IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received 27 March 2024, accepted 11 April 2024, date of publication 15 April 2024, date of current version 26 April 2024.

Digital Object Identifier 10.1109/ACCESS.2024.3389295

RESEARCH ARTICLE

Decision-Making Approach Based on Bipolar Complex Fuzzy Uncertain Linguistic Aggregation Operators

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This work was supported by King Saud University, Riyadh, Saudi Arabia, through the Researchers Supporting Project under Grant RSPD2024R749.

ABSTRACT There are many computer vision technologies available, each having advantages and disadvantages of its own. Effective visual data analysis requires choosing the best technology and giving its deployment top priority. A strong framework for decision-making (DM) is required, one that can manage the ambiguities, imprecision, dual aspects, and linguistic terms present in real-world computer vision applications. Thus, in this script, we investigate the DM framework under the structure of a bipolar complex fuzzy uncertain linguistic set (BCFULS). The theory of BCFULS is also devised in this manuscript, which can model the data that contains ambiguities, extra fuzzy information, dual aspects, and linguistic terms simultaneously. For this DM framework, we inaugurate averaging/geometric aggregation operators (AOs) within the structure of BCFULS and analyze the related properties. After that, we employ the inaugurated DM framework to prioritize the numerous types of computer vision by considering artificial data and achieve that "Feature Matching" is the finest computer vision. Finally, this script contains a comparative analysis of this work with numerous current works to depict the supremacy and advantages of the invented work.

INDEX TERMS Computer vision, aggregation operators, decision making, fuzzy theory.

I. INTRODUCTION

Since Zadeh's fuzzy set (FS) [1] extended the classical set theory and interpreted by the satisfaction degree confined in [0, 1]. FS is the influential substitute to the theory of probability to describe the vagueness, imprecision, and ambiguities in numerous areas. FS is also a basic and powerful technique in computational intelligence and can interpret the solution to numerous dilemmas of computer vision, control, reasoning planning, etc. Progressively, it has been noticed that in various cases the features of objects or things are not described properly by the satisfaction degree due to the intricacy of the information and the uncertainty of the human brain. To conquer this, Zhang [2] invented the bipolar FS (BFS) in 1994 which is interpreted by a positive satisfaction degree confined in [0, 1] and a negative satisfaction degree confined in [−1, 0]. The concept of FS received impressive success in numerous areas to cope with uncertainties but with more and more ambiguities and imprecision in the genuine-life dilemmas, various modifications of FS have been constructed by numerous researchers. A complex FS (CFS) is one of the modifications of FS invented by Ramot et al. [3] in 2002. CFS is a powerful technique to deal with two-dimensional information which has ambiguities and uncertainties. The Cartesian form of satisfaction degree in the structure of CFS was invented by Tamir et al. [4]. Consider a CEO of a multi-national company who wants to decide about the development and launching of a new product and whether

The associate editor coordinating the review of this manuscript and approving it for publication was Mohammad Shorif Uddin.

he should launch a new product or not. For making this decision, he must consider 4 various aspects that are product earning, product loss, positive effect on the customers, and negative effect on the customers. To cope with this sort of situation, Mahmood and Ur Rehman [5] invented a bipolar CFS (BCFS) which is interpreted by a positive satisfaction degree confined in $[0,\ 1]+\iota\ [0,\ 1]$ and a negative satisfaction degree confined in $[-1,\ 0]+\iota\ [-1,\ 0]$. However, in various genuine-life dilemmas, it is simpler and appropriate for the experts to present qualitative assessments instead of quantitative descriptions. Thus, the concept of linguistic variables (LVs) investigated by Zadeh [6] has developed as a great technique for experts to interpret qualitative assessment data. After that, Xu [7] invented the notion of uncertain linguistic variables (ULVs).

A. LITERATURE REVIEW

The field of computer vision in artificial intelligence (AI) makes up the system and computers to gather valuable data from digital pictures, movies, and other visual inputs and to perform operations or offer ideas about that information. If AI enables systems to comprehend, computer vision enables them to see, observe, and comprehend. Except for the advantage of human eyesight, computer vision performs identically to human vision. Human eyes have the advantage of knowing how to discern among objects, calculate their range from the observer, detect whether they are moving, and assess if a picture is real with a context history. Unlike human perception, which needs cells in the retina, nerve cells, and a visible brain, computer vision allows robots to perform identical tasks in an even shorter period by utilizing a camera, information, and programs. Because it can evaluate hundreds of products or processes per minute while picking up on minor faults or errors, a system that has been trained to verify products or manage an industrial asset can quickly outperform human skills. Voulodimos et al. [8] analyzed deep learning for computer vision. Baumgart [9] investigated polyhedron descriptions of computer vision. Weinstein [10] discussed animal ecology based on computer vision.

A vast spectrum of technical innovation is enabled by computer vision. It enables face detection programs to connect images of people's faces to their identities, auto cars to safely explore highways and roads, and virtual or augmented software to blend virtual elements with natural photographs. Applications of computer vision are utilized in various fields to enhance customer satisfaction, reduce expenses, and increase safety. It helps manufacturers identify flawed goods in the production plant and stop them from being supplied to customers. To evaluate car damage and decrease fraud throughout the claims process, insurance adjusters employ it. It is utilized by medical experts to discover health problems by analyzing X-rays, Magnetic resonance imaging (MRI), and ultrasounds. Before completing major transactions, banks utilize it to authenticate the identity of their customers. The study of cell biology relying on computer vision was described by Danuser [11]. Freeman et al. [12] analyzed interactive computer graphics based on computer vision. Xu et al. [13] studied the role of computer vision techniques in the field of construction. Meer [14] investigated the various techniques for computer vision. Meer et al. [15] studied the regression mechanism for computer vision. Veeraraghavan et al. [16] invented algorithms based on computer vision for intersection monitoring. Tahani and Keller [17] utilized fuzzy integral to study the information combination in computer vision. Huntsberger et al. [18] described the ambiguities in computer vision utilizing FS. FS in computer vision was discussed by Sobrevilla and Montseny [19].

Poulik et al. [20] analyzed the Randic index of graphs with fuzzy information. Hussain et al. [21] utilized fuzzy logic inference to devise an efficient power management algorithm for electric vehicle parking lots. Hussain et al. [22] devised a charging scheme based on fuzzy logic. Zhang et al. [23] analyzed reallocation-based level flash translation layers for smart cells. He et al. [24] investigated an image classification model for breast cancer. Hussain et al. [25] utilized a fuzzy inference system to investigate the waiting time for electric vehicles. Further, the improvement of the performance of the power grid in electric vehicle parking with the assistance of fuzzy logic was analyzed by Hussain et al. [26]. Keller and Krishnapuram [27] studied FS mechanisms in computer vision. Keller and Matsakis [28] described high-level computer vision employing the notion of FS. Hussain et al. [29] devised a two-layer decentralized charging technique relying on fuzzy data fusion and Hussain et al. [30] discussed a heuristic charging cost algorithm. The translation layer for NAND was investigated by Luo et al. [31]. Zhao et al. [32] discussed the SSD-based KV storage framework. Pajares et al. [33] employed fuzzy cognitive maps in computer vision. Montseny and Sobrevilla [34] described various algorithms in computer vision based on the application of fuzzy approaches. Ikidid et al. [35] investigated a fuzzy inference-based system for traffic lights. By employing fuzzy logic, the hybrid coordination scheme for electric vehicles was anticipated by Hussain et al. [36], and by using a fuzzy integer linear program, Hussain et al. [37] enhanced the efficiency of electric vehicle charging stations. Anari et al. [38] discussed trapezoidal truth grades in mining fuzzy association rules relying on learning automata. Mingwei et al. [39] deduced AOs under the picture fuzzy information. Das et al. [40] discussed picture fuzzy $\phi-$teolerance graphs. Lin et al. [41] analyzed correlation coefficient measures for Pythagorean FS. Numerous authors utilized bipolar fuzzy (BF) information to deduce various theories. Jana et al. [42] employed the model of BFS and Dombi t-norm and conorm to deduce Dombi AOs for BFS. Wei et al. [43] employed the structure of BFS and Hamacher t-norm and conorm to analyze Hamacher AOs for BFS. Riaz et al. [44] inaugurated sine trigonometric AOs for BFS. The two processes of MADM that is TOPSIS and

ELECTRE-I for BFS were Akram et al. [45]. Akram and Akmal [46] deduced the application of BFS in the graph structure. Poulik and Ghorai [47] devised the notion of connectivity in bipolar fuzzy graphs. Tamir et al. [48] discussed CFS and complex fuzzy logic. Yazdanbakhsh and Dick [49] gave a review of CFS and logic. Bi et al. [50] analyzed averaging AOs and Bi et al. [51] discussed geometric AOs for CFS. Mahmood et al. [52] studied Aczel-Alsina AOs, and Mahmood et al. [53] studied AOs based on BCFS. Ur Rehman and Mahmood [54] utilized the notion of BCFS in medical diagnosis and pattern recognition. Mahmood and Ur Rehman [55] invented Dombi AOs.

B. MOTIVATION

Various researchers constructed various uncertain linguistic sets (ULS) for instance, Liu and Jin [56] investigated intuitionistic ULS (IULS), Wu and Wei [57] analyzed picture ULS (PULS), Naeem et al. [58] established interval-valued PULS (IVPULS), Qiyas et al. [59] described spherical ULS (SULS), Wang and Ullah [60] analyzed T-spherical, Gao et al. [61] invented interval-valued bipolar ULS (IVBULS). From this discussion, we noted that the concept of ULS and BCFS achieved significant success in the past few years. But with more and more ambiguities and imprecision in genuine-life dilemmas, we can face a situation where separately these concepts would be failed, for instance, the condition of a house may be worse than "extremely good" $(\hat{s}_5)$ and better than "average" $(\hat{s}_3)$. The positive satisfaction degree of $[\hat{s}_3,\ \hat{s}_5]$ is $0.7+\iota\,0.3$ and the negative satisfaction degree of $[\hat{s}_3,\ \hat{s}_5]$ is $-0.3-\iota\,0.5$. This information would be modeled as $([\hat{s}_3,\ \hat{s}_5],\ (0.7+\iota\,0.3,\ -0.3-\iota\,0.5))$ which cannot be handled by any of the prevailing concepts and not by the ULS and BCFS. To overcome this research gap, in this script, we invent the structure of BCFULS along with basic operations. The main contribution of this manuscript is devised as follows.

- We construct the theory of BCFULS and devise its associated laws.
- We invent aggregation operators (AOs) such as bipolar complex fuzzy (BCF) uncertain linguistic weighted averaging (BCFULWA), BCF uncertain linguistic ordered weighted averaging (BCFULOWA), BCF uncertain linguistic hybrid averaging (BCFULHA), BCF uncertain linguistic weighted geometric (BCFULWG), BCF uncertain linguistic ordered weighted geometric (BCFULOWG) and BCF uncertain linguistic hybrid geometric (BCFULHG) in the environment of BCFULS.
- We devise a technique of decision-making (DM) under BCFULS by employing the invented operators.
- After that, we prioritize various computer vision techniques by utilizing the invented technique of DM.
- Finally, this script contains a comparative analysis of this work with numerous current works to depict the supremacy and advantages of the invented work.

The invented BCFULS is the generalization of various structures such as bipolar complex fuzzy linguistic set (BCFLS), complex fuzzy ULS (CFULS), complex fuzzy linguistic set (CFLS), bipolar fuzzy ULS (BFULS), bipolar fuzzy linguistic set (BFLS), etc. The flowchart in Figure 1 shows this generalization.

C. ORGANIZATION OF THE WORK

The script is constructed as: In Section II, we overview the vital notions and their operations. In Section III, we invent a new concept called BCFULS along with some essential properties. In Section IV, we invented AOs for the model of BCFULS that is BCFULWA, BCFULOWA, BCFULHA, BCFULWG, BCFULOWG, and BCFULHG operators. In Section V, we analyze the various forms of computer vision and investigate a numerical example to show the benefits of the proposed work in the field of computer vision. Section VI contains the comparative study and Section VII contains the conclusion.

II. PRELIMINARIES

This part contains certain fundamental definitions of various prevailing notions and their operations.

Definition 1 [5]: The underneath structure

$$\mathfrak{X} = \left\{\left(\acute{\alpha},\ \left(\mathfrak{F}^{P}_{\mathfrak{X}}(\acute{\alpha}),\ \mathfrak{F}^{N}_{\mathfrak{X}}(\acute{\alpha})\right)\right)\ |\ \acute{\alpha}\in \mathcal{A}_u\right\} = \left\{\left(\acute{\alpha},\ \begin{pmatrix}\mathfrak{F}^{RP}_{\mathfrak{X}}(\acute{\alpha})+\iota\,\mathfrak{F}^{IP}_{\mathfrak{X}}(\acute{\alpha}),\\ \mathfrak{F}^{RN}_{\mathfrak{X}}(\acute{\alpha})+\iota\,\mathfrak{F}^{IN}_{\mathfrak{X}}(\acute{\alpha})\end{pmatrix}\right)\ |\ \acute{\alpha}\in \mathcal{A}_u\right\} \quad (1)$$

Would be considered as the structure of BCFS. Where $\mathfrak{F}^{P}_{\mathfrak{X}}(\acute{\alpha})$ depict the positive satisfaction degree and $\mathfrak{F}^{N}_{\mathfrak{X}}(\acute{\alpha})$ depict the negative satisfaction degree. Both $\mathfrak{F}^{P}_{\mathfrak{X}}(\acute{\alpha}), \mathfrak{F}^{N}_{\mathfrak{X}}(\acute{\alpha})$ will be confined in the unit square of a complex plane and $\iota=\sqrt{-1}$. The bipolar complex fuzzy number (BCFN) would be considered as $\mathfrak{X}=\left(\mathfrak{F}^{P}_{\mathfrak{X}},\ \mathfrak{F}^{N}_{\mathfrak{X}}\right)=\left(\mathfrak{F}^{RP}_{\mathfrak{X}}+\iota\,\mathfrak{F}^{IP}_{\mathfrak{X}},\ \mathfrak{F}^{RN}_{\mathfrak{X}}+\iota\,\mathfrak{F}^{IN}_{\mathfrak{X}}\right)$.

Definition 2 [55]: The structures

$$\mathcal{S}(\mathfrak{X}) = \frac{1}{4}\begin{pmatrix}2+\mathfrak{F}^{RP}_{\mathfrak{X}}+\mathfrak{F}^{IP}_{\mathfrak{X}}\\ +\mathfrak{F}^{RN}_{\mathfrak{X}}+\mathfrak{F}^{IN}_{\mathfrak{X}}\end{pmatrix},\quad \mathcal{S}(\mathfrak{X})\in[0,\ 1] \quad (2)$$

$$\mathcal{H}(\mathfrak{X}) = \frac{\mathfrak{F}^{RP}_{\mathfrak{X}}+\mathfrak{F}^{IP}_{\mathfrak{X}}-\mathfrak{F}^{RN}_{\mathfrak{X}}-\mathfrak{F}^{IN}_{\mathfrak{X}}}{4},\quad \mathcal{H}(\mathfrak{X})\in[0,\ 1] \quad (3)$$

Would determine the score and accuracy values of a BCFN $\mathfrak{X}=\left(\mathfrak{F}^{P}_{\mathfrak{X}},\ \mathfrak{F}^{N}_{\mathfrak{X}}\right)$ respectively.

