha(M+l)+\beta M]}{\theta_m}\left(e^{\theta_m t_1}-1\right)\right.\\
&\left.+\frac{(\alpha+\beta)}{\theta_m^2}\left(e^{\theta_m t_1}-\theta_m t_1-1\right)\right\}\\
&+cI_c\left\{\frac{(1-\lambda)P-D}{\theta_f^2}\left(e^{-\theta_f t_1}+\theta_f t_1-1\right)\right.\\
&+\frac{\left(e^{-\theta_f t_1}-e^{-\theta_f N}\right)}{\theta^2}\left\{(1-\lambda)Pe^{\theta_f t_1}\right.\\
&\left.\left.-[(1-\lambda)P-D]\right\}-\frac{D}{\theta_f}(N-t_1)\right\}\Big\}\\
&-\frac{1}{T(t_1)}\{\\
&+p_c(\hat{c}+\lambda\hat{k})]P+\frac{\left(h_1+p_c\hat{h}_1\right)[(1-\lambda)P-D]}{\theta_f}\\
&+(h_2+p_c\hat{h}_2)\lambda Pt_1\\
&+\frac{\left(h_m+p_c\hat{h}_m\right)rP}{\theta_m}(e^{\theta_m t_1}-1)\\
&+c_mI_crP\left\{[\alpha(M+l)+\beta M]e^{\theta_m t_1}\right.\\
&\left.+\frac{(\alpha+\beta)}{\theta_m}(e^{\theta_m t_1}-1)\right\}\\
&\left.+\frac{cI_c(1-\lambda)P}{\theta_f}[1-e^{-\theta_f(N-t_1)}]\right\}=0. \qquad (31)
\end{aligned}
$$

Subsequently, the following theorem is obtained:

Theorem 5: The total profits per unit time $TP_{CC_5}(t_1)$ is concave and reaches its local maximum at point t_{15}; that is, the point that satisfies Eq.(31) and $t_{15}\leq N\leq T(t_{15})+N\leq M$.

Proof: Please see Appendix E.

Situation 6. $T\leq N\leq M\leq T+N$.

The necessary condition for maximizing total profits per unit time $TP_{CC_6}(t_1)$ is $\frac{dTP_{CC_6}(t_1)}{dt_1}=0$, from which the following equation can be derived:

$$
\begin{aligned}
&\frac{dTP_{CC_6}(t_1)}{dt_1}\\
&=\frac{1}{[T(t_1)]^2}\left(\frac{dT}{dt_1}\right)\left\{(S+A)+p_c(\hat{S}+\hat{A})\right.\\
&+\frac{(c_m+p_c\hat{c}_m)rP}{\theta_m}(e^{\theta_m t_1}-1)\\
&+[(c+\lambda k)+p_c(\hat{c}+\lambda\hat{k})]Pt_1
\end{aligned}
$$

$$+\frac{(h_1+cI_c+p_c\hat{h}_1)[(1-\lambda)P-D]t_1}{\theta_f^2}$$
$$+\frac{(h_2+p_c\hat{h}_2)\lambda Pt_1^2}{2}$$
$$+\frac{(h_m+p_c\hat{h}_m)rP}{\theta_m^2}(e^{\theta_m t_1}-\theta_m t_1-1$$
$$+c_mI_crP\left\{\frac{[\alpha(M+l)+\beta M]}{\theta_m}(e^{\theta_m t_1}-1)\right.$$
$$\left.+\frac{(\alpha+\beta)}{\theta_m^2}(e^{\theta_m t_1}-\theta_m t_1-1)\right\}$$
$$\left.-\frac{\gamma sI_eD(M-N)^2}{2}\right\}-\frac{1}{T(t_1)}+[(c+\lambda k)$$
$$+p_c(\hat{c}+\lambda\hat{k})]P$$
$$+\frac{(h_1+cI_c+p_c\hat{h}_1)[(1-\lambda)P-D]}{\theta_f}$$
$$+(h_2+p_c\hat{h}_2)\lambda Pt_1$$
$$+\frac{(h_m+p_c\hat{h}_m)rP}{\theta_m}(e^{\theta_m t_1}-1)$$
$$+c_mI_crP\left\{[\alpha(M+l)+\beta M]e^{\theta_m t_1}\right.$$
$$\left.\left.+\frac{(\alpha+\beta)}{\theta_m}(e^{\theta_m t_1}-1)\right\}\right\}=0. \tag{32}$$

Subsequently, the following theorem is obtained:

Theorem 6: The total profits per unit time $TP_{CC_6}(t_1)$ is concave and reaches its local maximum at point t_{16}; that is, the point that satisfies Eq.(32) and $T(t_{16}) \leq N \leq M \leq T(t_{16})+N$.

Proof: Please see Appendix F.

Situation 7. $T \leq N \leq T+N \leq M$.

The necessary condition for maximizing total profits per unit time $TP_{CC_7}(t_1)$ is $\frac{dTP_{CC_7}(t_1)}{dt_1}=0$, from which the following equation can be derived:

$$\frac{dTP_{CC_7}(t_1)}{dt_1}$$
$$=-\frac{\gamma sI_e}{2}\left(\frac{dT}{dt_1}\right)+\frac{1}{[T(t_1)]^2}\left(\frac{dT}{dt_1}\right)\{(S+A)$$
$$+p_c(\hat{S}+\hat{A})+\frac{(c_m+p_c\hat{c}_m)\,rP}{\theta_m}(e^{\theta_m t_1}-1)$$
$$+[(c+\lambda k)+p_c(\hat{c}+\lambda\hat{k})]Pt_1$$
$$+\frac{\left(h_1+cI_c+p_c\hat{h}_1\right)[(1-\lambda)\,P-D]\,t_1}{\theta_f^2}$$
$$+\frac{\left(h_2+p_c\hat{h}_2\right)\lambda Pt_1^2}{2}$$
$$+\frac{\left(h_m+p_c\hat{h}_m\right)rP}{\theta_m^2}(e^{\theta_m t_1}-\theta_m t_1-1)$$
$$+c_mI_crP\left\{\frac{[\alpha\,(M+l)+\beta M]}{\theta_m}\left(e^{\theta_m t_1}-1\right)\right.$$
$$\left.+\frac{(\alpha+\beta)}{\theta_m^2}\left(e^{\theta_m t_1}-\theta_m t_1-1\right)\right\}-\frac{1}{T\,(t_1)}\{$$
$$+p_c(\hat{c}+\lambda\hat{k})]P+\frac{(h_1+p_c\hat{h}_1)[(1-\lambda)P-D]}{\theta_f}$$
$$+(h_2+p_c\hat{h}_2)\lambda Pt_1+\frac{(h_m+p_c\hat{h}_m)rP}{\theta_m}(e^{\theta_m t_1}-1)$$
$$+c_mI_crP\left\{[\alpha(M+l)+\beta M]e^{\theta_m t_1}\right.$$
$$\left.\left.+\frac{(\alpha+\beta)}{\theta_m}(e^{\theta_m t_1}-1)\right\}\right\}=0. \tag{33}$$

Subsequently, the following theorem is obtained:

Theorem 7: The total profits per unit time $TP_{CC_7}(t_1)$ is concave and reaches its local maximum at point t_{17}; that is, the point that satisfies Eq.(33) and $T(t_{17}) \leq N \leq T(t_{17})+N \leq M$.

