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RESEARCH ARTICLE

Renewable Energy Power's Investment and Production Strategies for a Class of Noncooperative Game Models of Power Sales With Risk Avoidance

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ABSTRACT This paper studies the computational problems of the optimal renewable energy power's investment and production strategies for a class of noncooperative game models of power sales with risk avoidance considering renewable energy power production quotas and Green Power Certificate System. To accommodate the problems, the following steps are implemented: firstly, according to the objects of sales of the power produced by the energy suppliers and whether the players have characteristic of the risk avoidance, the models are subdivided into four kinds of models. Secondly, the employment of the gradient-like optimization method and the setting of auxiliary systems verify the existence and uniqueness of the Nash equilibrium for each of models (ie., the optimal renewable energy utilization rates) that is globally exponentially stable. Then, the definition of the complete information static game is employed to compute the Nash equilibriums and then the optimal power production quantities and the optimal revenue functions would be calculated by relevant equations. Finally, the convergent validations for renewable energy utilization rates, the numerical validations for the optimal calculations for the four proposed models are carried out through a specific numerical example. Moreover, the impacts of players' characteristic of the risk avoidance are investigated and remarks on the impacts are provided.

INDEX TERMS Renewable energy power, investment and production strategies, risk avoidance, renewable energy power production quotas, green power certificate system, Nash equilibrium.

I. INTRODUCTION

Since the Industrial Revolution, increasing fossil fuels have been burned, which leads to a large amount of greenhouse gases emissions where carbon dioxide is the mainstay of greenhouse gases, accounting for about 77% of the total [1]. The assessment report issued by Intergovernmental Panel on Climate Change has shown that global warming caused by carbon dioxide emissions has affected the survival and development of human beings seriously. In response to this,

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China put forward the goals of “peaking carbon dioxide emissions” by 2030 and “carbon neutrality” by 2060. Since the two-carbon goal was proposed, global consensus on carbon neutrality has been further reached and Carbon Emission Reduction has become a hot topic of the social attention. It's understood that renewable energy can make an indelible contribution to reducing carbon emissions [2], [3], [4] and thus this stimulates the research interest of many scholars on renewable energy.

Much research on renewable energy has been done, such as the optimal planning of renewable energy [5], [6], [7], [8], the integration and optimal dispatch of renewable energy

[9], [10], [11], [12], [13], [14], [15], [16], the generation technologies for renewable energy [17], [18], [19], [20], the storage technologies for renewable energy [21], [22], [23], [24], the renewable energy and e-mobility [25], [26], [27], the renewable energy and energy buildings [28], [29], [30], the renewable energy policies [31], [32], [33], the renewable energy and social impacts [34], [35], [36], the renewable energy and climate change [37], [38], [39], the renewable energy and energy transition [40], [41], [42], to name just a few. However, it's recognized that few results concerning the investment and production of the renewable energy power have been reported, which inspired the emergence of [43] where the authors constructed a multi-stage game model of the renewable energy power's investment and production considering renewable energy power production quotas and green power certificate system. According to the objects of sales of the power produced by the energy suppliers, the model was subdivided into two scenarios, namely the scenario of direct sale of power and the scenario of purchase and sale by power grids. The optimal renewable energy power's investment and production strategies for the two scenarios based on an analysis for the timeline of decisions, were developed.

However, it should be pointed that [43] has done the following works.

Existing works
The power price sold to the power users is a variable, which is viewed from a short-term planning perspective.
Only revenues of power sales of energy suppliers to the grid are considered in the renewable energy interest chain in the scenario of purchase and sale by power grids.
Players have the characteristic of the risk neutrality, which illustrates that they make benefit decisions without considering the impact of the risk on revenues.

Motivated by the above observations and with some preliminary findings in [44] that considers the centralized noncooperative game model of power sales with grid's risk avoidance, this paper has done the following works and then compared with existing works, the improvements and contributions are summarized as follows:

- 1) This paper constructs four kinds of noncooperative game models of power sales with risk avoidance considering renewable energy power production quotas and Green Power Certificate System, namely the distributed noncooperative game model of power sales with energy suppliers' risk avoidance and the centralized noncooperative game models of power sales with energy suppliers' risk avoidance, with grid's risk avoidance and with risk avoidance for grid and energy suppliers, and studies their optimal renewable energy power's investment and production strategies.
- 2) Firstly, by utilizing the gradient-like optimization method and setting up auxiliary systems, the existence and uniqueness of the Nash equilibrium for each of models (i.e., the optimal renewable energy utilization

rates) would be proven and their stability that the Nash equilibriums are globally exponentially stable would be obtained. Secondly, by utilizing the definition of the complete information static game, the values of the Nash equilibriums are computed. Thirdly, by correlating equations, the optimal power production quantities and the optimal revenue functions are calculated. Finally, the convergence of renewable energy utilization rates and calculated values for the Nash equilibriums, the optimal power production quantities and the optimal revenue functions for the models would be verified. Moreover, the impacts of risk-aversion coefficients on the optimal renewable energy power's investment and production strategies (the optimal power price and the optimal total power production quantity) are investigated and remarks on the impacts are provided.

3)

Improvements
The power prices sold to the power users are exogenous and constants, which is viewed from a long-term planning perspective.
Not only revenues of power sales of energy suppliers to the grid but also revenues of power sales of the grid to the power users are considered in the renewable energy interest chain for a class of centralized noncooperative game models of power sales with risk avoidance.
Players are considered as rational decision makers and respectively tend to show the characteristic of the risk avoidance where energy suppliers' risks are costs and prices volatility, uncertain power market's demand, high innovation costs of renewable energy technologies and uncertainty, policy adjustments and grid's risks are cost and price volatility, uncertain power market's demand.
Contributions
The models are simplified, they require less computation.
The models have advantage for comprehensive consideration.
The models are more realistic.

The structures of the paper are listed below. Section II gives Notations. Section III provides Models Descriptions and Assumptions. The Main Results for models are given in Section IV. Section V is Theorems-to-Validity. Section VI highlights Conclusions.

II. NOTATIONS

$A p_c$	Renewable energy power production quotas, Green power certificate price.
$c_g c_t c_n$	Costs of the grid, the traditional and new energy suppliers.
$p_t p_n$	Power prices sold to the power users respectively by the traditional and new energy suppliers.

w, p	Power prices sold to the grid by the energy suppliers and sold to the power users by the grid.
l_t, l_n	Renewable energy utilization rates of traditional and new energy suppliers.
M	Power market's initial total capacity.
$q_{dt}, q_{dn}, E(\pi_{dt}^a), E(\pi_{dn}^a)$	Power production quantities, revenue functions of the traditional and new energy suppliers for the distributed noncooperative game model of power sales with energy suppliers' risk avoidance.
$q, q_{ct}, q_{cn}, E^1(\pi_{cg}^n), E^1(\pi_{ct}^a), E^1(\pi_{cn}^a), E^2(\pi_{cg}^a), E^2(\pi_{ct}^n), E^2(\pi_{cn}^n), E^3(\pi_{cg}^a), E^3(\pi_{ct}^a), E^3(\pi_{cn}^a)$	Power production quantities and revenue functions of the grid, the traditional and new energy suppliers for a class of centralized noncooperative game models of power sales with risk avoidance.

III. MODELS DESCRIPTIONS AND ASSUMPTIONS

This paper considers a class of noncooperative game models of power sales with risk avoidance shown in Fig. 1 where players, namely the government, the energy suppliers, the grid and the power users are involved. According to the objects of sales of the power produced by the energy suppliers, two kinds of models, namely the distributed and centralized noncooperative game models of power sales are developed where the object of power sales for the distributed noncooperative game model of power sales is the power users (sale prices p_t and p_n , power production quantities q_{dt}, q_{dn}) while the objects of power sales for the centralized noncooperative game model of power sales is the grid (sale price w , power production quantities q_{ct}, q_{cn}) and then the power users (sale price p , total power production quantity q). Which player the characteristic of the risk avoidance is attributed to would be further investigated with the result that the two kinds of models are further subdivided into four kinds of models, namely the distributed noncooperative game model of power sales with energy suppliers' risk avoidance, the centralized noncooperative game models of power sales with energy suppliers' risk avoidance, with grid's risk avoidance and with risk avoidance for energy suppliers and grid. In the models, players' functions respectively are as follows:

The function of the government: it issues renewable energy power production quotas A to the energy suppliers (with risk avoidance) and correspondingly establishes Green Power Certificate System whose effect is that let them purchase or sell the quotas at green power certificate price p_c when the renewable energy power quantities produced by the energy suppliers (with risk avoidance) mismatch the quotas where p_c is the long-term average green power certificate price, that is, it's exogenous and a constant.

The functions of energy suppliers (with risk avoidance): the duopoly oligopoly energy suppliers (with risk avoidance),

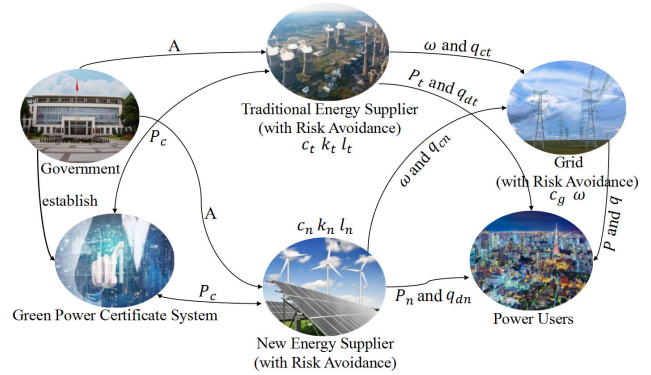


FIGURE 1. A class of noncooperative game models of power sales with risk avoidance.

namely the traditional and new energy suppliers (with risk avoidance) are considered in this paper. The traditional energy supplier (with risk avoidance) mainly utilizes thermal power generation method to provide non-renewable energy power while the new energy supplier (with risk avoidance) mainly utilizes photovoltaic power generation method, wind power generation method, etc. to provide renewable energy power.

For sustainable development of the power in the future, they are required to produce a certain amount of the renewable energy power to meet renewable energy power production quotas where the renewable energy power production quantities are determined by power production quantities ($q_{dt}, q_{dn}, q_{ct}, q_{cn}$) and renewable energy utilization rates (l_t, l_n). When the renewable energy power production quantities cannot meet the quotas, the measures taken by the energy suppliers (with risk avoidance) are: 1) Raising investment costs of renewable energy technologies to improve renewable energy utilization rates; 2) Boosting power production quantities. However, noticing that assumption 2, it can be learned that the essence of boosting power production quantities from their own perspective is to raise investment costs of renewable energy technologies; 3) Purchasing the quotas at green power certificate price p_c on Green Power Certificate System for balancing the quotas. When the renewable energy power production quantities exceed the quotas, the measure taken by the energy suppliers (with risk avoidance) is selling the excess quotas at green power certificate price p_c on Green Power Certificate System for obtaining returns. It's worth mentioning that they would face extremely high fines if the energy suppliers (with risk avoidance) do not complete the task of quotas. Obviously, compared with extremely high fines, they are more willing to undertake investment costs of renewable energy technologies or purchase the quotas on Green Power Certificate System.

The function of the grid (with risk avoidance): it works under a class of centralized noncooperative game models of power sales with risk avoidance and its working mechanism is formulating the power price w and purchasing power

production quantities of traditional and new energy suppliers (q_{ct}, q_{cn}) with costs (c_t, c_n).

The functions of power users: they purchase power production quantities q_{dt}, q_{dn} from the traditional and new energy suppliers with costs (c_t, c_n) respectively at the price p_t and p_n or total power production quantity q from the grid with cost c_g at the price p where p_t, p_n and p are the long-term average power prices, that is, they are exogenous and constants.

Based on the above description, it can be integrated that 1) revenue functions of energy suppliers (with risk avoidance) consist of benefits from selling the power to the power users or selling the power to the grid (with risk avoidance), costs of innovations in renewable energy technologies, gains or losses from matching renewable energy power production quotas on the Green Power Certificate System (and losses caused by the energy suppliers' risk avoidance). The question of how much power to produce, how much to invest in renewable energy technologies and how much to match quotas on the Green Power Certificate System (and how to control risky losses) is an important one for energy suppliers (with risk avoidance) to consider and it can be transformed into the problem of maximizing revenues of energy suppliers (with risk avoidance). 2) revenue function of the grid (with risk avoidance) consists of benefits from sales of the power to the power users (and the losses caused by grid's risk avoidance). Obviously, there also exists a problem of maximizing revenues of the grid (with risk avoidance).

The objective of this paper is to solve the problems of maximizing revenues of players and obtain the Nash equilibrium (i.e., the optimal renewable energy utilization rates), the optimal power production quantities, the optimal revenue functions for a class of noncooperative game models of power sales with risk avoidance.

Noticing that since the renewable energy utilization rates l_t, l_n and the power price w are coupled in the power production quantities, revenue functions are associated with the control variables of all players. i.e., $E(\pi_{dt}^a)(l_t, l_n)$ and $E(\pi_{dn}^a)(l_n, l_t)$, $E^1(\pi_{ct}^a)(l_t, l_n, w)$ and $E^1(\pi_{cn}^a)(l_n, l_t, w)$ and $E^1(\pi_{cg}^a)(w, l_t, l_n)$, $E^2(\pi_{ct}^a)(l_t, l_n, w)$ and $E^2(\pi_{cn}^a)(l_n, l_t, w)$ and $E^2(\pi_{cg}^a)(w, l_t, l_n)$, $E^3(\pi_{ct}^a)(l_t, l_n, w)$ and $E^3(\pi_{cn}^a)(l_n, l_t, w)$ and $E^3(\pi_{cg}^a)(w, l_t, l_n)$. **the solution to problems of maximizing revenues of players requires not only regulating its own control variable, but also taking into account control variables of other players.**

The definition of the Nash equilibrium is given below.

Definition 1: An action profile $\mathbf{L}^* = (l_t^*, l_n^*)$ is the Nash equilibrium [45] if

$$\begin{cases} E(\pi_{dt}^a)(l_t^*, l_n^*) \geq E(\pi_{dt}^a)(l_t, l_n^*), \\ E(\pi_{dn}^a)(l_n^*, l_t^*) \geq E(\pi_{dn}^a)(l_n, l_t^*). \end{cases} \quad (1)$$

or

$$\begin{cases} E^1(\pi_{ct}^a)(l_t^*, l_n^*, w^*) \geq E^1(\pi_{ct}^a)(l_t, l_n^*, w^*), \\ E^1(\pi_{cn}^a)(l_n^*, l_t^*, w^*) \geq E^1(\pi_{cn}^a)(l_n, l_t^*, w^*). \end{cases} \quad (2)$$

or

$$\begin{cases} E^2(\pi_{ct}^a)(l_t^*, l_n^*, w^*) \geq E^2(\pi_{ct}^a)(l_t, l_n^*, w^*), \\ E^2(\pi_{cn}^a)(l_n^*, l_t^*, w^*) \geq E^2(\pi_{cn}^a)(l_n, l_t^*, w^*). \end{cases} \quad (3)$$

or

$$\begin{cases} E^3(\pi_{ct}^a)(l_t^*, l_n^*, w^*) \geq E^3(\pi_{ct}^a)(l_t, l_n^*, w^*), \\ E^3(\pi_{cn}^a)(l_n^*, l_t^*, w^*) \geq E^3(\pi_{cn}^a)(l_n, l_t^*, w^*). \end{cases} \quad (4)$$

According to (1)-(4), the Nash equilibrium should satisfy

$$\begin{cases} \frac{\partial E(\pi_{dt}^a)(\mathbf{L}^*)}{\partial l_t} = 0, \quad \frac{\partial E(\pi_{dn}^a)(\mathbf{L}^*)}{\partial l_n} = 0, \\ \frac{\partial^2 E(\pi_{dt}^a)(\mathbf{L}^*)}{\partial l_t^2} < 0, \quad \frac{\partial^2 E(\pi_{dn}^a)(\mathbf{L}^*)}{\partial l_n^2} < 0. \end{cases} \quad (5)$$

or

$$\begin{cases} \frac{\partial E^1(\pi_{ct}^a)(\mathbf{L}^*, w^*)}{\partial l_t} = 0, \quad \frac{\partial E^1(\pi_{cn}^a)(\mathbf{L}^*, w^*)}{\partial l_n} = 0, \\ \frac{\partial^2 E^1(\pi_{ct}^a)(\mathbf{L}^*, w^*)}{\partial l_t^2} < 0, \quad \frac{\partial^2 E^1(\pi_{cn}^a)(\mathbf{L}^*, w^*)}{\partial l_n^2} < 0. \end{cases} \quad (6)$$

or

$$\begin{cases} \frac{\partial E^2(\pi_{ct}^a)(\mathbf{L}^*, w^*)}{\partial l_t} = 0, \quad \frac{\partial E^2(\pi_{cn}^a)(\mathbf{L}^*, w^*)}{\partial l_n} = 0, \\ \frac{\partial^2 E^2(\pi_{ct}^a)(\mathbf{L}^*, w^*)}{\partial l_t^2} < 0, \quad \frac{\partial^2 E^2(\pi_{cn}^a)(\mathbf{L}^*, w^*)}{\partial l_n^2} < 0. \end{cases} \quad (7)$$

or

$$\begin{cases} \frac{\partial E^3(\pi_{ct}^a)(\mathbf{L}^*, w^*)}{\partial l_t} = 0, \quad \frac{\partial E^3(\pi_{cn}^a)(\mathbf{L}^*, w^*)}{\partial l_n} = 0, \\ \frac{\partial^2 E^3(\pi_{ct}^a)(\mathbf{L}^*, w^*)}{\partial l_t^2} < 0, \quad \frac{\partial^2 E^3(\pi_{cn}^a)(\mathbf{L}^*, w^*)}{\partial l_n^2} < 0. \end{cases} \quad (8)$$

Note that the action profile of the Nash equilibrium is formulated under the condition that the grid has formulated the optimal power price w^* for the energy suppliers.

