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RESEARCH ARTICLE

Extended CSF-CoCoSo Method: A Novel Approach for Optimizing Logistics in the Oil and Gas Supply Chain

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ABSTRACT In this research, we introduce a significant advancement in decision-making methodologies by proposing the Combine Compromise Solution Approach. This innovative method addresses the complexities inherent in multi-criteria decision-making scenarios, particularly in optimizing logistics within the oil and gas supply chain. Through our exploration of circular spherical fuzzy sets, we investigate various algebraic operations without detailing the research methodology. Our primary contribution lies in the practical application and effectiveness of the Combine Compromise Solution Approach, exemplified through a case study on logistics optimization. By presenting insights gleaned from this approach and conducting a comparative analysis against alternative methods, we demonstrate its utility and potential impact in real-world contexts. This research not only offers immediate practical solutions but also paves the way for future investigations into the broader applications of circular spherical fuzzy sets in decision-making processes, thus advancing the field significantly.

INDEX TERMS Circular spherical fuzzy sets, CoCoSo method, decision-making, oil and gas supply chain.

I. INTRODUCTION

Decision-making (DM) is a complicated procedure that involves choosing among several possibilities according to a range of standards and criteria. To address the ambiguity and inaccuracy inherent in the DM process, fuzzy set theory is widely used [1]. Fuzzy set theory is frequently utilised in more adaptable and durable decision-making frameworks because it permits decision-makers to handle both qualitative and quantitative data. Fuzzy analysis, in particular, offers a methodical way to compare solutions based on several aspects while accounting for both objective and subjective data [2] in group decision making. Decision making has been the focus of countless research papers in the past. Examples include evaluating the AlzheimerŠs disease [3], selection of material for the nitrogen tank [4], healthcare and problems in the medical department [5], selection of electric vehicles [6] and supplier selection [7]. This article's goal is to expand on the fuzzy set theory by introducing a novel idea called the circular spherical fuzzy set and exhibiting its usefulness with a case study on risk expansion from manufacturers. In certain scenarios involving a group decision-making process, decision-makers may choose to deliver their linguistic assessments of alternatives using imbalanced linguistic word sets that are not evenly and symmetrically distributed [8]. The circular spherical set theory and its aggregations operator are incorporated into the suggested method to offer a thorough and efficient approach for decision making.

In order to handle unclear and inadequate data, Zadeh created the Fuzzy set (FS) theory [9]. It extends the notion of the membership value of a FS to the notion of the

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characteristic function, which establishes whether an element refers to an universal set. In the initial adaption of TOPSIS technique in an uncertain environment, fuzzy triangular numbers were employed to express Linguistic factors based on their ratings and relevance [10]. In FS theory, a variety of Decision making (DM) techniques are employed. As a result, determining the chance of future failures may be done simply and effectively [11]. One possible use of a TOPSIS fuzzy technique for DM process calculation is [12]. The study integrated the benefits of fuzzy TOPSIS and the analytic hierarchy process (AHP) to assess the significance of risk factors. A new ELECTRE method is also given for decision making for the purpose of vehicle project in Istanbul [13]. The grey theory and flexible evidence-based reasoning were used to improve the efficiency of traditional failure modes and effectiveness analysis (FMEA) methodologies [14]. A hybrid Multi Criteria Decision Making (MCDM) method integrating the modified fuzzy MAIRCA and the fuzzy analytical hierarchy process was proposed by Boral [15]. For decision making within the sort of FS known as a picture FS, a CODAS approach is applied [16]. Akram et al. give the VIKOR method for group DM [17]. There are some also method which is used for group decision making in hesitant fuzzy environment [18]. Threshold-based value-driven technique to solve consensus reaching in multicriteria group decision making problems [19]. Hesitant fuzzy set with linguistic term is also used for group decision making technique [18].

As a result, several academics wonder why fuzzy sets do not include the non-membership component. Atanassov created intuitionistic FS (IFS), which combines membership and non-membership, to put the following idea into practice [20]. Then, as time goes on, more FS kinds are added to help with DM problems. A subset of FS theory known as interval-valued fuzzy sets (IVFS) demonstrates the uncertainty in the assigned membership degrees by using an illustration between intervals to represent the belonging degree [21]. Hesitant FSs (HFSs) are an evaluation of FS with various memberships, as introduced by Torra [22]. Pythagorean fuzzy sets (PyFSs), along with an extended area for membership and non-belonging levels, were introduced by Yager [23]. Q-rung orthopair fuzzy sets (q-ROFSs) are the generalisation of PyFSs and IFSs, introduced by Yager [24]. One kind of FS that is explained in n-dimensional space on a sphere is a spherical FS (SFS) [25]. FS theory is expanding annually, as shown in Figure 1.

By including this component within the SFS methodology, Gndogdu generated the interval-valued SFS (IV-SFS) model [26]. We see that the individual aspects of the evaluations pertinent to SFSs are represented by correctly positioned rectangles, rather than by solitary pairs of integers. We see that the individual aspects of the evaluations pertinent to [27], Robust online tensor completion for IoT streaming data recovery [28], Integrated modeling for retired mechanical product genes in remanufacturing [29], Bus-trajectory-based street-centric routing for message delivery in

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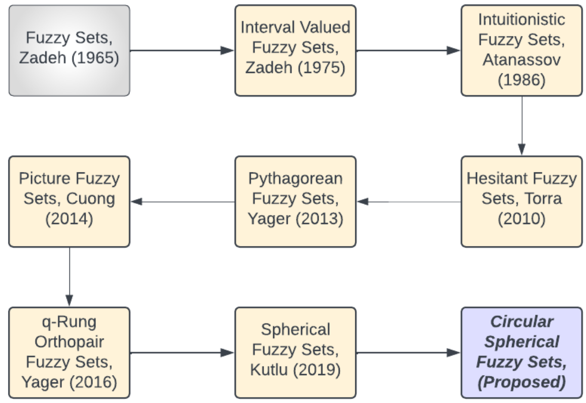

FIGURE 1. **Developmentation in fuzzy set theory.**

urban vehicular ad hoc networks [30] and multiagent-based supply chain systems [31].

Understanding private car aggregation effect [32], A deep neural network-based assistive decision method [33], Picture fuzzy decision-making theories and methodologies [34], System identification of nonlinear state-space models [35] and compact constraint incremental method for random weight networks [36].

Motivation

The two exciting areas among works of literature that provided motivation for the study suggested in this article are the CoCoSo [37] approach to MCDM and C-SFSs. Their combination yields an innovative approach known as circular spherical fuzzy, which allows us to perform pertinent assessments for DM. The new model permits independent looseness in the creation of MD, indeterminacy, and NMD, while IV-SFS allows for a certain amount of give around the orthopair created by the membership degree (MD), indeterminacy, and non-membership degree (NMD). Slackness is provided by the radius attached to the model, which goes by the designation C-SFS. Thus, the primary motivations behind this paper are as follows:

- To examine the ratings of possible failures and alarm symbols, C-SFS provides a broad range of membership, non-membership, and indeterminacy degrees with radius in the evaluation data depiction.
- To handle more complicated situations, C-SFSs incorporate both the radius of the supplied value and the capabilities of the SFS.
- When managing programs with multiple evaluation values of the data, the new C-SF approach allows for the creation of conclusions that are more accurate and dependable.
- The necessity for increased precision and stability in weight computations is the driving force behind the introduction of this novel technique. We expect a more complete depiction of decision considerations

by incorporating regret-based components into the conventional framework, which will ultimately produce more effective and refined outcomes.

