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## RESEARCH ARTICLE

# Application of the Terminal Synergetic Control for Biological Control of Sugarcane Borer

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**ABSTRACT** Sugarcane is an important agricultural commodity in economics that has been harmed by the invasion of sugarcane borer. Establishing a biological pest control strategy for sugarcane using their natural enemies can both protect agricultural products from pest invasion and the environment from chemical toxicity. In this regard, feedback control emerges as a practical and feasible approach to effectively implement the biological control strategy for managing the sugarcane borer. In this work, the terminal synergetic controller (TSC) was designed to develop a control strategy containing multiple inputs. The controller design was conducted based on the pest-parasitoid model. In the design procedure, the auxiliary system was employed to compensate for the input saturation effects. The control stability was conducted through the Lyapunov stability theorem. To confirm the capability and performance of the proposed strategy, the simulation results demonstrate that it can effectively regulate pest population densities at the desired level, comparable to both the conventional sliding mode control (SMC) and verticum-type control (VC) strategies. However, what sets it apart is that the terminal synergetic controller provides the preferable characteristics for controlling the sugarcane borer population which are the finite-time convergence of the control system, and the absence of chattering phenomena in the control inputs.

**INDEX TERMS** Agroecosystem, biological pest control, input saturation, locally finite-time stability, predator, sugarcane borer, terminal synergetic control.

### NOMENCLATURE

EIJ	Economic injury threshold.
SC	Synergetic control.
SMC	Sliding mode control.
TSC	Terminal synergetic control.
VC	Verticum-type control.
$r$	The rate of intrinsic oviposition of female sugarcane borer.
$K$	The potential maximum rate of oviposition of female sugarcane borer.
$m_1$	The rate of mortality corresponds to the population density of the egg.

$m_2$	The rate of mortality corresponds to the population density of the egg parasitoid.
$m_3$	The rate of mortality corresponds to the population density of the larvae.
$m_4$	The rate of mortality corresponds to the population density of the larvae parasitoid.
$n_1$	The fraction of the sugar borer larvae population emerging from the egg.
$n_3$	The fraction of the un-parasitized sugar borer larvae from emerging pupae.
$\alpha$	The rate of intrinsic parasitism of the egg.
$\beta$	The rate of intrinsic parasitism of the larvae of the parasitoid.
$\gamma_1$	The rate of survival of parasitized eggs.
$\gamma_2$	The rate of survival of larvae to adult age.

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## I. INTRODUCTION

Sugarcane is a major economic crop in many countries such as Brazil, India, China, Thailand, Pakistan, and Mexico. Sugarcane is not only an important source of raw materials for sugar and food production but also a source for ethanol fuel production [1], [2], [3], [4], [5], [6]. Invasion of sugarcane borer can cause damage to both sugarcane plantations and related food processing and ethanol industries [3], [7]. *Diatraea saccharalis* and *D. flavipennella* are the main sugarcane borers that invade the sugarcane plantation and need to be considered in Brazil. However, only invasion of *D. saccharalis* can be found in all regions of Brazil [3], [4], [5], [6], [8]. The sugarcane stalk is pierced by the larvae of the borer. This piercing causes the dead heart inside the sugarcane stalk. Apart from being a cause of death of the sugarcane, this stalk damage affects the growth of sugarcane and leads to fungal infection which affects the quality and quantity of sugar and ethanol production [3], [4], [5], [6], [7], [8]. In biological control in agroecosystems, natural enemies are used for pest population control to reduce the damage caused by pests [9], [10], [11], [12]. According to this concept, sufficient food production and environmental safety can be achieved despite using pesticides which are harmful to the environment and native species [11]. Natural enemies available for pest regulation include predators, pathogens, and parasitoids. These natural enemies can be manipulated in different categories including conservation of local natural enemies, introduction of new natural enemies, and augmentation of natural enemies. These methods collectively aim to maintain pest populations at levels aligned with the economic injury threshold (EIJ) [9], [10], [11], [12], [13]. The biological pest control or biological control policy for *D. saccharalis* is conducted by augmentation of egg and larvae parasitoids which are *Trichogramma galloi* and *Cotesia flavipes*, respectively so that the borer population is regulated at the desired level [3], [4], [14].

The dynamics of the ecosystem can be represented in the form of a set of continuous differential equations such as the Lotka-Volterra model and the cascade model. These mathematical models typically describe the predator-prey relationship or interaction in an ecosystem [4], [5], [6], [8], [13], [15], [16], [17], [18], [19], [20], [21]. Specifically, the dynamical models describing the interaction between parasitoid and sugarcane borer populations in agroecosystems have been developed in the form of Lotka-Volterra models and other types of models as seen in previous works [4], [5], [6], [13], [15], [16], [17], [18], [19], [20], [21], [22].

These models have been employed for dynamic analysis and for formulating control strategies. According to Puebla et al. [18], feedback control is a feasible approach that can formulate efficiently the biological control strategy for various ecosystems including predator-prey systems and parasitoid-pest ecosystems as comparing feedback control approach to other feasible approaches including the dynamic optimization based on Pontryagin's maximum principle and impulsive control strategy. Consequently, different feedback

control techniques have been applied to set the biological control strategy for various ecosystem systems as did in controlling other biological systems, for example, the treatment of cancer and HIV, or epidemic control [23], [24], [25], [26], [27]. For the ecosystem, these control approaches were formulated to control the target populations to converge to the desired levels such as EIJ, and satisfy some further preferable characteristics of the control systems such as chaotic control, robustness, and finite-time or fixed-time stability as presented in [8], [13], [15], [16], [17], [18], [19], [20], [26], [28], [29], [30], [31], [32], [33], and [34].

Nonlinear feedback controls such as sliding mode control (SMC), linear optimal control, and Lyapunov-based control, have been employed for establishing biological control strategies for pest population control in ecosystems. As presented in Boonyaprapasorn et al. [30], [32], synergetic control (SC) is also an appropriate control method for this application. The SC method, initially introduced by Kolesnikov et al. in the 1990s [35], [36], [37], [38], [39], [40], [41], [42], [43], is formulated by a combination of the following concepts including self-organization in an open system, modern mathematics, optimal control, and synergetic [38], [39]. Moreover, using the SC allows the designer to synthesize a controller for the dynamical system with desired properties including insensitivity to parameter variations and effective noise suppression. Notably, the SC method can avoid the undesirable property of the chattering phenomenon in control input, a drawback frequently associated with the SMC method [40], [41], [43]. Consequently, the SC has been employed for controlling several engineering systems, for example, power systems [38], [39], [40], [41], [44], [45], robotics [43], [46], satellite [47], and biological systems [24], [29], [48], [49], [50]. Furthermore, this feedback control approach can be exploited for controlling nonlinear high-order systems efficiently. According to [35], [36], [37], [38], [39], [40], [41], and [43], the key steps of the controller design can be concluded as follows. First, the set of macro variables is defined in agreement with the dimension of control inputs and specified control objectives to formulate the corresponding manifold. In the SC method, these control inputs are considered external controls. Second, the dynamic evolution for each macro variable is defined for constraining each macro variable. At this step, the convergence rate of each macro variable to the constructed manifold depends on the selection of the dynamic evolution. Finally, the set of the control inputs, or external controls, is determined by solving the set of defined dynamic evolutions associated with the considered dynamic models [35], [36], [37], [38], [39], [40], [41], [42], [43]. Moreover, according to the concept of the SC, both external and internal controls are utilized to synthesize the control law for the case when some subsystems cannot be controlled by external controls due to the structure of the model and/or the control objective of the considered dynamic system [46], [51].

Convergence time is an important characteristic of the control system to achieve the desired control objective in

the specified period as mentioned in [52]. Thus, concepts of finite time stability and other related stability properties have been developed and utilized in feedback controllers to serve this need [31], [43], [49], [53], [54], [55], [56], [57], [58]. The finite time or terminal synergetic control (TSC) is the improved version of the SC method of convergence time. In the finite time stable system, the convergence of the system occurs in finite time. Constructing the SC method based on finite-time stability results in the TSC method which was employed for various control applications [44], [49], [59], [60], [61], [62], [63]. Finite-time stability of the control system under the TSC approach can be achieved [43], [60], [63]. Consequently, the rate of convergence can be increased [44], [60], [63]. Still, using the classical SC method can avoid the chattering phenomenon in the control input. This phenomenon is the drawback characteristic of the SMC method. In the TSC method, various dynamic evolutions and/or macro variables, which hold the property of finite time stability, are selected [43], [44], [49], [53], [60], [63]. Apart from the conventional engineering systems, the different versions of the SC method stated above have been employed for biological systems [23], [24], [29], [31], [48], [49], [50]. Specifically, biological pest control strategies have been developed through the SC design shown in previous works [29], [31]. Considering the model's structure and control objectives, the potential for singularity arises within the formulated biological control strategy using the TSC method. This can be caused by the derivative of the macro variable with a fractional-order exponent, which is comparable to situations observed in the conventional terminal SMC method [55].

In general, control inputs are constrained or under saturation. Thus, the effect of input saturation can be compensated through the auxiliary system with the assumption that the difference between the actual control and the nominal controls is sufficiently small as presented in previous works [64], [65].

In this study, according to the preferable characteristics of the SC method stated above and the advantage of the finite time stability, the TSC approach design was employed to set the biological control strategy for the considered pest-parasitoid system. The verticum-type model of the pest-parasitoid system presented by Molnár et al. [8], [16] was used to formulate the biological pest control strategy. This model contains multiple control inputs and is a high-order system. Consequently, formulating the biological control strategy is considered a challenge of nonlinear feedback control problems. To the best of the authors' knowledge, applying terminal synergetic control for this system has not been presented. The specific contributions are presented as follows:

(a) The concept of the TSC approach with internal and external controls has been applied to set the biological control strategy for the considered pest-parasitoid agroecosystem.

