IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received 26 March 2024, accepted 7 April 2024, date of publication 15 April 2024, date of current version 24 April 2024.

Digital Object Identifier 10.1109/ACCESS.2024.3389104

RESEARCH ARTICLE

Performance Analysis of a Self-Organized Network Dynamics Model for Public Opinion Information

ZHUO YANG[1], YAN GUO[2], HONGYU PANG[2], AND FULIAN YIN[2], (Member, IEEE)
[1]School of Public Security Information Technology and Intelligence, Criminal Investigation Police University of China, Shenyang 110854, China
[2]School of Information and Communication Engineering, Communication University of China, Beijing 100024, China

Corresponding author: Yan Guo (guoyangy@cuc.edu.cn)

This work was supported in part by Fundamental Research Funds for the Universities in Liaoning Province under Grant JYTMS20231417, and in part by the Fundamental Research Funds for the Central Universities under Grant C2023011.

ABSTRACT With the rise of social networks, various types of information have emerged in the vision field in a complex manner, making it crucial to analyze the propagation patters of online public opinion to effectively guide information dissemination. To elucidate the dynamics of information dissemination, this paper proposes a directed network information based on self-organized network information dissemination scenario. This model takes into account the influence of networks formed by users spontaneously and distinguishes the dissemination population based on the in- and out-degree of user nodes in the network. To assess the model's performance, it is evaluated using real retweets from Chinese Sina Weibo, considering the impact of user interactions on information dissemination. Comparing real data with model-fitted data, the proposed model-based evaluation and numerical analysis demonstrate that the forwarding and transfer probabilities align with actual information dissemination. Furthermore, the evaluation sensitivity analyses describe the key factors influencing information dissemination, aiding decision-making in formulating strategies to guide public opinion. To quantify the importance of these factors, assessment metrics are introduced, such as the propagation regeneration number.

INDEX TERMS Information propagation, self-organized network, dynamic model, directed network, performance analysis.

I. INTRODUCTION

With the advent of the mobile self-media era, online public opinion information spreads globally at astonishing speed, giving rise to rapidly changing topics on the Internet. This constant stream profoundly affects all aspects of people's lives and society. Therefore, building a healthy network environment necessitates to study the law of network public opinion dissemination. Meanwhile, focusing on the analysis and evaluation of the direction of public opinion and making the information dissemination model conform to reality by assessing the performance of the established model is required.

The associate editor coordinating the review of this manuscript and approving it for publication was Yun Lin.

On social media, typical scenarios of self-organizing networks emerge from user-generated content, sharing, commenting, and forwarding behaviors. Information spreads rapidly through interconnections and interactions between users, giving rise to hot topics, trends, and network effects. This spontaneous mode of information dissemination holds particular significant on social media platforms, where user behavior and feedback directly shape the path of information dissemination and influence. When users engage in interactive behaviors, their actions form propagation paths that spreads from one user to another, resulting in a directed propagation network. Within directed networks, node interactions are asymmetric, indicating that users can exert a reverse influence on each other. Self-organizing networks illustrate the spontaneous structure and interaction patterns of information dissemination. Together, these elements shape

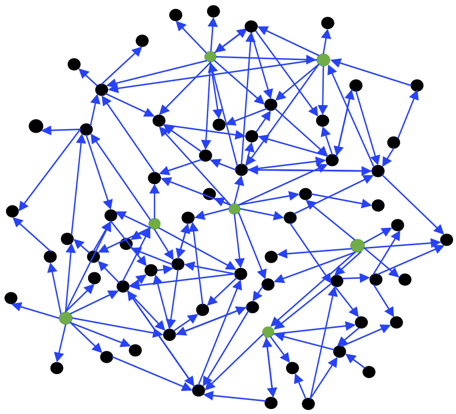

FIGURE 1. Directed network information dissemination map where the green dots represent the users who published the information, the black dots denotes the users who forwarded the information, and the blue arrows indicate for the direction of information dissemination.

the behavior of information dissemination in networks. Moreover, Figure 1 depicts information dissemination in directed networks. Notably, there is no major opinion leader during the dissemination process; instead, the network incorporates the influence of mutual discussion among ordinary users. Individual users can influence one another not only by browsing, reading, and forwarding others' Weibo posts but also by posting their own Weibo content on various topics. Subsequently, users may be influenced by others in turn, aligning with the principle of directed online public opinion dissemination.

Being an important research field, theoretical and applied significance of complex networks has garnered continuously recognition and expansion. Many real-world practical problems find abstraction into complex network models for research purposed, including the spread of infectious diseases and the dissemination of public opinion information. Self-organizing networks represent a kind of complex network structure formed through self-organization. They are characterized by spontaneous behaviors among nodes and hold great importance understanding network characteristics and optimizing performance, especially concerning the law of information dissemination within such networks.

Self-organizing networks typically exhibit dynamic evolution, with changes in network topology and node properties over time, adding complexity to the research process. One of the key challenges lies in constructing accurate dynamic models that capture the dynamics of the information dissemination process. This entails considering factors such as interactions between nodes, information dissemination rules, and environmental context.

Regarding information dissemination guided by self-organizing networks, Gen et al. [1] considered individual self-organizing behavior and developed a network-based knowledge infection model that integrates static resources and information dissemination into individual interactions of knowledge infection. They conceptualized information dissemination as a mediator for individual behavior influenced by the social environment, and theoretically analyzed fundamental aspects of the model such as the basic reproduction number, equilibrium point, and global stability rules. Moreover, Huang et al. [2] defined attitude update rules for nodes based on non-Bayesian social learning rules from an individual perspective, establishing an inter-node game matrix based on attitude values. Numerical analysis and simulation experiments confirmed the validity of the proposed model, highlighting the significant role of different attitudes among nodes in information dissemination.

In addition, Deng et al. [3] investigated the role of user self-organization in accordance with the characteristics of the pervasive Internet, emphasizing the importance of allowing people to freely express themselves while effectively managing speech on the network. Furthermore, Du et al. [4] analyzed the network structure of network media based on self-organizing networks and chaotic fractal theory. They underscored that network media information transmission network is self-organized, requiring four necessary conditions and certain driving forces for its realization. At the same time, scholars use self-organization charts to study the spread of disease [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], being very similar to the spread of information. In summary, user self-organized network information dissemination is prevalent across various social platforms, and understanding self-organized network characteristics and user attributes is crucial for studying dissemination and guiding information governance.

To enhance the understanding of network structure and dynamic behavior, the study of self-organized networks is complemented by directed networks. A directed network is a topological structure wherein edges connecting nodes in the network possess directionality, divided into in-degree and out-degree connections. In the propagation process of directed network, node interactions are asymmetric, allowing for reverse influence among users.

