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RESEARCH ARTICLE

Power Allocation Between a Distributed Multistatic Radar Network and a Smart Jammer Based on Non-Cooperative Game Theory

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ABSTRACT For the problem of power allocation between a distributed multistatic radar network and a smart jammer, the application of non-cooperative game theory is employed to address the issue in this paper. Consequently, three scenarios of power allocation games are examined. The first two game scenarios, characterized by information asymmetry, are categorized under the Stackelberg game framework, while the final scenario, with information symmetry, is classified as a non-cooperative game. Through the power allocation analyses of the three game scenarios, it is observed that both the radar system and the jammer possess a first-mover advantage. Additionally, the existence and uniqueness of the Nash equilibrium in the games are demonstrated. Based on the best response strategies within the games, three corresponding power allocation game algorithms are proposed. Ultimately, the convergence and performance comparison of the three power allocation game algorithms are validated through simulation experiments.

INDEX TERMS Radar, jammer, power allocation, game theory, Nash equilibrium.

I. INTRODUCTION

With the rapid development of modern radar technology, a distributed multistatic radar system is regarded as a unified radar system that is partitioned into transmitters and receivers at various bases, and has been embraced by many countries [1]. The effectiveness of a multistatic radar system in detecting, locating, and tracking targets is well recognized. However, interferences, such as cross-channel interference, barrage jamming, false target jamming, and noise jamming, are encountered, impacting the radar system's performance significantly [2]. Consequently, an optimal power allocation strategy that maximizes the radar signal-to-interference-plus-noise ratio (SINR) and mitigates interferences is deemed necessary [3]. In this paper, the focus is placed on investigating power allocation strategies between a multistatic radar network and a smart jammer. Game theory is considered one of the most appropriate and effective mathematical models and methods for analyzing the power allocation problems in countermeasures.

Game theory, as a branch of mathematics, finds wide application in finance, economics, biology, computer science, political science, military strategy, and various other disciplines [4]. Recently, significant utilization of game theory has been witnessed in wireless communication systems. The focus lies on employing game theory techniques to analyze and study the cooperation and non-cooperation among players in spectrum allocation, power allocation, and beamforming within wireless communication contexts [5], [6], [7], [8], [9], [10], [11]. In [6], a Stackelberg Bayesian game framework model is employed to investigate the power allocation problem of a two-tier downlink network. Furthermore, a multi-player game model of a multi-unit multi-user network is analyzed and researched, leading to the proposal of a corresponding game theory beamforming algorithm [8], [9], [10]. Game theory analysis, as depicted

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in [11], is conducted on the power allocation strategy between a wireless network and a smart jammer, with a related Stackelberg game method put forth. A cooperative and confrontational game relationship exists between radar and jammer, prompting the introduction of game theory analysis by numerous scholars into radar research [12], [13], [14], [15], [16], [17], [18], [19], [20]. In [12], an adaptive radar jamming game is explored, with the Bayesian risk function considered as the utility function of the game. Various forms of game utility functions, incorporating penalty items, are introduced, encompassing both supermodular and submodular games, each possessing a pure strategy Nash equilibrium. Authors in [13] delve into a game polarization design strategy under distributed multiple-input multiple-output (MIMO) radar, showcasing enhanced target detection ability compared to single vertical or horizontal polarizations. The utilization of the Stackelberg game and the zero-sum game is observed in [14] to analyze the countermeasure game between MIMO radar and the jammer. A non-cooperative game method is applied in radar network coding design to maximize the SINR of each active radar, with various coding strategies proposed [15]. In the realm of electronic warfare (EW) time allocation, authors in [16] employ a game theory model to analyze the optimal operational time of radars during peace and war. Non-cooperative and partially cooperative game-theoretic approaches, combined with convex optimization methods, address problems related to distributed beamforming and resource allocation in the presence of multiple targets [17]. A novel two-step water-filling method is utilized to investigate the Stackelberg game between MIMO radar and the target amid clutter [18]. Game theory techniques are harnessed in [19] and [20] to optimize radar transmit waveforms, thereby enhancing radar system performance. For power allocation issues, cooperative and non-cooperative game models are implemented to analyze power allocation between radar and jammer [21], [22], [23], [24], [25], [26], [27], [28], [29], [30], [31], [32]. In [21], an optimal power allocation algorithm is proposed based on cooperative game theory, showcasing superior performance compared to a single uniform or random power allocation algorithm. Subsequently, in alignment with [21], an alternative optimal power allocation strategy is introduced by authors in [22] utilizing a cooperative game approach, demonstrating enhanced power allocation performance. Strategic game-theoretic power allocation is investigated in [24], with a Nash equilibrium analysis conducted for a MIMO radar network. An iterative Nash bargaining algorithm is explored for the power allocation problem in low probability radar networks for interception, employing a cooperative game theoretical strategy [26]. Authors in [29] delve into a problem of non-cooperative game-theoretic power allocation for distributed multiple-radar architecture in a spectrum sharing environment, where multiple radars coexist with a communication system in the same frequency band. In practical terms, modern radar systems necessitate the allocation and scheduling of multiple resources for search and tracking tasks [33], [34], [35], [36]. Authors in [33] investigate array radar resource management, providing an overview of automated techniques for managing the operation and resources of an electronically steered array. Additionally, the evolution from adaptive to cognitive radar resource management is reviewed by authors in [36], emphasizing the significance of cognitive radar resource management.