We are unable to locate the radius of a circle in SFS if we would like to inspect it. Authors are therefore delighted to be able to fulfill this requirement. To address this kind of problem, we have to use C-SFS. This is a shift to all prediction algorithms that can handle membership, non-membership, and indeterminacy in any form, including circular radius. They are useful when we need to calculate the radius of SFS. It raises the question: why is the radius of any set calculated? The reason for this is that, once the radius has been determined, we can use it to determine where the oversetting values fall within the radius, which helps us see our findings.

In this study, we put forth the concepts of C-SFS. They are the extension of C-IFS [38] as well as C-PyFs [39]. The geometrical representations of SFS and C-SFS are shown in Figures 2 and 3, respectively.

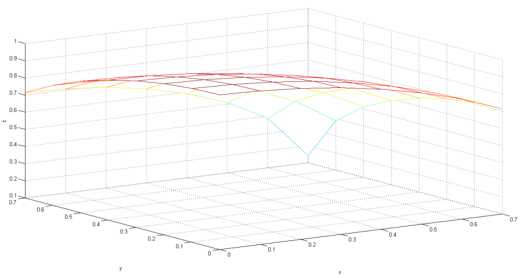

FIGURE 2. Representation in geometry of SFS.

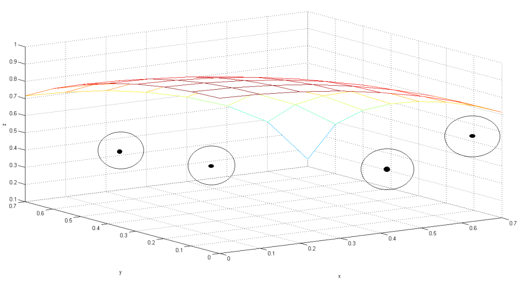

FIGURE 3. Representation in geometry of C-SFS.

Findings

By contrasting it with the current techniques, the research demonstrates the model's utility and feasibility. Our computational presentations show that the developed model is superior relevant to the undertaking's actual status and has a higher reference value. Our results demonstrate the ability of C-SFSs with CoCoSo of accurately interpreting the divergent viewpoints among diverse decision-makers as well as the decision-making problems.

Practical Application

The supply chain operations of the oil and gas industry, which is a vital component of the world's energy production, face numerous challenges. Logistics optimisation stands out as a crucial area of focus among these difficulties. The smooth flow of goods and materials throughout the oil and gas supply chain is crucial for efficiency, economy, and sustainability. It is not just a logistical consideration. Our case study's actual applicability goes beyond a theoretical comprehension of logistics optimisation. It summarises the tales of businesses that have mastered the complex web of obstacles and used creative thinking to improve operations and financial results. Our goal in telling these stories is to extract priceless lessons and best practises that can direct researchers and industry practitioners alike towards a more effective and sustainable future.

Originality

We use the CoCoSo method to develop a distinct approach in the flexible C-SF setting. It assesses risk factors and professional judgments about C-SFSs. We offer evidence that the recommended approach has markedly enhanced the reliability of the results and the integrity of the data used in the expert assessment.

Need

Moreover, if you want to examine a circle in SFS, you cannot find its radius. Because of this, writers are delighted to be able to fulfill the need. Therefore, to solve this kind of issue, we need to use C-SFS. Although we see the circular Pythagorean fuzzy set in this set, the degree of indeterminacy is missing, so we cannot find out the degree of indeterminacy of our set. This represents a change from all earlier algorithms that were capable of handling all membership, non-membership, and indeterminacy scenarios, including circular radii. They are useful in determining the radius of SFS.

The CoCoSo method becomes a vital instrument in the analysis of complicated scenarios in the pursuit of a thorough and methodical approach to decision-making. Our application of the CoCoSo approach is especially relevant when discussing the concept of C-SFS, where a systematic review becomes crucial. This approach is quite helpful when criteria are interdependent and the case study does not clearly specify the weight assignment. We carefully determine the weights of each criterion using CoCoSo, taking into account how dependent each is on the alternatives. This ensures that decision-making is educated and nuanced.

The circular spherical fuzzy set notions are expanded upon in this paper. C-SF data should be aggregated and the intersection and union of C-SFSs defined. Moreover, a CoCoSo approach for MCDM problems is provided to handle data of the circular spherical fuzzy set type. Applying the suggested technique to evaluate its dependability and efficiency involved analyzing the Performance Analysis of Optimising Logistics in the Oil and Gas Supply Chain. This work aims to propose the following concepts: (a) circular spherical fuzzy sets (b) A comparison section for C-SFS is also included; (c) To manage C-SFS, the CoCoSo technique is also provided: (d) To demonstrate how the recommended

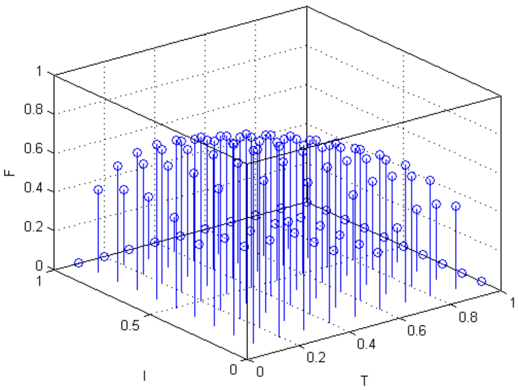

FIGURE 4. Spherical fuzzy set 3-D.

method is successful and dependable, a comparative section is provided.

II. PRELIMINARIES

The basic ideas pertaining to picture fuzzy set (PFS), spherical fuzzy set, C-SFS, and Disc spherical fuzzy set (D-SFS), as well as their operators and operations, are succinctly summarized in this section. In addition, we discuss some other well-known ideas that are relevant to the analysis that follows.

