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RESEARCH ARTICLE

Short-Term Load Foresting Using Combination of Linear and Non-Linear Models

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ABSTRACT Numerous short-term load forecasting models are available in the literature. However, the improvement in forecast accuracy using the combination models has yet to be analyzed on a daily rolling basis for a very long test period. In this paper, the characteristics of a combination of the Seasonal Autoregressive Integrated Moving Average (SARIMA) – a linear model and Radial Basis Function networks (RBFN) – a non-linear model have been studied in two different modeling frameworks, namely single series (SS) and variable segmented series (VSS). The hourly load data from the Ontario Electricity Market (OEM) and the Iberian Electricity Market (MIBEL) are used for the analysis. This dataset spans 12 years for OEM and one year for MIBEL. The impact on prediction accuracy by the size of training data and the combining individual forecasts has been studied for both markets. To achieve the empirical objective, a large number of models(1,447,740 in number) are estimated to produce load forecasts on a daily rolling basis. The forecast performance has been compared with the other models proposed in the literature. Among the linear models, for all window sizes of training data, the forecast accuracy of the combination model is better than the model selected with the minimum Akaike information criterion (AIC) and Bayesian information criterion (BIC) in both frameworks. Moreover, the ensemble of RBFN and linear models produces the best forecast. The results pinpointed that the proposed model's precision and stability are higher than the earlier forecasting models proposed for both markets. The novelty in the model is that only a single hourly time series is used for forecasting, and there is no need for other explanatory variables.

INDEX TERMS Artificial intelligence, electricity market, load forecasting, radial basis function, single series, variable segmentation.

NOMENCLATURES AND ABBREVIATIONS

The notation used throughout the paper is provided below.

ACF	Autocorrelation function
AIC	Akaike information criterion.
ANNs	Artificial neural networks.
AR	Autoregressive
ARMA	Autoregressive moving average
BIC	Bayesian information criterion.
DNN	Deep neural networks.
FL	Fuzzy logic.
LSTM	Long-short-term memory.
MAPE	Mean absolute percentage error.
MA	Moving average.
OEM	Ontario electricity market.
PACF	Partial autocorrelation function.
RBFN	Radial basis function networks.
SARIMA	Seasonal autoregressive integrated moving average.
STLF	Short-term load forecasting.
SS	Single series.
SVM	Support vector machines.
VSS	Variable segmented series.
L_t	Actual load at time t.
B	Backward shift operator.
α_t, α_{t-1}	Error components in load series after removing seasonality for SARIMA model.
D_1 to D_{14}	Input layer variables used for RBFN model.
$\phi_1, \phi_2, ., \phi_p$	Parameters of non-seasonal AR (p) model.
$\theta_1, \theta_2, .\theta_q$	Parameters of MA (q) model.

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Ψ_j	Gaussian radian function value at each hidden layer neuron in RBFN.
$\Phi_1, \Phi_2, \ldots$	Parameters of seasonal AR (Ps) model.
$\Theta_1, \Theta_2, ..$	Parameters of seasonal MA (Qs) model.
$(1 - B^s)$	Seasonal differencing operator with periodicity s.
T	Target / Forecast value of demand.

I. INTRODUCTION

The load forecasting is extremely useful for electric sector as it helps in efficient power grid operation and planning [1]. The load forecasting can be categorized into long-term, medium-term, short-term and very short-term forecasting [2], [3]. STLF typically one hour to a couple of days ahead is required by the utilities for their daily operations mainly for the unit commitment (UC), the load dispatch and the energy transfer scheduling activities [4]. Though on a longer time frame, the electricity demand is affected by many exogenous factors such as economic, demographic, technological, natural and social factors; yet, on a short-term basis, it is affected by weather variables, calendar, holiday and festival effects etc. Moreover, certain unexpected events are also there which make STLF a difficult exercise [5].

A. LITERATURE REVIEW

Several methods for the load forecasting have been reported in the last decades [6], [7], [8], [9], [10], [11]. In these models, three different paradigms of STLF have emerged: (i) statistical analysis, (ii) artificial intelligence (AI) and (iii) hybrid techniques. A model based on statistical analysis comprises either a single load series (univariate) or load series as a function of many exogenous variables i.e., multivariate framework. The univariate models are: autoregressive (AR) [12], moving average (MA) [13], ARMA [14], [15], [16], adaptive ARMA [17], SARMA [18], [19], [20], [21], [22], [23], [24], [25], [26], threshold AR [27], exponential smoothing models [28], fractional ARIMA [29] and smooth transition periodic autoregressive (STPAR) model [30]. It has been observed that electricity load data is not only affected by the previous lags of data but also by the weather variables like temperature, wind speed, cloud cover, and humidity etc. So multivariate weather-based models have been reported such as multiple regression [32], [33], [34], [35], [36], transfer function (TF) model [37], and ARMAX models [38], [39]. Recently, a new class of non-parametric models based on AI techniques have been proposed, which mainly include FL [40], [41], ANNs [42], [43], [44], [45], SVM [46], RBFN [47], [48] and transfer learning [49]. Hybrid techniques integrate the advantages of each of the models and may improve the prediction accuracy in two ways. For the first category, electricity load is predicted by the different models [50], [51], [52], and the final forecasting value is obtained by their combination. For the second category, electricity load is decomposed into several components, and the final forecasting value is the sum of predicted value of each component [53], [54]. Most of the statistical methods are based on linear models that make some assumptions about the characteristics of the load series. The idea is conceptually based on the understanding that a load pattern is a time series signal with seasonal, weekly, daily and a few hourly periodicities. The AR models were first presented by Yule [12] in 1926. Subsequently, Slutzky introduced MA models [13]. The combination of the AR and MA models, ARMA, was first implemented by Wold [55], which showed that ARMA processes can be used to model stationary time series data. Researchers applied least square estimation (LSE) based autoregressive integrated moving average (ARIMA) model for many a forecasting tasks [56], [57]. Seasonal ARIMA [58], [59] and double SARIMA [60], [61], [62] are powerful linear models extensively used to deal with various forecasting issues.

A variety of different forecasting models are available to forecast demand data, and it is essential to realize that no single model is universally applicable [63]. The fusion of SARIMA and non-linear models can overcome the shortage of adopting only one kind of model and provide more accurate results [64], [65], [66]. Karthika et. al proposed [54] hybrid model ARIMA-SVM to predict the hourly demand of southern region of India. Authers used ARIMA to predict the demand after correcting the outliers using percentage error method and its deviation is corrected using SVM. Kao et.al. [64] proposed ensemble empirical mode decomposition(EEMD)-ARIMA-genetic algorithm (GA)-SVR to predict the primary energy consumption of Taiwan. Based on their findings, the results obtained when using the hybrid model were far superior to those obtained when only using the ARIMA or ARIMA-SVR models. The main question in STLF is: whether to use a univariate or a multivariate model. In many studies the superiority of univariate models has been proved [67]. Recently, Makridakis and Hibon [68], deduced that statistically sophisticated or complex methods do not necessarily provide more accurate forecasts than the simpler ones. The development of a multivariate model depends on the availability of accurate weather forecasts. The utilization of forecast weather variables introduces more uncertainty in the STLF model. Therefore, the univariate methods are supposed to be sufficient for short lead times because the weather variables tend to change in a smooth fashion over short time frames, and this will be captured in the changes in the demand series itself. Moreover, the univariate methods are the only option when meteorological forecasts are either unavailable or too costly [69].