(b) The preferable characteristic of finite-time stability is achieved according to the proposed biological control

strategy. Thus, the convergence of the control agroecosystem occurs in finite time. This improves the convergence of the control agroecosystem. However, this characteristic cannot be achieved by using the linear optimal control presented in Rafikov et al. [4], [13], since this control system can only guarantee asymptotic stability.

(c) Using the proposed TSC policy allows the designer to avoid the chattering phenomena which is the main drawback of the SMC method [40], [41], [42], [43], [66], [67].

(d) The biological pest control strategy under the effect of input saturation is also synthesized with the aid of an auxiliary system. The proposed control strategy can handle the physical limitation of the control inputs. Also, the requirement of positive control input can be achieved by setting the lower bound of the input saturation greater than zero. This characteristic is preferable for pest population control in agroecosystems as mentioned in [8] and [16].

Apart from the introduction part in section I stated above, the rest of this paper is divided into the following sections: Section II includes the mathematical model of the pest-parasitoid ecosystem and mathematical preliminaries. Then, the controller design procedure formulating the biological control strategy is presented in section III. Section IV consists of simulation results of the control system and discussion. Finally, the conclusion of this study is stated.

## II. PRELIMINARIES

### A. MATHEMATICAL PRELIMINARIES

The lemmas required to design the controller and to prove the stability of the control system are summarized below.

*Lemma 1 ([48], [49]):* Consider the system in (1):

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}), \quad (1)$$

where  $\mathbf{x}$  denotes a state vector and  $\mathbf{x} \in \mathbb{R}^n$ . If there exists a positive-definite and continuous Lyapunov function with the following inequality

$$\dot{V}(t) \leq -\alpha V^\eta(t), \quad \forall t \geq t_0, V(t_0) \geq 0, \quad (2)$$

where  $\alpha$  is a positive constant and  $\eta$  is a constant exponent with  $0 < \eta < 1$ . Then, the following inequality

$$V^{1-\eta} \leq V^{1-\eta}(t_0) - \alpha(1-\eta)(t-t_0), \quad t_0 < t < T_s, \quad (3)$$

holds for given an initial time  $t_0$ , and

$$V(t) = 0, \quad \forall t \geq T_s, \quad (4)$$

where  $T_s$  is determined as

$$T_s = t_0 + \frac{V^{1-\eta}(t_0)}{\alpha(1-\eta)} \quad (5)$$

*Lemma 2 ([55]):* Suppose that  $\sigma_1, \sigma_2, \dots, \sigma_n$  are positive real numbers and  $0 < p \leq 1$ . Then, the following inequality holds.

$$(\sigma_1 + \sigma_2 + \dots + \sigma_n)^p \leq (\sigma_1^p + \sigma_2^p + \dots + \sigma_n^p). \quad (6)$$

## B. PEST-PARASITOID SYSTEM

The mathematical model explaining the dynamics of the interaction between pest and parasitoid with the control inputs was presented by Molnár et al. [8], [16] as shown in (7):

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} rx_1 \left(1 - \frac{x_1}{K}\right) - m_1x_1 - n_1x_1 - \alpha x_1x_2 \\ \alpha\gamma_1x_1x_2 - m_2x_2 + U_2 \\ n_1x_1 - m_3x_3 - n_3x_3 - \beta x_3x_4 \\ \beta\gamma_2x_3x_4 - m_4x_4 + U_4 \end{bmatrix}, \quad (7)$$

where the state  $x_1$  denotes the population density of sugarcane borer's unparasitized egg. The state  $x_2$  represents the population density of the egg parasitoid *Trichogramma galloi*. The state  $x_3$  denotes the population density of the unparasitized larvae of the sugarcane borer. The state  $x_4$  represents the population density of the larvae parasitoid *Cotesia flavipes*. The parameters specified in the nomenclature correspond to those of the model described in (7).

It is worth to emphasis that the units of parameters are annotated appropriately such that the unit of each term that appeared on the right-hand side is agreed with the unit of the term on the left-hand side which is density/unit of time. According to [8] and [16], to satisfy the requirement of positive control, the control inputs in (5) are modified as  $U_2 = u_2^* + u_2(t)$  and  $U_4 = u_4^* + u_4(t)$ , where  $|u_2(t)| \leq u_2^*$  and  $|u_4(t)| \leq u_4^*$ .  $U_2$  and  $U_4$  are the rate of density of egg and larvae parasitoids, respectively.

Let define the set that the system (7) is well-defined  $\Omega \triangleq \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 | x_1 \neq 0, x_3 \neq 0\}$ . Moreover, we define  $x \triangleq [x_1, x_2, x_3, x_4]^T \in \Omega$  as a vector of the states of the system, and note that  $t \in [0, \infty)$ , then the pest-parasitoid system is rewritten as:

$$\begin{aligned} \dot{x}_1 &= f_1(x) + g_1(x)x_2, \\ \dot{x}_2 &= f_2(x) + u_2^* + u_2(t), \\ \dot{x}_3 &= f_3(x) + g_3(x)x_4, \\ \dot{x}_4 &= f_4(x) + u_4^* + u_4(t), \end{aligned} \quad (8)$$

where  $f_i : \Omega \times [0, \infty) \rightarrow \mathbb{R}$ ,  $\forall i = 1, \dots, 4$ , and

$$\begin{aligned} f_1 &\triangleq (x)rx_1 \left(1 - \frac{x_1}{K}\right) - m_1x_1 - n_1x_1, \\ f_2 &\triangleq (x)\alpha\gamma_1x_1x_2 - m_2x_2, \\ f_3 &\triangleq (x)n_1x_1 - m_3x_3 - n_3x_3, \\ f_4 &\triangleq (x)\beta\gamma_2x_3x_4 - m_4x_4. \\ g_i &: \Omega \times [0, \infty) \rightarrow \mathbb{R} \setminus \{0\}, \quad \forall i = 1, 3, \\ g_1 &\triangleq (x) - \alpha x_1, \quad \text{and } g_3 \triangleq (x) - \beta x_3. \end{aligned}$$

This model facilitates the designer to formulate biological control strategies based on the feedback control approach. Readers can find further details of this model from the [8] and [16].

## C. AUXILIARY SYSTEM

To reduce the effect of saturated input, the biological control strategy was formulated through the TSC design with an auxiliary system.

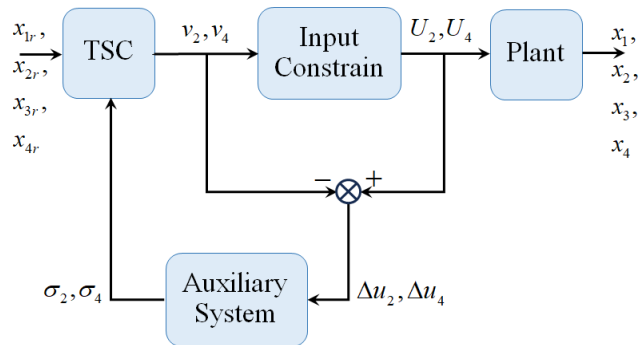


FIGURE 1. The control system block diagram.

As mentioned in [64] and [65], using the feedback controller with the auxiliary system to handle the input saturation effect is feasible under the assumption that the difference between the actual control input associated with the bounds of the input saturation and the nominal control input must be sufficiently small.

The main idea of auxiliary system design is to introduce an extra dynamic to dominate the excess input signal when the saturated input situation occurs [64], [65]. To illustrate this concept, the auxiliary system is designed as the following structure:

$$\dot{\sigma} = w(\sigma) + \Delta u,$$

where  $\sigma \in \mathbb{R}$  is the state of the auxiliary system,  $w(\sigma) : \mathbb{R} \setminus \{0\} \rightarrow (0, -\infty)$ ,  $w(0) = 0$  whenever  $\sigma = 0$  i.e. the negative definite function of  $\sigma$ , and  $\Delta u \in \mathbb{R}$  is the difference between the input that is determined by the TSC approach and the constraints of the input.

The function  $w(\sigma)$  is designed to dominate the term  $\Delta u$  to guarantee the convergence of  $\sigma$ . Hence, the greater  $|\Delta u|$  is required more robustness of the term  $w(\sigma)$ . More details of the auxiliary system will be discussed in Section III-C.

## III. BIOLOGICAL PEST CONTROL STRATEGY

### A. CONTROL OBJECTIVE

According to [8] and [16], the control objective of the biological control strategy for the pest-parasitoid system is to control both population densities of un-parasitized egg ( $x_1$ ) and un-parasitized larvae ( $x_3$ ) to the desired level  $x_{1r}$  and  $x_{3r}$  in the pre-specified settling time. This can be expressed mathematically as

$$\lim_{t \rightarrow T_s} x_i = x_{ir} \quad (9)$$

and

$$x_i(t) = x_{ir}, t \geq T_s \text{ for } i = 1, 3. \quad (10)$$

The reference signal  $x_{ir}$  represents desired levels associated with the nonzero equilibrium points of the pest-parasitoid system. The value of  $x_{ir}$  is designated to harmful pest level.

**B. TERMINAL SYNERGETIC CONTROLLER**

The biological pest control for the pest-parasitoid system is formulated based on the TSC design procedure. The design procedure can be summarized as follows [29], [31], [35], [36], [37], [38], [39], [40], [41], [42], [43], [46], [48], [51]:

1) Selection of the macro variables: according to the control objective and dimension of the control inputs, the first set of macro variables is selected as

$$\psi_i = x_i - x_{ir}, \text{ for } i = 1, 3. \tag{11}$$

The second set of macro variables associated with the difference between the required population densities of both egg parasitoid population density ( $\varphi_2$ ) and larvae parasitoid population density ( $\varphi_4$ ) for regulating the population densities of the egg and larval sugarcane borer and the corresponding state variables are defined as (12):

$$\psi_i = x_i - \varphi_i, \tag{12}$$

where  $\varphi_i$  for  $i = 2, 4$  are considered as the internal controls.