Definition 3 [55]: Consider two BCFNs

$$\mathfrak{X}_1 = \left(\mathfrak{F}^{P}_{\mathfrak{X}_1},\ \mathfrak{F}^{N}_{\mathfrak{X}_1}\right) = \left(\mathfrak{F}^{RP}_{\mathfrak{X}_1}+\iota\,\mathfrak{F}^{IP}_{\mathfrak{X}_1},\ \mathfrak{F}^{RN}_{\mathfrak{X}_1}+\iota\,\mathfrak{F}^{IN}_{\mathfrak{X}_1}\right)$$

and $\mathfrak{X}_2=\left(\mathfrak{F}^{P}_{\mathfrak{X}_2},\ \mathfrak{F}^{N}_{\mathfrak{X}_2}\right)=\left(\mathfrak{F}^{RP}_{\mathfrak{X}_2}+\iota\,\mathfrak{F}^{IP}_{\mathfrak{X}_2},\ \mathfrak{F}^{RN}_{\mathfrak{X}_2}+\iota\,\mathfrak{F}^{IN}_{\mathfrak{X}_2}\right)$, and $\hbar\geq 0$, then

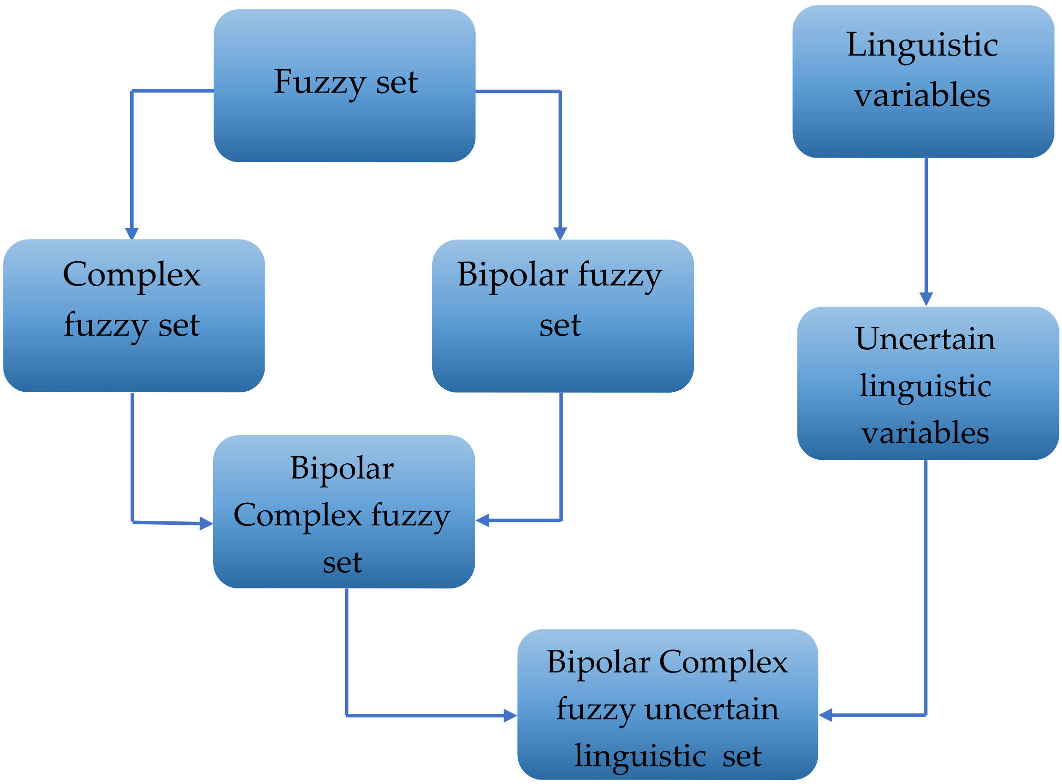

FIGURE 1. The flowchart of the invented theory.

1)

$$\text{Ӿ}_1 \oplus \text{Ӿ}_2 = \begin{pmatrix} \digamma^{RP}_{\text{Ӿ}_1} + \digamma^{RP}_{\text{Ӿ}_2} - \digamma^{RP}_{\text{Ӿ}_1}\digamma^{RP}_{\text{Ӿ}_2} \\ +\iota\left(\digamma^{IP}_{\text{Ӿ}_1} + \digamma^{IP}_{\text{Ӿ}_2} - \digamma^{IP}_{\text{Ӿ}_1}\digamma^{IP}_{\text{Ӿ}_2}\right), \\ -\left(\digamma^{RN}_{\text{Ӿ}_1}\digamma^{RN}_{\text{Ӿ}_2}\right) + \iota\left(-\left(\digamma^{IN}_{\text{Ӿ}_1}\digamma^{IN}_{\text{Ӿ}_2}\right)\right) \end{pmatrix}$$

2)

$$\text{Ӿ}_1 \otimes \text{Ӿ}_2 = \begin{pmatrix} \digamma^{RP}_{\text{Ӿ}_1}\digamma^{RP}_{\text{Ӿ}_2} + \iota\,\digamma^{IP}_{\text{Ӿ}_1}\digamma^{IP}_{\text{Ӿ}_2}, \\ \digamma^{RN}_{\text{Ӿ}_1} + \digamma^{RN}_{\text{Ӿ}_2} + \digamma^{RN}_{\text{Ӿ}_1}\digamma^{RN}_{\text{Ӿ}_2} \\ \iota\left(\digamma^{IN}_{\text{Ӿ}_1} + \digamma^{IN}_{\text{Ӿ}_2} + \digamma^{IN}_{\text{Ӿ}_1}\digamma^{IN}_{\text{Ӿ}_2}\right) \end{pmatrix}$$

3)

$$\hbar\text{Ӿ}_1 = \begin{pmatrix} 1 - \left(1 - \digamma^{RP}_{\text{Ӿ}_1}\right)^{\hbar} \\ +\iota\left(1 - \left(1 - \digamma^{IP}_{\text{Ӿ}_1}\right)^{\hbar}\right), \\ -\left|\digamma^{RN}_{\text{Ӿ}_1}\right|^{\hbar} + \iota\left(-\left|\digamma^{IN}_{\text{Ӿ}_1}\right|^{\hbar}\right) \end{pmatrix}$$

4)

$$\text{Ӿ}_1^{\hbar} = \begin{pmatrix} \left(\digamma^{RP}_{\text{Ӿ}_1}\right)^{\hbar} + \iota\left(\digamma^{IP}_{\text{Ӿ}_1}\right)^{\hbar}, \\ -1 + \left(1 + \digamma^{RN}_{\text{Ӿ}_1}\right)^{\hbar} \\ +\iota\left(-1 + \left(1 + \digamma^{IN}_{\text{Ӿ}_1}\right)^{\hbar}\right) \end{pmatrix}.$$

Definition 4 [6]: Take a set $S = \{\hat{s}_\phi \mid \phi = 0, 1, \ldots, ƙ - 1\}$ of odd cardinality, $ƙ$ would be assumed as a linguistic term set (LTS). Each $\hat{s}_\phi$ signifies the imaginable argument of the LV. Further, the set $\hat{s} = \left\{\hat{s}_\delta \mid \delta \in \mathbb{R}^+\right\}$ can be assumed as continue LTS, if this set satisfies the underneath axioms.

1) The LTS is ordered: $\hat{s}_\delta < \hat{s}_\varrho$ iff $\delta < \varrho$,
2) Negation operator: $Neg\left(\hat{s}_\delta\right) = \hat{s}_\varrho$ such that $\varrho = ƙ - \delta$
3) Max operator: if $\delta \leq \varrho$, then $\max\left(\hat{s}_\delta, \hat{s}_\varrho\right) = \hat{s}_\varrho$
4) Min operator: if $\delta \geq \varrho$ then $\min\left(\hat{s}_\delta, \hat{s}_\varrho\right) = \hat{s}_\varrho$

For example, S can be represented as

$$S = \begin{Bmatrix} \hat{s}_0 = \textit{Exceptionally poor}, \hat{s}_1 = \textit{poor}, \\ \hat{s}_2 = \textit{Slightly poor}, \hat{s}_3 = \textit{average}, \hat{s}_4 = \textit{good}, \\ \hat{s}_5 = \textit{extremly good}, \hat{s}_6 = \textit{excellent} \end{Bmatrix}$$

Definition 5 [7]: The $\hat{s}^*$ would be known as an uncertain linguistic variable (ULV), where $\hat{s}^* = \left[\hat{s}_\delta, \hat{s}_\varrho\right]$, $\hat{s}_\delta, \hat{s}_\varrho \in S$, $\hat{s}_\delta$ signifies the lower limit and $\hat{s}_\varrho$ signifies the upper limit.

III. BCF UNCERTAIN LINGUISTIC SET

To deal with uncertain linguistic variables (ULVs) and BCF information at the same time, here, we publicize the theory of BCF uncertain linguistic set (BCFULS). Further, we analyze certain properties of the publicized BCFULS.

Definition 6: Assume that $\text{Ά}_{\mathfrak{u}}$ as a universal set. A BCFULS over $\text{Ά}_{\mathfrak{u}}$ would be the following form

$$\text{Ӿ} = \left\{\left(\acute{\alpha}, \begin{pmatrix} \left[\hat{s}^{\mathcal{L}}_{\delta(\acute{\alpha})}, \hat{s}^{\mathcal{U}}_{\delta(\acute{\alpha})}\right], \\ \left(\digamma^{P}_{\text{Ӿ}}(\acute{\alpha}), \digamma^{N}_{\text{Ӿ}}(\acute{\alpha})\right) \end{pmatrix}\right) \mid \acute{\alpha} \in \text{Ά}_{\mathfrak{u}}\right\}$$

$$= \left\{ \left(\acute{\alpha}, \left(\begin{array}{c} \left[\hat{s}^{\mathcal{L}}_{\delta(\acute{\alpha})}, \hat{s}^{\mathcal{U}}_{\delta(\acute{\alpha})} \right], \\ \left(\begin{array}{c} \mathcal{F}^{RP}_{\mathfrak{X}}(\acute{\alpha}) + \iota \mathcal{F}^{IP}_{\mathfrak{X}}(\acute{\alpha}), \\ \mathcal{F}^{RN}_{\mathfrak{X}}(\acute{\alpha}) + \iota \mathcal{F}^{IN}_{\mathfrak{X}}(\acute{\alpha}) \end{array} \right) \end{array} \right) \right) \mid \acute{\alpha} \in \acute{A}_{u} \right\} \quad (4)$$

where, $\hat{s}^{\mathcal{L}}_{\delta(\acute{\alpha})}, \hat{s}^{\mathcal{U}}_{\delta(\acute{\alpha})} \in S$, $\mathcal{F}^{P}_{\mathfrak{X}}(\acute{\alpha})$ determine the positive satisfaction degree and $\mathcal{F}^{N}_{\mathfrak{X}}(\acute{\alpha})$ determine the negative satisfaction degree of every element $\acute{\alpha} \in \acute{A}_{u}$ to the linguistic term $\hat{s}_{\delta(\acute{\alpha})}$. Further, both positive and negative satisfaction degrees are confined in the unit square of a complex plane. $\mathfrak{X} = \left(\left[\hat{s}^{\mathcal{L}}_{\delta}, \hat{s}^{\mathcal{U}}_{\delta} \right], \left(\mathcal{F}^{P}_{\mathfrak{X}}, \mathcal{F}^{N}_{\mathfrak{X}} \right) \right) = \left(\left[\hat{s}^{\mathcal{L}}_{\delta}, \hat{s}^{\mathcal{U}}_{\delta} \right], \left(\mathcal{F}^{RP}_{\mathfrak{X}} + \iota \mathcal{F}^{IP}_{\mathfrak{X}}, \mathcal{F}^{RN}_{\mathfrak{X}} + \iota \mathcal{F}^{IN}_{\mathfrak{X}} \right) \right)$ would be considered BCFUL number (BCFULN).

Definition 7: Consider two BCFULNs that is

$$\begin{aligned} \mathfrak{X}_{\phi} &= \left(\left[\hat{s}^{\mathcal{L}}_{\delta_{\phi}}, \hat{s}^{\mathcal{U}}_{\delta_{\phi}} \right], \left(\mathcal{F}^{P}_{\mathfrak{X}_{\phi}}, \mathcal{F}^{N}_{\mathfrak{X}_{\phi}} \right) \right) \\ &= \left(\left[\hat{s}^{\mathcal{L}}_{\delta_{\phi}}, \hat{s}^{\mathcal{U}}_{\delta_{\phi}} \right], \left(\mathcal{F}^{RP}_{\mathfrak{X}_{\phi}} + \iota \mathcal{F}^{IP}_{\mathfrak{X}_{\phi}}, \mathcal{F}^{RN}_{\mathfrak{X}_{\phi}} + \iota \mathcal{F}^{IN}_{\mathfrak{X}_{\phi}} \right) \right), \ \phi = 1, 2 \end{aligned}$$

and $\hbar \geq 0$, then the operational laws of BCFULNs are below

1)

$$\mathfrak{X}_{1} \oplus \mathfrak{X}_{2} = \left(\begin{array}{c} \left[\hat{s}^{\mathcal{L}}_{\delta_{1}+\delta_{2}-\frac{\delta_{1}\delta_{2}}{\mathfrak{k}}}, \hat{s}^{\mathcal{U}}_{\delta_{1}+\delta_{2}-\frac{\delta_{1}\delta_{2}}{\mathfrak{k}}} \right], \\ \left(\begin{array}{c} \mathcal{F}^{RP}_{\mathfrak{X}_{1}} + \mathcal{F}^{RP}_{\mathfrak{X}_{2}} - \mathcal{F}^{RP}_{\mathfrak{X}_{1}} \mathcal{F}^{RP}_{\mathfrak{X}_{2}} \\ + \iota \left(\mathcal{F}^{IP}_{\mathfrak{X}_{1}} + \mathcal{F}^{IP}_{\mathfrak{X}_{2}} - \mathcal{F}^{IP}_{\mathfrak{X}_{1}} \mathcal{F}^{IP}_{\mathfrak{X}_{2}} \right), \\ - \left(\mathcal{F}^{RN}_{\mathfrak{X}_{1}} \mathcal{F}^{RN}_{\mathfrak{X}_{2}} \right) \\ + \iota \left(- \left(\mathcal{F}^{IN}_{\mathfrak{X}_{1}} \mathcal{F}^{IN}_{\mathfrak{X}_{2}} \right) \right) \end{array} \right) \end{array} \right)$$

2)

$$\mathfrak{X}_{1} \otimes \mathfrak{X}_{2} = \left(\begin{array}{c} \left[\hat{s}^{\mathcal{L}}_{\frac{\delta_{1}\delta_{2}}{\mathfrak{k}}}, \hat{s}^{\mathcal{U}}_{\frac{\delta_{1}\delta_{2}}{\mathfrak{k}}} \right], \\ \left(\begin{array}{c} \mathcal{F}^{RP}_{\mathfrak{X}_{1}} \mathcal{F}^{RP}_{\mathfrak{X}_{2}} \\ + \iota \mathcal{F}^{IP}_{\mathfrak{X}_{1}} \mathcal{F}^{IP}_{\mathfrak{X}_{2}}, \\ \mathcal{F}^{RN}_{\mathfrak{X}_{1}} + \mathcal{F}^{RN}_{\mathfrak{X}_{2}} + \mathcal{F}^{RN}_{\mathfrak{X}_{1}} \mathcal{F}^{RN}_{\mathfrak{X}_{2}} \\ \iota \left(\mathcal{F}^{IN}_{\mathfrak{X}_{1}} + \mathcal{F}^{IN}_{\mathfrak{X}_{2}} + \mathcal{F}^{IN}_{\mathfrak{X}_{1}} \mathcal{F}^{IN}_{\mathfrak{X}_{2}} \right) \end{array} \right) \end{array} \right)$$