In almost all research discussed above, the forecast period is from one hour to nearly one year. Work has yet to be done to study the characteristics of SARIMA models for STLF by utilizing time-series data in multiple frameworks on a daily rolling basis. It has also been observed that the performance of the forecast combinations of SARIMA and RBF network has not been explored in case of load-series exhibiting different volatility levels. Considering these gaps, this paper aims to study the STLF accuracy using a combination of SARIMA and RBF network. The forecasting performance has been done for real data sets of OEM and MIBEL by utilizing parameter estimation periods of different window sizes. The main contributions of the paper are summarised as follows:

- The proposed models preferred to minimize parameter dependency by using only single load series and its previous lags values at different intervals. The proposed forecasting model may help to deal with the situation where lack of weather related variables and ultimately contribute to the reduction of the load forecasting error.
- An improved, simple strategy to produce better load forecast by combination of a linear and a non-linear model has been proposed. In order to improve accuracy and reduce forecast volatility, individual models have been trained with training data of multiple sizes and then forecasts have been combined. The effect of size of training data for each model is also presented.
- The probability of accuracy increases by simple averages of SARIMA and RBFN models forecast. The strength and effectiveness of individual models and their ensemble for all window sizes have been validated for a very long period of most volatile OEM from 2007 to 2018 and MIBEL for 2016.
- The forecasting performance of the proposed model is compared with the benchmarks, individual SARIMA, other ensemble models and the reported works [44].
- A comparative study among the benchmark, individual models, and recent techniques in literature is performed. The results reveal that all the models give better performance in comparison to the benchmark models for both markets. The combination of SARIMA SS and RBFN has been found to be best among all the models for OEM. The combination of SARIMA SS, VSS and RBFN has been found to be best among all the models for MIBEL demand.

To evaluate and reach these objectives, this paper is divided into the following sections: Section II describes the problem formulation; Sections III and IV present the complete forecasting methodology and the computational implementation respectively. The numerical results together with interpretations are presented in section V. Finally, conclusions are given in the last section.

II. PROBLEM FORMULATION AND MODELS STRUCTURE

Theoretically, any function approximation model can be represented as:

$$T = f(D_t) + \epsilon_t \tag{1}$$

where, T is the target output, D_t is the input vector, and ϵ_t is the error series which is assumed to be homoscedastic (and possibly normally distributed). In case of linear models like SARIMA, f is a linear function; whereas, in non-linear models like RBFN, this is a non linear function of input variables [70].

A. SARIMA MODEL STRUCTURE

The load time series L_t is regarded as the realisation of a non-static stochastic process. General form of ARIMA model [71] is as:

$$\phi(B)\,(1-B)^d\,L_t = \theta(B)\,a_t \tag{2}$$

where, B is the backward shift operator that defines $B^n L_t = L_{t-n}$;

$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p \tag{3}$$

is a polynomial in B of degree p; ϕ_1, ϕ_2, ….., ϕ_p are the parameters of non-seasonal AR (p) model;

$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q, \tag{4}$$

is a polynomial in B of degree q; $\theta_1, \theta_2, \ldots, \theta_q$ are the parameters of the non-seasonal MA(q) model and a_t should be independently and identically distributed (iid). For the process to be stationary, the characteristic equation $\phi(B) = 0$ has all its roots outside the unit circle. Similarly, for the invertibility of the process, the roots of $\theta(B) = 0$ should lie outside the unit circle. $1 - B$ is the ordinary differencing operator and it removes the trend in load-series data, d specifies the degree of non-seasonal integration. In particular, load series are often well represented by the models in which one or more roots of the characteristic equation are unity. The load series exhibits several levels of seasonality; therefore, the load at a given hour is dependent on the load values at the previous hour, at the same hour on the previous day and week i.e.,$(t-1)$, $(t-24)$ and $(t-168)$. Since L_t is a correlated time series, the forecasting model can be assumed by the SARIMA model. The seasonality in the L_t can be removed by a model of the form:

$$\Phi(B^s)(1-B^s)L_t = \Theta(B^s)\alpha_t \tag{5}$$

Here, $(1-B^s)$ seasonal differencing operator with periodicity s. Seasonal differencing removes seasonality in the observed load series in the same way as ordinary differencing (d) removes a polynomial trend.

$$\Phi(B^s) = 1 - \Phi_1 B^s - \Phi_2 B^{2s} - \ldots - \Phi_P B^{Ps}, \tag{6}$$

$$\Theta(B^s) = 1 - \Theta_1 B^s - \Theta_2 B^{2s} - \ldots - \Theta_Q B^{Qs}, \tag{7}$$

$\Phi(B^s)$, $\Theta(B^s)$ are polynomial of degree P and Q respectively. Model (8) is used to link the current behaviour for L_{t-1} with previous load observations.

$$\Phi(B^s)(1-B^s)L_{t-1} = \Theta(B^s)\alpha_{t-1} \tag{8}$$

Now the error components α_t , α_{t-1}, would be in general correlated and are related by ARIMA model (9):

$$\phi(B)\,(1-B)^d\,\alpha_t = \theta(B)\,a_t, \tag{9}$$

Substituting (9) in (5), general multiplicative linear model is [71]:

$$\Phi(B^s)(1-B^s)\,\phi(B)(1-B)^d L_t = \Theta(B^s)\,\theta(B)a_t \tag{10}$$

In standard notion, (10) is SARIMA $(p, d, q) \times (P, 1, Q)$ model for load time series L_t.

TABLE 1. Input layer variables used for RBFN model.

1.	Forecast hour index in the form of sine curves ($D_1 - D_2$) $\sin(n\pi/24)$, n=1,2,3,.......,24 $\cos(n\pi/24)$, n=1,2,3,.......,24
2.	Hourly demand data of past 7 i.e.,$D - 1$ to $D - 7$ days ($D_3 - D_9$)
3.	Hourly average of past 7 days D_{10}
4.	Moving Averages: MA(24) (D_{11}), MA(168) (D_{12})
5.	Hourly demand data of past 14, 21 days (D_{13}, D_{14})

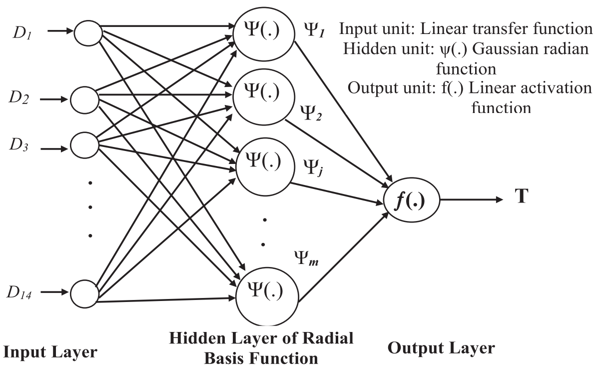

FIGURE 1. A diagram of radial basis function neural network (RBFN) model.

B. RBFN MODEL STRUCTURE

The RBF network consists of three layers: an input layer, a hidden layer and an output layer. The main forecast variables used as input neurons in the model are given in Table 1. The RBF networks are better at learning local data patterns. Each RBF network is a linear combination of non-linear functions known as radial basis function Ψ shown in figure (1). Input at m hidden neurons is $X_j = \|D - C_j\|\ ; j = 1, \ldots, m$ where $D = [D_1, D_2, \ldots.., D_{14}]$ is input vector and center, $C_j = [C_{j1}, C_{j2}, \ldots.., C_{j14}]$ is adjustable parameters can be determined by training algorithm. Hidden unit uses Gaussian radian function $\Psi(z) = e^{-\frac{(z-\mu)^2}{\sigma^2}}$, μ is mean, σ is standard deviation of input z and so output at each hidden layer neuron is given by

$$\psi_j = e^{-\frac{X_j^2}{\sigma_j^2}} \tag{11}$$

where spread (σ_j) is also an adjustable parameter for each neuron in hidden layer. The output of the RBF network is given as:

$$T = \sum_{j=1}^{m} w_j\, \psi_j \tag{12}$$

where the output T is represented as a sum of m radial basis functions Ψ_j, each associated with a different center C_j, and weighted by an appropriate coefficient w_j. Weights w_j are estimated through training. For the input variables near the center, the output Ψ_j is large.