The following assumptions, adapted from [43], are made to solve the problems of maximizing revenues of players for the models in the upcoming section.

Assumption 1: If the traditional and new energy suppliers need to improve their own renewable energy utilization rates, then they have to undertake investment costs of renewable energy technologies, which are respectively described as

$$\begin{cases} C_t = \frac{1}{2} k_t l_t^2, \\ C_n = \frac{1}{2} k_n l_n^2, \end{cases} \quad (9)$$

where $k_i > 0$ for $i \in \{t, n\}$ denotes the demand elasticity coefficient of investment costs of renewable energy technologies and $k_t > k_n$ holds.

Assumption 2: The power production quantities of traditional and new energy suppliers for the distributed

noncooperative game model of power sales with energy suppliers' risk avoidance are respectively described as

$$\begin{cases} q_{dt} = \alpha M - b'p_t + s'p_n + bl_t - sl_n, \\ q_{dn} = (1 - \alpha)M - b'p_n + s'p_t + bl_n - sl_t, \end{cases} \quad (10)$$

where $0 < \alpha < 1$ denotes the ratio of the traditional energy supplier in the power market's initial total capacity. $b' > 0$ and $s' > 0$ respectively represent the demand and substitution elasticity coefficients of power prices and $s' > b'$ holds. $b > 0$ and $s > 0$ respectively represent the demand and substitution elasticity coefficients of renewable energy utilization rates and $b > s$ holds. Moreover, for satisfying $q_{dt} > 0$ and $q_{dn} > 0$, the inequalities that $\alpha M - b'p_t + s'p_n > s$ and $(1 - \alpha)M - b'p_n + s'p_t > s$ hold.

The total power production quantity and the power production quantities of traditional and new energy suppliers for a class of centralized noncooperative game models of power sales with risk avoidance are respectively described as

$$\begin{cases} q_{ct} = \alpha M + \theta w + bl_t - sl_n, \\ q_{cn} = (1 - \alpha)M + \theta w + bl_n - sl_t, \\ q = M + 2\theta w + (b - s)(l_t + l_n), \end{cases} \quad (11)$$

where $\theta = s' - b' > 0$ denotes the demand elasticity coefficient of the power price. Moreover, for satisfying $q_{ct} > 0$ and $q_{cn} > 0$, the inequalities that $\alpha M > s - \theta \max\{c_t, c_n\}$ and $(1 - \alpha)M > s - \theta \max\{c_t, c_n\}$ hold where for the value of w , please refer to Remark 2.

Note that M proposed in (10)-(11) follows a normal distribution with mean value μ and standard deviation δ .

Assumption 3: After experiencing innovations in renewable energy technologies, the traditional and new energy suppliers would balance renewable energy power production quotas on Green Power Certificate System and the gains or losses are respectively described as

$$\begin{cases} P_t = (q_{dt}l_t - A)p_c, \\ P_n = (q_{dn}l_n - A)p_c. \end{cases} \quad (12)$$

$$\begin{cases} P_t = (q_{ct}l_t - A)p_c, \\ P_n = (q_{cn}l_n - A)p_c. \end{cases} \quad (13)$$

IV. MAIN RESULTS

Existence, uniqueness and stability of the Nash equilibriums for a class of noncooperative game models of power sales with risk avoidance are analyzed and the Nash equilibriums, the optimal power production quantities, the optimal revenue functions are computed in this section. Moreover, the impacts of risk-aversion coefficients on the optimal renewable energy power's investment and production strategies (the optimal power price and the optimal total power production quantity) are investigated.

A. THE DISTRIBUTED NONCOOPERATIVE GAME MODEL OF POWER SALES WITH ENERGY SUPPLIERS' RISK AVOIDANCE

The characteristics of the distributed noncooperative game model of power sales with energy suppliers' risk avoidance shown in Fig. 2 are as follows:

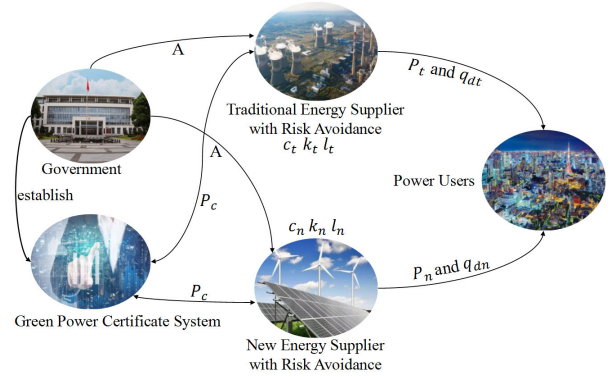


FIGURE 2. The distributed noncooperative game model of power sales with energy suppliers' risk avoidance.

- The power produced by the energy suppliers is sold directly to the power users.
- After innovations in renewable energy technologies, the energy suppliers match renewable energy power production quotas issued by the government on Green Power Certificate System.
- The energy suppliers have the characteristic of the risk avoidance.

By respectively adjusting their respective renewable energy utilization rates, their revenue functions are maximized with the following expressions.

$$\begin{cases} \max_{l_t} E(\pi_{dt}^a) = (p_t - c_t)q_{dt} - C_t + P_t + R_{dt}, \\ s.t. 0 \leq l_t \leq 1. \end{cases} \quad (14)$$

$$\begin{cases} \max_{l_n} E(\pi_{dn}^a) = (p_n - c_n)q_{dn} - C_n + P_n + R_{dn}, \\ s.t. 0 < l_n \leq 1. \end{cases} \quad (15)$$

where $R_{dt} < 0$ and $R_{dn} < 0$ respectively represent the losses of traditional and new energy suppliers caused by energy suppliers' risk avoidance and their expressions are designed as $R_{dt} = -\alpha\eta_{dt}\delta(|p_t - c_t| + p_cl_t)$ and $R_{dn} = -(1 - \alpha)\eta_{dn}\delta(|p_n - c_n| + p_cl_n)$ by utilizing the mean variance method and introducing the risk-aversion coefficients η_{dt} and η_{dn} . Moreover, for satisfying meaning of renewable energy utilization rates in reality, the constraints that $0 \leq l_t \leq 1$ and $0 < l_n \leq 1$ are constructed and since the new energy supplier mainly produce renewable energy power, l_n would be further constrained to $0 < l_n \leq 1$. In addition, for ensuring that the profit margins of traditional and new energy suppliers are positive, the inequalities that $p_t > c_t$ and $p_n > c_n$ hold.

1) ANALYSIS OF EXISTENCE, UNIQUENESS AND STABILITY OF THE NASH EQUILIBRIUM

Theorem 1: Suppose that $k_i - 2p_c b > p_c s$ for $i \in \{t, n\}$. Then, there exists a unique Nash equilibrium (l_t^*, l_n^*) that satisfies (1) and the Nash equilibrium is globally exponentially stable.

Proof: Motivated by [45], the gradient-like optimization method is utilized to analyze the existence, uniqueness and stability of the Nash equilibrium and an auxiliary system is designed as

$$\dot{\mathbf{L}} = \frac{\partial G(\mathbf{L})}{\partial \mathbf{L}}, \tag{16}$$

where $\mathbf{L} = [l_t, l_n]^T$ and $\frac{\partial G(\mathbf{L})}{\partial \mathbf{L}} = [\frac{\partial E(\pi_{dt}^a)(\mathbf{L})}{\partial l_t}, \frac{\partial E(\pi_{dn}^a)(\mathbf{L})}{\partial l_n}]^T$. Linearizing (16) at \mathbf{L}^* gives

$$\begin{aligned} \frac{d\mathbf{L}}{dt} &= \begin{bmatrix} \frac{\partial^2 E(\pi_{dt}^a)(\mathbf{L}^*)}{\partial l_t^2} & \frac{\partial^2 E(\pi_{dt}^a)(\mathbf{L}^*)}{\partial l_t \partial l_n} \\ \frac{\partial^2 E(\pi_{dn}^a)(\mathbf{L}^*)}{\partial l_n \partial l_t} & \frac{\partial^2 E(\pi_{dn}^a)(\mathbf{L}^*)}{\partial l_n^2} \end{bmatrix} (\mathbf{L} - \mathbf{L}^*) \\ &= - \begin{bmatrix} k_t - 2p_c b & p_c s \\ p_c s & k_n - 2p_c b \end{bmatrix} (\mathbf{L} - \mathbf{L}^*) \end{aligned} \tag{17}$$

Suppose that $k_i > 2p_c b$ and $k_i - 2p_c b > p_c s$ for $i \in \{t, n\}$,

which declares that $\begin{bmatrix} \frac{\partial^2 E(\pi_{dt}^a)(\mathbf{L}^*)}{\partial l_t^2} & \frac{\partial^2 E(\pi_{dt}^a)(\mathbf{L}^*)}{\partial l_t \partial l_n} \\ \frac{\partial^2 E(\pi_{dn}^a)(\mathbf{L}^*)}{\partial l_n \partial l_t} & \frac{\partial^2 E(\pi_{dn}^a)(\mathbf{L}^*)}{\partial l_n^2} \end{bmatrix}$ is strictly

diagonally dominant with all the diagonal elements being negative, it is Hurwitz by Gershgorin circle theorem [46]. According to Corollary 4.3 in [47], the equilibrium point \mathbf{L}^* is exponentially stable under (16). Moreover, since the auxiliary system in (16) is a linear system, the equilibrium point \mathbf{L}^* is globally exponentially stable. In addition, since $E(\pi_{dt}^a)$ and $E(\pi_{dn}^a)$ are quadratic functions, the equilibrium point \mathbf{L}^* is unique. Furthermore, since the unique equilibrium point \mathbf{L}^* satisfy (5), it's the Nash equilibrium that satisfies (1). To this end, the proof is completed. \square

2) COMPUTATION OF THE UNIQUE NASH EQUILIBRIUM

Theorem 2: Suppose that Assumptions 1-3 hold and the parameters satisfy

$$\begin{cases} \eta_{dt} \leq \frac{b(p_t - c_t) + p_c(\alpha M - b'p_t + s'p_n - s)}{\alpha \delta p_c}, \\ \eta_{dn} < \frac{b(p_n - c_n) + p_c((1 - \alpha)M - b'p_n + s'p_t - s)}{(1 - \alpha)\delta p_c}, \\ \frac{b(p_t - c_t) + p_c(\alpha M - b'p_t + s'p_n) - \alpha \eta_{dt} \delta p_c}{k_t - 2p_c b} \leq 1, \\ \frac{b(p_n - c_n) + p_c((1 - \alpha)M - b'p_n + s'p_t) - (1 - \alpha)\eta_{dn} \delta p_c}{k_n - 2p_c b} \leq 1. \end{cases} \tag{18}$$

Then, there exists a unique Nash equilibrium (l_t^*, l_n^*) for the distributed noncooperative game model of power sales

with energy suppliers' risk avoidance and its expression is followed as

$$(l_t^*, l_n^*) = (\frac{q_2 v_1 - q_3 v_2}{q_1 q_2 - q_3^2}, \frac{q_1 v_2 - q_3 v_1}{q_1 q_2 - q_3^2}), \tag{19}$$

where $q_1 = k_t - 2p_c b$, $q_2 = k_n - 2p_c b$, $q_3 = p_c s$, $v_1 = b(p_t - c_t) + p_c(\alpha M - b'p_t + s'p_n) - \alpha \eta_{dt} \delta p_c$ and $v_2 = b(p_n - c_n) + p_c((1 - \alpha)M - b'p_n + s'p_t) - (1 - \alpha)\eta_{dn} \delta p_c$.

Proof: Suppose that $\frac{\partial^2(E(\pi_{dt}^a))}{\partial l_t^2} = -k_t + 2p_c b < 0$. Let $\frac{\partial(E(\pi_{dt}^a))}{\partial l_t} = 0$ and then the optimal renewable energy utilization rate of the traditional energy supplier can be obtained: $l_t(l_n) = \frac{b(p_t - c_t) + p_c(\alpha M - b'p_t + s'p_n) - \alpha \eta_{dt} \delta p_c}{k_t - 2p_c b}$, whose value is related to the renewable energy utilization rate of the new energy supplier. Similarly, through analysis for revenue function of the new energy supplier, the optimal renewable energy utilization rate of the new energy supplier can be obtained: $l_n(l_t) = \frac{b(p_n - c_n) + p_c((1 - \alpha)M - b'p_n + s'p_t) - (1 - \alpha)\eta_{dn} \delta p_c}{k_n - 2p_c b}$ which is related to the renewable energy utilization rate of the traditional energy supplier.

Note that for ensuring that the value of $l_t(l_n)$ is greater than or equal to 0 and the value of $l_n(l_t)$ is greater than 0, the following inequalities should be satisfied that

$$\begin{cases} \eta_{dt} \leq \frac{b(p_t - c_t) + p_c(\alpha M - b'p_t + s'p_n - s)}{\alpha \delta p_c}, \\ \eta_{dn} < \frac{b(p_n - c_n) + p_c((1 - \alpha)M - b'p_n + s'p_t - s)}{(1 - \alpha)\delta p_c}, \end{cases} \tag{20}$$

and for ensuring that the values of $l_t(l_n)$ and $l_n(l_t)$ are less than or equal to 1, the following inequalities should be satisfied that

$$\begin{cases} l_t(l_n) \leq \frac{b(p_t - c_t) + p_c(\alpha M - b'p_t + s'p_n) - \alpha \eta_{dt} \delta p_c}{k_t - 2p_c b} \leq 1, \\ l_n(l_t) \leq \frac{b(p_n - c_n) + p_c((1 - \alpha)M - b'p_n + s'p_t) - (1 - \alpha)\eta_{dn} \delta p_c}{k_n - 2p_c b} \leq 1. \end{cases} \tag{21}$$

It's worth pointing out that $k_t - 2p_c b - p_c s \geq b(p_t - c_t) + p_c(\alpha M - b'p_t + s'p_n - s) - \alpha \eta_{dt} \delta p_c \geq 0$, $k_n - 2p_c b - p_c s \geq b(p_n - c_n) + p_c((1 - \alpha)M - b'p_n + s'p_t - s) - (1 - \alpha)\eta_{dn} \delta p_c > 0$ and $k_t > k_n$, which demonstrates that there exists a unique Nash equilibrium for the model according to Theorem 1.

Apparently, $l_t(l_n)$ and $l_n(l_t)$ are intertwined in terms of renewable energy utilization rates. Then, suppose that the model is under the complete information static game circumstance, that is, the energy suppliers understand each other's renewable energy utilization rates and simultaneously develop their respective renewable energy utilization rates and by linking $l_t(l_n)$ and $l_n(l_t)$, the unique Nash equilibrium (i.e., the optimal renewable energy utilization rates) can be

obtained as follows:

$$(l_t^*, l_n^*) = \left(\frac{q_2 v_1 - q_3 v_2}{q_1 q_2 - q_3^2}, \frac{q_1 v_2 - q_3 v_1}{q_1 q_2 - q_3^2} \right), \quad (22)$$

where $q_1 = k_t - 2p_c b$, $q_2 = k_n - 2p_c b$, $q_3 = p_c s$, $v_1 = b(p_t - c_t) + p_c(\alpha M - b'p_t + s'p_n) - \alpha \eta_{dt} \delta p_c$ and $v_2 = b(p_n - c_n) + p_c((1 - \alpha)M - b'p_n + s'p_t) - (1 - \alpha)\eta_{dn} \delta p_c$. To this end, the proof is completed. \square

3) COMPUTATION OF THE OPTIMAL POWER PRODUCTION QUANTITIES AND THE OPTIMAL REVENUE FUNCTIONS

Theorem 3: When the renewable energy utilization rates reach the unique Nash equilibrium for the distributed noncooperative game model of power sales with energy suppliers' risk avoidance, the optimal power production quantities and the optimal revenue functions are:

$$\left\{ \begin{aligned} q_{dt}^* &= \alpha M - b'p_t + s'p_n + \frac{q_2 b v_1 - q_1 s v_2 - q_3 v_3}{q_1 q_2 - q_3^2}, \\ q_{dn}^* &= (1 - \alpha)M - b'p_n + s'p_t + \frac{q_1 b v_2 - q_2 s v_1 - q_3 v_4}{q_1 q_2 - q_3^2}, \\ E^*(\pi_{dt}^a) &= (p_t - c_t)(\alpha M - b'p_t + s'p_n) - p_c A \\ &\quad - \alpha \eta_{dt} \delta (p_t - c_t) + \frac{q_2 v_5 - q_1 s (p_t - c_t) v_2 - q_3 v_6}{q_1 q_2 - q_3^2} \\ &\quad + \frac{(q_2 v_1 - q_3 v_2)(q_2 v_7 - q_1 s v_2 - q_3 v_8)}{(q_1 q_2 - q_3^2)^2}, \\ E^*(\pi_{dn}^a) &= (p_n - c_n)((1 - \alpha)M - b'p_n + s'p_t) - p_c A \\ &\quad - (1 - \alpha)\eta_{dn} \delta (p_n - c_n) + \frac{q_1 v_9 - q_2 s (p_n - c_n) v_1 - q_3 v_{10}}{q_1 q_2 - q_3^2} \\ &\quad + \frac{(q_1 v_2 - q_3 v_1)(q_1 v_{11} - q_2 p_c s v_1 - q_3 v_{12})}{(q_1 q_2 - q_3^2)^2}, \end{aligned} \right. \quad (23)$$

where $v_3 = b v_2 - s v_1$, $v_4 = b v_1 - s v_2$, $v_5 = v_1^2$, $v_6 = v_1 v_2 - s(p_t - c_t)v_1$, $v_7 = (-\frac{1}{2}k_t + p_c b)v_1$, $v_8 = (-\frac{1}{2}k_t + p_c b)v_2 - p_c s v_1$, $v_9 = v_2^2$, $v_{10} = v_1 v_2 - s(p_n - c_n)v_2$, $v_{11} = (-\frac{1}{2}k_n + p_c b)v_2$ and $v_{12} = (-\frac{1}{2}k_n + p_c b)v_1 - p_c s v_2$.

