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TOPICAL REVIEW

Beyond Bits: A Review of Quantum Embedding Techniques for Efficient Information Processing

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ABSTRACT The existing body of research on quantum embedding techniques is not only confined in scope but also lacks a comprehensive understanding of the intricacies of the quantum embedding process. To address this critical issue, this article explores quantum encoding schemes, uncovering valuable insights into their encoding algorithms from theoretical foundations to a mathematical perspective, as well as practical applications. Initially, the article briefly overviews classical computing and the limitations associated with classical bits in representing and processing complex information. Next, the article scrutinizes a variety of quantum embedding patterns, including basis encoding, amplitude encoding, Qsample encoding, angle encoding, quantum associative memory encoding, quantum random access memory, superdense encoding, Hamiltonian encoding, and others. In addition, each technique is accompanied by mathematical formulas and examples illustrating how each strategy can be applied. Finally, the article provides a comparative analysis of different quantum embedding/encoding methods, outlining their strengths and limitations. Overall, this insightful article highlights the potential of quantum encoding techniques for efficient information processing beyond classical bits, thereby facilitating scientists and design engineers in selecting the most appropriate encoding technique to develop smart algorithms for revolutionizing the field of quantum computing.

INDEX TERMS Encoding patterns, qubits, quantum computing, quantum information processing, quantum circuits.

I. INTRODUCTION

In today's digital world, one of the primary aims of information theory is to encode information for quantification, storage, or transmission. In the context of classical information, a bit has been the fundamental unit for information science for decades. However, the advent of quantum computing (QC) has brought about a paradigm shift in how we approach data processing and storage [1], [2]. This is because traditional classical computing (CC) operates on the manipulation of binary bits, which are confined to taking on values of either 0 or 1. Each bit in CC acts independently, representing the most basic form of data. In stark contrast, quantum computing (QC) leverages the capabilities of quantum bits, or qubits. Unlike classical bits, qubits possess the unique ability to exist not only in the states of 0 and 1 but also in a superposition of these states. This phenomenon allows a single qubit to represent multiple values simultaneously [3], [4]. Fig. 1 vividly illustrates this fundamental divergence.

Moreover, QC utilizes another quantum property known as entanglement, where the state of one qubit is intrinsically connected to the state of another, irrespective of distance [5], [6]. Entanglement between qubits, a profound form of quantum correlation, enables operations to be executed on multiple qubits simultaneously. The correlations between entangled particles surpass any classical correlation, underpinning the power of QC [7]. This capability facilitates an exponential speedup in processing power for certain types of computations [8], [9]. Thereby, superposition and entanglement are potent features of QC that make it vastly different from CC. These unique properties enable quantum computers to perform certain tasks or calculations much faster and

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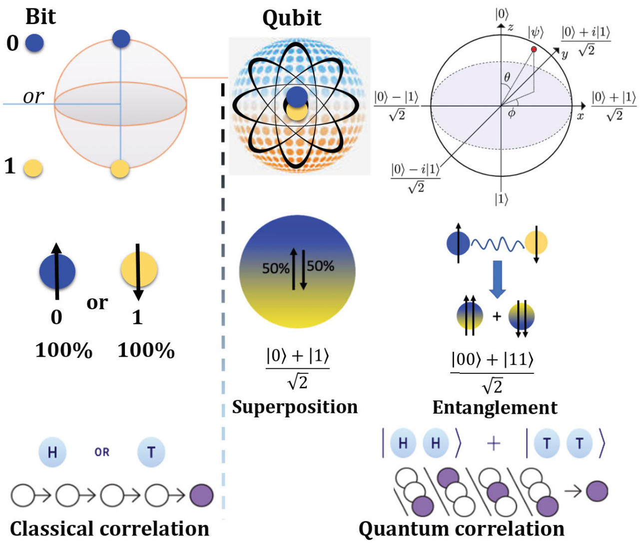


FIGURE 1. A diagrammatic portrayal showcasing the juxtaposition of classical and quantum bit designs.

