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RESEARCH ARTICLE

Event-Triggered $\mathcal{L}_2 - \mathcal{L}_\infty$ Exponential Consensus of Leader-Follower Multi-Agent Systems

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ABSTRACT This paper investigates the $\mathcal{L}_2 - \mathcal{L}_\infty$ exponential consensus control problem of leader-follower multi-agent systems based on an event-triggered strategy. It begins by establishing an error system and provides a sufficient condition guaranteeing exponential stability of the error system while satisfying the $\mathcal{L}_2 - \mathcal{L}_\infty$ performance index. Subsequently, utilizing the condition, a design method for the $\mathcal{L}_2 - \mathcal{L}_\infty$ controller is presented. Finally, through a numerical example, this paper discusses the relationship between optimal $\mathcal{L}_2 - \mathcal{L}_\infty$ performance index and the maximum sampling period under different topological structures. The effectiveness of the proposed theoretical framework is validated through the numerical example.

INDEX TERMS Leader-follower, multi-agent systems, event-triggered, $\mathcal{L}_2 - \mathcal{L}_\infty$, exponential consensus.

I. INTRODUCTION

Multi-Agent Systems (MASs) refer to systems composed of multiple interacting agents, holding a crucial position in contemporary control science and engineering. The research on MASs focuses on the interactions, collaborative behaviors, and optimization of overall performance among intelligent agents [1], [2], [3], [4], [5], [6]. Currently, the study of MASs has become a hotspot in the field of control, covering a broad spectrum from theoretical exploration to practical applications.

MAS consensus control is a critical research area aimed at achieving a coherent state among interacting agents during their evolution. This consistency involves mutual communication and adjustment strategies among agents to ensure their states or behaviors remain consistent in certain aspects. One of the key challenges in this field is to design effective control strategies that lead the collective exponential convergence of agents to a consistent state [7], [8], [9]. In numerous scenarios, the imperative for prompt responsiveness of MASs necessitates the prevalent demand of exponential consensus stability. Research approaches include the distributed control algorithms, event-triggered strategies, and mechanisms for local information sharing to achieve exponential consensus across the system. In [7], the authors addressed the leader-following exponential consensus problem of general linear MASs via event-triggered control. In [8], based on the dynamic event-triggered strategy, the exponential consensus of MASs of impulsive PDEs with switching topology was discussed. In [9], the authors researched the exponential convergence for heterogeneous linear MASs over unbalanced digraphs.

Building upon the above introduction, we know that event-triggered strategy is frequently employed in the control of MASs [7], [8]. Event-triggered strategy, as a significant component in the research of MASs, introduces a new paradigm aiming to optimize the utilization of communication and computational resources. The strategy's advantage lies in triggering communication only when specific conditions in the system state are met, reducing communication frequency and lowering the system's energy consumption. Ongoing research is primarily concentrated on the design, analysis, and performance optimization of event-triggered strategies in various application scenarios. For example, a distributed control approach for consensus of MASs based on event-triggered strategy was proposed

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in [3]. In [10], an adaptive event-triggered communication was adopted in the consensus control of leader-follower MASs. Recent, many works about dynamic event-triggered consensus such as dynamic event-triggered consensus of general linear multi-agent systems with adaptive strategy, and dynamic event-triggered consensus of multi-agent systems under directed topology have been addressed. The unique point of dynamic event-triggered strategy is that the triggered threshold is adaptive rather than static. This means that the conditions under which agents communicate and update their states are based on the system's current state and can change over time. The adaptiveness helps in reducing communication demands while ensuring that the agents can still reach consensus efficiently. Dynamic event-triggered strategy is part of a growing field of research that seeks to optimize how and when agents in a network communicate so that collective goals are achieved efficiently and resources such as energy or bandwidth are conserved. Obviously, We should keep an eye on dynamic event-triggered strategy, as it is a research trend in this field.

External disturbances are prevalent in control systems. Both $\mathcal{H}_\infty$ control and $\mathcal{L}_2-\mathcal{L}_\infty$ control methods are highly effective in suppressing the impact of external disturbances on the system. Wang developed an adaptive $\mathcal{H}_\infty$ control scheme to ensure the consensus of the nonlinear second-Order MASs in [11]. In [12], the $\mathcal{H}_\infty$ consensus control for MASs with input delay and directed topology was discussed. Similar problem for MASs with linear coupling dynamics and communication delays was studied in [13].

However, to the best of our knowledge, very few authors have researched the $\mathcal{L}_2-\mathcal{L}_\infty$ consensus problem for MASs recently. $\mathcal{L}_2-\mathcal{L}_\infty$ control stands as a control methodology, aiming to achieve stability and performance optimization. This method considers the system's performance from a global perspective by defining the $\mathcal{L}_2$ and $\mathcal{L}_\infty$ norms between the external disturbances and estimated signals. Current research focuses on the theoretical derivation and practical applications of $\mathcal{L}_2-\mathcal{L}_\infty$ control, providing a new perspective for the control of MASs.

Motivated by the above discussions, this paper investigates the event-triggered $\mathcal{L}_2-\mathcal{L}_\infty$ consensus problems for the leader-follower MASs with the disturbances. The main contributions of this paper can be summarized as follows.

1) A closed-loop error model for the consensus of leader-follower MASs is established. Based on the error nodel of MASs and event-triggered strategy, a sufficient condition in terms of linear matrix inequalities is proposed to guarantee the consensus stability and $\mathcal{L}_2-\mathcal{L}_\infty$ attenuation level.
2) An $\mathcal{L}_2-\mathcal{L}_\infty$ consensus controller gain matrix is given for leader-follower MASs.
3) Based on the proposed algorithm, we discuss the relationship between optimal $\mathcal{L}_2-\mathcal{L}_\infty$ attenuation level γ^* and the maximum allowed sampling period δ^* with different topological structures.

The rest of this paper is organized as follows. The closed-loop error MASs is established in Section II. The main results on the $\mathcal{L}_2-\mathcal{L}_\infty$ consensus are proposed in Section III. Section IV provides a numerical example to illustrate the effectiveness of the results given in Section III. Section V draws the conclusions.