In this paper, a non-cooperative game-theoretic analysis of power allocation between a distributed multistatic radar network and a smart jammer is undertaken. Three types of game scenarios, in which power allocation is carried out by the radar system and the jammer, are examined. All power allocation games lead to the Nash equilibrium. In cases of information asymmetry, two scenarios arise. One party identifies the other party first to enhance its own power allocation strategy formulation. Conversely, the latter adopts a passive approach by following the power allocation strategy of the former to devise its strategy, resulting in the former transmitting less power and gaining more benefits. Both the radar system and the jammer can assume roles as leaders or followers, aligning with the Stackelberg game framework. In scenarios with symmetric information, both parties dynamically adjust their strategies based on their best response strategy and the opposing power allocation strategy. The power allocation strategies of both parties are directly linked to their respective gains.

The subsequent sections of this paper are structured as follows: Section II presents the system model. Section III introduces game theory and establishes the existence and uniqueness of the Nash equilibrium. Section IV outlines the proposed algorithms. Section V details simulation experiments conducted to validate the convergence of the proposed algorithm. Finally, Section VI concludes the paper.

II. SYSTEM MODEL

As depicted in Figure 1, a multistatic radar network system model is initially considered, comprising K widely separated

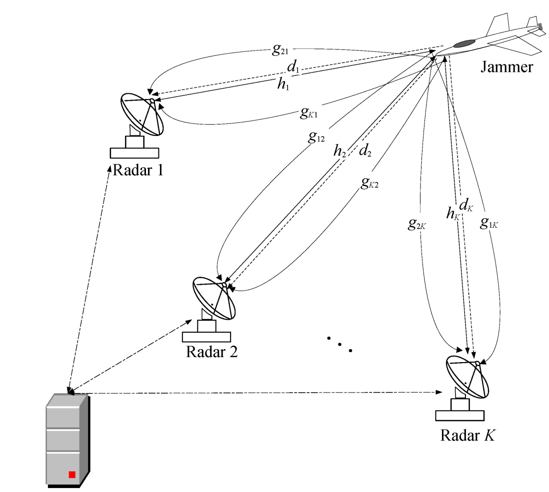

FIGURE 1. A multistatic radar network with a control center and a smart jammer.

radars and a control center. Within the context of the distributed radar network's non-cooperative setting, each radar is focused on maximizing its own SINR, leveraging its private information, which encompasses a comprehensive understanding of its individual channel gain realization. Conversely, it is assumed that each radar possesses solely the distribution of the cross-radar channel gains as shared knowledge. The scenario is set in a far-field environment where a smart flying jammer interacts with a multistatic radar network [3], exerting a significant impact on the radar network system's performance.

In the presence of a smart jammer, the received signal for the k^{th} radar is obtained by

$$y_k = \sqrt{h_k p_{Rk}}\mathbf{s}_k + \sum_{\substack{i=1\\ i\neq k}}^{K} \sqrt{g_{ik} p_{Ri}}\mathbf{s}_i + \sqrt{d_k p_{Jk}} + n_k$$
$$k = 1, 2, \cdots, K \tag{1}$$

where N defines the number of signal return samples that the radars receive at each time step, $s_k = \psi_k a_k$ is the transmitted signal from the k^{th} radar, $a_k = \left[1, e^{j2\pi f_k}, \cdots, e^{j2\pi(N-1)f_k}\right]^{\mathrm{T}}$ is the steering vector of the k^{th} radar for the desired target, ψ_k denotes the predesigned transmitted waveform from the k^{th} radar and meets orthogonality, f_k describes the normalized Doppler shift of the k^{th} radar, h_k is the channel gain at the direction of the desired target, g_{ik} denotes the cross-channel gain between the k^{th} radar and the i^{th} radar, p_{Rk} describes the transmit power of the k^{th} radar, d_k denotes the interference channel gain, p_{Jk} describes the jammer transmit power, $n_k \sim \mathcal{CN}\left(0, \sigma_n^2\right)$ represents the noise received by the k^{th} radar. Therefore, the SINR of the k^{th} radar is defined by the following formulation as

$$\mathrm{SINR}_k = \frac{K h_k p_{Rk}}{\sum\limits_{\substack{i=1\\ i\neq k}}^{K} g_{ik} p_{Ri} + d_k p_{Jk} + \sigma_n^2} \tag{2}$$

In this section, we have constructed the system model and the SINR formulation. In the next section, game theoretic formulation and problems are introduced.

III. GAME THEORY

In this section, a non-cooperative game theory framework for power allocation is established. Firstly, the utility functions of the radar and jammer are introduced, and their best response functions are calculated. Subsequently, the existence of the Nash equilibrium in the non-cooperative game model of power allocation is proven using the Nash equilibrium existence theorem [3], [4]. The uniqueness of the Nash equilibrium of the power allocation game is demonstrated by applying the definition of the standard function. Finally, the non-cooperative power allocation is characterized as a dynamic game process in which power iteration formulations of the radar system and the smart jammer are introduced, respectively.