Proof: Please see Appendix G.

Next, the following algorithm is applied to ensure that t_1 and T are within the feasible range and determine whether the optimal solution for the entire problem has been identified. Numerous examples are examined using the Mathematica 12.0 software (Wolfram Research, Inc., Champaign, IL) to identify the optimal solution under given parameters.

C. ALOGRITHM

Step 1. The value of t_{11} is determined by solving Eq. (27) and substituting t_{11} into Eq. (8) to obtain the value of T (denoted by T_1).

Step 1-1. When $M \leq t_{11}$, t_{11} is a feasible solution, and $t_1 = t_{11}$ is set and substituted into Eq. (26) to calculate $TP_{CC_1}(t_1)$.

Step 1-2. Otherwise, t_{11} is not a feasible solution, and $TP_{CC_1}(t_1)$ is set to equal 0.

Step 2. The value of t_{12} is determined by solving Eq. (28) and substituting t_{12} into Eq. (8) to obtain T (denoted by T_2).

Step 2-1. When $N \leq t_{12} \leq M \leq T_2+N$, t_{12} is a feasible solution, and $t_1 = t_{12}$ is set and substituted into Eq. (26) to calculate $TP_{CC_2}(t_1)$.

Step 2-2. Otherwise, t_{12} is not a feasible solution, and $TP_{CC_2}(t_1)$ is set to equal 0.

Step 3. The value of t_{13} is determined by solving Eq. (29) and substituting t_{13} into Eq. (8) to obtain T (denoted by T_3).

Step 3-1. When $N \leq t_{13} \leq T_3 + N \leq M$, t_{13} is a feasible solution, and $t_1 = t_{13}$ is substituted into Eq. (26) to calculate $TP_{CC_3}(t_1)$.

Step 3-2. Otherwise, t_{13} is not a feasible solution, and $TP_{CC_3}(t_1)$ is set to 0.

Step 4. The value of t_{14} is determined by solving Eq. (30) and substituting t_{14} into Eq. (8) to obtain T (denoted by T_4).

Step 4-1. When $t_{14} \leq N \leq M \leq T_4 + N$, t_{14} is a feasible solution, and $t_1 = t_{14}$ is set and substituted into Eq. (26) to calculate $TP_{CC_4}(t_1)$.

Step 4-2. Otherwise, t_{14} is not a feasible solution, and $TP_{CC_4}(t_1)$ is set to equal 0.

Step 5. The value of t_{15} is determined by solving Eq. (31) and substituting t_{15} into Eq. (8) to obtain T (denoted by T_5).

Step 5-1. When $t_{15} \leq N \leq T_5 + N \leq M$, t_{15} is a feasible solution, and $t_1 = t_{15}$ is set and substituted into Eq. (26) to calculate $TP_{CC_5}(t_1)$.

Step 5-2. Otherwise, t_{15} is not a feasible solution, and $TP_{CC_5}(t_1)$ is set to equal 0.

Step 6. The value of t_{16} is determined by solving Eq. (32) and substituting t_{16} into Eq. (8) to obtain T (denoted by T_6).

Step 6-1. When $T_1 \leq N \leq M \leq T_1 + N$, t_{16} is a feasible solution, and $t_1 = t_{16}$ is set and substituted into Eq. (26) to calculate $TP_{CC_6}(t_1)$.

Step 6-2. Otherwise, t_{16} is not a feasible solution, and $TP_{CC_6}(t_1)$ is set to equal 0.

Step 7. The value of t_{17} is determined by solving Eq. (33) and substituting t_{17} into Eq. (8) to obtain T (denoted by T_7).

Step 7-1. When $T_7 \leq N \leq T_7 + N \leq M$, t_{17} is a feasible solution, and $t_1 = t_{17}$ is set and substituted into Eq. (26) to calculate $TP_{CC_7}(t_1)$.

Step 7-2. Otherwise, t_{17} is not a feasible solution, and $TP_{CC_7}(t_1)$ is set to equal 0.

Step 8. $TP_{CCi}(t_1)$ is determined. When $TP_{CCi}(t_1) = TP_{CCi}(t_{1i})$, $(t_1^*, T^*) = (t_{1i}, T_i)$ is the optimal solution, where i= 1, 2, …, 7.

Step 9. The algorithm is terminated.

V. NUMERICAL EXAMPLES

In this section, the aforementioned theoretical results are verified, and the proposed algorithm is applied to the following numerical examples.

Example 1. First, we consider a more realistic example. Because numerous parameters are considered in the proposed model, we simplify the process by first classifying them into four categories, namely production or demand related, inventory related, trade credit, and carbon emission parameters. The values of the parameters for each category are as follows:

Production/demand-related parameters	P=5000 unit/year D=2000 unit/year S=\$500/time A=\$300/time c_m=\$5/unit c=\$10/unit s=\$30/unit λ=0.05 r= 1 k=\$2/unit
Inventory-related parameters	h_1=\$2/unit/year h_2=\$1/unit/year h_m=\$0.5/unit/year θ_f=0.05 θ_m=0.03
Trade credit parameters	α=0.2 β=0.3 γ=0.5, I_c=0.03 I_e=0.01 l=15/365 years M=55/365 years N=30/365 years
Carbon emission parameters	$\hat{S}$=300 kg/time $\hat{A}$=100 kg/time $\hat{c}$=1 kg/unit $\hat{c}_m$=0.5 kg/unit $\hat{h}_1$=0.05 kg/unit/year $\hat{h}_2$=0.05 kg/unit/year $\hat{h}_m$=0.03 kg/unit/year p_c=\$1/unit ϖ=500 kg

Through the application of the aforementioned algorithm, the optimal solutions are obtained with $t_1^* = t_{11} = 0.317043$, $T^* = T_1 = 0.74493$, $q_m^* = 1592.78$, $E(t_1^*, T^*) = 3766.65$, and $TP_{CC}(t_1^*, T^*) = 18{,}320.6$ in Scenario 1.