Proof: By substituting l_t^* , l_n^* into (10) (14)-(15), q_{dt}^* , q_{dn}^* , $E^*(\pi_{dt}^a)$ and $E^*(\pi_{dn}^a)$ can be respectively calculated. To this end, the proof is completed. \square

4) IMPACTS OF RISK-AVERSION COEFFICIENTS ON THE OPTIMAL RENEWABLE ENERGY POWER'S INVESTMENT AND PRODUCTION STRATEGY

Theorem 4: The optimal renewable energy utilization rate of the traditional energy supplier l_t^* and the optimal power production quantity of the traditional energy supplier q_{dt}^* are negatively correlated with the risk-aversion coefficient η_{dt} and are positively correlated with the risk-aversion coefficient η_{dn} . The optimal renewable energy utilization rate of the new energy supplier l_n^* and the optimal power production quantity of the new energy supplier q_{dn}^* are negatively correlated with the risk-aversion coefficient

η_{dn} and are positively correlated with the risk-aversion coefficient η_{dt} .

Proof: Noticing that $q_1 > q_2 > q_3 > 0$, one can obtain that

$$\left\{ \begin{aligned} \frac{\partial l_t^*}{\partial \eta_{dt}} &= -\frac{\alpha \delta p_c q_2}{q_1 q_2 - q_3^2} < 0, \quad \frac{\partial l_t^*}{\partial \eta_{dn}} = \frac{(1 - \alpha) \delta p_c q_3}{q_1 q_2 - q_3^2} > 0, \\ \frac{\partial l_n^*}{\partial \eta_{dt}} &= \frac{\alpha \delta p_c q_3}{q_1 q_2 - q_3^2} > 0, \quad \frac{\partial l_n^*}{\partial \eta_{dn}} = -\frac{(1 - \alpha) \delta p_c q_1}{q_1 q_2 - q_3^2} < 0, \\ \frac{\partial q_{dt}^*}{\partial \eta_{dt}} &= -\frac{\alpha \delta p_c (b q_2 + s q_3)}{q_1 q_2 - q_3^2} < 0, \\ \frac{\partial q_{dt}^*}{\partial \eta_{dn}} &= \frac{(1 - \alpha) \delta p_c (b q_3 + s q_1)}{q_1 q_2 - q_3^2} > 0, \\ \frac{\partial q_{dn}^*}{\partial \eta_{dt}} &= \frac{\alpha \delta p_c (b q_3 + s q_2)}{q_1 q_2 - q_3^2} > 0, \\ \frac{\partial q_{dn}^*}{\partial \eta_{dn}} &= -\frac{(1 - \alpha) \delta p_c (b q_1 + s q_3)}{q_1 q_2 - q_3^2} < 0. \end{aligned} \right. \quad (24)$$

To this end, the proof is completed. \square

Remark 1: The control variable method is adopted to analyze the impact of each of risk-aversion coefficients in Theorem 4 and then the following conclusions are obtained: 1) When the traditional energy supplier focuses on risk avoidance, to satisfy its own risk-avoidance needs, it would reduce its own renewable energy utilization rate. As the renewable energy utilization rate of the traditional energy supplier decreases, to capture power market's share, the new energy supplier would improve its own renewable energy utilization rate. This leads to the power quantity produced by the traditional energy supplier being decreased while the power quantity produced by the new energy supplier being increased. 2) A similar analysis for the new energy supplier is conducted and is omitted here.

This section investigates the distributed noncooperative game model of power sales with energy suppliers' risk avoidance. In the following, the centralized noncooperative game model of power sales with energy suppliers' risk avoidance will be taken into account.

B. THE CENTRALIZED NONCOOPERATIVE GAME MODEL OF POWER SALES WITH ENERGY SUPPLIERS' RISK AVOIDANCE

The characteristics of the centralized noncooperative game model of power sales with energy suppliers' risk avoidance shown in Fig. 3 are as follows:

- The power users get the power from the grid, which gets the power from the energy suppliers.
- After innovations in renewable energy technologies, the energy suppliers match renewable energy power production quotas issued by the government on Green Power Certificate System.

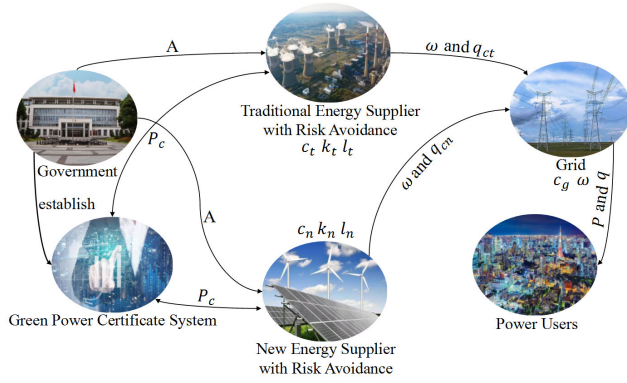


FIGURE 3. The centralized noncooperative game model of power sales with energy suppliers' risk avoidance.

- The energy suppliers have the characteristic of the risk avoidance.

By respectively adjusting their respective control variables, their revenue functions are maximized with the following expressions.

$$\begin{cases} \max_{l_t} E^1(\pi_{ct}^a) = (\omega - c_t)q_{ct} - C_t + P_t + R_{ct}, \\ s.t. 0 \leq l_t \leq 1. \end{cases} \quad (25)$$

$$\begin{cases} \max_{l_n} E^1(\pi_{cn}^a) = (\omega - c_n)q_{cn} - C_n + P_n + R_{cn}, \\ s.t. 0 < l_n \leq 1. \end{cases} \quad (26)$$

$$\begin{cases} \max_w E^1(\pi_{cg}^n) = (p - c_g - \omega)q, \\ s.t. 0 \leq w < p - c_g. \end{cases} \quad (27)$$

where $R_{ct} < 0$ and $R_{cn} < 0$ respectively represent losses of traditional and new energy suppliers due to energy suppliers' risk avoidance and their expressions are designed as $R_{ct} = -\alpha\eta_{ct}\delta(|w - c_t| + p_c l_t)$ and $R_{cn} = -(1 - \alpha)\eta_{cn}\delta(|w - c_n| + p_c l_n)$ by utilizing the mean variance method and introducing the risk-aversion coefficients η_{ct} η_{cn} . Moreover, it can be seen in (25)-(26) that for achieving fairness and justice, there is no price discrimination, that is, the power price w set by the grid for the traditional and new energy suppliers is the same. In addition, for ensuring that the profit margin of the grid is positive, the constraint that $0 \leq w < p - c_g$ is constructed.

Remark 2: It can be learned from (25)-(26) that the profit margins of energy suppliers are influenced by the power price w set by the grid. If $0 \leq w \leq \max\{c_t, c_n\}$, which indicates that the profit margins of at least one of energy suppliers are non-positive, the profitability of the energy supplier with non-positive profit margin is contingent on the gains earned from the sale of excess renewable energy power production quotas. To be more specific, noticing that $E^1(\pi_{ci}^a) = -(c_i - w)q_{ci} - C_i - p_c A + p_c q_{ci} l_i + R_{ci}$ for $i \in \{t, n\}$ reorganized by (25)-(26), the profitable segment of the energy supplier with non-positive profit margin is

determined primarily by the green power certificate price set by Green Power Certificate System followed by the renewable energy utilization rate determined by itself, which is not allowed because of poor subjective decisions and strong objective decisions on power sales by the energy supplier with non-positive profit margin. There is no doubt that this would result in it being reluctant to sell the power to the grid. This constraint would lead to the collapse of the centralized noncooperative game model of power sales with energy suppliers' risk avoidance. If and only if $w > \max\{c_t, c_n\}$, the energy suppliers with positive profit margins have stronger subjective decisions on power sales and could take the initiative in terms of the profitable segment. Therefore, w would be further constrained to $\max\{c_t, c_n\} < w < p - c_g$. Note that this constraint is established from the perspective of the grid and the energy suppliers and is utilized for subsequent analysis.

The event sequence for solving the problem of maximizing revenues of players and then obtaining the Nash equilibrium, the optimal power production quantities, the optimal revenue functions for the model is given as follows:

- 1) The grid determines the optimal power price w^* ($\max\{c_t, c_n\} < w^* < p - c_g$) for the energy suppliers.
- 2) The traditional and new energy suppliers observe w^* and then develop the Nash equilibrium.
- 3) The optimal power production quantities and the optimal revenue functions can be calculated according to the optimal power price and the Nash equilibrium.

1) ANALYSIS OF EXISTENCE, UNIQUENESS AND STABILITY OF THE NASH EQUILIBRIUM

Theorem 5: Suppose that $k_i - 2p_c b > p_c s$ for $i \in \{t, n\}$. Then, there exists a unique Nash equilibrium (l_t^*, l_n^*) that satisfies (2) and the Nash equilibrium is globally exponentially stable.

Proof: One performs the following analysis under the condition that the grid has formulated the optimal power price w^* for the energy suppliers. An auxiliary system is designed as

$$\dot{\mathbf{L}} = \frac{\partial G^1(\mathbf{L}, w^*)}{\partial \mathbf{L}}, \quad (28)$$

where $\mathbf{L} = [l_t, l_n]^T$ and $\frac{\partial G^1(\mathbf{L}, w^*)}{\partial \mathbf{L}} = \left[\frac{\partial E^1(\pi_{ct}^a)(\mathbf{L}, w^*)}{\partial l_t}, \frac{\partial E^1(\pi_{cn}^a)(\mathbf{L}, w^*)}{\partial l_n} \right]^T$.

Linearizing (28) at \mathbf{L}^* gives

$$\frac{d\mathbf{L}}{dt} = B(\mathbf{L} - \mathbf{L}^*), \quad (29)$$

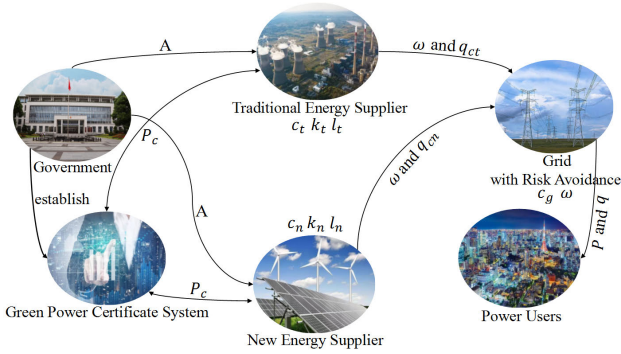


FIGURE 4. The centralized noncooperative game model of power sales with grid's risk avoidance.

where $B = \begin{bmatrix} \frac{\partial^2 E^1(\pi_{ct}^a)(\mathbf{L}^*, w^*)}{\partial l_t^2} & \frac{\partial^2 E^1(\pi_{ct}^a)(\mathbf{L}^*, w^*)}{\partial l_t \partial l_n} \\ \frac{\partial^2 E^1(\pi_{cn}^a)(\mathbf{L}^*, w^*)}{\partial l_n \partial l_t} & \frac{\partial^2 E^1(\pi_{cn}^a)(\mathbf{L}^*, w^*)}{\partial l_n^2} \end{bmatrix} = - \begin{bmatrix} k_t - 2p_c b & p_c s \\ p_c s & k_n - 2p_c b \end{bmatrix}$. The remaining analysis follows that of Theorem 1 and hence, is omitted. \square

2) COMPUTATION OF THE UNIQUE NASH EQUILIBRIUM, THE OPTIMAL POWER PRODUCTION QUANTITIES AND THE OPTIMAL REVENUE FUNCTIONS

Theorem 6: Suppose that Assumptions 1-3 hold and the parameters satisfy

$$\begin{cases} \max\{c_t, c_n\} < w^* < p - c_g, \\ \text{where } w^* = \frac{1}{2}(p - c_g) \\ \frac{M(q_1 q_2 - q_3^2)}{4\theta(q_1 q_2 - q_3^2) + 2(b-s)(b+p_c\theta)(q_1+q_2-2q_3) + (b-s)(q_1-q_3)(bc_n - p_c(1-\alpha)M + (1-\alpha)\eta_{cn}\delta p_c)} + \frac{4\theta(q_1 q_2 - q_3^2) + 2(b-s)(b+p_c\theta)(q_1+q_2-2q_3)}{(b-s)(q_2-q_3)(bc_t - p_c\alpha M + \alpha\eta_{ct}\delta p_c)}, \\ \eta_{ct} \leq \frac{4\theta(q_1 q_2 - q_3^2) + 2(b-s)(b+p_c\theta)(q_1+q_2-2q_3)}{b(w^* - c_t) + p_c(\alpha M + \theta w^* - s)}, \\ \eta_{cn} < \frac{\alpha\delta p_c}{b(w^* - c_n) + p_c((1-\alpha)M + \theta w^* - s)}, \\ \frac{(1-\alpha)\delta p_c}{b(w^* - c_t) + p_c(\alpha M + \theta w^*) - \alpha\eta_{ct}\delta p_c} \leq 1, \\ \frac{k_t - 2p_c b}{b(w^* - c_n) + p_c((1-\alpha)M + \theta w^*) - (1-\alpha)\eta_{cn}\delta p_c} \leq 1, \end{cases} \quad (30)$$

Then, there exists a unique Nash equilibrium (l_t^*, l_n^*) for the centralized noncooperative game model of power sales with energy suppliers' risk avoidance and its expression is followed as

$$(l_t^*, l_n^*) = \left(\frac{q_2\epsilon_3 - q_3\epsilon_4}{q_1 q_2 - q_3^2}, \frac{q_1\epsilon_4 - q_3\epsilon_3}{q_1 q_2 - q_3^2} \right), \quad (31)$$

where, (32) as shown at the bottom of the next page.