Notations: In this paper, $\mathbb{Z}_+$ denotes the set of non-negative integers; $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of all $n\times m$ real matrices, respectively; I and 0 denote identity matrix and zero matrix with appropriate dimensions, respectively; $\otimes$ denotes the Kronecker product between two matrices; sym(A) stands for $A+A^T$; the superscript "T" stands for matrix transpose. The Laplacian matrix of a multi-agent system is denoted as $L=[l_{ij}]_{N\times N}$, where $l_{ii}=\sum_{j=1,j\neq i}^N w_{ij}$, and $l_{ij}=-w_{ij}$ for $i\neq j$. w_{ij} represents the link quality from agent j to agent i and satisfies $w_{ii}=0$, $w_{ij}\geq 0$. The leader adjacency matrix is denoted by $M=\text{diag}\{m_1,m_2,\ldots,m_N\}$, where $m_i\geq 0$ for $i=1,2,\ldots,N$. Note that if there is an edge from the leader to agent i, then $m_i>0$, otherwise, $m_i=0$.

II. PROBLEM FORMULATION

In this article, the N followers in MAS are described by

$$\begin{cases}\dot{x}_i(t)=Ax_i(t)+Bu_i(t)+D\omega_i(t),\\ x_i(t_0)=x_i^0,\ \ t_0\leq t,\ \ i=1,\cdots,N,\end{cases}\tag{1}$$

where $x_i(t)\in\mathbb{R}^n$, $u_i(t)\in\mathbb{R}^n$ and $\omega_i(t)\in\mathbb{R}^n$ are the state, control input and external disturbance of the ith agent, respectively. A, B and D are the system matrices with appropriate dimensions.

Let $x_0(t)$ be the leader, whose dynamical behavior is formulated as follows

$$\begin{cases}\dot{x}_0(t)=Ax_0(t)+D\omega_0(t),\\ x_0(t_0)=x_0^0,\ \ t_0\leq t,\end{cases}\tag{2}$$

The controlled output is defined as is

$$z_i(t)=C\big(x_i(t)-x_0(t)\big).\tag{3}$$

The control strategy adopts an event-triggered mechanism. Let δ_s denotes the sampling instants with a constant sampling period δ ($s\in\mathbb{Z}_+$). Meantime, let t_k^i ($k\in\mathbb{Z}_+$) denotes the event-triggered instants of the agent i. For simplicity, we set $\delta_0=t_0^i=0$.

Then, the controller is designed as

$$u_i(t)=-K\Bigg(\sum_{j\in\mathcal{N}(i)}w_{ij}\Big[x_i(t_k^i)-x_j(t_{k'}^j)\Big]+m_i\Big[x_i(t_k^i)-x_0(\delta_s)\Big]\Bigg),\quad t\in[\delta_s,\delta_{s+1}),\tag{4}$$

where t_k^i and $t_{k'}^j$ denote the latest event-triggered instants for the agent i and agent j, and $K\in\mathbb{R}^{n\times n}$ denotes the control gain matrix to be given. $\mathcal{N}(i)$ represents the neighbor set of agent i, e.g., for agent i and agent j, $j\in\mathcal{N}(i)$ if $w_{ij}>0$.

Here, for agent i, we define an event-triggered function as:

$$f(\delta_s)=\xi_i^T(\delta_s)\xi_i(\delta_s)-\epsilon_i e_i^T(\delta_s)e_i(\delta_s),\tag{5}$$

where $\xi_i(\delta_s) = x_i(\delta_s) - x_i(t_k^i)$, $e_i(\delta_s) = x_i(\delta_s) - x_0(\delta_s)$, t_k^i denotes the event-triggered instant closest to sampling instant δ_s, and ϵ_i is a constant threshold.

The triggering instants for i are defined as follows:

$$t_{k+1}^i = \inf\{\delta_s > t_k^i : f(\delta_s) > 0\}. \tag{6}$$

Noting $x_i(t_k^i) = x_i(\delta_s) - \xi_i(\delta_s)$, we can rewrite (1) as

$$\begin{aligned}\dot{x}_i(t) = Ax_i(t) - BK \\ &\times \Big(\sum_{j\in\mathcal{N}(i)} w_{ij}\Big[x_i(\delta_s) - \xi_i(\delta_s) - x_j(\delta_s) + \xi_j(\delta_s)\Big] \\ &+ m_i\Big[x_i(\delta_s) - \xi_i(\delta_s) - x_0(\delta_s)\Big]\Big) \\ &+ D\omega_i(t), \quad t \in [\delta_s, \delta_{s+1}).. \end{aligned} \tag{7}$$

Define $e_i(t) = x_i(t) - x_0(t)$. Then, we get the error system

$$\begin{aligned}\dot{e}_i(t) = Ae_i(t) - BK \sum_{j\in\mathcal{N}(i)} w_{ij}\Big[e_i(\delta_s) - e_j(\delta_s)\Big] \\ &+ BK \sum_{j\in\mathcal{N}(i)} w_{ij}\Big[\xi_i(\delta_s) - \xi_j(\delta_s)\Big] \\ &- BKm_i\Big[e_i(\delta_s) - \xi_i(\delta_s)\Big] \\ &+ D\Big[\omega_i(t) - \omega_0(t)\Big]. \end{aligned} \tag{8}$$

Let

$$\begin{aligned} e(t) &= [e_1^T(t) \ \cdots \ e_N^T(t)]^T \in \mathbb{R}_1^{Nn} \\ \xi(t) &= [\xi_1^T(t) \ \cdots \ \xi_N^T(t)]^T \in \mathbb{R}_1^{Nn}, \\ \omega(t) &= [\omega_1^T(t) \ \cdots \ \omega_N^T(t), \omega_0^T(t)]^T \in \mathbb{R}_1^{(N+1)n} \\ z(t) &= [z_1^T(t) \ \cdots \ z_N^T(t)]^T \in \mathbb{R}_1^{Nn}. \end{aligned} \tag{9}$$

Based on (8) and (3), we have

$$\begin{cases} \dot{e}(t) = [I_N \otimes A]e(t) - [H \otimes BK]e(\delta_s) \\ +[H \otimes BK]\xi(\delta_s) + [I_N \otimes D]\mathbb{D}_\omega\omega(t), \\ z(t) = [I_N \otimes C]e(t) \end{cases} \tag{10}$$

where $H = L + M$ and

$$\mathbb{D}_\omega = \begin{bmatrix} I & 0 & \dots & 0 & -I \\ 0 & I & \dots & 0 & -I \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & I & -I \end{bmatrix}_{N\times(N+1)}.$$

Define $\varepsilon(t) = t - \delta_s$ for $t \in [\delta_s, \delta_{s+1})$. $\varepsilon(t)$ can be regarded as a time-varying delay with $\dot{\varepsilon}(t) = 1$ and $\varepsilon(t) \le \delta$. We transform (10) into

$$\begin{cases} \dot{e}(t) = \mathbb{A}e(t) - \mathbb{A}_K e(t - \varepsilon(t)) + \mathbb{A}_K \xi(t - \varepsilon(t)) \\ +\mathbb{D}\mathbb{D}_\omega\omega(t), \\ z(t) = \mathbb{C}e(t), \end{cases} \tag{11}$$

where

$$\mathbb{A} = I_N \otimes A, \quad \mathbb{A}_K = H \otimes BK,$$
$$\mathbb{D} = I_N \otimes D, \quad \mathbb{C} = I_N \otimes C.$$

Next, we give two definitions and a lemma, which can help us understand the results.