In the dissemination of public opinion information, the connections between nodes in self-organizing networks formed through spontaneous user organization are based on social relations. Therefore, the node establishment of such self-organizing networks is characterized by being a directed network. In such networks, the connections between nodes are no longer bidirectional; however, there is still a directional flow of information between nodes, resulting in an increased complexity of the analysis. Moreover, the node's influence may depend on its position in the network and the nature of its directional connections; therefore, the modeling process of dynamic evolution considers different changes in inter-node relationships over time.

Recent years have witnessed significant research results in propagation dynamics within directed networks. For instance, Karsai et al. [15] investigated the weakening phenomenon of strong connections in directed networks, shedding light on the influence of time variation on connection strength. Moreover, Huang et al. [16] studied the problem of distributed constrained nonlinear Least Squares (LS) estimation in directed networks, devising a novel projected LS estimation algorithm and validating it through numerical examples. In addition, Liu et al. [17] studied behavior propagation in the online social network experiment and found the interaction between network structure and propagation dynamics. Moreover, Wei et al. [18] proposed an improved Susceptible-Infected-Recovered (SIR) model based on the traditional SIR, focusing mainly on infectious disease dynamics wherein some of the ignorant users are directly replaced by immune ones, designed a directed network combined with Weibo network topology, constructing a Weibo network public opinion propagation model and analyzing factors influencing public opinion propagation. Consequently, the application of directed networks in information dissemination models has spurred scholars interest in elucidating the penetration and influence of social media, summarizing the form and interactive role of information exchange [19], [20], [21], [22], [23], [24], [25], [26].

Introducing dynamic modeling into the information dissemination process allows for a more nuanced understanding of changes and evolution within the dissemination process. By incorporating analysis of self-organized networks and directed networks into dynamic model construction, parameters become more comprehensive, covering more factors influencing information dissemination, thus improving fitting and prediction ability of the model.

The recognition of similarities between the process of network information diffusion and the one of disease transmission, as noted by Daley and Kendall [27], led to the development of the Daley-Kendall (DK) rumor propagation model. Since then, many information propagation dynamics models have emerged from biomathematics, leveraging insights from disease transmission dynamics. Moreover, Wang et al. [28] constructed a spatio-temporal evolution model, analyzing influencing factors. Based on the optimal segmentation, spatial autocorrelation analysis, and text analysis methods, the evolution of online public opinion was analyzed over time. Furthermore, Cooper et al. [29] used the SIR model to simulate COVID-19 transmission, exploring the impact of changes in susceptible individuals. In addition, Chen et al. [30] proposed a time-varying SIR model, capable of tracking the propagation rate and recovery rate at an given time. Added to that, Wang et al. [31]studied a Spreader1-Spreader2-Ignorant-Hesitant-Stifler1-Stifler2 (2SIH2R) rumor propagation model, incorporating a confrontation mechanism to quantify the competition between rumors and truth, revealing its influence on propagation dynamics. Finally, Zhu et al. [32] introduced a saturation treatment function to model the specific influence of rumor propagation control on regulators. Researchers have developed a variety of dynamic models to describe how information spreads in social networks. These models help to understand the underlying mechanisms of information dissemination and to predict dissemination trends. The effects of network topology on the efficiency of information dissemination have been revealed by analyzing the properties of network structure, and these studies provide a theoretical basis for designing efficient dissemination strategies.

Based on our current understanding, there appears to be a gap between a suitable modeling framework using directed network dynamics to analyze the impact of interactions among forwarding users on information dissemination dynamics and emerging public opinion. Additionally, most of the existing information dissemination models tend to overlook the crucial role of ordinary users in the dissemination process. This prompts us to consider how to more accurately capture the dissemination path of information in networks. In response to these challenges and referring to the self-organized network information propagation scenario, we propose a directed network information propagation dynamics model for the self-organized network propagation spontaneously formed by users. To assess the performance of the model, the main objectives consist of:

1. analyzing the topology structure of a directed network and its impact on information propagation;
2. predicting the development trend of public opinion;
3. formulating intervention policies.

To validate the effectiveness of the mode, we plan to analyze the propagation of public opinion topics on the Weibo platform, utilizing it as a typical case for data fitting to determine the trend of information dissemination in directed networks.

The organizational structure of this paper is as follows: In Section II, based on the scenario of self-organized network propagation, building the information propagation dynamics model of directed networks, meanwhile defining the information propagation evaluation indexes. In Section III, the numerical simulation experiments and parametric sensitivity analysis were performed to verify the effects of different conditions on information spread. Finally, the content of this article is summarized in Section IV.

II. MODELING THE DYNAMICS OF INFORMATION PROPAGATION IN A DIRECTED NETWORK

Based on the scenarios of self-organizing network propagation, the information dissemination dynamics model of directed networks is proposed in this work. As shown in Figure 2, nodes represent users in different states whereas edges indicate the dissemination paths of the information between users. In the directed network, the edge pointing to the user node highlight that the user is affected other users, and whereas the edge emanating from the user node indicates users' ability to spread information outward.

Moreover, we assume that the multi-information topic spreads in a closed and stable environment and only users that can be reached in the topic's spread process are considered. The total number of people (N) remains unchanged throughout the study. For each information part, we focus on the diffusion of information generated by users' forwarding behavior, assuming that the same user can choose whether to forward the information after reading it; moreover, a user can forward each piece of information once. At the same

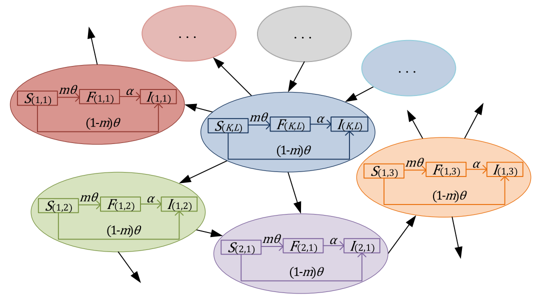

FIGURE 2. Dynamic model of a directed network.

TABLE 1. Description of each state.

State	Interpretation
$S_{(k,l)}(t)$	The total number of individuals in susceptible situations who have not yet been exposed to information; yet, they have the opportunity to access it in the future. They are susceptible to the influence of information and may exhibit forward-thinking behavior.
$F_{(k,l)}(t)$	The total number of individuals in susceptible situations who have already forwarded information and are still in the exposure period of the information. They can make individuals who are in susceptible situations aware of the information and engaged in forward behavior.
$I_{(k,l)}(t)$	The total number of individuals' immune to information. It consists of two parts: the first represents the total number of individuals who have forwarded the information and exceeded the exposure period of the information over time. They can no longer influence other users to access the content of the information. The second part regroups the total number of individuals who are in a susceptible state and are directly immune to the information due to their subjective lack of interest in the information after being exposed.

time, any individual in the crowd can be categorized in one of the following three states: Susceptible (S), Forwarding (F), or Immune (I). In this paper, the in-degree of network nodes k and the out-degree of network nodes l are introduced. Therefore, categorizing the population in each state based on the in-degree and out-degree of the different nodes in the network, is displayed in Table 1.