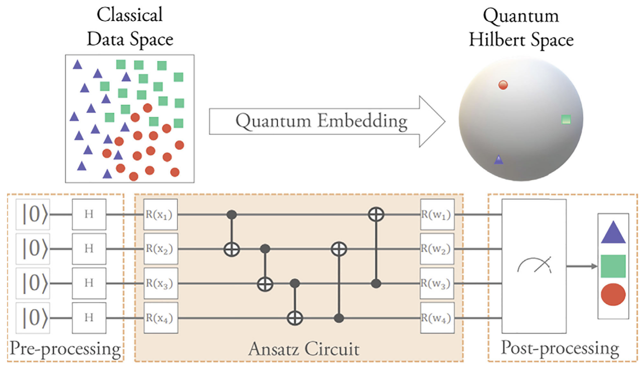


FIGURE 2. Schematic of classical data embedding into quantum Hilbert space [38].

more effectively than classical computers, a phenomenon referred to as "Quantum Supremacy" [10], [11]. QC gets empowered by exploiting the merits of superposition, quantum entanglement, and interference, among others, enabling it to solve specific problems faster than CC by applying various quantum algorithms, such as those developed by Shor and Grover, and proof-of-principle demonstrations of quantum computational advantage by entities like IBM, Google, Xanadu, etc., [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22]. Moreover, quantum communication protocols that rely on entangled states, e.g., Quantum Key Distribution (QKD), transfer encrypted data as classical bits through networks, while the keys to decode the information are transmitted and encoded in a quantum state using qubits [23], [24]. In 2016, China launched the world's first quantum communication satellite-to-ground entanglement-based QKD [25], [26]. The "Micius" satellite was designed to establish a secure and unhackable communication channel between Beijing and Vienna using QKD. Quantum Xchange also plans to launch about 500 miles of fiber optic cable to create "Phio" the first-of-its-kind commercial QKD network in the U.S [27], [28]. Furthermore, researchers at Toshiba have successfully transmitted quantum information using QKD over a 600-kilometer optical fiber network [29], [30].

The juxtaposition shown in Fig. 1 between the linear, singular state of classical bits and the multifaceted, superposed states of qubits encapsulates the transformative potential of QC in handling more intricate and voluminous data representations. Nevertheless, quantum information theory delves into the representation and manipulation of quantum variables or states (qubits) and harnesses the inherent advantages of quantum mechanics for communication, computation, and cryptography [31], [32]. Quantum computers require embedding or encoding techniques for the following crucial reasons:

1) **No-cloning theorem**: In QC, quantum objects—qubits cannot be copied because of quantum mechanics [33], [34].
2) **Quantum version of random access memory (QRAM)**: The way QC handles data is fundamentally different from CC. Current first-generation quantum computers, for instance, Noisy Intermediate-Scale Quantum (NISQ) devices do not have access to a database primarily because of their hardware limitations. There is no direct concept of RAM or storage in the traditional sense for quantum computers [35], [36]. Therefore, an additional step called state preparation is required for initializing a quantum register and preparing the qubits in a desired initial state before performing computations or applying quantum circuits/gates [37], as depicted in Fig. 2 and Fig. 3. Consequently, data has to be loaded by encoding into the state of the qubits before it can be used by quantum computers.
3) **Enhanced computational power**: Quantum embedding allows for representing complex information using quantum states, leveraging the unique properties of quantum mechanics such as superposition and entanglement [3], [18].
4) **Quantum algorithms**: Quantum embedding enables the implementation of quantum algorithms. Algorithms, such as Shor's algorithm [56] for integer factorization and Grover's algorithm for searching [57], rely on quantum embedding to realize their potential [58].
5) **Quantum communication**: Quantum encoding techniques are crucial for quantum communication protocols like quantum teleportation [26], [34], quantum secure direct communication (QSDC) [59], and QKD [60], [61], [62]. By encoding information in quantum states, secure and efficient communication channels can be established, ensuring the transmission of information with quantum-level security and fidelity [63], [64], [65].
6) **Quantum simulations and data models**: QC holds the potential for polynomial or exponential speed-up in solving specific problems compared to CC. However, practical implementation faces several fundamental technical challenges. Among these challenges is the

TABLE 1. Related prominent surveys on quantum encoding techniques.