III. FORECASTING METHODOLOGY

The methodology used for the load forecasting by linear SARIMA model can be described by the following four main steps:

1) Identifying trends and applying data transformations
   - Identifying polynomial and seasonal trends
   - Eliminating exponential trends
   - Testing for data stationarity: Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test [72], Dickey-Fuller test [73],
2) Model selection and estimation of parameters
   - Computing autocorrelation function (ACF) and partial autocorrelation function (PACF)
   - Tentative model selection
   - Parameters estimation using maximum likelihood estimation (MLE)
   - Selecting a model with the lowest Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC) values
3) Model validation using residual analysis
   - Infer residual from the fitted model
   - Testing residuals for Normality
   - Q-Q plot of residuals
   - Observing ACF and PACF of residuals, Plotting graph of standardized residuals
4) Forecasting
   - Forecast demand from selected model with minimum AIC and BIC values
   - Calculate combination forecast from all the estimated models.

The RBF network approach consists of the following steps:

1) model selection
2) data normalization
3) updating model parameters using training data of rolling windows
4) producing forecasts with multiple models
5) combining forecasts.

The proposed COM model calculates the forecast by averaging the predictions of individual models across all window sizes. MATLAB has been used as the programming environment and the code was run using Intel i7, 8 GB RAM 2.90 GHz system (www.mathworks.com).

A. BENCHMARK MODELS

For comparison purpose, the following two benchmarks models have been selected:

Benchmark 1: Last day model ($D1$)

$$\hat{Y}_{h,k} = Y_{h,k-1} \tag{13}$$

Benchmark 2:Last week model ($D7$)

$$\hat{Y}_{h,k} = Y_{h,k-7} \tag{14}$$

where, $h = 1, 2, ..24$ hours of a day, $Y_{h,k-1}$ is h^{th} hour load of $(k-1)^{th}$ day, Y_{k-7} is h^{th} hour load of $(k-7)^{th}$ day and $\hat{Y_{h,k}}$ is forecast load of h^{th} hour of k^{th} day.

B. METRICS FOR ACCURACY ASSESSMENT

The mean absolute percentage error (MAPE) is employed to evaluate the performance of the various models. If L_t is the actual load at time t and $\hat{L_t}$ is the forecast for the same period, then the MAPE for N observations [74] is as:

$$MAPE = \frac{1}{N}\sum_{t=1}^{N}\left|\frac{L_t - \hat{L_t}}{L_t}\right| \times 100\,\% \tag{15}$$

IV. CASE STUDY AND COMPUTATIONAL IMPLEMENTATION

For computational implementation, the historical hourly demand data of the OEM (https://www.ieso.ca/power-data) from March, 2006 to December, 2018 and MIBEL (https//www.mibel.com/en/home_en/) from January, 2015 to December, 2016 have been collected. For comparing the variability of demand of OEM and MIBEL markets,various statistical measures for 2016 presented in table 2. The value of measures of dispersion, i.e., range and the standard deviation, is comparatively significant for OEM demand.The coefficient of skewness for MIBEL and OEM are 0.07 and 0.38, respectively.The coefficient of skewness for OEM is relatively large.In addition to above measure of central tendency, dispersion and skewness, one more measure, kurtosis enables authers to have an idea about the peakedness of the data. For both markets, its value is less than 3 which indicates that frequency curve of demand is flatter than the normal curve. Coefficient of variation which is 100 times the coefficient of dispersion based upon standard deviation is slightly greater for MIBEL for first four months.

A complete description from model selection to model forecast for OEM has been presented in this section. In similar ways, models for MIBEL demand have been analyzed. It can be observed from Figure (2) that the load data series is non-stationary in nature. In the short term, the load series depends on the changing nature of the weather variables, calendar, holiday effects, and electricity market conditions. Since most of the demand in the electricity market is settled on a day-ahead basis, the impact of minor price variations on load demand is minimal.

Initially the demand data is converted in two different frameworks, i.e. single series (SS) and variable segmented series (VSS). In SS model, complete 24 hourly demand data is used. This model computes one-day-ahead forecasts using forecast horizon of 24 hours.

A. FORECASTING STEPS IMPLEMENTATION IN SS

This section briefly describes all the steps for forecasting one day using the SARIMA model by dividing demand into a single-series framework.

1) IDENTIFICATION TRENDS AND APPLYING DATA TRANSFORMATION

In order to identify the suitable tentative linear model, the properties of load series i.e., correlated observation, non-stationary, daily and weekly seasonality are examined. The

TABLE 2. Comparision of basic characteristics.

	Month	Mean (MWh)	Max (MWh)	Min (MWh)	SD (MWh)	CV	KR	SK
MIBEL Demand 2016	Jan.	5929	7895	3509	1090	18.38	1.71	-0.15
	Feb.	6065	8084	4116	1023	16.85	1.69	-0.15
	March.	5868	7810	3786	982	16.73	1.75	-0.10
	April.	5554	7253	3684	853	15.36	1.75	-0.06
	May.	5235	6650	3581	809	15.45	1.62	-0.14
	June.	5377	6868	3562	828	15.39	1.68	-0.07
	July.	5740	7301	3890	874	15.23	1.69	-0.03
	Aug.	5422	6900	3821	789	14.54	1.71	-0.08
	Sept.	5557	7315	3950	826	14.87	1.73	-0.05
	Oct.	5296	6944	3783	820	15.48	1.70	-0.01
	Nov.	5689	7881	3718	996	17.5	1.80	-0.04
	Dec.	5843	7919	3671	1011	17.3	1.84	-0.01
	2016	5629	8084	3509	948	16.83	1.95	0.07
OEM Demand 2016	Jan.	16646	20836	12116	1962	11.79	2.22	-0.18
	Feb.	16482	20766	12533	1798	10.91	2.24	-0.19
	March.	15192	20063	11717	1717	11.3	2.39	0.16
	April.	14458	17821	11286	1545	10.68	2.17	-0.03
	May.	14074	19885	10461	2060	14.64	3.02	0.62
	June.	15419	21692	10596	2358	15.29	2.45	0.24
	July.	16798	22659	10985	2940	17.50	1.86	0.03
	Aug.	17625	23100	12287	2783	15.79	1.90	-0.09
	Sept.	15378	23213	10855	2587	16.82	3.03	0.58
	Oct.	14138	18189	10663	1748	12.36	1.92	-0.13
	Nov.	14858	19369	11211	1816	12.22	2.14	-0.03
	Dec.	16099	20688	11684	1997	12.40	2.25	-0.08
	2016	15597	23213	10461	2417	15.49	2.71	0.38

SD:standard deviation, CV:Coefficient of variation
KR:Kurtosis, SK: Coefficient of skewness

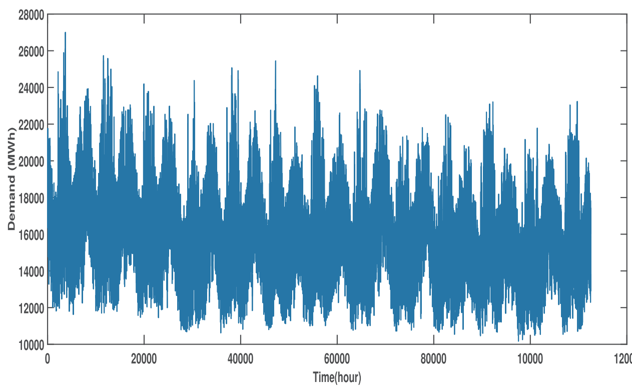

FIGURE 2. Hourly demand data of OEM (March 2006 to Dec. 2018).