At this point, the optimal power production quantities and the optimal revenue functions are:

$$\begin{cases} q_{ct}^* = \alpha M + \frac{1}{2}\theta(p - c_g) + \frac{q_2 b \epsilon_3 - q_1 s \epsilon_4 - q_3 \epsilon_5}{\theta M(q_1 q_2 - q_3^2)} \\ \frac{4\theta(q_1 q_2 - q_3^2) + 2(b-s)(b+p_c\theta)(q_1+q_2-2q_3)}{\theta(b-s)(q_2-q_3)(bc_t - p_c\alpha M + \alpha\eta_{ct}\delta p_c)} \\ + \frac{4\theta(q_1 q_2 - q_3^2) + 2(b-s)(b+p_c\theta)(q_1+q_2-2q_3)}{\theta(b-s)(q_1-q_3)(bc_n - p_c(1-\alpha)M + (1-\alpha)\eta_{cn}\delta p_c)}, \\ q_{cn}^* = (1-\alpha)M + \frac{1}{2}\theta(p - c_g) + \frac{q_1 b \epsilon_4 - q_2 s \epsilon_3 - q_3 \epsilon_6}{q_1 q_2 - q_3^2} \\ \frac{4\theta(q_1 q_2 - q_3^2) + 2(b-s)(b+p_c\theta)(q_1+q_2-2q_3)}{\theta(b-s)(q_2-q_3)(bc_t - p_c\alpha M + \alpha\eta_{ct}\delta p_c)} \\ + \frac{4\theta(q_1 q_2 - q_3^2) + 2(b-s)(b+p_c\theta)(q_1+q_2-2q_3)}{\theta(b-s)(q_1-q_3)(bc_n - p_c(1-\alpha)M + (1-\alpha)\eta_{cn}\delta p_c)}, \\ q^* = M + \theta(p - c_g) + \frac{(b-s)(q_1\epsilon_4 + q_2\epsilon_3 - q_3(\epsilon_3 + \epsilon_4))}{q_1 q_2 - q_3^2} \\ \frac{2\theta(q_1 q_2 - q_3^2) + (b-s)(b+p_c\theta)(q_1+q_2-2q_3)}{\theta(b-s)(q_2-q_3)(bc_t - p_c\alpha M + \alpha\eta_{ct}\delta p_c)} \\ + \frac{2\theta(q_1 q_2 - q_3^2) + (b-s)(b+p_c\theta)(q_1+q_2-2q_3)}{\theta(b-s)(q_1-q_3)(bc_n - p_c(1-\alpha)M + (1-\alpha)\eta_{cn}\delta p_c)}, \\ E^{1*}(\pi_{ct}^a) = H_t(\alpha M + \theta(H_t + c_t)) - p_c A \\ - \alpha\eta_{ct}\delta H_t + \frac{q_2\epsilon_7 - q_1 s H_t \epsilon_4 - q_3 \epsilon_8}{q_1 q_2 - q_3^2} \\ + \frac{(q_2\epsilon_3 - q_3\epsilon_4)(q_2\epsilon_9 - q_1 p_c s \epsilon_4 - q_3 \epsilon_{10})}{(q_1 q_2 - q_3^2)^2}, \\ E^{1*}(\pi_{cn}^a) = H_n((1-\alpha)M + \theta(H_n + c_n)) - p_c A \\ - (1-\alpha)\eta_{cn}\delta H_n + \frac{q_1\epsilon_{11} - q_2 s H_n \epsilon_3 - q_3 \epsilon_{12}}{q_1 q_2 - q_3^2} \\ + \frac{(q_1\epsilon_4 - q_3\epsilon_3)(q_1\epsilon_{13} - q_2 p_c s \epsilon_3 - q_3 \epsilon_{14})}{(q_1 q_2 - q_3^2)^2}, \\ E^{1*}(\pi_{cg}^n) = H_g(M + 2\theta(p - c_g - H_g) \\ + (b-s)\frac{q_1\epsilon_4 + q_2\epsilon_3 - q_3(\epsilon_3 + \epsilon_4)}{q_1 q_2 - q_3^2}), \end{cases} \quad (33)$$

where $H_t = w^* - c_t$, $H_n = w^* - c_n$, $H_g = p - c_g - w^*$, $\epsilon_5 = b\epsilon_4 - s\epsilon_3$, $\epsilon_6 = b\epsilon_3 - s\epsilon_4$, $\epsilon_7 = \epsilon_3^2$, $\epsilon_8 = \epsilon_3\epsilon_4 - sH_t\epsilon_3$, $\epsilon_9 = (-\frac{1}{2}k_t + p_c b)\epsilon_3$, $\epsilon_{10} = (-\frac{1}{2}k_t + p_c b)\epsilon_4 - p_c s\epsilon_3$, $\epsilon_{11} = \epsilon_4^2$, $\epsilon_{12} = \epsilon_3\epsilon_4 - sH_n\epsilon_4$, $\epsilon_{13} = (-\frac{1}{2}k_n + p_c b)\epsilon_4$ and $\epsilon_{14} = (-\frac{1}{2}k_n + p_c b)\epsilon_3 - p_c s\epsilon_4$.

Proof: Under the condition that the grid has formulated the optimal power price w^* for the energy suppliers, one performs the following analysis. Suppose that $\frac{\partial^2(E^1(\pi_{ct}^a)|_{w=w^*})}{\partial l_t^2} = -k_t + 2p_c b < 0$. Let $\frac{\partial(E^1(\pi_{ct}^a)|_{w=w^*})}{\partial l_t} = 0$ and then the optimal renewable energy utilization rate of the traditional energy supplier can be obtained:

$l_t(l_n, w^*) = \frac{b(w^* - c_t) + p_c(\alpha M + \theta w^* - s_t) - \alpha \eta_{ct} \delta p_c}{k_t - 2p_c b}$. Similarly, through analysis for the revenue function of the new energy supplier, the optimal renewable energy utilization rate of the new energy supplier can be obtained: $l_n(l_t, w^*) = \frac{b(w^* - c_n) + p_c((1 - \alpha)M + \theta w^* - s_t) - (1 - \alpha)\eta_{cn} \delta p_c}{k_n - 2p_c b}$.

Note that for satisfying meaning of renewable energy utilization rates in reality, the following inequalities should be satisfied that

$$\begin{cases} \eta_{ct} \leq \frac{b(w^* - c_t) + p_c(\alpha M + \theta w^* - s)}{\alpha \delta p_c}, \\ \eta_{cn} < \frac{b(w^* - c_n) + p_c((1 - \alpha)M + \theta w^* - s)}{(1 - \alpha)\delta p_c}, \\ \frac{b(w^* - c_t) + p_c(\alpha M + \theta w^*) - \alpha \eta_{ct} \delta p_c}{k_t - 2p_c b} \leq 1, \\ \frac{b(w^* - c_n) + p_c((1 - \alpha)M + \theta w^*) - (1 - \alpha)\eta_{cn} \delta p_c}{k_n - 2p_c b} \leq 1. \end{cases} \quad (34)$$

It's worth pointing out that $k_i - 2p_c b - p_c s > 0$ for $i \in \{t, n\}$, which demonstrates that there exists a unique Nash equilibrium for the model according to Theorem 5.

Apparently, $l_t(l_n, w^*)$ and $l_n(l_t, w^*)$ are intertwined in terms of renewable energy utilization rates. Then, suppose that the model is under the complete information static game circumstance and by linking $l_t(l_n, w^*)$ and $l_n(l_t, w^*)$, the unique Nash equilibrium can be obtained as follows:

$$(l_t^*(w^*), l_n^*(w^*)) = \left(\frac{q_2 \epsilon_1 - q_3 \epsilon_2}{q_1 q_2 - q_3^2}, \frac{q_1 \epsilon_2 - q_3 \epsilon_1}{q_1 q_2 - q_3^2} \right), \quad (35)$$

where $\epsilon_1 = b(w^* - c_t) + p_c(\alpha M + \theta w^*) - \alpha \eta_{ct} \delta p_c$ and $\epsilon_2 = b(w^* - c_n) + p_c((1 - \alpha)M + \theta w^*) - (1 - \alpha)\eta_{cn} \delta p_c$. It can be seen in (35) that the unique Nash equilibrium is determined by the optimal power price w^* offered by the grid. Next, w^* would be found.

By Substituting $l_t^*(w^*)$ and $l_n^*(w^*)$ into (11), one can compute $q^*(w^*)$ based on which the expression

of the revenue function of the grid is integrated as $E^1(\pi_{cg}^n)(w) = E^1(\pi_{cg}^n)(w^*)|_{w^*=w} = (p - c_g - \omega)(M + 2\theta w + (b - s) \frac{(q_1 - q_3)(b(w - c_n) + p_c((1 - \alpha)M + \theta w) - (1 - \alpha)\eta_{cn} \delta p_c)}{q_1 q_2 - q_3^2} + \frac{(q_2 - q_3)(b(w - c_t) + p_c(\alpha M + \theta w) - \alpha \eta_{ct} \delta p_c)}{q_1 q_2 - q_3^2})$. Noticing that $b > s > 0$ proposed in Assumption 2 and $q_1 > q_2 > q_3 > 0$, it can be obtained that $\frac{\partial^2 E^1(\pi_{cg}^n)(w)}{\partial w^2} = -4\theta - 2(b - s)(b + p_c \theta) \frac{q_1 + q_2 - 2q_3}{q_1 q_2 - q_3^2} < 0$, which declares that $E^1(\pi_{cg}^n)(w)$ is a strictly concave function of w and there exists an optimal w^* whose acquisition requires $\frac{\partial E^1(\pi_{cg}^n)(w)}{\partial w}|_{w=w^*} = 0$ and then the result can be obtained as follows:

$$\begin{cases} w^* = \frac{1}{2}(p - c_g) \\ - \frac{M(q_1 q_2 - q_3^2)}{4\theta(q_1 q_2 - q_3^2) + 2(b - s)(b + p_c \theta)(q_1 + q_2 - 2q_3)} \\ + \frac{(b - s)(q_1 - q_3)(bc_n - p_c(1 - \alpha)M + (1 - \alpha)\eta_{cn} \delta p_c)}{4\theta(q_1 q_2 - q_3^2) + 2(b - s)(b + p_c \theta)(q_1 + q_2 - 2q_3)} \\ + \frac{4\theta(q_1 q_2 - q_3^2) + 2(b - s)(b + p_c \theta)(q_1 + q_2 - 2q_3)}{(b - s)(q_2 - q_3)(bc_t - p_c \alpha M + \alpha \eta_{ct} \delta p_c)} \\ + \frac{4\theta(q_1 q_2 - q_3^2) + 2(b - s)(b + p_c \theta)(q_1 + q_2 - 2q_3)}{4\theta(q_1 q_2 - q_3^2) + 2(b - s)(b + p_c \theta)(q_1 + q_2 - 2q_3)}, \end{cases} \quad (36)$$

where the optimal power price w^* must satisfy the constraint that $\max\{c_t, c_n\} < w^* < p - c_g$ according to Remark 2.

By substituting w^* into (35), the unique Nash equilibrium can be calculated as follows:

$$(l_t^*, l_n^*) = \left(\frac{q_2 \epsilon_3 - q_3 \epsilon_4}{q_1 q_2 - q_3^2}, \frac{q_1 \epsilon_4 - q_3 \epsilon_3}{q_1 q_2 - q_3^2} \right), \quad (37)$$

Note that the values of ϵ_3 and ϵ_4 are omitted here due to space limitations of the paper.

By substituting w^* , l_t^* , l_n^* into (11) (25)-(27), q_{ct}^* , q_{cn}^* , q^* , $E^{1*}(\pi_{ct}^a)$, $E^{1*}(\pi_{cn}^a)$ and $E^{1*}(\pi_{cg}^n)$ can be respectively calculated. To this end, the proof is completed. \square

$$\begin{cases} \epsilon_3 = \frac{(b + p_c \theta)(p - c_g)}{2} - bc_t + p_c \alpha M - \alpha \eta_{ct} \delta p_c \\ - \frac{M(b + p_c \theta)(q_1 q_2 - q_3^2)}{4\theta(q_1 q_2 - q_3^2) + 2(b - s)(b + p_c \theta)(q_1 + q_2 - 2q_3)} \\ + \frac{(b + p_c \theta)(b - s)(q_1 - q_3)(bc_n - p_c(1 - \alpha)M + (1 - \alpha)\eta_{cn} \delta p_c)}{4\theta(q_1 q_2 - q_3^2) + 2(b - s)(b + p_c \theta)(q_1 + q_2 - 2q_3)} \\ + \frac{(b + p_c \theta)(b - s)(q_2 - q_3)(bc_t - p_c \alpha M + \alpha \eta_{ct} \delta p_c)}{4\theta(q_1 q_2 - q_3^2) + 2(b - s)(b + p_c \theta)(q_1 + q_2 - 2q_3)}, \\ \epsilon_4 = \frac{(b + p_c \theta)(p - c_g)}{2} - bc_n + p_c(1 - \alpha)M - (1 - \alpha)\eta_{cn} \delta p_c \\ - \frac{M(b + p_c \theta)(q_1 q_2 - q_3^2)}{4\theta(q_1 q_2 - q_3^2) + 2(b - s)(b + p_c \theta)(q_1 + q_2 - 2q_3)} \\ + \frac{(b + p_c \theta)(b - s)(q_1 - q_3)(bc_n - p_c(1 - \alpha)M + (1 - \alpha)\eta_{cn} \delta p_c)}{4\theta(q_1 q_2 - q_3^2) + 2(b - s)(b + p_c \theta)(q_1 + q_2 - 2q_3)} \\ + \frac{(b + p_c \theta)(b - s)(q_2 - q_3)(bc_t - p_c \alpha M + \alpha \eta_{ct} \delta p_c)}{4\theta(q_1 q_2 - q_3^2) + 2(b - s)(b + p_c \theta)(q_1 + q_2 - 2q_3)}, \end{cases} \quad (32)$$

3) IMPACTS OF RISK-AVERSION COEFFICIENTS ON THE OPTIMAL RENEWABLE ENERGY POWER'S INVESTMENT AND PRODUCTION STRATEGY, THE OPTIMAL POWER PRICE AND THE OPTIMAL TOTAL POWER PRODUCTION QUANTITY

Theorem 7: The optimal renewable energy utilization rate of the traditional energy supplier l_t^* and the optimal power production quantity of the traditional energy supplier q_{ct}^* are negatively correlated with the risk-aversion coefficient η_{ct} and are positively correlated with the risk-aversion coefficient η_{cn} . The optimal renewable energy utilization rate of the new energy supplier l_n^* and the optimal power production quantity of the new energy supplier q_{cn}^* are negatively correlated with the risk-aversion coefficient η_{cn} and are positively correlated with the risk-aversion coefficient η_{ct} . The optimal power price w^* is positively correlated with the risk-aversion coefficients η_{ct} η_{cn} . The optimal total power production quantity q^* is negatively correlated with the risk-aversion coefficients η_{ct} η_{cn} .

Proof: Noticing that $b > s > 0$ and $q_1 > q_2 > q_3 > 0$, one can obtain that, as in (38), shown at the bottom of the next page.

To this end, the proof is completed. \square

Remark 3: The control variable method is adopted to analyze the impact of each of risk-aversion coefficients in Theorem 7 and then the following conclusions are obtained:

1) When the traditional energy supplier focuses on risk avoidance, to satisfy its own risk-avoidance needs, it would reduce its own renewable energy utilization rate. As the renewable energy utilization rate of the traditional energy supplier decreases, to capture power market's share, the new energy supplier would improve its own renewable energy utilization rate. However, due to $\frac{\partial l_t^*}{\partial \eta_{ct}} + \frac{\partial l_n^*}{\partial \eta_{cn}} = \frac{\delta \alpha p_c (q_2 - q_3) (\frac{(b-s)(b+p_c\theta)(q_1+q_2-2q_3)}{4\theta(q_1q_2-q_3^2)+2(b-s)(b+p_c\theta)(q_1+q_2-2q_3)} - 1)}{q_1q_2-q_3^2} < 0$, which declares that the total power production quantity would be decreased from the perspective of energy suppliers, to ensure the power demands of power users, the grid would raise the power price to increase the total power production quantity from the perspective of the grid. However, since the power production quantities are associated with renewable energy utilization rates and the power price, further calculations are required to obtain the correlations between the risk-aversion coefficient of the traditional energy supplier and power production quantities with the calculations that with an increase in the risk-aversion coefficient of the traditional energy supplier, the power production quantity of the traditional energy supplier and the total power production quantity decrease and the power production quantity of the new energy supplier increases. 2) A similar analysis for the new energy supplier is conducted and is omitted here.

This section provides the centralized noncooperative game model of power sales with energy suppliers' risk avoidance. Next, the centralized noncooperative game model of power sales with grid's risk avoidance will be provided.

C. THE CENTRALIZED NONCOOPERATIVE GAME MODEL OF POWER SALES WITH GRID'S RISK AVOIDANCE

The characteristics of the centralized noncooperative game model of power sales with grid's risk avoidance shown in Fig. 4 are as follows:

- The power of power users is provided by the grid while the power of the grid is provided by the energy suppliers.
- After innovations in renewable energy technologies, the energy suppliers match renewable energy power production quotas issued by the government on Green Power Certificate System.
- The grid has the characteristic of the risk avoidance.

By respectively adjusting their respective control variables, their revenue functions are maximized and the expressions are as follows:

$$\begin{cases} \max_{l_t} E^2(\pi_{ct}^n) = (\omega - c_t)q_{ct} - C_t + P_t, \\ s.t. 0 \leq l_t \leq 1. \end{cases} \quad (39)$$

$$\begin{cases} \max_{l_n} E^2(\pi_{cn}^n) = (\omega - c_n)q_{cn} - C_n + P_n, \\ s.t. 0 < l_n \leq 1. \end{cases} \quad (40)$$

$$\begin{cases} \max_w E^2(\pi_{cg}^a) = (p - c_g - \omega)q + R_{cg}, \\ s.t. \max\{c_t, c_n\} < w < p - c_g. \end{cases} \quad (41)$$

where R_{cg} denotes the losses of the grid caused by grid's risk avoidance and its expression is designed as $R_{cg} = -\eta_{cg}\delta(p - c_g - w)$ by utilizing the mean variance method and introducing the risk-aversion coefficient η_{cg} . Note that the event sequence for solving the problem of maximizing revenues of players for the model is the same as the event sequence in the previous section and is omitted here.