A. NON-COOPERATIVE POWER ALLOCATION GAME

Due to the countermeasure relationship between the radar system and the jammer, the interaction can be redefined as a non-cooperative game relationship. Given the assumption that all radars belong to the same organization, the game scenario lacks competitiveness, and intentional interference among the radars is absent. The radar system engages solely with the jammer, making the players in the game the radar system and the jammer, with their respective transmit power sets denoted as p_R and p_J. The primary objective of the radar unilateral game is to maximize the target SINR under specified total power constraints. The non-cooperative power allocation game (NPAG) model is outlined as follows:

$$\mathcal{G}_{NPAG} = \{\mathcal{P}, \mathcal{S}, \mathcal{U}\} \tag{3}$$

a) **Player set**: $\mathcal{P} = \{Radars, Jammers\}$;

b) **Strategy set**: $\mathcal{S} = \mathcal{S}_R \times \mathcal{S}_J$, $\mathcal{S}_R = \{\mathbf{p}_R\}$, $\mathcal{S}_J = \{\mathbf{p}_J\}$, $\mathbf{p}_R = [p_{R1}, p_{R2}, \cdots, p_{RK}]^{\mathrm{T}}$ and $\mathbf{p}_J = [p_{J1}, p_{J2}, \cdots, p_{JK}]^{\mathrm{T}}$;

c) **Utility function set**: $\mathcal{U} = \{u_R, u_J\}$.

According to the maximization of the radar utility function u_{Rk}, the best response strategy constructs the following optimization model

$$\begin{aligned} \max_{\mathbf{p}_R} \min_{\mathbf{p}_J} \;& u_R\left(\mathbf{p}_R, \mathbf{p}_J\right) \\ \text{s.t.} \quad & \mathbf{1}^{\mathrm{T}}\mathbf{p}_R \leq p_R^{Tot} \\ & 0 \leq p_{Rk} \leq p_{Rk}^{Max} \\ & \mathbf{1}^{\mathrm{T}}\mathbf{p}_J \leq p_J^{Tot} \\ & 0 \leq p_{Jk} \leq p_{Jk}^{Max} \end{aligned} \tag{4}$$

where $\mathbf{1}^{\mathrm{T}}$ represents a vector of all 1, p_R^{Tot} is the total radar transmit power, p_{Rk}^{Max} describes the maximum transmit power of the k^{th} radar, p_J^{Tot} is the total jammer power, p_{Jk}^{Max} describes the maximum power of the jammer corresponding to the k^{th} radar.

In addition, the target SINR of the k^{th} radar does not include the energy reflected by other radars through target jammer onto the k^{th} radar, which can be also written as

$$\gamma_k = \frac{h_k p_{Rk}}{\sum\limits_{\substack{i=1\\ i\neq k}}^{K} g_{ik} p_{Ri} + d_k p_{Jk} + \sigma_n^2} \tag{5}$$

In this case, the utility function of the k^{th} radar can be defined as the following form

$$u_{Rk}\left(\mathbf{p}_R, \mathbf{p}_J\right) = \frac{\gamma_k}{\gamma_k + \eta} - \tau p_{Rk} \tag{6}$$

where $\eta(\eta \geq 1)$ is a constant for all radar channel and is defined as adjustable parameter. τ is a pricing factor, τp_{Rk} describes an independent linear pricing function.

Thus, we can get the total utility function of the radar system as follows

$$u_R\left(p_R, p_J\right) = \sum_{k=1}^{K} \frac{\gamma_k}{\gamma_k + \eta} - \tau \sum_{k=1}^{K} p_{Rk} \tag{7}$$

On the other hand, the jammer is smart and can adjust its transmit power towards the radar. The utility function of the jammer is as follows [11]

$$u_{Jk}\left(\mathbf{p}_R,\mathbf{p}_J\right)=-\frac{h_k p_{Rk}}{d_k p_{Jk}+\sigma_n^2}-\mu p_{Jk} \tag{8}$$

where μ is the jammer cost of per unit power.

Meanwhile, we can also get the total utility function of the jammer as follows

$$u_J\left(p_R,p_J\right)=-\sum_{k=1}^{K}\frac{h_k p_{Rk}}{d_k p_{Jk}+\sigma_n^2}-\mu\sum_{k=1}^{K}p_{Jk} \tag{9}$$

The solution to a non-cooperative game is termed the Nash equilibrium. In the non-cooperative power allocation game, the power allocation Nash equilibrium represents a stable point where no player can profit from unilaterally altering its own power allocation strategy. The Nash equilibrium is characterized by the strategy sets $\mathbf{p}_R^*$ and $\mathbf{p}_J^*$, where

$$u_{Rk}\left(p_R^*,p_J^*\right)\geq u_{Rk}\left(p_R,p_J\right) \tag{10}$$

$$u_{Jk}\left(p_R^*,p_J^*\right)\geq u_{Jk}\left(p_R,p_J\right) \tag{11}$$

B. EXISTENCE AND UNIQUENESS OF THE NASH EQUILIBRIUM

Both the radar system and the jammer want to maximize their utility with minimal power consumption. According to equation (6) and (8), the best power allocation strategy of the k^{th} radar and the jammer are as follows

$$p_{Rk}^*=\arg\ max\left\{u_{Rk}\left(p_R,p_J\right)\right\} \tag{12}$$

$$p_{Jk}^*=\arg\ max\left\{u_{Jk}\left(p_R,p_J\right)\right\} \tag{13}$$

Using equation (6) to solve the best response function of the radar power allocation. In equation (6), the first-order partial derivative of u_{Rk} with respect to p_{Rk} and makes it zero, then there is

$$\frac{\partial u_{Rk}}{\partial p_{Rk}}=\frac{\partial u_{Rk}}{\partial \gamma_k}\frac{\partial \gamma_k}{\partial p_{Rk}}-\tau=\frac{\phi_k\eta}{\left(\gamma_k+\eta\right)^2}-\tau=0 \tag{14}$$

where

$$\phi_k=\frac{\gamma_k}{p_{Rk}}=\frac{h_k}{\sum_{\substack{i=1\\i\neq k}}^{K}g_{ik}p_{Ri}+d_k p_{Jk}+\sigma_n^2} \tag{15}$$

The best power response function of the k^{th} radar is obtained by equation (14) as follows

$$BR_{Rk}\left(p_J\right)=p_{Rk}^*=\left[\sqrt{\frac{\eta}{\tau\phi_k}}-\frac{\eta}{\phi_k}\right]^+ \tag{16}$$

where $[x]^+=max\{x,0\}$.