It is worth noting that subscript (k, l) represents the state with an in-degree k and an out-degree l at time t.

A. MODEL CONSTRUCTION

Our model takes the form in Equation (1)-(5), and the definition for parameters is listed in Table 2.

$$S_{(k,l)}(t) + F_{(k,l)}(t) + I_{(k,l)}(t) = N_{(k,l)}(t) \tag{1}$$

$$\mathrm{d}S_{(k,l)}(t)/dt = -kS_{(k,l)}(t)\sum\nolimits_{i}^{K}\sum\nolimits_{\mathrm{j}}^{L} p((i,j)|(k,l)) \, F_{(i,j)}(t)/N_{(i,j)}(t) \tag{2}$$

$$\mathrm{dF}_{(k,l)}(t)/dt = \mathrm{m}kS_{(k,l)}(t)\sum\nolimits_{i}^{K}\sum\nolimits_{j}^{L} p((i,j)|(k,l)) \, F_{(i,j)}(t)/N_{(i,j)}(t) - \alpha F_{(k,l)}(t) \tag{3}$$

$$\mathrm{dI}_{(k,l)}(t)/dt = (1-\mathrm{m})kS_{(k,l)}(t)\sum\nolimits_{i}^{K}\sum\nolimits_{j}^{L} p((i,j)|(k,l)) \, F_{(i,j)}(t)/N_{(i,j)}(t) + \alpha F_{(k,l)}(t) \tag{4}$$

TABLE 2. Interpretation of directed network model parameters.

Parameters	Interpretation
k	The in-degree of nodes in the network, *i.e.*, the number of edges points to user nodes in the network. It represents the number of users in the network who are affected by other users per unit of time, $k \in [1, K]$.
l	The out-degree of nodes in the network, *i.e.*, the number of edges emanating outward from each node in the network. It highlights the number of users in the network who can influence other users per unit of time, $l \in [1, L]$.
K	The maximum value of node in-degree in the network. It indicates the maximum number of users in the network affected by other users per unit of time.
L	The maximum value of node out-degree in the network indicates the maximum number of users in the network able to influence other users per unit of time.
m	The average probability of forwarding a message. It represents the average probability of forwarding a message after exposure to an individual who is in a susceptible state towards the message, $m \in$ [0,1].
α	The average rate of immunization of a message. It denotes the average rate at which an individual moves from the forwarding state to the immunized state, where $1/\alpha$ denotes the average exposure period of the message, $1/\alpha \in$ [0,48h].

We assumed that the total number of users in each state remained constant, as indicated in Equation (5):

$$\sum\nolimits_{k=1}^{K}\sum\nolimits_{l=1}^{L} S_{(k,l)}(t) + \sum\nolimits_{k=1}^{K}\sum\nolimits_{l=1}^{L} F_{(k,l)}(t) + \sum\nolimits_{k=1}^{K}\sum\nolimits_{l=1}^{L} I_{(k,l)}(t) = N \tag{5}$$

The initial value of each state is defined as: $S_{(k,l)}(0) = N_{(k,l)}(0)$, $F_{(k,l)}(0) = C_k(0)$, $I_{(k,l)}(0) = 0$.

Moreover, through Equations (2)-(4), $p((i,j)|(k,l))$ denotes the conditional probability that any edge in the network is emitted by a node with a degree (i, j) and points to a node with a degree (k, l). Therefore, $\sum_{i}^{K}\sum_{j}^{L} p((i,j)|(k,l))F_{(i,j)}(t)/N_{(i,j)}(t)$ represents the probability that a node user with degree (k, l) is connected to a user in a forwarding state at time t. Thus, the amount of change in the susceptible population $S_{(k,l)}(t)$ can be expressed using Equation (2). Similarly, Equation (3) represents the amount of change in the population of $F_{(k,l)}(t)$ states, consisting of two components (*e.g.*, $S_{(k,l)}(t)$ that knows the information and forwards it to become the $F_{(k,l)}(t)$ and $F_{(k,l)}(t)$ that loses interest in the information over time to become the $I_{(k,l)}(t)$). Finally, Equation (4) denotes the amount of change in the population of $I_{(k,l)}(t)$ state, also consisting of two parts (*e.g.*, $S_{(k,l)}(t)$ state knows the information but is not interested in it, so it shifts directly into the $I_{(k,l)}(t)$ state, and $F_{(k,l)}(t)$ state loses interest in the information over time and shifts into the $I_{(k,l)}(t)$ state).

$$p((i,j)|(k,l)) = jp(i,j)/\langle l\rangle \tag{6}$$

Assuming that the directed network is an unrelated directed network, the conditional probability is only related to the degree of the upstream node (i, j). As indicated in Equation (6), $p(k,l)$ represents the joint probability distribution of nodes (k,l) in the network, denoting the probability of randomly selecting a node with an in-degree k and an out-degree l in the network. It is also equivalent to the ratio of the total number of nodes N_{kl}. Therefore, the degree distribution of any point in the network with degree (k, l) is shown in Equation (7).

$$p(k,l) = N_{kl}/N \tag{7}$$

where, referring to Equation (6), the $\langle l\rangle$ symbol represents the average degree of a directed network. This can be explained by the fact that, in any directed network, each edge leaves one node and enters another. Therefore, the average in-degree is equal to the average out-degree, as represented in Equation (8).

$$\langle k\rangle = \langle l\rangle = \sum\nolimits_{i}^{K}\sum\nolimits_{j}^{L} kp(k,l) = \sum\nolimits_{i}^{K}\sum\nolimits_{j}^{L} lp(k,l) \tag{8}$$

Thus, $\theta(t)$ represents the probability that a node user with degree (k,l) is connected to the forwarding user per unit time, as highlighted in Equation (9).

$$\theta(t) = \sum\nolimits_{i}^{K}\sum\nolimits_{\mathrm{j}}^{L} jp(i,j)F_{(i,j)}(t)/(\langle l\rangle N_{(k,l)}(t)) \tag{9}$$

In the dynamics of directed network propagation, a user susceptible to information is affected by an average of k individuals per unit of time. Since the conditional probability that a user of degree (k, l) is connected to another user of degree (i, j) in a degree-uncorrelated network is $p((i,j)|(k,l))$, the probability that a user in a susceptible state with degree (k,l) contacts a user in a forwarding state per unit time is defined as $k\sum_{i}^{K}\sum_{j}^{L} p((i,j)|(k,l))F_{(i,j)}(t)/N_{(i,j)}(t)$. According to the user's interest in the information, when $kS_{(k,l)}(t)\sum_{i}^{K}\sum_{j}^{L} p((i,j)|(k,l))F_{(i,j)}(t)/N_{(i,j)}(t)$ users are exposed, $mkS_{(k,l)}(t)\sum_{i}^{K}\sum_{j}^{L} p((i,j)|(k,l))F_{(i,j)}(t)/N_{(i,j)}(t)$ represents a portion who will choose to forward the information. They switch from $S_{(k,l)}$ state to $F_{(k,l)}$ state as presented previously. Moreover, $(1-m)kS_{(k,l)}(t)\sum_{i}^{K}\sum_{j}^{L} p((i,j)|(k,l))$ $F_{(i,j)}(t)/N_{(i,j)}(t)$ individuals will not forward the information. They will move from $S_{(k,l)}$ state to $I_{(k,l)}$ state. When the group that has forwarded the message has exceeded the message exposure period $(1/\alpha)$, it will no longer be able of influencing other users to be exposed to the content of this message; therefore, the user changes from state $F_{(k,l)}$ to state $I_{(k,l)}$ at an average immunization rate α.