Example 2. To compare the proposed model with the models of other studies, we consider the optimal solutions, total annual carbon emissions, and total annual profits under various payment arrangements in the presence or absence of a carbon emission policy (i.e., p_c=1 or $p_c = 0$), and adjust the proportional parameters α, β, and γ on the basis of the parameter values presented in Example 1. The comparison of the optimal solutions is displayed in Table 2.

Table 2 indicates that in both the presence and absence of a carbon reduction policy, the implementation of payment terms involving full trade credit (i.e., $\gamma = 1$) results in the longest optimal production period and replenishment cycle, the largest optimal quantity of raw materials, and the lowest annual carbon emission and highest total annual profit. When payment terms are not considered (i.e., $\alpha = 1$) and a carbon emission policy is implemented, the production period and replenishment cycle are shortest, the quantity of raw materials is smallest, the annual carbon emissions are highest, and the

TABLE 2. Comparison of multiple payment types in the presence or absence of a carbon emission policy.

Cases	Types	t_1^*	T^*	q_m^*	E^*	TP_{CC}^*
$p_c = 1$	$\alpha = 1$	0.31642	0.74348	1589.62	3767.56	18,266.3
	$\beta = 1$	0.31643	0.74351	1589.69	3767.54	18,275.1
	$\gamma = 1$	0.31766	0.74636	1595.89	3765.76	18,369.6
	$\alpha = 0$ $\beta = 0.4$ $\gamma = 0.6$	0.31717	0.74522	1593.41	3766.47	18,331.8
$p_c = 0$	$\alpha = 1$	0.26427	0.62202	1326.58	3860.83	21,575.3
	$\beta = 1$	0.26428	0.62205	1326.64	3860.81	21,584.0
	$\gamma = 1$	0.26575	0.62549	1334.07	3857.61	21,676.1
	$\alpha = 0$ $\beta = 0.4$ $\gamma = 0.6$	0.26516	0.62412	1331.10	3858.88	21,639.3
The proposed model		0.31704	0.74493	1592.78	3766.65	18,320.6

total annual profits are lowest. In addition, when $\beta = 1$, it implies the traditional cash on delivery scenario. Based on the above, Table 2 indicates that our proposed model is a more of a general-purpose model relative to the other cases, which are all special cases of the proposed model.

VI. SENSITIVITY ANALYSIS

In this section, the effects of model parameters on optimal solutions are examined by using Example 1. Each parameter is increased or reduced by 25% or 50% while the remaining parameters are kept constant. Because numerous parameters are discussed, the four categories used in the previous section are applied.

A. SENSITIVITY ANALYSIS OF PRODUCTION- OR DEMAND-RELATED PARAMETERS

This subsection discusses the effects of changes to production- or demand-related parameters (P, S, A, c_m, c, λ, r, and k) on the values of t_1^*, T^*, q_m^*, E^* and TP_{CC}^*. The results of the sensitivity analysis are presented in Table 3.

Table 3 reveals several notable findings. First, an increase in production rate, unit production cost, unit purchase of raw materials, or unit disposal cost reduces the production period, replenishment cycle, order quantity of raw materials, and total annual profits. However, it increases annual carbon emissions; in particular, the effect of production rate changes is substantial. Second, an increase in setup cost or order cost increases the production period, replenishment cycle, and order quantity of raw materials but reduces total carbon emissions and total annual profits. Third, improvements in the material feeding effect reduce the number of materials used, thereby increasing total annual profits and lowering total carbon emissions. Finally, increases in demand and unit price lead to a proportional increase in total profits and carbon emissions and a shortening of the replenishment cycle. Specifically, an increase in demand increases the length of the production cycle and the order quantity of raw materials, whereas an increase in price has the opposite effect. Furthermore, carbon emissions are more sensitive to changes in demand than changes in unit price. However, total profits are more sensitive to changes in unit price than changes in demand.

TABLE 3. Sensitivity analysis of production- or demand-related parameters.

Parameters	Change (%)	Change (%) t_1^*	T^*	q_m^*	E^*	TP_{CC}^*
P	-50	157.6442	29.7243	29.7938	-3.5589	4.0916
	-25	43.1468	7.5761	7.5811	-1.0983	1.2663
	+25	-22.9701	-3.8193	-3.8179	0.6218	-0.7303
	+50	-37.2896	-6.1053	-6.1013	1.0197	-1.2107
S	-50	-11.0029	-10.8992	-11.0498	1.5349	1.9372
	-25	-5.3435	-5.2899	-5.3679	0.6949	0.9405
	+25	5.0756	5.0192	5.1012	-0.5851	-0.8935
	+50	9.9204	9.8052	9.9726	-1.0847	-1.7461
A	-50	-6.4483	-6.3845	-6.4774	0.8496	1.1353
	-25	-3.1712	-3.1387	-3.1863	0.4017	0.5578
	+25	3.0750	3.0415	3.0902	-0.3624	-0.5415
	+50	6.0623	5.9941	6.0925	-0.6911	-1.0671
c_m	-50	3.4528	3.4151	3.4700	-0.4054	29.3484
	-25	1.6805	1.6623	1.6882	-0.2012	14.6715
	+25	-1.5960	-1.5792	-1.6035	0.1983	-14.6676
	+50	-3.1141	-3.0822	-3.1285	0.3942	-29.3304
c	-50	3.6692	3.6291	3.6873	-0.4296	58.0980
	-25	1.7843	1.7653	1.7931	-0.2135	29.0460
	+25	-1.6925	-1.6748	-1.7008	0.2105	-29.0411
	+50	-3.3005	-3.2667	-3.3156	0.4187	-58.0771
λ	-50	-2.2344	0.3147	-2.2451	-2.2163	5.5047
	-25	-1.1282	0.1607	-1.1339	-1.1227	2.7881
	+25	1.1509	-0.1677	1.1565	1.1530	-2.8634
	+50	2.3252	-0.3426	2.3368	2.3376	-5.8049
r	−50	7.0918	7.0114	-46.4361	-15.1509	32.7839
	-25	3.3573	3.3207	-22.4695	-7.5635	16.3821
	+25	-3.0403	-3.0089	21.1818	7.5422	-16.3646
	+50	-5.8103	-5.7522	41.2455	15.0651	-32.7129
k	-50	0.7141	0.7064	0.7170	-0.0865	11.6153
	-25	0.3552	0.3513	0.3566	-0.0433	5.8077
	+25	-0.3514	-0.3477	-0.3535	0.0430	-5.8077
	+50	-0.6990	-0.6917	-0.7025	0.0860	-11.6153
D	-50	-35.3353	28.2844	-35.4443	-45.5261	-53.5444
	-25	-17.3775	9.8094	-17.4456	-22.4297	-27.1443
	+25	17.8111	-5.5200	17.9109	21.9395	27.6940
	+50	36.8578	-8.3506	37.0981	43.4730	55.8475
s	-50	0.6122	0.6057	0.6153	-0.0743	-100.0000
	-25	0.0104	0.0104	0.0107	-0.0013	-81.8771
	+25	-0.0104	-0.0105	-0.0106	0.0014	81.8767
	+50	-0.0211	-0.0208	-0.0213	0.0025	163.7534

B. SENSITIVITY ANALYSIS OF INVENTORY-RELATED PARAMETERS

This subsection discusses the effects of changes to inventory-related parameters (h_1, h_2, h_m, θ_f, and θ_m) on t_1^*, T^*, q_m^*, E^*, and TP_{CC}^*. The results of the sensitivity analysis are displayed in Table 4.