Dynamic evolution selection: the dynamic evolutions corresponding to the macro variables in the previous step are specified so that the macro variables converge to zero within a finite time. Thus, the dynamic evolution presented by Hachana and Harmas [49] so that finite-time convergence of each macro variable is achieved. Dynamic evolution is defined as:

$$\dot{\psi}_i + T_i \dot{\psi}_i^{p_c/q_c} = 0, \tag{13}$$

where the coefficients  $T_i > 0$  is the tuning parameter for settling time. The numerator  $p_c$  and denominator  $q_c$  are positive real numbers such that the exponent terms are denoted as  $1 < p_c/q_c < 2$ .

2) Finding the internal and external controls: both types of controls can be determined by solving the dynamic evolutions associated with the pest-parasitoid ecosystems in (8).

First, the internal control can be determined as follows. Dynamic evolutions (14) can be written as

$$\dot{\psi}_i = W_i, \tag{14}$$

where  $W_i = \left(-\frac{1}{T_i} \dot{\psi}_i\right)^{q_c/p_c}$  for  $i = 1, 3$  are derived from (13). Based on the defined macro variable and dynamic systems, (14) can be expressed as

$$\begin{aligned} f_1(x) + g_1(x)x_2 - \dot{x}_{1r} &= W_1, \\ f_3(x) + g_3(x)x_4 - \dot{x}_{3r} &= W_3. \end{aligned} \tag{15}$$

Recall that  $x_i = \psi_i + \varphi_i$  for  $i = 2, 4$ ,

$$\begin{aligned} f_1(x) + g_1(x)(\psi_2 + \varphi_2) - \dot{x}_{1r} &= W_1, \\ f_3(x) + g_3(x)(\psi_4 + \varphi_4) - \dot{x}_{3r} &= W_3. \end{aligned} \tag{16}$$

Then,

$$\begin{aligned} \varphi_2 &= g_1(x)^{-1} (W_1 - f_1(x) + \dot{x}_{1r} - g_1(x)\psi_2), \\ \varphi_4 &= g_3(x)^{-1} (W_3 - f_3(x) + \dot{x}_{3r} - g_3(x)\psi_4). \end{aligned} \tag{17}$$

Next, the external control can be determined from the specified dynamic evolution as follows:

$$\begin{aligned} f_2(x) + u_2 + u_2^* - \dot{\varphi}_2 &= W_2, \\ f_4(x) + u_4 + u_4^* - \dot{\varphi}_4 &= W_4. \end{aligned} \tag{18}$$

Thus, the external controls can be determined as

$$\begin{aligned} u_2 &= W_2 - f_2(x) - u_2^* + \dot{\varphi}_2, \\ u_4 &= W_4 - f_4(x) - u_4^* + \dot{\varphi}_4. \end{aligned} \tag{19}$$

From the preliminary setting we have mentioned in (11)–(29), we now can show that the proposed controller in (19) can locally stabilize the systems (8) in finite time.

*Theorem 1:* The nonlinear system (8) can be locally stabilized by the controller (19) within the finite settling time:

$$T_s = t_0 + \frac{V^{1-\eta}(t_0)}{\alpha(1-\eta)},$$

where  $V$  is defined as  $V \triangleq \frac{1}{2} \sum_{i=1,\dots,4} \psi_i^2$ ,  $\alpha \triangleq 2^\eta \cdot \min_{1 \leq i \leq 4} \left\{ \frac{1}{T_i} \right\}$ ,  $\psi_i, \forall i \in \{1, \dots, 4\}$  are defined in (11) and (12),  $T_i > 0, \forall i \in \{1, \dots, 4\}$  are the tuning parameter for settling time as in (13),  $\eta = \frac{p_c+q_c}{2p_c}$ , and  $t_0$  is the initial time.

The proof of stability or Theorem 1 can be found in the Appendix.

From the preliminary setting in (11)-(14), we can alleviate the saturated input effect by introducing an auxiliary system and modifying (12) to be associated with the state of the auxiliary system as we will elaborate in the next section.

**C. TERMINAL SYNERGETIC CONTROLLER UNDER INPUT SATURATION**

The design of the terminal synergetic with an auxiliary system follows the approach outlined in Section III-B. In this study, the auxiliary systems are designed as

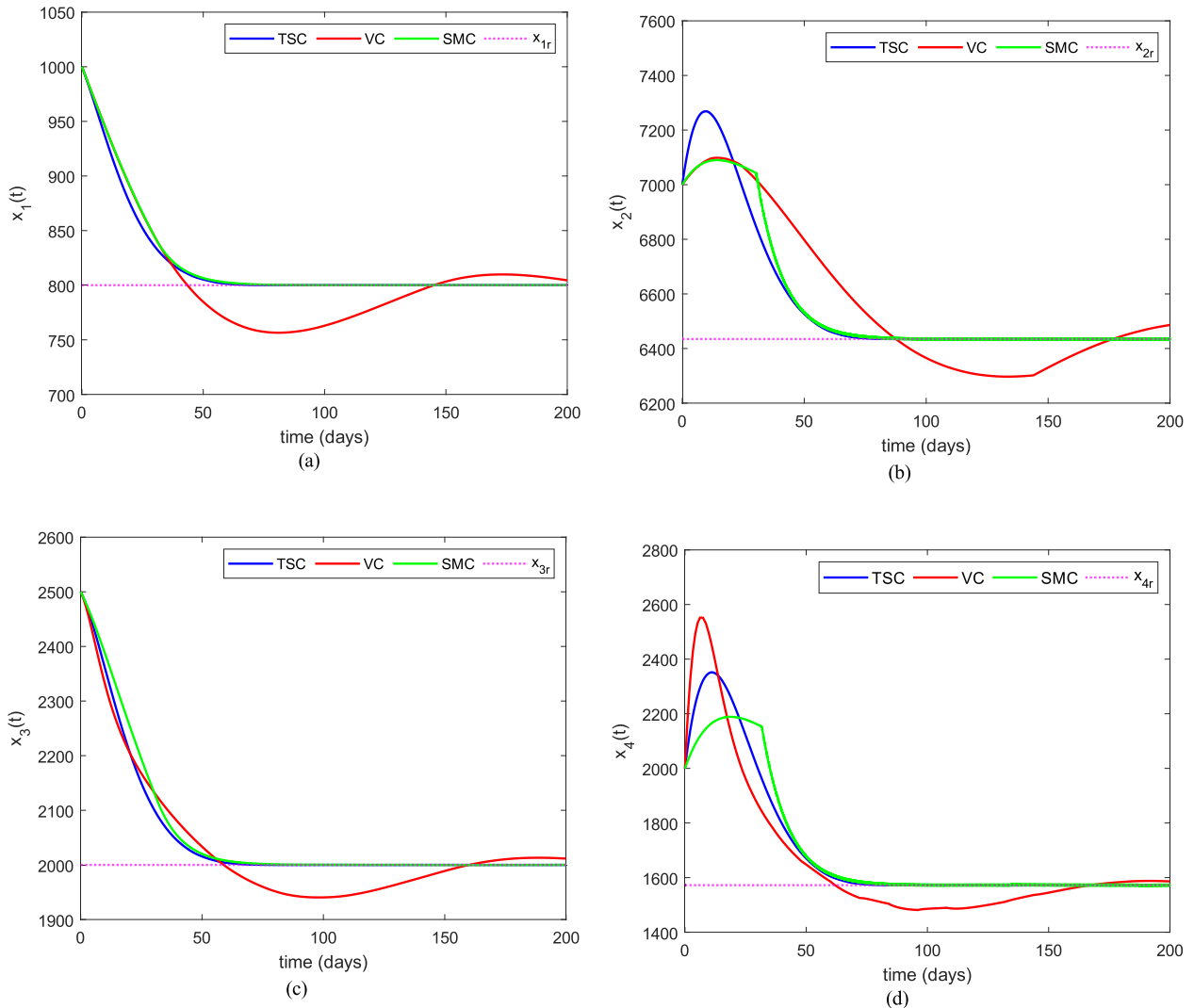
$$\dot{\sigma}_i = w_{\sigma i} + \Delta u_i, \tag{20}$$

where  $w_{\sigma i} = \left(-\frac{1}{T_{\sigma i}} \sigma_i\right)^{q_\sigma/p_\sigma}$  and  $\Delta u_i = u_i - v_i$  for  $i = 2, 4$ . The control variable,  $v_i$ , denotes the  $i^{th}$  nominal control, while the external control input,  $u_i$ , is considered as actual control inputs and constrained  $-u_{i,\min} \leq u_i \leq u_{i,\max}$  for  $i = 2, 4$ . According to [8], the these upper and lower bounds are set as  $u_{i,\min} = u_{i,\max} = u_i^*$  so that the control input  $U_i \geq 0$  for  $i = 2, 4$ . The related parameters of the auxiliary system,  $p_\sigma$  and  $q_\sigma$ , are positive real numbers such that  $1 < p_\sigma/q_\sigma < 2$ . The coefficient for tuning the settling time of the auxiliary system,  $T_{\sigma i}$ , is a real positive number for  $i = 2, 4$ .

Similar to the previous case, the controller design procedure can be summarized as:

1) Selection of macro variable: the macro variables corresponding to the control objective are specified as presented in the non-saturation case as (21)

$$\psi_i = x_i - x_{ir}, \text{ for } i = 1, 3. \tag{21}$$



**FIGURE 2.** Comparing the time responses of the control ecosystem states (a)–(d) under the control strategies based on the TSC, VC, and SMC methods, using the first initial condition (IC1).