Augmented Dickey-Fuller and KPSS tests are conducted on the training demand time-series. For identification of SS models for January 2007, the demand data of 4 September to 31 December 2006 has been considered as the sample period and is denoted by $\{S_t, t = 1, 2 \ldots, 2856\}$.

2) MODEL SELECTION AND ESTIMATION OF PARAMETERS

The model selection techniques rely on the ACF and PACF analysis. These functions are systematic and helpful in the determination of SARIMA model order, in the preliminary estimation of model parameters, and in diagnostic checking. From Figure 3, it is clear that S_t is non-stationary in nature, since the ACF dies down slowly with oscillations. Here, a tendency for the sample ACF not to die out quickly is

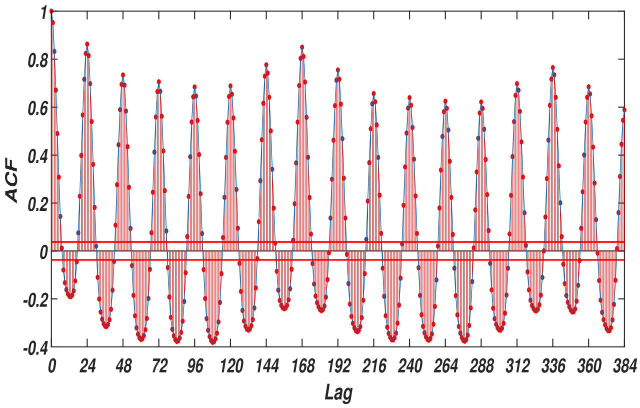

FIGURE 3. ACF for the sample period of OEM.

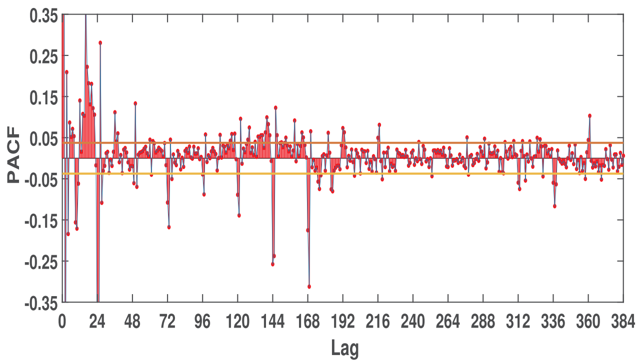

FIGURE 4. PACF for the sample period of OEM.

taken as an indication that a root close to unity may exist and it is confirmed by the KPSS and the augmented Dickey-Fuller tests. The PACF shown in Figure 4 is also typical of a non-stationary series with periodicities and with large spikes at 1, 2, 23, 24, 144,145,167,168 lags. In order to remove trend, a non-seasonal transformation $(1-B)S_t$ of sample data S_t is applied and is denoted by dS_t. When ACF and PACF of dS_t are studied, it shows the presence of seasonal pattern at lags 24. To remove this seasonality at lags 24 a seasonal transformation $(1-B^{24})dS_t$ of demand data dS_t is applied and is denoted by $D24dS_t$. The study of ACF (Figure 5) and PACF (Figure 6) of $D24dS_t$ shows the presence of one more seasonal behaviour. A seasonal transformation $(1-B^{168})S_t$ of demand data S_t is applied and is denoted by $D168S_t$.

The analysis of ACF (Figure 7) and PACF (Figure 8) of $D168dS_t$ demonstrate seasonal MA lags at (24, 48 and 72) and seasonal AR lags at (168) respectively as shown in equation(16).

$$\begin{aligned}(1-B)(1-B^{168})(1-\Phi_1B^{168})\,S_t\\ =(1-\Theta_1B^{24}-\Theta_2B^{48}-\Theta_3B^{72})\,\alpha_t\end{aligned} \tag{16}$$

Now the error components α_t, α_{t-1} are correlated and assumed to be related by ARMA model. To identify the non-seasonal MA lags, it can be observed from (Figure 7)

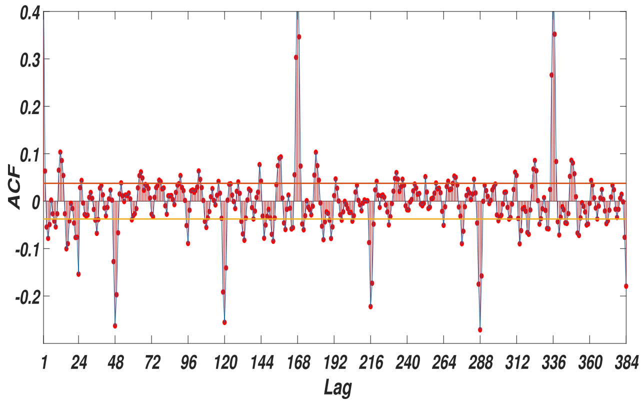

FIGURE 5. ACF of D24d S_t for the sample period of OEM.

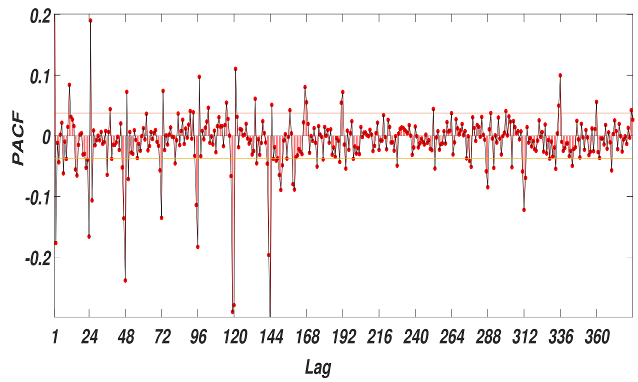

FIGURE 6. PACF of D24dS_t for the sample period of OEM.

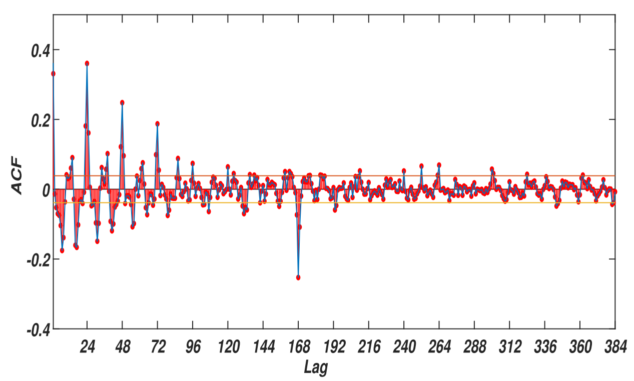

FIGURE 7. ACF of D168 dS_t for the sample period of OEM.

that ACF spikes also appear at lags 1, 7, 14, 16, 23. The order of non-seasonal AR model can be examined from the PACF (Figure 8) which shows significant spikes at lags (1, 2, 4, 5, 24, 48). So, ARMA model (17) for α_t is:

$$\begin{aligned}(1-\phi_1B-\phi_2B^2-\phi_4B^4-\phi_5B^5-\phi_{24}B^{24}-\phi_{48}B^{48})\alpha_t\\ =(1-\theta_1B-\theta_7B^7-\theta_{14}B^{14}-\theta_{16}B^{16}-\theta_{23}B^{23})\,a_t\end{aligned} \tag{17}$$

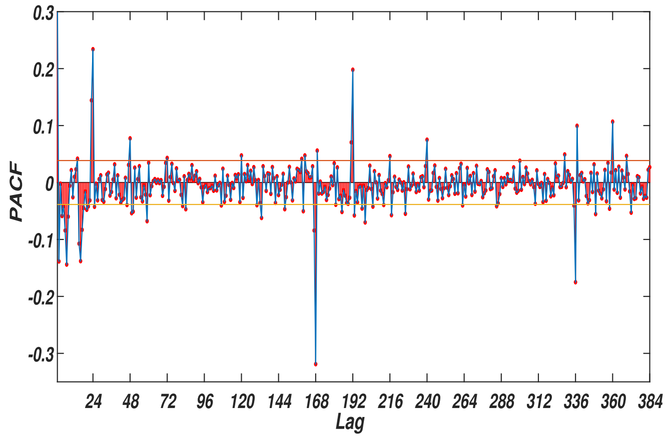

FIGURE 8. PACF of D168dS_t for the sample period of OEM.