TABLE 4. Sensitivity analysis of inventory-related parameters.

Parameters	Change (%)	Change (%)				
		t_1^*	T^*	q_m^*	E^*	TP_{CC}^*
h_1	-50	16.8778	16.6696	16.9716	-1.7166	2.5403
	-25	7.4655	7.3807	7.5039	-0.8381	1.2210
	+25	-6.1068	-6.0460	-6.1339	0.8013	-1.1424
	+50	-11.2215	-11.1156	-11.2689	1.5696	-2.2199
h_2	-50	0.2681	0.2654	0.2693	-0.0327	0.0459
	-25	0.1337	0.1324	0.1344	-0.0165	0.0229
	+25	-0.1331	-0.1318	-0.1337	0.0162	-0.0229
	+50	-0.2659	-0.2631	-0.2675	0.0324	-0.0464
h_m	-50	2.8040	2.7734	2.8178	-0.3315	0.4683
	-25	1.3724	1.3574	1.3788	-0.1648	0.2325
	+25	-1.3169	-1.3030	-1.3228	0.1631	-0.2298
	+50	-2.5817	-2.5552	-2.5942	0.3248	-0.4558
θ_f	-50	6.9823	7.5141	7.0179	-1.3455	1.2145
	-25	3.3040	3.5515	3.3200	-0.6584	0.5961
	+25	-2.9895	-3.2073	-3.0036	0.6327	-0.5769
	+50	-5.7125	-6.1229	-5.7384	1.2430	-1.1359
θ_m	-50	0.9264	0.9165	0.6887	-0.1805	0.1556
	-25	0.4605	0.4556	0.3428	-0.0903	0.0775
	+25	-0.4555	-0.4506	-0.3397	0.0897	-0.0775
	+50	-0.9059	-0.8962	-0.6762	0.1792	-0.1550

Table 4 indicates that an increase in the holding cost of non-defective products, defective products, raw materials, the deterioration rate of finished products or raw materials reduces the production period, replenishment cycle, order quantity of raw materials, and total annual profits but increases the annual carbon emissions. Optimization results are most sensitive to changes in the holding cost of non-defective products but are relatively insensitive to changes in the holding cost or the deterioration rate of raw materials.

C. SENSITIVITY ANALYSIS OF TRADE CREDIT PARAMETERS

This subsection discusses the effects of changes to trade credit parameters (I_c, I_e, l, N, and M) on t_1^*, T^*, q_m^*, E^*, and TP_{CC}^*. The results of the sensitivity analysis are presented in Table 5.

As shown in Table 5, an increase in interest charged, interest earned or length of payment in advance decreases the production period, replenishment cycle, and order quantity of raw materials. However, an increase in the length of the trade credit period provided to the retailer or length of the trade credit period offered by the supplier increase the length of the production period, length of the replenishment cycle, and order quantity of raw materials. A higher interest charged or interest earned, longer length of payment in advance, shorter trade credit period provided to the retailer or trade credit period offered by the supplier leads to higher total carbon emissions. Finally, an increase in interest earned positively affects total annual profits, whereas an increase in interest charged, the length of the trade credit period provided to the retailer, or the length of the trade credit period offered by the supplier or the length of payment in advance negatively affects total profits.

TABLE 5. Sensitivity analysis of trade credit parameters.

Parameters	Change (%)	Change (%)				
		t_1^*	T^*	q_m^*	E^*	TP_{CC}^*
I_c	-50	0.6523	0.6453	0.6555	-0.0791	0.1725
	-25	0.3239	0.3204	0.3252	-0.0396	0.0862
	+25	-0.3195	-0.3163	-0.3215	0.0390	-0.0862
	+50	-0.6352	-0.6285	-0.6385	0.0781	-0.1719
I_e	-50	0.0211	0.0209	0.0213	-0.0027	-0.0038
	-25	0.0104	0.0105	0.0107	-0.0013	-0.0022
	+25	-0.0104	-0.0105	-0.0107	0.0013	0.0016
	+50	-0.0211	-0.0208	-0.0213	0.0024	0.0033
l	-50	0.0004	0.0004	0.0003	-0.00005	0.0044
	-25	0.0002	0.0002	0.0001	-0.00003	0.0022
	+25	-0.0002	-0.0002	-0.0003	0.00002	-0.0027
	+50	-0.0004	-0.0004	-0.0005	0.00005	-0.0049
N	-50	-0.1397	-0.1381	-0.1406	0.0170	0.0246
	-25	-0.0744	-0.0736	-0.0747	0.0090	0.0131
	+25	0.0836	0.0827	0.0835	-0.0104	-0.0147
	+50	0.1760	0.1742	0.1770	-0.0215	-0.0311
M	-50	-0.1158	-0.1145	-0.1161	0.0141	0.0202
	-25	-0.0606	-0.0597	-0.0609	0.0074	0.0104
	+25	0.0656	0.0648	0.0659	-0.0080	-0.0120
	+50	0.1363	0.1346	0.1369	-0.0167	-0.0246

D. SENSITIVITY ANALYSIS OF CARBON EMISSION PARAMETERS

In this subsection, the effects of changes to carbon emission parameters ($\hat{S}$, $\hat{A}$, $\hat{c}$, $\hat{c}_m$, $\hat{h}_1$, $\hat{h}_2$, $\hat{h}_m$, p_c, and ϖ) on t_1^*, T^*, q_m^*, E^*, and TP_{CC}^* are explored, and the results of the sensitivity analysis are presented in Table 6.