Definition 1 [40]: Let U represent the universe's set. Then

$$\beta = \{\langle \hat{u}, \mu_\beta(\hat{u}), \eta_\beta(\hat{u}), \nu_\beta(\hat{u}) | \hat{u} \in U\rangle\} \tag{1}$$

is reportedly a PFS, where $\mu_\beta(\hat{u}) : U \rightarrow [0, 1], \eta_\beta(\hat{u}) : U \rightarrow [0, 1]$ *and* $\nu_\beta(\hat{u}) : U \rightarrow [0, 1]$ *are called to be degree belonging of* $\hat{u}$ *in U, neutral belonging of* $\hat{u}$ *in U and non belonging degree of* $\hat{u}$ *in U correspondingly. Additionally,* μ_β, η_β *and* ν_β *meet the requirements listed below:*

$$(\forall\, \hat{u} \in U)(0 \leq (\mu_\beta(\hat{u})) + (\eta_\beta(\hat{u})) + (\nu_\beta(\hat{u})) \leq 1) \tag{2}$$

Definition 2 [41]: Let U represent the universe's set. Then

$$\beta = \{\langle \hat{u}, \mu_\beta(\hat{u}), \eta_\beta(\hat{u}), \nu_\beta(\hat{u}) | \hat{u} \in U\rangle\} \tag{3}$$

is reportedly a SFS, where $\mu_\beta(\hat{u}) : U \rightarrow [0, 1], \eta_\beta(\hat{u}) : U \rightarrow [0, 1]$ *and* $\nu_\beta(\hat{u}) : U \rightarrow [0, 1]$ *are called to be degree belonging of* $\hat{u}$ *in U, neutral belonging of* $\hat{u}$ *in U and non belonging degree of* $\hat{u}$ *in U correspondingly. Additionally,* μ_β, η_β *and* ν_β *meet the requirements listed below:*

$$(\forall\, \hat{u} \in U)(0 \leq (\mu_\beta(\hat{u}))^2 + (\eta_\beta(\hat{u}))^2 + (\nu_\beta(\hat{u}))^2 \leq 1) \tag{4}$$

For SFS $\{\langle \hat{u}, \mu_\beta(\hat{u}), \eta_\beta(\hat{u}), \nu_\beta(\hat{u}) | \hat{u} \in U\rangle\}$*, which is a triple component.*

$$\langle \mu_\beta(\hat{u}), \eta_\beta(\hat{u}), \nu_\beta(\hat{u})\rangle$$

are regarded as SFN, and $e = \langle \mu_e, \eta_e, \nu_e\rangle$ *denotes each spherical number, where* $\mu_e, \eta_e, \nu_e \in [0, 1]$*, under the condition that* $0 \leq \mu_e^2 + \eta_e^2 + \nu_e^2 \leq 1$*. Figure 5 shows the graphical representation of SFS.*

Definition 3 [42]: Assume that β *is a subset of* Ξ*, which is a fixed universe. This is*

$$\beta = \{\langle \hat{\alpha}, \mu_\beta(\hat{\alpha}), \eta_\beta(\hat{\alpha}), \nu_\beta(\hat{\alpha}); \hat{r} | \hat{\alpha} \in \Xi\rangle\} \tag{5}$$

is said to be a C-SFS, where $\mu_\beta(\hat{\alpha}) : \Xi \rightarrow [0, 1]$*,* $\eta_\beta(\hat{\alpha}) : \Xi \rightarrow [0, 1]$*,* $\nu_\beta(\hat{\alpha}) : \Xi \rightarrow [0, 1]$ *are represented the degrees of belonging of* $\hat{\alpha}$ *in* Ξ*, neutral-belonging degree of* $\hat{\alpha}$ *in* Ξ *and non belonging degree of* $\hat{\alpha}$ *in* Ξ *correspondingly. Furthermore* μ_β, η_β *and* ν_β *satisfy the following conditions:*

$$(\forall\, \hat{\alpha} \in \Xi)(0 \leq (\mu_\beta(\hat{\alpha}))^2 + (\eta_\beta(\hat{\alpha}))^2 + (\nu_\beta(\hat{\alpha}))^2 \leq 1) \tag{6}$$

The surrounding circle's radius is $\hat{r}$ *the point* $(\mu_\beta(\hat{\alpha}), \eta_\beta(\hat{\alpha}), \nu_\beta(\hat{\alpha}))$ *on the sphere. Circle here stands for the belonging, non belonging, and the indeterminacy degree of* $\hat{\alpha} \in \Xi$*.*

Unlike ordinary SFSs, which represent each element as a point in the spherical fuzzy interpretation triangle, this C-SFS represents each element as a circle with a radius $\hat{r}$ *and a centre* $(\mu_\beta(\hat{\alpha}), \eta_\beta(\hat{\alpha}), \nu_\beta(\hat{\alpha}))$*.*

Since the form is a part of every standard SFS, the new types of sets are an improvement above the norm.

$$\beta = \beta_o = \{\langle \hat{\alpha}, \mu_\beta(\hat{\alpha}), \eta_\beta(\hat{\alpha}), \nu_\beta(\hat{\alpha}); 0\rangle\}$$

Nevertheless, a typical SFS cannot coexist with the C-SFS with $\hat{r} > 0$*.*

The evaluation for $\hat{\alpha}, a = \langle \hat{\alpha}, \mu_a(\hat{\alpha}), \eta_a(\hat{\alpha}), \nu_a(\hat{\alpha}); \hat{r}\rangle$*, symbolizes a circle with radius* $\hat{r}$ *at center* $(\mu_a(\hat{\alpha}), \eta_a(\hat{\alpha}), \nu_a(\hat{\alpha}))$ *called C-SFS. In an abstract setting, a C-SFV is written as* $(\mu_a, \eta_a, \nu_a; \hat{r})$ *rather than the* $(\hat{\alpha}, \mu_a(\hat{\alpha}), \eta_a(\hat{\alpha}), \nu_a(\hat{\alpha}); \hat{r})$ *expression associated with an alternative* $\hat{\alpha}$*.*

Definition 4 [43]: Assume that β *is a subset of a fixed universe* Ξ*. This is*

$$\beta = \{\langle \hat{\alpha}, \mu_\beta(\hat{\alpha}), \eta_\beta(\hat{\alpha}), \nu_\beta(\hat{\alpha}); \hat{r}(\hat{\alpha}) | \hat{\alpha} \in \Xi\rangle\} \tag{7}$$

is said to be a D-SFS, where $\mu_\beta(\hat{\alpha}) : \Xi \rightarrow [0, 1]$*,* $\eta_\beta(\hat{\alpha}) : \Xi \rightarrow [0, 1]$*,* $\nu_\beta(\hat{\alpha}) : \Xi \rightarrow [0, 1]$ *are apparently degrees of belonging of* $\hat{r}(\hat{\alpha})$ *in* Ξ*, neutral-belonging degree of* $\hat{r}(\hat{\alpha})$ *in* Ξ *and non belonging degree of* $\hat{r}(\hat{\alpha})$ *in* Ξ *correspondingly. Also* μ_β, η_β *and* ν_β *fulfil the requirements listed below:*

$$(\forall\, \hat{\alpha} \in \Xi)(0 \leq (\mu_\beta(\hat{\alpha}))^2 + (\eta_\beta(\hat{\alpha}))^2 + (\nu_\beta(\hat{\alpha}))^2 \leq 1) \tag{8}$$

The surrounding circle's radius is $\hat{r}$ *the point* $(\mu_\beta(\hat{\alpha}), \eta_\beta(\hat{\alpha}), \nu_\beta(\hat{\alpha}))$ *on the sphere. Circle here stands for the belonging, non belonging, and the indeterminacy degree of* $\hat{\alpha} \in \Xi$*.*