1) ANALYSIS OF EXISTENCE, UNIQUENESS AND STABILITY OF THE NASH EQUILIBRIUM

Theorem 8: Suppose that $k_i - 2p_c b > p_c s$ for $i \in \{t, n\}$. Then, there exists a unique Nash equilibrium (l_t^*, l_n^*) that satisfies (3) and the Nash equilibrium is globally exponentially stable.

Proof: The remaining analysis follows that of Theorem 5 by replacing an auxiliary system therein with

$$\dot{\mathbf{L}} = \frac{\partial G^2(\mathbf{L}, w^*)}{\partial \mathbf{L}}, \quad (42)$$

where $\mathbf{L} = [l_t, l_n]^T$ and $\frac{\partial G^2(\mathbf{L}, w^*)}{\partial \mathbf{L}} = [\frac{\partial E^2(\pi_{ct}^n)(\mathbf{L}, w^*)}{\partial l_t}, \frac{\partial E^2(\pi_{cn}^n)(\mathbf{L}, w^*)}{\partial l_n}]^T$ and is omitted here. \square

2) COMPUTATION OF THE UNIQUE NASH EQUILIBRIUM, THE OPTIMAL POWER PRODUCTION QUANTITIES AND THE OPTIMAL REVENUE FUNCTIONS

Theorem 9: Suppose that Assumptions 1-3 hold and the parameters satisfy, as in (43), shown at the bottom of page 13.

Then, there exists a unique Nash equilibrium (l_t^*, l_n^*) for the centralized noncooperative game model of power sales with

$$\begin{aligned}
 \frac{\partial l_t^*}{\partial \eta_{ct}} &= \frac{\alpha \delta p_c (q_2 - q_3) \left(\frac{(b-s)(b+p_c\theta)(q_2-q_3)}{4\theta(q_1q_2-q_3^2)+2(b-s)(b+p_c\theta)(q_1+q_2-2q_3)} - 1 \right)}{q_1q_2 - q_3^2} - \frac{\alpha \delta p_c q_3}{q_1q_2 - q_3^2} < 0, \\
 \frac{\partial l_t^*}{\partial \eta_{cn}} &= \frac{(1-\alpha)\delta p_c(b-s)(b+p_c\theta)(q_1-q_3)(q_2-q_3)}{(q_1q_2 - q_3^2)(4\theta(q_1q_2 - q_3^2) + 2(b-s)(b+p_c\theta)(q_1+q_2-2q_3))} \\
 &+ \frac{(1-\alpha)\delta p_c q_3}{q_1q_2 - q_3^2} > 0, \\
 \frac{\partial l_n^*}{\partial \eta_{ct}} &= \frac{\alpha \delta p_c (b-s)(b+p_c\theta)(q_1-q_3)(q_2-q_3)}{(q_1q_2 - q_3^2)(4\theta(q_1q_2 - q_3^2) + 2(b-s)(b+p_c\theta)(q_1+q_2-2q_3))} \\
 &+ \frac{\alpha \delta p_c q_3}{q_1q_2 - q_3^2} > 0, \\
 \frac{\partial l_n^*}{\partial \eta_{cn}} &= \frac{(1-\alpha)\delta p_c (q_1 - q_3) \left(\frac{(b-s)(b+p_c\theta)(q_1-q_3)}{4\theta(q_1q_2-q_3^2)+2(b-s)(b+p_c\theta)(q_1+q_2-2q_3)} - 1 \right)}{q_1q_2 - q_3^2} \\
 &- \frac{(1-\alpha)\delta p_c q_3}{q_1q_2 - q_3^2} < 0, \\
 \frac{\partial q_{ct}^*}{\partial \eta_{ct}} &= -\frac{\theta \alpha \delta p_c (3b+s)(q_2-q_3)}{4\theta(q_1q_2 - q_3^2) + 2(b-s)(b+p_c\theta)(q_1+q_2-2q_3)} \\
 &- \frac{\alpha \delta p_c (b-s)(b+p_c\theta)(q_2-q_3)(b(q_1+q_2-2q_3) + (b+s)(q_1-q_3))}{(q_1q_2 - q_3^2)(4\theta(q_1q_2 - q_3^2) + 2(b-s)(b+p_c\theta)(q_1+q_2-2q_3))} \\
 &- \frac{\alpha \delta p_c (b+s)q_3}{q_1q_2 - q_3^2} < 0, \\
 \frac{\partial q_{ct}^*}{\partial \eta_{cn}} &= \frac{\theta(1-\alpha)\delta p_c(b-s)(q_1-q_3)}{4\theta(q_1q_2 - q_3^2) + 2(b-s)(b+p_c\theta)(q_1+q_2-2q_3)} \\
 &+ \frac{(1-\alpha)\delta p_c b(b-s)(b+p_c\theta)(q_1-q_3)(q_2-q_3)}{(q_1q_2 - q_3^2)(4\theta(q_1q_2 - q_3^2) + 2(b-s)(b+p_c\theta)(q_1+q_2-2q_3))} \\
 &+ \frac{(1-\alpha)\delta p_c s(q_1 - q_3) \left(1 - \frac{(b-s)(b+p_c\theta)(q_1-q_3)}{4\theta(q_1q_2-q_3^2)+2(b-s)(b+p_c\theta)(q_1+q_2-2q_3)} \right)}{q_1q_2 - q_3^2} \\
 &+ \frac{(1-\alpha)\delta p_c (b+s)q_3}{q_1q_2 - q_3^2} > 0, \\
 \frac{\partial q_{cn}^*}{\partial \eta_{ct}} &= \frac{\theta \alpha \delta p_c (b-s)(q_2-q_3)}{4\theta(q_1q_2 - q_3^2) + 2(b-s)(b+p_c\theta)(q_1+q_2-2q_3)} \\
 &+ \frac{\alpha \delta p_c b(b-s)(b+p_c\theta)(q_1-q_3)(q_2-q_3)}{(q_1q_2 - q_3^2)(4\theta(q_1q_2 - q_3^2) + 2(b-s)(b+p_c\theta)(q_1+q_2-2q_3))} \\
 &+ \frac{\alpha \delta p_c s(q_2 - q_3) \left(1 - \frac{(b-s)(b+p_c\theta)(q_2-q_3)}{4\theta(q_1q_2-q_3^2)+2(b-s)(b+p_c\theta)(q_1+q_2-2q_3)} \right)}{q_1q_2 - q_3^2} \\
 &+ \frac{\alpha \delta p_c (b+s)q_3}{q_1q_2 - q_3^2} > 0, \\
 \frac{\partial q_{cn}^*}{\partial \eta_{cn}} &= -\frac{\theta(1-\alpha)\delta p_c(3b+s)(q_1-q_3)}{4\theta(q_1q_2 - q_3^2) + 2(b-s)(b+p_c\theta)(q_1+q_2-2q_3)} \\
 &- \frac{(1-\alpha)\delta p_c(b-s)(b+p_c\theta)(q_1-q_3)(b(q_1+q_2-2q_3) + (b+s)(q_2-q_3))}{(q_1q_2 - q_3^2)(4\theta(q_1q_2 - q_3^2) + 2(b-s)(b+p_c\theta)(q_1+q_2-2q_3))} \\
 &- \frac{(1-\alpha)\delta p_c (b+s)q_3}{q_1q_2 - q_3^2} < 0, \\
 \frac{\partial w^*}{\partial \eta_{ct}} &= \frac{\alpha \delta p_c (b-s)(q_2-q_3)}{4\theta(q_1q_2 - q_3^2) + 2(b-s)(b+p_c\theta)(q_1+q_2-2q_3)} > 0, \\
 \frac{\partial w^*}{\partial \eta_{cn}} &= \frac{(1-\alpha)\delta p_c (b-s)(q_1-q_3)}{(1-\alpha)\delta p_c (b-s)(q_1-q_3)} > 0, \\
 \frac{\partial q^*}{\partial \eta_{cn}} &= \frac{4\theta(q_1q_2 - q_3^2) + 2(b-s)(b+p_c\theta)(q_1+q_2-2q_3)}{\alpha \delta p_c (b-s)(q_2-q_3)} < 0, \\
 \frac{\partial \eta_{ct}}{\partial q^*} &= -\frac{2(q_1q_2 - q_3^2)}{(1-\alpha)\delta p_c (b-s)(q_1-q_3)} < 0, \\
 \frac{\partial \eta_{cn}}{\partial q^*} &= -\frac{(1-\alpha)\delta p_c (b-s)(q_1-q_3)}{2(q_1q_2 - q_3^2)} < 0,
 \end{aligned} \tag{38}$$

grid's risk avoidance and its expression is followed as:

$$(l_t^*, l_n^*) = \left(\frac{q_2 s_3 - q_3 s_4}{q_1 q_2 - q_3^2}, \frac{q_1 s_4 - q_3 s_3}{q_1 q_2 - q_3^2} \right), \quad (44)$$

where, as in (45), shown at the bottom of the next page.

At this point, the optimal power production quantities and the optimal revenue functions are, as in (46), shown at the bottom of the next page, where $z_t = w^* - c_t$, $z_n = w^* - c_n$, $z_g = p - c_g - w^*$, $s_5 = b s_4 - s s_3$, $s_6 = b s_3 - s s_4$, $s_7 = s_3^2$, $s_8 = s_3 s_4 - s z_t s_3$, $s_9 = (-\frac{1}{2} k_t + p_c b) s_3$, $s_{10} = (-\frac{1}{2} k_t + p_c b) s_4 - p_c s s_3$, $s_{11} = s_4^2$, $s_{12} = s_3 s_4 - s z_n s_4$, $s_{13} = (-\frac{1}{2} k_n + p_c b) s_4$ and $s_{14} = (-\frac{1}{2} k_n + p_c b) s_3 - p_c s s_4$.

The Proof follows that of Theorem 6 and hence, is omitted.

3) IMPACTS OF RISK-AVERSION COEFFICIENTS ON THE OPTIMAL RENEWABLE ENERGY POWER'S INVESTMENT AND PRODUCTION STRATEGY, THE OPTIMAL POWER PRICE AND THE OPTIMAL TOTAL POWER PRODUCTION QUANTITY

Theorem 10: The optimal power price w^* , the optimal total power production quantity q^* and the Nash equilibrium (l_t^*, l_n^*) are positively correlated with the risk-aversion coefficient η_{cg} . The optimal power production quantity of the traditional energy supplier q_{ct}^* is positively correlated with the risk-aversion coefficient η_{cg} under certain condition. The optimal power production quantity of the new energy supplier q_{cn}^* is positively correlated with the risk-aversion coefficient η_{cg} .

Proof: Noticing that $b > s > 0$ and $q_1 > q_2 > q_3 > 0$, one can obtain that, as in (47), shown at the bottom of page 15.

To this end, the proof is completed. \square

Remark 4: Theorem 10 declares that as the risk-aversion coefficient of the grid increases, the power price of the grid would be improved with the aim to hedge the risk and reduce the losses caused by the risk avoidance and the grid reduces the impact of the risk on its own revenue by utilizing a method of small profit but rapid turnover. For the energy suppliers, their profit margins and grid's purchasing demands for the total power production quantity are improved. Thus, they would seize the opportunity to boost their power production quantities from their own perspective

with the measure that they invest more in renewable energy technologies to improve the renewable energy utilization rates. However, since the power production quantities of traditional and new energy suppliers are negatively correlated with each other's renewable energy utilization rates, further calculations are needed to conclude the correlation between the risk-aversion coefficient of the grid and their power production quantities. By further calculations, the conclusion that with an increase in the risk-aversion coefficient of the grid, the power production quantity of the traditional energy supplier under certain condition increases and the power production quantity of the new energy supplier increases is drawn.

The centralized noncooperative game model of power sales with grid's risk avoidance is analyzed in this section. In the following, the centralized noncooperative game model of power sales with risk avoidance for grid and energy suppliers will be considered.

D. THE CENTRALIZED NONCOOPERATIVE GAME MODEL OF POWER SALES WITH RISK AVOIDANCE FOR GRID AND ENERGY SUPPLIERS

The characteristics of the centralized noncooperative game model of power sales with risk avoidance for grid and energy suppliers shown in Fig. 5 are as follows:

- The power users get the power from the grid whose power comes from the energy suppliers.
- After innovations in renewable energy technologies, the energy suppliers match renewable energy power production quotas issued by the government on Green Power Certificate System.
- The energy suppliers and the grid have the characteristic of the risk avoidance.

By respectively adjusting their respective control variables, their revenue functions are maximized and the expressions are as follows:

$$\begin{cases} \max_{l_t} E^3(\pi_{ct}^a) = (\omega - c_t)q_{ct} - C_t + P_t + R_{ct}, \\ s.t. 0 \leq l_t \leq 1. \end{cases} \quad (48)$$

$$\begin{cases} \max\{c_t, c_n\} < w^* < p - c_g, \\ \text{where } w^* = \frac{1}{2}(p - c_g) \\ + \frac{(\eta_{cg}\delta - M)(q_1 q_2 - q_3^2)}{4\theta(q_1 q_2 - q_3^2) + 2(b - s)(b + p_c\theta)(q_1 + q_2 - 2q_3)} \\ + \frac{(b - s)((q_1 - q_3)(bc_n - p_c(1 - \alpha)M) + (q_2 - q_3)(bc_t - p_c\alpha M))}{4\theta(q_1 q_2 - q_3^2) + 2(b - s)(b + p_c\theta)(q_1 + q_2 - 2q_3)}, \\ \frac{b(w^* - c_t) + p_c(\alpha M + \theta w^*)}{k_t - 2p_c b} \leq 1, \\ \frac{b(w^* - c_n) + p_c((1 - \alpha)M + \theta w^*)}{k_n - 2p_c b} \leq 1, \end{cases} \quad (43)$$

$$\begin{cases} \max_{l_n} E^3(\pi_{cn}^a) = (\omega - c_n)q_{cn} - C_n + P_n + R_{cn}, \\ \text{s.t. } 0 < l_n \leq 1. \end{cases} \quad (49)$$

event sequence in the previous two sections and is omitted here.

$$\begin{cases} \max_w E^3(\pi_{cg}^a) = (p - c_g - \omega)q + R_{cg}, \\ \text{s.t. } \max\{c_t, c_n\} < w < p - c_g. \end{cases} \quad (50)$$

1) ANALYSIS OF EXISTENCE, UNIQUENESS AND STABILITY OF THE NASH EQUILIBRIUM

Theorem 11: Suppose that $k_i - 2p_c b > p_c s$ for $i \in \{t, n\}$. Then, there exists a unique Nash equilibrium (l_t^*, l_n^*) that satisfies (4) and the Nash equilibrium is globally exponentially stable.

