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RESEARCH ARTICLE

Research on Optimization Model of Public Transport Dispatch Based on Combination Mode

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ABSTRACT To solve the conflict of interests between citizens' travel and public transportation enterprises, and alleviate the pressure of passenger flow at morning and evening peak bus stop, a multimodal combination optimization model for public transportation scheduling was proposed. Because the public transport enterprise adopted the conventional dispatching mode, there was a mismatch between passenger flow and transport capacity input. By establishing the public transport dispatching optimization model of the combination of the conventional bus and the inter-district bus, the conventional bus, and the large station express bus, solving the public transport dispatching algorithm, using the vehicle control method of the large passenger flow regional bus stop, we could obtain the control relationship of the arrival time of controllable vehicles. The experimental results show that the Combinatorial optimization scheduling of controllable vehicles enables controllable vehicles to reach the sixth station at the same time, the congestion cost of passengers on the vehicle is zero, and the waiting time of citizens is reduced by 11.15%, which can alleviate the congestion of citizens, improve the public transport capacity, and solve the travel problems of citizens.

INDEX TERMS Combination mode, bus scheduling, optimization model, vehicle control, controllable scheduling, public transportation capacity.

I. INTRODUCTION

To solve the problems of high travel cost for citizens in large cities and high transportation capacity pressure on public transportation enterprises, starting from the prediction of public transportation passenger flow and the scheduled departure time interval, combined with the actual need of Guangzhou's public transportation operation, a combined mode of the conventional bus and the large station express bus, the conventional bus and the inter-district bus scheduling optimization model is established to meet the need of citizens' travel. During peak hours in the morning and evening, on the one hand, the large station express bus in high passenger flow areas stop at selected high passenger flow area station from the route station, timely evacuating crowded passengers at these stations. On the other hand, there is a significant proportion of passenger flow in a certain riding section of the bus route, which is significantly higher than the passenger flow in other sections of the route. For this distribution of passenger flow at large station, auxiliary dispatch can be carried out in the form of the inter-district bus to timely evacuate passenger flow, solve the problem of citizen travel, and improve the quality and operational efficiency of bus services.

According to the optimal scheduling control theory of intelligent transportation, real-time adjustment of controllable bus stops and controllable bus vehicles can be achieved. Chien et al. [2] optimized the operational service model and

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related service frequencies for transportation routes with heterogeneous demands, minimizing total cost. Ulusoy et al. [3] considered a bus dispatch service model and frequency that minimizes the overall cost of transferring demand elasticity. Cristian proposed that the inter-district vehicle strategy may generate significant benefits in reducing total cost, and the factor that reduces profit is the additional cost incurred by empty driving on certain road sections [4]. Therefore, the coordinated departure plan of intelligent public transportation combinations is an important planning tool for intelligent public transportation [5], [6]. Ceder has developed a schedule for the average maximum passenger load, creating a schedule based on user satisfaction and convenience by maximizing the number of vehicles arriving at transfer stations [7], [8]. Hassald proposed a new method for Multi Vehicle type Vehicle Scheduling Problem (MVT-VSP), which minimizes the expected waiting time of passengers and reduces service differences among passengers with different need [9]. Weng et al. [10] proposed a combined scheduling model where multiple routes shared passenger and vehicle resource to improve the matching degree of transportation capacity supply and demand and the efficiency of public transportation operation. Feng et al. [11] proposed an optimization method for overlapping bus route scheduling considering passenger transfer. A multi-objective emergency bus dispatch optimization model was constructed with the parking capacity, vehicle transportation capacity, dispatching capacity, and passenger tolerance time of emergency bus stop as constraint [12]. Wang et al. [13] proposed the goal of minimizing transfer waiting time and the number of lost passengers, with a lower-level optimization model for driving plan and the goal of minimizing bus operating cost. Yao et al. [14] proposed a load balancing dynamic scheduling algorithm for heterogeneous rail transit clusters based on ant colony algorithm. A multi-objective emergency bus dispatch optimization model was constructed with the parking capacity, vehicle transportation capacity, dispatching capacity, and passenger tolerance time of emergency bus stop as constraint [12]. To improve the operational efficiency of urban conventional public transportation, Lai et al. [15] proposed a bus scheduling optimization model based on simulated annealing adaptive cuckoo algorithm. Based on analyzing the layout model of urban bus stop, Fu et al. [16] proposed the concepts of the cost of urban and rural bus operators and the time value cost of bus passengers. With the goal of minimizing the sum of the two, a layout model of urban and rural bus stop was established and optimized. Zhang et al. [17] proposed using the BL model to allocate passenger flow, considering the differences in departure schedules and travel times between the large station express bus and the full course bus. Based on passenger allocation, while keeping public transportation resource constant, they comprehensively considered the benefits and losses of passengers from skipping station, and established an improved model for setting up the large station express bus with the goal of maximizing total revenue. Based on the characteristics of the model, a solution process was designed using genetic algorithm. Hu et al. [18] applied the theory of inverse function, considering factors such as multiple time periods and station balance, with the goal of minimizing the overall fleet size. By slightly adjusting the departure time, they proposed a joint scheduling method for the mutual borrowing of bus numbers between the large station express bus and the full course bus. Luo [19] aimed to improve the efficiency of bus stop operations by constructing a vehicle combination scheduling model with the goals of passenger time, comfort cost, and operator operating cost, and proposing dynamic control strategies. This is of great significance for improving the planning and design theory of urban public transportation systems and promoting the research of intelligent operation theory of public transportation.

To improve the operation efficiency of bus rapid transit and minimize the travel time, Ren et al. [20] proposed a new bus rapid transit schedule optimization model, which optimized the schedule of bus station along the route and the passive signal priority control of intersection. To alleviate traffic congestion during peak travel time, Beijing, Guangzhou, and other cities had opened customized bus routes. Because customized bus routes need to reach a certain number of reservations before they could be operated, they are more suitable for commuting and bus travel [21]. When setting the objective function of static scheduling, the initial research mainly goals at the lowest operating cost of enterprise [22]. To improve passenger travel satisfaction, some scholars considered the maximum passenger travel benefit as the goal [23]. However, the consideration of the goal is still relatively simple, and only the enterprise or passenger benefit factors are considered separately to establish the model. To get a better scheduling scheme, Ayadi et al. [24] based on the study of demand-responsive static scheduling problem, considered both enterprise and passenger benefit, and established a model with the lowest vehicle operating cost and the best service quality as the goal, to improve passenger travel satisfaction on the basis of ensuring the lowest vehicle operating cost. Dynamic scheduling considers both the benefits of enterprises and passengers when setting the objective function. For example, Lu et al. [25] considered both the benefit of enterprises and passengers at the same time, and built a multi-objective route optimization model with the lowest vehicle operating cost and passenger travel cost. This model can not only save the operating cost of enterprises, but also minimize the travel time cost of passengers. Pei et al. [26] studied the demand-responsive scheduling problem of dynamic classification of stations, considering the coverage of many stations and the limitation of the number of stations, which can effectively improve the income of enterprises. Wang [27] studied the demand-responsive bus connection scheduling problem, and adopted a variety of vehicle models for coordination, which can improve the operating efficiency of the system and the level of vehicle service. Zhang et al. [28] studied the optimization problem of demand responsive routes in suburban areas, considering the coordination between renting vehicles and demand responsive routes in special circumstances to meet the diverse travel need of passengers. Zhao et al. [29] jointly optimized the conventional bus and demand-responsive bus, and used demand-responsive buses at specific time intervals, while shortening the routes of the conventional bus, which can effectively reduce the travel time cost of passengers.

In response to the fundamental problem of maximizing the benefit between passenger waiting time, congestion cost, and operating cost of public transportation enterprises, the bus scheduling strategies, models, and algorithms proposed by the scholars have not been addressed. Therefore, a combination mode bus scheduling optimization strategy, model, and algorithm have been proposed to solve the contradiction between citizen travel and public transportation enterprises operation, and to achieve multi-objective optimal planning of bus scheduling.

The purpose of optimizing a multimodal public transportation scheduling system is to enhance the urban transportation environment, improve the travel need of citizens, and enhance the economic and social benefit of public transportation enterprises. Public transportation scheduling is an optimization and planning problem that needs to meet the interests of both citizens and public transportation enterprises. The interests between the two are not consistent and form a contradictory unity. Therefore, the optimization of public transportation scheduling based on multi-mode combination is a multi-objective and optimal planning problem.

II. RELATED WORKS

A. THE COMBINED DYNAMIC DISPATCHING OPTIMIZATION MODEL OF CONVENTIONAL BUSES AND INTER-DISTRICT BUSES

1) PROBLEM ANALYSIS

According to the actual passenger flow of public transportation routes in Guangzhou, there are three different forms of inter-district bus dispatch, the agreement is as follows.