Therefore, the total power allocation of the radar system is

$$BR_R\left(p_J\right)=\sum_{k=1}^{K}\left[\sqrt{\frac{\eta}{\tau\phi_k}}-\frac{\eta}{\phi_k}\right]^+ \tag{17}$$

Further the second-order partial derivative of u_{Rk} with respect to p_{Rk} is

$$\frac{\partial^2 u_{Rk}}{\partial p_{Rk}^2}=\frac{\partial\left(\frac{\phi_k\eta}{(\gamma_k+\eta)^2}-\tau\right)}{\partial p_{Rk}}=-\frac{2\phi_k^2\eta}{\left(\gamma_k+\eta\right)^3}<0 \tag{18}$$

According to the Nash equilibrium theorem, the set of radar power allocation is both non-empty and compact in the Euclidean space. As per equation (18), the radar utility function u_{Rk} is a continuously and strictly concave function of p_{Rk}. Hence, the existence of the corresponding power allocation solution is ensured.

In addition, we analyze the existence of the Nash equilibrium of the jammer, and the first-order partial derivative of the jammer utility function u_{Jk} with respect to p_{Jk} and makes it zero as

$$\frac{\partial u_{Jk}}{\partial p_{Jk}}=\frac{h_k d_k p_{Rk}}{\left(d_k p_{Jk}+\sigma_n^2\right)^2}-\mu=0 \tag{19}$$

The best response function of the jammer is calculated by equation (19), then we obtain

$$BR_{Jk}\left(p_R\right)=p_{Jk}^*=\left[\sqrt{\frac{h_k p_{Rk}}{\mu d_k}}-\frac{\sigma_n^2}{d_k}\right]^+ \tag{20}$$

Therefore, the total power allocation of the jammer is

$$BR_J\left(p_R\right)=\sum_{k=1}^{K}\left[\sqrt{\frac{h_k p_{Rk}}{\mu d_k}}-\frac{\sigma_n^2}{d_k}\right]^+ \tag{21}$$

Further the second-order partial derivative of u_{Jk} with respect to p_{Jk} is

$$\frac{\partial^2 u_{Jk}}{\partial p_{Jk}^2}=\frac{\partial\left(\frac{h_k d_k p_{Rk}}{(d_k p_{Jk}+\sigma_n^2)^2}-\mu\right)}{\partial p_{Jk}}=-\frac{2h_k d_k^2 p_{Rk}}{\left(d_k p_{Jk}+\sigma_n^2\right)^3}<0 \tag{22}$$

The jammer utility function u_{Jk} is a continuous function of p_{Jk}, and it is known from equation (22) that u_{Jk} is a strictly concave function of p_{Jk}. Therefore, the corresponding jammer power allocation solution must exist.

On the other hand, if the radar best response function of the game satisfies the necessary properties of the following standard function, the Nash equilibrium of the game will be unique. The three properties of the standard function [3], [10]:

a) Positivity: The function is strictly positive, $F(x)>0$.

b) Monotonicity: If $x\geq x'$, then $F\left(x'\right)\geq F(x)$.

c) Scalability: For all $a>1$, it has $aF(x)\geq F(ax)$.

The radar best response function satisfies the three properties of the standard function:

a) Positivity: According to the power allocation, the every transmit power is positive, so $BR_R\left(\mathbf{p}_J\right)>0$.

b) Monotonicity: According to $BR_R\left(p_{Jk}\right)>0$, if $p_{Jk}\leq p_{Jk}'$, then $\phi_k\geq\phi_k'$. Also, we have

$u_{Rk}(p_R, p_J) = \frac{\gamma_k}{\gamma_k+\eta} - \tau p_{Rk} \geq 0 \Rightarrow \frac{\phi_k}{p_{Rk}\phi_k+\eta} \geq \tau \Rightarrow \frac{1}{\tau} \geq \frac{p_{Rk}\phi_k+\eta}{\phi_k} = p_{Rk} + \frac{\eta}{\phi_k} \geq \frac{\eta}{\phi_k}$.