Definition 1 (see [7]): The agents of (1) and (2) are said to be exponential consensus if there exist positive-definite constants $\kappa > 0$, $\beta > 0$ and $T > 0$, the following condition holds:

$$\|x_i(t) - x_0(t)\| \le \kappa e^{-\beta t}$$

for all $t > T$, $\omega_i(t) = 0$ and any $x_i(0) \in \mathbb{R}^n$ $(i = 1, 2, \cdots, N)$. β is called the convergence rate.

Definition 2 (see [14]): Given a scalar γ, the multi-agent system composed of (1), (2) and (3) is said to be exponential consensus with a prescribed $\mathcal{L}_2 - \mathcal{L}_\infty$ performance level γ if system (11) is exponentially stable, and under zero initial condition ($e(0) = 0$), $\|z(t)\|_\infty < \gamma\|\omega(t)\|_2$ for all nonzero $\omega(t) \in L_2[0, \infty)$, where $\|z(t)\|_\infty = \sup_t \sqrt{z(t)^T z(t)}$.

Lemma 1 (see [10]): For a matrix $Q > 0$ and any Y with appropriate dimensions, the following inequality holds:

$$-Y^T Q^{-1} Y \le -Y^T - Y + Q.$$

Remark 1: According to Definition 1 and 2, the exponential consensus of agents of (1), (2) and the exponential stability of system (11) are equivalent. Moreover, the $\mathcal{L}_2 - \mathcal{L}_\infty$ exponential consensus controller of multi-agent system composed of (1), (2) and (3) can be obtained by system (11).

III. MAIN RESULTS

The sufficient condition and the controller design method for the $\mathcal{L}_2 - \mathcal{L}_\infty$ consensus of the multi-agent system composed of (1), (2) and (3) with the event-triggered condition (5) will be provided in this section.

Theorem 1: The error system (11) is exponentially stable with a prescribed $\mathcal{L}_2 - \mathcal{L}_\infty$ performance γ under the event-triggered condition (5) if there exist positive-definite matrices $\mathbb{W}_1 > 0$, $\mathbb{W}_2 > 0$, $\mathbb{W}_3 > 0$, positive scalar $\beta > 0$ and matrix K such that the following inequalities hold:

$$\tilde{\Psi}_\omega = \begin{bmatrix} \tilde{\Psi} & \tilde{\Psi}_{\omega D} \\ \tilde{\Psi}_{\omega D}^T & -\gamma^2 I \end{bmatrix} < 0 \tag{12}$$

and

$$\begin{bmatrix} \mathbb{W}_1 & \mathbb{C}^T \\ * & I \end{bmatrix} > 0, \tag{13}$$

where

$$\begin{aligned} \tilde{\Psi}_{\omega D} &= \begin{bmatrix} \mathbb{D}_\omega^T \mathbb{D}^T & 0 & 0 & 0 \end{bmatrix}^T, \\ \tilde{\Psi} &= \tilde{\Psi}_1 + \tilde{\Psi}_2 + \delta e^{\beta\delta} F_0^T \mathbb{W}_3 F_0, \\ \tilde{\Psi}_1 &= \Big(\mathrm{sym}\big(F_1^T \mathbb{W}_1 F_0\big) + \beta F_1^T \mathbb{W}_1 F_1\Big) \\ &+ \Big(F_1^T e^{\beta\delta} \mathbb{W}_2 F_1 - F_3^T \mathbb{W}_2 F_3\Big) \\ &+ \Big(F_2^T F_2 - F_4^T \mathcal{I}_\epsilon^{-1} F_4\Big), \end{aligned}$$

$$\tilde{\Psi}_2=-\frac{1}{\delta}\tilde{\Gamma}^T\begin{bmatrix}\mathbb{W}_3 & 0\\ 0 & \mathbb{W}_3\end{bmatrix}\tilde{\Gamma},$$
$$\tilde{\Gamma}=\begin{bmatrix}I & -I & 0 & 0\\ 0 & I & -I & 0\end{bmatrix},$$
$$F_0=[\mathbb{A}\ \ -\mathbb{A}_K\ \ 0\ \ \mathbb{A}_K],$$
$$F_1=[I,0,0,0],\ F_2=[0,I,0,0],$$
$$F_3=[0,0,I,0],\ F_4=[0,0,0,I],$$
$$\mathcal{I}_\epsilon=\mathrm{diag}\{\epsilon_1,\epsilon_2,\ldots,\epsilon_N\}\otimes I_n.$$

and the convergence rate is $\frac{\beta}{2}$.

Proof: First, define

$$\gamma(t)\triangleq[e^T(t),e^T(t-\varepsilon(t)),e^T(t-\delta),\xi^T(t-\varepsilon(t))]^T,$$
$$\gamma_1(t)\triangleq[\gamma^T(t),\omega^T(t)]^T.$$

To establish the asymptotical stability condition of the system (11) under the condition (5), we consider $\omega(t)=0$. Then, an LKF is chosen as

$$V(t)=\sum_{l=1}^{3}V_l(t),\quad t\in[\delta_s,\delta_{s+1}) \tag{14}$$

where

$$V_1(t)=e^T(t)\mathbb{W}_1e(t), \tag{15}$$
$$V_2(t)=\int_{t-\delta}^{t}e^{\beta(\mu-t+\delta)}e^T(\mu)\mathbb{W}_2e(\mu)\mathrm{d}\mu, \tag{16}$$
$$V_3(t)=\int_{-\delta}^{0}\int_{t+v}^{t}e^{\beta(\mu-t+\delta)}\dot{e}^T(\mu)\mathbb{W}_3\dot{e}(\mu)\mathrm{d}\mu\mathrm{d}v, \tag{17}$$

with $\mathbb{W}_1>0$, $\mathbb{W}_2>0$, $\mathbb{W}_3>0$. Considering (11), we get the time derivative of $V_1(t)$

$$\dot{V}_1(t)=\gamma^T(t)\mathrm{sym}\big(F_1^T\mathbb{W}_1F_0\big)\gamma(t) \tag{18}$$

which indicates that

$$\dot{V}_1(t)+\beta V_1(t)=\gamma^T(t)\Big[\mathrm{sym}\big(F_1^T\mathbb{W}_1F_0\big)+\beta F_1^T\mathbb{W}_1F_1\Big]\gamma(t). \tag{19}$$