(1) The starting point is the same as the full course bus, and a turn back is made at a certain intermediate station.

(2) The starting point and turning point are both at a certain intermediate station.

(3) The starting point is at a certain station in the middle, and the ending point is the turning point.

To ensure the standardization, universality, and operability of the model data, the agreement is as follows.

(1) The models of the full course bus and the inter-district bus are the same, with the same ticket price, number of seats, and passenger capacity as the inter-district bus.

(2) The running time of the full course bus and the inter-district bus is fixed.

(3) There is no transfer between the full course bus and the inter-district bus.

2) MODEL PARAMETER

i : The serial number of bus stops, $i = 1,2,3, \cdots n$ is the total number of bus stops.

j : Sequence number of the bus stop where the inter-district bus departs.

k : The sequence number of the inter-district bus returning to the bus stop, $j < k \leq n$, $k = n$, when it means that the inter-district bus runs to the terminal and returns.

m : The inter-district bus multi-mode combined scheduling optimization parameters, m means that a full-range vehicle is combined with the inter-district bus for departure.

L_i : The distance between i station and $i+1$ station, the unit is km.

L : The mileage of a single journey of the full course bus, the unit is km.

L_s : The length of the one-way operation of the inter-district bus, $L_S = \sum_{i=j}^{k} L_i$, the unit is km.

Δt : The length of the model study period, the unit is min.

h_a : The interval time between the last full course bus and the next full course bus, the unit is min.

h_b : The interval time between the last full course bus and the next inter-district bus, the unit is min.

h_c : The interval time between the last inter-district bus and the next full-course bus, the unit is min.

t_b : The time for individual passengers to board the bus.

F : The total number of departures in a certain period are related to the length of the period and the interval between departures, and the unit is vehicle/h.

r_{il} : The average arrival rate of passengers getting on the bus at the station and the terminal station is the first station, the unit is person/min, $1 \leq i < l \leq n$.

r_i : The average arrival rate of passengers at the station, $r_i = \sum_{l=i+1}^{n} r_{il}$, in person/min.

P_i : The number of passengers boarding the bus when it arrives station i.

Q_i : Number of passengers getting off the bus when it arrives station i.

N_i : The number of passengers on the bus when the bus leaves station i, the number of passengers accumulated on the section of the bus route between station i and station $i-1$.

C : The number of full passengers per bus, the unit is person/vehicle.

A_i : The full load rate of the bus station i.

A_1: Maximum full load rate of bus.

A_0 : Minimum full load rate of bus.

μ : Passenger's unit waiting time cost, unit yuan/ min.

β : The unit fuel consumption of the bus, the unit is yuan/km.

T : The average running time of the full course bus in the morning and evening peak hour, the unit is min.

T_S : The average running time of one-way bus in the morning and evening peak hour, the unit is min.

$U_{\min}$: The minimum number of vehicles allocated on the route, in units of vehicle.

3) OBJECTIVE FUNCTION

a: PASSENGER COST

The cost of waiting time for passengers and the time cost of getting on and off the bus constitute passenger travel cost, and a, b, c is different superscript, represent the passenger time cost of each bus in different situation.

If both the first bus and the second bus are scheduled to send the full course bus, the total passenger cost of the second full course bus is as shown in formula (1).

$$E^a = \sum_{i=1}^{n-1} (W_i^a + Y_i^a) \tag{1}$$

W_i^a is the cost of waiting time for passengers at the bus stop of this route, Since the departure time interval of the two vehicles is h_a, and there is no passenger detention, the number of passengers waiting for the bus station i is shown in formula (2).

$$P_i^a = \sum\nolimits_{l=i+1}^{n} r_{il} h_a \tag{2}$$

Since passengers arrive at the bus stop obey the uniform distribution law, the unit time of waiting for the bus is $\frac{h_a}{2}$, and the cost of waiting for the next bus at the bus stop is obtained, as shown in formula (3).

$$W_i^a = \mu \frac{h_a^2}{2} \sum\nolimits_{l=i+1}^{n} r_{il} \tag{3}$$

Y_i^a is for the waiting cost of passengers getting on and off the bus, passengers who get on the bus station i and get off the bus station l need to wait for the time for passengers to get on and off the non-destination station $P_i^a = \sum_{l=i+1}^{n} r_{il} h_a$, as shown in formula (4).

$$\sum\nolimits_{l=i+2}^{n} \sum\nolimits_{q=i+1}^{l-1} r_{iq} h_a \max\{P_q, Q_q\} t_b \tag{4}$$

Obtain the cost of passengers getting on and off at the station waiting for passengers at the non-destination station to the bus station i and get off the bus station l, as shown in formula (5).

$$Y_i^a = \mu \sum\nolimits_{l=i+1}^{n} \sum\nolimits_{q=i+1}^{l-1} r_{iq} h_a \max\{P_q, Q_q\} t_b \tag{5}$$

Therefore, the total cost of passengers taking this bus is as shown in formula (6).

$$E^a = \mu \sum_{i=1}^{n-1} (\frac{h_a^2}{2} \sum\nolimits_{l=i+1}^{n} r_{il} + \sum\nolimits_{l=i+1}^{n} \sum\nolimits_{q=i+1}^{l-1} r_{iq} h_a \max\{P_q, Q_q\} t_b) \tag{6}$$

2) If the first trip is arranged to send the full course bus, and the second trip is arranged to send one inter-district bus, the total cost of passengers taking this bus is shown in formula (7).

$$E^b = \sum\nolimits_{i=j}^{k-1} (W_i^b + Y_i^b) \tag{7}$$

W_i^b is for the total cost of passengers taking this bus, the departure interval of the first bus for the full course bus and the second bus for the inter-district bus is h_b, so the number of passengers waiting station i is $P_i^b = \sum_{l=i+1}^{k} r_{il} h_b$, and the average waiting time of passengers is $P_i^b = \sum_{l=i+1}^{k} r_{il} h_b$, the cost of a bus trip is as shown in formula (8).

$$W_i^b = \mu \frac{h_b^2}{2} \sum\nolimits_{l=i+1}^{k} r_{il} \tag{8}$$

Y_i^b is for the waiting cost of passengers getting on and off the bus on this bus, passengers who get on the station i and get off the station l need to wait for the time for passengers to get on and off the non-destination station, as shown in formula (9).

$$\sum\nolimits_{l=i+2}^{k} \sum\nolimits_{q=i+1}^{l-1} r_{iq} h_b \max\{P_q, Q_q\} t_b \tag{9}$$

From formula (9), the cost of passengers who boarded and got off the station waiting for passengers to get on and off the non-destination station is obtained $P_i^b = \sum_{l=i+1}^{k} r_{il} h_b$, as in Formula (10).

$$Y_i^b = \mu \sum\nolimits_{l=i+1}^{k} \sum\nolimits_{q=i+1}^{l-1} r_{iq} h_b \max\{P_q, Q_q\} t_b \tag{10}$$

Therefore, the total cost of passengers taking this bus is as shown in formula (11).

$$E^b = \mu \sum\nolimits_{i=j}^{k-1} (\frac{h_b^2}{2} \sum\nolimits_{l=i+1}^{k} r_{il} + \sum\nolimits_{l=i+1}^{k} \sum\nolimits_{q=i+1}^{l-1} r_{iq} h_b \max\{P_q, Q_q\} t_b) \tag{11}$$

3) If the inter-district bus is sent on the first trip and the full course bus on the second trip, the total cost of passengers taking this bus is shown in formula (12).

$$E^c = \sum\nolimits_{i=1}^{j-1} W_{it_1}^c + \sum\nolimits_{i=j}^{k-1} W_{is}^c + \sum\nolimits_{i=k}^{n-1} W_{it_2}^c + \sum\nolimits_{i=1}^{n-1} Y_i^c \tag{12}$$

W_{it}^c represents the waiting time cost of passengers at the station where no the inter-district bus passes by taking this bus under the above circumstances. The average waiting time for passengers at the station where no the inter-district bus passes is $\frac{h_b+h_c}{2}$, the number of passengers waiting to board the bus station it_1 (the station before the inter-district bus start) is $P_{it_1}^c = \sum_{l=i+1}^{n} r_{il}(h_b + h_c)$, the number of passengers waiting to board the bus station it_2 (the inter-district bus turning point and after) is $P_{it_2}^c = \sum_{l=i+1}^{n} r_{il}(h_b + h_c)$, and the waiting cost of the passenger who passes the bus station it_2 is $P_{it_2}^c = \sum_{l=i+1}^{n} r_{il}(h_b + h_c)$, and the waiting cost of the passenger who passes the bus station at the non-inter-district bus is obtained, as shown in formula (13).