The event sequence for solving the problem of maximizing revenues of players for the model is the same as the

$$\begin{cases} s_3 = \frac{1}{2}(b + p_c\theta)(p - c_g) - bc_t + p_c\alpha M \\ \quad + \frac{(b + p_c\theta)(\eta_{cg}\delta - M)(q_1q_2 - q_3^2)}{4\theta(q_1q_2 - q_3^2) + 2(b - s)(b + p_c\theta)(q_1 + q_2 - 2q_3)} \\ \quad + \frac{(b + p_c\theta)(b - s)((q_1 - q_3)(bc_n - p_c(1 - \alpha)M) + (q_2 - q_3)(bc_t - p_c\alpha M))}{4\theta(q_1q_2 - q_3^2) + 2(b - s)(b + p_c\theta)(q_1 + q_2 - 2q_3)}, \\ s_4 = \frac{1}{2}(b + p_c\theta)(p - c_g) - bc_n + p_c(1 - \alpha)M \\ \quad + \frac{(b + p_c\theta)(\eta_{cg}\delta - M)(q_1q_2 - q_3^2)}{4\theta(q_1q_2 - q_3^2) + 2(b - s)(b + p_c\theta)(q_1 + q_2 - 2q_3)} \\ \quad + \frac{(b + p_c\theta)(b - s)((q_1 - q_3)(bc_n - p_c(1 - \alpha)M) + (q_2 - q_3)(bc_t - p_c\alpha M))}{4\theta(q_1q_2 - q_3^2) + 2(b - s)(b + p_c\theta)(q_1 + q_2 - 2q_3)}. \end{cases} \quad (45)$$

$$\begin{cases} q_{ct}^* = \alpha M + \frac{1}{2}\theta(p - c_g) + \frac{q_2bs_3 - q_1s_4 - q_3s_5}{q_1q_2 - q_3^2} \\ \quad + \frac{\theta(\eta_{cg}\delta - M)(q_1q_2 - q_3^2)}{4\theta(q_1q_2 - q_3^2) + 2(b - s)(b + p_c\theta)(q_1 + q_2 - 2q_3)} \\ \quad + \frac{\theta(b - s)((q_1 - q_3)(bc_n - p_c(1 - \alpha)M) + (q_2 - q_3)(bc_t - p_c\alpha M))}{4\theta(q_1q_2 - q_3^2) + 2(b - s)(b + p_c\theta)(q_1 + q_2 - 2q_3)}, \\ q_{cn}^* = (1 - \alpha)M + \frac{1}{2}\theta(p - c_g) + \frac{q_1bs_4 - q_2s_3 - q_3s_6}{q_1q_2 - q_3^2} \\ \quad + \frac{\theta(\eta_{cg}\delta - M)(q_1q_2 - q_3^2)}{4\theta(q_1q_2 - q_3^2) + 2(b - s)(b + p_c\theta)(q_1 + q_2 - 2q_3)} \\ \quad + \frac{\theta(b - s)((q_1 - q_3)(bc_n - p_c(1 - \alpha)M) + (q_2 - q_3)(bc_t - p_c\alpha M))}{4\theta(q_1q_2 - q_3^2) + 2(b - s)(b + p_c\theta)(q_1 + q_2 - 2q_3)}, \\ q^* = M + \theta(p - c_g) + \frac{(b - s)(q_1s_4 + q_2s_3 - q_3(s_3 + s_4))}{q_1q_2 - q_3^2} \\ \quad + \frac{\theta(\eta_{cg}\delta - M)(q_1q_2 - q_3^2)}{2\theta(q_1q_2 - q_3^2) + (b - s)(b + p_c\theta)(q_1 + q_2 - 2q_3)} \\ \quad + \frac{\theta(b - s)((q_1 - q_3)(bc_n - p_c(1 - \alpha)M) + (q_2 - q_3)(bc_t - p_c\alpha M))}{2\theta(q_1q_2 - q_3^2) + (b - s)(b + p_c\theta)(q_1 + q_2 - 2q_3)}, \\ E^{2*}(\pi_{ct}^n) = z_t(\alpha M + \theta(z_t + c_t)) - p_cA \\ \quad + \frac{q_2s_7 - q_1sz_t s_4 - q_3s_8}{q_1q_2 - q_3^2} + \frac{(q_2s_3 - q_3s_4)(q_2s_9 - q_1p_c s_4 - q_3s_{10})}{(q_1q_2 - q_3^2)^2}, \\ E^{2*}(\pi_{cn}^n) = z_n((1 - \alpha)M + \theta(z_n + c_n)) - p_cA \\ \quad + \frac{q_1s_{11} - q_2sz_n s_3 - q_3s_{12}}{q_1q_2 - q_3^2} + \frac{(q_1s_4 - q_3s_3)(q_1s_{13} - q_2p_c s_3 - q_3s_{14})}{(q_1q_2 - q_3^2)^2}, \\ E^{2*}(\pi_{cg}^a) = z_g(M + 2\theta(p - c_g - z_g) - \eta_{cg}\delta \\ \quad + (b - s)\frac{q_1s_4 + q_2s_3 - q_3(s_3 + s_4)}{q_1q_2 - q_3^2}), \end{cases} \quad (46)$$

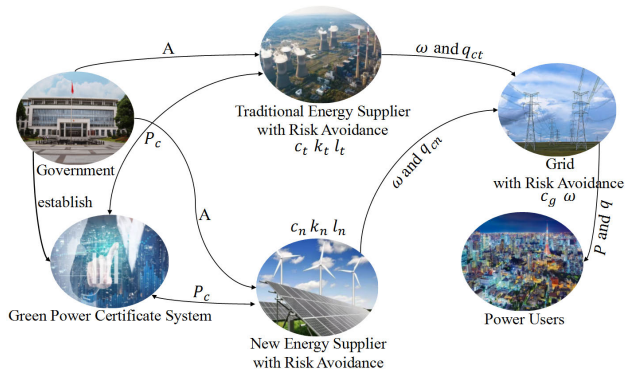


FIGURE 5. The centralized noncooperative game model of power sales with risk avoidance for grid and energy suppliers.

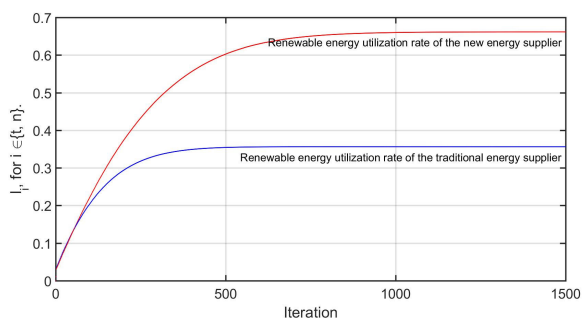


FIGURE 6. The trajectories of l_i for $i \in \{t, n\}$ for the distributed noncooperative game model of power sales with energy suppliers' risk avoidance.

Proof: The remaining analysis follows that of Theorem 5 by replacing an auxiliary system therein with

$$\dot{\mathbf{L}} = \frac{\partial G^3(\mathbf{L}, w^*)}{\partial \mathbf{L}}, \quad (51)$$

where $\mathbf{L} = [l_t, l_n]^T$ and $\frac{\partial G^3(\mathbf{L}, w^*)}{\partial \mathbf{L}} = \left[\frac{\partial E^3(\pi_{ct}^a)(\mathbf{L}, w^*)}{\partial l_t}, \frac{\partial E^3(\pi_{cn}^a)(\mathbf{L}, w^*)}{\partial l_n} \right]^T$ and is omitted here. \square

2) COMPUTATION OF THE UNIQUE NASH EQUILIBRIUM, THE OPTIMAL POWER PRODUCTION QUANTITIES AND THE OPTIMAL REVENUE FUNCTIONS

Theorem 12: Suppose that Assumptions 1-3 hold and the parameters satisfy

$$\left\{ \max\{c_t, c_n\} < w^* < p - c_g, \right. \quad (52)$$

$$\left\{ \begin{aligned} &\text{where } w^* = \frac{1}{2}(p - c_g) \\ &+ \frac{(\eta_{cg}\delta - M)(q_1q_2 - q_3^2)}{4\theta(q_1q_2 - q_3^2) + 2(b-s)(b+p_c\theta)(q_1+q_2-2q_3)} \\ &+ \frac{(b-s)(q_1-q_3)(bc_n - p_c(1-\alpha)M + (1-\alpha)\eta_{cn}\delta p_c)}{4\theta(q_1q_2 - q_3^2) + 2(b-s)(b+p_c\theta)(q_1+q_2-2q_3)} \\ &+ \frac{4\theta(q_1q_2 - q_3^2) + 2(b-s)(b+p_c\theta)(q_1+q_2-2q_3)}{(b-s)(q_2-q_3)(bc_t - p_c\alpha M + \alpha\eta_{ct}\delta p_c)}, \\ &\eta_{ct} \leq \frac{b(w^* - c_t) + p_c(\alpha M + \theta w^* - s)}{\alpha\delta p_c}, \\ &\eta_{cn} < \frac{b(w^* - c_n) + p_c((1-\alpha)M + \theta w^* - s)}{(1-\alpha)\delta p_c}, \\ &\frac{b(w^* - c_t) + p_c(\alpha M + \theta w^*) - \alpha\eta_{ct}\delta p_c}{k_t - 2p_c b} \leq 1, \\ &\frac{b(w^* - c_n) + p_c((1-\alpha)M + \theta w^*) - (1-\alpha)\eta_{cn}\delta p_c}{k_n - 2p_c b} \leq 1. \end{aligned} \right. \quad (53)$$

Then, there exists a unique Nash equilibrium for the centralized noncooperative game model of power sales with risk avoidance for grid and energy suppliers and its expression is followed as

$$(l_t^*, l_n^*) = \left(\frac{q_2\mu_3 - q_3\mu_4}{q_1q_2 - q_3^2}, \frac{q_1\mu_4 - q_3\mu_3}{q_1q_2 - q_3^2} \right), \quad (54)$$

where, as in (55), shown at the bottom of the next page.

$$\left\{ \begin{aligned} &\frac{\partial w^*}{\partial \eta_{cg}} = \frac{\delta(q_1q_2 - q_3^2)}{4\theta(q_1q_2 - q_3^2) + 2(b-s)(b+p_c\theta)(q_1+q_2-2q_3)} > 0, \\ &\frac{\partial q^*}{\partial \eta_{cg}} = \frac{\delta}{2} > 0, \\ &\frac{\partial \eta_{cg}}{\partial l_t^*} = \frac{\delta(b+p_c\theta)(q_2-q_3)}{4\theta(q_1q_2 - q_3^2) + 2(b-s)(b+p_c\theta)(q_1+q_2-2q_3)} > 0, \\ &\frac{\partial \eta_{cg}}{\partial l_n^*} = \frac{\delta(b+p_c\theta)(q_1-q_3)}{4\theta(q_1q_2 - q_3^2) + 2(b-s)(b+p_c\theta)(q_1+q_2-2q_3)} > 0, \\ &\frac{\partial \eta_{cg}}{\partial q_{ct}^*} = \frac{\delta}{4} \left(1 - \frac{\delta(b+s)(b+p_c\theta)(q_1-q_2)}{2\theta(q_1q_2 - q_3^2) + (b-s)(b+p_c\theta)(q_1+q_2-2q_3)} \right) > 0 \\ &\text{if and only if } \frac{2\theta(q_1q_2 - q_3^2) + (b-s)(b+p_c\theta)(q_1+q_2-2q_3)}{(b+s)(b+p_c\theta)(q_1-q_2)} < 1, \\ &\frac{\partial q_{cn}^*}{\partial \eta_{cg}} = \frac{\delta}{4} \left(1 + \frac{\delta(b+s)(b+p_c\theta)(q_1-q_2)}{2\theta(q_1q_2 - q_3^2) + (b-s)(b+p_c\theta)(q_1+q_2-2q_3)} \right) > 0, \end{aligned} \right. \quad (47)$$

At this point, the optimal power production quantities and the optimal revenue functions are:

$$\begin{aligned}
 q_{ct}^* &= \alpha M + \frac{\theta(p-c_g)}{2} + \frac{q_2 b \mu_3 - q_1 s \mu_4 - q_3 \mu_5}{q_1 q_2 - q_3^2} \\
 &+ \frac{\theta(\eta_{cg} \delta - M)(q_1 q_2 - q_3^2)}{4\theta(q_1 q_2 - q_3^2) + 2(b-s)(b+p_c \theta)(q_1 + q_2 - 2q_3)} \\
 &+ \frac{\theta(b-s)(q_2 - q_3)(bc_t - p_c \alpha M + \alpha \eta_{ct} \delta p_c)}{4\theta(q_1 q_2 - q_3^2) + 2(b-s)(b+p_c \theta)(q_1 + q_2 - 2q_3)} \\
 &+ \frac{\theta(b-s)(q_1 - q_3)(bc_n - p_c(1-\alpha)M + (1-\alpha)\eta_{cn} \delta p_c)}{4\theta(q_1 q_2 - q_3^2) + 2(b-s)(b+p_c \theta)(q_1 + q_2 - 2q_3)}, \\
 q_{cn}^* &= (1-\alpha)M + \frac{\theta(p-c_g)}{2} + \frac{q_1 b \mu_4 - q_2 s \mu_3 - q_3 \mu_6}{q_1 q_2 - q_3^2} \\
 &+ \frac{\theta(\eta_{cg} \delta - M)(q_1 q_2 - q_3^2)}{4\theta(q_1 q_2 - q_3^2) + 2(b-s)(b+p_c \theta)(q_1 + q_2 - 2q_3)} \\
 &+ \frac{\theta(b-s)(q_2 - q_3)(bc_t - p_c \alpha M + \alpha \eta_{ct} \delta p_c)}{4\theta(q_1 q_2 - q_3^2) + 2(b-s)(b+p_c \theta)(q_1 + q_2 - 2q_3)} \\
 &+ \frac{\theta(b-s)(q_1 - q_3)(bc_n - p_c(1-\alpha)M + (1-\alpha)\eta_{cn} \delta p_c)}{4\theta(q_1 q_2 - q_3^2) + 2(b-s)(b+p_c \theta)(q_1 + q_2 - 2q_3)}, \\
 q^* &= M + \theta(p-c_g) + \frac{(b-s)(q_1 \mu_4 + q_2 \mu_3 - q_3(\mu_3 + \mu_4))}{q_1 q_2 - q_3^2} \\
 &+ \frac{\theta(\eta_{cg} \delta - M)(q_1 q_2 - q_3^2)}{2\theta(q_1 q_2 - q_3^2) + (b-s)(b+p_c \theta)(q_1 + q_2 - 2q_3)} \\
 &+ \frac{\theta(b-s)(q_2 - q_3)(bc_t - p_c \alpha M + \alpha \eta_{ct} \delta p_c)}{2\theta(q_1 q_2 - q_3^2) + (b-s)(b+p_c \theta)(q_1 + q_2 - 2q_3)} \\
 &+ \frac{\theta(b-s)(q_1 - q_3)(bc_n - p_c(1-\alpha)M + (1-\alpha)\eta_{cn} \delta p_c)}{2\theta(q_1 q_2 - q_3^2) + (b-s)(b+p_c \theta)(q_1 + q_2 - 2q_3)}, \\
 E^{3*}(\pi_{ct}^a) &= x_t(\alpha M + \theta(x_t + c_t)) \\
 &+ \frac{(q_2 \mu_3 - q_3 \mu_4)(q_2 \mu_9 - q_1 p_c s \mu_4 - q_3 \mu_{10})}{(q_1 q_2 - q_3^2)^2} - p_c A \\
 &+ \frac{q_2 \mu_7 - q_1 s x_t \mu_4 - q_3 \mu_8}{q_1 q_2 - q_3^2} - \alpha \eta_{ct} \delta x_t,
 \end{aligned} \tag{56}$$

$$\begin{aligned}
 E^{3*}(\pi_{cn}^a) &= x_n((1-\alpha)M + \theta(x_n + c_n)) \\
 &+ \frac{(q_1 \mu_4 - q_3 \mu_3)(q_1 \mu_{13} - q_2 p_c s \mu_3 - q_3 \mu_{14})}{(q_1 q_2 - q_3^2)^2} - p_c A \\
 &+ \frac{q_1 \mu_{11} - q_2 s x_n \mu_3 - q_3 \mu_{12}}{q_1 q_2 - q_3^2} - (1-\alpha)\eta_{cn} \delta x_n, \\
 E^{3*}(\pi_{cg}^a) &= x_g(M + 2\theta(p-c_g-x_g) - \eta_{cg} \delta \\
 &+ (b-s)\frac{q_1 \mu_4 + q_2 \mu_3 - q_3(\mu_3 + \mu_4)}{q_1 q_2 - q_3^2}),
 \end{aligned} \tag{57}$$

where $x_t = w^* - c_t$, $x_n = w^* - c_n$, $x_g = p - c_g - w^*$, $\mu_5 = b\mu_4 - s\mu_3$, $\mu_6 = b\mu_3 - s\mu_4$, $\mu_7 = \mu_3^2$, $\mu_8 = \mu_3\mu_4 - sx_t\mu_3$, $\mu_9 = (-\frac{1}{2}k_t + p_c b)\mu_3$, $\mu_{10} = (-\frac{1}{2}k_t + p_c b)\mu_4 - p_c s\mu_3$, $\mu_{11} = \mu_4^2$, $\mu_{12} = \mu_3\mu_4 - sx_n\mu_4$, $\mu_{13} = (-\frac{1}{2}k_n + p_c b)\mu_4$ and $\mu_{14} = (-\frac{1}{2}k_n + p_c b)\mu_3 - p_c s\mu_4$.

The Proof follows that of Theorem 6 and hence, is omitted. Moreover, it's found that the correlations between the risk-aversion coefficients and the optimal renewable energy power's investment and production strategy, the optimal power price and the optimal total power production are calculated with the same results presented in (38) and (47). Therefore, the analysis of impacts of each of risk-aversion coefficients on them can be referred to the previous two sections and is omitted here.