Similarly, the derivatives of $V_2(t)$ and $V_3(t)$

$$\begin{aligned}\dot{V}_2(t)&=e^{\beta\delta}e^T(t)\mathbb{W}_2e(t)-e^T(t-\delta)\mathbb{W}_2e(t-\delta)\\&\quad-\beta\int_{t-\delta}^{t}e^{\beta(\mu-t+\delta)}e^T(\mu)\mathbb{W}_2e(\mu)\mathrm{d}\mu\\&=\gamma^T(t)\Big(F_1^Te^{\beta\delta}\mathbb{W}_2F_1-F_3^T\mathbb{W}_2F_3\Big)\gamma(t)\\&\quad-\beta V_2(t).\end{aligned} \tag{20}$$

$$\begin{aligned}\dot{V}_3(t)&=\delta e^{\beta\delta}\gamma^T(t)F_0^T\mathbb{W}_3F_0\gamma(t)\\&\quad-\int_{t-\delta}^{t}e^{\beta(\mu-t+\delta)}\dot{e}^T(\mu)\mathbb{W}_3\dot{e}(\mu)\mathrm{d}\mu\\&\quad-\beta V_3(t).\end{aligned} \tag{21}$$

Further, we obtain

$$\dot{V}_2(t)+\beta V_2(t)=\gamma^T(t)\Big(F_1^Te^{\beta\delta}\mathbb{W}_2F_1-F_3^T\mathbb{W}_2F_3\Big)\gamma(t), \tag{22}$$

and

$$\dot{V}_3(t)+\beta V_3(t)=\delta e^{\beta\delta}\gamma^T(t)F_0^T\mathbb{W}_3F_0\gamma(t)-\int_{t-\delta}^{t}e^{\beta(\mu-t+\delta)}\dot{e}^T(\mu)\mathbb{W}_3\dot{e}(\mu)\mathrm{d}\mu. \tag{23}$$

As $e^{\beta(\mu-t+\delta)}\geq 1$ for $\mu\in[t-\delta,\ \ t]$, we know that

$$-\int_{t-\delta}^{t}e^{\beta(\mu-t+\delta)}\dot{e}^T(\mu)\mathbb{W}_3\dot{e}(\mu)\mathrm{d}\mu\leq-\int_{t-\delta}^{t}\dot{e}^T(\mu)\mathbb{W}_3\dot{e}(\mu)\mathrm{d}\mu$$

i.e.,

$$\begin{aligned}&-\int_{t-\delta}^{t}e^{\beta(\mu-t+\delta)}\dot{e}^T(\mu)\mathbb{W}_3\dot{e}(\mu)\mathrm{d}\mu\\&\leq-\int_{t-\delta}^{t-\varepsilon(t)}\dot{e}^T(\mu)\mathbb{W}_3\dot{e}(\mu)\mathrm{d}\mu\mathrm{d}\mu\\&\quad-\int_{t-\varepsilon(t)}^{t}\dot{e}^T(\mu)\mathbb{W}_3\dot{e}(\mu)\mathrm{d}\mu\mathrm{d}\mu.\end{aligned} \tag{24}$$

Then, applying Jensen inequality [15] to (24), we get

$$\begin{aligned}&-\int_{t-\delta}^{t}e^{\beta(\mu-t+\delta)}\dot{e}^T(\mu)\mathbb{W}_3\dot{e}(\mu)\mathrm{d}\mu\\&\leq-\gamma^T(t)\frac{1}{\delta}\tilde{\Gamma}^T\begin{bmatrix}\mathbb{W}_3 & 0\\ 0 & \mathbb{W}_3\end{bmatrix}\tilde{\Gamma}\gamma(t),\end{aligned} \tag{25}$$

Noting that eq. (5) implies that

$$\gamma^T(t)\Big(F_2^TF_2-F_4^T\mathcal{I}_\epsilon^{-1}F_4\Big)\gamma(t)\geq 0. \tag{26}$$

Combining (19), (20), (22), (23), (25) and (26) yields

$$\dot{V}(t)+\beta V(t)\leq\gamma^T(t)\tilde{\Psi}\gamma(t), \tag{27}$$

Obviously, $\tilde{\Phi}_\omega<0$ results in $\tilde{\Phi}<0$, which represents $\dot{V}(t)+\beta V(t)\leq 0$. It means that, if (12) holds, the system (11) is exponentially stable under condition (5). Further, it implies that the convergence rate is $\beta/2$ [16].

Next, we introduce an index to establish the $\mathcal{L}_2-\mathcal{L}_\infty$ performance condition for the system (11) under the zero-initial condition.

$$J=V(t)-\int_0^t\gamma^2\omega^T(\mu)\omega(\mu)\mathrm{d}\mu, \tag{28}$$

where $V(t)$ is defined in (14). Noting zero-initial condition ($V(0)=0$), we get

$$J=\int_0^t[\dot{V}(\mu)-\gamma^2\omega^T(\mu)\omega(\mu)]\mathrm{d}\mu,$$

Due to $\beta V(t)>0$, it can be concluded that

$$J\leq\int_0^t\gamma_1(\mu)^T\tilde{\Phi}_\omega\gamma_1(\mu)\mathrm{d}\mu,$$

$\tilde{\Phi}_\omega<0$ in Theorem 1 implies $J<0$ for $\omega(t)\neq 0$ and $t>0$. Therefore, we have $V(t)\leq\gamma^2\|\omega\|^2$. Moreover, from (13) and (14), we have,

$$e^T(t)\mathbb{W}_1e(t)<V(t)\leq\gamma^2\|\omega\|^2 \tag{29}$$

and

$$\mathbb{C}^T\mathbb{C}\leq\mathbb{W}_1 \tag{30}$$

Finally, combining (29) and (30), we have

$$z^T(t)z(t)=e^T(t)\mathbb{C}^T\mathbb{C}e(t)\leq V(t)\leq\gamma^2\|\omega\|^2 \tag{31}$$

Hence, we obtain that $\|z(t)\|_\infty \leq \gamma\|\omega\|_2$ for any nonzero $\omega(t)$. This completes the proof. □

Now, based on Theorem 1, we provide an LMI design method of the $\mathcal{L}_2-\mathcal{L}_\infty$ consensus controller for the system (11) under condition (5).