Using (17) in (16), the SARIMA model is selected (18):

$$\begin{aligned}&(1-\phi_1B-\phi_2B^2-\phi_4B^4-\phi_5B^5-\phi_{24}B^{24}-\phi_{48}B^{48})\\&\quad\times(1-B)(1-\Phi_1B^{168})(1-B^{168})S_t\\&=(1-\theta_1B-\theta_7B^7-\theta_{14}B^{14}-\theta_{16}B^{16}-\theta_{23}B^{23})\\&\quad\times(1-\Theta_1B^{24}-\Theta_2B^{48}-\Theta_3B^{72})a_t\end{aligned}\tag{18}$$

From the analysis of Figures 7 and 8, one more SARIMA model (19) is identified:

$$\begin{aligned}&(1-\phi_1B-\phi_5B^5-\phi_7B^7)(1-B)(1-B^{168})\\&\quad\times(1-\Phi_1B^{24}-\Phi_2B^{168})S_t\\&=(1-\theta_1B)(1-\Theta_1B^{24}-\Theta_2B^{48}-\Theta_3B^{72}-\Theta_4B^{168})a_t\end{aligned}\tag{19}$$

Now the parameters coefficients of models (18) and (19) are estimated in MATLAB using maximum likelihood estimation (MLE) method for the training data of 17 weeks. On comparison of AIC and BIC values, the model (18) has been selected as first tentative model for January and is named SSJAN1. Similarly the analysis of ACF (Figure 5) and PACF (Figure 6) plots for seasonal and non-seasonal differences of demand S_t has been carried out. The ACF and PACF for sample period after double seasonal differencing and single non-seasonal differencing $dD24D168S_t$ are also observed. Based on the analysis of these transformed series, two more tentative models are identified for January: SSJAN2 (20) and SSJAN3 (21).

$$\begin{aligned}&(1-\phi_1B)(1-B)(1-\Phi_1B^{24}-\Phi_2B^{48}-\Phi_3B^{72}-\Phi_4B^{96}\\&\quad-\Phi_5B^{120}-\Phi_6B^{144})(1-B^{24})S_t=(1-\theta_1B)(1-\Theta_1B^{24}\\&\quad-\Theta_2B^{48}-\Theta_3B^{120}-\Theta_3B^{168}-\Theta_4B^{216}-\Theta_5B^{288})a_t\end{aligned}\tag{20}$$

$$\begin{aligned}&(1-\phi_1B-\phi_2B^2)(1-B)(1-B^{24})(1-\Phi_1B^{24}-\Phi_2B^{48}-\Phi_3B^{72}\\&\quad-\Phi_4B^{96}-\Phi_5B^{120}-\Phi_6B^{144})S_t=(1-\theta_1B)(1-\Theta_1B^{24}\\&\quad-\Theta_2B^{48}-\Theta_3B^{120}-\Theta_4B^{168}-\Theta_5B^{216}-\Theta_6B^{288})a_t\end{aligned}\tag{21}$$

3) MODEL VALIDATION USING RESIDUAL ANALYSIS

The adequacy of all the three SARIMA models (18), (20) and (21) is tested prior to using them for forecasting. The model validation has been done based on residual and forecast error analysis. The residuals are calculated on the estimation set, while forecast errors are calculated based on the test set. The residuals for the estimated SARIMA models are calculated using the infer function in MATLAB. The diagnostic checking is performed by the residual analysis. This is done using (i) residual plot, (ii) checking for normality of the residuals, (iii) quantile-quantile (Q-Q) plot, (iv) analysis of ACF and PACF of residual, (v) plot of Standardized residual,(vi) Durbin-Watson statistic test, and (vii) Ljung Box Q test. All the three models have been found to be adequate for forecasting. The diagnostic checking for predicting load of 5 January, 2007 by the model (18) using training window size of 17 weeks is shown in Figures 9 and 10. The observation of the standardized residuals (white noise) and QQ plots (straight line) for 2856 residuals in Figure 9 confirms the validation of the model (18). The ACF and PACF tests for residual conducted to verify the selected model. Figure 10, demonstrates ACF and PACF of the residuals are smaller than the absolute value 0.07. As the values of ACF and PACF are less than the absolute value 0.1 so no autocorrelation and partial autocorrelation exist within the residuals. The validation is performed randomly for prediction corresponding to different days and the satisfactory validation results by all the models are obtained. Similarly, for identification of SS models for the month of February, the demand for previous 17 weeks have been used. This process of identification and diagnostic checking is repeated for the selection of SS models for every month of the different test period. Initially, a total of $12 \times 3 = 36$ models are selected as the tentative SARIMA models for forecast of daily demand by the SS models.

4) FORECASTING

For one day forecast using only single series of load, the parameters of all the three SARIMA models have been

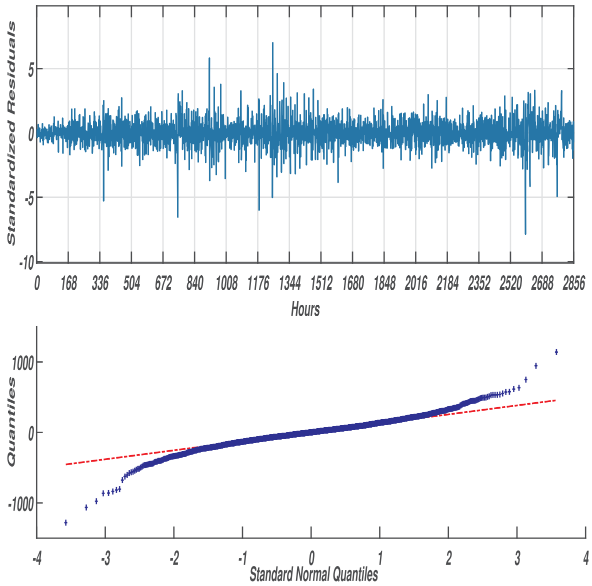

FIGURE 9. Residual analysis for prediction 5 January 2007 (SSJAN1 model).

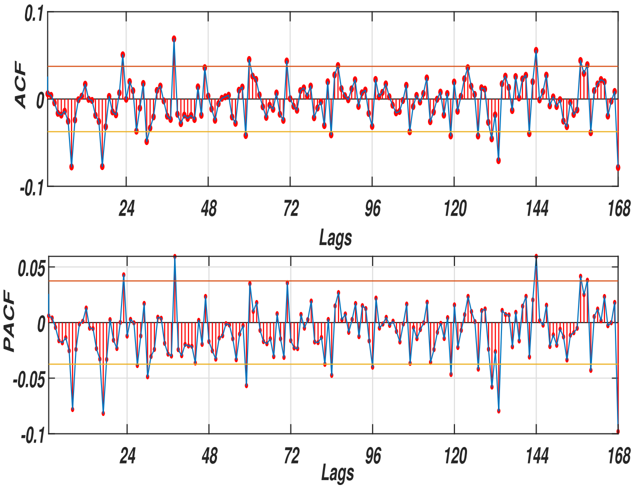

FIGURE 10. ACF and PACF of residual for predicting 5 January 2007 (SSJAN1 model).

estimated for a given window size of estimation data. Then, the model with the lowest values of AIC and BIC has been selected among them for forecast. This selected model is named as SS*min* model. The average of forecast from each of these SARIMA models is also calculated and named as SS*ave* model forecast. Repeating the same steps, 12 years of demand have been forecasted using parameter estimation window sizes of 63, 70, 77,..., and 140 days on a rolling basis. The authors want to highlight the large number of models estimated. To achieve our objective of assessing the forecast accuracy over a very long time period (12 years), a total of $(365 \times 12 + 3) \times 3 \times 12 = 157,788$ models are estimated by the SS model alone for twelve different parameter estimation window sizes.