3)

$$\hbar \mathfrak{X}_{1} = \left(\begin{array}{c} \left[\hat{s}^{\mathcal{L}}_{\mathfrak{k}-\mathfrak{k}\left(1-\frac{\delta_{1}}{\mathfrak{k}}\right)^{\hbar}}, \hat{s}^{\mathcal{U}}_{\mathfrak{k}-\mathfrak{k}\left(1-\frac{\delta_{1}}{\mathfrak{k}}\right)^{\hbar}} \right], \\ \left(\begin{array}{c} 1 - \left(1 - \mathcal{F}^{RP}_{\mathfrak{X}_{1}} \right)^{\hbar} \\ + \iota \left(1 - \left(1 - \mathcal{F}^{IP}_{\mathfrak{X}_{1}} \right)^{\hbar} \right), \\ - \left| \mathcal{F}^{RN}_{\mathfrak{X}_{1}} \right|^{\hbar} + \iota \left(- \left| \mathcal{F}^{IN}_{\mathfrak{X}_{1}} \right|^{\hbar} \right) \end{array} \right) \end{array} \right)$$

4)

$$\mathfrak{X}_{1}^{\hbar} = \left(\begin{array}{c} \left[\hat{s}^{\mathcal{L}}_{\mathfrak{k}\left(\frac{\delta_{1}}{\mathfrak{k}}\right)^{\hbar}}, \hat{s}^{\mathcal{U}}_{\mathfrak{k}\left(\frac{\delta_{1}}{\mathfrak{k}}\right)^{\hbar}} \right], \\ \left(\begin{array}{c} \left(\mathcal{F}^{RP}_{\mathfrak{X}_{1}} \right)^{\hbar} + \iota \left(\mathcal{F}^{IP}_{\mathfrak{X}_{1}} \right)^{\hbar}, \\ -1 + \left(1 + \mathcal{F}^{RN}_{\mathfrak{X}_{1}} \right)^{\hbar} \\ + \iota \left(-1 + \left(1 + \mathcal{F}^{IN}_{\mathfrak{X}_{1}} \right)^{\hbar} \right) \end{array} \right) \end{array} \right).$$

Definition 8: Consider a BCFULN $\mathfrak{X} = \left(\hat{s}_{\delta}, \left(\mathcal{F}^{P}_{\mathfrak{X}}, \mathcal{F}^{N}_{\mathfrak{X}} \right) \right) = \left(\hat{s}_{\delta}, \left(\mathcal{F}^{RP}_{\mathfrak{X}} + \iota \mathcal{F}^{IP}_{\mathfrak{X}}, \mathcal{F}^{RN}_{\mathfrak{X}} + \iota \mathcal{F}^{IN}_{\mathfrak{X}} \right) \right)$, then the score value of $\mathfrak{X}$ will be figured out by the below formula

$$\mathcal{S}(\mathfrak{X}) = \frac{1}{4} \left(\begin{array}{c} 2 + \mathcal{F}^{RP}_{\mathfrak{X}} + \mathcal{F}^{IP}_{\mathfrak{X}} \\ + \mathcal{F}^{RN}_{\mathfrak{X}} + \mathcal{F}^{IN}_{\mathfrak{X}} \end{array} \right) \times \left(\hat{s}^{\mathcal{L}}_{\delta} + \hat{s}^{\mathcal{U}}_{\delta} \right) \quad (5)$$

Definition 9: Consider a BCFULN $\mathfrak{X} = \left(\hat{s}_{\delta}, \left(\mathcal{F}^{P}_{\mathfrak{X}}, \mathcal{F}^{N}_{\mathfrak{X}} \right) \right) = \left(\hat{s}_{\delta}, \left(\mathcal{F}^{RP}_{\mathfrak{X}} + \iota \mathcal{F}^{IP}_{\mathfrak{X}}, \mathcal{F}^{RN}_{\mathfrak{X}} + \iota \mathcal{F}^{IN}_{\mathfrak{X}} \right) \right)$, then the accuracy value of $\mathfrak{X}$ will be figured out by the below formula

$$\mathcal{H}(\mathfrak{X}) = \frac{\left(\begin{array}{c} \mathcal{F}^{RP}_{\mathfrak{X}} + \mathcal{F}^{IP}_{\mathfrak{X}} \\ - \mathcal{F}^{RN}_{\mathfrak{X}} - \mathcal{F}^{IN}_{\mathfrak{X}} \end{array} \right)}{4} \times \left(\hat{s}^{\mathcal{L}}_{\delta} + \hat{s}^{\mathcal{U}}_{\delta} \right) \quad (6)$$

Theorem 1: Consider two BCFULNs that is

$$\begin{aligned} \mathfrak{X}_{\phi} &= \left(\left[\hat{s}^{\mathcal{L}}_{\delta_{\phi}}, \hat{s}^{\mathcal{U}}_{\delta_{\phi}} \right], \left(\mathcal{F}^{P}_{\mathfrak{X}_{\phi}}, \mathcal{F}^{N}_{\mathfrak{X}_{\phi}} \right) \right) \\ &= \left(\left[\hat{s}^{\mathcal{L}}_{\delta_{\phi}}, \hat{s}^{\mathcal{U}}_{\delta_{\phi}} \right], \left(\mathcal{F}^{RP}_{\mathfrak{X}_{\phi}} + \iota \mathcal{F}^{IP}_{\mathfrak{X}_{\phi}}, \mathcal{F}^{RN}_{\mathfrak{X}_{\phi}} + \iota \mathcal{F}^{IN}_{\mathfrak{X}_{\phi}} \right) \right), \ \phi = 1, 2 \end{aligned}$$

and $\hbar_{1}, \hbar_{2} \geq 0$, then the following holds.

1) $\mathfrak{X}_{1} \oplus \mathfrak{X}_{2} = \mathfrak{X}_{2} \oplus \mathfrak{X}_{1}$
2) $\mathfrak{X}_{1} \otimes \mathfrak{X}_{2} = \mathfrak{X}_{2} \otimes \mathfrak{X}_{1}$
3) $\hbar_{1} (\mathfrak{X}_{1} \oplus \mathfrak{X}_{2}) = \hbar_{1} \mathfrak{X}_{1} \oplus \hbar_{1} \mathfrak{X}_{2}$
4) $(\mathfrak{X}_{1} \otimes \mathfrak{X}_{2})^{\hbar_{1}} = \mathfrak{X}_{1}^{\hbar_{1}} \otimes \mathfrak{X}_{2}^{\hbar_{1}}$
5) $\hbar_{1} \mathfrak{X}_{1} \oplus \hbar_{2} \mathfrak{X}_{1} = (\hbar_{1} + \hbar_{2}) \mathfrak{X}_{1}$
6) $\mathfrak{X}_{1}^{\hbar_{1}} \otimes \mathfrak{X}_{1}^{\hbar_{2}} = \mathfrak{X}_{1}^{\hbar_{1} + \hbar_{2}}$

IV. BCFUL AGGREGATION OPERATORS

This Section of the article contains the AOs such as BCFULWA, BCFULOWA, BCFULHA, BCFULWG, BCFULOWG, and BCFULHG in the setting of BCFUL information.

BCFULWA operator is straightforward and easy to recognize. It is designed by adding up all the values after scalar multiplication with weight. It is accessible and utilized widely in many different fields because of its simplicity.

Definition 10: Consider an assembly of BCFULNs

$$\begin{aligned} \mathfrak{X}_{\phi} &= \left(\left[\hat{s}^{\mathcal{L}}_{\delta_{\phi}}, \hat{s}^{\mathcal{U}}_{\delta_{\phi}} \right], \left(\mathcal{F}^{P}_{\mathfrak{X}_{\phi}}, \mathcal{F}^{N}_{\mathfrak{X}_{\phi}} \right) \right) \\ &= \left(\left[\hat{s}^{\mathcal{L}}_{\delta_{\phi}}, \hat{s}^{\mathcal{U}}_{\delta_{\phi}} \right], \left(\mathcal{F}^{RP}_{\mathfrak{X}_{\phi}} + \iota \mathcal{F}^{IP}_{\mathfrak{X}_{\phi}}, \mathcal{F}^{RN}_{\mathfrak{X}_{\phi}} + \iota \mathcal{F}^{IN}_{\mathfrak{X}_{\phi}} \right) \right), \\ &\quad \phi = 1, 2, \ldots, \mathfrak{b}, \end{aligned}$$

then

$$BCFULWA\left(Ӿ_1,\ Ӿ_2,\ldots,\ Ӿ_ƀ\right)=\bigoplus_{\phi=1}^{ƀ}\vartheta_{\mathfrak{WV}-\phi}Ӿ_\phi \quad (7)$$

revealed the BCFULWA operator, where $\vartheta_{\mathfrak{WV}}=\left(\vartheta_{\mathfrak{WV}-1},\ \vartheta_{\mathfrak{WV}-2},\ \ldots,\ \vartheta_{\mathfrak{WV}-ƀ}\right)$ is a weight vector, $\vartheta_{\mathfrak{WV}-\phi}\in[0,\ 1]$ and $\sum_{\phi=1}^{ƀ}\vartheta_{\mathfrak{WV}-\phi}=1$.

Theorem 2: Consider an assembly of BCFULNs

$$\begin{aligned}Ӿ_\phi&=\left(\left[\hat{s}^{\mathcal{L}}_{\delta_\phi},\ \hat{s}^{\mathcal{U}}_{\delta_\phi}\right],\left(Ƒ^{P}_{Ӿ_\phi},\ Ƒ^{N}_{Ӿ_\phi}\right)\right)\\&=\left(\left[\hat{s}^{\mathcal{L}}_{\delta_\phi},\ \hat{s}^{\mathcal{U}}_{\delta_\phi}\right],\left(Ƒ^{RP}_{Ӿ_\phi}+\iota\,Ƒ^{IP}_{Ӿ_\phi},\ Ƒ^{RN}_{Ӿ_\phi}+\iota\,Ƒ^{IN}_{Ӿ_\phi}\right)\right),\\&\quad\phi=1,\ 2,\ \ldots,ƀ,\end{aligned}$$

then aggregating this assembly with the help of Eq. (7), we get BCFULN and

$$BCFULWA\left(Ӿ_1,\ Ӿ_2,\ldots,\ Ӿ_B\right)$$

$$=\left(\begin{array}{c}\left[\hat{s}^{\mathcal{L}}_{Ꝁ-Ꝁ\prod_{\phi=1}^{ƀ}\left(1-\frac{\delta_\phi}{Ꝁ}\right)^{\vartheta_{\mathfrak{WV}-\phi}}},\ \hat{s}^{\mathcal{U}}_{Ꝁ-Ꝁ\prod_{\phi=1}^{ƀ}\left(1-\frac{\delta_\phi}{Ꝁ}\right)^{\vartheta_{\mathfrak{WV}-\phi}}}\right],\\ \left(\begin{array}{c}1-\prod_{\phi=1}^{ƀ}\left(1-Ƒ^{RP}_{Ӿ_\phi}\right)^{\vartheta_{\mathfrak{WV}-\phi}}\\+\iota\left(1-\prod_{\phi=1}^{ƀ}\left(1-Ƒ^{IP}_{Ӿ_\phi}\right)^{\vartheta_{\mathfrak{WV}-\phi}}\right),\\-\prod_{\phi=1}^{ƀ}\left|Ƒ^{RN}_{Ӿ_\phi}\right|^{\vartheta_{\mathfrak{WV}-\phi}}\\+\iota\left(-\prod_{\phi=1}^{ƀ}\left|Ƒ^{IN}_{Ӿ_\phi}\right|^{\vartheta_{\mathfrak{WV}-\phi}}\right)\end{array}\right)\end{array}\right) \quad (8)$$

Proof: See Appendix.

The BCFULWA operator owns the three properties.

1) Consider an assembly of BCFULNs

$$\begin{aligned}Ӿ_\phi&=\left(\left[\hat{s}^{\mathcal{L}}_{\delta_\phi},\ \hat{s}^{\mathcal{U}}_{\delta_\phi}\right],\left(Ƒ^{P}_{Ӿ_\phi},\ Ƒ^{N}_{Ӿ_\phi}\right)\right)\\&=\left(\left[\hat{s}^{\mathcal{L}}_{\delta_\phi},\ \hat{s}^{\mathcal{U}}_{\delta_\phi}\right],\left(Ƒ^{RP}_{Ӿ_\phi}+\iota\,Ƒ^{IP}_{Ӿ_\phi},\ Ƒ^{RN}_{Ӿ_\phi}+\iota\,Ƒ^{IN}_{Ӿ_\phi}\right)\right),\\&\quad\phi=1,\ 2,\ \ldots,ƀ,\end{aligned}$$

then if $\forall\,\phi\ Ӿ_\phi=Ӿ$, then

$$BCFULWA\left(Ӿ_1,\ Ӿ_2,\ldots,\ Ӿ_ƀ\right)=Ӿ$$

This property of the BCFULWA operator would be called idempotency.

2) Consider two assemblies of BCFULNs

$$\begin{aligned}Ӿ_\phi&=\left(\hat{s}_{\delta_\phi},\left(Ƒ^{P}_{Ӿ_\phi},\ Ƒ^{N}_{Ӿ_\phi}\right)\right)\\&=\left(\hat{s}_{\delta_\phi},\left(Ƒ^{RP}_{Ӿ_\phi}+\iota\,Ƒ^{IP}_{Ӿ_\phi},\ Ƒ^{RN}_{Ӿ_\phi}+\iota\,Ƒ^{IN}_{Ӿ_\phi}\right)\right)\end{aligned}$$

and

$$\begin{aligned}Ӿ^{\#}_\phi&=\left(\hat{s}_{\delta_\phi},\left(Ƒ^{P}_{Ӿ^{\#}_\phi},\ Ƒ^{N}_{Ӿ^{\#}_\phi}\right)\right)\\&=\left(\hat{s}_{\delta_\phi},\left(Ƒ^{RP}_{Ӿ^{\#}_\phi}+\iota\,Ƒ^{IP}_{Ӿ^{\#}_\phi},\ Ƒ^{RN}_{Ӿ^{\#}_\phi}+\iota\,Ƒ^{IN}_{Ӿ^{\#}_\phi}\right)\right),\\&\quad\phi=1,\ 2,\ldots,\ ƀ\end{aligned}$$

if $\forall\,\phi\ Ƒ^{RP}_{Ӿ_\phi}\le Ƒ^{RP}_{Ӿ^{\#}_\phi}, Ƒ^{IP}_{Ӿ_\phi}\le Ƒ^{IP}_{Ӿ^{\#}_\phi}, Ƒ^{RN}_{Ӿ_\phi}\le Ƒ^{RN}_{Ӿ^{\#}_\phi}, Ƒ^{IN}_{Ӿ_\phi}\le Ƒ^{IN}_{Ӿ^{\#}_\phi}$ then,

$$\begin{aligned}&BCFULWA\left(Ӿ_1,\ Ӿ_2,\ldots,\ Ӿ_ƀ\right)\\&\quad\le BCFULWA\left(Ӿ^{\#}_1,\ Ӿ^{\#}_2,\ldots,\ Ӿ^{\#}_ƀ\right)\end{aligned}$$

This property of the BCFULWA operator would be called monotonicity.