Instead of a point in the spherical fuzzy interpretation triangle as in normal SFSs, each element in this D-SFS is represented by a circle with a radius $\hat{r}(\hat{\alpha})$ *and a centre* $(\mu_\beta(\hat{\alpha}), \eta_\beta(\hat{\alpha}), \nu_\beta(\hat{\alpha}))$ *and a circle with a radius* $\hat{r}(\hat{\alpha})$*.*

C-SFSs are D-SFSs with all $\alpha, \hat{\alpha} \in \Xi$. The assessment for α in a D-SFS is $a = (\alpha, \mu_a(\alpha), \eta_a(\alpha), \nu_a(\alpha); \hat{r}(\alpha))$. It now resembles a circle with three points at its center: $(\mu_a(\alpha), \eta_a(\alpha), \nu_a(\alpha))$. The radius of the circle, $\hat{r}(\alpha)$, may vary depending on the choice made. As a result, this assessment is also a spherical fuzzy value. We maintain that these C-SFVs, unlike C-SFSs, may have varied radii when the alternative changes. In any case, C-SFV-dependent activities will be able to work with each of the D-SFSs and C-SFSs. When dealing with the D-SFS issue alone in later parts, we will apply this capability.

A. RULES OF COMPARISON FOR C-SFSS

The functions that are presented in this section and are essential to the ranking of C-SFSs include the following:

Definition 5: Let $\underset{\sim}{D} = \langle \mu_{\underset{\sim}{D}}, \eta_{\underset{\sim}{D}}, \nu_{\underset{\sim}{D}}; \hat{r}_{\underset{\sim}{D}} \rangle$ *exist any C-SFSs. After that*

1) *Score function:-* $\S(\underset{\sim}{D}) = \frac{1}{4}(\mu_{\underset{\sim}{D}} - \eta_{\underset{\sim}{D}} - \nu_{\underset{\sim}{D}} + \sqrt{2\hat{r}}(2p-1))$ *where* $\S(\underset{\sim}{D}) \in [-1, 1]$ *and p can take any value in the interval [0, 1].*
2) *Accuracy function:-* $\hat{D}(\underset{\sim}{D}) = \mu^2_{\underset{\sim}{D}} + \eta^2_{\underset{\sim}{D}} + \nu^2_{\underset{\sim}{D}}$ *where* $\hat{D}(\underset{\sim}{D}) \in [0, 1]$

Taking into account the C-SFNs $\underset{\sim}{Q}$ *and* $\underset{\sim}{G}$ *with these two definitions.*

- $\underset{\sim}{Q}$ *is greater to* $\underset{\sim}{G}$ *if* $\S(\underset{\sim}{Q}) > \S(\underset{\sim}{G})$
- $\underset{\sim}{Q}$ *is less to* $\underset{\sim}{G}$ *if* $\S(\underset{\sim}{Q}) < \S(\underset{\sim}{G})$

If $\S(\underset{\sim}{Q}) = \S(\underset{\sim}{G})$ *for two D-SFNs. After that*

- $\underset{\sim}{Q}$ *is greater to* $\underset{\sim}{G}$ *if* $\hat{D}(\underset{\sim}{Q}) > \hat{D}(\underset{\sim}{G})$
- $\underset{\sim}{Q}$ *is less to* $\underset{\sim}{G}$ *if* $\hat{D}(\underset{\sim}{Q}) < \hat{D}(\underset{\sim}{G})$
- $\underset{\sim}{Q}$ *is equivalent to* $\underset{\sim}{G}$ *If* $\hat{D}(\underset{\sim}{Q}) = \hat{D}(\underset{\sim}{G})$

III. CIRCULAR SPHERICAL FUZZY MCDM METHOD

The set of m alternatives is denoted by $L = \Re_1, \Re_2, \ldots, \Re_m$. The set of n attributes is represented by $C = \psi_1, \psi_2, \ldots, \psi_n$. The weight vector $\partial_j = \partial_1, \partial_2, \ldots, \partial_n$ is assigned to the set of attributes, where $0 \leq \partial_j \leq 1$ and $\sum_{j=1}^{n} \partial_j = 1$. Assume that the circular spherical fuzzy (C-SF) matrix $S^k_{[m\times n]}$ represents the evaluation of alternative L_i in relation to criterion C_j.

$$S^k_{[m\times n]} = \begin{bmatrix} & \psi_1 & \psi_2 & .. & \psi_n \\ \Re_1 & ((T^k_{\text{Ł}_{11}}, I^k_{\text{Ł}_{11}}, F^k_{\text{Ł}_{11}}); \hat{r}_{\text{Ł}_{11}}) & .. & .. & ((T^k_{\text{Ł}_{1n}}, I^k_{\text{Ł}_{1n}}, F^k_{\text{Ł}_{1n}}); \hat{r}_{\text{Ł}_{1n}}) \\ \Re_2 & ((T^k_{\text{Ł}_{21}}, I^k_{\text{Ł}_{21}}, F^k_{\text{Ł}_{21}}); \hat{r}_{\text{Ł}_{21}}) & .. & .. & ((T^k_{\text{Ł}_{2n}}, I^k_{\text{Ł}_{2n}}, F^k_{\text{Ł}_{2n}}); \hat{r}_{\text{Ł}_{2n}}) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \Re_m & ((T^k_{\text{Ł}_{m1}}, I^k_{\text{Ł}_{m1}}, F^k_{\text{Ł}_{m1}}); \hat{r}_{\text{Ł}_{m1}}) & .. & .. & ((T^k_{\text{Ł}_{mn}}, I^k_{\text{Ł}_{mn}}, F^k_{\text{Ł}_{mn}}); \hat{r}_{\text{Ł}_{mn}}) \end{bmatrix}$$

A. METHOD FOR CALCULATING COMBINED WEIGHT

Assume that, based on the *jth* attribute, $s^k_{[ij]}(i = 1, 2, \ldots, m; j = 1, 2 \ldots, n)$ represents the C-SF preference value of the *ith* alternative. The *jth* criterion's subjective weight, objective weight, and combined weight are denoted by the symbols w_j, ω_j, and ϖ_j, respectively.

The steps for calculating C-SF combined weights are set down in the subsequent.

Step 1. The matrix of decisions S will be normalised based on the benefit and cost criteria provided.

$$S^k_{[m\times n]} = \begin{cases} ((T^k_{\text{Ł}_{ij}}, I^k_{\text{Ł}_{ij}}, F^k_{\text{Ł}_{ij}}); \hat{r}_{\text{Ł}_{ij}}) & \text{for benefit criteria} \\ ((F^k_{\text{Ł}_{ij}}, I^k_{\text{Ł}_{ij}}, T^k_{\text{Ł}_{ij}}); \hat{r}_{\text{Ł}_{ij}}) & \text{for cost criteria} \end{cases}$$

Step 2. Calculate the function for scoring $\S = (\S_{ij})_{m\times n}$ by using the definition of 5.