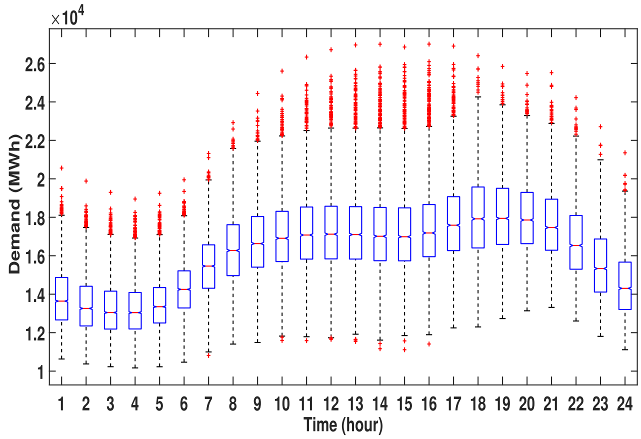

FIGURE 11. OEM demand box plot for 24-hour subseries for each hour March 2006 to december 2018.

B. FORECASTING STEPS IMPLEMENTATION IN VSS MODELS

In VSS model, the hourly demand data has been split into 24 hourly sub-series, one corresponding to each hour of the day. A box plot of these 24 hourly sub-series is shown in Figure (11). It can be observed that the peak hours sub-series have wider range as compared to off-peak hours. The VSS model computes one day ahead forecast using one-step ahead forecast for each hour of the next day using the respective hourly sub-series. Six tentative models have been identified based on the ACF and PACF of each hour load series. The graph in Figure 12 presents the non-seasonal and seasonal ($s = 7$) differenced subseries of hour1 for training data of 119 days. It indicates that the transformed hour1 demand can be assumed to be a stationary time series. The process is repeated with different days in the estimation series to identify six tentative SARIMA models for hour1. Once the tentative models are formulated, the related model parameters are estimated using the MLE algorithm. Then, a one step ahead forecast for each hour of the next day for a given window size of training data has been made by selecting one model having the lowest AIC and BIC values.This selected model is named as VSSmin model. We also calculated the average forecast for each hour from six tentative SARIMA models and named it as VSSave model forecast. For one day forecast, the parameters of all the SARIMA models have been estimated for a given window size of training data. A total of $(365 \times 12+3) \times 144 = 631,152$ models have been estimated for a test period of twelve years using 126 days window size of training data.

C. FORECASTING STEPS IMPLEMENTATION IN RBF NETWORK MODELS

In RBFNs, the main issues are: selection of training data, number of hidden neurons in hidden layer. The hidden layer has been set with radial basis function and output layer has a linear activation function. To reduce training data bias, each network trained with a past data of 14, 21, 28, 35, 42, 63 and

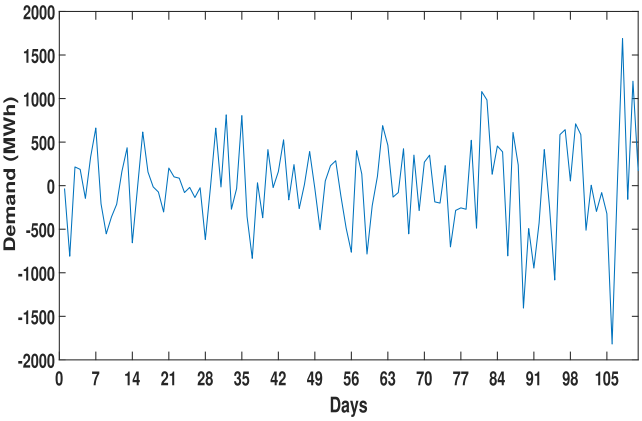

FIGURE 12. OED for hour1 after seasonal (s=7) and non-seasonal differences.

84 days before D-day. The entire training data divided in the ratio of 85:15 and a repeated random sub-sampling applied. The number of random initializations l taken as 10 and then averaged to reduce the random initialization bias. After multiple experiments, the number of hidden neurons has been taken as 20, 20, 20, 25 and 25 for the training window size of 7, 14, 21, 28, 35 days respectively. The spread factor is also fixed at 3.5.

V. NUMERICAL RESULTS AND DISCUSSION

In this section, the forecast performance of various models is compared and the impact of varying window size of parameter estimation data sets has been analysed.

A. COMPARISON OF INDIVIDUAL MODEL PERFORMANCE

Linear demand forecast has been obtained using two approaches: (i) by the model having lowest AIC and BIC value, and (ii) by averaging forecast of all the SARIMA models using all parameter estimation window datasets. Better results have been obtained by the SS model, so forecast for this model has been shown for all the window sizes. Apart from this, demand also forecasted by the VSS model for different window sizes; but, no significant differences in the forecast performance were observed. So the best forecast results of the VSS have been presented for the window size of 126 days respectively. Non-linear demand forecast have been obtained using RBF network with different size of training data 14, 21, 28, 35, 42, 56 days and different number of neurons in hidden layer.

The forecast MAPE performance of the SS, VSS, and RBF network models using parameter estimation of window size of 140, 126, and 28 days, respectively, is compared with benchmark models in Table 4. The following models have been compared:

VSS1: VSS*min* model using window size 126 days,
VSS2: VSS*ave* model using window size 126 days,
SS1: SS*min* model using window size 140 days,
SS2: SS*ave* model using window size 140 days,
RBFN: RBF network model using window size 28 days

TABLE 3. Count of days having best performance among six models for OEM.

Model	D7	D1	VSS2	SS2	RBFN	COM
Count of days	389	475	552	1076	989	902

TABLE 4. Yearly MAPE comparision OEM.

Model	*D7*	*D1*	*VSS1*	*VSS2*	*SS1*	*SS2*	*RBFN*	*COM*
2007	6.63	5.63	3.84	3.69	3.07	2.98	3.07	2.39
2008	5.49	4.93	3.39	3.26	2.81	2.79	2.77	2.24
2009	5.28	5.08	3.25	3.17	2.84	2.75	2.71	2.24
2010	6.36	4.89	3.57	3.43	2.94	2.85	2.88	2.32
2011	5.36	4.95	3.55	3.44	2.92	2.85	2.81	2.29
2012	5.92	5.04	3.89	3.73	3.16	3.06	2.61	2.51
2013	7.08	5.05	3.75	3.65	3.03	3.01	2.72	2.57
2014	5.84	4.90	3.71	3.59	2.95	2.92	2.50	2.41
2015	6.73	5.01	3.89	3.78	3.14	3.06	2.73	2.61
2016	7.46	5.44	4.33	4.17	3.42	3.38	2.87	2.78
2017	6.63	5.28	4.047	3.95	3.42	3.37	2.84	2.75
2018	7.43	5.15	4.51	4.33	3.81	3.68	2.96	2.95
12 Years	6.35	5.11	3.81	3.48	3.12	3.06	2.75	**2.50**

COM: combination of SS2 and RBFN forecast

It can be observed that all the proposed linear models give better performance in comparison to D1 and D7 models. However, VSS2 gives smaller MAPE than VSS1 each year (Table 4). Same results are obtained on comparison of SS1 and SS2 models. These observations conclude that average forecast is better than the forecast obtained by the model which has lowest AIC and BIC values. The SS2 outperforms among all the other linear models. The accuracy of combination SS2 is better than D7, D1 and VSS2 by 51.96%, 40.33%, and 12.40% respectively. The non-linear RBFN model performs better than the SS2. Overall, best performance is obtained by the combination of linear and non-linear models. The accuracy of COM model is better than D7, D1, VSS2, SS2 and RBFN by 60%, 51%, 28.16%, 18.30% and 9.09% respectively. Tables 5 and 6 compare the relative change in mape value yearly with different window sizes of training data by SS*min* and SS*avg* models, respectively.