Year	Authors	Contributions
2023	M. Zajac et al., [39]	Addresses the methodologies in encoding and managing data models for quantum computing
2023	B. Bhattaraprot et al., [40]	Hybrid quantum encoding: combining amplitude and basis encoding for enhanced data storage and processing in quantum computing
2022	S. Ashhab [41]	Quantum state preparation protocol for encoding classical data into the amplitudes of a quantum information processing
2022	N. Mahmud et al., [42]	Efficient data encoding and decoding for quantum computing
2022	M. Beisel et al., [43]	Encoding patterns specific to quantum error handling
2021	A. Gilliam et al., [44]	Foundational patterns for efficient quantum computing
2021 2020	M. Weigold et al., [45] M. Weigold et al., [46]	Overview of data encoding patterns A brief survey on encoding patterns for quantum algorithms
2021	Y. Shee et al., [47]	Qubit-efficient encoding scheme for quantum simulations of electronic structure
2020	S. Lloyd et al., [48] I. Gianani et al., [49]	Quantum embeddings for machine learning Experimental quantum embeddings for machine learning
2020	R. LaRose et al., [50]	Study on data encodings for binary quantum classification
2019	A. Ferraro et al., [51]	Promising ways to encode and manipulate quantum information
2019	D.K. Park et al., [52]	Circuit-based quantum random access memory for classical data
2018	J. Cortese et al., [53]	Loading classical data into a quantum computer
2017	J. Romero et al., [54]	Quantum autoencoders for efficient compression of quantum data
2007	E. Noam et al., [55]	Optimal encoding of classical information in a quantum medium

loading of data into quantum computers and the associated encoding, given their inability to directly access database systems. Quantum embedding allows for the simulation of complex quantum systems and phenomena [39], [66]. By encoding the relevant information into quantum states—making it accessible in a suitable format—researchers can study quantum processes, diverse data models/structures, and properties that are difficult or infeasible to simulate using CC methods.

Thus, to fully leverage the potential of QC, it is essential to understand input data processing for developing novel encoding techniques that allow us to efficiently represent and manipulate information using qubits. The reason is that the choice of an encoding technique has a significant impact on the accuracy and efficiency of quantum data processing.

In the corpus of extant literature, discussions pertaining to quantum embedding techniques (referenced in Table 1) frequently exhibit a paucity of comprehensive insights, are scattered among various publications with divergent objectives, or are limited to specific encoding paradigms tailored for particular applications. Notably, the works of Zajac et al. [39], Bhattaraprot and Smanchat [40], and Gilliam et al. [44] elucidate foundational methodologies in the realms of encoding and hybrid quantum encoding strategies. These investigations, however, mainly focus on basis and amplitude encoding, thereby omitting an expansive survey of alternative data encoding methodologies. Similarly, the studies delineated in [50] and [67] are centered on the utilization of angle encoding techniques for quantum classifiers. Furthermore, a broader review of data encoding methods, particularly within the quantum machine learning (QML) domain, can be found in [38], [41], [49], and [68], whereas the research presented in [42] and [43] delve into encoding modalities relevant to quantum error correction and NISQ systems. Although the contributions of Weigold et al. [45], [46] and the research collectives under Lloyd et al. [48] have significantly enriched our understanding of data encoding patterns, comprehensive analyses addressing runtime complexity or scalability of all pertinent encoding patterns remain sparse.

Despite the substantial advancements each cited study within Table 1 offers to the QC discourse, it invariably manifests certain notable gaps along with its own set of limitations. These range from a lack of providing a detailed overview of quantum predominant encoding techniques to an incomplete scrutiny of the mathematical frameworks, requisite qubit allocations, computational efficiency, and potential practical applications. In contrast, our survey addresses these gaps by furnishing an in-depth comparative analysis of the most prominent encoding schemes, with a particular emphasis on their mathematical formulations, qubit requirements, runtime complexities, and applicability in practical scenarios, which have hitherto been inadequately explored in the prevailing literature. Beyond mere aggregation of extant knowledge, our work aspires to contribute critical evaluations and delineate prospective avenues for future inquiry within the domain of quantum encoding.

In summary, this paper provides a panoramic overview of various quantum encoding patterns, encompassing basis encoding, amplitude encoding, Qsample encoding, angle encoding, quantum associative memory encoding, Hamiltonian encoding, quantum random access memory, superdense encoding, and others. We delve into the potential applications of each technique, along with numerical equations and examples to reinforce our findings. This article's primary contributions are as follows:

i. Focus on how to process information in QC—data embedding.
ii. Provide a comparative analysis of encoding schemes to highlight their merits and demerits in terms of the number of qubits required and runtime complexity.
iii. Facilitate scientists and design engineers in selecting the best-suited encoding strategy for their specific needs.