$$\sum\nolimits_{i=1}^{j-1} W_{it_1}^c + \sum\nolimits_{i=k}^{n-1} W_{it_2}^c = \mu \frac{(h_b + h_c)^2}{2} (\sum\nolimits_{i=1}^{j-1} \sum\nolimits_{l=i+1}^{j} r_{il} + \sum\nolimits_{i=k}^{n-1} \sum\nolimits_{l=i+1}^{n} r_{il}) \tag{13}$$

W_{is}^c represents the waiting cost for passengers at the station where the inter-district bus passes. Because the last bus is one inter-district bus, some passengers whose destination stations are not in the inter-district bus station will be stranded. Passengers who take the full course bus will be stranded. When the inter-district bus arrives at the station i, the number of people in this part of the station is $\sum_{l=k+1}^{n} r_{il}h_b$, when the first inter-district bus arrives, the average waiting time of passengers is $\frac{h_b}{2}$, and the average waiting time until the full course bus arrives at the station is $\frac{h_b}{2}+h_c$, to get the waiting time of this part of passengers, as in formula (14).

$$\frac{h_b+2h_c}{2}\mu \sum_{l=k+1}^{n} r_{il}h_b \quad (14)$$

The other part of the passengers is the passengers who arrived at the station i between the arrival of the last inter-district bus and the arrival of the full course bus. The number of passengers in this part is $\sum_{l=j+1}^{n} r_{il}h_c$, the average waiting time for the bus is $\frac{h_c}{2}$, and the cost of waiting for the bus, as shown in formula (15).

$$\frac{h_c^2}{2}\mu \sum_{l=j+1}^{k} r_{il} \quad (15)$$

In summary, the number of passengers boarding at the station i in the interval is $\sum_{l=k+1}^{n} r_{il}h_b+\sum_{l=j+1}^{n} r_{il}h_c$, and the time cost of waiting for passengers at the station i is as shown in formula (16).

$$\sum_{i=j}^{k-1} W_{is}^{c}=\sum_{i=j}^{k-1} (\frac{h_b+2h_c}{2}\mu \sum_{l=k+1}^{n} r_{il}h_b + \frac{h_c^2}{2}\mu \sum_{l=j+1}^{n} r_{il}) \quad (16)$$

$\sum_{i=1}^{n-1} Y_i^c$ is for the time cost of passengers on this bus waiting for passengers to get on and off at non-destination bus station, Y_i^c represents the passengers getting on the station i and off the station l need to wait for the non-destination station cost of passengers getting off the bus. Since the previous one was one inter-district bus, passengers will be stranded at non-inter-district stations, which is also classified and considered based on the relationship between the passenger's destination station l and the inter-district station.

1) When $1 \le i \le j-1$, the number of passengers at this part of the station is $\sum_{l=i+1}^{n} r_{il}(h_b+h_c)$, and the waiting time for passengers at the target station l is $\sum_{q=i+1}^{l-1} \max\{P_q, Q_q\}t_b$, so the waiting cost of passengers is as shown in formula (17).

$$\sum_{i=1}^{j-1} \sum_{l=i+1}^{n} r_{il}(h_b+h_c) \sum_{q=i+1}^{l-1} \max\{P_q, Q_q\}t_b \quad (17)$$

2) At that time $j \le i \le k-1$, the number of people at this part of the station is $\sum_{l=k+1}^{n} r_{il}h_b+\sum_{l=j+1}^{n} r_{il}h_c$, and the passenger cost, as shown in formula (18).

$$\sum_{i=j}^{k-1} (\sum_{l=k+1}^{n} r_{il}h_b+\sum_{l=j+1}^{n} r_{il}h_c) \sum_{q=i+1}^{l-1} \max\{P_q, Q_q\}t_b \quad (18)$$

3) At that time $k \le i \le n-1$, the number of passengers at this part of the station is $\sum_{l=k+1}^{n} r_{il}(h_b+h_c)$, and the passenger cost, as shown in formula (19).

$$\sum_{i=k}^{n-1} \sum_{l=k+1}^{n} r_{il}(h_b+h_c) \sum_{q=i+1}^{l-1} \max\{P_q, Q_q\}t_b \quad (19)$$

b: PUBLIC TRANSPORT OPERATING COST

Public transport fixed cost and variable cost constitute bus operating cost. The fixed cost is related to the size of the bus fleet and station management. If the existing fleet size and station yards and other basic conditions can meet the adjusted need, only it is necessary to consider the impact of variable cost on public transport operating cost. Variable cost of public transport is mainly related to departure schedules and distance traveled. In practice, conditions such as the length of a specific bus route are certain, and vehicle failures, traffic accidents and other factors are not considered. Variable cost in all aspects is mainly related to departure schedules, and only consider the impact of bus departure schedules on variable cost. Set the variable cost coefficient γ_1 of the inter-district bus schedule to F_1 and the variable cost coefficient γ_2 of the full course bus schedule to F_2. The operating cost of the bus is shown in formula (20).

$$E_d=\gamma_1 F_1+\gamma_2 F_2 \quad (20)$$

c: OBJECTIVE FUNCTION

1) For the scheduling form of one full course bus and one inter-district bus, the objective function is as in formula (21).

$$E=\omega_1(E_bF_1+E_cF_2)+\omega_1(\gamma_1F_1+\gamma_2F_2), F_1=F_2 =\frac{\tau}{h_b+h_c} \quad (21)$$

In formula (21), ω_1 represents the weight value of the time consumption value of passengers and the consumption value during the operating time of the bus company. τ represents the total interval time between the departure of the full course bus and the departure of the inter-district bus. 2) For the scheduling form where two full course buses and one inter-district bus depart at intervals, the objective function is as shown in formula (22).

$$E=\omega_1(E_a\frac{F_2}{2}+E_bF_1+E_c\frac{F_2}{2})+\omega_1(\gamma_1F_1+\gamma_2F_2), \quad 2F_1=F_2=\frac{2\tau}{h_b+h_b+h_c} \quad (22)$$

3) For the scheduling form of m full course buses and one inter-district bus at interval, the objective function is as in formula (23).

$$E=\omega_1(E_a\frac{(m-1)F_2}{m}+E_bF_1+E_c\frac{F_2}{m})+\omega_1(\gamma_1F_1+\gamma_2F_2), \quad mF_1=F_2=\frac{m\tau}{h_b+h_b+h_c} \quad (23)$$

d: CONSTRAINT

In actual bus operation, limited by the size of the fleet, there will be a minimum departure time interval $h_{\min}$. on the one hand, it is necessary to increase the departure interval and reduce the cost while ensuring a certain level of bus service, so the departure interval cannot be greater than the maximum departure interval $h_{\max}$, then $h_{\min} \leq h_a \leq h_{\max}, h_{\min} \leq h_b \leq h_{\max}, h_{\min} \leq h_c \leq h_{\max}$.

On the other hand, to ensure the utilization of vehicles and the comfort of passengers in the vehicle, the bus is restricted by the full load rate during the driving process. The number of passengers between station i and station i-1 is as follows.

1) If the previous one is one full course bus, then the current one is also one full course bus, $i = 1, 2, 3, \cdots\cdots, n-1$, as shown in formula (24).

$$N_i^a = N_{i-1}^a + \sum_{l=i+1}^{n} r_{il} h_a - \sum_{g=1}^{i-1} r_{gi} h_a \tag{24}$$

2)The last one vehicle is one full course bus, and this one vehicle is one inter-district bus, $i = j, j+1, \cdots\cdots, k-1$, as shown in formula (25).

$$N_i^b = N_{i-1}^b + \sum_{l=i+1}^{k} r_{il} h_b - \sum_{g=j}^{i} r_{gi} h_b \tag{25}$$

3)The previous one vehicle is one inter-district bus, and this one vehicle is one full course bus, $i = 1, 2, 3, \cdots\cdots, j-1$,as in formula (26).

$$N_i^c = N_{i-1}^c + \sum_{l=i+1}^{n} r_{il}(h_b + h_c) - \sum_{g=1}^{i-1} r_{gi}(h_b + h_c) \tag{26}$$

When $i = j$, as in formula (27).

$$N_j^c = N_{j-1}^c + \sum_{l=i+1}^{n} r_{il} h_b + \sum_{l=i+1}^{n} r_{il} h_c - \sum_{g=1}^{j-1} r_{gj}(h_b + h_c) \tag{27}$$

When $I = j+1, j+2, \cdots\cdots, k-1$, as in formula (28).

$$N_i^c = N_{i-1}^c + \sum_{l=k+1}^{n} r_{il} h_b + \sum_{l=j+1}^{n} r_{il} h_c - \sum_{g=1}^{i-1} r_{gj}(h_b + h_c) - \sum_{g=j}^{i-1} r_{gi} h_c \tag{28}$$