$$dC_{(k,l)}(t)/dt = mkS_{(k,l)}(t)\theta(t) \tag{10}$$

$$C(t) = \sum\nolimits_{k=1}^{K}\sum\nolimits_{l=1}^{L}\int_{0}^{t} mkS_{(k,l)}(t)\theta(t)dt \tag{11}$$

The forwarding cumulative amount can be directly obtained from Sina Weibo. In the proposed model, the rate of change of the cumulative amount of users' forwarding over time with degree (k,l) is represented in Equation (10). Therefore, it is possible to obtain the forwarding accumulation $C(t)$ for a single message, as indicated in Equation (11).

B. ASSESSMENT OF INDICATORS

In a typical information dissemination process, the total number of forwarding groups varies over time following a bell-shaped curve, whereas the cumulative amount of forwarding follows the S-shaped incremental curve form, which eventually stabilizes. Based on these laws, a series of information dissemination indicators is determined, as depicted in Figure 3.

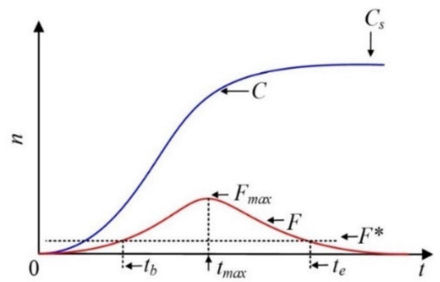

FIGURE 3. Index map for the relevance of information dissemination based on forwarding.

1) PEAK INFORMATION DISSEMINATION

F_{max} represents the maximum value of the instantaneous total number of current forwarding groups per unit of time. It reflects the maximum number of active users in the information dissemination process; moreover, these users can influence other individuals to access the information and forward it [33].

2) FINAL SCALE OF INFORMATION DISSEMINATION

The final Scale of Information Dissemination (C_s) denotes the final stabilized value of the cumulative forwarding curve C. It is used to measure the extent to which information dissemination can eventually spread [33].

3) INFORMATION DISSEMINATION TIME

Four key indicators are mainly defined. They include: (1) information dissemination outbreak time t_b, representing the time when public opinion starts to break out and satisfying the condition $F(t_b) = F^*$, where F^* denotes the threshold value, usually $F^* = 0.1 \times F_{\max}$; (2) information dissemination end time t_e, representing the time when public opinion tends to die out, satisfying the condition $F(t_e) = F^*$; (3) information dissemination duration time $t_i = t_e - t_b$, defined as the time from the beginning of the outbreak of public opinion to the tendency to die out; and (4) information dissemination climax time t_{max}, defined as the time when public opinion breaks out reaching the highest point and satisfying the condition $F(t_{\max}) = F_{\max}$.[34]

4) RATE OF INFORMATION DISSEMINATION

Similarly, three metrics representing the rate of information dissemination are defined. They include: (1) the average outbreak rate of information dissemination $V_o = (F_{\max} - F^*)/(t_{\max} - t_b)$, representing the average speed between the beginning of the outbreak of public opinion and the spread of public opinion until reaching the highest point; (2) the average decay rate of information dissemination $V_d = (F_{\max} - F^*)/(t_e - t_{\max})$, defined as being the average speed of public opinion between getting to its highest point and its tendency to die out; and (3) the average rate of information dissemination $V_a = (F_{\max} - F^*)/(t_e - t_b)$, defined as the average speed of public opinion between the beginning of its outburst and its tendency to die out [33].

5) PROPAGATION REPRODUCTION NUMBER

In the network dynamics model of information dissemination in directed networks, determining whether information bursts depends on the magnitude of the instantaneous forwarding user's rate of change over time $dF(t)/dt$ is represented in Equation (12). If $dF(t)/dt \prec 0$, the information will not burst as the number of susceptible users decreases [35].

$$dF(t)/dt = d\sum_{k=1}^{K}\sum_{l=1}^{L} F_{(k,l)}(t)/dt \tag{12}$$

$$\theta(t) = \sum_{k=1}^{K}\sum_{l=1}^{L} jp(i,j)F_{(i,j)}(t)/(\langle l\rangle N_{(k,l)}(t)) \tag{13}$$

When a network is determined, the average in- and out-degree of the network are respectively $\langle k\rangle$ and $\langle l\rangle$. Moreover, the total number of nodes $N_{(k,l)}(t)$ are fixed. Thus Equation (9) can be rewritten as shown in Equation (13).

At the same time, both sides of Equation (3) are simultaneously multiplied by $lp(k,l)/(\langle l\rangle N_{(k,l)}(t))$ to obtain Equation (14).

$$\begin{aligned} &dF_{(k,l)}(t)lp(k,l)/(\langle l\rangle N_{(k,l)}(t))dt \\ &= mkS_{(k,l)}(t)\theta(t)lp(k,l)/(\langle l\rangle N_{(k,l)}(t)) \\ &\quad - \alpha F_{(k,l)}(t)lp(k,l)/(\langle l\rangle N_{(k,l)}(t)) \end{aligned} \tag{14}$$

Summing both sides of Equation (14) over (k, l) yields Equation (15):

$$\begin{aligned} &d\theta(t)dt \\ &= \sum_{k=1}^{K}\sum_{l=1}^{L}\Big[mkS_{(k,l)}(t)\theta(t)lp(k,l)/(\langle l\rangle N_{(k,l)}(t)) \\ &\quad -\alpha F_{(k,l)}(t)lp(k,l)/(\langle l\rangle N_{(k,l)}(t))\Big] \\ &= \theta(t)\{\sum_{k=1}^{K}\sum_{l=1}^{L}[mkS_{(k,l)}(t)lp(k,l)/(\langle l\rangle N_{(k,l)}(t))] - \alpha\} \\ &= \theta(t)\{m\sum_{k=1}^{K}\sum_{l=1}^{L}[kS_{(k,l)}(t)lp(k,l)/(\langle l\rangle N_{(k,l)}(t))] - \alpha\} \end{aligned} \tag{15}$$

As $lp(k,l)/(\langle l\rangle N_{(k,l)}(t))$ is always positive, we can let $dF(t)/dt \prec 0$ be equivalent to $d\theta(t)/dt \succ 0$. For $t = 0$, $S_{(k,l)}(0) = N_{(k,l)}(0)$. Based on the numerical characterization of the joint probability distribution of directed networks, one can be obtained Equation (16) as follows:

$$\sum_{k=1}^{K}\sum_{l=1}^{L} klp(k,l) = \langle kl\rangle \tag{16}$$

$$\Re_0 = m\langle kl\rangle/\alpha\langle l\rangle \tag{17}$$

Based on the above, the propagation reproduction number $\Re_0$ is expressed using Equation (17). Moreover, if $\Re_0 \prec 1$, the public opinion does not erupt, whereas, if $\Re_0 \succ 1$, the public opinion is bound to eruption. When the average forwarding probability m as well as $\langle kl\rangle$ increase, the value of $\Re_0$ increases as well, and the public opinion eruption is accelerated. When the average immunization rate α and the average degree of the directed network increase, $\Re_0$ decreases, resulting in slowing down the public opinion eruption.