Table 6 indicates that an increase in carbon emissions (generated by production setup and material orders) increases the length of the production period, length of the replenishment cycle, and order quantity of raw materials. An increase in the carbon emissions generated by production, purchasing materials, holding products or raw materials leads to an increase in the length of the production period, length of the replenishment cycle, and order quantity of raw materials. Furthermore, annual carbon emissions increase but total annual profits decrease under increases in the carbon emissions generated

TABLE 6. Sensitivity analysis of carbon emission parameters.

Parameters	Change (%)	Change (%) t_1^*	T^*	q_m^*	E^*	TP_{CC}^*
$\hat{S}$	-50	-6.4483	-6.3845	-6.4774	-4.8608	1.1353
	-25	-3.1712	-3.1387	-3.1863	-2.3578	0.5578
	+25	3.0750	3.0415	3.0902	2.2317	-0.5415
	+50	6.0623	5.9941	6.0925	4.3524	-1.0671
$\hat{A}$	-50	-2.1029	-2.0810	-2.1127	-1.5568	0.3701
	-25	-1.0459	-1.0350	-1.0510	-0.7712	0.1839
	+25	1.0352	1.0243	1.0403	0.7569	-0.1823
	+50	2.0603	2.0379	2.0700	1.5008	-0.3630
$\hat{c}$	-50	0.3552	0.3513	0.3566	-28.2922	5.8077
	-25	0.1769	0.1752	0.1777	-14.1460	2.9038
	+25	-0.1760	-0.1742	-0.1770	14.1452	-2.9038
	+50	-0.3514	-0.3477	-0.3535	28.2899	-5.8077
$\hat{c}_m$	-50	0.2583	0.2555	0.2593	-14.2235	2.9175
	-25	0.1290	0.1275	0.1293	-7.1116	1.4585
	+25	-0.1284	-0.1270	-0.1293	7.1111	-1.4590
	+50	-0.2561	-0.2536	-0.2574	14.2222	-2.9180
$\hat{h}_1$	-50	0.3372	0.3337	0.3390	-0.3289	0.0590
	-25	0.1681	0.1665	0.1689	-0.1641	0.0295
	+25	-0.1675	-0.1655	-0.1683	0.1635	-0.0295
	+50	-0.3340	-0.3304	-0.3359	0.3266	-0.0590
$\hat{h}_2$	-50	0.0132	0.0133	0.0132	-0.0130	0.0022
	-25	0.0066	0.0066	0.0063	-0.0064	0.0011
	+25	-0.0066	-0.0066	-0.0069	0.0064	-0.0011
	+50	-0.0132	-0.0132	-0.0138	0.0127	-0.0027
$\hat{h}_m$	-50	0.1618	0.1600	0.1626	-0.1548	0.0273
	-25	0.0807	0.0799	0.0810	-0.0773	0.0136
	+25	-0.0804	-0.0796	-0.0810	0.0770	-0.0142
	+50	-0.1609	-0.1591	-0.1620	0.1542	-0.0278
p_c	-50	-7.6577	-7.5826	-7.6915	1.0243	8.9648
	-25	-3.7106	-3.6726	-3.7274	0.4732	4.4693
	+25	3.5017	3.4633	3.5191	-0.4106	-4.4469
	+50	6.8174	6.7402	6.8522	-0.7709	-8.8742
ϖ	-50	0.0000	0.0000	0.0000	0.0000	-1.3646
	-25	0.0000	0.0000	0.0000	0.0000	-0.6823
	+25	0.0000	0.0000	0.0000	0.0000	0.6823
	+50	0.0000	0.0000	0.0000	0.0000	1.3646

by production setup, material orders, production, purchasing materials, holding products or raw materials.

In addition, when the trading price for carbon emissions increases, the optimal length of the production period, length of the replenishment cycle, and order quantity of raw materials increase, whereas total annual carbon emissions and total annual profits decrease. Further, although a change in carbon emission quota does not affect the optimal solution results or annual carbon emissions, an increase in the quota leads to an increase in total profits because of the increased sellable carbon rights. Finally, the results in Table 6 show that changes in carbon emissions caused by production are significantly sensitive to changes in annual total carbon emissions.

VII. CONCLUSION

From the manufacturer's perspective, we examine herein a two-stage trade credit system in which an ACC payment is offered by the supplier and a delayed payment is provided to the retailer. We also consider a carbon cap-and-trade policy and establish a production–inventory model for deteriorating items with defective products. Several theorems are proposed to verify the optimization for multiple scenarios and ensure model rigorousness. Furthermore, an algorithm is employed to identify optimal solutions, and numerical examples are presented for model testing. Through an examination of the effects of parameter changes on the optimal solution results, we obtain the following main conclusions.

(a) When a full trade credit payment model is implemented, the optimal production period and replenishment cycle are the longest, the optimal quantity of raw materials is the largest, the annual carbon emission is lowest and total annual profit is the highest. Relatively, when payment terms are not considered and a carbon emission policy is implemented, the production period and replenishment cycle are shortest, the quantity of raw materials is smallest, the annual carbon emissions are highest, and the total annual profits are lowest. Nonetheless, when a full trade credit model is adopted in practice, the risk of bad debts may increase. Therefore, the general model we propose is more feasible in practice. The other cases are all special cases relating to our model.

(b) Relative to those in scenarios in which carbon emission policies are present, the production period and replenishment cycle are shorter, the order quantity of raw materials is lower, and the annual carbon emissions and total annual profits are relatively higher in scenarios where such policies are absent.

(c) Improvements to the feeding effect reduce the amount of materials used, thereby increasing total profits and lowering carbon emissions.

(d) Both increases in demand or unit price increase carbon emissions. However, carbon emissions are more sensitive to changes in demand than changes in unit price, whereas total profits are more sensitive to changes in unit price than changes in demand.

(e) When the deterioration rates of finished products and raw materials are simultaneously considered, regardless of whether the deterioration rate of materials or finished products increases, the production period, replenishment cycle, order quantity of raw materials, and total annual profits decrease while the total annual carbon emissions increases. The main difference is that the change in the deterioration rate of the finished product is significantly more sensitive to the optimal solutions than the change in the deterioration rate of the raw materials.

(f) For a production-inventory model that utilizes a two-stage trade credit system and is subjected to a carbon cap-and-trade policy, a lower interest charged or interest earned, longer length of payment in advance, longer trade credit period provided to the retailer or trade credit period offered by the supplier leads to higher carbon emissions.

(g) An increase in fixed carbon emissions generated by the production–inventory process increases the length of the production period, length of the replenishment cycle, and order quantity of raw materials, whereas an increase in unit carbon emissions reduces the length of the production period, length of the replenishment cycle, and order quantity of raw materials. Furthermore, changing the carbon emission quota does not affect annual carbon emissions, but increasing the trading price of carbon emissions does, thereby encouraging companies to reduce their annual carbon emissions under a carbon cap-and-trade policy.