3) Consider an assembly of BCFULNs

$$\begin{aligned}Ӿ_\phi&=\left(\left[\hat{s}^{\mathcal{L}}_{\delta_\phi},\ \hat{s}^{\mathcal{U}}_{\delta_\phi}\right],\left(Ƒ^{P}_{Ӿ_\phi},\ Ƒ^{N}_{Ӿ_\phi}\right)\right)\\&=\left(\left[\hat{s}^{\mathcal{L}}_{\delta_\phi},\ \hat{s}^{\mathcal{U}}_{\delta_\phi}\right],\left(Ƒ^{RP}_{Ӿ_\phi}+\iota\,Ƒ^{IP}_{Ӿ_\phi},\ Ƒ^{RN}_{Ӿ_\phi}+\iota\,Ƒ^{IN}_{Ӿ_\phi}\right)\right),\\&\quad\phi=1,\ 2,\ \ldots,ƀ,\end{aligned}$$

and assume

$$Ӿ^{-}=\left(\begin{array}{c}\min_{\phi}\left\{Ƒ^{RP}_{Ӿ_\phi}\right\}+\iota\,\min_{\phi}\left\{Ƒ^{IP}_{Ӿ_\phi}\right\},\\ \max_{\phi}\left\{Ƒ^{RN}_{Ӿ_\phi}\right\}+\iota\,\max_{\phi}\left\{Ƒ^{IN}_{Ӿ_\phi}\right\}\end{array}\right),$$

and

$$Ӿ^{+}=\left(\begin{array}{c}\max_{\phi}\left\{Ƒ^{RP}_{Ӿ_\phi}\right\}+\iota\,\max_{\phi}\left\{Ƒ^{IP}_{Ӿ_\phi}\right\},\\ \min_{\phi}\left\{Ƒ^{RN}_{Ӿ_\phi}\right\}+\iota\,\min_{\phi}\left\{Ƒ^{IN}_{Ӿ_\phi}\right\}\end{array}\right),$$

then

$$Ӿ^{-}\le BCFULWA\left(Ӿ_1,\ Ӿ_2,\ldots,\ Ӿ_ƀ\right)\le Ӿ^{+}$$

This property of the BCFULWA operator would be called boundedness.

BCFULOWA operator is similar to the BCFULWA operator but In the BCFULOWG the data which is considered must be ordered.

Definition 11: Consider an assembly of BCFULNs

$$\begin{aligned}Ӿ_\phi&=\left(\left[\hat{s}^{\mathcal{L}}_{\delta_\phi},\ \hat{s}^{\mathcal{U}}_{\delta_\phi}\right],\left(Ƒ^{P}_{Ӿ_\phi},\ Ƒ^{N}_{Ӿ_\phi}\right)\right)\\&=\left(\left[\hat{s}^{\mathcal{L}}_{\delta_\phi},\ \hat{s}^{\mathcal{U}}_{\delta_\phi}\right],\left(Ƒ^{RP}_{Ӿ_\phi}+\iota\,Ƒ^{IP}_{Ӿ_\phi},\ Ƒ^{RN}_{Ӿ_\phi}+\iota\,Ƒ^{IN}_{Ӿ_\phi}\right)\right),\\&\quad\phi=1,\ 2,\ \ldots,ƀ,\end{aligned}$$

then

$$BCFULOWA\left(Ӿ_1,\ Ӿ_2,\ldots,\ Ӿ_ƀ\right)=\bigoplus_{\phi=1}^{ƀ}\vartheta_{\mathfrak{WV}-¥(\phi)}Ӿ_{¥(\phi)} \quad (9)$$

revealed the BCFULOWA operator, where $\vartheta_{\mathfrak{WV}}=\left(\vartheta_{\mathfrak{WV}-1},\ \vartheta_{\mathfrak{WV}-2},\ \ldots,\ \vartheta_{\mathfrak{WV}-ƀ}\right)$ is a weight vector, $\vartheta_{\mathfrak{WV}-\phi}\in[0,\ 1]$ and $\sum_{\phi=1}^{ƀ}\vartheta_{\mathfrak{WV}-\phi}=1$. Further, $\left(¥(1),\ ¥(1),\ \ldots,\ ¥(ƀ)\right)$ is a permutation such that $¥(\phi-1)\ge ¥(\phi)\ \forall\,\phi=2,\ 3,\ldots,\ ƀ$.

Theorem 3: Consider an assembly of BCFULNs

$$\text{Ӿ}_\phi = \left(\left[\hat{s}^{\mathcal{L}}_{\delta_\phi},\ \hat{s}^{\mathfrak{U}}_{\delta_\phi}\right], \left(\text{Ƒ}^{P}_{\text{Ӿ}_\phi},\ \text{Ƒ}^{N}_{\text{Ӿ}_\phi}\right)\right) = \left(\left[\hat{s}^{\mathcal{L}}_{\delta_\phi},\ \hat{s}^{\mathfrak{U}}_{\delta_\phi}\right], \left(\text{Ƒ}^{RP}_{\text{Ӿ}_\phi} + \iota\, \text{Ƒ}^{IP}_{\text{Ӿ}_\phi},\ \text{Ƒ}^{RN}_{\text{Ӿ}_\phi} + \iota\, \text{Ƒ}^{IN}_{\text{Ӿ}_\phi}\right)\right), \quad \phi = 1,\ 2,\ \ldots, \text{ƀ},$$

then aggregating this assembly with the help of Eq. (9), we get BCFULN and

$$BCFULOWA\left(\text{Ӿ}_1,\ \text{Ӿ}_2, \ldots,\ \text{Ӿ}_{\text{ƀ}}\right) = \left(\begin{array}{c} \left[\begin{array}{c} \hat{s}^{\mathcal{L}}_{\text{ƙ}-\text{ƙ}\prod_{\phi=1}^{\text{ƀ}}\left(1-\frac{\delta_{\text{¥}(\phi)}}{\text{ƙ}}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-\phi}}}, \\ \hat{s}^{\mathfrak{U}}_{\text{ƙ}-\text{ƙ}\prod_{\phi=1}^{\text{ƀ}}\left(1-\frac{\delta_{\text{¥}(\phi)}}{\text{ƙ}}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-\phi}}} \end{array}\right], \\ \left(\begin{array}{c} 1 - \prod_{\phi=1}^{\text{ƀ}}\left(1 - \text{Ƒ}^{RP}_{\text{Ӿ}_{\text{¥}(\phi)}}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-\phi}} \\ +\iota\left(1 - \prod_{\phi=1}^{\text{ƀ}}\left(1 - \text{Ƒ}^{IP}_{\text{Ӿ}_{\text{¥}(\phi)}}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-\phi}}\right), \\ -\prod_{\phi=1}^{\text{ƀ}}\left|\text{Ƒ}^{RN}_{\text{Ӿ}_{\text{¥}(\phi)}}\right|^{\vartheta_{\mathfrak{W}\mathfrak{V}-\phi}} \\ +\iota\left(-\prod_{\phi=1}^{\text{ƀ}}\left|\text{Ƒ}^{IN}_{\text{Ӿ}_{\text{¥}(\phi)}}\right|^{\vartheta_{\mathfrak{W}\mathfrak{V}-\phi}}\right) \end{array}\right) \end{array}\right) \tag{10}$$

The BCFULOWA operator holds the idempotency, boundedness, and monotonicity properties.

Definition 12: Consider an assembly of BCFULNs

$$\text{Ӿ}_\phi = \left(\left[\hat{s}^{\mathcal{L}}_{\delta_\phi},\ \hat{s}^{\mathfrak{U}}_{\delta_\phi}\right], \left(\text{Ƒ}^{P}_{\text{Ӿ}_\phi},\ \text{Ƒ}^{N}_{\text{Ӿ}_\phi}\right)\right) = \left(\left[\hat{s}^{\mathcal{L}}_{\delta_\phi},\ \hat{s}^{\mathfrak{U}}_{\delta_\phi}\right], \left(\text{Ƒ}^{RP}_{\text{Ӿ}_\phi} + \iota\, \text{Ƒ}^{IP}_{\text{Ӿ}_\phi},\ \text{Ƒ}^{RN}_{\text{Ӿ}_\phi} + \iota\, \text{Ƒ}^{IN}_{\text{Ӿ}_\phi}\right)\right), \quad \phi = 1,\ 2,\ \ldots, \text{ƀ},$$

then

$$BCFULHA\left(\text{Ӿ}_1,\ \text{Ӿ}_2, \ldots,\ \text{Ӿ}_{\text{ƀ}}\right) = \bigoplus_{\phi=1}^{\text{ƀ}} \theta_{\mathfrak{W}\mathfrak{V}-\phi}\text{Ӿ}^{*}_{\text{¥}(\phi)} \tag{11}$$

revealed the BCFULHA operator, where $\theta_{\mathfrak{W}\mathfrak{V}} = \left(\theta_{\mathfrak{W}\mathfrak{V}-1},\ \theta_{\mathfrak{W}\mathfrak{V}-2},\ \ldots,\ \theta_{\mathfrak{W}\mathfrak{V}-\text{ƀ}}\right)$ is a linked weight vector, $\theta_{\mathfrak{W}\mathfrak{V}-\phi} \in [0,\ 1]$ and $\sum_{\phi=1}^{\text{ƀ}} \theta_{\mathfrak{W}\mathfrak{V}-\phi} = 1$, $\text{Ӿ}^{*}_{\text{¥}(\phi)}$ is the ϕ th biggest argument of the BCFULNs $\text{Ӿ}^{*}_\phi\ \left(\text{Ӿ}^{*}_\phi = (\text{ƀ}\vartheta_{\mathfrak{W}\mathfrak{V}-\text{ƀ}})\,\text{Ӿ}_\phi\right)$, and ƀ is the balancing coefficient.

Theorem 4: Consider an assembly of BCFULNs

$$\text{Ӿ}_\phi = \left(\left[\hat{s}^{\mathcal{L}}_{\delta_\phi},\ \hat{s}^{\mathfrak{U}}_{\delta_\phi}\right], \left(\text{Ƒ}^{P}_{\text{Ӿ}_\phi},\ \text{Ƒ}^{N}_{\text{Ӿ}_\phi}\right)\right) = \left(\left[\hat{s}^{\mathcal{L}}_{\delta_\phi},\ \hat{s}^{\mathfrak{U}}_{\delta_\phi}\right], \left(\text{Ƒ}^{RP}_{\text{Ӿ}_\phi} + \iota\, \text{Ƒ}^{IP}_{\text{Ӿ}_\phi},\ \text{Ƒ}^{RN}_{\text{Ӿ}_\phi} + \iota\, \text{Ƒ}^{IN}_{\text{Ӿ}_\phi}\right)\right), \quad \phi = 1,\ 2,\ \ldots, \text{ƀ},$$

then aggregating this assembly with the help of Eq. (11), we get BCFULN and

$$BCFULHA\left(\text{Ӿ}_1,\ \text{Ӿ}_2, \ldots,\ \text{Ӿ}_{\text{ƀ}}\right) = \left(\begin{array}{c} \left[\hat{s}^{*\mathcal{L}}_{\text{ƙ}-\text{ƙ}\prod_{\phi=1}^{\text{ƀ}}\left(1-\frac{\delta_{\text{¥}(\phi)}}{\text{ƙ}}\right)^{\theta_{\mathfrak{W}\mathfrak{V}-\phi}}},\ \hat{s}^{*\mathfrak{U}}_{\text{ƙ}-\text{ƙ}\prod_{\phi=1}^{\text{ƀ}}\left(1-\frac{\delta_{\text{¥}(\phi)}}{\text{ƙ}}\right)^{\theta_{\mathfrak{W}\mathfrak{V}-\phi}}}\right], \\ \left(\begin{array}{c} 1 - \prod_{\phi=1}^{\text{ƀ}}\left(1 - \text{Ƒ}^{*RP}_{\text{Ӿ}_{\text{¥}(\phi)}}\right)^{\theta_{\mathfrak{W}\mathfrak{V}-\phi}} \\ +\iota\left(1 - \prod_{\phi=1}^{\text{ƀ}}\left(1 - \text{Ƒ}^{*IP}_{\text{Ӿ}_{\text{¥}(\phi)}}\right)^{\theta_{\mathfrak{W}\mathfrak{V}-\phi}}\right), \\ -\prod_{\phi=1}^{\text{ƀ}}\left|\text{Ƒ}^{*RN}_{\text{Ӿ}_{\text{¥}(\phi)}}\right|^{\theta_{\mathfrak{W}\mathfrak{V}-\phi}} \\ +\iota\left(-\prod_{\phi=1}^{\text{ƀ}}\left|\text{Ƒ}^{*IN}_{\text{Ӿ}_{\text{¥}(\phi)}}\right|^{\theta_{\mathfrak{W}\mathfrak{V}-\phi}}\right) \end{array}\right) \end{array}\right) \tag{12}$$

The BCFULHA operator owns the property of idempotency, monotonicity, and boundedness.

The BCFULWG operator is a statistical measure that aggregates the data by taking the scalar power of the values and then multiplying the values. It is very useful in various fields such as environmental science, computer science, etc.

Definition 13: Consider an assembly of BCFULNs

$$\text{Ӿ}_\phi = \left(\left[\hat{s}^{\mathcal{L}}_{\delta_\phi},\ \hat{s}^{\mathfrak{U}}_{\delta_\phi}\right], \left(\text{Ƒ}^{P}_{\text{Ӿ}_\phi},\ \text{Ƒ}^{N}_{\text{Ӿ}_\phi}\right)\right) = \left(\left[\hat{s}^{\mathcal{L}}_{\delta_\phi},\ \hat{s}^{\mathfrak{U}}_{\delta_\phi}\right], \left(\text{Ƒ}^{RP}_{\text{Ӿ}_\phi} + \iota\, \text{Ƒ}^{IP}_{\text{Ӿ}_\phi},\ \text{Ƒ}^{RN}_{\text{Ӿ}_\phi} + \iota\, \text{Ƒ}^{IN}_{\text{Ӿ}_{\backslash\phi}}\right)\right), \quad \phi = 1,\ 2,\ \ldots, \text{ƀ},$$

then

$$BCFULWG\left(\text{Ӿ}_1,\ \text{Ӿ}_2, \ldots,\ \text{Ӿ}_{\text{ƀ}}\right) = \bigotimes_{\phi=1}^{\text{ƀ}} \left(\text{Ӿ}_\phi\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-\phi}} \tag{13}$$

revealed the BCFULWG operator, where $\vartheta_{\mathfrak{W}\mathfrak{V}} = \left(\vartheta_{\mathfrak{W}\mathfrak{V}-1},\ \vartheta_{\mathfrak{W}\mathfrak{V}-2},\ \ldots,\ \vartheta_{\mathfrak{W}\mathfrak{V}-\text{ƀ}}\right)$ is a weight vector, $\vartheta_{\mathfrak{W}\mathfrak{V}-\phi} \in [0,\ 1]$ and $\sum_{\phi=1}^{\text{ƀ}} \vartheta_{\mathfrak{W}\mathfrak{V}-\phi} = 1$.

Theorem 5: Consider an assembly of BCFULNs

$$\text{Ӿ}_\phi = \left(\left[\hat{s}^{\mathcal{L}}_{\delta_\phi},\ \hat{s}^{\mathfrak{U}}_{\delta_\phi}\right], \left(\text{Ƒ}^{P}_{\text{Ӿ}_\phi},\ \text{Ƒ}^{N}_{\text{Ӿ}_\phi}\right)\right) = \left(\left[\hat{s}^{\mathcal{L}}_{\delta_\phi},\ \hat{s}^{\mathfrak{U}}_{\delta_\phi}\right], \left(\text{Ƒ}^{RP}_{\text{Ӿ}_\phi} + \iota\, \text{Ƒ}^{IP}_{\text{Ӿ}_\phi},\ \text{Ƒ}^{RN}_{\text{Ӿ}_\phi} + \iota\, \text{Ƒ}^{IN}_{\text{Ӿ}_\phi}\right)\right), \quad \phi = 1,\ 2,\ \ldots, \text{ƀ},$$

then aggregating this assembly with the help of Eq. (13), we get BCFULN and

$$BCFULWG\left(\text{Ӿ}_1,\ \text{Ӿ}_2, \ldots,\ \text{Ӿ}_{\text{ƀ}}\right)$$

$$= \left(\begin{array}{c} \left[\hat{s}^{\mathcal{L}}_{\mathfrak{k}\prod_{\phi=1}^{\mathfrak{b}}\left(\frac{\delta_\phi}{\mathfrak{k}}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-\phi}}}, \hat{s}^{\mathcal{U}}_{\mathfrak{k}\prod_{\phi=1}^{\mathfrak{b}}\left(\frac{\delta_\phi}{\mathfrak{k}}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-\phi}}} \right], \\ \left(\begin{array}{c} \prod_{\phi=1}^{\mathfrak{b}} \left(\mathcal{F}^{RP}_{\mathfrak{x}_\phi}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-\phi}} \\ +\iota \prod_{\phi=1}^{\mathfrak{b}} \left(\mathcal{F}^{IP}_{\mathfrak{x}_\phi}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-\phi}}, \\ -1+\prod_{\phi=1}^{\mathfrak{b}}\left(1+\mathcal{F}^{RN}_{\mathfrak{x}_\phi}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-\phi}} \\ +\iota\left(-1+\prod_{\phi=1}^{\mathfrak{b}}\left(1+\mathcal{F}^{RN}_{\mathfrak{x}_\phi}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-\phi}}\right) \end{array} \right) \end{array} \right) \quad (14)$$

Proof: See Appendix

The BCFULWG operator holds the idempotency, boundedness, and monotonicity properties.