B. DAILY PERFORMANCE COMPARISON

The proposed model is more accessible as it utilizes only previous values of available load series only. Table 7 depicts the daily performance of the individual models for one of the most volatile weeks. Linear model (SS2) performs better for four days, non-linear model (RBFN) for one day, and proposed for two days. But for a complete week, the proposed model performance is best. Count of days having minimum MAPE among all six models carried out for the 12 years, and results are presented in Table 3. It shows a similar probabilistic count of days as in Table 7.

C. IMPLEMENTATION OF METHODOLOGY FOR MIBEL DEMAND

The forecast performance of the models using parameter estimation of ten window sizes of 21, 28, 35,…, 84 days is

TABLE 5. Yearly MAPE comparison of OEM demand by SS*min* model using different window sizes (WS).

WS	63	77	91	98	105	112	119	126	133	140	Avg
2007	3.18	3.13	3.05	3.09	3.16	3.13	3.14	3.06	3.09	3.07	3.00
2008	2.97	2.85	2.89	2.93	2.84	2.88	2.87	2.86	2.83	2.81	2.78
2009	2.96	2.87	2.78	2.81	2.80	2.82	2.77	2.80	2.80	2.84	2.75
2010	3.01	3.00	2.95	2.98	2.95	2.97	2.92	2.98	2.93	2.94	2.88
2011	3.02	2.94	2.96	2.95	2.94	3.0	2.97	2.94	2.92	2.92	2.88
2012	3.17	3.17	3.12	3.16	3.12	3.12	3.11	3.14	3.15	3.16	3.06
2013	3.30	3.18	3.11	3.09	3.12	3.16	3.18	3.19	3.06	3.03	3.07
2014	3.08	2.95	3.01	2.94	3.02	2.99	2.99	2.94	2.92	2.95	2.90
2015	3.35	3.25	3.20	3.27	3.22	3.23	3.18	3.16	3.16	3.14	3.11
2016	3.54	3.53	3.47	3.47	3.44	3.45	3.46	3.43	3.43	3.42	3.37
2017	3.49	3.40	3.42	3.41	3.38	3.42	3.34	3.39	3.39	3.42	3.39
2018	3.75	3.76	3.76	3.76	3.76	3.78	3.82	3.81	3.78	3.81	3.69
Avg	3.23	3.17	3.14	3.15	3.14	3.16	3.14	3.13	3.12	3.13	3.07

TABLE 6. Yearly MAPE comparision of OEM demand by SSavg model using different window sizes (WS).

WS	63	77	91	98	105	112	119	126	133	140	Avg
2007	3.08	3.06	3.02	2.97	3.16	3.13	3.14	2.99	3.02	2.98	2.99
2008	2.83	2.80	2.80	2.82	2.78	2.81	2.81	2.80	2.82	2.79	2.78
2009	3.10	2.79	2.76	2.73	2.75	2.75	2.74	2.77	2.76	2.75	2.75
2010	2.91	2.88	2.87	2.87	2.86	2.83	2.82	2.87	2.88	2.85	2.84
2011	3.44	2.88	2.87	2.86	2.88	2.88	2.88	2.89	2.85	2.85	2.85
2012	3.13	3.11	3.04	3.06	3.08	3.05	3.06	3.06	3.07	3.06	3.04
2013	3.14	3.12	3.05	3.05	3.06	3.08	3.05	3.04	3.04	3.01	3.05
2014	2.96	2.95	2.93	2.93	2.93	2.94	2.94	2.93	2.95	2.92	2.90
2015	3.11	3.09	3.08	3.08	3.09	3.08	3.08	3.09	3.12	3.06	3.07
2016	3.44	3.45	3.41	3.41	3.38	3.39	3.39	3.38	3.40	3.38	3.37
2017	3.39	3.36	3.36	3.37	3.35	3.36	3.32	3.38	3.39	3.37	3.33
2018	3.69	3.70	3.67	3.70	3.69	3.67	3.70	3.69	3.68	3.68	3.65
Avg	3.19	3.09	3.07	3.07	3.07	3.07	3.07	3.07	3.08	3.06	3.05

TABLE 7. Forecasting accuracy (MAPE) comparison of OEM demand for one week.

Models Year 2007	D7	D1	SS2	VSS2	RBFN	COM
Jan 3	6.187	2.819	**1.441**	3.517	2.576	2.006
Jan 4	4.635	1.948	**1.117**	1.175	3.091	2.181
Jan 5	2.411	2.259	**1.397**	1.891	2.325	1.619
Jan 6	3.283	8.37	**1.545**	1.739	4.260	2.547
Jan 7	2.289	2.465	4.124	4.021	3.278	**2.211**
Jan 8	17.235	12.437	9.525	13.04	**2.563**	6.112
Jan 9	6.965	2.124	2.139	2.576	2.436	**1.106**
Avg.	6.144	4.63	3.041	3.994	2.933	**2.540**

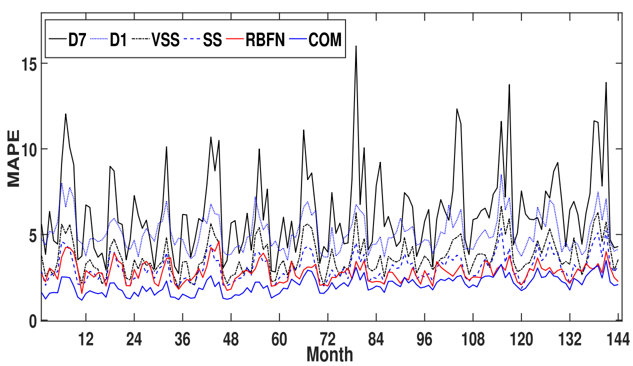

FIGURE 13. Comparison of monthly MAPE OEM for long period.

calculated for MIBEL demand of 2016. A total of (658, 800 SARIMA models are estimated using ten different window sizes for MIBEL one year forecast. Results of average forecast of individual models and com model are presented in Table 9. The following models have been compared for MIBEL demand forecast:

VSS3: average forecast all window size of VSS*min* model,
VSS4: average forecast all window size of VSS*ave* model,
SS3: average forecast all window size of SS*min* model,
SS4: average forecast all window size of SS*ave* model,
RBFN: average forecast all window size of RBF network model

COM2: combination of VSS4, SS4 and RBFN forecast. SARIMA models perform better then RBFN model for MIBEL demand. VSS*ave* model performance is best among the individual models. The accuracy of COM2 model is better than D7, D1, VSS4, SS4 and RBFN by 63%, 74%, 13 %, 19% and 30 % respectively. It is remarkably observed in Tables4, 8, 9 that the com model is ameliorating MAPE compared to single models for both markets.

TABLE 8. Monthly MAPE comparison of OEM demand for the year 2012.