III. CASE ASSESSMENT ANALYSIS

In this paper, we consider the self-organized network communication topic #*Should we get COVID-19 vaccine boosters?*# related to the COVID-19 vaccine on Weibo serving as a typical case to validate the effectiveness of the dynamic model of directed network information dissemination. The topic has no official opinion leaders participants, and it is related to people's interests. With strong discussion, more ordinary users post on the topic data and they interact with each other, forming an information dissemination network for directed network. Therefore, this topic is used to obtain real data to estimate the parameters of the directed network model and the initial susceptible population, used to carry out the validity verification of the model.

A. NUMERICAL FITTING

Concerning a specific message, relevant data is collected through Weibo API interface, including the content of the message text and the forwarding time of each user. As the topic contains multiple messages, all required message data is collected. Firstly, the forwarding data of all messages is sorted according to the forwarding time, followed by data pre-processing. Similarly, the start time of all forwarding data for this topic's multiple messages is set to zero and the sampling frequency is set to one hour. Finally, the multi-message spreading forwarding is generated to accumulate volume over the whole time. The specific data is displayed in Table 3, where T represents the sampling time and C denotes the Cumulative forwarded quantity.

To fit the proposed model with real data from the Chinese Sina microblog, the LS (Least Squares) method is applied to estimate the model parameters and the initial susceptible population. The parameter vector is set as

TABLE 3. Topics # should we get COVID-19 vaccine boosters ?# multi-message spreading forwarding to accumulate volume.

T(h)	0	1	2	3	4	5
C	501	1047	1551	2007	2304	2622
T(h)	6	7	8	9	10	11
C	2827	2886	2969	3051	3093	3103
T(h)	12	13	14	15	16	17
C	3124	3142	3159	3169	3174	3177
T(h)	18	19	20	21	22	23
C	3178	3179	3180	3181	3182	3183
T(h)	24	25				
C	3183	3183				

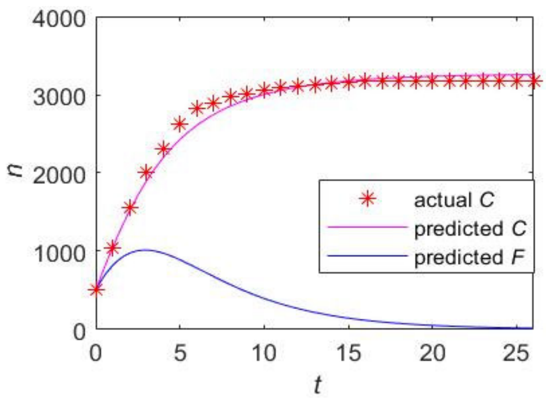

FIGURE 4. Topics # Should we get COVID-19 vaccine boosters ?# numerical fitting results.

$\theta = (\alpha, m, S_0, N, K, L)$. Thus, the LS error function is displayed in Equation (18).

$$LS = \sum_{w=0}^{T} |f_c(w, \theta) - ID_w|^2 \tag{18}$$

where $f_C(w, \Theta)$ represents the numerical result of $C(t)$ of the topic under the condition of parameter vector Θ, ID_w denotes the actual forwarding accumulation amount of directed network information dissemination, and w indicates the sampling time, $w = 0, 1, 2, \ldots T$.

Referring to Figure 4, data fitting is performed on real data. The real forwarding accumulation of the topic as well as the estimated forwarding accumulation drawn from the parameters estimation of the directed network information propagation dynamics model are displayed. It is evident that, for the cumulative forwarding, the trend of the predicted data is similar to that of the real observations, and the predicted trend for the instantaneous forwarding is aligned with the bell-shaped trend of the instantaneous forwarding of common information.

Based on the results of parameter estimation in Table 4, it is clear that the average forwarding probability parameter $m = 0.91$ is large, highlighting that the topic # Should we get COVID-19 vaccine boosters # is significantly attracting users, and most of them participate in forwarding, exploding quickly the information. Moreover, the average probability of

TABLE 4. Topics # should we get COVID-19 vaccine boosters ?# parameter estimation results.

Parameters	α	m	S_0	N	K	L
Estimate	0.3407	0.91	4300	4500	40	2

immunization $\alpha = 0.3407$ represents the average user active time during three hours. Meanwhile, the network maximum access degree ($K = 40$, and $L = 2$) is relative to the overall number of users; therefore, it is small. On behalf of the information publisher, the participants do not have a high fan base, and relative to the opinion leaders, their ability to contact and influence other users is relatively small. This implies that ordinary users are forwarding information to participate in the discussion of each other and to promote the dissemination of information in line with the law of information dissemination in the directed network.

Referring to the fitting curves and parameter estimation results, the proposed directed network information dynamics model can well estimate parameters for these events based on multi-information, and both the feasibility and reliability of the model are improved.

B. SENSITIVITY ANALYSIS

Partial Rank Correlation Coefficients (PRCCs) were used to better discern the different parameters responsible for the model. The simulation was based on 1000 samples for various input parameters against the threshold condition to evaluate the model's sensitivity. When the value PRCC > 0, there is a positive correlation effect between the index and the parameter. However, when the value PRCC < 0, the parameter plays a negative role. The p-value was deployed as the probability of observing the current PRCC value in the absence of a correlation among parameters. A lower p-value implies that the observed PRCC value is unlikely to occur; therefore, a significant correlation between the parameters can be inferred. Typically, the significance level p-value is set to 0.01 and it is considered as the criterion for determining whether a parameter is significant or not. In more detail, if the p-value is less than 0.01, the correlation is considered significant and the results of the parameter's sensitivity analysis are statistically significant. When using real-world data, we set the initial value of each state to $S_0 = 4.300\times10^3$, $F_0 = C_0 = 501$, and $I_0 = 0$. Moreover, Figures 5-7 generate the PRCC results along with histograms and scatter plot, showing the effect of six parameters $(\alpha, m, S_0, N, K, L)$ on the model indices $\Re_0$, C_s, F_{max}, t_b, t_i, t_{max}, V_o, V_d, and V_a, respectively.