When $i = k$, as in formula (29).

$$N_k^c = N_{k-1}^c + \sum_{l=k+1}^{n} r_{kl}(h_b + h_c) - \sum_{g=1}^{j} r_{gi}(h_b + h_c) - \sum_{g=j}^{k} r_{gi} h_c \tag{29}$$

When $i = k+1, k+2, \cdots\cdots, n-1$, as in formula (30).

$$N_i^c = N_{i-1}^c + \sum_{l=i+1}^{n} r_{il}(h_b + h_c) - \sum_{g=1}^{j-1} r_{gi}(h_b + h_c) - \sum_{g=j}^{k-1} r_{gi} h_c - \sum_{g=k}^{n-1} r_{gi}(h_b + h_c) \tag{30}$$

B. COMBINED DYNAMIC DISPATCHING OPTIMIZATION MODEL OF CONVENTIONAL BUSES AND EXPRESS BUS AT MAJOR STATIONS

1) PROBLEM ANALYSIS

During the actual operation of Guangzhou public transport, it was found that the passenger flow of some bus routes is far greater than other bus stops, which often results in passengers being stranded at the bus stops. Due to overcrowding on the buses, passengers are getting on and off the bus. An accident occurred due to congestion in China. In response to the above problems, it is proposed to use the large station express bus to assist the full-course bus for combined bus dispatching to meet the travel need of passengers. The assumptions are as follows.

1) The full course bus is the same as the large station express.

2) There is no difference when the passengers at the station where the large station express buses stop when choosing a bus to take the large station express bus or the full course bus.

3) Passengers reach the station evenly.

4) The bus runs at a constant speed.

5) No other auxiliary dispatch forms such as the inter-district bus.

6) The cost of bus operation is only related to the departure schedule.

7) The bus departure interval is fixed within a certain period.

2) MODEL PARAMETERS

M : The total number of stations on the route.

i, j: The label of the site.

N : The total number of departures within a certain period.

L : Certain research period.

F_K, F_Q : Respectively indicate the departure schedule of the large station express bus and the full course bus, if all the schedules depart continuously at even interval.

Δt : Bus departure interval.

$$\Delta t = \frac{L}{F_K + F_Q} \tag{31}$$

x_{ij} : The arrival rate of passengers who boarded station i and got off the station j within a certain unit period.

x_i : The arrival rate of passengers who boarded the bus station i in a certain unit period.

$$x_i = \sum\nolimits_{j=i+1}^{M} x_{ij} \tag{32}$$

U_i : The number of passengers boarding when the bus arrives station i.

$$U_i = \sum\nolimits_{j=i+1}^{M} x_{ij} \Delta t \tag{33}$$

D_i : The number of passengers alighting when the bus arrives station i.

$$D_i = \sum\nolimits_{k=1}^{i-1} x_{ki} \Delta t \tag{34}$$

C_i : The number of people in the bus when the bus leaves station i.

$$C_i = C_{i-1} + \sum\nolimits_{j=i+1}^{M} x_{ij}\Delta t - \sum\nolimits_{k=1}^{i-1} x_{ki}\Delta t \quad (35)$$

ω_1 : If the station i is the stop of the large station express bus, then $\phi_i = 1$, otherwise, $\phi_i = 0$.

t_b : The time taken for a single passenger to board the bus.

v : The average running speed of the bus in a certain period.

c_1, c_h: Respectively the minimum and maximum full-load rate restrictions given by the bus company.

t_1, t_h: Respectively the minimum and maximum departure time intervals of buses.

ω_1, ω_2: Passenger time consumption value, the weight of the consumption value during the operation period of the bus company, $\omega_1 + \omega_2 = 1$.

C : The rated passenger capacity of the vehicle.

L_{ij} : The length of the bus route from station i to station j (km).

μ : Conversion coefficient between time and price in the studied city.

3) CONSTRAINTS

1) Full load rate constraint

When the large station express bus is used to assist the entire bus for public transportation dispatch, it is constrained by the full load rate of the vehicle. To ensure the utilization of vehicles, the full load rate of vehicles during the study period is satisfied, as shown in formula (36).

$$N(M-1)C_1 \le L\sum\nolimits_{i=1}^{M-1}\sum\nolimits_{j=i+1}^{M} x_{ij} \quad (36)$$

2) Total number of vehicles dispatched during the study period

Since the main purpose of the large station express is to meet the need of passengers in time and evacuate the passenger flow, the departure schedule must meet the need of passengers at each station to avoid long-term waiting for passengers. The number of vehicles sent should satisfy the formula (37).

$$L * \frac{\sum_{i=1}^{M-1}\sum_{j=i+1}^{M} x_{ij}}{C_h} \le N = F_Q + F_K \le \frac{L}{t_1} \quad (37)$$

Except for the passengers carried by the large station express bus, the entire bus should be able to meet the need of the remaining passengers, as shown in formula (38).

$$S - L * \frac{F_K}{F_Q + F_K}\sum\nolimits_{i=1}^{M-1}\sum\nolimits_{j=i+1}^{M} \omega_1\omega_2 x_{ij} \le C_h F_Q \quad (38)$$

Among them, is all passengers waiting to board the bus (39).

$$S = L * \sum\nolimits_{i=1}^{M-1}\sum\nolimits_{j=i+1}^{M} x_{ij} \quad (39)$$

3) According to the principle of selection of stations for major station express buses, the number of stations for the large station express bus should be at least two stations, and should not exceed the total number of stations on the route (40).

$$2 \le \sum\nolimits_{i=1}^{M} \omega_1 < M \quad (40)$$

a: OBJECTIVE FUNCTION

The common goal of passenger travel time cost and public transportation enterprise operating cost is the optimization of the combined dispatching model of the conventional bus and the large station express bus.

(1) Passenger travel cost Passenger travel cost is reflected in the loss of passenger travel time, which mainly includes waiting time and boarding time.

1) Waiting time

The waiting time is equal to the time that passengers wait for the bus to arrive at the station plus the time for passengers to get on and off the bus. This part of time is related to the bus departure time interval. Since the bus frequency is divided into two types: large station expresses and the full course bus, it is necessary to consider the two types of stops belonging to the large station express and non-major express stops separately.

For the large station express bus stop, the waiting time of passengers is only related to the total departure interval, as in formula (41).

$$T_1^K = \frac{1}{2(F_K + F_Q)}\sum\nolimits_{i=1}^{M-1}\sum\nolimits_{j=i+1}^{M} \omega_1\omega_2 x_{ij}\Delta t \quad (41)$$

Passengers need to take the full course bus when they stop at the conventional bus stop. When the next bus waiting is the full course bus, the passenger needs to wait for the departure time of the full course bus to be the waiting time for passengers at the conventional bus stop, as shown in formula (42).

$$T_1^Q = (\frac{1}{2F_Q} + \frac{1}{2F_K})(S - \sum\nolimits_{i=1}^{M-1}\sum\nolimits_{j=i+1}^{M} \omega_1\omega_2 x_{ij})\Delta t \quad (42)$$

Therefore, the total waiting time is $T_1 = T_1^K + T_1^Q$.

2) Waiting time in the bus for passengers at non-destination stations to board the bus Passenger travel time includes waiting time in the bus for passengers at non-destination stations to board the bus and bus travel time.

Different from the full course bus, due to the existence of the large station express, there is a significant difference in waiting time and boarding time between the bus station where the large station express stop and the passenger at the station where the large station express does not stop. Therefore, for taking the large station express the waiting time of the passenger unit in a certain unit period, as shown in formula (43).

$$T_2^K = \frac{F_K t_b}{F_K + F_Q}\sum\nolimits_{i=1}^{M-1}\sum\nolimits_{j=i+1}^{M} \omega_1\omega_2 x_{ij}\Delta t\sum\nolimits_{q=i+1}^{j-1} x_{iq}\Delta t \quad (43)$$

The unit waiting time of passengers taking the full course bus, as in formula (44).

$$T_2^Q = \frac{t_b}{F_Q}(S - \frac{F_K}{F_K + F_Q} \sum_{i=1}^{M-1} \sum_{j=i+1}^{M} \omega_1 \omega_2 x_{ij}) \Delta t \tag{44}$$

Considering all passengers, the total waiting time in the vehicle for passengers at non-destination stations to board the vehicle, as shown in formula (45).

$$T_2 = T_2^K + T_2^Q \tag{45}$$

Combining the above two parts of time consumption, it can be obtained that the total waiting time consumption of passengers choosing public transportation is converted into capital consumption, as shown in formula (46).