$$\begin{aligned} &BR_{Rk}(\mathbf{p}_J) - BR_{Rk}(\mathbf{p}'_J) \\ &= \left(\sqrt{\frac{\eta}{\tau\phi_k}} - \frac{\eta}{\phi_k}\right) - \left(\sqrt{\frac{\eta}{\tau\phi'_k}} - \frac{\eta}{\phi'_k}\right) \\ &= \sqrt{\frac{1}{\tau}}\left(\sqrt{\frac{\eta}{\phi_k}} - \sqrt{\frac{\eta}{\phi'_k}}\right) - \left(\frac{\eta}{\phi_k} - \frac{\eta}{\phi'_k}\right) \\ &\geq \sqrt{\frac{\eta}{\phi_k}}\left(\sqrt{\frac{\eta}{\phi_k}} - \sqrt{\frac{\eta}{\phi'_k}}\right) - \left(\frac{\eta}{\phi_k} - \frac{\eta}{\phi'_k}\right) \\ &= \frac{\eta}{\sqrt{\phi'_k}}\left(\frac{1}{\sqrt{\phi'_k}} - \frac{1}{\sqrt{\phi_k}}\right) \geq 0 \end{aligned} \tag{23}$$

c) Scalability: For $\forall a > 1$, we make $\phi_a = \frac{h_k}{\sum_{i=1, i\neq k}^{K} g_{ik}p_{Ri} + ad_k p_{Jk} + \sigma_n^2}$, then we have $\phi_a < \phi_k$. Further we have

$$\begin{aligned} &aBR_{Rk}(\mathbf{p}_J) - BR_{Rk}(a\mathbf{p}_J) \\ &= a\left(\sqrt{\frac{\eta}{\tau\phi_k}} - \frac{\eta}{\phi_k}\right) - \left(\sqrt{\frac{\eta}{\tau\phi_a}} - \frac{\eta}{\phi_a}\right) \\ &= \sqrt{\frac{1}{\tau}}\left(a\sqrt{\frac{\eta}{\phi_k}} - \sqrt{\frac{\eta}{\phi_a}}\right) - \left(\frac{a\eta}{\phi_k} - \frac{\eta}{\phi_a}\right) \\ &\geq \sqrt{\frac{\eta}{\phi_k}}\left(a\sqrt{\frac{\eta}{\phi_k}} - \sqrt{\frac{\eta}{\phi_a}}\right) - \left(\frac{a\eta}{\phi_k} - \frac{\eta}{\phi_a}\right) \\ &= \frac{\eta\left(\phi_k - \sqrt{\phi_k\phi_a}\right)}{\phi_k\phi_a} > 0 \end{aligned} \tag{24}$$

Therefore, the power best response function of the radar system satisfies the three properties of the standard function. As a result, the Nash equilibrium of the game exists and is unique.

C. POWER ITERATION FORMULATION

Power allocation serves as an effective means for the radar system and the jammer to engage in strategic interactions. Within the power allocation game, three scenarios are delineated:

a) Information asymmetry 1: The radar system initially detects the jammer and devises corresponding power allocation strategies, while the jammer formulates its strategy based on the radar's strategies. In this case, the radar system assumes the role of a leader, and the intelligent jammer acts as a follower.

b) Information asymmetry 2: The intelligent jammer first acquires layout information about the radars and devises its own corresponding power allocation strategy, prompting the radar system to formulate its strategy in response to the jammer's approach. Here, the intelligent jammer takes the lead, and the radar system follows suit.

c) Information symmetry: Both the radar and the target jammer simultaneously detect each other and devise their respective power allocation strategies. This case represents a non-cooperative game.

The first two scenarios correspond to the Stackelberg game, with the radar system and the jammer assuming leadership roles in each, respectively. The third scenario represents a non-cooperative game when information is symmetric. To address these three power allocation games, this paper introduces a fixed-step iterative formulation that proposes three corresponding game algorithms. As detailed in [29], [30], and [31], the power iteration formulation of the radar system is as follows:

$$p_{Rk}(n+1) = \frac{\gamma_k^*}{\gamma_k} p_{Rk}(n) \tag{25}$$

The power iteration formulation of the smart jammer is

$$p_{Jk}(n+1) = \frac{\gamma_k^*}{\gamma_k} p_{Jk}(n) - \lambda p_{Jk}(n) \tag{26}$$

where γ_k^* is the actual expect SINR, and λ is the power cost factor of the smart jammer.

Above all, the non-cooperative power allocation game model is established. Moreover, based on the proof, the existence and uniqueness of the Nash equilibrium of the power allocation game are demonstrated. The power iteration formulation of the radar system and the smart jammer is introduced and employed in various algorithms in the subsequent section.

IV. THE PROPOSED ALGORITHMS

Drawing upon the analysis and investigation of the aforementioned background information, solutions tailored to diverse power allocation game challenges are put forward. In addressing the initial information asymmetry game, a corresponding power allocation game algorithm 1 is proposed to address the power allocation game dilemma. Consequently, a more rational power allocation game resolution is furnished for the radar system, enhancing its capacity to effectively counter jammer threats. Within algorithm 1, the radar system is designated as the leader, and the jammer as the follower, with a focus on iterating the power allocation strategy. The sequential steps of the algorithm are outlined as follows:

Algorithm 1 : The Radar System Is the Leader and the Jammer Is the Follower

1: Initial Data: Iteration number $n = 1$. Set initialization radar power $\mathbf{p}_R = \left[p_{R1}^{(1)}, p_{R2}^{(1)}, \cdots, p_{RK}^{(1)}\right]^T$, the radar channel gain matrix $\mathbf{H}$, the interference channel gain $\mathbf{d}_k$, the actual expect SINR γ_k^*, the noise power σ_n^2, constants η, τ, μ, λ. Calculate the initial SINR γ_k of the target.

2: Repeat $n = n + 1$. Calculate the updated power $\mathbf{p}_R$ of the radars using equation (23). Update the jammer power $\mathbf{p}_J$ with equation (20), and update the target SINR γ_k. Equations (6) and (8) update the utility function values of the radars and the jammer, respectively.

3: While $\max\{|u_R(n+1) - u_R(n)|, |u_J(n+1) - u_J(n)|\} < \varepsilon$, stop iteration.