Step 3. Ascertain the Renyi entropy E_j of score function § by equation (9).

$$E_j = \frac{1}{1-\alpha} log \left(\sum_{i=1}^{m} \S^{\alpha}_{ij} \right) \tag{9}$$

Step 4. Find the objective weight ω_j by using equation (10).

$$\omega_j = \frac{1 - E_j}{\sum_{j=1}^{n}(1 - E_j)} \tag{10}$$

Step 5. Calculate ϖ_j, the combined weight, using equation (11).

$$\varpi_j = \frac{w_j \omega_j}{\sum_{j=1}^{n} w_j \omega_j} \tag{11}$$

B. CIRCULAR SPHERICAL FUZZY MCDM BASED ON COCOSO

In general, the C-SF-CoCoSo approach consists of several essential stages meant to accomplish its objectives. These steps function as a well-organized framework that leads practitioners through a structured and thorough process. By using this approach, people can handle the complexity of their work with a clear and efficient plan.

Step 1. Using Table 1, obtain the linguistic matrix.

TABLE 1. Terms from linguistics to evaluate alternatives.

Linguistic term	Shorthand Used	C-SFN
Extremely low	EL	(0.0, 0.0 ,0.1)
Very low	VL	(0.1, 0.3 ,0.6)
Low	L	(0.2, 0.3 ,0.5)
Middle low	ML	(0.3, 0.3 ,0.3)
Below middle	BM	(0.4, 0.2 ,0.3)
Middle	M	(0.5, 0.2 ,0.2)
Above high	AH	(0.6, 0.2 ,0.2)
Middle high	MH	(0.7, 0.1 ,0.1)
High	H	(0.8, 0.1 ,0.1)
Very high	VH	(0.9, 0.0 ,0.1)
Extremely high	EH	(1,0.0,0.0)

Step 2. Make the C-SF matrix out of the linguistic matrix.
Step 3. Normalized the C-SF matrix into a standard C-SF matrix by using step 1 in III-A.
Step 4. Utilizing the definition of 5, calculate the score function.
Step 5. Using (11), find the combined weight ϖ.
Step 6. Determine the sum of the weighted comparability sequence, or S_i, for each option.

$$S_i = \sum_{j=1}^{n} w_j \S_{ij} \tag{12}$$

Step 7. Determine the total power weight (P_i) of the comparison sequences for each option:

$$P_i = \sum_{j=1}^{n} (\S_{ij})^{w_j} \tag{13}$$

Step 8. Apply the aggregation techniques by using equations (14) to (16)

$$k_{ia} = \frac{P_i + S_i}{\sum_{i=1}^{m}(P_i + S_i)} \tag{14}$$

$$k_{ib} = \frac{S_i}{\min\limits_i S_i} + \frac{P_i}{\min\limits_i P_i} \tag{15}$$

$$k_{ic} = \frac{\lambda S_i + (1-\lambda)P_i}{\lambda \max\limits_i S_i + (1-\lambda)\max\limits_i P_i}, 0 \leq \lambda \leq 1 \tag{16}$$

Step 9. Enter the assessment value k_i and use equation (17) to calculate it.

$$k_i = (k_{ia}k_{ib}k_{ic})^{\frac{1}{3}} + \frac{k_{ia} + k_{ib} + k_{ic}}{3} \tag{17}$$

Step 10. Sort alternatives according to the assessed values $k_i(i = 1, 2, \ldots, m)$ decreasing values.

IV. CASE STUDY

Optimizing Logistics in the Oil and Gas Supply Chain

The global economy is based on the oil and gas sector, which supplies vital energy resources to many other industries. This sector has a very complex supply chain that includes distribution, refining, extraction, and exploration. To keep a competitive edge in the market, raw materials and completed goods must be moved with efficiency.

They have five attributes $U = \{\Re_1, \Re_2, \Re_3, \Re_4, \Re_5\}$ These five attributes are: $\Re_1$ = Transportation Infrastructure (TIS), $\Re_2$ = Inventory Management (IM), $\Re_3$ = Technology Integration (TI), $\Re_4$ = Regulatory Compliance (RC) and $\Re_5$ = Supplier and Vendor Relationships (SVR). Assume the selection process takes into account the following four criteria:ψ_1 (Cost Efficiency (CE)), ψ_2 (Risk Mitigation (RM)), ψ_3 (Operational Performance (OP)), and ψ_4 (Sustainability (SU)). Moreover, the equivalent weight information $w = (0.40, 0.10, 0.20, 0.30)$ is determined by the knowledge and experience of financial experts.

A. TRANSPORTATION INFRASTRUCTURE $\Re_1$

The current oil and gas supply chain transportation infrastructure consists of a system of pipelines, seaports, and land transportation. While marine shipping manages the movement of refined products to various regions, pipelines are essential for moving crude oil and natural gas from extraction points to refineries. Road transportation is frequently used for last-mile deliveries and shorter distances. Examine the shipping routes' and pipelines' ability to accommodate present and future volumes. Consider possible disruptions and maintenance concerns when assessing how dependable a particular mode of transportation is. Analyze the cost per unit and delivery time efficiency of various transportation routes.

B. INVENTORY MANAGEMENT $\Re_2$

Inventory control in the oil and gas supply chain is striking a balance between eliminating excess inventory and the requirement to maintain sufficient stock levels. Considering the erratic nature of demand and the strategic value of maintaining adequate reserves during volatile market periods, this is imperative. Examine how well inventory control procedures maintain a steady supply to satisfy demand and avoid stockouts. Evaluate the efficiency of inventory management in minimizing surplus inventory to maximize working capital. Analyze how accurate demand projections are and how they affect stock levels.

C. TECHNOLOGY INTEGRATION $\Re_3$

Technology integration can have a major positive impact on the oil and gas supply chain. This covers the utilization of advanced tracking systems for improved visibility, data analytics for decision-making, and Internet of Things (IoT) devices for real-time monitoring. Examine the ways in which technology improves supply chain visibility for product movement. Evaluate a product's capacity to be tracked back to its manufacturer, guaranteeing legal compliance and quality assurance. Consider how real-time information and data analytics help make better decisions.

D. REGULATORY COMPLIANCE $\Re_4$

Numerous rules, ranging from environmental compliance to safety standards, apply to the oil and gas business. Respecting these rules is not only the law, but also necessary to keep the public's confidence and reduce the possibility of incurring expensive fines. Evaluate the possible hazards and repercussions of breaking national and international laws. Analyze how operational procedures are affected by regulatory compliance and determine whether it results in any additional expenses or bottlenecks. Analyze how well the supply chain can adjust to changing legal requirements.