However, the second set of the macro variables is modified according to the state of the auxiliary system:

$$\psi_i = x_i - \varphi_i - \sigma_i, \text{ for } i = 2, 4, \quad (22)$$

where  $\sigma_i$  refers to the  $i^{th}$  state variables of the auxiliary system. Like the case of the non-input saturation, the dynamic evolutions are defined as:

$$\psi_i + T_i \dot{\psi}_i^{p_c/q_c} = 0, \quad (23)$$

where the coefficients  $T_i > 0$ . The numerator  $p_c$  and denominator  $q_c$  are positive real numbers such that the exponent terms are denoted as  $1 < p_c/q_c < 2$ .

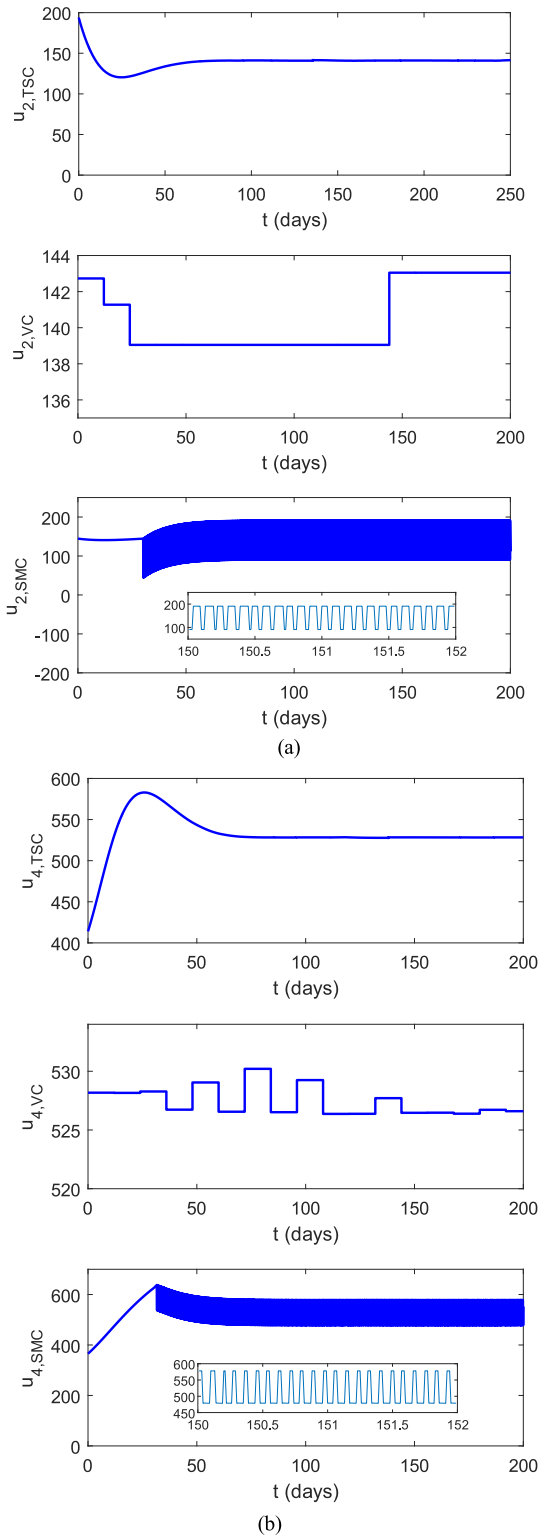
2) Finding the internal and external controls: the set of internal and nominal external controls are determined from the dynamic evolutions in (23) as conducted in the non-input saturation case.

**TABLE 1.** Parameters of the pest-parasitoid ecosystem.

Parameters	Value (Units)
$r$	0.19 (day) <sup>-1</sup>
$K$	25000 (numbers of population/ha)
$m_1, m_2$	0.03566 (day) <sup>-1</sup>
$m_3$	0.00256 (day) <sup>-1</sup>
$m_4$	1 (day) <sup>-1</sup>
$n_1$	1 (day) <sup>-1</sup>
$n_3$	0.02439 (day) <sup>-1</sup>
$\alpha$	0.0000075 (ha/(numbers of population · day))
$\beta$	0.0000083 (ha/(numbers of population · day))
$\gamma_1$	2.29
$\gamma_2$	40

First, the internal control can be determined from the dynamic evolutions as follows.

$$\dot{\psi}_i = W_i, \quad (24)$$



**FIGURE 3.** Control inputs of the control ecosystem using the first initial condition (IC1) under the control strategy based on the TSC, VC, and SMC methods: (a)  $u_2$  and (b)  $u_4$ .

where  $W_i = \left(-\frac{1}{T_i} \psi_i\right)^{q_c/p_c}$  for  $i = 1, 3$  is due to (23). With the defined macro variable and dynamic systems, (24) can be

expressed as

$$\begin{aligned} f_1(x) + g_1(x)x_2 - \dot{x}_{1r} &= W_1, \\ f_3(x) + g_3(x)x_4 - \dot{x}_{3r} &= W_3. \end{aligned} \quad (25)$$

Recall that  $x_i = \psi_i + \varphi_i$ , for  $i = 2, 4$ ,

$$\begin{aligned} f_1(x) + g_1(x)(\psi_2 + \varphi_2) - \dot{x}_{1r} &= W_1, \\ f_3(x) + g_3(x)(\psi_4 + \varphi_4) - \dot{x}_{3r} &= W_3. \end{aligned} \quad (26)$$

Then,

$$\begin{aligned} \varphi_2 &= g_1(x)^{-1} (W_1 - f_1(x) + \dot{x}_{1r} - g_1(x)\psi_2), \\ \varphi_4 &= g_3(x)^{-1} (W_3 - f_3(x) + \dot{x}_{3r} - g_3(x)\psi_4). \end{aligned} \quad (27)$$

Next, equivalent to determining external control for the case of non-input saturation, the nominal controls including  $v_1$  and  $v_2$  can be determined from the specified dynamic evolution (24) and the macro variables associated with the state variables (22) of the auxiliary system as follows:

$$\begin{aligned} f_2(x) + u_2 + u_2^* - \dot{\varphi}_2 - \dot{\sigma}_2 &= W_2, \\ f_4(x) + u_4 + u_4^* - \dot{\varphi}_4 - \dot{\sigma}_4 &= W_4. \end{aligned} \quad (28)$$

Based on the dynamics of the state variables of the auxiliary, (28) becomes

$$\begin{aligned} f_2(x) + u_2 + u_2^* - \dot{\varphi}_2 - (-w_{\sigma 2} + \Delta u_2) &= W_2, \\ f_4(x) + u_4 + u_4^* - \dot{\varphi}_4 - (-w_{\sigma 4} + \Delta u_4) &= W_4. \end{aligned} \quad (29)$$

Recall that  $v_i = u_i - \Delta u_i$  for  $i = 2, 4$ , then (29) becomes:

$$\begin{aligned} f_2(x) + v_2 + u_2^* - \dot{\varphi}_2 + w_{\sigma 2} &= W_2, \\ f_4(x) + v_4 + u_4^* - \dot{\varphi}_4 + w_{\sigma 4} &= W_4. \end{aligned} \quad (30)$$

Thus, the external controls can be determined as

$$\begin{aligned} v_2 &= W_2 - f_2(x) - u_2^* + \dot{\varphi}_2 - w_{\sigma 2}, \\ v_4 &= W_4 - f_4(x) - u_4^* + \dot{\varphi}_4 - w_{\sigma 4}. \end{aligned} \quad (31)$$

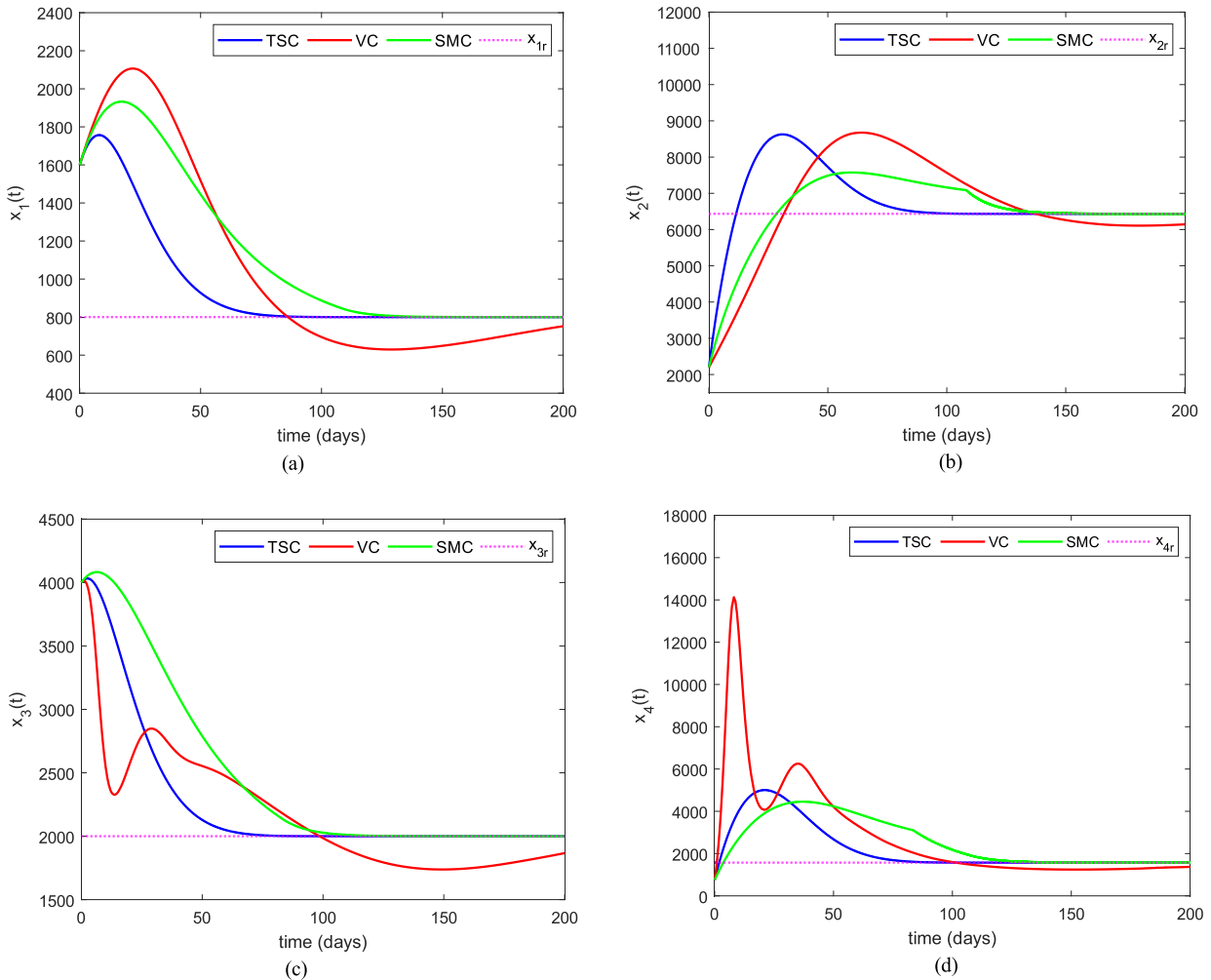
The simulation results of the saturated input case will be shown in section IV.