In addressing the second information asymmetry game, the corresponding power allocation game algorithm 2 is additionally suggested to tackle the power allocation game issue. This algorithm can offer an improved power allocation solution for the radar system concerning the jammer's priority attack strategy. Within algorithm 2, the jammer is designated as the leader, and the radar system as the follower, with a focus on iterating the power allocation strategy. The sequential steps of the algorithm are detailed as follows:

Algorithm 2 : The Jammer Is the Leader and the Radar System Is the Follower

1: Initial Data: Iteration number $n = 1$. Set initialization jammer power $\mathbf{p}_J = \left[p_{J1}^{(1)}, p_{J2}^{(1)}, \cdots, p_{JK}^{(1)}\right]^{\mathrm{T}}$, the radar channel gain matrix $\mathbf{H}$, the interference channel gain $\mathbf{d}_k$, the actual expect SINR γ_k^*, the noise power σ_n^2, constants η, τ, μ, λ. Calculate the initial SINR γ_k of the target.

2: Repeat $n = n + 1$. Calculate the updated power $\mathbf{p}_J$ of the jammer using equation (24). Update the radars power $\mathbf{p}_R$ with equation (16), and update the target SINR γ_k. Equations (6) and (8) update the utility function values of the radars and the jammer, respectively.

3: While $\max\{|u_R(n+1) - u_R(n)|, |u_J(n+1) - u_J(n)|\} < \varepsilon$, stop iteration.

Furthermore, a power allocation game algorithm 3 is put forward to furnish the radar system with an optimal response strategy and aid in enhancing the radar's capability to effectively mitigate the jamming interference introduced by the jammer. Within algorithm 3, both the radar system and the jammer engage in simultaneous power allocation strategies. The sequential steps of the algorithm are delineated as follows:

Algorithm 3 : The Radar System and the Smart Jammer Simultaneously Allocate the Power

1: Initial Data: Iteration number $n = 1$. Set initialization radar power $\mathbf{p}_R = \left[p_{R1}^{(1)}, p_{R2}^{(1)}, \cdots, p_{RK}^{(1)}\right]^{\mathrm{T}}$ and jammer power $\mathbf{p}_J = \left[p_{J1}^{(1)}, p_{J2}^{(1)}, \cdots, p_{JK}^{(1)}\right]^{\mathrm{T}}$, the radar channel gain matrix $\mathbf{H}$, the interference channel gain $\mathbf{d}_k$, the actual expect SINR γ_k^*, the noise power σ_n^2, constants η, τ, μ, λ. Calculate the initial SINR γ_k of the target.

2: Repeat $n = n + 1$. Calculate the updated power $\mathbf{p}_R$ of the radars using equation (16) and (24), Update the jammer power $\mathbf{p}_J$ with equation (20) and (23),, and update the target SINR γ_k. Equations (6) and (8) update the utility function values of the radars and the jammer, respectively.

3: While $\max\{|u_R(n+1) - u_R(n)|, |u_J(n+1) - u_J(n)|\} < \varepsilon$, stop iteration.

In conclusion, these three algorithms involve power allocation based on the mutual attack sequence between the radar systems and the jammer. Algorithm 1 entails the radar initiating a power allocation strategy to counter the jammer, utilizing the radar's power iteration formula for power allocation. Algorithm 2 involves the jammer initiating a power allocation strategy to disrupt the radar's normal operation, employing the jammer's power iteration formula for power allocation. Algorithm 3 entails a power allocation game where both the radar and the jammer simultaneously engage in power allocation, utilizing the power iteration formulas of both entities.

V. SIMULATION RESULTS AND PERFORMANCE COMPARISON

In this section, the convergence of the power allocation strategy between the multistatic radar system and the jammer is confirmed through simulation experiments. The non-cooperative power game models involving $K = 3$ radars with a control center and a smart jammer are examined. It is assumed that each radar acts independently and is unable to exert direct influence on the others. Furthermore, the cross-radar channel gains are shared as common information among all the radars.

A. POWER ALLOCATION GAME IN INFORMATION ASYMMETRY

In this subsection, simulation experiments are conducted on the scenarios involving two types of information asymmetry. Algorithm 1 represents a Stackelberg game algorithm in which the radar system assumes the role of a leader, while the smart jammer acts as a follower. Initially, the radar system identifies the jammer and acquires the jammer's information, subsequently formulating its own power allocation strategy. The jammer, on the other hand, devises its strategy based solely on the transmit power strategy of the radar system. The parameters are configured as follows: The actual expected SINR $\gamma_k^* = 20$ dB, noise power $\sigma_n^2 = 1.5$, constants $\eta = 1$, $\tau = 1.1$, $\mu = 0.5$, $\lambda = 0.0005$, $\varepsilon = 10^{-15}$, or the iteration termination step $n = 30$, the radar channel gain matrix $\mathbf{H} = \begin{bmatrix} 1.8 & 0.9 & 2.7 \\ 2.4 & 1.2 & 3.6 \\ 1.8 & 0.9 & 2.7 \end{bmatrix}$, and the interference channel gain $\mathbf{d}_k = [10, 10, 10]$. Figure 2 illustrates the SINR variations during the radar and jammer game process, ultimately converging to the actual expected SINR γ_k^*, indicating the convergence of the non-cooperative power allocation game algorithm. Figure 3 displays the transmit power allocation of the radar system and the jammer. The figure distinctly reveals that the jammer's power consumption in the equilibrium state significantly surpasses that of the radar system. Comparing the utility function values, the absolute values of the utility function are considered. Figure 4 demonstrates that the radar's transmit power is lower than the jammer's transmit power, yet the total utility function values of the radars exceed those of the jammer. Therefore, in instances of information asymmetry, the radar system that acts first employs less transmit power to achieve a greater benefit.