E. SUPPLIER AND VENDOR RELATIONSHIPS $\Re_5$

In the oil and gas supply chain, cooperation and solid rapport with vendors and suppliers are essential. Appropriate and prompt access to services and raw materials is necessary to fulfill client demand and maintain production schedules. Evaluate the efficiency of the channels used for vendor and supplier communication. Analyze joint efforts to minimize interruptions and optimize procedures. Examine methods for locating and reducing supplier and vendor-related risks. This thorough background prepares the ground for a thorough examination of the oil and gas supply chain, with an emphasis on these characteristics to pinpoint areas in need of enhancement and optimization.

Likewise, specifics of the criteria,s that rely on these attributes.

F. COST EFFICIENCY (CE) ψ_1

Analyze the supply chain's total cost-effectiveness while taking into account expenditures for technology purchases, inventory keeping, transportation, and compliance.

G. RISK MITIGATION (RM) ψ_2

Examine the methods used to detect, track, and reduce risks in the supply chain, such as operational, regulatory, and geopolitical threats.

H. OPERATIONAL PERFORMANCE(OP) ψ_3

Evaluate the efficacy and efficiency of daily operations, taking into account lead times, order fulfillment percentages, and inventory prediction accuracy.

I. SUSTAINABILITY (SU) ψ_4

Evaluate how supply chain operations affect the environment, taking into account compliance with environmental regulations, initiatives to reduce carbon footprints, and sustainability goals.

Afterwards, the presented C-SF-CoCoSo algorithm ($\lambda = 0.5, \alpha = 5$) is used to select the best alternative for our decision making. Figure 5 shows the framework of C-SF decision making.

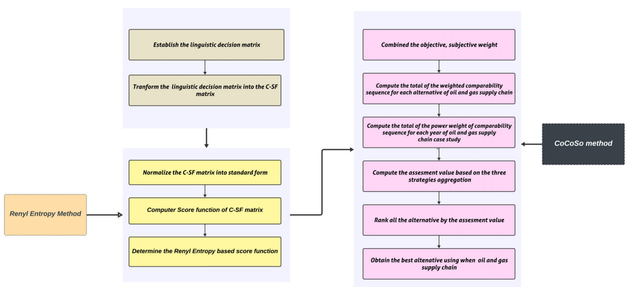

FIGURE 5. Circular spherical fuzzy decision making for analyze the oil and gas supply chain.

Step 1. You can find the linguistic matrix in Table 2.

TABLE 2. Information linguistic term.

	ψ_1	ψ_2	ψ_3	ψ_4
$\Re_1$	BM	EL	AH	M
$\Re_2$	EH	BM	L	VH
$\Re_3$	EL	EH	ML	M
$\Re_4$	M	VH	L	BM
$\Re_5$	VH	MH	EH	BM

Step 2. Convert the linguistic matrix in Table 3 into a C-SF matrix..
Step 3. There is no requirement to convert because it is a benefit attribute..
Step 4. Score function $\S = \S_{5\times4}$ is computed by definition 5 as.

$$\S_{[5\times4]} = \begin{bmatrix} -0.0372 & -0.0472 & 0.0253 & -0.0022 \\ 0.2184 & -0.0241 & -0.1066 & 0.1684 \\ -0.3000 & 0.2000 & -0.0725 & -0.0075 \\ 0.0201 & 0.1766 & -0.0974 & -0.0149 \\ 0.1684 & 0.0859 & 0.2184 & -0.0241 \end{bmatrix}$$

TABLE 3. Information linguistic term.

	ψ_1	ψ_2
$\Re_1$	((0.4 ,0.2 ,0.3) ; 0.4)	((0.0 ,0.0 ,0.1) ; 0.4)
$\Re_2$	((1 ,0.0 ,0.0) ; 0.2)	((0.4 ,0.2 ,0.3) ; 0.2)
$\Re_3$	((0.0 ,0.0 ,1) ; 0.5)	((1 ,0.0 ,0.0) ; 0.5)
$\Re_4$	((0.5 ,0.2 ,0.2) ; 0.1)	((0.9 ,0.0 ,0.1) ; 0.1)
$\Re_5$	((0.9 ,0.0 ,0.1) ; 0.2)	((0.7 ,0.1 ,0.1) ; 0.2)
	ψ_3	ψ_4
$\Re_1$	((0.6 ,0.2 ,0.2) ; 0.4)	((0.5 ,0.2 ,0.2) ; 0.4)
$\Re_2$	((0.2 ,0.3 ,0.5) ; 0.2)	((0.9 ,0.0 ,0.1) ; 0.2)
$\Re_3$	((0.3 ,0.3 ,0.3) ; 0.5)	((0.5 ,0.2 ,0.2) ; 0.5)
$\Re_4$	((0.2 ,0.3 ,0.5) ; 0.1)	((0.4 ,0.2 ,0.3) ; 0.1)
$\Re_5$	((1 ,0.0 ,0.0) ; 0.2)	((0.4 ,0.2 ,0.3) ; 0.2)

Step 5. Use (11) to get the combined weight ϖ.

$$\varpi_1 = 0.67, \varpi_2 = 0.09, \varpi_3 = 0.18, \varpi_4 = 0.05$$

Step 6. Compile the entire sequence of weighted comparabilities S_i as
$S_1 = -0.0250, S_2 = 0.1343, S_3 = -0.1965, S_4 = 0.0118, S_5 = 0.1595$
Step 7. Ascertain the total number of sequences that are power-weighted comparable P_i as
$P_1 = 2.0989, P_2 = 2.6419, P_3 = 2.6994, P_4 = 2.3798, P_5 = 2.6770$
Step 8. The three score strategies k_{ia}, k_{ib} and k_{ic} are presented as
$k_{1a} = 0.1648, k_{2a} = 0.2207, k_{3a} = 0.1989, k_{4a} = 0.1901, k_{5a} = 0.2255$
$k_{1b} = 1.1273, k_{2b} = 0.5755, k_{3b} = 2.2861, k_{4b} = 1.0736,$
$k_{5b} = 0.4638$
$k_{1c} = 0.7254, k_{2c} = 0.9711, k_{3c} = 0.8755, k_{4c} = 0.8365, k_{5c} = 0.9922$
Step 9. Diagnose the corresponding assessment value k_i as $k_1 = 0.7175, k_2 = 0.6302, k_3 = 1.2529, k_4 = 0.7570, k_5 = 0.5951$
Step 10. Rank the alternative of oil and gas supply chain is given below.

$$\Re_3 > \Re_4 > \Re_1 > \Re_2 > \Re_5$$

In the context of optimizing logistics in the Oil and Gas supply chain, one of the most crucial attributes among the ones you've listed would likely be $\Re_3$ = Technology Integration. Automation and real-time tracking of logistics processes are made possible by technology integration. By lowering paperwork, cutting down on manual errors, and improving transit routes, this can result in cost savings. Our alternatives are ranked in Figure 6.