*Remark:* By viewing  $\Delta u_i$  for  $i = 2, 4$  as a disturbance, the auxiliary systems are intentionally designed such that the term  $w_{\sigma i}$  for  $i = 2, 4$  can dominate  $\Delta u_i$  for  $i = 2, 4$  respectively to guarantee that the states  $\sigma_i$  for  $i = 2, 4$  tend to zero and (22) will turn to be (12).

#### IV. SIMULATION

To demonstrate the capability and performance of the proposed control strategy, the proposed biological pest control strategy was applied to the simulation example of the pest-parasitoid ecosystem presented by Molnár et al. [8], [16], and Rafikov [68]. The parameters of the pest-parasitoid ecosystem in (8) are referred to in the literature. The numerical parameters of the ecosystem are presented in Table 1.

It is worth to note that  $\gamma_i \forall i \in \{1, 2\}$  are unitless. The simulation was conducted using MATLAB software. The function ode23s was used for numerical integration. The simulation time was specified from the initial time of 0 day to the final



**FIGURE 4.** Comparing the time responses of the control ecosystem states (a)–(d) under the control strategies based on the TSC, VC, and SMC methods, using the second initial condition (IC2).

time of 200 days discretization time step of 0.01 day. The controller parameters of the biological control strategy based on terminal synergetic controller were set as follows:  $T_1 = T_2 = T_3 = T_4 = 8$ ,  $p_c = 17$ , and  $q_c = 15$ .

Additionally, the controller parameters corresponding to the specified auxiliary system were selected as:

$$T_{c1} = T_{c2} = T_{c3} = T_{c4} = 0.001, p_\sigma = 17, \text{ and } q_\sigma = 15.$$

To evaluate the performance of the proposed biological control, two control approaches were used as benchmarks, including the SMC and VC methods. First, the biological control strategy based on SMC method [64], [65]. was determined from the backstepping SMC and presented as:

$$u_{2,SMC} = -f_2(x) - u_2^* - \beta_{s2}s_2 - \lambda_{s2}sign(s_2), \quad (32)$$

and

$$u_{4,SMC} = -f_4(x) - u_4^* + \dot{\varphi}_4 - \beta_{s4}s_4 - \lambda_{s4}sign(s_4), \quad (33)$$

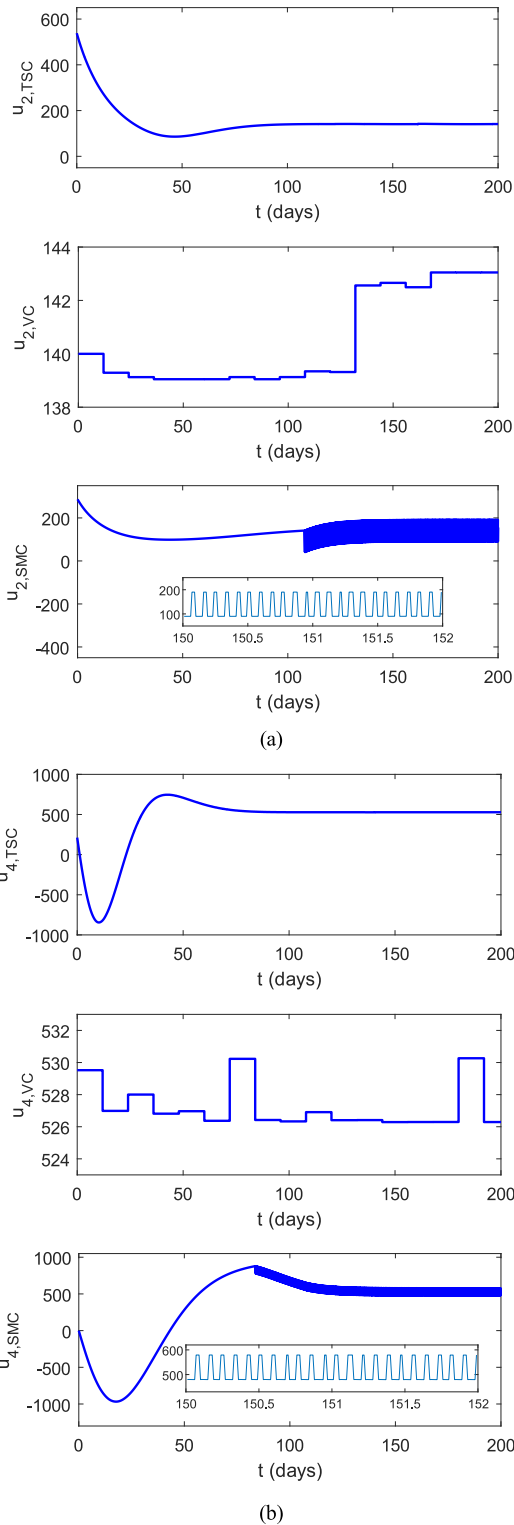
where the sliding surface is denoted by the errors associated with the control objective and virtual control as  $s_2 = x_2 - \varphi_{s2}$ ,  $s_4 = x_4 - \varphi_{s4}$ ,  $z_1 = x_1 - x_{1r}$ , and  $z_3 = x_3 - x_{3r}$ . The virtual controls were found

as  $\varphi_{s2} = (g_1(x))^{-1}(-\alpha_{s2}z_1 + \dot{x}_{1r} - f_1(x))$ , and  $\varphi_{s4} = (g_3(x))^{-1}(-\alpha_{s4}z_3 + \dot{x}_{3r} - f_3(x))$ . The controller parameters,  $\alpha_{s2}$ ,  $\alpha_{s4}$ ,  $\beta_{s2}$ ,  $\beta_{s4}$ ,  $\lambda_{s2}$ , and  $\lambda_{s4}$  are positive real numbers. The numerical values of the controller parameters were selected as follows:  $\alpha_{s2} = \alpha_{s4} = 0.095$ ,  $\beta_{s2} = \beta_{s4} = 0.01$ ,  $\lambda_{s2} = 50$ , and  $\lambda_{s4} = 50$ . These parameters were chosen so that the occurrence of the control system convergence close to that of the proposed biological pest control.

The second benchmark is the VC method, formulated based on (3.12) presented by Molnár et al. [8]. In this method, we allow the input function to be a piece-wise constant function with  $|u_i(t)| \leq 2$ ,  $\forall i \in \{2, 4\}$  i.e.  $u_i^* - 2 \leq U_i \leq u_i^* + 2$ ,  $\forall i \in \{2, 4\}$ .

The simulation of the control ecosystem was divided into two parts. First, simulating the control ecosystem under the proposed biological control strategy using (19) with varying initial conditions. Second, the simulation of the control system manipulated by the proposed biological control strategy composited with the specified auxiliary system in (31) under input saturations.





**FIGURE 5.** Control inputs of the control ecosystem using the second initial condition (IC2) under the control strategy based on the TSC, VC, and SMC methods: (a)  $U_2$  and (b)  $U_4$ .