BCFULOWG operator is similar to the BCFULWG operator but In the BCFULOWG the data which is considered must be ordered.

Definition 14: Consider an assembly of BCFULNs

$$\mathfrak{x}_\phi = \left(\left[\hat{s}^{\mathcal{L}}_{\delta_\phi}, \hat{s}^{\mathcal{U}}_{\delta_\phi}\right], \left(\mathcal{F}^{P}_{\mathfrak{x}_\phi}, \mathcal{F}^{N}_{\mathfrak{x}_\phi}\right)\right) = \left(\left[\hat{s}^{\mathcal{L}}_{\delta_\phi}, \hat{s}^{\mathcal{U}}_{\delta_\phi}\right], \left(\mathcal{F}^{RP}_{\mathfrak{x}_\phi}+\iota\,\mathcal{F}^{IP}_{\mathfrak{x}_\phi}, \mathcal{F}^{RN}_{\mathfrak{x}_\phi}+\iota\,\mathcal{F}^{IN}_{\mathfrak{x}_\phi}\right)\right), \quad \phi = 1, 2, \ldots, \mathfrak{b},$$

then

$$BCFULOWG\left(\mathfrak{x}_1, \mathfrak{x}_2, \ldots, \mathfrak{x}_{\mathfrak{b}}\right) = \bigotimes_{\phi=1}^{\mathfrak{b}} \left(\mathfrak{x}_{\yen(\phi)}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-\phi}} \quad (15)$$

revealed the BCFULOWG operator, where $\vartheta_{\mathfrak{W}\mathfrak{V}} = \left(\vartheta_{\mathfrak{W}\mathfrak{V}-1}, \vartheta_{\mathfrak{W}\mathfrak{V}-2}, \ldots, \vartheta_{\mathfrak{W}\mathfrak{V}-\mathfrak{b}}\right)$ is a weight vector, $\vartheta_{\mathfrak{W}\mathfrak{V}-\phi} \in [0, 1]$ and $\sum_{\phi=1}^{\mathfrak{b}} \vartheta_{\mathfrak{W}\mathfrak{V}-\phi} = 1$. Further, $(\yen(1), \yen(1), \ldots, \yen(\mathfrak{b}))$ is a permutation such that $\yen(\phi-1) \geq \yen(\phi)\ \forall\, \phi = 2, 3, \ldots, \mathfrak{b}$.

Theorem 6: Consider an assembly of BCFULNs

$$\mathfrak{x}_\phi = \left(\left[\hat{s}^{\mathcal{L}}_{\delta_\phi}, \hat{s}^{\mathcal{U}}_{\delta_\phi}\right], \left(\mathcal{F}^{P}_{\mathfrak{x}_\phi}, \mathcal{F}^{N}_{\mathfrak{x}_\phi}\right)\right) = \left(\left[\hat{s}^{\mathcal{L}}_{\delta_\phi}, \hat{s}^{\mathcal{U}}_{\delta_\phi}\right], \left(\mathcal{F}^{RP}_{\mathfrak{x}_\phi}+\iota\,\mathcal{F}^{IP}_{\mathfrak{x}_\phi}, \mathcal{F}^{RN}_{\mathfrak{x}_\phi}+\iota\,\mathcal{F}^{IN}_{\mathfrak{x}_\phi}\right)\right), \quad \phi = 1, 2, \ldots, \mathfrak{b},$$

then aggregating this assembly with the help of Eq. (15), we get BCFULN and

$$BCFULOWG\left(\mathfrak{x}_1, \mathfrak{x}_2, \ldots, \mathfrak{x}_{\mathfrak{b}}\right)$$

$$= \left(\begin{array}{c} \left[\hat{s}^{\mathcal{L}}_{\mathfrak{k}\prod_{\phi=1}^{\mathfrak{b}}\left(\frac{\delta_{\yen(\phi)}}{\mathfrak{k}}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-\phi}}}, \hat{s}^{\mathcal{U}}_{\mathfrak{k}\prod_{\phi=1}^{\mathfrak{b}}\left(\frac{\delta_{\yen(\phi)}}{\mathfrak{k}}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-\phi}}} \right], \\ \left(\begin{array}{c} \prod_{\phi=1}^{\mathfrak{b}} \left(\mathcal{F}^{RP}_{\mathfrak{x}_{\yen(\phi)}}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-\phi}} \\ +\iota \prod_{\phi=1}^{\mathfrak{b}} \left(\mathcal{F}^{IP}_{\mathfrak{x}_{\yen(\phi)}}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-\phi}}, \\ -1+\prod_{\phi=1}^{\mathfrak{b}}\left(1+\mathcal{F}^{RN}_{\mathfrak{x}_{\yen(\phi)}}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-\phi}} \\ +\iota\left(-1+\prod_{\phi=1}^{\mathfrak{b}}\left(1+\mathcal{F}^{RN}_{\mathfrak{x}_{(\phi)}}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-\phi}}\right) \end{array} \right) \end{array} \right) \quad (16)$$

The BCFULOWG operator holds the idempotency, boundedness, and monotonicity properties.

Definition 15: Consider an assembly of BCFULNs

$$\mathfrak{x}_\phi = \left(\left[\hat{s}^{\mathcal{L}}_{\delta_\phi}, \hat{s}^{\mathcal{U}}_{\delta_\phi}\right], \left(\mathcal{F}^{P}_{\mathfrak{x}_\phi}, \mathcal{F}^{N}_{\mathfrak{x}_\phi}\right)\right) = \left(\left[\hat{s}^{\mathcal{L}}_{\delta_\phi}, \hat{s}^{\mathcal{U}}_{\delta_\phi}\right], \left(\mathcal{F}^{RP}_{\mathfrak{x}_\phi}+\iota\,\mathcal{F}^{IP}_{\mathfrak{x}_\phi}, \mathcal{F}^{RN}_{\mathfrak{x}_\phi}+\iota\,\mathcal{F}^{IN}_{\mathfrak{x}_\phi}\right)\right), \quad \phi = 1, 2, \ldots, \mathfrak{b},$$

then

$$BCFULHG\left(\mathfrak{x}_1, \mathfrak{x}_2, \ldots, \mathfrak{x}_{\mathfrak{b}}\right) = \bigotimes_{\phi=1}^{\mathfrak{b}} \left(\mathfrak{x}^{*}_{\yen(\phi)}\right)^{\theta_{\mathfrak{W}\mathfrak{V}-\phi}} \quad (17)$$

revealed the BCFULHG operator, where $\theta_{\mathfrak{W}\mathfrak{V}} = \left(\theta_{\mathfrak{W}\mathfrak{V}-1}, \theta_{\mathfrak{W}\mathfrak{V}-2}, \ldots, \theta_{\mathfrak{W}\mathfrak{V}-\mathfrak{b}}\right)$ is a linked weight vector, $\theta_{\mathfrak{W}\mathfrak{V}-\phi} \in [0, 1]$ and $\sum_{\phi=1}^{\mathfrak{b}} \theta_{\mathfrak{W}\mathfrak{V}-\phi} = 1$, $\mathfrak{x}^{*}_{\yen(\phi)}$ is the ϕ th biggest argument of the BCFULNs $\mathfrak{x}^{*}_{\phi}\left(\mathfrak{x}^{*}_{\phi} = \left(\mathfrak{b}\vartheta_{\mathfrak{W}\mathfrak{V}-\mathfrak{b}}\right)\mathfrak{x}_\phi\right)$, and $\mathfrak{b}$ is the balancing coefficient.

Theorem 7: Consider an assembly of BCFULNs

$$\mathfrak{x}_\phi = \left(\left[\hat{s}^{\mathcal{L}}_{\delta_\phi}, \hat{s}^{\mathcal{U}}_{\delta_\phi}\right], \left(\mathcal{F}^{P}_{\mathfrak{x}_\phi}, \mathcal{F}^{N}_{\mathfrak{x}_\phi}\right)\right) = \left(\left[\hat{s}^{\mathcal{L}}_{\delta_\phi}, \hat{s}^{\mathcal{U}}_{\delta_\phi}\right], \left(\mathcal{F}^{RP}_{\mathfrak{x}_\phi}+\iota\,\mathcal{F}^{IP}_{\mathfrak{x}_\phi}, \mathcal{F}^{RN}_{\mathfrak{x}_\phi}+\iota\,\mathcal{F}^{IN}_{\mathfrak{x}_\phi}\right)\right), \quad \phi = 1, 2, \ldots, \mathfrak{b},$$

then aggregating this assembly with the help of Eq. (17), we get BCFULN and

$$BCFULHG\left(\mathfrak{x}_1, \mathfrak{x}_2, \ldots, \mathfrak{x}_{\mathfrak{b}}\right)$$

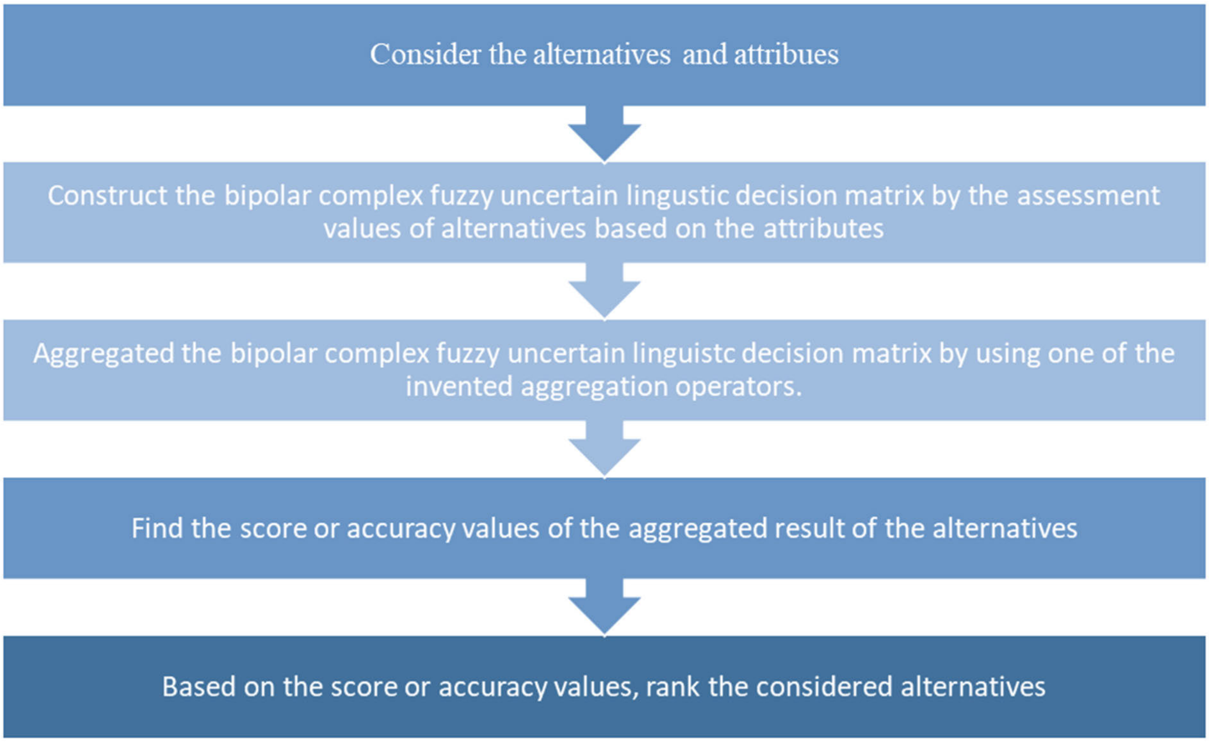

FIGURE 2. **The flowchart of the devised DM framework.**

$$= \left(\begin{array}{c} \left[\hat{s}^{*\mathcal{L}}_{\mathfrak{k} \prod_{\phi=1}^{ƀ} \left(\frac{\delta_{\yen(\phi)}}{\mathfrak{k}}\right)^{\theta_{\mathfrak{W}\mathfrak{B}-\phi}}}, \hat{s}^{\mathcal{U}}_{\mathfrak{k} \prod_{\phi=1}^{ƀ} \left(\frac{\delta_{\yen(\phi)}}{\mathfrak{k}}\right)^{\theta_{\mathfrak{W}\mathfrak{B}-\phi}}} \right], \\ \left(\begin{array}{c} \prod_{\phi=1}^{ƀ} \left(Ϝ^{*RP}_{X_{\yen(\phi)}}\right)^{\theta_{\mathfrak{W}\mathfrak{B}-\phi}} \\ +\iota \prod_{\phi=1}^{ƀ} \left(Ϝ^{*IP}_{X_{\yen(\phi)}}\right)^{\theta_{\mathfrak{W}\mathfrak{B}-\phi}}, \\ -1 + \prod_{\phi=1}^{ƀ} \left(1 + Ϝ^{*RN}_{X_{\yen(\phi)}}\right)^{\theta_{\mathfrak{W}\mathfrak{B}-\phi}} \\ +\iota \left(-1 + \prod_{\phi=1}^{ƀ} \left(1 + Ϝ^{*RN}_{X_{\yen(\phi)}}\right)^{\theta_{\mathfrak{W}\mathfrak{B}-\phi}}\right) \end{array} \right) \end{array} \right) \tag{18}$$

The BCFULHG operator owns the property of idempotency, monotonicity, and boundedness.

V. APPLICATION (PRIORITIZATION OF THE VARIOUS TYPES OF COMPUTER VISION)

The prioritization of various types of computer vision can be thought of as a DM problem, where the objective is to determine which method is best suited for a particular task. Image segmentation, edge detection, image classification, and other computer vision techniques can all be taken into consideration as possible solutions in this situation. Evaluating these forms according to standards including accuracy, computational efficiency, robustness, and suitability for specific applications is part of the DM process. Every technique is a potential alternative, and the decision-maker needs to consider the preferences and trade-offs of each. The choice of the best computer vision technique for a given use case is ultimately determined by the employment of machine learning models, optimization algorithms, and performance measures as key DM tools. Moreover, when there are dual aspects (positive and negative aspects) and linguistic uncertainty involved along with extra fuzzy information in the process of prioritization of types of computer vision, then there is no technique of DM, that can handle this DM dilemma. Thus, following we interpret an approach of DM to cope with such sort of dilemmas.

Assume an assembly of ƀ alternative

$$X_{\mathfrak{A}\mathfrak{B}} = \{X_{\mathfrak{A}\mathfrak{B}-1}, X_{\mathfrak{A}\mathfrak{B}-2}, \ldots, X_{\mathfrak{A}\mathfrak{B}-ƀ}\}$$

and an assembly of đ attributes

$$\mathfrak{T}_{\mathfrak{A}\mathfrak{T}} = \left\{\mathfrak{T}_{\mathfrak{A}\mathfrak{T}-1}, \mathfrak{T}_{\mathfrak{A}\mathfrak{T}-2}, \ldots, \mathfrak{T}_{\mathfrak{A}\mathfrak{T}-đ}\right\}.$$

Further, consider that

$$\vartheta_{\mathfrak{W}\mathfrak{B}} = \left(\vartheta_{\mathfrak{W}\mathfrak{B}-1}, \vartheta_{\mathfrak{W}\mathfrak{B}-2}, \ldots, \vartheta_{\mathfrak{W}\mathfrak{B}-đ}\right)$$

is a weight vector,

$$\vartheta_{\mathfrak{W}\mathfrak{B}-\phi} \in [0, 1] \text{ and } \sum_{\phi=1}^{đ} \vartheta_{\mathfrak{W}\mathfrak{B}-\phi} = 1$$

associated with the attributes. Next, consider that the decision-maker or an expert interpreted his/her opinion in the shape of a positive satisfaction degree and negative satisfaction degree to construct the BCFUL decision matrix

$$\mathcal{Z}_{dm} = \left(X_{\phi\varphi}\right)_{ƀ\times đ}$$

TABLE 1. The evaluated arguments are interpreted by the experts.