$$E_1 = \mu(T_1 + T_2) \tag{46}$$

$$\begin{aligned} E_1 = \mu(&\frac{1}{2(F_K + F_Q)} \sum_{i=1}^{M-1} \sum_{j=i+1}^{M} \omega_1 \omega_2 x_{ij} \Delta t \\ &+ (\frac{1}{2F_Q} + \frac{1}{2F_K})(S - \sum_{i=1}^{M-1} \sum_{j=i+1}^{M} \omega_1 \omega_2 x_{ij}) \Delta t \\ &+ \frac{F_K t_b}{F_K + F_Q} \sum_{i=1}^{M-1} \sum_{j=i+1}^{M} \omega_1 \omega_2 x_{ij} \Delta t \\ &\sum_{q=i+1}^{j-1} x_{iq} \Delta t + \frac{t_b}{F_Q}(S - \frac{F_K}{F_K + F_Q} \sum_{i=1}^{M-1} \\ &\sum_{j=i+1}^{M} \omega_1 \omega_2 x_{ij}) \Delta t) \end{aligned} \tag{47}$$

Simplified, as in formula (48).

$$\begin{aligned} E_1 = \mu(&\frac{2t_b + 1}{2(F_K + F_Q)} \sum_{i=1}^{M-1} \sum_{j=i+1}^{M} \omega_1 \omega_2 x_{ij} \Delta t \\ &+ (\frac{1}{2F_Q} + \frac{1}{2F_K})(S - \sum_{i=1}^{M-1} \sum_{j=i+1}^{M} \omega_1 \omega_2 x_{ij}) \Delta t \\ &+ \frac{t_b}{F_Q}(S - \frac{F_K}{F_K + F_Q} \sum_{i=1}^{M-1} \sum_{j=i+1}^{M} \omega_1 \omega_2 x_{ij}) \Delta t)) \end{aligned} \tag{48}$$

(2) Public transport operating cost. When the fixed cost of bus operation can be ignored, the variable cost of public transport operation is only related to the departure frequency. The variable cost coefficient of bus operation for the full course bus is γ_1, and the variable cost coefficient of express public transport operation for the large station express bus is γ_2. The total variable cost of public transport operation is shown in formula (49).

$$E_2 = \gamma_1 F_Q + \gamma_2 F_K \tag{49}$$

In summary, the objective function is as shown in formula (50).

$$\min E = \phi_1 E_1 + \phi_2 E_2 \tag{50}$$

$$\begin{aligned} \min E = \phi_1 \mu(&\frac{2t_b + 1}{2(F_K + F_Q)} \sum_{i=1}^{M-1} \sum_{j=i+1}^{M} \omega_1 \omega_2 x_{ij} \Delta t \\ &+ (\frac{1}{2F_Q} + \frac{1}{2F_K})(S - \sum_{i=1}^{M-1} \sum_{j=i+1}^{M} \omega_1 \omega_2 x_{ij}) \Delta t \\ &+ \frac{t_b}{F_Q}(S - \frac{F_K}{F_K + F_Q} \sum_{i=1}^{M-1} \sum_{j=i+1}^{M} \omega_1 \omega_2 x_{ij}) \Delta t)) \\ &+ \phi_2(\gamma_1 F_Q + \gamma_2 F_K) \end{aligned} \tag{51}$$

C. SOLVING THE OPTIMIZATION MODEL FOR PUBLIC TRANSPORT DISPATCH

This article takes the BRT B3 route on Zhongshan Avenue in Guangzhou as an example, with a total of 31 bus stops and a total length of about 23km. The operation time of public transportation is from 6:00 to 22:00, with a daily operation time of 960 minutes. According to the distribution of passenger flow during different time periods, the operation time is divided into 8 time periods, with 120 minutes each. The departure time interval is controlled within ($4 < \Delta t_i < 10$). Select 8 stations on B3 route (from Huangcun Station to Shangshe Station) and conduct statistics on passenger flow at different time periods on weekdays, as shown in Table 1.

The solution of the public transportation scheduling model is to determine the departure time interval of public transportation vehicles based on the optimal interests of travelers and public transportation companies, as shown in formulas (52) and (53).

$$\arg\min_{h_i} \alpha C_p + \beta C_b \tag{52}$$

$$s.t. h_{i\min} \le h_i \le h_{i\max} \tag{53}$$

In formula (52), $C_P = \phi(h_i)$ is the travel cost of the traveler; $C_b = \varphi(h_i)$ is the operating cost of the public transportation company. α is the weighted coefficient of passenger waiting costs, and β is the weighted coefficient of operating costs for public transportation companies, used to adjust the interest relationship between travelers and the bus company.

In formula (53), h_i is the departure time interval of the i period, $h_{i\min}$ is the minimum departure time interval, and $h_{i\max}$ is the maximum departure time interval.

To ensure the normal production and operation of the bus company, the bus load factor is controlled between 0.5 and 1.2. To balance the interests of travelers and the bus company, a weighted coefficient is introduced and the objective function is defined, as shown in formulas (54) and (55).

$$\begin{aligned} \text{Min} C = \alpha \times \mu_1 \times \sum_{i=1}^{m} \sum_{j=1}^{n} T_i \times \omega_{ij} \times 1/2 \times \Delta t_i + \beta \times \mu_2 \\ \times \sum_{i=1}^{m} T_i / \Delta t_i \end{aligned} \tag{54}$$

In formula (54), T_i: Total duration of the i period, ω_{ij}: passenger arrival rate of station j in the i period, μ_1: Passengers' cost per minute of waiting time, yuan/min. person. μ_2: The bus company's operating cost per time, yuan/time. Δt_i: Time

TABLE 1. Statistics of passenger flow at eight stations in each period of bus B3 route.

Time Slot \ Bus Stop	1	2	3	4	5	6	7	8
06:00~08:00	450	435	456	416	463	426	459	437
08:00~10:00	243	228	252	206	264	222	258	230
10:00~12:00	351	320	392	299	396	201	401	331
12:00~14:00	266	244	294	223	298	228	307	249
14:00~16:00	312	301	3261	278	333	282	320	308
16:00~18:00	381	356	359	341	372	319	392	363
18:00~20:00	438	407	452	382	466	387	460	418
20:00~22:00	284	270	289	251	307	248	302	279

interval of the i period.

$$\text{st.}\begin{cases}\Delta t_{\min} \leq \Delta t_i \leq \Delta t_{\max} \\ 0.5 \leq \dfrac{\sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n}\mu_{ij}}{Q\times\sum\limits_{i=1}^{m}T_i/\Delta t_i} \leq 1.2 \\ \alpha+\beta=1 \\ 0<\alpha,\beta<1\end{cases} \tag{55}$$

In formula (55), $\Delta t_{\min}$: Tower limit of departure time interval, $\Delta t_{\max}$: Upper limit of departure time interval. Q: number of passengers loaded on the vehicle, μ_{ij}: Total number of passengers boarding from station j during i period.

To solve the optimization model of public transportation scheduling, establish a Lagrangian function as shown in formula (56).

$$L=\alpha\times\mu_1\times\sum_{i=1}^{m}\sum_{j=1}^{n}T_i\times\omega_{ij}\times 1/2\times\Delta t_i+\beta\times\mu_2 \times\sum_{i=1}^{m}T_i/\Delta t_i+\lambda\times(\alpha+\beta-1) \tag{56}$$

Among them, λ represents the Lagrange multiplier. To Δt_i, α, β, λ take the partial derivative and obtain formula (57) to (60).

$$\frac{\partial L}{\partial\Delta t_i}=\alpha\times\mu_1\times\sum_{i=1}^{m}\sum_{j=1}^{n}T_i\times\omega_{ij}\times 1/2-\beta\times\mu_2 \times\sum_{i=1}^{m}T_i/\Delta t_i\times\Delta t_i \tag{57}$$

$$\frac{\partial L}{\partial\alpha}=\mu_1\times\sum_{i=1}^{m}\sum_{j=1}^{n}T_i\times\omega_{ij}\times 1/2\times\Delta t_i+\lambda \tag{58}$$

$$\frac{\partial L}{\partial\beta}=\mu_2\times\sum_{i=1}^{m}T_i/\Delta t_i+\lambda \tag{59}$$

$$\frac{\partial L}{\partial\lambda}=\alpha+\beta-1 \tag{60}$$

Let formula (57) to (60) be equal to zero, calculate the weighting coefficients, and obtain a peak departure time interval of 5 minutes in the morning and evening, and a peak departure time interval of 7 minutes in the evening. In the process of solving the model, it was found that the weight was different during different time periods. When the passenger flow changed, the weight also changed, so the departure time interval was more accurate. The schedule and departure time of the public transportation plan have been adjusted, and the range of schedule adjustments in Table 2 is related to the interval between the schedule and departure time of the public transportation plan.