J. SENSITIVE ANALYSIS

In order to better discuss and analyze the influence of parameter in combined weight information, the sensitivity analysis is constructed in Figure 7. To enhance discourse and examination of the impact of parameter α in the combined weight data, the sensitivity analysis is built into Figure 7. The

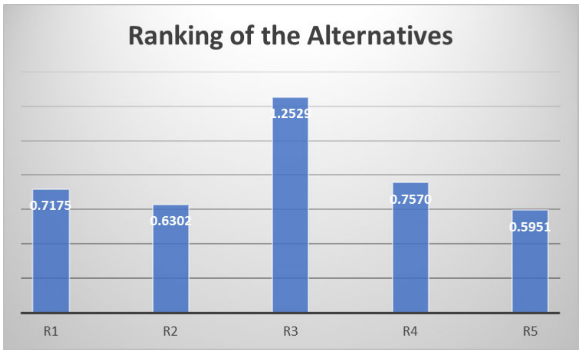

FIGURE 6. Ranking of the alternatives.

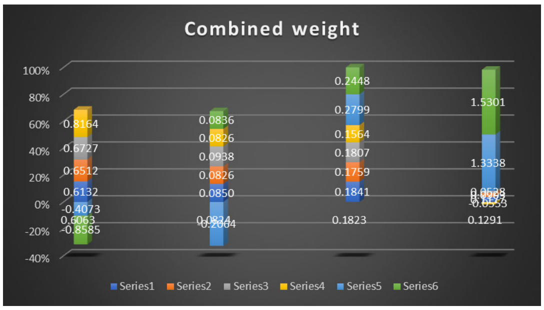

FIGURE 7. Sensitive analysis proposed combined weights.

values of combined weight are shown in the x-axis, and the different values of α are shown in the y-axis. Figure 7 tells us how our weight varies after changing the values of α.

V. COMPARATIVE ANALYSIS

A detailed comparison between the present method and our case study example is provided in Appendix A. We then implement our proposed methodology and give an alternative approach to deciding the weights of our criteria. Once conclusions are reached, we carefully compare our weighted results, which we computed ourselves, with the weights that were determined by the traditional technique. Ranking of the alternatives compare with other method are shown in Table 4.

TABLE 4. Ranking of the alternatives.

Authors	Ranking orders	Best
Regret based [45]	$\Re_3 > \Re_4 > \Re_1 > \Re_2 > \Re_5$	$\Re_3$
CODAS (C-PyFS) [39]	$\Re_3 > \Re_4 > \Re_1 > \Re_2 > \Re_5$	$\Re_3$
VIKOR (PyFs)[46]	$\Re_3 > \Re_4 > \Re_1 > \Re_2 > \Re_5$	$\Re_3$
CoCoSo (Proposed)	$\Re_3 > \Re_4 > \Re_1 > \Re_2 > \Re_5$	$\Re_3$

DISCUSSION

In this study, we introduce an innovative approach for decision-making by using the CoCoSo approach in addition to C-SFSs. We solve the difficulties in managing uncertainty by introducing circular spherical fuzzy sets into the process of making decisions. In complicated and uncertain contexts, the CoCoSo technique improves precision and guarantees solid decision outcomes. In order to evaluate the feasibility of our suggested approach, we carried out a case study centered on logistics optimization within the Oil and Gas supply chain. This industry's inherent uncertainties and complexity offer the perfect setting for assessing C-SFSs and the CoCoSo method's efficacy. We found that "Technology Integration" was a crucial factor affecting the case study's decision-making results. By including this feature as the third dimension, it was possible to conduct a thorough analysis of the options and gain important insight into the technological factors that were taken into account during the decision-making process.

We added a second weight calculation approach to our system to increase its resilience. We used this approach, which we modified from [44], as a standard for allocating weights in our decision model. We performed a comparative analysis by using this alternate method in addition to our suggested methodology in order to evaluate the advantages and disadvantages of each. The comparison analysis's findings confirm the effectiveness of C-SFSs and the CoCoSo technique. In addition to keeping us ahead of the competition, our method also demonstrates how flexible it may be used in situations where other weight calculating techniques might be taken into account. Evaluation of Accuracy and Ordering of Options By using our suggested approach on the case study, we were able to thoroughly evaluate its accuracy. The outcomes show that our method offers a trustworthy rating of options and well captures decision nuances. This illustrates the CoCoSo method's and C-SFSs's resilience in handling the complexities of real-world decision scenarios, particularly those with multiple, connected qualities and uncertainty.

VI. CONCLUSION

By combining C-SFSs with the CoCoSo technique, we have presented a fresh approach to decision-making in this paper. Our methodology offers a strong foundation for assessing alternatives in MCDM problems by addressing the inherent challenges of ambiguity and complexity in decision scenarios. Develop advanced algorithms or computational techniques to enhance the efficiency and scalability of the CoCoSo method, especially for handling large-scale decision-making problems or real-time applications. We defined C-SFSs, in which a fixed-radius circle describes each choice. Naturally, in the case where all radii coincide, the model produces a C-SFS. Lastly, a circular spherical fuzzy environment is employed in a method of option ranking that is based on these operators. The utility and reliability of the proposed technique are demonstrated with a descriptive example. To ascertain the superiority of the suggested approach, tests have also been conducted on its validity and reliability.

Promising outcomes have been observed when C-SFSs are applied to the oil and gas supply chain's logistics optimization. The addition of the attribute "Technology Integration" as a crucial component emphasizes how adaptable and thorough our methodology is in capturing the nuances of real-world decision contexts.

By presenting a different approach to calculating weight and performing a comparative study, we have reinforced the basis of our methodology. The outcomes demonstrate not only how competitive our strategy is, but also how flexible it is in comparison to other approaches.

To sum up, we have made two contributions to decision-making methodologies: first, we have integrated the CoCoSo approach for robust decision outcomes, and we have introduced C-SFSs as a potent representation of uncertainty. We see our methodology continuing to develop and be used in the future, advancing decision science and providing decision-makers with the means to effectively traverse the complicated and ever-changing environment. Some suggested future studies based on the content of the article:

- Integration of Advanced Technologies
- Dynamic Optimization Models
- Sustainability and Environmental Impact Assessment
- Supply Chain Resilience and Risk Management
- Cross-Sector Collaboration and Integration
- Long-Term Strategic Planning
- Case Studies and Real-World Implementations

COMPETING INTERESTS

The authors are here with confirm that there are no competing interests between them.

APPENDIX A ESTABLISHING ATTRIBUTE WEIGHT

Consider an MADM problem with a set of g criteria, denoted as $\psi = \{\psi_1, \psi_2, \ldots, \psi_g\}$, and a set of h attributes, denoted as $\Re = \{\Re_1, \Re_2, \ldots, \Re_h\}$.