	$\mathfrak{T}_{\mathfrak{AT}-1}$	$\mathfrak{T}_{\mathfrak{AT}-2}$	$\mathfrak{T}_{\mathfrak{AT}-3}$	$\mathfrak{T}_{\mathfrak{AT}-4}$
$\mathrm{X}_{\mathfrak{AB}-1}$	$\left([\hat{s}_1,\hat{s}_5],\begin{pmatrix}0.21+\iota\,0.34\\-0.56-\iota\,0.89\end{pmatrix}\right)$	$\left([\hat{s}_3,\hat{s}_4],\begin{pmatrix}0.76+\iota\,0.99\\-0.35-\iota\,0.47\end{pmatrix}\right)$	$\left([\hat{s}_1,\hat{s}_3],\begin{pmatrix}0.11+\iota\,0.21\\-0.22-\iota\,0.90\end{pmatrix}\right)$	$\left([\hat{s}_4,\hat{s}_6],\begin{pmatrix}0.77+\iota\,0.87\\-0.76-\iota\,0.76\end{pmatrix}\right)$
$\mathrm{X}_{\mathfrak{AB}-2}$	$\left([\hat{s}_3,\hat{s}_4],\begin{pmatrix}0.31+\iota\,0.80\\-0.72-\iota\,0.09\end{pmatrix}\right)$	$\left([\hat{s}_5,\hat{s}_6],\begin{pmatrix}0.32+\iota\,0.29\\-0.67-\iota\,0.31\end{pmatrix}\right)$	$\left([\hat{s}_2,\hat{s}_4],\begin{pmatrix}0.22+\iota\,0.76\\-0.45-\iota\,0.32\end{pmatrix}\right)$	$\left([\hat{s}_4,\hat{s}_5],\begin{pmatrix}0.65+\iota\,0.45\\-0.72-\iota\,0.76\end{pmatrix}\right)$
$\mathrm{X}_{\mathfrak{AB}-3}$	$\left([\hat{s}_1,\hat{s}_2],\begin{pmatrix}0.15+\iota\,0.29\\-0.43-\iota\,0.34\end{pmatrix}\right)$	$\left([\hat{s}_2,\hat{s}_3],\begin{pmatrix}0.14+\iota\,0.78\\-0.42-\iota\,0.21\end{pmatrix}\right)$	$\left([\hat{s}_4,\hat{s}_6],\begin{pmatrix}0.49+\iota\,0.86\\-0.53-\iota\,0.33\end{pmatrix}\right)$	$\left([\hat{s}_5,\hat{s}_6],\begin{pmatrix}0.89+\iota\,0.76\\-0.67-\iota\,0.98\end{pmatrix}\right)$
$\mathrm{X}_{\mathfrak{AB}-4}$	$\left([\hat{s}_4,\hat{s}_6],\begin{pmatrix}0.97+\iota\,0.61\\-0.21-\iota\,0.35\end{pmatrix}\right)$	$\left([\hat{s}_3,\hat{s}_4],\begin{pmatrix}0.13+\iota\,0.91\\-0.17-\iota\,0.45\end{pmatrix}\right)$	$\left([\hat{s}_4,\hat{s}_5],\begin{pmatrix}0.44+\iota\,0.21\\-0.23-\iota\,0.66\end{pmatrix}\right)$	$\left([\hat{s}_3,\hat{s}_6],\begin{pmatrix}0.90+\iota\,0.42\\-0.59-\iota\,0.35\end{pmatrix}\right)$

TABLE 2. The aggregated arguments after using invented operators.

	$\mathrm{X}_{\mathfrak{AB}-1}$	$\mathrm{X}_{\mathfrak{AB}-2}$	$\mathrm{X}_{\mathfrak{AB}-3}$	$\mathrm{X}_{\mathfrak{AB}-4}$
BCFULWA	$\left([\hat{s}_{6.9797},\hat{s}_{6.9986}],\begin{pmatrix}0.555+\iota\,0.846\\-0.379-\iota\,0.724\end{pmatrix}\right)$	$\left([\hat{s}_{6.9971},\hat{s}_{69997}],\begin{pmatrix}0.39+\iota\,0.617\\-0.596-\iota\,0.331\end{pmatrix}\right)$	$\left([\hat{s}_{6.9936},\hat{s}_{6.9996}],\begin{pmatrix}0.566+\iota\,0.778\\-0.512-\iota\,0.381\end{pmatrix}\right)$	$\left([\hat{s}_{6.9966},\hat{s}_{6.9999}],\begin{pmatrix}0.715+\iota\,0.628\\-0.263-\iota\,0.471\end{pmatrix}\right)$
BCFULOWA	$\left([\hat{s}_{6.981},\hat{s}_{6.9986}],\begin{pmatrix}0.498+\iota\,0.82\\-0.41-\iota\,0.736\end{pmatrix}\right)$	$\left([\hat{s}_{6.997},\hat{s}_{6.9997}],\begin{pmatrix}0.35+\iota\,0.622\\-0.618-\iota\,0.261\end{pmatrix}\right)$	$\left([\hat{s}_{6.994},\hat{s}_{6.9996}],\begin{pmatrix}0.43+\iota\,0.739\\-0.478-\iota\,0.325\end{pmatrix}\right)$	$\left([\hat{s}_{6.997},\hat{s}_{6.999}],\begin{pmatrix}0.774+\iota\,0.628\\-0.236-\iota\,0.467\end{pmatrix}\right)$
BCFULWG	$\left([\hat{s}_{1.876},\hat{s}_{3.713}],\begin{pmatrix}0.322+\iota\,0.478\\-0.481-\iota\,0.804\end{pmatrix}\right)$	$\left([\hat{s}_{3.189},\hat{s}_{4.708}],\begin{pmatrix}0.33+\iota\,0.52\\-0.627-\iota\,0.448\end{pmatrix}\right)$	$\left([\hat{s}_{2.923},\hat{s}_{4.314}],\begin{pmatrix}0.346+\iota\,0.706\\-0.531-\iota\,0.699\end{pmatrix}\right)$	$\left([\hat{s}_{3.454},\hat{s}_{5.036}],\begin{pmatrix}0.417+\iota\,0.423\\-0.322-\iota\,0.508\end{pmatrix}\right)$
BCFULOWG	$\left([\hat{s}_{1.611},\hat{s}_{3.843}],\begin{pmatrix}0.301+\iota\,0.457\\-0.482-\iota\,0.818\end{pmatrix}\right)$	$\left([\hat{s}_{3.144},\hat{s}_{4.478}],\begin{pmatrix}0.315+\iota\,0.508\\-0.644-\iota\,0.36\end{pmatrix}\right)$	$\left([\hat{s}_{2.3},\hat{s}_{3.591}],\begin{pmatrix}0.254+\iota\,0.629\\-0.493-\iota\,0.551\end{pmatrix}\right)$	$\left([\hat{s}_{3.366},\hat{s}_{5.098}],\begin{pmatrix}0.446+\iota\,0.436\\-0.273-\iota\,0.505\end{pmatrix}\right)$

$$
=\begin{pmatrix}\left[\hat{s}^{\mathcal{L}}_{\delta_{\phi\varphi}},\ \hat{s}^{\mathcal{U}}_{\delta_{\phi\varphi}}\right],\\ \left(\mathrm{F}^{P}_{\mathrm{X}_{\phi\varphi}},\ \mathrm{F}^{N}_{\mathrm{X}_{\phi\varphi}}\right)\end{pmatrix}_{\mathfrak{b}\times\mathfrak{d}}
=\begin{pmatrix}\left[\hat{s}^{\mathcal{L}}_{\delta_{\phi\varphi}},\ \hat{s}^{\mathcal{U}}_{\delta_{\phi\varphi}}\right],\\ \begin{pmatrix}\mathrm{F}^{RP}_{\mathrm{X}_{\phi\varphi}}+\iota\,\mathrm{F}^{IP}_{\mathrm{X}_{\phi\varphi}},\\ \mathrm{F}^{RN}_{\mathrm{X}_{\phi\varphi}}+\iota\,\mathrm{F}^{IN}_{\mathrm{X}_{\phi\varphi}}\end{pmatrix}\end{pmatrix}_{\mathfrak{b}\times\mathfrak{d}}.
$$

To overcome this DM dilemma, we give the underneath process based on the invented operators.

Step 1: The information of the decision matrix would be aggregated with the assistance of one of the invented operators.

Step 2: The score values of the aggregated information would be invented by utilizing Eq. (5) to compare the alternatives.

Step 3: The ranking order would be achieved based on the above-mentioned score values to find the most superb alternative.

Step 4: End

The flowchart of the invented DM approach is interpreted in Figure 2.

A. NUMERICAL EXAMPLE

Considering that the experts in AI want to prioritize the form of computer vision the presented 4 forms of computer vision are

$\mathrm{X}_{\mathfrak{AB}-1}$: **Image segmentation:** Image segmentation is another key aspect of computer vision. It is the process of separating an image into various sections based on pixel characteristics to detect objects or boundaries to simplify and more quickly analyze the image. Numerous sectors, from the film business to the realm of medicine, are impacted by segmentation. Green screen software, for example, uses picture segmentation to crop off the front and set it on a backdrop for scenes that cannot or would be risky to photograph in real life.

$\mathrm{X}_{\mathfrak{AB}-2}$: **Edge detection:** Edge detection is a beneficial method in computer vision that automatically detects the limits between objects. With these limits, it is simple to divide the image (split it into discrete objects or sections that may then be identified separately).

$\mathrm{X}_{\mathfrak{AB}-3}$: **Image classification:** The process of classifying and labeling clusters of pixel intensities or dimensions in an image based on specific principles is known as image classification. It serves as the foundation for more computer vision issues. Medical imaging, object recognition in

TABLE 3. The score values of the aggregated outcomes.

	$\mathcal{S}(\mathrm{X}_{\mathfrak{AB}-1})$	$\mathcal{S}(\mathrm{X}_{\mathfrak{AB}-2})$	$\mathcal{S}(\mathrm{X}_{\mathfrak{AB}-3})$	$\mathcal{S}(\mathrm{X}_{\mathfrak{AB}-4})$
BCFULWA	8.032	7.275	8.572	9.13
BCFULOWA	7.591	7.325	8.277	9.444
BCFULWG	2.117	3.506	3.295	4.266
BCFULOWG	1.988	3.467	2.709	4.451

TABLE 4. The ranking relies on the score values.

	Ranking
BCFULWA	$\mathrm{X}_{\mathfrak{AB}-4} > \mathrm{X}_{\mathfrak{AB}-3} > \mathrm{X}_{\mathfrak{AB}-1} > \mathrm{X}_{\mathfrak{AB}-2}$
BCFULOWA	$\mathrm{X}_{\mathfrak{AB}-4} > \mathrm{X}_{\mathfrak{AB}-3} > \mathrm{X}_{\mathfrak{AB}-1} > \mathrm{X}_{\mathfrak{AB}-2}$
BCFULWG	$\mathrm{X}_{\mathfrak{AB}-4} > \mathrm{X}_{\mathfrak{AB}-2} > \mathrm{X}_{\mathfrak{AB}-3} > \mathrm{X}_{\mathfrak{AB}-1}$
BCFULOWG	$\mathrm{X}_{\mathfrak{AB}-4} > \mathrm{X}_{\mathfrak{AB}-2} > \mathrm{X}_{\mathfrak{AB}-3} > \mathrm{X}_{\mathfrak{AB}-1}$

satellite observations, traffic surveillance systems, braking signal detection, machine vision, and other applications use image classification applications.

$\mathrm{X}_{\mathfrak{AV}-4}$: **Feature matching:** Feature matching is the method of locating similar features in two comparable photographs using a search vision. The feature-matching approach is applied to either discover or deduce and transfer qualities from the source to the desired image. One of these images is conceived of as the source, the other as the target.

Next, consider 4 attributes or characteristics that are $\mathfrak{T}_{\mathfrak{AT}-1} = \textit{Faster process}$, $\mathfrak{T}_{\mathfrak{AT}-2} = \textit{Simpler process}$, $\mathfrak{T}_{\mathfrak{AT}-3} = \textit{Cost reduction}$, and $\mathfrak{T}_{\mathfrak{AT}-4} = \textit{Better product and services}$, based on these attributes the expert will evaluate the interpreted forms of the computer vision. For each attribute, the experts gave the weight that is (0.13, 0.27, 0.36, 0.24) according to his/her preference. The evaluated arguments interpreted by the experts are displayed in Table 1.

Step 1: We aggregated the information of the decision matrix by utilizing the invented operators one by one and the required outcomes are revealed in Table 2.

Step 2: We accomplished the score values revealed in Table 3 for comparing the alternatives.

Step 3: We accomplished the ranking order revealed in Table 4.

Table 4 exposed the prioritization of the form of computer vision. Where $\mathrm{X}_{\mathfrak{AV}-4}$ is the first one and $\mathrm{X}_{\mathfrak{AV}-1}$ or $\mathrm{X}_{\mathfrak{AV}-2}$ is the last one. As the operators BCFU LWA, BCFULOWA, BCFULWG, and BCFULOWG operators are independent operators and thus the ranking can be different. In a DM dilemma, one can employ any one of them.

Step 4: End.

VI. COMPARATIVE ANALYSIS

In this part, we are going to compare a few current works such as Liu and Jin [56], Gao et al. [61], Lan et al. [62], Jana et al. [42], Wei et al. [43], Riaz et al. [44], Bi et al. [50], Bi et al. [51] and Mahmood et al. [53] with our invented work to expose the dominance and benefits of the invented work. The brief description of the current work is as follows

- Liu and Jin [56] studied geometric AOs in the setting of intuitionistic ULVs (IULVs) and provided a group DM approach based on the geometric AOs.
- Gao et al. [61] analyzed the averaging and geometric AOs in the environment of interval-valued bipolar uncertain linguistic (IVBUL) information. Further Gao et al. [61] provided a MADM mechanism for tackling IVBUL information.
- Lan et al. [62] interpreted a CODAS approach for group DM in the environment of IVBUL information.
- Jana et al. [42] deduced Dombi AOs and the MADM approach for BFS.
- Wei et al. [43] diagnosed Hamacher AOs and the MADM approach for BFS.
- Riaz et al. [44] studied sine trigonometric AOs and SIR technique for BFS.
- Bi et al. [50] discussed averaging AOs for CFS.
- Bi et al. [51] deduced geometric AOs for CFS.
- Mahmood et al. [53] analyzed the averaging and geometric AOs for BCFS and investigated a DM approach based on the invented operators.

For this comparison, we are taking the data in the model of BCFULS which is described in Table 1. Let us apply the above current and the invented operators and DM approaches to the data in Table 1. The required outcomes are exposed in Tables 5 and 6.