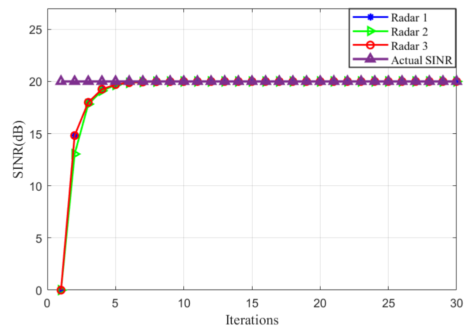

FIGURE 2. SINR convergence for algorithm 1.

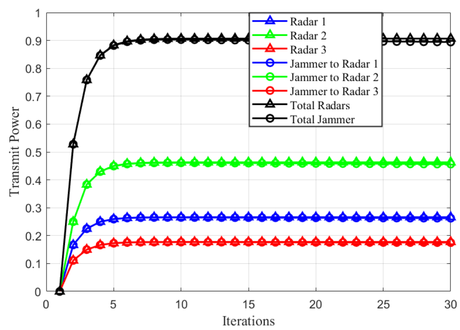

FIGURE 3. Power allocation convergence for algorithm 1.

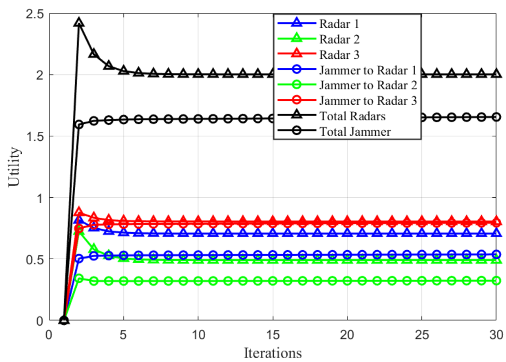

FIGURE 4. Utility value convergence for algorithm 1.

Algorithm 2 represents another Stackelberg game in which the jammer assumes the role of a leader, and the radar system acts as a follower. Initially, the jammer identifies the radar system, acquires their information, and subsequently formulates its own power allocation strategy. The radar system, in turn, devises its strategy based solely on the jammer's transmit power strategy. The parameters remain the same as those in Algorithm 1. Figure 5 illustrates the SINR variations during the radar and jammer game process. As the jammer first establishes a power allocation strategy, the radars remain in a passive state. Consequently, the SINR of the radars experiences some degradation, preventing it from converging to the actual expected SINR γ_k^*. Figure 6 depicts the transmit power of the radar system and the jammer. It is evident from the figure that the power consumed by the jammer in the equilibrium state is lower than that of the entire radar system. Figure 7 demonstrates that the jammer's transmit power is lower than the radar's transmit power, yet the absolute values of the jammer's utility function exceed those of the radar system. Therefore, in scenarios of information asymmetry, the jammer that acts first can achieve a greater benefit.

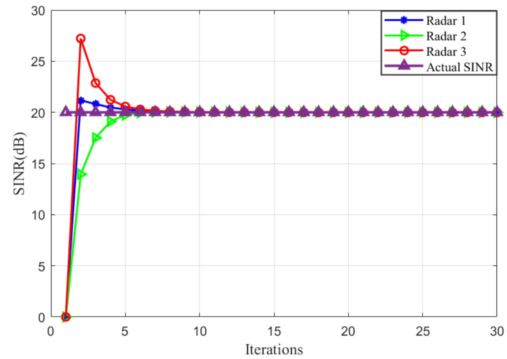

FIGURE 5. SINR convergence for algorithm 2.

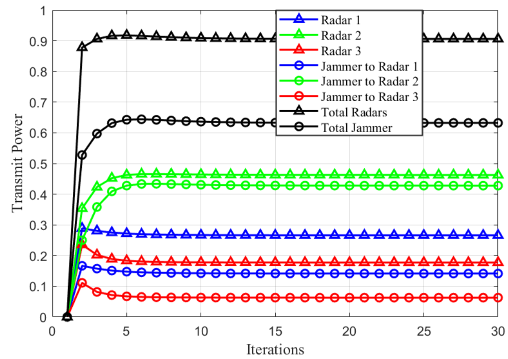

FIGURE 6. Power allocation convergence for algorithm 2.

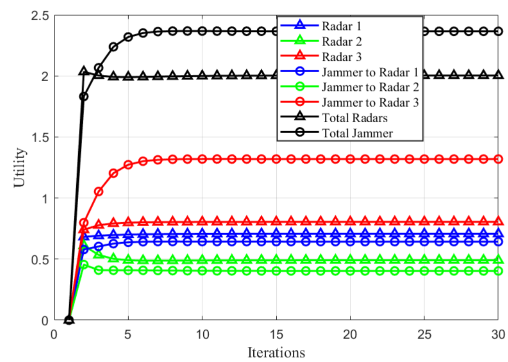

FIGURE 7. Utility value convergence for algorithm 2.