Additionally, every single thing in the C-SF matrix can be viewed as a C-SFS defined over the attribute set $\Re$. In mathematical terms, this can be expressed as

$$ß = \{\langle \hat{\alpha}, \mu_ß(\hat{\alpha}), \eta_ß(\hat{\alpha}), \nu_ß(\hat{\alpha}); \hat{r}|\hat{\alpha} \in \Re\rangle\}$$

Definition 6: If we have a set of C-SFS defined on the attribute set $\Re = \{\Re_1, \Re_2, \ldots, \Re_h\}$ within a C-SF matrix, denoted as $\psi = \{\psi_1, \psi_2, \ldots, \psi_g\}$, then the C-SF mean of the C-SF matrix is calculated as follows:

$$M^* = \left\{\langle \Re_t, \mu^*(\Re_t), \eta^*(\Re_t), \nu^*(\Re_t)|\Re_t \in \Re\right\} \tag{18}$$

where
$\mu^*(\Re_t) = \sqrt{\frac{1}{g}\sum_{u=1}^{g} \mu_{ut}^2}$, $\eta^*(\Re_t) = \sqrt{\frac{1}{g}\sum_{u=1}^{g} \eta_{ut}^2}$,
$\nu^*(\Re_t) = \sqrt{\frac{1}{g}\sum_{u=1}^{g} \nu_{ut}^2}$

Definition 7: If we have a set of C-SFS defined on the attribute set $\Re = \{\Re_1, \Re_2, \ldots, \Re_h\}$ within a C-SF decision matrix, denoted as $\psi = \{\psi_1, \psi_2, \ldots, \psi_s\}$, then, with regard to the entity ψ_s, the C-SF deviation of the attribute $\Re_h$ is defined as: $DV_{\psi_s}(\Re_h) =$

$$[(\mu_{ut})^2 - (\mu^*(\Re_t))^2] - [(\eta_{ut})^2 - (\eta^*(\Re_t))^2] - [(\nu_{ut})^2 - (\nu^*(\Re_t))^2] \tag{19}$$

Definition 8: If we have a set of C-SF $\psi = \{\psi_1, \psi_2, \ldots, \psi_s\}$ defined on the attributes $\Re = \{\Re_1, \Re_2, \ldots, \Re_h\}$ in a C-SF decision matrix then attribute $\Re_h$ can be defined as follows:

$$VR(\Re_h) = \frac{1}{g-1}\sum_{s=1}^{g}\left(DV_{\psi_s}(\Re_h)\right)^2 \tag{20}$$

Definition 9: If we have a set of C-SFSs $\psi = \{\psi_1, \psi_2, \ldots, \psi_s\}$ defined on the attributes $\Re = \{\Re_1, \Re_2, \ldots, \Re_h\}$ in a C-SF decision matrix, then the C-SF covariance between attributes $\Re_t$ and $\Re_u$ can be described as:

$$CV(\Re_t, \Re_u) = \frac{1}{g-1}\sum_{u=1}^{g} DV_{\psi_s}(\Re_t) DV_{\psi_s}(\Re_u). \tag{21}$$

Proposition 1: If we consider a set of C-SFSs $\psi = \{\psi_1, \psi_2, \ldots, \psi_s\}$ defined on the attributes $\Re = \{\Re_1, \Re_2, \ldots, \Re_h\}$ within a C-SF decision matrix, then

1) $VR(\Re_t) > 0, \ \forall \ \Re_t \in \Re$
2) $CV(\Re_t, \Re_t) = VR(\Re_t)$.
3) $CV(\Re_t, \Re_u) = CV(\Re_u, \Re_t)$.

Definition 10: If we have a set of C-SFSs $\psi = \{\psi_1, \psi_2, \ldots, \psi_s\}$ defined on the attributes $\Re = \{\Re_1, \Re_2, \ldots, \Re_h\}$ within a C-SF DM, then the C-SF covariance matrix of the C-SF DM can be represented as:

$$✠_{CSF} = \begin{pmatrix} VR(\Re_1) & CV(\Re_1, \Re_2) & \cdots & CV(\Re_1, \Re_h) \\ CV(\Re_2, \Re_1) & VR(\Re_2) & \cdots & CV(\Re_2, \Re_h) \\ \vdots & \vdots & \ddots & \vdots \\ CV(\Re_h, \Re_1) & CV(\Re_h, \Re_2) & \cdots & VR(\Re_h) \end{pmatrix}$$

The significance of attributes in decision-making is determined by their respective weights. These weight values indicate the relative importance of each attribute, a factor that can differ across attributes. In this investigation, we present a novel technique for evaluating attribute weight, that draws from the CRITIC method [44]. This involves calculating the degree of correlation between two attributes, denoted as $\Re_t$ and $\Re_u$ as demonstrated in equation given below:

$$CR_{CSF}(\Re_t, \Re_u) = \frac{CV(\Re_t, \Re_u)}{\sqrt{VR(\Re_t)}\sqrt{VR(\Re_u)}}. \tag{22}$$

The degree of contrast in an attribute is directly proportional to its C-SF variance value. This implies that the contribution of an attribute to the DM process is encapsulated in its C-SF variance. Conversely, a higher C-SF correlation coefficient for an attribute compared to others signifies a higher level of agreement. Consequently, the following formula quantifies the information generated by attribute ζ_t:

$$\$(\Re_t) = VR(\Re_t)\sum_{u=1}^{h}|CR(\Re_t, \Re_u)|. \tag{23}$$

The attribute $\Re_t$ holds greater significance in the decision-making framework when the value of $\$(\Re_t)$ is larger. Ultimately, the attribute weights, normalised, are computed using the equation (24):

$$\varpi = \frac{\$(\Re_t)}{\sum_{t=1}^{h} \$(\Re_t)} \tag{24}$$

A. CALCULATION

By using equation 24 we calculate the weights of our case study. The calculate weights are

$$\varpi_1 = 0.56,\ \varpi_2 = 0.16,\ \varpi_3 = 0.19\ \varpi_4 = 0.09$$

The entire sequence S_i of weighted comparabilities as
$S_1 = -0.0238, S_2 = 0.1133, S_3 = -0.1505, S_4 = 0.0199, S_5 = 0.1474$

The total of the comparability sequences with weighted power P_i as
$P_1 = 1.8461, P_2 = 2.4830, P_3 = 2.5337, P_4 = 2.1978, P_5 = 2.5080$

The three score strategies k_{ia}, k_{ib} and k_{ic} are presented as
$k_{1a} = 0.1561, k_{2a} = 0.2224, k_{3a} = 0.2041, k_{4a} = 0.1899, k_{5a} = 0.2274$
$k_{1b} = 1.1582, k_{2b} = 0.5917, k_{3b} = 2.3724, k_{4b} = 1.0584, k_{5b} = 0.3791$
$k_{1c} = 0.6797, k_{2c} = 0.9684, k_{3c} = 0.8889, k_{4c} = 0.8271, k_{5c} = 0.9904$

Determine the matching assessment value k_i as
$k_1 = 0.7056, k_2 = 0.6366, k_3 = 1.2987, k_4 = 0.7473, k_5 = 0.5608$

Rank the alternative of oil and gas supply chain for the purpose of best alternative.

$$\Re_3 > \Re_4 > \Re_1 > \Re_2 > \Re_5$$

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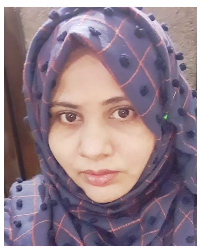

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