Theorem 2: The error system (11) is exponentially stable with a convergence rate $\frac{\beta}{2}$ and a prescribed $\mathcal{L}_2-\mathcal{L}_\infty$ performance γ under the event-triggered condition (5) if there exist positive-definite matrices $\hat{W}_1>0$, $\hat{\mathbb{W}}_2>0$, $\hat{\mathbb{W}}_3>0$ and matrix $\hat{K}$, such that the following inequality holds:

$$\Delta+\Pi<0, \tag{32}$$

$$\begin{bmatrix}\hat{\mathbb{W}}_1 & \hat{\mathbb{W}}_1\mathbb{C}^T\\ * & I\end{bmatrix}>0, \tag{33}$$

where

$$\Delta=\begin{bmatrix}\Delta_{11} & -\hat{\Theta} & 0 & \hat{\Theta} & \hat{\mathbb{W}}_1\mathbb{A}^T & \hat{\mathbb{W}}_1\mathbb{D}\mathbb{D}_\omega & \hat{\mathbb{W}}_1 & 0\\ * & 0 & 0 & 0 & -\hat{\Theta}^T & 0 & 0 & \hat{\mathbb{W}}_1\\ * & * & \Delta_{33} & 0 & 0 & 0 & 0 & 0\\ * & * & * & \Delta_{44} & \hat{\Theta}^T & 0 & 0 & 0\\ * & * & * & * & \Delta_{55} & 0 & 0 & 0\\ * & * & * & * & * & -\gamma^2 I & 0 & 0\\ * & * & * & * & * & * & \Delta_{77} & 0\\ * & * & * & * & * & * & * & -I\end{bmatrix}$$

$$\Delta_{11}=\mathrm{sym}(\mathbb{A}\hat{\mathbb{W}}_1)+\beta\hat{\mathbb{W}}_1,\quad \Delta_{33}=-\mathrm{sym}(\hat{\mathbb{W}}_1)+\hat{\mathbb{W}}_2,$$
$$\Delta_{44}=-\mathrm{sym}(\hat{\mathbb{W}}_1)+\mathcal{I}_\epsilon,\ \Delta_{55}=-\frac{1}{\delta e^{\beta\delta}}\hat{\mathbb{W}}_3,$$
$$\Delta_{77}=-\frac{1}{e^{\beta\delta}}\hat{\mathbb{W}}_2,\quad \hat{\Theta}=H\otimes B\hat{K},\quad \hat{\mathbb{W}}_1=I_N\otimes\hat{W}_1,$$
$$\Pi=\frac{1}{\delta}\hat{\Gamma}^T\begin{bmatrix}-\mathrm{sym}(\hat{\mathbb{W}}_1)+\hat{\mathbb{W}}_3 & 0\\ 0 & -\mathrm{sym}(\hat{\mathbb{W}}_1)+\hat{\mathbb{W}}_3\end{bmatrix}\hat{\Gamma},$$
$$\hat{\Gamma}=\begin{bmatrix}I & -I & 0 & 0\ 0\ 0\ 0\ 0\ 0\\ 0 & I & -I & 0\ 0\ 0\ 0\ 0\ 0\end{bmatrix}.$$

Then, the $\mathcal{L}_2-\mathcal{L}_\infty$ consensus controller K is given by

$$K=\hat{K}\hat{W}_1^{-1}. \tag{34}$$

Proof: In (12), we set $\mathbb{W}_1$ as: $\mathbb{W}_1=I_N\otimes W_1$. At the same time, let $\hat{W}_1=W_1^{-1}$ and set $\hat{\mathbb{W}}_1=I_N\otimes\hat{W}_1$. It's easy to get

$$\hat{\mathbb{W}}_1\mathbb{W}_1=I. \tag{35}$$

Applying Shur complement to (12), we know that (12) is equivalent to

$$\Psi_1+\Psi_2<0, \tag{36}$$

where

$$\Psi_1=\begin{bmatrix}\Sigma & -\mathbb{W}_1\mathbb{A}_K & 0 & \mathbb{W}_1\mathbb{A}_K & \mathbb{A}^T & \mathbb{D}\mathbb{D}_\omega\\ * & I & 0 & 0 & -\mathbb{A}_K^T & 0\\ * & * & -\mathbb{W}_2 & 0 & 0 & 0\\ * & * & * & -\mathcal{I}_\epsilon^{-1} & \mathbb{A}_K^T & 0\\ * & * & * & * & -\frac{1}{\delta e^{\beta\delta}}\mathbb{W}_3^{-1} & 0\\ * & * & * & * & * & -\gamma^2 I\end{bmatrix},$$
$$\Sigma=\mathrm{sym}(\mathbb{W}_1\mathbb{A})+\beta\mathbb{W}_1+e^{\beta\delta}\mathbb{W}_2,$$
$$\mathcal{I}_\epsilon=diag\{\epsilon_1,\epsilon_2,\cdots,\epsilon_N\}\otimes I_n,$$
$$\Psi_2=-\frac{1}{\delta}\Gamma^T\begin{bmatrix}\mathbb{W}_3 & 0\\ 0 & \mathbb{W}_3\end{bmatrix}\Gamma,$$
$$\Gamma=\begin{bmatrix}I & -I & 0 & 0\ 0\ 0\\ 0 & I & -I & 0\ 0\ 0\end{bmatrix}.$$

Defining $J=\mathrm{diag}\{\hat{\mathbb{W}}_1,\hat{\mathbb{W}}_1,\hat{\mathbb{W}}_1,\hat{\mathbb{W}}_1,I,I\}$. Pre- and post-multiplying (36) with J, we have,

$$J\Psi_1J+J\Psi_2J<0. \tag{37}$$

Let $\hat{\Psi}_1=J\Psi_1J$ and $\hat{\Psi}_2=J\Psi_2J$, then we obtain

$$\hat{\Psi}_1=\begin{bmatrix}\hat{\Sigma}_{11} & -\mathbb{A}_K\hat{\mathbb{W}}_1 & 0 & \mathbb{A}_K\hat{\mathbb{W}}_1 & \hat{\mathbb{W}}_1\mathbb{A}^T & \hat{\mathbb{W}}_1\mathbb{D}\mathbb{D}_\omega\\ * & \hat{\mathbb{W}}_1\hat{\mathbb{W}}_1 & 0 & 0 & -\hat{\mathbb{W}}_1\mathbb{A}_K^T & 0\\ * & * & -\hat{\mathbb{W}}_1\mathbb{W}_2\hat{\mathbb{W}}_1 & 0 & 0 & 0\\ * & * & * & -\hat{\mathbb{W}}_1\mathcal{I}_\epsilon^{-1}\hat{\mathbb{W}}_1 & \hat{\mathbb{W}}_1\mathbb{A}_K^T & 0\\ * & * & * & * & -\frac{1}{\delta}\mathbb{W}_3^{-1} & 0\\ * & * & * & * & * & -\gamma^2 I\end{bmatrix} \tag{38}$$

where

$$\hat{\Sigma}_{11}=\mathrm{sym}(\mathbb{A}\hat{\mathbb{W}}_1)+\beta\hat{\mathbb{W}}_1+e^{\beta\delta}\hat{\mathbb{W}}_1\mathbb{W}_2\hat{\mathbb{W}}_1$$