In more detail, Figure 5 represents how the values of $\Re_0$, C_s, and F_{max} are affected by parameters. There is data to show that $\Re_0$, C_s, and F_{max} are strongly and positively affected by the average probability of forwarding message m, the initial value S_0, and the maximum value of the in-degree node K. The average rate of message immunization α, the total number of people N, and the maximum value of out-degree node L have a negative effect on $\Re_0$, C_s, and F_{max}, indicating that

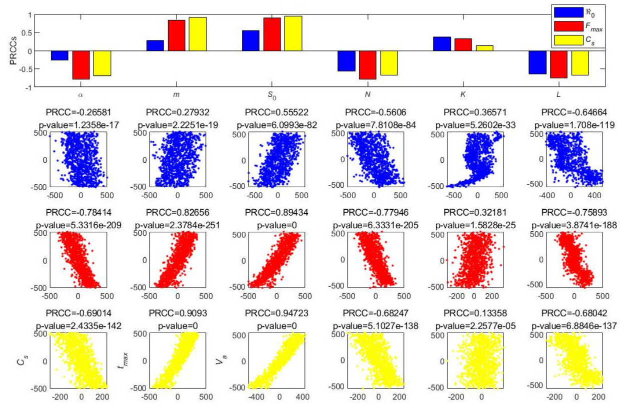

FIGURE 5. PRCC results with indices $\Re_o$, C_s, and F_{max} of different parameters.

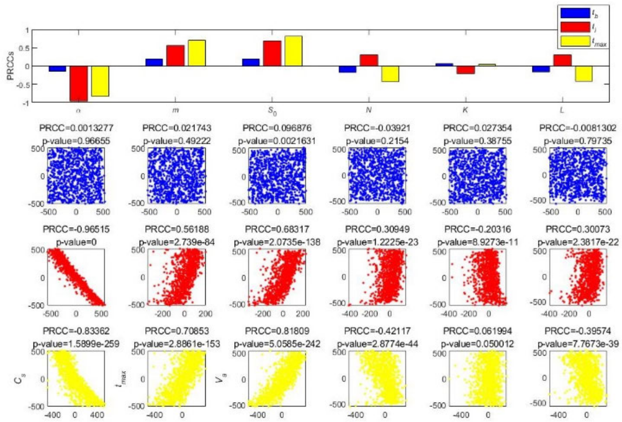

FIGURE 6. PRCC results with indices t_b, t_i, and t_{max} of different parameters.

prolonged exposure time of information and increase the ability of users to receive information conducive to the information outbreak. Meanwhile, to increase the final scale of information dissemination as well as the peak of information outbursts, we can reduce the number of users influencing others.

Moreover, Figure 6 illustrates how the values of t_b, t_i, and t_{max} are affected by parameters. For the information dissemination outbreak time t_b, all parameters have no significant influence on it. In addition, the start time of information dissemination is largely subjective to the user's discretion. Regarding the information dissemination duration time t_i and the information dissemination climax time t_{max}, parameters N, K, and L have no significant influence on it, while m and S_0 have a positive influence; however, α have a negative effect on t_i and t_{max}. This suggests that if a message needs to ferment quickly in the short term, it is required to increase the probability of forwarding by users so that more people are exposed to the message.

Finally, Figure 7 shows how the values of V_o, V_d, and V_a are affected by parameters. It is evident that parameters α, m, and S_0 play a significant role in influencing V_o, among which α has a significant positive effect on it, whereas m and S_0 significantly affect it negatively. In addition, each parameter

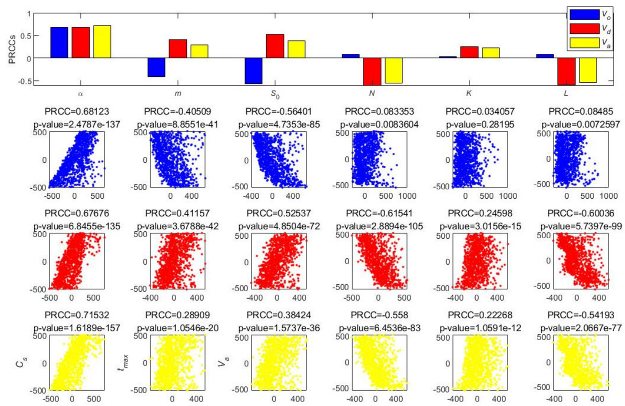

FIGURE 7. PRCC results with indices V_o, V_d, and V_a of different parameters.

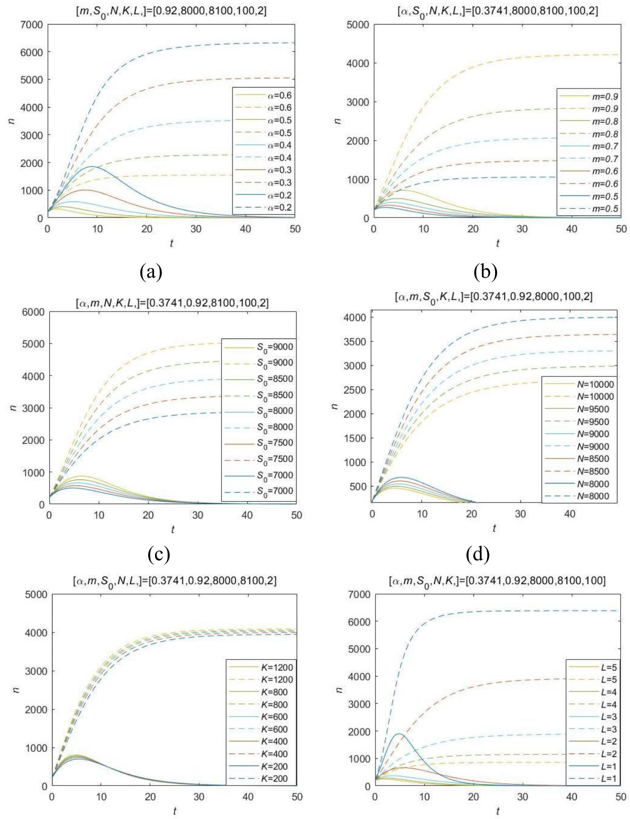

FIGURE 8. Influence on directed network dissemination with the change of a single parameter where: (a) only α changes; (b) only m changes; (c) only s_0 changes; (d) only N changes; (e) only K changes; (f) only L changes.

has almost the same effect on V_d and V_a, where parameters α, m, S_0, and K have positive effects on both, contrarily to parameters N and L. Moreover, high population density and increased frequency of contact between people augment the rate of information dissemination.