V. THEOREMS-TO-VALIDITY

Through the simulations by MATLAB for auxiliary systems in (16), (28), (42) and (51), the convergent results of renewable energy utilization rates are obtained and numerical verifications for the Nash equilibriums, the optimal power production quantities and the optimal revenue functions are realized for a class of noncooperative game models of power sales with risk avoidance with the aim to verify the validity of Theorems. For the subsequent simulations, the following parameters' values are provided:

$$\begin{aligned}
 \mu_3 &= \frac{(b+p_c \theta)(p-c_g)}{2} - bc_t + p_c \alpha M - \alpha \eta_{ct} \delta p_c \\
 &+ \frac{(b+p_c \theta)(\eta_{cg} \delta - M)(q_1 q_2 - q_3^2)}{4\theta(q_1 q_2 - q_3^2) + 2(b-s)(b+p_c \theta)(q_1 + q_2 - 2q_3)} \\
 &+ \frac{(b+p_c \theta)(b-s)(q_1 - q_3)(bc_n - p_c(1-\alpha)M + (1-\alpha)\eta_{cn} \delta p_c)}{4\theta(q_1 q_2 - q_3^2) + 2(b-s)(b+p_c \theta)(q_1 + q_2 - 2q_3)} \\
 &+ \frac{(b+p_c \theta)(b-s)(q_2 - q_3)(bc_t - p_c \alpha M + \alpha \eta_{ct} \delta p_c)}{4\theta(q_1 q_2 - q_3^2) + 2(b-s)(b+p_c \theta)(q_1 + q_2 - 2q_3)}, \\
 \mu_4 &= \frac{(b+p_c \theta)(p-c_g)}{2} - bc_n + p_c(1-\alpha)M - (1-\alpha)\eta_{cn} \delta p_c \\
 &+ \frac{(b+p_c \theta)(\eta_{cg} \delta - M)(q_1 q_2 - q_3^2)}{4\theta(q_1 q_2 - q_3^2) + 2(b-s)(b+p_c \theta)(q_1 + q_2 - 2q_3)} \\
 &+ \frac{(b+p_c \theta)(b-s)(q_1 - q_3)(bc_n - p_c(1-\alpha)M + (1-\alpha)\eta_{cn} \delta p_c)}{4\theta(q_1 q_2 - q_3^2) + 2(b-s)(b+p_c \theta)(q_1 + q_2 - 2q_3)} \\
 &+ \frac{(b+p_c \theta)(b-s)(q_2 - q_3)(bc_t - p_c \alpha M + \alpha \eta_{ct} \delta p_c)}{4\theta(q_1 q_2 - q_3^2) + 2(b-s)(b+p_c \theta)(q_1 + q_2 - 2q_3)},
 \end{aligned} \tag{55}$$

$A = 15$ billion kW·h	$p_c = 0.02$ CNY/kW·h
$c_g = 0.1$ CNY/kW·h	$c_t = 0.2$ CNY/kW·h
$c_n = 0.3$ CNY/kW·h	$p = 0.9$ CNY/kW·h
$p_t = 0.65$ CNY/kW·h	$p_n = 0.7$ CNY/kW·h
$M = 50$ billion kW·h	$\alpha = 0.6, \delta = 5$
$\eta_{dt} = \eta_{dn} = 5$	$\eta_{ct} = \eta_{cn} = \eta_{cg} = 5$
$k_t = 120$	$k_n = 60$

Note that to achieve convergent verification for renewable energy utilization rates and numerical validations for the optimal calculations for the four proposed models, discretizations of auxiliary systems are needed.

A. THE DISTRIBUTED NONCOOPERATIVE GAME MODEL OF POWER SALES WITH ENERGY SUPPLIERS' RISK AVOIDANCE

The demand and substitution elasticity coefficients of power prices are respectively set as $b' = 40$ and $s' = 120$. The demand and substitution elasticity coefficients of renewable energy utilization rates are respectively set as $b = 90$ and $s = 20$. By (23), the optimal renewable energy power's investment and production strategy and the optimal revenue functions are respectively calculated as $(l_t^*, l_n^*) = (0.3582, 0.657)$, $(q_{dt}^*, q_{dn}^*) = (107.0994, 121.9686)$ billion kW·h and $(E^*(\pi_{dt}^a), E^*(\pi_{dn}^a)) = (34.1051, 33.008)$ billion CNY. With the discretization stepsize being 0.006, the simulation results generated by (16) are depicted as Fig. 6-8 which respectively plot the trajectories of l_i, q_{di} and $E(\pi_{di}^a)$ for $i \in \{t, n\}$ from which it's clear that according to Fig. 6, the renewable energy utilization rates converge to (l_t^*, l_n^*) and according to Fig. 7-8, the power production quantities and the revenue functions respectively converge to (q_{dt}^*, q_{dn}^*) and $(E^*(\pi_{dt}^a), E^*(\pi_{dn}^a))$, which declares that the convergent conclusion of renewable energy utilization rates is proven and the numerical verifications for the optimal calculations are completed. Therefore, Theorem 1-3 are verified.

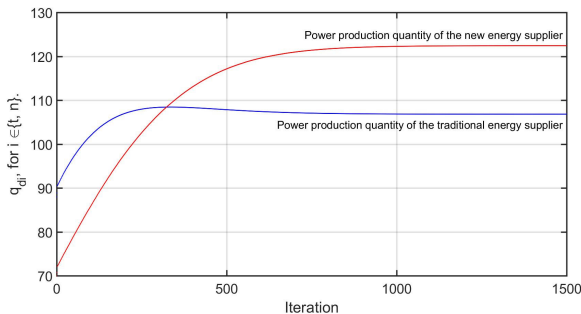


FIGURE 7. The trajectories of q_{di} for $i \in \{t, n\}$ for the distributed non-cooperative game model of power sales with energy suppliers' risk avoidance.

B. THE CENTRALIZED NONCOOPERATIVE GAME MODEL OF POWER SALES WITH ENERGY SUPPLIERS' RISK AVOIDANCE

The demand elasticity coefficient of the power price is set as $\theta = 80$. The demand and substitution elasticity coefficients of renewable energy utilization rates are respectively set as

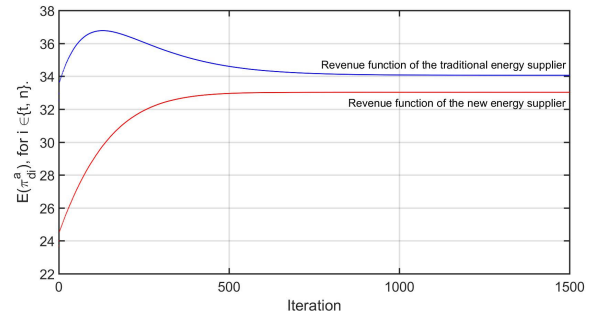


FIGURE 8. The trajectories of $E(\pi_{di}^a)$ for $i \in \{t, n\}$ for the distributed non-cooperative game model of power sales with energy suppliers' risk avoidance.

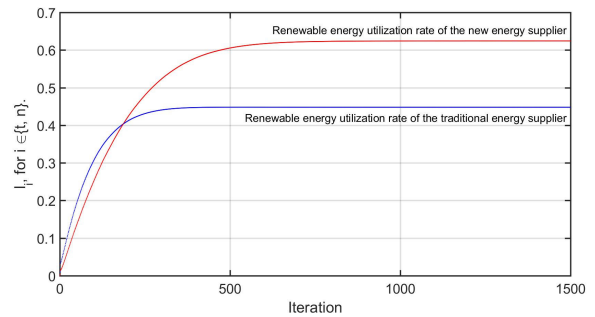


FIGURE 9. The trajectories of l_i for $i \in \{t, n\}$ for the centralized non-cooperative game model of power sales with energy suppliers' risk avoidance.

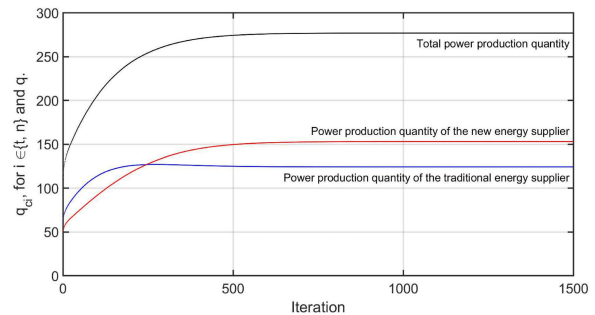


FIGURE 10. The trajectories of q_{ci} for $i \in \{t, n\}$ and q for the centralized non-cooperative game model of power sales with energy suppliers' risk avoidance.

$b = 180$ and $s = 40$. By (33), the optimal renewable energy utilization rates, the optimal power production quantities and the optimal revenue functions are respectively calculated as $(l_t^*, l_n^*) = (0.4497, 0.6205)$, $(q_{ct}^*, q_{cn}^*, q^*) = (124.4173, 151.9945, 276.4117)$ billion kW·h and $(E^{1*}(\pi_{ct}^a), E^{1*}(\pi_{cn}^a), E^{1*}(\pi_{cg}^a)) = (19.0392, 15.2783, 88.8257)$ billion CNY. With the discretization stepsize being 0.0001, the simulation results generated by (28) are shown in Fig. 9-11 which respectively illustrate the trajectories of l_i, q_{ci} for $i \in \{t, n\}$ and $q, E^1(\pi_{ci}^a)$ for $i \in \{t, n\}$ and $E^1(\pi_{cg}^a)$. Obviously, the renewable energy utilization rates, the power production quantities and the revenue functions are respectively convergent to $(l_t^*, l_n^*), (q_{ct}^*, q_{cn}^*, q^*)$ and

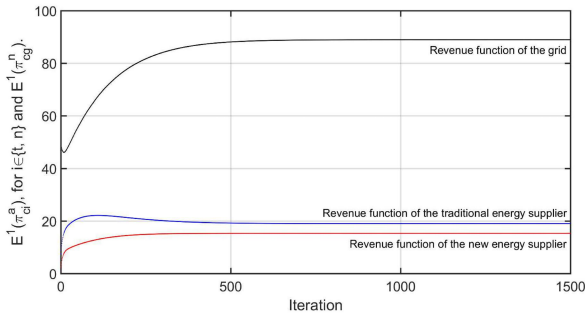


FIGURE 11. The trajectories of $E^1(\pi_{ci}^a)$ for $i \in \{t, n\}$ and $E^1(\pi_{cg}^a)$ for the centralized noncooperative game model of power sales with energy suppliers' risk avoidance.

$(E^{1*}(\pi_{ct}^a), E^{1*}(\pi_{cn}^a), E^{1*}(\pi_{cg}^a))$. Therefore, Theorem 5-6 are verified.

C. THE CENTRALIZED NONCOOPERATIVE GAME MODEL OF POWER SALES WITH GRID'S RISK AVOIDANCE

With the settings of simulation parameters that are same as the centralized noncooperative game model of power sales with energy suppliers' risk avoidance and by (46), $(l_t^*, l_n^*) = (0.4746, 0.6721)$, $(q_{ct}^*, q_{cn}^*, q^*) = (127.9477, 161.4107, 289.3584)$ billion kW·h and $(E^{2*}(\pi_{ct}^a), E^{2*}(\pi_{cn}^a), E^{2*}(\pi_{cg}^a)) = (24.8476, 19.4152, 81.2478)$ billion CNY are respectively calculated. With the discretization stepsize being 0.0001, the simulation results generated by (42) are given in Fig. 12-14, which plot the trajectories of l_i , q_{ci} for $i \in \{t, n\}$ and q and $E^2(\pi_{ci}^a)$ for $i \in \{t, n\}$ and $E^2(\pi_{cg}^a)$ from which it can be seen that the renewable energy utilization rates, the power production quantities and the revenue functions are respectively convergent to the corresponding optimal calculations. Therefore, Theorem 8-9 are verified.

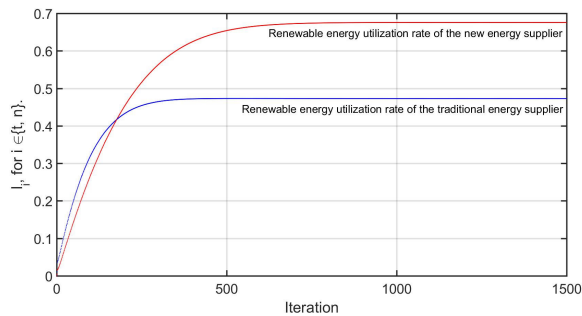


FIGURE 12. The trajectories of l_i for $i \in \{t, n\}$ for centralized noncooperative game model of power sales with grid's risk avoidance.

D. THE CENTRALIZED NONCOOPERATIVE GAME MODEL OF POWER SALES WITH RISK AVOIDANCE FOR GRID AND ENERGY SUPPLIERS

With the settings of the same simulation parameters as previous two simulations, $(l_t^*, l_n^*) = (0.4727, 0.6701)$, $(q_{ct}^*, q_{cn}^*, q^*) = (127.7425, 161.1693, 288.9117)$ billion

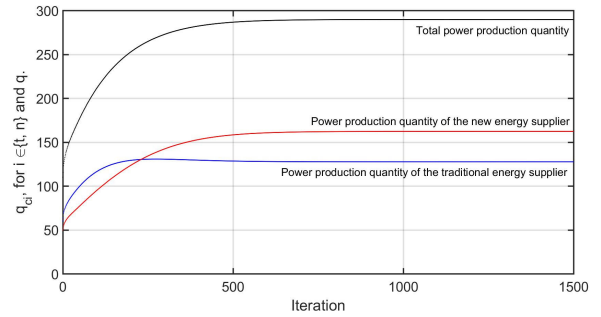


FIGURE 13. The trajectories of q_{ci} for $i \in \{t, n\}$ and q for centralized noncooperative game model of power sales with grid's risk avoidance.

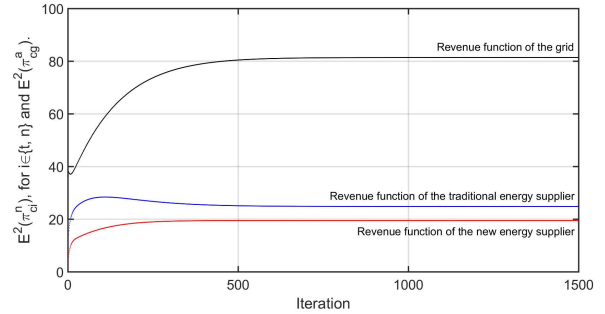


FIGURE 14. The trajectories of $E^2(\pi_{ci}^a)$ for $i \in \{t, n\}$ and $E^2(\pi_{cg}^a)$ for centralized noncooperative game model of power sales with grid's risk avoidance.

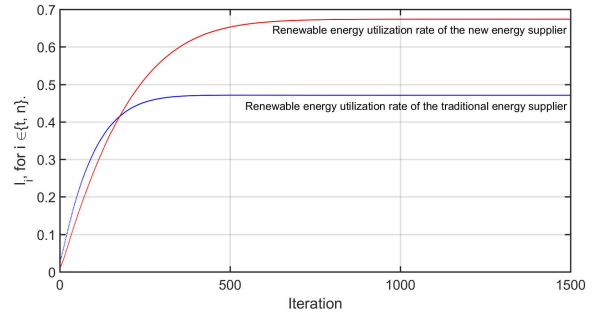


FIGURE 15. The trajectories of l_i for $i \in \{t, n\}$ for centralized noncooperative game model of power sales with risk avoidance for grid and energy suppliers.

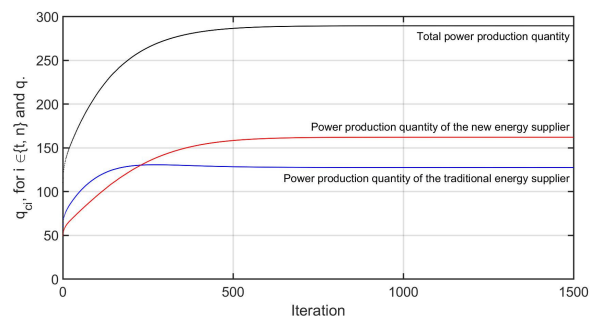


FIGURE 16. The trajectories of q_{ci} for $i \in \{t, n\}$ and q for centralized noncooperative game model of power sales with risk avoidance for grid and energy suppliers.

kW·h and $(E^{3*}(\pi_{ct}^a), E^{3*}(\pi_{cn}^a), E^{3*}(\pi_{cg}^a)) = (20.4106, 17.4564, 80.9735)$ billion CNY are respectively calculated under (56)-(57). With the discretization stepsize being 0.0001, the simulation results generated by (51) are given

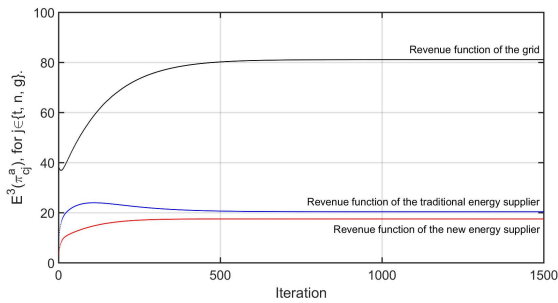


FIGURE 17. The trajectories of $E^3(p_j^a)$ for $j \in \{t, n, g\}$ for centralized noncooperative game model of power sales with risk avoidance for grid and energy suppliers.

in Fig. 15-17. Obviously, the renewable energy utilization rates, the power production quantities and the revenue functions respectively converge to the corresponding optimal calculations. Therefore, Theorem 11-12 are verified.

VI. CONCLUSION

A class of noncooperative game models of power sales with risk avoidance under the consideration of renewable energy power production quotas and Green Power Certificate System are constructed in this paper, namely the distributed noncooperative game model of power sales with energy suppliers' risk avoidance and the centralized noncooperative game models of power sales with energy suppliers' risk avoidance, with grid's risk avoidance and with risk avoidance for grid and energy suppliers, and their computational problems of the optimal renewable energy power's investment and production strategies are investigated.

By utilizing the gradient-like optimization method and setting up the auxiliary systems, it's proven that there exists a unique Nash equilibrium for each of models (i.e., the optimal renewable energy utilization rates) and the stable conclusions that the Nash equilibriums are globally exponentially stable can be obtained. Moreover, by employing the definition of the complete information static game, the Nash equilibriums can be calculated and then by correlating equations, the optimal power production quantities and the optimal revenue functions can be computed.