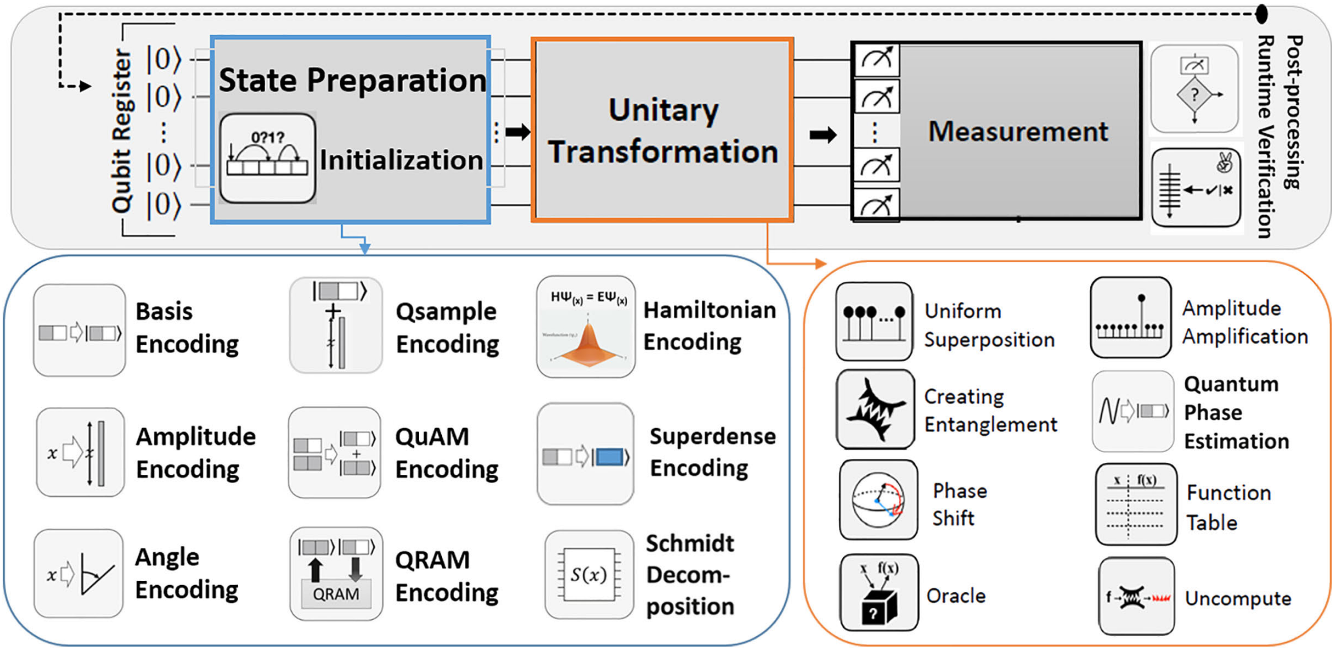


FIGURE 3. Steps to execute quantum algorithms including non-trivial initialization (state preparation) via data encoding or embedding strategies.

The remainder of this paper is organized as follows: Section II describes the fundamentals of quantum embedding. Section III provides an in-depth overview of key encoding patterns for quantum algorithms, in conjunction with comparative analysis. Sections IV and V present the challenges and future research directions, and paper conclusion, respectively.

II. BACKGROUND

In classical information processing, techniques are generally categorized into analog or digital depending on how data is represented. Typically, a binary digital system is used in CC, where information is represented by a bit, which can be either 0 or 1, also known as a 2-bit system. These bits are then processed using classical logic gates, such as AND and OR gates, to perform arithmetic operations. For instance, 8085 and 8086-based microprocessors (1976-1979) launched by Intel using NMOS technology store/process the data in the form of 8-bits and 16-bits, respectively [69]. Contrary to that, quantum information processing (QIP) relies on the principles of quantum mechanics to encode information in quantum states [32]. A qubit is the basic unit of information that can be either in a state of 0, 1, or a superposition of both states, illustrated by a linear combination of 0 and 1 (Fig. 1).

Furthermore, qubits can be entangled, meaning they are ubiquitously non-separable in the sense that the state of one qubit is dependent on the state of the other. These two peculiar properties of qubits enable QC to represent complex data sets and perform certain calculations exponentially faster than classical computers in specific problem domains. This capability is one of the key reasons that QIP is emerging as a field of research that has the potential to transform the way we perform data computing and communication tasks [14], [70]. Nonetheless, to fully realize this significant impact, it is imperative to recognize that the direct utilization of classical data in QIP is not viable. This stems from the fact that quantum computers operate under fundamentally different principles from CC. This distinction necessitates a specific approach to data loading or handling. Consequently, QC demands the transformation of classical data into a quantum-compatible format. This transformation ensures that quantum computers are compatible with the data, adhering to quantum principles like superposition and entanglement. Additionally, quantum experiments, which abide by quantum laws may still yield noisy data [71]. Therefore, embedding techniques are required to refine the data and convert it into a format suitable for processing by quantum algorithms.