Furthermore, Figure 8 highlights the effect of changes in the single-parameter average immunization rate α, the average forwarding probability m, the initial value of the total number of susceptible groups S_0, the total number of users N, the maximum value of node ingress in the network K, and the maximum value of the out-degree node L regarding the instantaneous number of forwarding users $F(t)$ and the cumulative number of forwarding users $C(t)$. When parameters α and N decrease, $F(t)$ becomes steeper, peaks higher, and reaches its peak more quickly. Similarly, $C(t)$ will stabilize earlier and reach a larger final size. On the contrary, as the m, S_0, K, and L increases, the burst of information becomes faster, $F(t)$ peak becomes higher, and $C(t)$ also increases.

C. DECISION-MAKING SUPPORT

Aiming at the self-organized network communication scenario formed spontaneously by users, a directed network information dissemination dynamics model is proposed. The process of generating, developing, and influencing public opinion is efficiently analyzed, revealing the changing rules and paths of public opinion information dissemination through self-organized networks. Based on our proposed model, the government and related departments can better understand the reasons of public opinion events, predict the evolution trend based on the existing data, and consequently formulate corresponding policies and measures to quickly assess the advantages and disadvantages of decision-making.

Combined with multi-parameter sensitivity analysis carried out by PRCCs, it is evident that, in the directed network model, changes in parameters, such as the average forwarding probability m and initial value of the total number of susceptible groups S_0, have different degrees of influence on the indicators of online public opinion dissemination, such as $\Re_0$, C_s, and F_{max}. Specifically, to expedite the rapid expansion and engagement of spontaneously formed topics, leading to more forwarding and discussions, we can make the content of the topic entries richer and more interesting to attract more users to browse, read, and participate in the discussion to increase the initial value of the total number of susceptible groups S_0. Moreover, as the rapid increase in topic popularity moves on, it will keep users active for a long time, leading to increase the average forwarding probability m and decrease the average immunization rate α resulting in a higher peak dissemination and a larger final size of the message. Similarly, if the objective consists of making the discussion of a negative impacts topic decrease rapidly, opinion leaders' interventions can be deployed, leading to increase the average immunization rate α and the maximum value of node incidence in the network K to render the duration of information dissemination t_i decrease sharply and stop the spread of the negative public opinion.

IV. CONCLUSION

To sum up, our research explores the intricate dynamics of information dissemination within directed networks, considering the unique behavior patterns inherent in user-generated networks and the varying information contact rates that arise from these directed structures. We meticulously analyzed the directional flow of communication between network nodes, distinguishing dissemination pathways through edges. Additionally, we categorize users based on their connections, such as in-degree and out-degree, which reflect their influence and reach within the network.

We present a dynamics model of information dissemination in directed networks. Through performance analysis of the model, we utilize typical topic information from Sina Weibo, characterized by spontaneously dissemination and discussion nature, to verify its adherence to actual information propagation law and proves the influence of directed network characteristics on information propagation. According to results analysis, we found that the in-degree and out-degree of directed networks and other parameters have different impact degrees on the information dissemination stability value, peak value, time, and rate.

By enhancing topic content, increasing the size of the initial susceptible population, and engaging in discussions to augment the average probability of forwarding, rapid topic explosion will be facilitated and more attention and discussion could be generated. In summary, our research provides important theoretical and practical guidance for information dissemination in mixed scenarios.

In the realm of online information dissemination, users' dissemination behavior is subject to a variety of factors, including personal characteristics, social environment, and cultural background, leading to the diversity and uncertainty of behavior. Therefore, it is difficult to achieve a comprehensive range of factors considered during the modeling process, leading to a lack of understanding of the whole picture of information dissemination. Meanwhile, the application of the model to real-world scenarios and guidance in policy formulation is not consistent, and there is a lack in the prediction trends after information interventions.

Therefore, future research can consider more factors that can influence information dissemination into the model, add intervention strategies, and construct an information dissemination dynamics model based on complex behaviors. This will help in describing, more accurately, the trend of information dissemination.

REFERENCES

[1] S.-G. Liao and S.-P. Yi, "Modeling and dynamic analysis of knowledge transmission process: A model considering individual perception of knowledge value," *Commun. Nonlinear Sci. Numer. Simul.*, vol. 95, Apr. 2021, Art. no. 105598.

[2] H. Huang, T. Wang, M. Hu, M. Dong, and L. Lai, "Node attitude aware information dissemination model based on evolutionary game in social networks," *Mobile Netw. Appl.*, vol. 26, no. 1, pp. 114–129, Feb. 2021.

[3] X. Deng, "Misconduct and regulation of information dissemination in network media," *Sci. J. Econ. Manag. Res.*, vol. 3, no. 1, pp. 105–109, 2021.

[4] Y. Du and M. Zhou, "Analysis of online media transmission based on the theory of giant fluctuation of self-organizing network," in *Proc. Int. Conf. Intell. Media, Big Data Knowl. Mining (IMBDKM)*, Mar. 2023, pp. 31–36.

[5] D. Galvan, L. Effting, H. Cremasco, and C. A. Conte-Junior, "The spread of the COVID-19 outbreak in Brazil: An overview by Kohonen self-organizing map networks," *Medicina*, vol. 57, no. 3, p. 235, Mar. 2021.

[6] P. Melin, J. C. Monica, D. Sanchez, and O. Castillo, "Analysis of spatial spread relationships of coronavirus (COVID-19) pandemic in the world using self organizing maps," *Chaos, Solitons Fractals*, vol. 138, Sep. 2020, Art. no. 109917.

[7] P. Melin and O. Castillo, "Spatial and temporal spread of the COVID-19 pandemic using self organizing neural networks and a fuzzy fractal approach," *Sustainability*, vol. 13, no. 15, p. 8295, Jul. 2021.

[8] Y. Lin, M. Wang, X. Zhou, G. Ding, and S. Mao, "Dynamic spectrum interaction of UAV flight formation communication with priority: A deep reinforcement learning approach," *IEEE Trans. Cognit. Commun. Netw.*, vol. 6, no. 3, pp. 892–903, Sep. 2020.

[9] Q. Xuan, J. Wang, M. Zhao, J. Yuan, C. Fu, Z. Ruan, and G. Chen, "Subgraph networks with application to structural feature space expansion," *IEEE Trans. Knowl. Data Eng.*, vol. 33, no. 6, pp. 2776–2789, Jun. 2021.

[10] J. Zhou, C. Hu, J. Chi, J. Wu, M. Shen, and Q. Xuan, "Behavior-aware account de-anonymization on Ethereum interaction graph," *IEEE Trans. Inf. Forensics Security*, vol. 17, pp. 3433–3448, 2022.

[11] J. Zhou, Z. Chen, M. Du, L. Chen, S. Yu, G. Chen, and Q. Xuan, "RobustECD: Enhancement of network structure for robust community detection," *IEEE Trans. Knowl. Data Eng.*, vol. 35, no. 1, pp. 842–856, Jan. 2023.