In summary, the proposed model fills the research gap for supply chain production inventory models by using more realistic assumptions for modeling. The presented research results can help multinational supply chain facility decision-makers develop practical applications and strategies when facing carbon emission reduction policies and sustainable development issues. In future research, it can be expanded in several directions. For example, carbon reduction policies other than the cap-and-trade policy (e.g., carbon taxation and carbon offset) can be discussed in the future. Furthermore, future studies could introduce variable demand and out-of-stock systems or discuss integrated supply chain production–inventory models.

APPENDIX A
PROOF OF THEOREM 1

The following equation is obtained by taking the second derivative of $TP_{CC_1}(t_1)$ with respect to t_1 and substituting $t_1 = t_{11}$:

$$
\begin{aligned}
&\left.\frac{d^2TP_{CC_1}(t_1)}{dt_1^2}\right|_{t_1=t_{11}} \\
&= \frac{1}{[T(t_{11})]^2}\left(\left.\frac{d^2T}{dt_1^2}\right|_{t=t_{11}}\right)\{S + A \\
&\quad + p_c(\hat{S} + \hat{A}) + \frac{(c_m + p_c\hat{c}_m)\, rP(e^{\theta_m t_{11}} - 1)}{\theta_m} \\
&\quad + \left[(c + \lambda k) + p_c\left(\hat{c} + \lambda\hat{k}\right)\right] Pt_{11} \\
&\quad + \frac{\left(h_1 + p_c\hat{h}_1\right)[(1-\lambda)P - D]\, t_{11}}{\theta_f^2} \\
&\quad + \frac{\left(h_2 + p_c\hat{h}_2\right)\lambda P t_{11}^2}{2} \\
&\quad + \frac{\left(h_m + p_c\hat{h}_m\right) rP}{\theta_m^2}\left(e^{\theta_m t_{11}} - \theta_m t_{11} - 1\right) \\
&\quad + c_m I_c rP\left\{\frac{[\alpha(M + l) + \beta M]}{\theta_m}\left(e^{\theta_m t_{11}} - 1\right)\right. \\
&\quad + \frac{(\alpha + \beta)}{\theta_m^2}\left(e^{\theta_m t_{11}} - \theta_m t_{11} - 1\right) \\
&\quad \left. + \frac{\gamma}{\theta_m^2}\left[e^{\theta_m(t_{11} - M)} - \theta_m(t_{11} - M) - 1\right]\right\} \\
&\quad + \frac{cI_c[(1-\lambda)P - D]}{\theta_f^2}\left(e^{-\theta_f N} + \theta_f N - 1\right) \\
&\quad \left. - \frac{\gamma s I_e D(M - N)^2}{2}\right\} - \frac{1}{T(t_{11})}\{(\theta_m c_m + \theta_m p_c\hat{c}_m \\
&\quad + h_m + p_c\hat{h}_m) rPe^{\theta_m t_{11}} + \left(h_2 + p_c\hat{h}_2\right)\lambda P \\
&\quad + c_m I_c rPe^{\theta_m t_{11}}\{[\alpha(M + l) + \beta M]\theta_m \\
&\quad + (\alpha + \beta) + \gamma e^{-\theta_m M}\}\}.
\end{aligned}
$$

Because $\left(\left.\frac{d^2T}{dt_1^2}\right|_{t=t_{11}}\right) < 0$, $\left.\frac{d^2TP_{CC_1}(t_1)}{dt_1^2}\right|_{t_1=t_{11}} < 0$; therefore, the proof is completed.

APPENDIX B
PROOF OF THEOREM 2

The following equation is obtained by taking the second derivative of $TP_{CC_2}(t_1)$ with respect to t_1 and substituting $t_1 = t_{12}$:

$$
\begin{aligned}
&\left.\frac{d^2TP_{CC_2}(t_1)}{dt_1^2}\right|_{t_1=t_{12}} \\
&= \frac{1}{[T(t_{12})]^2}\left(\left.\frac{d^2T}{dt_1^2}\right|_{t=t_{12}}\right)\{S + A \\
&\quad + p_c(\hat{S} + \hat{A}) + \frac{(c_m + p_c\hat{c}_m)\, rP(e^{\theta_m t_{12}} - 1)}{\theta_m} \\
&\quad + \left[(c + \lambda k) + p_c\left(\hat{c} + \lambda\hat{k}\right)\right] Pt_{12} \\
&\quad + \frac{\left(h_1 + p_c\hat{h}_1\right)[(1-\lambda)P - D]\, t_{12}}{\theta_f^2} \\
&\quad + \frac{\left(h_2 + p_c\hat{h}_2\right)\lambda P t_{12}^2}{2} \\
&\quad + \frac{\left(h_m + p_c\hat{h}_m\right) rP}{\theta_m^2}\left(e^{\theta_m t_{12}} - \theta_m t_{12} - 1\right) \\
&\quad + c_m I_c rP\left\{\frac{[\alpha(M + l) + \beta M]}{\theta_m}\left(e^{\theta_m t_{12}} - 1\right)\right. \\
&\quad + \frac{(\alpha + \beta)}{\theta_m^2}\left(e^{\theta_m t_{12}} - \theta_m t_{12} - 1\right) \\
&\quad + \frac{cI_c[(1-\lambda)P - D]}{\theta_f^2}(e^{-\theta_f N} + \theta_f N - 1) \\
&\quad \left. - \frac{\gamma s I_e D(M - N)^2}{2}\right\} - \frac{1}{T(t_{12})}(\theta_m c_m \\
&\quad + \theta_m p_c\hat{c}_m + h_m + p_c\hat{h}_m) rPe^{\theta_m t_{12}}
\end{aligned}
$$

$$+ \left(h_2 + p_c\hat{h}_2\right) \lambda P + c_m I_c r P e^{\theta_m t_{12}} \{[\alpha(M+l) + \beta M]\theta_m + (\alpha+\beta)\}\}.$$

Because $\left(\left.\frac{d^2T}{dt_1^2}\right|_{t=t_{12}}\right) < 0$, $\left.\frac{d^2TP_{CC_2}(t_1)}{dt_1^2}\right|_{t_1=t_{12}} < 0$; therefore, the proof is completed.