Month	D7	D1	VSS2	ANN1	ANN2	SS2	RBFN	COM
Jan.	6.01	4.21	3.49	2.89	2.17	2.72	2.17	1.97
Feb.	4.21	3.49	2.61	2.76	2.05	2.48	2.28	1.98
March	5.78	4.99	3.46	2.80	2.53	3.08	3.44	2.67
April	3.96	4.82	2.89	2.49	2.31	3.02	2.59	2.19
May	5.12	5.61	3.72	2.67	2.17	3.08	2.40	2.07
June	11.01	6.59	5.48	3.48	2.49	4.13	2.94	2.59
July	8.2	6.914	5.61	3.83	2.78	4.27	3.11	2.87
Aug.	8.59	6.12	5.37	3.65	2.68	4.16	3.08	2.64
Sep.	6.38	6.40	4.22	3.03	2.72	3.03	3.26	2.59
Oct.	3.32	3.96	2.81	2.25	2.20	2.23	2.03	1.68
Nov.	4.30	3.70	2.01	2.23	2.11	2.09	2.09	1.73
Dec.	3.97	3.68	3.03	2.29	2.48	2.32	1.99	1.86
Avg.	5.92	5.04	3.73	2.91	2.38	3.04	2.62	2.24

TABLE 9. Forecasting accuracy (MAPE) comparison of MIBEL for 2016.

Models	D7	D1	SS3	SS4	VSS3	VSS4	RBFN	COM
Jan.	5.94	6.34	2.24	2.13	2.99	2.79	3.42	2.26
Feb.	4.05	6.89	2.45	2.27	1.97	1.9	2.01	1.65
March.	4.12	7.04	2.33	2.25	2.01	2.01	2.55	1.96
April.	5.39	5.84	2.28	2.15	1.96	1.99	2.66	1.68
May.	3.81	7.67	2.37	2.07	1.61	1.56	3.23	1.84
June.	3.86	6.43	2.34	2.06	2.01	2.28	2.8	1.84
July.	2.69	5.94	0.69	0.68	0.98	0.78	1.35	0.62
Aug.	6.24	5.35	1.74	1.71	1.34	1.27	1.94	1.17
Sept.	3.28	5.87	0.81	0.79	0.82	0.76	1.25	0.63
Oct.	2.63	7.32	2.12	2.02	1.61	1.39	2.01	1.56
Nov.	5.72	7.2	3.47	3.40	2.16	2.18	2.65	3.28
Dec.	7.1	7.61	2.93	2.88	3.30	3.30	4.14	4.06
2016	4.57	6.63	2.14	2.03	2.09	1.95	2.40	1.69

D. COMPARISON WITH MODELS IN LITERATURE

Table 8 depicts the monthly performance comparison for the year 2012 with the earlier models proposed for OEM [44]. Here ANN1 and ANN2 are the artificial neural network models forecasted demand without and with considering temperature data respectively. Here also, COM approach provides more accurate prediction with a less MAPE not only every month but also complete year 2012. According to Table 8, the proposed model achieved a significant

TABLE 10. Comparison of the proposed model with models in the literature.

Ref	Model	Market	EV	TD	PF	HF	MAPE (%)
[42]	ANN	GIPS	T, PD	1997-1999	2000	DA	1.75 to 3.04
[44]	ANN	OEM	T,PD	2007 to 2011	2012	DA	2.05 to 2.78
[48]	RBFN	PJM	WF,T,H	48 days	3 days	DA	DME 2.62 to 3.45:
[54]	Hybrid ARIMA-SVM	southern region, India	T,WF, TI	One year (2014)	2015	DA	5.16
[62]	Double SARIMA	MEK	PD	52 weeks	one month	hour ahead l step ahead	less than 5
[64]	Hybrid EEMD-ARIMA-GA-SVR	Taiwan	T	1965 to 2014	4 years	one-step-ahead	1.346 to 4.782
[69]	Two-level SAR	Brazilian		1990-1998	1999, 2000		1999: 3.08 2000: 3.56
[76]	DNN	KEM	T, H, SR, CC, WD	3 years	10 weeks	DA	2.19 to 2.27
[77]	Hybrid SVR, ARIMA	New England	T, DT, TI, PD	20 days	4 weeks	DA	1.53 to 2.51
[78]	LSTM	Victoria Australia	T, TI, PD	2015	2016	3DA	3.15
[79]	Hybrid stacked approach	Malaysia, New England	T, TI, DT, PD day, hour feature	2009-2011, 2003-2014	31 Dec 2009	24h, 48 h	2.69, 4.96
	SARIMA, RBFN, COM	OEM , MIBEL	PD	14 days to 140 days	13 years	DA	1.68 to 2.98

EV: Explanatory variables,TD: training data,PF:Period of forecast, HF:Horizon of Forecast, T:Temperature, WF:weather factor, H:humidity, SR:solar radiations, CC:cloud cover ,WD:wind speed, DA: day-ahead PD:previous lags value of demand, DT: Day Type, TI: Time Index ,GIPS:Greek intercontinental power system. PJM: Pennsylvania–New Jersey–Maryland

improvement in accuracy compared to single models D7, D1, ANN1, ANN2 by 62%,55%, 23% and 6% respectively. Table 10 indicates that long load forecast on a rolling basis using only a single load series is not predicted in the literature. In [69], Soares and Medeiros presented the results for only two years and MAPE changes from 3.08 to 3.56. The proposed model performance is consistent in the whole period of testing, with an absolute difference of MAPE of two consecutive years not exceed to 0.2. A large amount of the training data is required to estimate parameters by models ANN, DNN, and LSTM in Table 10, but in the proposed model, only data from some weeks is sufficient. In Summary, the following observations are made:

- VSS, SS, and RBFN models perform better than benchmark models.
- For OEM, the performance of SS model is better than VSS model. RBFN models is best among all the presented individual models. For MIBEL, the performance of VSS models is better than SS and RBFN models.
- For all the parameter estimation window sizes, the SSave performs better than the SSmin for both markets. The forecast accuracy of combination forecasts for all window size by SSave model (Table 6) and SSmin model (Table 5) are closely matched, although SSave is better linear model for OEM.
- The model performance varies with the window size of parameter estimation data sets. Linear model (SARIMA) forecast gets improved on increasing window sizes but for non-linear it is reverse. Forecast performance of RBFN is best when training data of 28 days is used.
- Monthly (Figure 13, Table 9) and the yearly (Table 4) MAPE are smaller for the forecast by using COM model. It shows that combination of models produced better forecast than the individual model [75].
- It can be observed that the probability of getting better forecast with the COM model is highest than the ANN1 and ANN2 models used in literature, Table 8.
- From the study of the daily MAPE of all models, the COM model reasonably reduces MAPE when the performance of individual models is comparable.

The empirical results have verified the feasibility of the proposed method. It combines the merits of every single models to overcome the limitations of low-precision prediction of single models. Therefore, it can be deduced that the proposed combination technique is perfectly tailored for STLF.

VI. CONCLUSION

In this paper, day-ahead load forecasting methodology for building SARIMA and RBF models has been discussed. The models have been developed using the demand series in two frameworks: (i) single series, (ii) variable segmented series. The metrics for accuracy assessment i.e., MAPE of all the models has been compared on daily, monthly, and yearly basis for two markets real data. The parameters and weights of all the models for each demand series have been estimated for different window size of estimation data. The performance of linear models has been compared on the basis of two approaches:(i) by the model having lowest AIC and BIC values, and (ii) by averaging forecast of all the SARIMA models.Results showed that all models outperformed compared to the benchmark models D7 and D1 in both markets. Forecast by SARIMA models (SSave, VSSave) gives smaller MAPE than (SSmin, VSSmin) respectively for both markets. Among all individual models, RBFN performance is good for OEM demand and linear models using VSS framework perform better for MIBEL demand. Overall, proposed model, COM produces best forecast than all other forecast for both real demand data. For longer durations, the performance of the model has been further validated and it is proved that simple average is a prudent strategy for combining the demand forecasts of multiple models. For future directions, the proposed COM model can be designed with auto-selection of best training data window based on volatility associated with the historical demand series.

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