Based on Lemma 1, we have

$$\begin{aligned}\hat{\Psi}_2&=-\frac{1}{\delta}\Gamma^T\begin{bmatrix}\hat{\mathbb{W}}_1 & 0\\ 0 & \hat{\mathbb{W}}_1\end{bmatrix}\begin{bmatrix}\mathbb{W}_3 & 0\\ 0 & \mathbb{W}_3\end{bmatrix}\begin{bmatrix}\hat{\mathbb{W}}_1 & 0\\ 0 & \hat{\mathbb{W}}_1\end{bmatrix}\Gamma\\ &\leq\frac{1}{\delta}\Gamma^T\begin{bmatrix}-\mathrm{sym}(\hat{\mathbb{W}}_1)+\mathbb{W}_3^{-1} & 0\\ 0 & -\mathrm{sym}(\hat{\mathbb{W}}_1)+\mathbb{W}_3^{-1}\end{bmatrix}\Gamma\end{aligned} \tag{39}$$

and

$$-\hat{\mathbb{W}}_1\mathbb{W}_2\hat{\mathbb{W}}_1\leq-\mathrm{sym}(\hat{\mathbb{W}}_1)+\mathbb{W}_2^{-1}, \tag{40}$$
$$-\hat{\mathbb{W}}_1\mathcal{I}_\epsilon^{-1}\hat{\mathbb{W}}_1\leq-\mathrm{sym}(\hat{\mathbb{W}}_1)+\mathcal{I}_\epsilon. \tag{41}$$

Integrating (38), (39), (40), (41) into (37), and setting $\hat{K}=K\hat{W}_1$, $\hat{\mathbb{W}}_2=\mathbb{W}_2^{-1}$, $\hat{\mathbb{W}}_3=\mathbb{W}_3^{-1}$, inequality (32) can be obtained. Consequently, we can conclude that inequality (12) holds if inequality (32) holds. Similarly, Pre- and post-multiplying (13) with $\mathrm{diag}\{\hat{\mathbb{W}}_1,I\}$ and noting (35), we can deduce that inequality (13) holds if (33) holds. This completes the proof. □

Next, an algorithm for optimization $\mathcal{L}_2 - \mathcal{L}_\infty$ attenuation level γ based on the Theorem 2 would be proposed.

Algorithm 1: Given the system parameters, the minimum attenuation level γ can be obtained by solving the following problem

$$\min \ \hat{\gamma}$$
$$\text{s.t. LMIs (32), (33)}$$

where $\hat{\gamma} = \gamma^2$. Then, the minimum attenuation level is calculated as $\gamma^* = \sqrt{\hat{\gamma}_{\min}}$, where $\hat{\gamma}_{\min}$ denotes the optimal value of $\hat{\gamma}$.

IV. NUMERICAL EXAMPLE

In this section, we will present five multi-agent systems with different topological structures, dynamics of which are described by (1) and (2). Three of these systems consist of one leader and three followers with various topological configurations, which are represented in Fig 1, 2, 3. One MAS comprises one leader and four followers, which is shown in Fig 4. The last MAS comprises one leader and five followers, which is shown in Fig 5. Here, x_0 denotes the leader agent. x_1, x_2, x_3, x_4 and x_5 are the follower agents.

The system matrices of (1) and (2) are set as

$$A = \begin{bmatrix} -1 & 0.1 \\ 0.1 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$D = \begin{bmatrix} 0 & 0.4 \\ -0.2 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} -0.2 & 0 \\ 0 & 0.5 \end{bmatrix},$$

the thresholds in event-triggered condition (5) are set as $\epsilon_1 = \epsilon_2 = \epsilon_3 = 1.0$ and the convergence rate is set as $\beta/2 = 0.5$.

From the topologies of the MASs, the Laplacian matrices and the leader adjacency matrices of the four MASs are got by

$$L_1 = \begin{bmatrix} 0&0&0 \\ 0&0&0 \\ 0&0&0 \end{bmatrix}, \quad M_1 = \begin{bmatrix} 1&0&0 \\ 0&1&0 \\ 0&0&1 \end{bmatrix}.$$
$$L_2 = \begin{bmatrix} 0&0&0 \\ -1&1&0 \\ -1&0&1 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 1&0&0 \\ 0&0&0 \\ 0&0&0 \end{bmatrix}.$$
$$L_3 = \begin{bmatrix} 0&0&0 \\ -1&1&0 \\ 0&-1&1 \end{bmatrix}, \quad M_3 = \begin{bmatrix} 1&0&0 \\ 0&0&0 \\ 0&0&0 \end{bmatrix}.$$
$$L_4 = \begin{bmatrix} 0&0&0&0 \\ -1&1&0&0 \\ 0&-1&1&0 \\ 0&0&-1&1 \end{bmatrix}, \quad M_4 = \begin{bmatrix} 1&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \end{bmatrix}.$$
$$L_5 = \begin{bmatrix} 0&0&0&0&0 \\ -1&1&0&0&0 \\ 0&-1&1&0&0 \\ 0&0&-1&1&0 \\ 0&0&0&-1&1 \end{bmatrix}, M_5 = \begin{bmatrix} 1&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \end{bmatrix}.$$

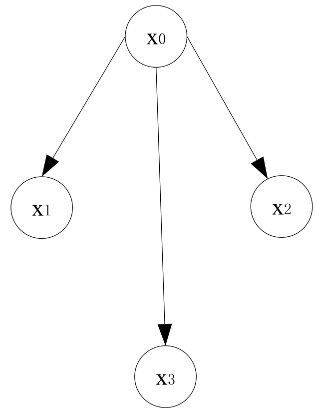

FIGURE 1. Topology of the MAS_1.