Convergent verifications for renewable energy utilization rates, numerical verifications for the optimal renewable energy power's investment and production strategies and the optimal revenue functions for the four proposed models are achieved through a specific numerical example. Moreover, the analysis of impacts of risk-aversion coefficients on the optimal renewable energy power's investment and production strategies (the optimal power price and the optimal total power production quantity) and remarks on impacts are provided.

REFERENCES

- [1] F. A. Rahman, M. M. A. Aziz, R. Saidur, W. A. W. A. Bakar, M. R. Hainin, R. Putrajaya, and N. A. Hassan, "Pollution to solution: Capture and sequestration of carbon dioxide (CO₂) and its utilization as a renewable energy source for a sustainable future," *Renew. Sustain. Energy Rev.*, vol. 71, pp. 112–126, May 2017, doi: [10.1016/j.rser.2017.01.011](https://doi.org/10.1016/j.rser.2017.01.011).
- [2] S. Adams and A. O. Acheampong, "Reducing carbon emissions: The role of renewable energy and democracy," *J. Cleaner Prod.*, vol. 240, Dec. 2019, Art. no. 118245, doi: [10.1016/j.jclepro.2019.118245](https://doi.org/10.1016/j.jclepro.2019.118245).
- [3] B. Doğan, O. M. Driha, D. Balsalobre Lorente, and U. Shahzad, "The mitigating effects of economic complexity and renewable energy on carbon emissions in developed countries," *Sustain. Develop.*, vol. 29, no. 1, pp. 1–12, Jan. 2021, doi: [10.1002/sd.2125](https://doi.org/10.1002/sd.2125).
- [4] H. Zheng, M. Song, and Z. Shen, "The evolution of renewable energy and its impact on carbon reduction in China," *Energy*, vol. 237, Dec. 2021, Art. no. 121639, doi: [10.1016/j.energy.2021.121639](https://doi.org/10.1016/j.energy.2021.121639).
- [5] S. Bahramara, M. P. Moghaddam, and M. R. Haghifam, "Optimal planning of hybrid renewable energy systems using HOMER: A review," *Renew. Sustain. Energy Rev.*, vol. 62, pp. 609–620, Sep. 2016, doi: [10.1016/j.rser.2016.05.039](https://doi.org/10.1016/j.rser.2016.05.039).
- [6] O. Hafez and K. Bhattacharya, "Optimal planning and design of a renewable energy based supply system for microgrids," *Renew. Energy*, vol. 45, pp. 7–15, Sep. 2012, doi: [10.1016/j.renene.2012.01.087](https://doi.org/10.1016/j.renene.2012.01.087).
- [7] W. Huang, N. Zhang, J. Yang, Y. Wang, and C. Kang, "Optimal configuration planning of multi-energy systems considering distributed renewable energy," *IEEE Trans. Smart Grid*, vol. 10, no. 2, pp. 1452–1464, Mar. 2019, doi: [10.1109/TSG.2017.2767860](https://doi.org/10.1109/TSG.2017.2767860).
- [8] L. Che, X. Zhang, M. Shahidehpour, A. Alabdulwahab, and A. Abusorrah, "Optimal interconnection planning of community microgrids with renewable energy sources," *IEEE Trans. Smart Grid*, vol. 8, no. 3, pp. 1054–1063, May 2017, doi: [10.1109/TSG.2015.2456834](https://doi.org/10.1109/TSG.2015.2456834).
- [9] X. Liang, "Emerging power quality challenges due to integration of renewable energy sources," *IEEE Trans. Ind. Appl.*, vol. 53, no. 2, pp. 855–866, Mar. 2017, doi: [10.1109/TIA.2016.2626253](https://doi.org/10.1109/TIA.2016.2626253).
- [10] T. Kataray, B. Nitesh, B. Yarram, S. Sinha, E. Cuce, S. Shaik, P. Vigneshwaran, and A. Roy, "Integration of smart grid with renewable energy sources: Opportunities and challenges—A comprehensive review," *Sustain. Energy Technol. Assessments*, vol. 58, Aug. 2023, Art. no. 103363, doi: [10.1016/j.seta.2023.103363](https://doi.org/10.1016/j.seta.2023.103363).
- [11] H. Ranjbar, M. Kazemi, N. Amjadi, H. Zareipour, and S. H. Hosseini, "Maximizing the utilization of existing grids for renewable energy integration," *Renew. Energy*, vol. 189, pp. 618–629, Apr. 2022, doi: [10.1016/j.renene.2022.03.035](https://doi.org/10.1016/j.renene.2022.03.035).
- [12] S. D. Ahmed, F. S. M. Al-Ismail, M. Shafiullah, F. A. Al-Sulaiman, and I. M. El-Amin, "Grid integration challenges of wind energy: A review," *IEEE Access*, vol. 8, pp. 10857–10878, 2020, doi: [10.1109/ACCESS.2020.2964896](https://doi.org/10.1109/ACCESS.2020.2964896).
- [13] S. Chen, C. Wang, and Z. Zhang, "Multitime scale active and reactive power coordinated optimal dispatch in active distribution network considering multiple correlation of renewable energy sources," *IEEE Trans. Ind. Appl.*, vol. 57, no. 6, pp. 5614–5625, Nov. 2021, doi: [10.1109/TIA.2021.3100468](https://doi.org/10.1109/TIA.2021.3100468).
- [14] M. Bell, F. Berkel, and S. Liu, "Real-time distributed control of low-voltage grids with dynamic optimal power dispatch of renewable energy sources," *IEEE Trans. Sustain. Energy*, vol. 10, no. 1, pp. 417–425, Jan. 2019, doi: [10.1109/TSTE.2018.2800108](https://doi.org/10.1109/TSTE.2018.2800108).
- [15] Y. Dong, H. Zhang, C. Wang, and X. Zhou, "Soft actor-critic DRL algorithm for interval optimal dispatch of integrated energy systems with uncertainty in demand response and renewable energy," *Eng. Appl. Artif. Intell.*, vol. 127, Jan. 2024, Art. no. 107230, doi: [10.1016/j.engappai.2023.107230](https://doi.org/10.1016/j.engappai.2023.107230).
- [16] W. Liu, T. Wang, S. Wang, E. Zhijun, and R. Fan, "Day-ahead robust optimal dispatching method for urban power grids containing high proportion of renewable energy," *Process Saf. Environ. Protection*, vol. 178, pp. 715–727, Oct. 2023, doi: [10.1016/j.psep.2023.08.025](https://doi.org/10.1016/j.psep.2023.08.025).
- [17] K.-C. Tseng and C.-C. Huang, "High step-up high-efficiency interleaved converter with voltage multiplier module for renewable energy system," *IEEE Trans. Ind. Electron.*, vol. 61, no. 3, pp. 1311–1319, Mar. 2014, doi: [10.1109/TIE.2013.2261036](https://doi.org/10.1109/TIE.2013.2261036).
- [18] A. Ajami, H. Ardi, and A. Farakhor, "A novel high step-up DC/DC converter based on integrating coupled inductor and switched-capacitor techniques for renewable energy applications," *IEEE Trans. Power Electron.*, vol. 30, no. 8, pp. 4255–4263, Aug. 2015, doi: [10.1109/TPEL.2014.2360495](https://doi.org/10.1109/TPEL.2014.2360495).
- [19] S. Miao, N. Lyu, X. Jiang, W. Liu, and Y. Jin, "A new fifth-order boost converter with dual operating modes and wider conversion ratios for renewable energy applications," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 69, no. 12, pp. 4914–4918, Dec. 2022, doi: [10.1109/TCSII.2022.3192641](https://doi.org/10.1109/TCSII.2022.3192641).

- [20] D. Yang and H. Yin, "Energy conversion efficiency of a novel hybrid solar system for photovoltaic, thermoelectric, and heat utilization," *IEEE Trans. Energy Convers.*, vol. 26, no. 2, pp. 662–670, Jun. 2011, doi: [10.1109/TEC.2011.2112363](https://doi.org/10.1109/TEC.2011.2112363).
- [21] M. D. Leonard, E. E. Michaelides, and D. N. Michaelides, "Energy storage needs for the substitution of fossil fuel power plants with renewables," *Renew. Energy*, vol. 145, pp. 951–962, Jan. 2020, doi: [10.1016/j.renene.2019.06.066](https://doi.org/10.1016/j.renene.2019.06.066).
- [22] S. Ould Amrouche, D. Rekioua, T. Rekioua, and S. Bacha, "Overview of energy storage in renewable energy systems," *Int. J. Hydrogen Energy*, vol. 41, no. 45, pp. 20914–20927, Dec. 2016, doi: [10.1016/j.ijhydene.2016.06.243](https://doi.org/10.1016/j.ijhydene.2016.06.243).
- [23] N. Pang, Q. Meng, and M. Nan, "Multi-criteria evaluation and selection of renewable energy battery energy storage system—A case study of Tibet, China," *IEEE Access*, vol. 9, pp. 119857–119870, 2021, doi: [10.1109/ACCESS.2021.3107192](https://doi.org/10.1109/ACCESS.2021.3107192).
- [24] F. Díaz-González, A. Sumper, O. Gomis-Bellmunt, and R. Villafáfila-Robles, "A review of energy storage technologies for wind power applications," *Renew. Sustain. Energy Rev.*, vol. 16, no. 4, pp. 2154–2171, May 2012, doi: [10.1016/j.rser.2012.01.029](https://doi.org/10.1016/j.rser.2012.01.029).
- [25] C. Jin, X. Sheng, and P. Ghosh, "Optimized electric vehicle charging with intermittent renewable energy sources," *IEEE J. Sel. Topics Signal Process.*, vol. 8, no. 6, pp. 1063–1072, Dec. 2014, doi: [10.1109/JSTSP.2014.2336624](https://doi.org/10.1109/JSTSP.2014.2336624).
- [26] D. Liu, L. Wang, W. Wang, H. Li, M. Liu, and X. Xu, "Strategy of large-scale electric vehicles absorbing renewable energy abandoned electricity based on master-slave game," *IEEE Access*, vol. 9, pp. 92473–92482, 2021, doi: [10.1109/ACCESS.2021.3091725](https://doi.org/10.1109/ACCESS.2021.3091725).
- [27] V. Mohan, J. G. Singh, and W. Ongsakul, "Sortino ratio based portfolio optimization considering EVs and renewable energy in microgrid power market," *IEEE Trans. Sustain. Energy*, vol. 8, no. 1, pp. 219–229, Jan. 2017, doi: [10.1109/TSTE.2016.2593713](https://doi.org/10.1109/TSTE.2016.2593713).
- [28] Z. Wu, X. Shi, F. Fang, G. Wen, and Y. Mi, "Co-optimization of building energy systems with renewable generations combining active and passive energy-saving," *Appl. Energy*, vol. 351, Dec. 2023, Art. no. 121514, doi: [10.1016/j.apenergy.2023.121514](https://doi.org/10.1016/j.apenergy.2023.121514).
- [29] X. Lü, T. Lu, S. Karirinne, A. Mäkiranta, and D. Clements-Croome, "Renewable energy resources and multi-energy hybrid systems for urban buildings in Nordic climate," *Energy Buildings*, vol. 282, Mar. 2023, Art. no. 112789, doi: [10.1016/j.enbuild.2023.112789](https://doi.org/10.1016/j.enbuild.2023.112789).
- [30] S. Christopher, M. P. Vikram, C. Bakli, A. K. Thakur, Y. Ma, Z. Ma, H. Xu, P. M. Cuce, E. Cuce, and P. Singh, "Renewable energy potential towards attainment of net-zero energy buildings status—A critical review," *J. Cleaner Prod.*, vol. 405, Jun. 2023, Art. no. 136942, doi: [10.1016/j.jclepro.2023.136942](https://doi.org/10.1016/j.jclepro.2023.136942).
- [31] Y. Wang and F. Taghizadeh-Hesary, "Green bonds markets and renewable energy development: Policy integration for achieving carbon neutrality," *Energy Econ.*, vol. 123, Jul. 2023, Art. no. 106725, doi: [10.1016/j.eneco.2023.106725](https://doi.org/10.1016/j.eneco.2023.106725).
- [32] B. Lin, J. Chen, and P. K. Wesseh, "Peak-valley tariffs and solar prosumers: Why renewable energy policies should target local electricity markets," *Energy Policy*, vol. 165, Jun. 2022, Art. no. 112984, doi: [10.1016/j.enpol.2022.112984](https://doi.org/10.1016/j.enpol.2022.112984).
- [33] Y. Yeo and I. Oh, "Evaluating the impacts of renewable energy promotion policies on sustainable development: A computable general equilibrium model approach," *J. Cleaner Prod.*, vol. 421, Oct. 2023, Art. no. 138360, doi: [10.1016/j.jclepro.2023.138360](https://doi.org/10.1016/j.jclepro.2023.138360).
- [34] G. Ellis, N. Schneider, and R. Wüstenhagen, "Dynamics of social acceptance of renewable energy: An introduction to the concept," *Energy Policy*, vol. 181, Oct. 2023, Art. no. 113706, doi: [10.1016/j.enpol.2023.113706](https://doi.org/10.1016/j.enpol.2023.113706).
- [35] F. J. Rodríguez-Segura, J. C. Osorio-Aravena, M. Frolova, J. Terrados-Cepeda, and E. Muñoz-Cerón, "Social acceptance of renewable energy development in southern Spain: Exploring tendencies, locations, criteria and situations," *Energy Policy*, vol. 173, Feb. 2023, Art. no. 113356, doi: [10.1016/j.enpol.2022.113356](https://doi.org/10.1016/j.enpol.2022.113356).
- [36] O. A. Shobande, L. Ogbefun, and A. K. Tiwari, "Re-evaluating the impacts of green innovations and renewable energy on carbon neutrality: Does social inclusiveness really matter?" *J. Environ. Manage.*, vol. 336, Jun. 2023, Art. no. 111670, doi: [10.1016/j.jenvman.2023.111670](https://doi.org/10.1016/j.jenvman.2023.111670).
- [37] A. G. Olabi and M. A. Abdelkareem, "Renewable energy and climate change," *Renew. Sustain. Energy Rev.*, vol. 158, Apr. 2022, Art. no. 112111, doi: [10.1016/j.rser.2022.112111](https://doi.org/10.1016/j.rser.2022.112111).
- [38] A. Suman, "Role of renewable energy technologies in climate change adaptation and mitigation: A brief review from Nepal," *Renew. Sustain. Energy Rev.*, vol. 151, Nov. 2021, Art. no. 111524, doi: [10.1016/j.rser.2021.111524](https://doi.org/10.1016/j.rser.2021.111524).
- [39] T. Akan, "Can renewable energy mitigate the impacts of inflation and policy interest on climate change?" *Renew. Energy*, vol. 214, pp. 255–289, Sep. 2023, doi: [10.1016/j.renene.2023.05.023](https://doi.org/10.1016/j.renene.2023.05.023).
- [40] A. Gulagi, S. Pathak, D. Bogdanov, and C. Breyer, "Renewable energy transition for the Himalayan countries Nepal and Bhutan: Pathways towards reliable, affordable and sustainable energy for all," *IEEE Access*, vol. 9, pp. 84520–84544, 2021, doi: [10.1109/ACCESS.2021.3087204](https://doi.org/10.1109/ACCESS.2021.3087204).
- [41] G. Sun, G. Li, A. Dilanchiev, and A. Kazimova, "Promotion of green financing: Role of renewable energy and energy transition in China," *Renew. Energy*, vol. 210, pp. 769–775, Jul. 2023, doi: [10.1016/j.renene.2023.04.044](https://doi.org/10.1016/j.renene.2023.04.044).
- [42] D. Werner and L. L. B. Lazaro, "The policy dimension of energy transition: The Brazilian case in promoting renewable energies (2000–2022)," *Energy Policy*, vol. 175, Apr. 2023, Art. no. 113480, doi: [10.1016/j.enpol.2023.113480](https://doi.org/10.1016/j.enpol.2023.113480).
- [43] M. Song, Y. Wang, and Y. Long, "Investment and production strategies of renewable energy power under the quota and green power certificate system," *Energies*, vol. 15, no. 11, p. 4110, Jun. 2022, doi: [10.3390/en15114110](https://doi.org/10.3390/en15114110).
- [44] D. Li, J. Xu, Y. Ye, K. Duan, and C. Liu, "Renewable energy power's investment and production strategy for centralized noncooperative game model of power sales with grid's risk avoidance," in *Proc. Annu. Conf. IEEE Ind. Electron. Soc.*, Nov. 2023, pp. 1–7, doi: [10.1109/IECON51785.2023.10311732](https://doi.org/10.1109/IECON51785.2023.10311732).
- [45] M. Ye and G. Hu, "Distributed Nash equilibrium seeking by a consensus based approach," *IEEE Trans. Autom. Control*, vol. 62, no. 9, pp. 4811–4818, Sep. 2017, doi: [10.1109/TAC.2017.2688452](https://doi.org/10.1109/TAC.2017.2688452).
- [46] R. Horn and C. Johnson, *Matrix Analysis*. Cambridge, U.K.: Cambridge Univ. Press, 1985.
- [47] H. Khalil, *Nonlinear Systems*. Upper Saddle River, NJ, USA: Prentice-Hall, 2002.



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