**A. THE SIMULATION OF THE CONTROL ECOSYSTEM UNDER THE EFFECT OF DIFFERENT INITIAL CONDITIONS**

The first part of the simulation was used to show the capability of the proposed biological control strategy formulated

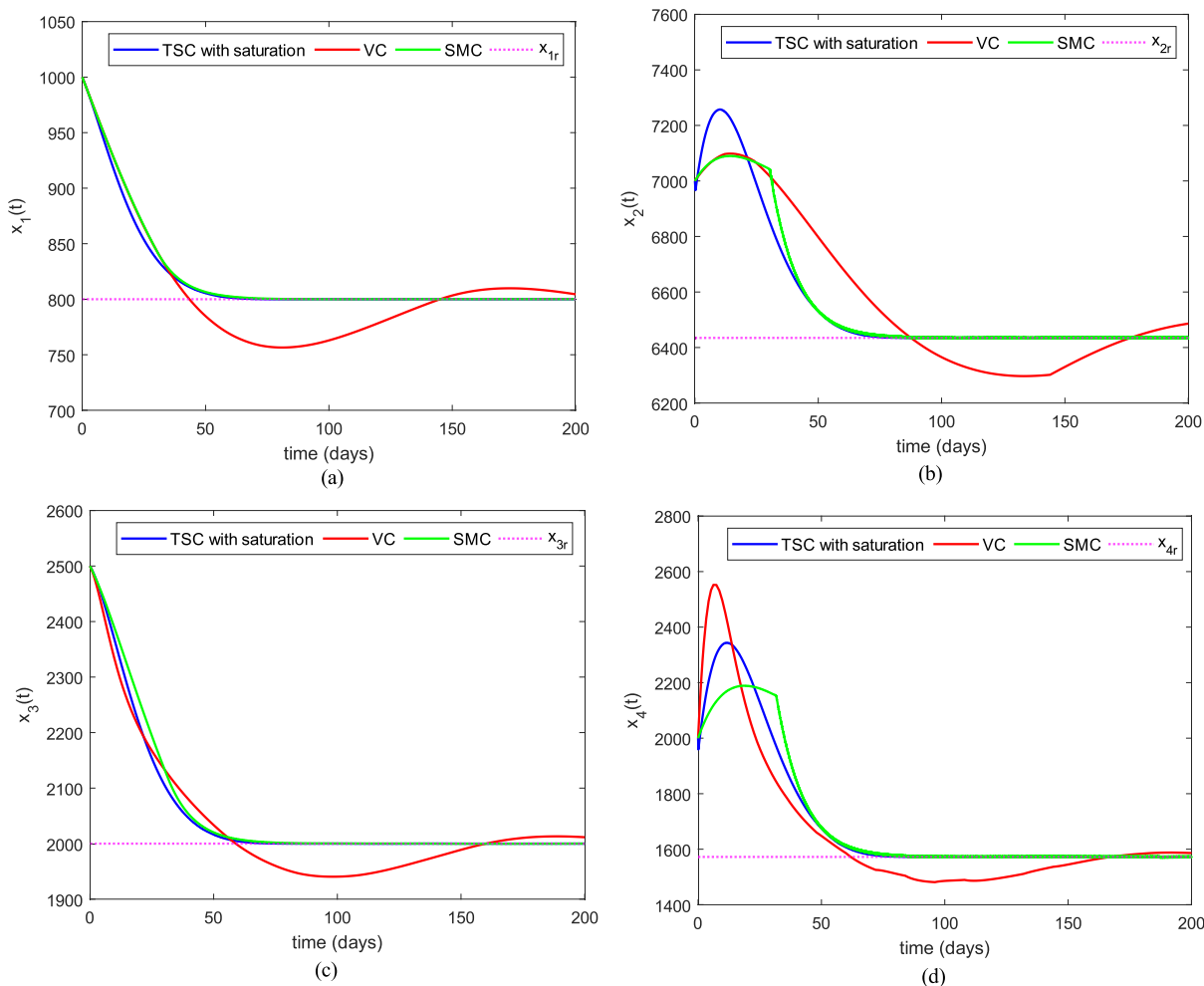
by the TSC method in terms of the finite-time convergence property through the simulation of the control ecosystem under the proposed biological control strategy in (19). Also, to evaluate the performance of the proposed biological control strategy, the simulation results of the pest-parasitoid ecosystem under the designed biological control strategy were presented and compared with those of the biological control strategy formulated by the SMC and VC methods. Two initial conditions were considered in this part of the simulation including  $x_{01} = [1000 \ 7000 \ 2500 \ 2000]^T$  (IC1) [8], [16] and  $x_{02} = [1600 \ 2200 \ 4000 \ 750]^T$  (IC2).

The first initial condition (IC1) represents the situation when the population density of each parasitoid is greater than the population density of each stage of the sugarcane borer. On the other hand, the second initial condition (IC2) represents the situation when the population density of each parasitoid is less than the population density of each stage of the sugarcane borer. According to the selected controller parameters and the considered initial conditions, the convergence time of the control ecosystem corresponding to each initial condition can be calculated based on (A9) as  $T_s = 131.28$  days and  $T_s = 156.94$  days, respectively.

For IC1, the time responses of the control ecosystem and control inputs of the proposed biological control (TSC) are presented in Fig. 2, while those of the biological control formulated by the SMC and VC methods are presented in Fig. 3. The graph in Fig. 3 illustrates the quantities of egg ( $U_2$ ) and larvae ( $U_4$ ) parasitoids per hectare for each specific day, comparing different control methods.

For IC2, the plots of the time responses are presented in Fig. 4 and Fig. 5, subsequently. As seen from Fig. 2 and Fig. 4, the proposed biological pest control strategy (TSC) can manipulate the state variables  $x_1$  and  $x_3$  associated with the egg and larvae sugarcane borer population densities from both initial conditions to the desired level of  $x_{1r}$  and  $x_{3r}$  respectively, and the state variables  $x_2$  and  $x_4$  associated with adult egg parasitoid and adult larvae parasitoid population densities converge to their corresponding equilibria within estimated finite time. Also, the control inputs of the proposed TSC method exhibit no signs of the chattering phenomenon. The time responses in Fig. 2 and Fig. 4 show that the biological pest control strategy formulated by the SMC and VC methods can manipulate all state variables from both initial conditions to the desired levels as well. However, using the SMC method, the chattering phenomenon occurs in each control input as shown in Fig. 3 and Fig. 5, respectively. These graphs demonstrate the rapid fluctuation of parasitoids' quantity, occurring approximately 22 times within a span of 2 days.

The annual quantities of egg ( $U_2$ ) and larvae ( $U_4$ ) parasitoids for one hectare can be derived by calculating the area under the curve of  $U_2$  and  $U_4$  graphs over 365 days. Table 2 presents comparisons of the annual quantities of parasitoids resulting from various control approaches. The annual quantities of parasitoids from the various approaches fall within a comparable range. Given the minimal variance in parasitoid quantities across diverse control approaches, it underscores



**FIGURE 6.** Time responses of the control ecosystem states (a) – (d) using the first initial condition (IC1) under input saturation of  $u_{i,max} = u_i^*$  for  $i = 2, 4$ .

**TABLE 2.** The annual quantities of egg ( $U_2$ ) and larvae ( $U_4$ ) parasitoids released per hectare without constraints.

Parasitoid	Controller	IC1	IC2
Egg ( $U_2$ )	TSC	51,044	53,422
	VC	51,396	48,096
	SMC	50,632	45,782
Larvae ( $U_4$ )	TSC	193,690	172,770
	VC	192,870	180,270
	SMC	193,210	150,270

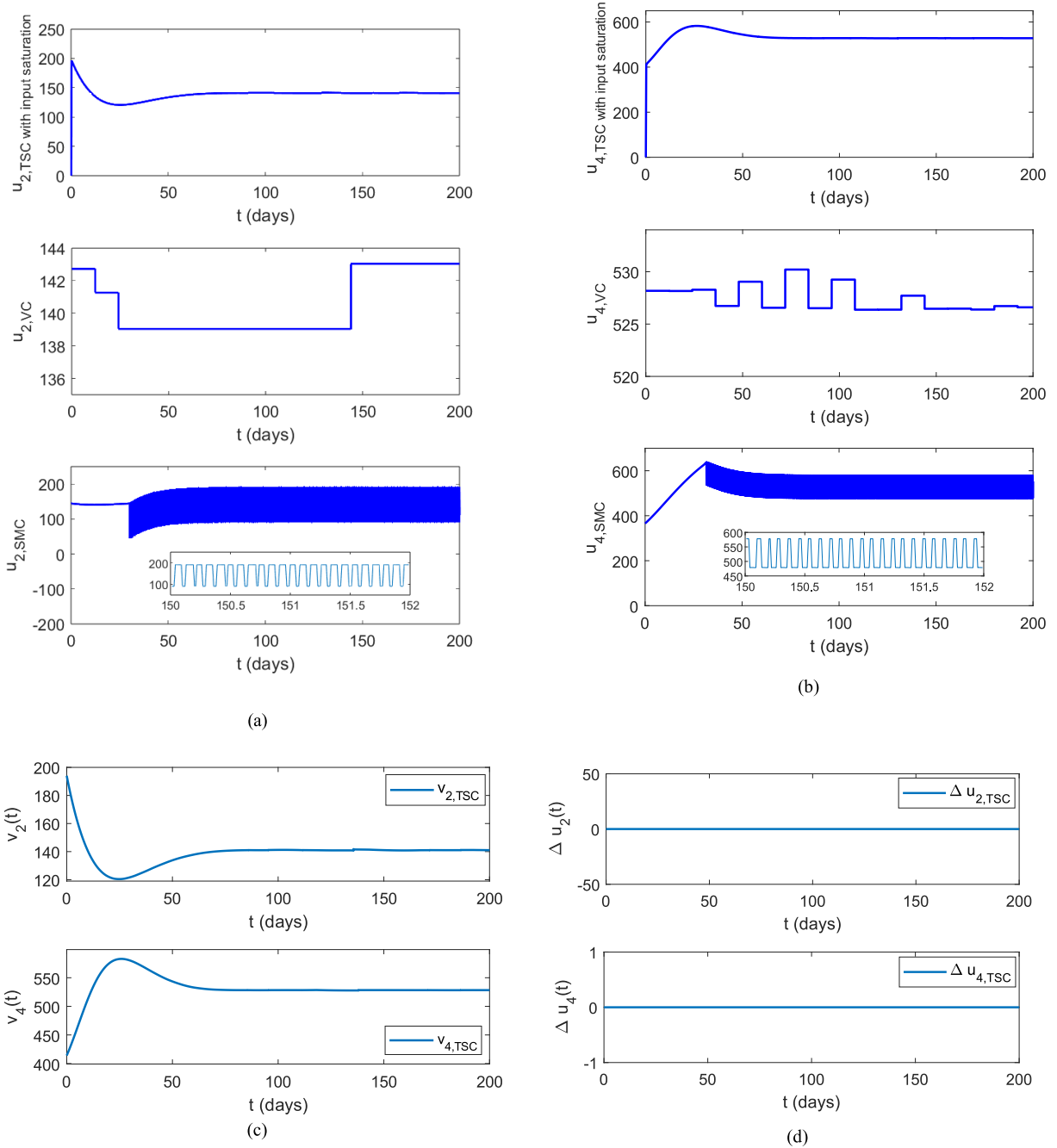
the crucial role of timing in releasing parasitoids for the effectiveness of biological pest control.