Tables 5 and 6 revealed that the current work interpreted by Liu and Jin [56], Gao et al. [61], Lan et al. [62], Jana et al. [42], Wei et al. [43], Riaz et al. [44], Bi et al. [50], Bi et al. [51] and Mahmood et al. [53] collide as applied to the information in the structure of BCFULS. The work of Liu and Jin [56] is in the model of IULVs and the work of Geo et al. [61] and Lan et al. [62] is in the structure of IVBUL information. Further, the research of Jana et al. [42], Wei et al. [43], and Riaz et al. [44] are in the structure of BFS, and as we know the BFS has no negative aspect and linguistic variable so these theories are not applicable to this information. But the developed work can handle the information in the model of BFS. Likewise, the work of Bi et al. [50] and Bi et al. [51] in the model of CFS can't deal with BCFUL information. Further, the work discussed by Mahmood et al. [53] is in the model of BCFS and is missing linguistic terms and hence can't cope with BCFUL information. This shows the importance and the need for

TABLE 5. The comparative analysis of the invented work with current work.

Foundation	$\mathcal{S}(\mathrm{X}_{\mathfrak{AB}-1})$	$\mathcal{S}(\mathrm{X}_{\mathfrak{AB}-2})$	$\mathcal{S}(\mathrm{X}_{\mathfrak{AB}-3})$	$\mathcal{S}(\mathrm{X}_{\mathfrak{AB}-4})$
Liu and Jin [56]	Failed	Failed	Failed	Failed
Gao et al. [61]	Failed	Failed	Failed	Failed
Lan et al. [62]	Failed	Failed	Failed	Failed
Jana et al. [42]	Failed	Failed	Failed	Failed
Wei et al. [43]	Failed	Failed	Failed	Failed
Riaz et al. [44]	Failed	Failed	Failed	Failed
Bi et al. [50]	Failed	Failed	Failed	Failed
Bi et al. [51]	Failed	Failed	Failed	Failed
Mahmood et al. [53]	Failed	Failed	Failed	Failed
BCFULWA	8.032	7.275	8.572	9.13
BCFULOWA	7.591	7.325	8.277	9.444
BCFULWG	2.117	3.506	3.295	4.266
BCFULOWG	1.988	3.467	2.709	4.451

TABLE 6. The ranking comparison between current work and invented work.

Foundation	Ranking
Liu and Jin [56]	Failed
Gao et al. [61]	Failed
Lan et al. [62]	Failed
Jana et al. [42]	Failed
Wei et al. [43]	Failed
Riaz et al. [44]	Failed
Bi et al. [50]	Failed
Bi et al. [51]	Failed
Mahmood et al. [53]	Failed
BCFULWA	$\mathrm{X}_{\mathfrak{AB}-4} > \mathrm{X}_{\mathfrak{AB}-3} > \mathrm{X}_{\mathfrak{AB}-1} > \mathrm{X}_{\mathfrak{AB}-2}$
BCFULOWA	$\mathrm{X}_{\mathfrak{AB}-4} > \mathrm{X}_{\mathfrak{AB}-3} > \mathrm{X}_{\mathfrak{AB}-1} > \mathrm{X}_{\mathfrak{AB}-2}$
BCFULWG	$\mathrm{X}_{\mathfrak{AB}-4} > \mathrm{X}_{\mathfrak{AB}-2} > \mathrm{X}_{\mathfrak{AB}-3} > \mathrm{X}_{\mathfrak{AB}-1}$
BCFULOWG	$\mathrm{X}_{\mathfrak{AB}-4} > \mathrm{X}_{\mathfrak{AB}-2} > \mathrm{X}_{\mathfrak{AB}-3} > \mathrm{X}_{\mathfrak{AB}-1}$

TABLE 7. The feature comparison of the invented and current work.

Foundation	Positive satisfaction degree	Negative satisfaction degree	2^{nd} dimension	ULV
Liu and Jin [56]	√	×	×	√
Gao et al. [61]	√	√	×	√
Lan et al. [62]	√	√	×	√
Jana et al. [42]	√	√	×	×
Wei et al. [43]	√	√	×	×
Riaz et al. [44]	√	√	×	×
Bi et al. [50]	√	×	√	×
Bi et al. [51]	√	×	√	×
Mahmood et al. [53]	√	√	√	×
Invented work	√	√	√	√

the invented work in the literature. The invented work generalized various concepts such as BCFS by neglecting the ULVs, BFULS by neglecting the imaginary parts in both positive satisfaction and negative satisfaction degrees, BFS by neglecting the ULV and neglecting the imaginary parts in both positive satisfaction and negative satisfaction degrees, fuzzy uncertain linguistic set by neglecting the imaginary part in positive satisfaction degree and neglecting the negative satisfaction degrees, etc. To elaborate on the comparison of the invented concept with current concepts we compare a few features of the invented work with prevailing work in Table 7.

VII. CONCLUSION

We pointed out that in recent years, the ideas of ULS and BCFSs have seen a lot of success. However, if real-life situations get more and more vague and imprecise, we may find that these two conceptions are incompatible. Motivated by this, in this script, we introduced a brand-new idea identified as BCFULS which combines the concepts of BCFS and ULS. The BCFULS concept is a crucial tool for dealing with complex and difficult information in real-world situations. For BCFULNs, we also examined basic operations, scoring, and accuracy functions. Additionally, we invented AOs relying on the newly developed concept BCFULS that is BCFULWA, BCFULOWA, BCFULHA, BCFULWG, BCFULOWG, and BCFULHG operators. Further, with its unique set of benefits and drawbacks, computer vision technologies come in a variety of forms. Selected top-tier technology and top-notch deployment are necessary for effective visual data analysis. Real-world computer vision applications include ambiguities, imprecision, dual features, and linguistic terms that must be managed by a robust DM framework. Therefore, in this script, we examined a DM framework inside the model of a BCFULS. Afterward, we used the established DM framework to rank the various computer vision techniques according to artificial data, ultimately determining that $Ӿ_{\mathfrak{A}\mathfrak{V}-4}$ i.e. "Feature Matching" is the best computer vision technique. Finally, to reveal the dominance and priority of the invented work we presented a comparative analysis of the invented work with the current work.

A. LIMITATIONS AND FUTURE DIRECTION

The information found in the structures of bipolar complex fuzzy soft sets [63], [64], complex hesitant FS [65], complex bipolar picture FS [66], etc. is beyond the capabilities of the inferred concepts that are AOs and DM framework under BCFULS. As a result, our goal going forward is to extend our work into these mathematical frameworks. Further, we also want to introduce some more notions and concepts within the model of BCFULS.

APPENDIX

Proof of Theorem 2:

(i) Let $ƀ = 2$. Then

$$\vartheta_{\mathfrak{W}\mathfrak{V}-1}Ӿ_1 = \begin{pmatrix} \left[\hat{s}^{\mathcal{L}}_{ƙ-ƙ\left(1-\frac{\delta_1}{ƙ}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-1}}},\ \hat{s}^{\mathcal{U}}_{ƙ-ƙ\left(1-\frac{\delta_1}{ƙ}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-1}}}\right], \\ \begin{pmatrix} 1-\left(1-Ƒ^{RP}_{Ӿ_1}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-1}} \\ +\iota\left(1-\left(1-Ƒ^{IP}_{Ӿ_1}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-1}}\right), \\ -\left|Ƒ^{RN}_{Ӿ_1}\right|^{\vartheta_{\mathfrak{W}\mathfrak{V}-1}} \\ +\iota\left(-\left|Ƒ^{IN}_{Ӿ_1}\right|^{\vartheta_{\mathfrak{W}\mathfrak{V}-1}}\right) \end{pmatrix} \end{pmatrix}$$

$$\vartheta_{\mathfrak{W}\mathfrak{V}-2}Ӿ_2 = \begin{pmatrix} \left[\hat{s}^{\mathcal{L}}_{ƙ-ƙ\left(1-\frac{\delta_2}{ƙ}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-2}}},\ \hat{s}^{\mathcal{U}}_{ƙ-ƙ\left(1-\frac{\delta_2}{ƙ}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-2}}}\right], \\ \begin{pmatrix} 1-\left(1-Ƒ^{RP}_{Ӿ_2}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-2}} \\ +\iota\left(1-\left(1-Ƒ^{IP}_{Ӿ_2}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-2}}\right), \\ -\left|Ƒ^{RN}_{Ӿ_2}\right|^{\vartheta_{\mathfrak{W}\mathfrak{V}-2}} \\ +\iota\left(-\left|Ƒ^{IN}_{Ӿ_2}\right|^{\vartheta_{\mathfrak{W}\mathfrak{V}-2}}\right) \end{pmatrix} \end{pmatrix}$$

$$\vartheta_{\mathfrak{W}\mathfrak{V}-1}Ӿ_1 \oplus \vartheta_{\mathfrak{W}\mathfrak{V}-2}Ӿ_2$$

$$= \begin{pmatrix} \left[\hat{s}^{\mathcal{L}}_{ƙ-ƙ\left(1-\frac{\delta_1}{ƙ}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-1}}},\ \hat{s}^{\mathcal{U}}_{ƙ-ƙ\left(1-\frac{\delta_1}{ƙ}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-1}}}\right], \\ \begin{pmatrix} 1-\left(1-Ƒ^{RP}_{Ӿ_1}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-1}} \\ +\iota\left(1-\left(1-Ƒ^{IP}_{Ӿ_1}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-1}}\right), \\ -\left|Ƒ^{RN}_{Ӿ_1}\right|^{\vartheta_{\mathfrak{W}\mathfrak{V}-1}} \\ +\iota\left(-\left|Ƒ^{IN}_{Ӿ_1}\right|^{\vartheta_{\mathfrak{W}\mathfrak{V}-1}}\right) \end{pmatrix} \end{pmatrix} \oplus \begin{pmatrix} \left[\hat{s}^{\mathcal{L}}_{ƙ-ƙ\left(1-\frac{\delta_2}{ƙ}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-2}}},\ \hat{s}^{\mathcal{U}}_{ƙ-ƙ\left(1-\frac{\delta_2}{ƙ}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-2}}}\right], \\ \begin{pmatrix} 1-\left(1-Ƒ^{RP}_{Ӿ_2}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-2}} \\ +\iota\left(1-\left(1-Ƒ^{IP}_{Ӿ_2}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-2}}\right), \\ -\left|Ƒ^{RN}_{Ӿ_2}\right|^{\vartheta_{\mathfrak{W}\mathfrak{V}-2}} \\ +\iota\left(-\left|Ƒ^{IN}_{Ӿ_2}\right|^{\vartheta_{\mathfrak{W}\mathfrak{V}-2}}\right) \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} \left[\hat{s}^{\mathcal{L}}_{ƙ-ƙ\left(1-\frac{\delta_1}{ƙ}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-1}}\left(1-\frac{\delta_2}{ƙ}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-2}}},\ \hat{s}^{\mathcal{U}}_{ƙ-ƙ\left(1-\frac{\delta_1}{ƙ}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-1}}\left(1-\frac{\delta_2}{ƙ}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-2}}}\right], \\ \begin{pmatrix} 1-\left(1-Ƒ^{RP}_{Ӿ_1}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-1}}\left(1-Ƒ^{RP}_{Ӿ_2}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-2}} \\ +\iota\left(1-\left(1-Ƒ^{IP}_{Ӿ_1}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-1}}\left(1-Ƒ^{IP}_{Ӿ_2}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-2}}\right), \\ -\left(\left(-\left|Ƒ^{RN}_{Ӿ_1}\right|^{\vartheta_{\mathfrak{W}\mathfrak{V}-1}}\right)\left(-\left|Ƒ^{RN}_{Ӿ_2}\right|^{\vartheta_{\mathfrak{W}\mathfrak{V}-2}}\right)\right) \\ +\iota\left(-\left(\left(-\left|Ƒ^{IN}_{Ӿ_1}\right|^{\vartheta_{\mathfrak{W}\mathfrak{V}-1}}\right)\left(-\left|Ƒ^{IN}_{Ӿ_2}\right|^{\vartheta_{\mathfrak{W}\mathfrak{V}-2}}\right)\right)\right) \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} \left[\hat{s}^{\mathcal{L}}_{ƙ-ƙ\prod_{\phi=1}^{2}\left(1-\frac{\delta_\phi}{ƙ}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-\phi}}},\ \hat{s}^{\mathcal{U}}_{ƙ-ƙ\prod_{\phi=1}^{2}\left(1-\frac{\delta_\phi}{ƙ}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-\phi}}}\right], \\ \begin{pmatrix} 1-\prod_{\phi=1}^{2}\left(1-Ƒ^{RP}_{Ӿ_\phi}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-\phi}} \\ +\iota\left(1-\prod_{\phi=1}^{2}\left(1-Ƒ^{IP}_{Ӿ_\phi}\right)^{\vartheta_{\mathfrak{W}\mathfrak{V}-\phi}}\right), \\ -\prod_{\phi=1}^{2}\left|Ƒ^{RN}_{Ӿ_\phi}\right|^{\vartheta_{\mathfrak{W}\mathfrak{V}-\phi}} \\ +\iota\left(-\prod_{\phi=1}^{2}\left|Ƒ^{IN}_{Ӿ_\phi}\right|^{\vartheta_{\mathfrak{W}\mathfrak{V}-\phi}}\right) \end{pmatrix} \end{pmatrix}$$

So, Eq. (8) is right for $\mathfrak{b}=2$.

(ii) Let Eq. (8) is right for $\mathfrak{b}=\mathfrak{p}$ that is

$$BCFULWA\left(\mathfrak{X}_1,\ \mathfrak{X}_2,\ldots,\ \mathfrak{X}_{\mathfrak{p}}\right)$$

$$=\left(\begin{array}{c}\left[\hat{s}^{\mathcal{L}}_{\mathfrak{k}-\mathfrak{k}\prod_{\phi=1}^{\mathfrak{p}}\left(1-\frac{\delta_\phi}{\mathfrak{k}}\right)^{\vartheta\mathfrak{wv}-\phi}},\ \hat{s}^{\mathcal{U}}_{\mathfrak{k}-\mathfrak{k}\prod_{\phi=1}^{\mathfrak{p}}\left(1-\frac{\delta_\phi}{\mathfrak{k}}\right)^{\vartheta\mathfrak{wv}-\phi}}\right],\\ \left(\begin{array}{c}1-\prod_{\phi=1}^{\mathfrak{p}}\left(1-F^{RP}_{\mathfrak{X}_\phi}\right)^{\vartheta\mathfrak{wv}-\phi}\\ +\iota\left(1-\prod_{\phi=1}^{\mathfrak{p}}\left(1-F^{IP}_{\mathfrak{X}_\phi}\right)^{\vartheta\mathfrak{wv}-\phi}\right),\\ -\prod_{\phi=1}^{\mathfrak{p}}\left|F^{RN}_{\mathfrak{X}_\phi}\right|^{\vartheta\mathfrak{wv}-\phi}\\ +\iota\left(-\prod_{\phi=1}^{\mathfrak{p}}\left|F^{IN}_{\mathfrak{X}_\phi}\right|^{\vartheta\mathfrak{wv}-\phi}\right)\end{array}\right)\end{array}\right)$$