[12] Z. Zheng and A. K. Bashir, "Graph-enabled intelligent vehicular network data processing," *IEEE Trans. Intell. Transp. Syst.*, vol. 23, no. 5, pp. 4726–4735, May 2022.

[13] Z. Zheng, B. Du, C. Zhao, and P. Xie, "Path merging based betweenness centrality algorithm in delay tolerant networks," *IEEE J. Sel. Areas Commun.*, vol. 41, no. 10, pp. 3133–3145, Oct. 2023.

[14] H. Zhang, S. Peng, J. Zhang, and Y. Lin, "Big data analysis and prediction of electromagnetic spectrum resources: A graph approach," *Sustainability*, vol. 15, no. 1, p. 508, Dec. 2022.

[15] M. Karsai, N. Perra, and A. Vespignani, "Time varying networks and the weakness of strong ties," *Sci. Rep.*, vol. 4, no. 1, p. 4001, Feb. 2014.

[16] Y. Huang, Z. Meng, J. Sun, and G. Wang, "Distributed constrained nonlinear least-squares estimation algorithm over unbalanced directed networks," *IEEE Trans. Netw. Sci. Eng.*, vol. 11, no. 1, pp. 982–993, Feb. 2024.

[17] Y.-Y. Liu, J.-J. Slotine, and A.-L. Barabási, "Controllability of complex networks," *Nature*, vol. 473, no. 7346, pp. 167–173, May 2011.

[18] J. Wei, H. Y. J. Huang, and P. Lin, "Public opinion transmission of microblog network based on improved SIR model," *Inf. Sci.*, vol. 37, no. 6, pp. 16–22, 2019.

[19] Q. Wu and L. He, "Network information dissemination model and development analysis under new media environment," in *Proc. 2nd Int. Conf. Inf. Sci. Educ. (ICISE-IE)*, Chongqing, China, Nov. 2021, pp. 54–57.

[20] F. Yin, Y. She, Q. Yang, H. Pang, Y. Huang, and J. Wu, "Modeling and analyzing dynamic information propagation on sina Weibo in a semi-directed network," *J. Biol. Syst.*, vol. 32, pp. 219–237, 2023.

[21] D. Wang and P. Zhu, "Graph reduction in a path information-based rough directed graph model," *Soft Comput.*, vol. 26, no. 9, pp. 4171–4186, May 2022.

[22] Z. Wang and C. Xia, "Co-evolution spreading of multiple information and epidemics on two-layered networks under the influence of mass media," *Nonlinear Dyn.*, vol. 102, no. 4, pp. 3039–3052, Dec. 2020.

[23] J. Yan, Y. Zhou, S. Wang, and J. Li, "To share or not to share? Credibility and dissemination of electric vehicle-related information on WeChat: A moderated dual-process model," *IEEE Access*, vol. 7, pp. 46808–46821, 2019.

[24] H. Liu, W. Liu, and Y. Li, "Private information dissemination and noise trading: Implications for price efficiency and market liquidity," *Sustainability*, vol. 14, no. 18, p. 11624, Sep. 2022.

[25] Z. Long, R. Alharthi, and A. E. Saddik, "NeedFull—A tweet analysis platform to study human needs during the COVID-19 pandemic in new York state," *IEEE Access*, vol. 8, pp. 136046–136055, 2020, doi: 10.1109/ACCESS.2020.3011123.

[26] B. Zhang, Y. Bai, Q. Zhang, J. Lian, and M. Li, "An opinion-leader mining method in social networks with a phased-clustering perspective," *IEEE Access*, vol. 8, pp. 31539–31550, 2020, doi: 10.1109/ACCESS.2020.2972997.

[27] D. J. Daley and D. G. Kendall, "Epidemics and rumors," *Nature*, vol. 204, no. 4963, p. 1118, 1964.

[28] J. Wang, X. Zhang, W. Liu, and P. Li, "Spatiotemporal pattern evolution and influencing factors of online public opinion—Evidence from the early-stage of COVID-19 in China," *Heliyon*, vol. 9, no. 9, Sep. 2023, Art. no. e20080.

[29] I. Cooper, A. Mondal, and C. G. Antonopoulos, "A SIR model assumption for the spread of COVID-19 in different communities," *Chaos, Solitons Fractals*, vol. 139, Oct. 2020, Art. no. 110057.

[30] Y.-C. Chen, P.-E. Lu, C.-S. Chang, and T.-H. Liu, "A time-dependent SIR model for COVID-19 with undetectable infected persons," *IEEE Trans. Netw. Sci. Eng.*, vol. 7, no. 4, pp. 3279–3294, Oct. 2020.

[31] Y. Wang, F. Qing, J.-P. Chai, and Y.-P. Ni, "Spreading dynamics of a 2SIH2R, rumor spreading model in the homogeneous network," *Complexity*, vol. 2021, pp. 1–9, Feb. 2021.

[32] L. Zhu, F. Yang, G. Guan, and Z. Zhang, "Modeling the dynamics of rumor diffusion over complex networks," *Inf. Sci.*, vol. 562, pp. 240–258, Jul. 2021.

[33] F. Yin, X. Shao, and J. Wu, "Nearcasting forwarding behaviors and information propagation in Chinese Sina-Microblog," *Math. Biosci. Eng.*, vol. 16, no. 5, pp. 5380–5394, 2019.

[34] F. Yin, X. Xia, Y. Pan, Y. She, X. Feng, and J. Wu, "Sentiment mutation and negative emotion contagion dynamics in social media: A case study on the Chinese sina microblog," *Inf. Sci.*, vol. 594, pp. 118–135, May 2022.

[35] X. Feng, X. Huo, B. Tang, S. Tang, K. Wang, and J. Wu, "Modelling and analyzing virus mutation dynamics of chikungunya outbreaks," *Sci. Rep.*, vol. 9, no. 1, p. 2860, Feb. 2019.

ZHUO YANG received the bachelor's and Ph.D. degrees from Harbin Engineering University, in 2006 and 2011, respectively. He is currently a Lecturer with the School of Public Security Information Technology and Intelligence, Criminal Investigation Police University of China. Before that, he was a Visiting Scholar with Carleton University, Ottawa, ON, Canada. His current research interests include dynamic systems, public opinion analysis, and data mining.

YAN GUO received the B.S. degree from the College of Information and Communication Engineering, Communication University of China, in 2023, where she is currently pursuing the M.S. degree. Her research interests include information propagation dynamics and data analysis.

HONGYU PANG received the M.S. degree, in 2022. Her research interests include information propagation dynamics and data analysis.

FULIAN YIN (Member, IEEE) received the bachelor's, master's, and Ph.D. degrees from Harbin Engineering University, in 2005, 2007, and 2010, respectively. She is currently a Professor and a Ph.D. Tutor with the College of Information and Communication Engineering, Communication University of China. Before that, she was a Visiting Scholar with York University, Toronto, ON, Canada. Her current research interests include dynamic systems, public opinion analysis, and data mining.