APPENDIX C
PROOF OF THEOREM 3

The following equation is obtained by taking the second derivative of $TP_{CC_3}(t_1)$ with respect to t_1 and substituting $t_1 = t_{13}$:

$$\left.\frac{d^2TP_{CC_3}(t_1)}{dt_1^2}\right|_{t_1=t_{13}} = \gamma s I_e \left(\left.\frac{d^2T}{dt_1^2}\right|_{t=t_{13}}\right) + \frac{1}{[T(t_{13})]^2}\left(\left.\frac{d^2T}{dt_1^2}\right|_{t=t_{13}}\right)\{S + A + p_c(\hat{S}+\hat{A}) + \frac{\left(c_m + p_c\hat{c}_m\right) rP(e^{\theta_m t_{13}}-1)}{\theta_m} + \left[(c+\lambda k) + p_c\left(\hat{c}+\lambda\hat{k}\right)\right]Pt_{13} + \frac{\left(h_1 + p_c\hat{h}_1\right)[(1-\lambda)P - D]t_{13}}{\theta_f^2} + \frac{\left(h_2 + p_c\hat{h}_2\right)\lambda P t_{13}^2}{2} + \frac{\left(h_m + p_c\hat{h}_m\right) rP}{\theta_m^2} \times \left(e^{\theta_m t_{13}} - \theta_m t_{13} - 1\right) + c_m I_c r P \times \left\{\frac{[\alpha(M+l)+\beta M]}{\theta_m}\left(e^{\theta_m t_{13}}-1\right) + \frac{(\alpha+\beta)}{\theta_m^2}\left(e^{\theta_m t_{13}} - \theta_m t_{13} - 1\right) + \frac{cI_c[(1-\lambda)P-D]}{\theta_f^2}(e^{-\theta_f N} + \theta_f N - 1)\right\} - \frac{1}{T(t_{13})}(\theta_m c_m + \theta_m p_c \hat{c}_m + h_m + p_c\hat{h}_m) \times rPe^{\theta_m t_{13}} + (h_2 + p_c\hat{h}_2)\lambda P + c_m I_c r P e^{\theta_m t_{13}} \times \{[\alpha(M+l)+\beta M]\theta_m + (\alpha+\beta)\}\}.$$

Because $\left(\left.\frac{d^2T}{dt_1^2}\right|_{t=t_{13}}\right) < 0$, $\left.\frac{d^2TP_{CC_3}(t_1)}{dt_1^2}\right|_{t_1=t_{13}} < 0$; therefore, the proof is completed.

APPENDIX D
PROOF OF THEOREM 4

The following equation is obtained by taking the second derivative of $TP_{CC_4}(t_1)$ with respect to t_1 and substituting $t_1 = t_{14}$:

$$\left.\frac{d^2TP_{CC_4}(t_1)}{dt_1^2}\right|_{t_1=t_{14}} = \frac{1}{[T(t_{14})]^2}\left(\left.\frac{d^2T}{dt_1^2}\right|_{t=t_4}\right)\{S + A + p_c(\hat{S}+\hat{A}) + \frac{\left(c_m + p_c\hat{c}_m\right) rP(e^{\theta_m t_{14}}-1)}{\theta_m} + \left[(c+\lambda k) + p_c\left(\hat{c}+\lambda\hat{k}\right)\right]Pt_{14} + \frac{\left(h_1 + p_c\hat{h}_1\right)[(1-\lambda)P - D]t_{14}}{\theta_f^2} + \frac{\left(h_2 + p_c\hat{h}_2\right)\lambda P t_{14}^2}{2} + \frac{\left(h_m + p_c\hat{h}_m\right) rP}{\theta_m^2} \times \left(e^{\theta_m t_{14}} - \theta_m t_{14} - 1\right) + c_m I_c r P \times \left\{\frac{[\alpha(M+l)+\beta M]}{\theta_m}\left(e^{\theta_m t_{14}}-1\right) + \frac{(\alpha+\beta)}{\theta_m^2}\left(e^{\theta_m t_{14}} - \theta_m t_{14} - 1\right)\right\} + cI_c\left\{\frac{(1-\lambda)P-D}{\theta_f^2}(e^{-\theta_f t_{14}} + \theta_f t_{14} - 1) + \frac{(e^{-\theta_f t_{14}} - e^{-\theta_f N})}{\theta^2}(1-\lambda)Pe^{\theta_f t_{14}} - [(1-\lambda)P-D]\} - \frac{D}{\theta_f}(N - t_{14})\right\} - \frac{\gamma s I_e D(M-N)^2}{2}\} - \frac{1}{T(t_{14})}(\theta_m c_m + \theta_m p_c \hat{c}_m + h_m + p_c\hat{h}_m)rP\theta_m e^{\theta_m t_{14}} + (h_2 + p_c\hat{h}_2)\lambda P + c_m I_c r P e^{\theta_m t_{14}} \times \{[\alpha(M+l)+\beta M]\theta_m + (\alpha+\beta)\} + cI_c(1-\lambda)Pe^{-\theta_f(N-t_{14})}\}.$$

Because $\left(\left.\frac{d^2T}{dt_1^2}\right|_{t=t_{14}}\right) < 0$, $\left.\frac{d^2TP_{CC_4}(t_1)}{dt_1^2}\right|_{t_1=t_{14}} < 0$; therefore, the proof is completed.

APPENDIX E
PROOF OF THEOREM 5

The following equation is obtained by taking the second derivative of $TP_{CC_5}(t_1)$ with respect to t_1 and substituting $t_1 = t_{15}$:

$$\left.\frac{d^2TP_{CC_5}(t_1)}{dt_1^2}\right|_{t_1=t_{15}} = \gamma s I_e \left(\left.\frac{d^2T}{dt_1^2}\right|_{t=t_{15}}\right)$$

$$+\frac{1}{[T(t_{15})]^2}\left(\left.\frac{d^2T}{dt_1^2}\right|_{t=t_{15}}\right)\{S+A$$
$$+p_c(\hat{S}+\hat{A})+\frac{\left(c_m+p_c\hat{c}_m\right)rP(e^{\theta_m t_{15}}-1)}{\theta_m}$$
$$+\left[(c+\lambda k)+p_c\left(\hat{c}+\lambda\hat{k}\right)\right]Pt_{15}$$
$$+\frac{\left(h_1+p_c\hat{h}_1\right)[(1-\lambda)P-D]t_{15}}{\theta_f^2}$$
$$+\frac{\left(h_2+p_c\hat{h}_2\right)\lambda Pt_{15}^2}{2}+\frac{\left(h_m+p_c\hat{h}_m\right)rP}{\theta_m^2}$$
$$\times\left(e^{\theta_m t_{15}}-\theta_m t_{15}-1\right)+c_mI_crP$$
$$\times\left\{\frac{[\alpha(M+l)+\beta M]}{\theta_m}\left(e^{\theta_m t_{15}}-1\right)\right.$$
$$+\frac{(\alpha+\beta)}{\theta_m^2}\left(e^{\theta_m t_{15}}-\theta_m t_{15}-1\right)$$
$$\left.+\frac{cI_c[(1-\lambda)P-D]}{\theta_f^2}(e^{-\theta_f N}+\theta_f N-1)\right\}$$
$$+\frac{(e^{-\theta_f t_{15}}-e^{-\theta_f N})}{\theta_f^2}(1-\lambda)Pe^{\theta_f t_{15}}$$
$$\left.-[(1-\lambda)P-D]\}-\frac{D}{\theta_f}(N-t_{15})\right\}$$
$$-\frac{1}{T(t_{15})}(\theta_m c_m+\theta_m p_c\hat{c}_m+h_m+p_c\hat{h}_m)$$
$$\times rPe^{\theta_m t_{15}}+(h_2+p_c\hat{h}_2)\lambda P+c_mI_crPe^{\theta_m t_{15}}$$
$$\times\{[\alpha(M+l)+\beta M]\theta_m+(\alpha+\beta)\}$$
$$+cI_c(1-\lambda)Pe^{-\theta_f(N-t_{15})}\}.$$