The scheduling of this combination mode is based on a uniform departure schedule, and the adjustment range of departure time is related to the departure interval. Generally, the longer the departure interval, the smaller the weight of the adjustable range of departure time. In specific applications, the adjustment of departure time is based on a standard provided by Ceder in his monograph [30]. The calculation method for the adjustment range of departure time is as follows.

$$t_{i,k}-\Delta_{(i,k,-)}\leq t_{i,k}\leq t_{i,k}+\Delta_{(i,k,+)} \tag{61}$$

In formula (61), $t_{i,k}$ is the departure time of the station k bus i number. $\Delta_{(i,k,-)}$ and $\Delta_{(i,k,+)}$ respectively represent the maximum advance and maximum delay of the station k bus departure time i number.

Taking the B3 bus route of the station k in Guangzhou as an example, the multi-mode combination scheduling process based on departure time adjustment is as follows.

Step1: Input parameters M, including n_l, $t_{i,k}$, $t_{j,l}$, $\Delta_{(i,k,-)}$, $\Delta_{(i,k,+)}$, $\Delta_{(j,l,-)}$, $\Delta_{(j,l,+)}$, $T_{i,k}$, $T_{j,l}$.

Step2: Enter the station k bus number $i=1$, traverse the bus numbers $j=1,\ldots,n$ issued by the station l, and adjust $t_{i,k}$ the bus number i to arrive at the station l between time $t_{j,l}$ and $t_{j-1,l}$.

$$t_{j-1,l}+\Delta_{(j-1,l,+)}\leq t_{i,k}\pm\Delta_{(i,k,\pm)}+T_{i,k}\leq t_{j,l}-\Delta_{(j,l,-)} \tag{62}$$

In formula (62), i and j respectively represent the bus numbers at the station k and station l. $t_{j,l}$ indicates the number of buses l departing from station j. $\Delta_{(j,l,-)}$ and $\Delta_{(j,l,+)}$ indicates the departure time of buses l departing from j station and respectively represent the maximum advance and maximum delay of station bus departure time, $T_{i,k}$ and $T_{j,l}$ represent the average running time of vehicles departing from station k and station l on the road segment $t_{i,k}$ and $t_{j,l}$ at the same time.

When formula (62) is satisfied, the bus number i issued by k station can be allocated by the bus number j issued by the station l at $t_{j,l}$ time. When formula (62) is not satisfied, proceed to step 4.

Step3: When the bus number i issued by the station k can be allocated according to the bus number l issued by the station j, it is found that the required number of vehicles at the station l is reduced by 1, that is $n_l=n_l-1$, the parameter set M is automatically updated.

Step4: Traverse the bus numbers $i=1,\ldots,n$ issued by the station k, and follow steps 2 and 3 to find the bus numbers that meet the conditions for allocation.

TABLE 2. Adjustment of scheduled bus schedule and departure time.

Serial number	Time interval H/min	Adjustment range percentage%	Adjustment duration range/min
1	<5	40	0.4H
2	5-10	30	0.3H
3	11-20	20	0.2H
4	>20	10	0.1H

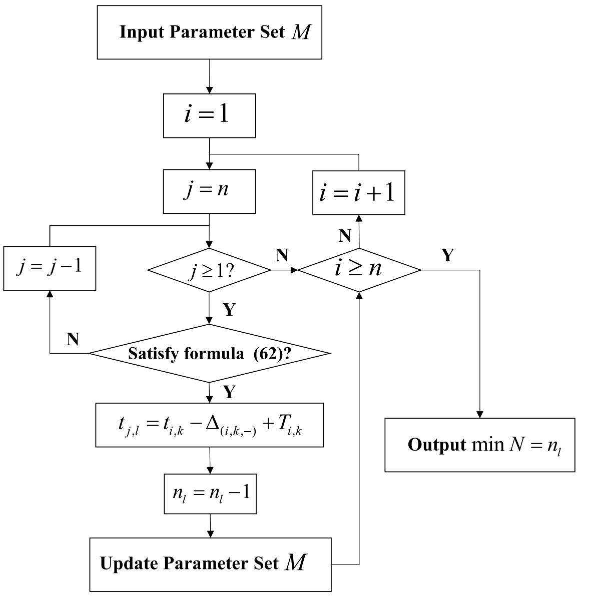

FIGURE 1. Combination scheduling flowchart for adjusting the departure time of station k.

Step5: Output the minimum number of vehicle sizes $\min N = n_l$, as shown in Figure 1.

III. EXPERIMENTAL DESIGN AND VERIFICATION OF BUS DISPATCH OPTIMIZATION

A. EXPERIMENTAL DESIGN OF BUS DISPATCH OPTIMIZATION

According to the method of multi-mode combination scheduling, it is necessary to adjust the arrival time of the large station express bus and the inter-district bus at major stations in real time, control the arrival and stopping time of bus sets at stations with high passenger flow, reasonably allocate the passenger load at this station, and ensure that the passenger load of bus sets is uniform while ensuring that the intervals between each bus set are uniform and normal. Control the vehicles at the decision-making station to stay at the station for a short period of time, ensuring sufficient transportation capacity at high passenger flow stations. Control the parking time of the bus group at a certain passenger flow reduction station, separate the control vehicles of the bus group, and avoid wasting transportation capacity. Ensure that the transportation capacity provided by the bus group meets the need of passengers at high peak hour stations, coordinate with the original departure plan after the separation of the bus group, ensure the rationality of the adjusted departure interval, and achieve optimization of multi-mode bus combination scheduling. When the unbalanced coefficient of the bus station and section of the road section exceeds the maximum threshold, the vehicle scheduling method is implemented, and the time control in the bus station is implemented to realize the variable scheduling operation plan, as shown in Figure 2.

The relationship between the current i vehicle dwell time at the station k and the previous vehicle's arrival at the station k, as shown in formula (63).

$$S_{i,k} = (1 - T_{i,k})u_{i,k} + (d_{i,k} - d_{i,k-1})T_{i,k} \tag{63}$$

In the formula(63), $d_{i,k}$, the moment when i vehicle leaves station k. $S_{i,k}$, the stopping time of i vehicle at the station k, min.

$u_{i,k}$, the controllable time of i vehicle at the station k, min.

$T_{i,k}$, Indicates the maximum allowed boarding rate.

The relationship between the stopping time of the current i vehicle at the station k and the arrival of a vehicle at the station k, as shown in formula (64).

$$S_{i,k} = a_{i,k+1} - a_{i,k} - Tr_{i,k} \tag{64}$$

In the formula (64), the time when i vehicle can be scheduled to arrive at the station k. $Tr_{i,k}$, the travel time from station k to station $k+1$, s.

When departing according to the original plan, due to lack of control over the vehicle's time.

$$S'_{i,k} = (d'_{i,k} - d'_{i-1,k})T'_{i,k} = a'_{i,k+1} - a'_{i,k} - Tr'_{i,k} \tag{65}$$

Definition: $T_{i,k} = T'_{i,k}, x_{i,k} = a_{i,k} - a'_{i,k}$

$x_{i,k}$,The difference between the actual arrival time of the i vehicle at the station k and the planned arrival time.

Solve $S_{s,k}$ for the time of arrival of the vehicle in the bus group.

When $d_{i,k} = d_{i-1,k}$, that is two vehicles have left the docking station one after another, as shown in formula (66) and (67).

$$S_{s,k} = (1 - T_{i,k})u_{i,k} \tag{66}$$

$$T_{i,-1,,k} = (1 - T_{i,k}) \tag{67}$$

According to the original schedule, the relationship between the stay time of $i-1$ bus at the station k and $i-2$ bus, as shown in formula (68).

$$S'_{i-1,k} = (d'_{i-1,k} - d'_{i-2,k})T'_{i-1,k} \tag{68}$$

According to the original schedule, the relationship between the stay time of $i-1$ bus at the station k and station $k+1$, as shown in formula (69).

$$S'_{i-1,k} = a'_{i-1,k+1} - a'_{i-1,k} - Tr'_{i-1,k} \tag{69}$$

The staying time of $i-1$ vehicle at the actual station of the dispatching group, as shown in formula (70).

$$S_{i-1,k} = T_{i-1,k}(d_{i-1,k} - d_{i-2,k}) + (1 - T_{i-1,k})u_{i-1,k} \tag{70}$$