**B. THE SIMULATION OF THE CONTROL ECOSYSTEM UNDER THE EFFECT OF THE INPUT SATURATION**

In the second part of the simulation, the simulation examined the effectiveness of the biological control strategy under input saturation. This was done by manipulating the system using the proposed control strategy (TSC) along with the

auxiliary system as outlined in (20). According to [8], the input saturation occurs due to the requirement of the positive biological control strategy,  $U_i = u_i + u_i^* \geq 0$ , the input saturation is set as  $u_{i,max} = u_i^*$  for  $i = 2, 4$ . This is a preferable characteristic of the biological control strategy mentioned in Meza et al. [28]. The simulation results of the ecosystem under input saturation with the IC1 are shown in Fig. 6 and Fig. 7, and those with the IC2 are shown in Fig. 8 and Fig. 9. For the IC1, the plot of the state variables of the control ecosystem and the specified auxiliary systems are in Fig. 6a and Fig. 6b, respectively. The actual control inputs, the nominal control inputs, and their differences are shown in Fig. 7a, Fig. 7b, and Fig. 7c, subsequently. Likewise, the plot of those state variables corresponding to the IC2 are shown in Fig. 8a and Fig. 8b. Those actual and nominal control inputs corresponding to the IC2 are presented in Fig. 9a and Fig. 9b. Their differences are shown in Fig. 9.

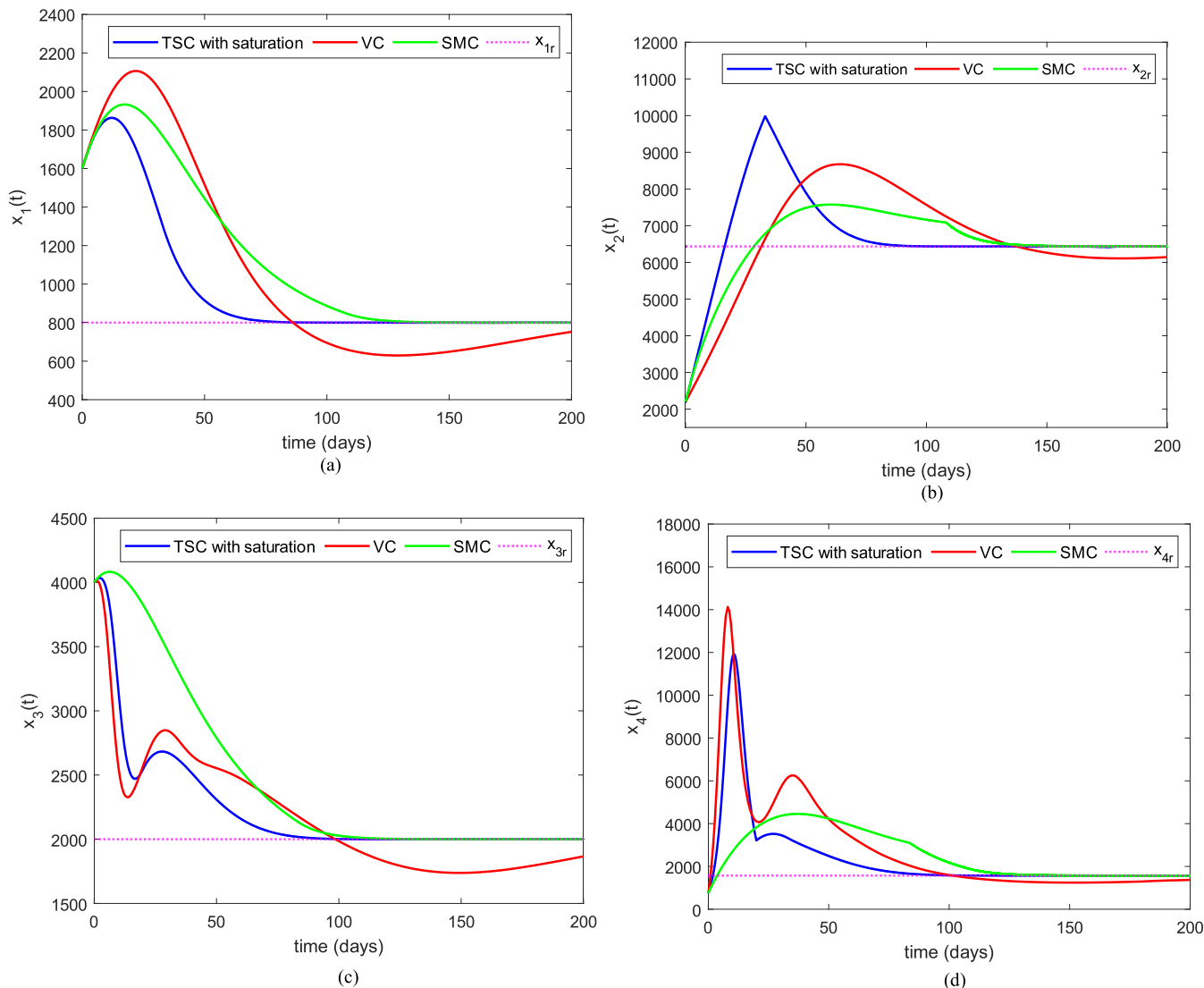
Figures 6a and 8a show that all state variables associated with the population densities of the egg and larvae sugarcane borer are manipulated from both initial conditions to the



**FIGURE 7.** Control inputs of the proposed TSC method with the specified auxiliary systems using the first initial condition (IC1) under the input saturation of  $u_{i,max} = u_i^*$  for  $i = 2, 4$ : (a) actual control inputs  $U_2$  and  $U_4$ , comparing between the TSC, VC, and SMC methods (c) nominal control inputs  $v_2$  and  $v_4$  the, and (d)  $\Delta u_2$  and  $\Delta u_4$ . both (c) and (d) are for the TSC approach.

desired levels, and those associated with adult egg and adult larvae parasitoids converge to their corresponding equilibria within calculated finite-times corresponding to each initial condition. The state variables of both specified auxiliary systems associated with both initial conditions converge to zero. The actual control inputs,  $U_2$  and  $U_4$ , of the IC1 are the same as those presented in the non-saturation case. Both actual and nominal control inputs are the same as shown in Fig. 3, Fig. 7a, and Fig. 7b. Consequently, the difference

between each actual and each nominal control input is zero as seen in Fig. 7c. However, for the IC2, the actual control inputs,  $u_2$  and  $u_4$ , are different from the control input of the non-saturation case, and different from the nominal control inputs,  $v_1$  and  $v_2$  as shown in Fig. 5, Fig. 9a, and Fig. 9b. The difference between each actual and each nominal control input is nonzero as presented in Fig. 9c. Comparing the results of both initial conditions explains that manipulating all state variables to the desired levels from the IC2 requires



**FIGURE 8.** Time responses of the control ecosystem using the second initial condition (IC2) under input saturation of  $u_{i,max} = u_i^*$  for  $i = 2, 4$ .