(iii) Let $\mathfrak{b}=\mathfrak{p}+1$. Then

$$BCFULWA\left(\begin{array}{c}\mathfrak{X}_1,\ \mathfrak{X}_2,\ldots,\\ \mathfrak{X}_{\mathfrak{p}},\ \mathfrak{X}_{B\mathfrak{p}+1}\end{array}\right)$$

$$=\left(\begin{array}{c}\left[\hat{s}^{\mathcal{L}}_{\mathfrak{k}-\mathfrak{k}\prod_{\phi=1}^{\mathfrak{p}}\left(1-\frac{\delta_\phi}{\mathfrak{k}}\right)^{\vartheta\mathfrak{wv}-\phi}},\ \hat{s}^{\mathcal{U}}_{\mathfrak{k}-\mathfrak{k}\prod_{\phi=1}^{\mathfrak{p}}\left(1-\frac{\delta_\phi}{\mathfrak{k}}\right)^{\vartheta\mathfrak{wv}-\phi}}\right],\\ \left(\begin{array}{c}1-\prod_{\phi=1}^{\mathfrak{p}}\left(1-F^{RP}_{\mathfrak{X}_\phi}\right)^{\vartheta\mathfrak{wv}-\phi}\\ +\iota\left(1-\prod_{\phi=1}^{\mathfrak{p}}\left(1-F^{IP}_{\mathfrak{X}_\phi}\right)^{\vartheta\mathfrak{wv}-\phi}\right),\\ -\prod_{\phi=1}^{\mathfrak{p}}\left|F^{RN}_{\mathfrak{X}_\phi}\right|^{\vartheta\mathfrak{wv}-\phi}\\ +\iota\left(-\prod_{\phi=1}^{\mathfrak{p}}\left|F^{IN}_{\mathfrak{X}_\phi}\right|^{\vartheta\mathfrak{wv}-\phi}\right)\end{array}\right)\end{array}\right)$$

$$\oplus\left(\begin{array}{c}\left[\hat{s}^{\mathcal{L}}_{\mathfrak{k}-\mathfrak{k}\left(1-\frac{\delta_{\mathfrak{p}+1}}{\mathfrak{k}}\right)^{\vartheta\mathfrak{wv}-\mathfrak{p}+1}},\ \hat{s}^{\mathcal{U}}_{\mathfrak{k}-\mathfrak{k}\left(1-\frac{\delta_{\mathfrak{p}+1}}{\mathfrak{k}}\right)^{\vartheta\mathfrak{wv}-\mathfrak{p}+1}}\right],\\ \left(\begin{array}{c}1-\left(1-F^{RP}_{\mathfrak{X}_{\mathfrak{p}+1}}\right)^{\vartheta\mathfrak{wv}-\mathfrak{p}+1}\\ +\iota\left(1-\left(1-F^{IP}_{\mathfrak{X}_{\mathfrak{p}+1}}\right)^{\vartheta\mathfrak{wv}-\mathfrak{p}+1}\right),\\ -\left|F^{RN}_{\mathfrak{X}_{\mathfrak{p}+1}}\right|^{\vartheta\mathfrak{wv}-\mathfrak{p}+1}\\ +\iota\left(-\left|F^{IN}_{\mathfrak{X}_{\mathfrak{p}+1}}\right|^{\vartheta\mathfrak{wv}-\mathfrak{p}+1}\right)\end{array}\right)\end{array}\right)$$

$$=\left(\begin{array}{c}\left[\hat{s}^{\mathcal{L}}_{\mathfrak{k}-\mathfrak{k}\prod_{\phi=1}^{\mathfrak{p}+1}\left(1-\frac{\delta_\phi}{\mathfrak{k}}\right)^{\vartheta\mathfrak{wv}-\phi}},\ \hat{s}^{\mathcal{U}}_{\mathfrak{k}-\mathfrak{k}\prod_{\phi=1}^{\mathfrak{p}+1}\left(1-\frac{\delta_\phi}{\mathfrak{k}}\right)^{\vartheta\mathfrak{wv}-\phi}}\right],\\ \left(\begin{array}{c}1-\prod_{\phi=1}^{\mathfrak{p}+1}\left(1-F^{RP}_{\mathfrak{X}_\phi}\right)^{\vartheta\mathfrak{wv}-\phi}\\ +\iota\left(1-\prod_{\phi=1}^{\mathfrak{p}+1}\left(1-F^{IP}_{\mathfrak{X}_\phi}\right)^{\vartheta\mathfrak{wv}-\phi}\right),\\ -\prod_{\phi=1}^{\mathfrak{p}+1}\left|F^{RN}_{\mathfrak{X}_\phi}\right|^{\vartheta\mathfrak{wv}-\phi}\\ +\iota\left(-\prod_{\phi=1}^{\mathfrak{p}+1}\left|F^{IN}_{\mathfrak{X}_\phi}\right|^{\vartheta\mathfrak{wv}-\phi}\right)\end{array}\right)\end{array}\right)$$

So Eq. (8) is right for $\mathfrak{b}=\mathfrak{p}+1$ and thus right for all $\mathfrak{b}$.

Proof of Theorem 5:

(i) Let $\mathfrak{b}=2$. Then

$$\left(\mathfrak{X}_1\right)^{\vartheta\mathfrak{wv}-1}$$

$$=\left(\begin{array}{c}\left[\hat{s}^{\mathcal{L}}_{\mathfrak{k}\left(\frac{\delta_1}{\mathfrak{k}}\right)^{\vartheta\mathfrak{wv}-1}},\ \hat{s}^{\mathcal{U}}_{\mathfrak{k}\left(\frac{\delta_1}{\mathfrak{k}}\right)^{\vartheta\mathfrak{wv}-1}}\right],\\ \left(\begin{array}{c}\left(F^{RP}_{\mathfrak{X}_1}\right)^{\vartheta\mathfrak{wv}-1}\\ +\iota\left(F^{IP}_{\mathfrak{X}_1}\right)^{\vartheta\mathfrak{wv}-1},\\ -1+\left(1+F^{RN}_{\mathfrak{X}_1}\right)^{\vartheta\mathfrak{wv}-1}\\ +\iota\left(-1+\left(1+F^{IN}_{\mathfrak{X}_1}\right)^{\vartheta\mathfrak{wv}-1}\right)\end{array}\right)\end{array}\right)$$

$$\left(\mathfrak{X}_2\right)^{\vartheta\mathfrak{wv}-2}$$

$$=\left(\begin{array}{c}\left[\hat{s}^{\mathcal{L}}_{\mathfrak{k}\left(\frac{\delta_1}{\mathfrak{k}}\right)^{\vartheta\mathfrak{wv}-2}},\ \hat{s}^{\mathcal{U}}_{\mathfrak{k}\left(\frac{\delta_1}{\mathfrak{k}}\right)^{\vartheta\mathfrak{wv}-2}}\right],\\ \left(\begin{array}{c}\left(F^{RP}_{\mathfrak{X}_2}\right)^{\vartheta\mathfrak{wv}-2}\\ +\iota\left(F^{IP}_{\mathfrak{X}_2}\right)^{\vartheta\mathfrak{wv}-2},\\ -1+\left(1+F^{RN}_{\mathfrak{X}_2}\right)^{\vartheta\mathfrak{wv}-2}\\ +\iota\left(-1+\left(1+F^{IN}_{\mathfrak{X}_2}\right)^{\vartheta\mathfrak{wv}-2}\right)\end{array}\right)\end{array}\right)$$

$$\left(\mathfrak{X}_1\right)^{\vartheta\mathfrak{wv}-1}\otimes\left(\mathfrak{X}_2\right)^{\vartheta\mathfrak{wv}-2}$$

$$=\left(\begin{array}{c}\left[\hat{s}^{\mathcal{L}}_{\mathfrak{k}\left(\frac{\delta_1}{\mathfrak{k}}\right)^{\vartheta\mathfrak{wv}-1}},\ \hat{s}^{\mathcal{U}}_{\mathfrak{k}\left(\frac{\delta_1}{\mathfrak{k}}\right)^{\vartheta\mathfrak{wv}-1}}\right],\\ \left(\begin{array}{c}\left(F^{RP}_{\mathfrak{X}_1}\right)^{\vartheta\mathfrak{wv}-1}\\ +\iota\left(F^{IP}_{\mathfrak{X}_1}\right)^{\vartheta\mathfrak{wv}-1},\\ -1+\left(1+F^{RN}_{\mathfrak{X}_1}\right)^{\vartheta\mathfrak{wv}-1}\\ +\iota\left(-1+\left(1+F^{IN}_{\mathfrak{X}_1}\right)^{\vartheta\mathfrak{wv}-1}\right)\end{array}\right)\end{array}\right)$$

$$
\otimes\left(\begin{array}{c}\left[\hat{s}^{\mathcal{L}}_{\Bbbk\left(\frac{\delta_1}{\Bbbk}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-2}},\ \hat{s}^{\mathcal{U}}_{\Bbbk\left(\frac{\delta_1}{\Bbbk}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-2}}\right],\\
\left(\begin{array}{c}\left(\digamma^{RP}_{\bar{X}_2}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-2}\\
+\iota\left(\digamma^{IP}_{\bar{X}_2}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-2},\\
-1+\left(1+\digamma^{RN}_{\bar{X}_2}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-2}\\
+\iota\left(-1+\left(1+\digamma^{IN}_{\bar{X}_2}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-2}\right)\end{array}\right)\end{array}\right)
$$

$$
=\left(\begin{array}{c}\left[\hat{s}^{\mathcal{L}}_{\Bbbk\left(\frac{\delta_1}{\Bbbk}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-1}\Bbbk\left(\frac{\delta_2}{\Bbbk}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-2}},\ \hat{s}^{\mathcal{U}}_{\Bbbk\left(\frac{\delta_1}{\Bbbk}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-1}\Bbbk\left(\frac{\delta_2}{\Bbbk}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-2}}\right]\\
\left(\begin{array}{c}\left(\digamma^{RP}_{\bar{X}_1}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-1}\left(\digamma^{RP}_{\bar{X}_2}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-2}\\
+\iota\left(\left(\digamma^{IP}_{\bar{X}_1}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-1}\left(\digamma^{IP}_{\bar{X}_2}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-2}\right),\\
\left(\begin{array}{c}-1+\left(1+\digamma^{RN}_{\bar{X}_1}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-1}\\
\left(1+\digamma^{RN}_{\bar{X}_2}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-2}\end{array}\right)\\
+\iota\left(\begin{array}{c}-1+\left(1+\digamma^{IN}_{\bar{X}_1}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-1}\\
\left(1+\digamma^{IN}_{\bar{X}_2}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-2}\end{array}\right)\end{array}\right)\end{array}\right)
$$

$$
=\left(\begin{array}{c}\left[\hat{s}^{\mathcal{L}}_{\Bbbk\prod_{\phi=1}^{2}\left(\frac{\delta_\phi}{\Bbbk}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-\phi}},\ \hat{s}^{\mathcal{U}}_{\Bbbk\prod_{\phi=1}^{2}\left(\frac{\delta_\phi}{\Bbbk}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-\phi}}\right],\\
\left(\begin{array}{c}\prod_{\phi=1}^{2}\left(\digamma^{RP}_{\bar{X}_\phi}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-\phi}\\
+\iota\prod_{\phi=1}^{2}\left(\digamma^{IP}_{\bar{X}_\phi}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-\phi},\\
-1+\prod_{\phi=1}^{2}\left(1+\digamma^{RN}_{\bar{X}_\phi}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-\phi}\\
+\iota\left(-1+\prod_{\phi=1}^{2}\left(1+\digamma^{RN}_{\bar{X}_\phi}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-\phi}\right)\end{array}\right)\end{array}\right)
$$

So, Eq. (14) is right for $\text{ь}=2$.

(ii) Let Eq. (14) is right for $\text{ь}=\text{ᵽ}$ that is

$$
BCFULWG\left(\bar{X}_1,\ \bar{X}_2,\ldots,\ \bar{X}_{\text{ᵽ}}\right)
=\left(\begin{array}{c}\left[\hat{s}^{\mathcal{L}}_{\Bbbk\prod_{\phi=1}^{\text{ᵽ}}\left(\frac{\delta_\phi}{\Bbbk}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-\phi}},\ \hat{s}^{\mathcal{U}}_{\Bbbk\prod_{\phi=1}^{\text{ᵽ}}\left(\frac{\delta_\phi}{\Bbbk}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-\phi}}\right],\\
\left(\begin{array}{c}\prod_{\phi=1}^{\text{ᵽ}}\left(\digamma^{RP}_{\bar{X}_\phi}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-\phi}\\
+\iota\prod_{\phi=1}^{\text{ᵽ}}\left(\digamma^{IP}_{\bar{X}_\phi}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-\phi},\\
-1+\prod_{\phi=1}^{\text{ᵽ}}\left(1+\digamma^{RN}_{\bar{X}_\phi}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-\phi}\\
+\iota\left(-1+\prod_{\phi=1}^{\text{ᵽ}}\left(1+\digamma^{RN}_{\bar{X}_\phi}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-\phi}\right)\end{array}\right)\end{array}\right)
$$

(iii) Let $\text{ь}=\text{ᵽ}+1$. Then

$$
BCFULWG\left(\begin{array}{c}\bar{X}_1,\ \bar{X}_2,\ldots,\\ \bar{X}_{\text{ᵽ}},\ \bar{X}_{\text{ᵽ}+1}\end{array}\right)
=\left(\begin{array}{c}\left[\hat{s}^{\mathcal{L}}_{\Bbbk\prod_{\phi=1}^{\text{ᵽ}}\left(\frac{\delta_\phi}{\Bbbk}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-\phi}},\ \hat{s}^{\mathcal{U}}_{\Bbbk\prod_{\phi=1}^{\text{ᵽ}}\left(\frac{\delta_\phi}{\Bbbk}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-\phi}}\right],\\
\left(\begin{array}{c}\prod_{\phi=1}^{\text{ᵽ}}\left(\digamma^{RP}_{\bar{X}_\phi}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-\phi}\\
+\iota\prod_{\phi=1}^{\text{ᵽ}}\left(\digamma^{IP}_{\bar{X}_\phi}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-\phi},\\
-1+\prod_{\phi=1}^{\text{ᵽ}}\left(1+\digamma^{RN}_{\bar{X}_\phi}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-\phi}\\
+\iota\left(-1+\prod_{\phi=1}^{\text{ᵽ}}\left(1+\digamma^{RN}_{\bar{X}_\phi}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-\phi}\right)\end{array}\right)\end{array}\right)
$$

$$
\otimes\left(\begin{array}{c}\left[\hat{s}^{\mathcal{L}}_{\Bbbk\left(\frac{\delta_1}{\Bbbk}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-\text{ᵽ}+1}},\ \hat{s}^{\mathcal{U}}_{\Bbbk\left(\frac{\delta_1}{\Bbbk}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-\text{ᵽ}+1}}\right],\\
\left(\begin{array}{c}\left(\digamma^{RP}_{\bar{X}_{\text{ᵽ}+1}}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-\text{ᵽ}+1}\\
+\iota\left(\digamma^{IP}_{\bar{X}_{\text{ᵽ}+1}}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-\text{ᵽ}+1},\\
-1+\left(1+\digamma^{RN}_{\bar{X}_{\text{ᵽ}+1}}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-\text{ᵽ}+1}\\
+\iota\left(-1+\left(1+\digamma^{IN}_{\bar{X}_{\text{ᵽ}+1}}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-\text{ᵽ}+1}\right)\end{array}\right)\end{array}\right)
$$

$$
=\left(\begin{array}{c}\left[\hat{s}^{\mathcal{L}}_{\Bbbk\prod_{\phi=1}^{\text{ᵽ}+1}\left(\frac{\delta_\phi}{\Bbbk}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-\phi}},\ \hat{s}^{\mathcal{U}}_{\Bbbk\prod_{\phi=1}^{\text{ᵽ}+1}\left(\frac{\delta_\phi}{\Bbbk}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-\phi}}\right],\\
\left(\begin{array}{c}\prod_{\phi=1}^{\text{ᵽ}+1}\left(\digamma^{RP}_{\bar{X}_\phi}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-\phi}\\
+\iota\prod_{\phi=1}^{\text{ᵽ}+1}\left(\digamma^{IP}_{\bar{X}_\phi}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-\phi},\\
-1+\prod_{\phi=1}^{\text{ᵽ}+1}\left(1+\digamma^{RN}_{\bar{X}_\phi}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-\phi}\\
+\iota\left(-1+\prod_{\phi=1}^{\text{ᵽ}+1}\left(1+\digamma^{RN}_{\bar{X}_\phi}\right)^{\vartheta\mathfrak{w}\mathfrak{v}-\phi}\right)\end{array}\right)\end{array}\right)
$$

So Eq. (14) is right for $\text{ь}=\text{ᵽ}+1$ and thus right for all ь.

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