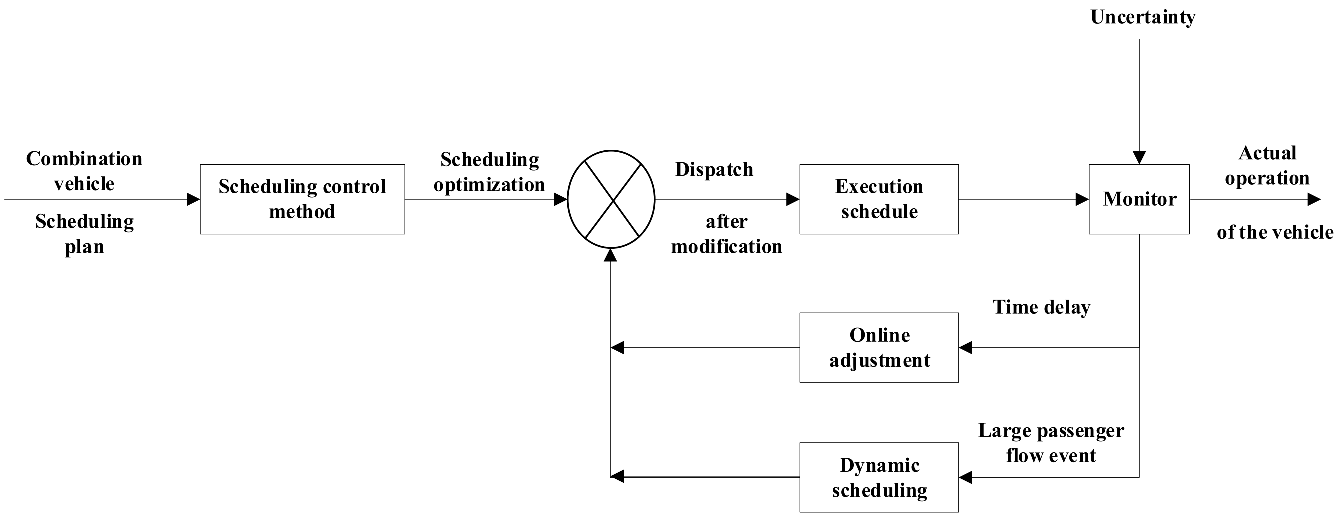

FIGURE 2. Vehicle dynamic dispatch control diagram at bus stop in large passenger flow area.

The $i-1$ bus and i bus depart at the same time at the k station, and the relationship with the arrival at the station $k+1$, as shown in formula (71), (72) and (73).

$$S_{i-1,k} = a_{(i-1,i),k+1} - a_{i,k} - Tr_{i-1,k} \tag{71}$$

$$S_{i-1,k} - S'_{i-1,k} = T_{i-1,k}(d_{i-1,k} - d_{i-2,k}) + (1 - T_{i-1,k})u_{i-1,k} - T'_{i-1,k}(d'_{i-1,k} - d'_{i-2,k}) \tag{72}$$

$$S_{i-1,k} - S'_{i-1,k} = x_{(i-1,i),k+1} - x_{i,k} - W_{i,k} \tag{73}$$

Assuming $W_{i,k} = 0$ that the passengers boarding at the start site of the implementation of the vehicle are evenly distributed in the vehicles of the vehicle group, then $T_{i-1,k} = \theta_{(i-1,j),k} T'_{i-1,k}$.

The formula can be simplified to:

$$T_{i-1,k}x_{i-1,k} - x_{i,k} - x_{(i-1,i),k+1} + (1 - T_{i-1,k})u_{i-1,k} = 0 \tag{74}$$

In summary, the relationship between the starting point of the vehicle group and the control of the arrival time of the controllable vehicle is obtained.

B. OPTIMIZATION VERIFICATION OF PUBLIC TRANSPORTATION SCHEDULING

Judged according to the passenger flow distribution characteristics of the Guangzhou bus B3 route and the bus station in the large passenger flow area. Set the number of the bus stop starting to implement dispatch control as $\alpha = 6$,and the number of the bus stop ending implementing dispatch control as $\beta = 11$.During the morning and evening peak hours, the number of buses can be controlled to 12 vehicles. The number of bus seats is 30, and the bus is full of 60 people. The passenger unit time cost is RMB 0.145, the critical full-load rate of the bus is 1.2, and the average time of one-way bus operation is 60 min.

According to the schedule requirements, the bus B3 route will be overloaded when they arrive from the 6th station to the 11th station during the morning and evening rush hours. Therefore, it is necessary to increase the number of assistance buses and evacuate passengers at stations in high passenger flow areas in time to ease the situation. Bus passenger flow at major stations to avoid some passengers being unable to board the bus. Therefore, it is necessary to adopt a multi-mode combined scheduling optimization method to increase the number of bus groups, improve the overall capacity of public transportation, optimize the departure time interval, and solve the problem that passengers cannot board the bus at the bus station in the large station area. To verify the speed, effectiveness, and rationality of the bus to the bus group, the following three options are evaluated:

Option1: Depart according to the bus schedule, with an interval of 3 minutes.

Option2: When buses depart at equal intervals every 3 minutes, only the capacity of the bus will be increased, but the departure time interval will remain the same.

Option3: Increase the capacity of buses. According to the time requirements for the controllable buses to reach the controllable bus station, the departure time interval and the arrival time of the controllable buses need to be optimized and adjusted.

In the third option, according to the analysis and prediction of bus passenger flow of Guangzhou bus B3 route, it is predicted that the overload will be the most serious at the ninth station. According to the rules of weighting matrix, the **A** weighting matrix has the largest value at the ninth station and its response speed is the fastest. Similarly, the **B** weighting matrix accounts for the largest proportion in the ninth station, which reduces the amplitude of the control component. Through the controllable bus and controllable bus stop to form a discrete system, the state matrix and

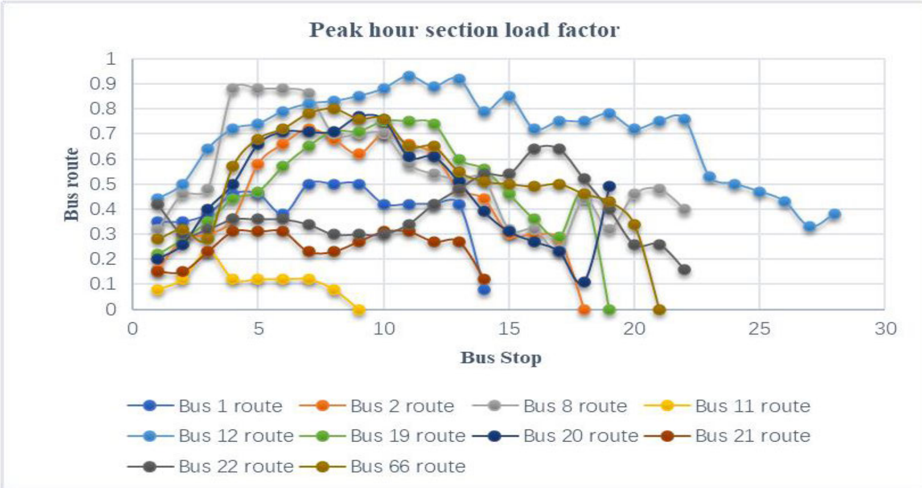

FIGURE 3. Peak hour section load factor of urban route.

TABLE 3. State matrix of controllable bus stop and controllable bus.

Number	Controllable bus stop					
	6	7	8	9	10	11
1	2.12	-0.16	0.00	0.00	0.00	0.00
2	-0.16	6.53	-0.74	0.06	0.00	0.00
3	0.00	-0.74	8.31	-1.12	0.08	0.00
4	0.00	0.06	-1.12	11.25	-0.61	0.15
5	0.00	0.00	0.08	-0.61	5.89	-0.06
6	0.00	0.00	0.00	0.15	-0.06	2.23

TABLE 4. Control matrix of controllable bus stop and controllable bus.

Number	Controllable bus stop					
	6	7	8	9	10	11
1	-1.83	0.91	-0.13	-0.20	0.00	0.00
2	0.19	-5.84	2.15	-0.41	0.03	0.00
3	0.00	1.69	-9.56	4.14	-0.35	0.00
4	0.00	-0.19	2.37	-11.02	1.03	0.00
5	0.00	0.00	-0.35	1.82	-3.65	0.07
6	0.00	0.00	0.03	-0.16	0.31	-2.38

control matrix of controllable bus at controllable bus stop can be obtained by using linear quadratic optimal control method and MATLAB programming, as shown in Table 3 and Table 4.

According to the maximum boarding rate of passengers when the bus is full, the controllable vehicle arrival time is adjusted without increasing the bus capacity. 0 means that the controllable vehicles arrive at the corresponding controllable bus stop according to the original schedule. a negative number means the time for the controllable vehicle to reach the controllable bus stop in advance, and a positive number means the controllable stay time of the bus at the stop.

TABLE 5. Optimization and evaluation of dispatching option.

Sequence number	Evaluation Index	Option1	Option2	Option3
1	Waiting time/sec	2795.3	2795.3	2483.6
2	Congestion cost/yuan	3154.7	0	0
3	Operating cost/yuan	712	776	776

By comparing the above three schemes, the following conclusions can be drawn from Table 5.