B. POWER ALLOCATION GAME IN INFORMATION SYMMETRY

Algorithm 3 represents a non-cooperative game under information symmetry. The radars and the jammer discover each other simultaneously and make their respective power allocation decisions. The parameters remain consistent with those in Algorithm 1 and Algorithm 2. Figure 8 illustrates the progression of the radar and jammer game process. As a result of the radar system's proactive approach, the SINR of the radars converges to the actual expected SINR γ_k^*. Figure 9 and Figure 10 reveal that the radars employ more transmit

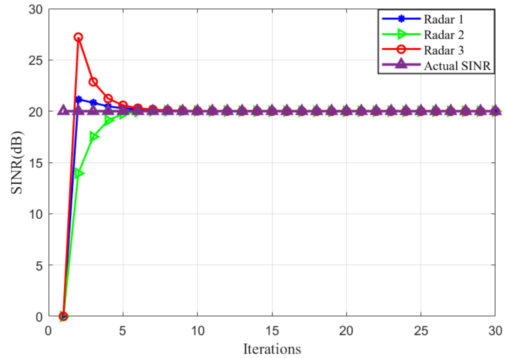

FIGURE 8. SINR convergence for algorithm 3.

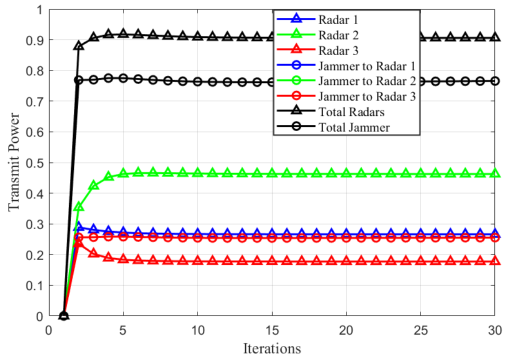

FIGURE 9. Power allocation convergence for algorithm 3.

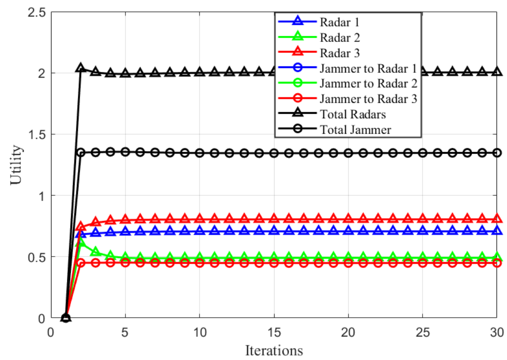

FIGURE 10. Utility value convergence for algorithm 3.

power, leading to higher utility values compared to those of the jammer. Conversely, the jammer utilizes less transmit power, resulting in lower utility values in comparison to the radar system. The radar power closely aligns with the jammer power, and the radar utility values closely correspond to the jammer utility values. Therefore, in scenarios of symmetrical information where both sides act simultaneously, we observe a positive correlation between transmit power and utility values.

Above all, the power allocation strategies of the radar system and the jammer, along with their respective utility values, are examined across three scenarios encompassing information symmetry and asymmetry. Figure 11 and

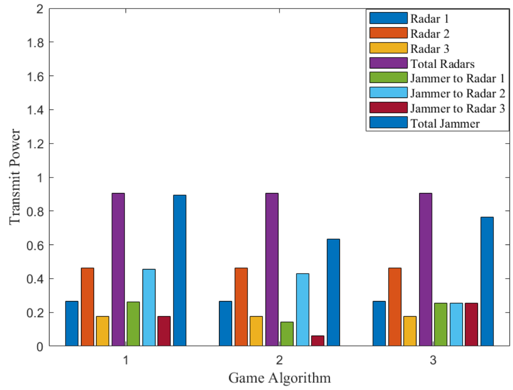

FIGURE 11. Nash equilibrium transmit power for three game algorithms.

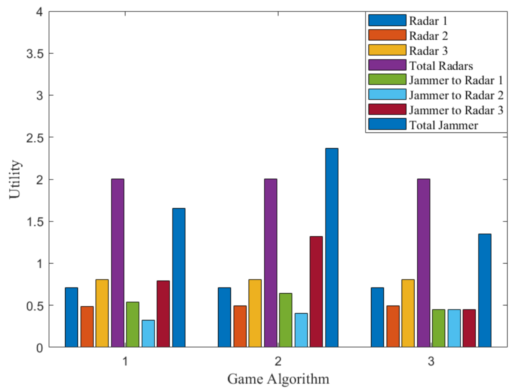

FIGURE 12. Nash equilibrium utility values for three game algorithms.

Figure 12 present comparisons of the transmit power and utility values of the three algorithms at equilibrium in the three games. The figures distinctly illustrate that in instances of asymmetric information, the side that acts first gains significant advantages in power allocation and benefits. Conversely, under conditions of symmetric information, there exists a direct positive correlation between the power allocation strategies and benefits of both sides.

VI. CONCLUSION

The non-cooperative power allocation games between a distributed multistatic radar network and a smart jammer have been investigated in this study. The radar system and the jammer have acted as a leader or a follower, respectively. Three game situations in which power is allocated by the radar system and the jammer have been analyzed, and the existence and uniqueness of Nash equilibrium in the games have been demonstrated. Three power allocation game algorithms have been proposed based on the three different games. The convergence of the algorithms and the existence of Nash equilibrium in the power allocation game have been confirmed through simulation experiments. In the future, game theory will be further applied to address resource management challenges in array radar and cognitive radar. The study of radar system target detection under future smart jamming attacks would also be meaningful.

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