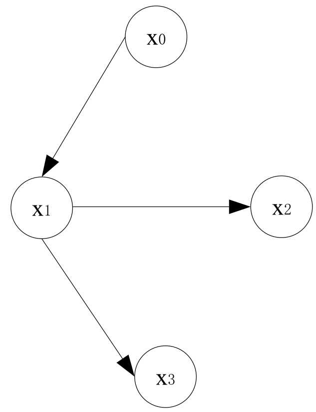

FIGURE 2. Topology of the MAS_2.

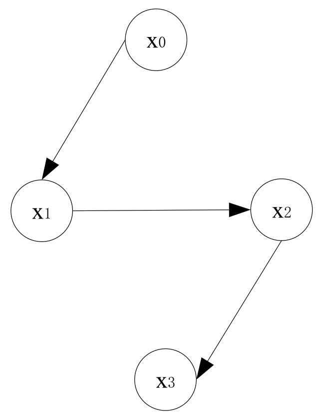

FIGURE 3. Topology of the MAS_3.

A. DISCUSSION ON THE RELATIONSHIP BETWEEN γ^* AND δ

Firstly, we illustrate the relationship between the optimization $\mathcal{L}_2 - \mathcal{L}_\infty$ attenuation level γ obtained based on **Algorithm 1** and the time delay δ for multi-agent systems with different topological structures.

Here, we incrementally increase the sampling period, δ, from 0.01 until Algorithm 1 becomes unsolvable, and

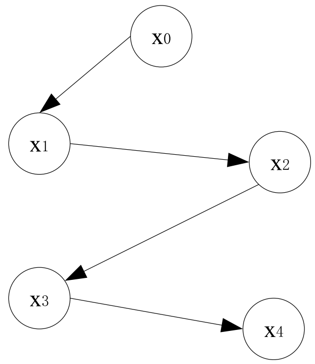

FIGURE 4. Topology of the MAS_4.

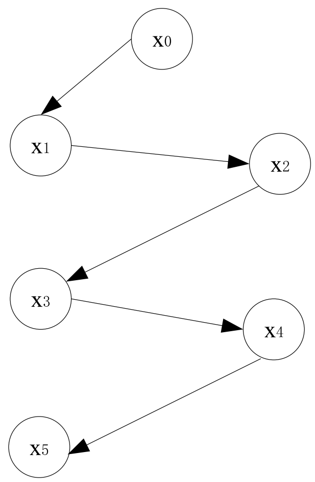

FIGURE 5. Topology of the MAS_5.

calculate the optimal attenuation level γ^*, which is illustrated in Fig.6

Remark 2: As we know, time-delay affects the stability and performance of systems. From Fig.6, it can be seen that when the time-delay reaches a certain level, the system performance sharply decreases.

Remark 3: From Fig.6, the comparative analysis reveals that, under identical conditions for other parameters, MAS_1 exhibits the minimum $\mathcal{L}_2 - \mathcal{L}_\infty$ attenuation ratio γ^*, while MAS_5 demonstrates the maximum. Conversely, the maximum allowable time delay δ^*, i.e., sampling period δ, is observed for MAS_1 and the minimum for MAS_5, as detailed in Table 1.

This phenomenon arises from the fact that, in the context of consensus control aligned with the leader, MAS_1's three follower agents are directly connected to the leader, enabling them to receive the leader's state information directly, enhancing their control decision effectiveness. In contrast, MAS_2 has only one follower, x_1, directly obtaining the leader's information, while other followers receive second-hand information. Also, MAS_3, MAS_4 and MAS_5 experience instances where some followers receive leader information second-hand or even third-hand, resulting in these MASs having less effective information.

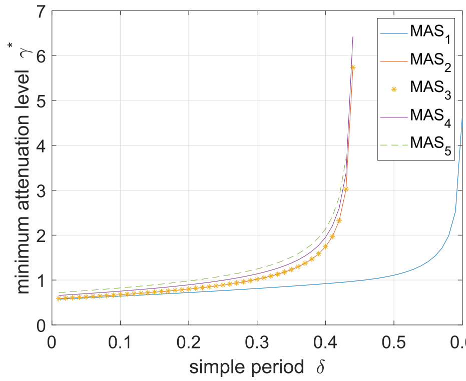

FIGURE 6. Minimum attenuation level γ^* with sample period δ for various MASs.

TABLE 1. The maximum allowed delays δ^* and corresponding γ^* for various MASs.

MAS	MAS_1	MAS_2	MAS_3	MAS_4	MAS_5
δ^*	0.6000	0.4400	0.4400	0.4400	0.4300
γ^*	4.6635	5.7142	5.7385	6.4238	3.7074

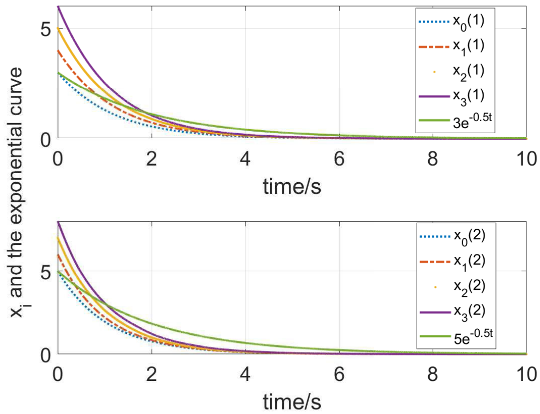

FIGURE 7. System states x_i and the exponential curve $y = [3, 5]^T e^{-\frac{\beta}{2}t}$.

It should be pointed out that although the curves for MAS_2 and MAS_3, line types of which are asterisk line and solid line, closely align on the Fig.6, their distinctions can be found in Table 1.

Especially, comparing MAS_3, MAS_4, and MAS_5, we observe similar topological structures. It can be asserted that by maintaining a topology similar to MAS_3, MAS_4, and MAS_5 and increasing the number of agents, the allowable time delay δ^* is expected to decrease, while the $\mathcal{L}_2 - \mathcal{L}_\infty$ attenuation ratio is anticipated to increase.

B. VERIFICATION OF EXPONENTIAL CONSENSUS

Without loss of generality, we choose MAS_3 for simulation. Setting $\delta = 0.05$, by Theorem 2, the $\mathcal{L}_2 - \mathcal{L}_\infty$ controller gain matrix is obtained as

$$K = \begin{bmatrix} 0.0225 & -0.1059 \\ -0.1217 & 0.5843 \end{bmatrix},$$

and the minimum attenuation level $\gamma^* = 0.6244$.