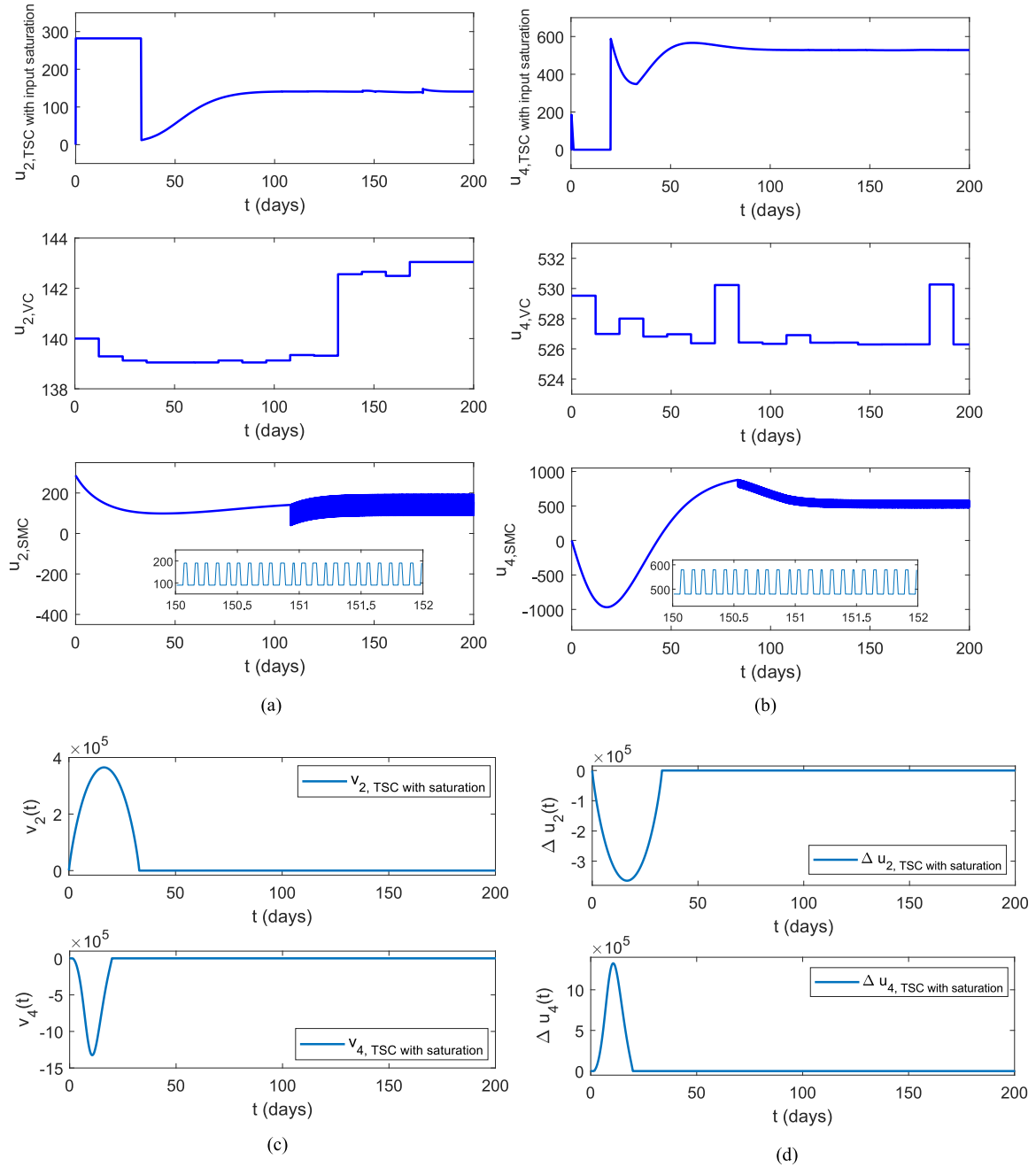
more control effort than manipulating them from the IC1 to the desired levels. Consequently, the nominal control input exceeds the upper or lower bound of the input saturation as seen in Fig. 9a and Fig. 9b. Nonetheless, for both initial conditions, the actual and nominal control inputs are free from the chattering phenomenon. Table 3 shows the annual quantities of parasitoids per hectare derived from TSC control approach under input saturation constraints.

The first part of the simulation confirms the capability of the proposed biological pest control, demonstrating its ability to achieve finite-time convergence and maintain a control input signal free from chattering. The first characteristic is beneficial for the designer to estimate or calculate the period for applying the biological control strategy through the calculation of the settling time. The second property is another superior property compared to the control strategy formulated by the conventional SMC method. This allows the control

**TABLE 3.** The annual quantities of egg ( $U_2$ ) and larvae ( $U_4$ ) parasitoids released per hectare under input saturation constraints.

Parasitoid	Controller	IC1	IC2
Egg ( $U_2$ )	TSC	51,032	53,010
Larvae ( $U_4$ )	TSC	193,580	180,680

strategy to be smooth and have no chattering phenomenon. The second part of the simulation shows the following facts. First, for the case when the nominal control inputs do not exceed the bounds of the corresponding saturation, the locally finite time stability of the control ecosystem can be guaranteed as the simulation associated with the IC1. Second, the simulation results associated with the IC2 confirm the fact mentioned by Chen et al. [64] that if the difference



**FIGURE 9.** Control inputs of the proposed TSC method with the specified auxiliary systems for the ecosystem using the second condition (IC2), under the input saturation of  $u_{i,max} = u_i^*$  for  $i = 2, 4$ : (a) actual control inputs  $U_2$  and  $U_4$ , comparing between the TSC, VC, and SMC methods (c) nominal control inputs  $v_2$  and  $v_4$  the, and (d)  $\Delta u_2$  and  $\Delta u_4$ . both (c) and (d) are for the TSC approach.

between the nominal control and the actual control inputs is sufficiently small, the preferable characteristic of the control system including the chattering-free control input of the control ecosystem can still be achieved as in the case of non-input saturation.

**V. CONCLUSION**

The results of this study can be concluded as follows. The biological strategy for the control pest-parasitoid ecosystem

based on the finite-time or TSC design procedure was proposed. In the design procedure, the dynamic evolution of the macro variables was selected so that macro variables associated with the control objective together with the difference between required internal controls and the corresponding state variables satisfy finite-time convergence characteristics without singularity. The proof of control system stability under the proposed strategy was conducted based on the Lyapunov stability theorem. Simulation results confirmed

that the egg and larvae population densities of the sugarcane borer were regulated at the desired levels within the calculated finite time. The control input signals were free from chattering phenomena. Moreover, the finite-time biological control for the ecosystem with input saturation was formulated based on the finite-time synergetic controller together with the specified auxiliary system. The stability proof for the control ecosystem was likewise conducted in a similar manner. This effect was confirmed and described through the simulation as well. It is feasible to achieve the finite-time convergence and chattering-free characteristic of the control inputs of the control ecosystem system when the difference between the nominal and the actual control inputs is sufficiently small. Hence, applying the TSC design to formulate the biological control strategy is advantageous, as it provides preferable characteristics which include finite-time stability and chattering-free control input signal. These attributes make the proposed biological pest control strategy well-suited for practical implementation. In future research, exploring state observer-based control methods aimed at reducing the cost of the control strategy, as well as investigating time delays in both the state and input, may impact the performance of the control strategy and could lead to instability issues.

**APPENDIX**

**A. PROOF OF THEOREM 1**

The Lyapunov function is selected as

$$V = \frac{1}{2}\psi_1^2 + \frac{1}{2}\psi_2^2 + \frac{1}{2}\psi_3^2 + \frac{1}{2}\psi_4^2. \tag{A1}$$

Recall that  $x_i = \psi_i + \varphi_i$  for  $i = 2, 4$ , the derivative of the Lyapunov function is determined as

$$\begin{aligned} \dot{V} &= \psi_1 \dot{\psi}_1 + \psi_2 \dot{\psi}_2 + \psi_3 \dot{\psi}_3 + \psi_4 \dot{\psi}_4, \\ &= \psi_1 (\dot{x}_1 - \dot{x}_{1r}) + \psi_2 (\dot{x}_2 - \dot{\varphi}_2) \\ &\quad + \psi_3 (\dot{x}_3 - \dot{x}_{3r}) + \psi_4 (\dot{x}_4 - \dot{\varphi}_4), \\ &= \psi_1 [f_1(x) + g_1(x)\psi_2 + g_1(x)\varphi_2 - \dot{x}_{1r}] \\ &\quad + \psi_2 [f_2(x) + u_2^* + u_2 - \dot{\varphi}_2] \\ &\quad + \psi_3 [f_3(x) + g_2(x)\psi_4 + g_2(x)\varphi_4 - \dot{x}_{3r}] \\ &\quad + \psi_4 [f_4(x) + u_4^* + u_4 - \dot{\varphi}_4]. \end{aligned} \tag{A2}$$

Substituting the internal and external controls from (17) and (19) into (A2) yields

$$\begin{aligned} \dot{V} &= \psi_1 W_1 + \psi_2 W_2 + \psi_3 W_3 + \psi_4 W_4 \\ &\quad + g_1(x)\psi_1 \psi_2 - g_1(x)\psi_1 \psi_2 \\ &\quad + g_3(x)\psi_3 \psi_4 - g_3(x)\psi_3 \psi_4. \end{aligned} \tag{A3}$$

Then,

$$\dot{V} = \psi_1 W_1 + \psi_2 W_2 + \psi_3 W_3 + \psi_4 W_4, \tag{A4}$$

where  $W_i = \left(-\frac{1}{T_i}\psi_i\right)^{q_c/p_c}$  for  $i = 1, \dots, 4$ . It can be obtained as follows.

$$\dot{V} = -\left(\frac{1}{T_1}\psi_1^{q_c/p_c+1}\right) - \left(\frac{1}{T_2}\psi_2^{q_c/p_c+1}\right) - \left(\frac{1}{T_3}\psi_3^{q_c/p_c+1}\right) - \left(\frac{1}{T_4}\psi_4^{q_c/p_c+1}\right). \tag{A5}$$

Let define  $\lambda_{\min} \triangleq \min_{1 \leq i \leq 4} \left\{\frac{1}{T_i}\right\}$  and  $\eta = \frac{p_c+q_c}{2p_c}$ , notice that  $0 < \eta < 1$  by construction then equation, (A5) can be expressed as

$$\begin{aligned} \dot{V} &\leq -\lambda_{\min} \left[\psi_1^{2\eta} + \psi_2^{2\eta} + \psi_3^{2\eta} + \psi_4^{2\eta}\right] \\ &= -2^\eta \lambda_{\min} \left[\left(\frac{\psi_1^2}{2}\right)^\eta + \left(\frac{\psi_2^2}{2}\right)^\eta + \left(\frac{\psi_3^2}{2}\right)^\eta + \left(\frac{\psi_4^2}{2}\right)^\eta\right]. \end{aligned} \tag{A6}$$

Based on Lemma 2, inequality (A6) can be determined as

$$\dot{V} \leq -2^\eta \lambda_{\min} \left[\frac{\psi_1^2}{2} + \frac{\psi_2^2}{2} + \frac{\psi_3^2}{2} + \frac{\psi_4^2}{2}\right]^\eta. \tag{A7}$$

According to (A1), inequality (A7) can be manipulated as:

$$\dot{V} \leq -\alpha V^\eta, \tag{A8}$$

where  $\alpha \triangleq 2^\eta \lambda_{\min}$  and note that  $\alpha, \eta > 0$  by construction. Based on Lemma 1, the settling time  $T_s$  is determined as

$$T_s = t_0 + \frac{V^{1-\eta}(t_0)}{\alpha(1-\eta)}. \tag{A9}$$

Then, the proof of locally finite-time stability is completed  $\square$

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