Because $\left(\left.\frac{d^2T}{dt_1^2}\right|_{t=t_{15}}\right)<0$, $\left.\frac{d^2TP_{CC_5}(t_1)}{dt_1^2}\right|_{t_1=t_{15}}<0$; therefore, the proof is completed.

APPENDIX F
PROOF OF THEOREM 6

The following equation is obtained by taking the second derivative of $TP_{CC_6}(t_1)$ with respect to t_1 and substituting $t_1=t_{16}$:

$$\left.\frac{d^2TP_{CC_6}(t_1)}{dt_1^2}\right|_{t_1=t_{16}}$$
$$=\frac{1}{[T(t_{16})]^2}\left(\left.\frac{d^2T}{dt_1^2}\right|_{t=t_{16}}\right)\{S+A$$
$$+p_c(\hat{S}+\hat{A})+\frac{\left(c_m+p_c\hat{c}_m\right)rP(e^{\theta_m t_{16}}-1)}{\theta_m}$$
$$+\left[(c+\lambda k)+p_c\left(\hat{c}+\lambda\hat{k}\right)\right]Pt_{16}$$
$$+\frac{\left(h_1+p_c\hat{h}_1\right)[(1-\lambda)P-D]t_{16}}{\theta_f^2}$$
$$+\frac{\left(h_2+p_c\hat{h}_2\right)\lambda Pt_{16}^2}{2}+\frac{\left(h_m+p_c\hat{h}_m\right)rP}{\theta_m^2}$$
$$\times\left(e^{\theta_m t_{16}}-\theta_m t_{16}-1\right)+c_mI_crP$$
$$\times\left\{\frac{[\alpha(M+l)+\beta M]}{\theta_m}\left(e^{\theta_m t_{16}}-1\right)\right.$$
$$\left.+\frac{(\alpha+\beta)}{\theta_m^2}\left(e^{\theta_m t_{16}}-\theta_m t_{16}-1\right)\right\}$$
$$\left.-\frac{\gamma sI_eD(M-N)^2}{2}\right\}-\frac{1}{T(t_{14})}(\theta_m c_m$$
$$+p_c\hat{c}_m)rP\theta_m e^{\theta_m t_{16}}+(h_2+p_c\hat{h}_2)\lambda P$$
$$+\left(h_m+p_c\hat{h}_m\right)rPe^{\theta_m t_{16}}+c_mI_crPe^{\theta_m t_{16}}$$
$$\times\{[\alpha(M+l)+\beta M]\theta_m+(\alpha+\beta)\}\}.$$

Because $\left(\left.\frac{d^2T}{dt_1^2}\right|_{t=t_{16}}\right)<0$, $\left.\frac{d^2TP_{CC_6}(t_1)}{dt_1^2}\right|_{t_1=t_{16}}<0$; therefore, the proof is completed.

APPENDIX G
PROOF OF THEOREM 7

The following equation is obtained by taking the second derivative of $TP_{CC_7}(t_1)$ with respect to t_1 and substituting $t_1=t_{17}$:

$$\left.\frac{d^2TP_{CC_7}(t_1)}{dt_1^2}\right|_{t_1=t_{17}}$$
$$=\gamma sI_e\left(\left.\frac{d^2T}{dt_1^2}\right|_{t=t_{17}}\right)$$
$$+\frac{1}{[T(t_{17})]^2}\left(\left.\frac{d^2T}{dt_1^2}\right|_{t=t_{17}}\right)\{S+A$$
$$+p_c(\hat{S}+\hat{A})+\frac{\left(c_m+p_c\hat{c}_m\right)rP(e^{\theta_m t_{17}}-1)}{\theta_m}$$
$$+\left[(c+\lambda k)+p_c\left(\hat{c}+\lambda\hat{k}\right)\right]Pt_{17}$$
$$+\frac{\left(h_1+p_c\hat{h}_1\right)[(1-\lambda)P-D]t_{17}}{\theta_f^2}$$
$$+\frac{\left(h_2+p_c\hat{h}_2\right)\lambda Pt_{17}^2}{2}+\frac{\left(h_m+p_c\hat{h}_m\right)rP}{\theta_m^2}$$
$$\times\left(e^{\theta_m t_{17}}-\theta_m t_{17}-1\right)+c_mI_crP$$
$$\times\left\{\frac{[\alpha(M+l)+\beta M]}{\theta_m}\left(e^{\theta_m t_{17}}-1\right)\right.$$
$$\left.+\frac{(\alpha+\beta)}{\theta_m^2}\left(e^{\theta_m t_{17}}-\theta_m t_{17}-1\right)\right\}$$
$$-\frac{1}{T(t_{17})}(\theta_m c_m+\theta_m p_c\hat{c}_m+h_m+p_c\hat{h}_m)$$
$$\times rPe^{\theta_m t_{17}}+\left(h_2+p_c\hat{h}_2\right)\lambda P$$
$$+c_mI_crPe^{\theta_m t_{17}}[\alpha(M+l)+\beta M]\theta_m$$
$$+(\alpha+\beta)\}\}.$$

Because $\left(\left. \frac{d^2T}{dt_1^2} \right|_{t=t_{17}} \right) < 0$, $\left. \frac{d^2TP_{CC_7}(t_1)}{dt_1^2} \right|_{t_1=t_{17}} < 0$; therefore, the proof is completed.

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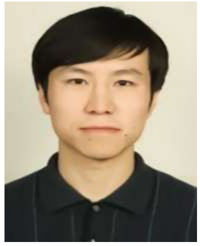

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