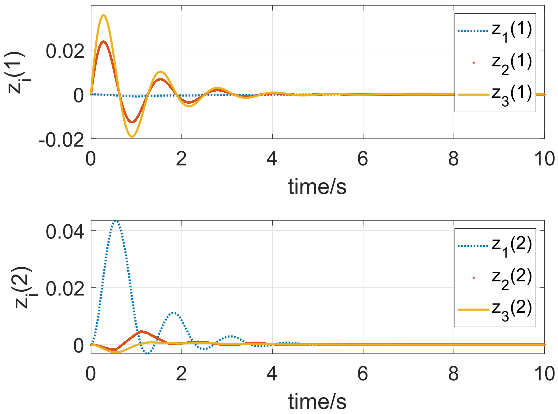

FIGURE 8. The controlled output z of the follower agents under the disturbance $\omega(t)$.

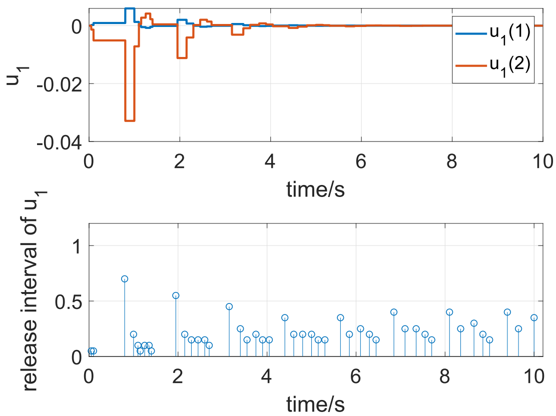

FIGURE 9. Control input, event-triggered instants and release intervals of u_1.

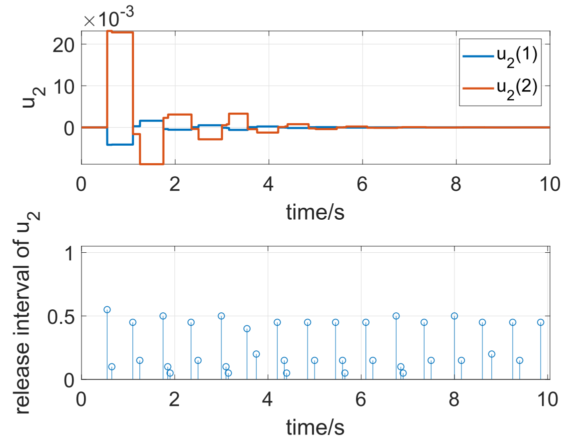

FIGURE 10. Control input, event-triggered instants and release intervals of u_2.

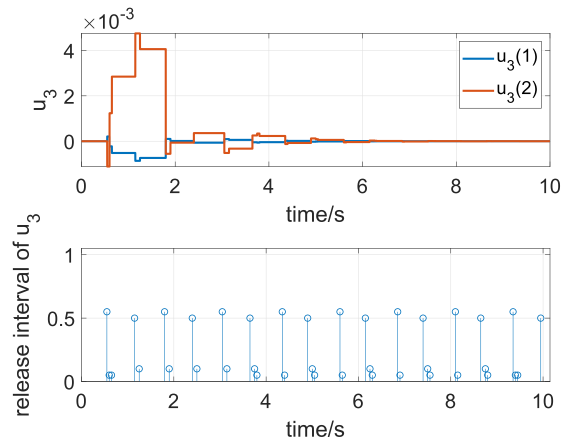

FIGURE 11. Control input, event-triggered instants and release intervals of u_3.

Now, we verify the exponential stability of the system. For clarity, we choose the initial state of the system as $x_0 = [3, 5]^T$, $x_1 = [4, 6]^T$, $x_2 = [5, 7]^T$, $x_3 = [6, 8]^T$. Meanwhile, as a comparison, we have added an exponential curve that conforms to the equation $y = [3, 5]^T e^{-\frac{\beta t}{2}}$ to the Figure. From the Fig.7, it can be seen that the convergence speed of system states meets the requirements of the specified convergence rate $\beta/2 = 0.5$.

C. VERIFICATION OF $\mathcal{L}_2 - \mathcal{L}_\infty$ PERFORMANCE

Next, the $\mathcal{L}_2 - \mathcal{L}_\infty$ performance index is verified. The disturbance input $\omega(t)$ is assumed to be

$$\omega_0(t) = \begin{bmatrix} e^{-t}\sin(5t) \\ e^{-t}\cos(5t) \end{bmatrix}, \quad \omega_1(t) = \begin{bmatrix} -e^{-t}\sin(5t) \\ e^{-t}\cos(5t) \end{bmatrix},$$
$$\omega_2(t) = \begin{bmatrix} e^{-t}\sin(5t) \\ -e^{-t}\cos(5t) \end{bmatrix}, \quad \omega_3(t) = \begin{bmatrix} e^{-t}\sin(5t) \\ -2e^{-t}\cos(5t) \end{bmatrix},$$

The controlled output $z(t)$ of the follower agents are drawn in Fig.8. At the same time, Fig. 9, Fig. 10 and Fig. 11 illustrate the event-triggered instants and release intervals for the three follower agents. Noting that the sampling period, $\delta = 0.05$, obviously, the frequency of controller changes is much lower than that of the sampling.

With zero-initial condition, we calculate $\|z(t)\|_\infty$ and $\gamma\|\omega\|_2$.

$$\|z(t)\|_\infty = 0.0517 < \gamma\|\omega\|_2 = 0.6244 * 1.2241 = 0.7643$$

which implies that system composed of (1), (2) and (3) with parameters setting above satisfies the $\mathcal{L}_2 - \mathcal{L}_\infty$ performance level $\gamma = 0.6244$.

V. CONCLUSION

This paper addressed the $\mathcal{L}_2 - \mathcal{L}_\infty$ exponential consensus control problem for multi-agent systems, employing an event-triggered strategy. The establishment of an error system allowed for the derivation of a sufficient condition ensuring the exponential stability of the error system while meeting the $\mathcal{L}_2 - \mathcal{L}_\infty$ performance index. The subsequent design of the $\mathcal{L}_2 - \mathcal{L}_\infty$ controller based on this condition provided a practical method for implementation. Through the numerical example, the investigation explored the correlation between $\mathcal{L}_2 - \mathcal{L}_\infty$ performance index and the maximum sampling period under diverse topology structures. The empirical results not only validated the effectiveness of the proposed theoretical framework but also offered insights into the interplay between control performance and sampling intervals in real-world applications of MASs. In future, I will apply the relevant results proposed in this paper to the Markovian jump systems. Then, I will compare the results with distributed optimal consensus of multi-agent systems with Markovian switching topologies: synchronous and asynchronous communications. Of course, dynamic event-triggered strategy is also my main research focus in future.

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