1) **Option 1**: Although the operating cost of the bus company is reduced, the passenger travel cost and the cost of in-bus congestion are increased, the ride comfort is reduced, and the user travel experience is poor.

2) **Option 2**: Under the premise that the scheduled departure time interval remains the same, although the bus company sends reinforcements to increase the capacity, the passenger waiting time is the same as in option 1, which is 2795.3 seconds, and the passenger congestion cost is zero. The ride is comfortable and feels good.

3) **Option 3**: Implement controllable and optimized dispatch of controllable buses. Public transport companies will send reinforcements to increase capacity, so that controllable buses can reach the sixth station at the same time. The cost of passenger congestion on the bus is zero, and the ride is comfortable good, to realize the controllable combined dispatching of buses to buses, save passengers' travel cost, improve the capacity of bus companies, and reduce waiting time by 11.15%.

In the optimization evaluation of dispatching scheme, the waiting time, congestion cost and operating cost are compared, and it is found that option 3 is obviously superior to option 1 and option 2, and it is innovative to implement the

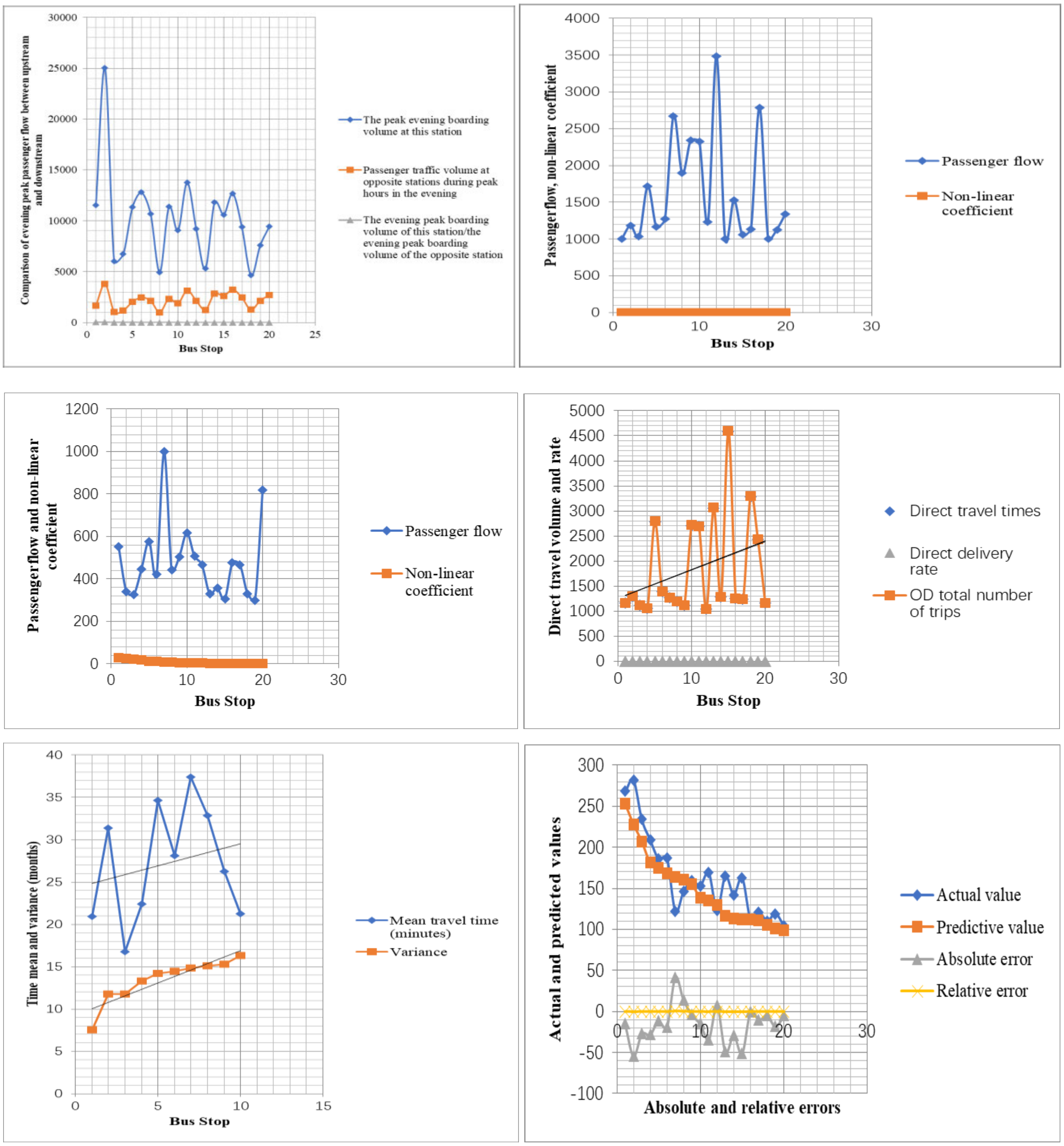

FIGURE 4. Comparison of tidal passenger flow analysis for public transport travel.

optimal dispatching strategy for controllable public transport vehicles.

Compared with the traditional bus scheduling method, the multi-mode combination bus scheduling model proposed in this paper is presented. The case analysis results show that the third scheme is the proposed mode, while the first and second schemes are the traditional bus scheduling methods. The third scheme is significantly better than the first and second schemes, which makes the passengers feel comfortable and comfortable, saves the passengers' travel cost, improves the capacity level of the bus company, and reduces the waiting time by 11.15%. Therefore, this multi-mode combination scheduling method has strong practical significance in bus optimization scheduling.

C. FULL LOAD VERIFICATION OF BUS DISPATCH SECTION

According to survey data, on May 11, 2020, the peak hour cross-sectional load factor of 10 urban bus routes in Guangzhou is calculated, as shown in Figure 3. Bus 8 route and 12 route have a high cross-sectional load factor during peak hours, resulting in overcrowding within the buses. Therefore, it is necessary to increase the number of trips and reduce departure intervals. The load capacity of bus 2, 19, 20, 22, and 66 route is moderate and does not require any adjustment. The cross section full load rate of bus 1, 11, and 21route is relatively low, with less passenger flow during peak hours. The main reason is that bus 1 route has just been adjusted, and the passenger flow on the route is increasing. Bus 11 route and 21 route are newly opened routes. After on-site investigation and passenger flow analysis, bus 21 route is a promising route, and the layout of bus 11 route is unreasonable, which needs further adjustment.

D. SIMULATION AND VERIFICATION OF PASSENGER FLOW IN PUBLIC TRANSPORTATION DISPATCH

According to the analysis of the tidal characteristics of urban public transportation, a monthly statistical analysis was conducted on the passenger flow of up and down buses at the same OD bus stop in Guangzhou on August 20, 2020. It was found that the morning peak passenger flow at the up station was significantly higher than that at the down station, which conforms to the law of urban public transportation OD passenger flow. In the morning, the passenger flow of citizens heading to the city center for work increased sharply. The comparison of the morning peak boarding volume at opposite stations, the top 20 pairs of bus stop with the highest nonlinear coefficient, the top 20 pairs of bus stop with the highest nonlinear coefficient monthly, the effect of bus OD on the direct travel rate, the monthly analysis of bus OD on the stability of workday travel time, and the prediction of passenger flow at bus stop were formed. Comparison, as shown in Figure 4.

The sample data of the experimental data in this paper is from 10 urban bus routes in Guangzhou, 24 stations in each route, namely 240 bus stations, to calculate the peak hour section full load rate on May 11, 2020. To analyze the tidal passenger flow of bus travel in Guangzhou on August 20, 2020, 20 pairs of OD passenger flow samples of bus stations were collected: 37145 direct travel trips, 37215 total OD trips, 3269 predicted passenger flows of bus B3 route from 14:00 to 15:00, 3274 actual passenger flows, 514,858 boarding volumes in the morning peak from 7:30 to 9:00, and 60,183 boarding volumes in the evening peak. 355,385 boarding volumes in the morning peak from 7:30 to 9:00 of bus B4 route, and 49,605 boarding volumes in the morning peak of opposite stations.

IV. CONCLUSION

The combination scheduling of public transportation is the core part of intelligent transportation scheduling. By establishing optimization models for the combination scheduling of conventional public transportation and express buses at large stations, as well as the combination scheduling optimization model of conventional public transportation and inter-district buses, the scheduling optimization control of public transportation stations in high passenger flow areas is carried out. Based on the research results of optimal control, an equal interval departure scheduling plan was adopted to optimize the passenger flow at the BRT area of Zhongshan avenue in Guangzhou, achieving a relative balance between passenger travel cost and bus enterprise cost, solving the problem of citizen travel, and effectively alleviating the traffic pressure on the surrounding area road network during peak hours. The results proved that the model and algorithm are practical and reliable.

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