Quantum embedding, also known as quantum encoding, is a technique used in QIP to represent classical information in the quantum state of qubits. More specifically, classical data is transformed into quantum states through a process called a quantum feature map, which is a mathematical function that maps classical data into a Hilbert space [72], [73]. Quantum feature map takes classical data as input and applies a series of quantum operations to project it to a quantum state, as depicted in Figs. 2 and 3. These operations are typically implemented using quantum gates and are designed to preserve certain properties of classical data. For example, Fig. 2 illustrates a QML framework mapping differently shaped and colored data points, each representing unique categories or features from the classical domain into the quantum Hilbert space. This transformation is executed through a quantum circuit that includes two main stages: (i) the pre-processing phase, which prepares/initializes the quantum system in a standard state, typically $|0\rangle$, and applies a Hadamard gate (H) to each qubit to generate a superposition, and (ii) an ansatz circuit that processes the

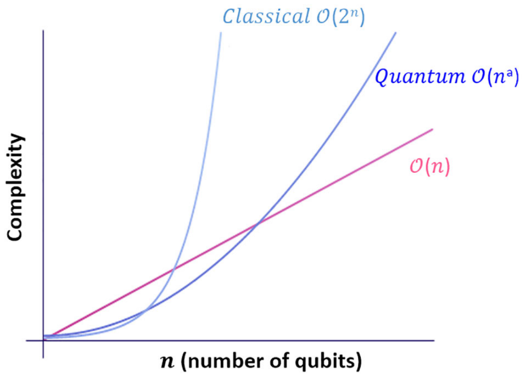


FIGURE 4. Expected scaling in simulating a quantum system blue [74].

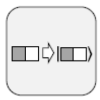
quantum states using data-parameterized rotations $R(x_i)$ and entanglement (via controlled-NOT gates) to delineate distinct regions in the Hilbert space corresponding to each data category [38]. As the central component of the QML model, the ansatz circuit's weights w are tunable parameters that are optimized during the learning process. After computation, the qubit states are measured, and the results are post-processed to assign a category to the new data points, effectively performing classification.

In the context of QC, loading data is a complex and nontrivial process. This complexity arises because there are different ways to encode or represent data, and the choice of data encoding depends on the specific requirements of a quantum algorithm's unitary transformation. Similar to how the ansatz circuit forms the core of QML models, the unitary transformation is the computational heart of quantum algorithms. Key operations that constitute unitary transformations are detailed in Fig. 3. Prior to that, the way that data is initially represented or "loaded" into the quantum system can significantly impact the efficiency and effectiveness of a quantum algorithm. In measuring the general speed of an algorithm, the computational complexity theory primarily focuses on the asymptotic complexity (mostly Big-O notation), which indicates the rate of growth of the runtime with the input size n [75], [76]. Although numerous encoding techniques can be used to represent information in a quantum system, the development of quantum algorithms theoretically aims to achieve polynomial or even exponential speed-ups over their classical counterparts [77], [78]. This potential is reflected in the quantum complexity curve depicted in Fig. 4, which illustrates less steep growth for linear $O(n)$ and polynomial $O(n^a)$ curves compared to the exponential $O(2^n)$ curve associated with certain CC. For example, Shor's algorithm is a quantum algorithm designed for factoring an integer P with a complexity of $O\left((\log P)^3\right)$, indicating polynomial time complexity. This enables it to potentially break RSA encryption in polynomial time [12], [79]. It is generally assumed that for algorithms offering significant speed-ups, the data loading phase should only take logarithmic or linear time [46], [80], [81]. Therefore, when applying an encoding scheme, it is necessary to weigh the trade-offs between (i) the number of qubits required, (ii) the nature of the data itself, (iii) circuit depth (i.e., the number of quantum gates required in state preparation), and (iv) the runtime complexity for the loading process. Consequently, the choice of an encoding technique has a significant impact on the accuracy and efficiency of quantum data processing within the decoherence time.

III. QUANTUM ENCODING TECHNIQUES

To tap into the power of QC, it is essential to transform classical data into a quantum format using specialized encoding methods. These advanced methods are designed to ensure compatibility with the unique and potent capabilities of QC, notably in efficiently handling complex calculations through superposition and entanglement. The encoding techniques employed in QC are diverse and complex, serving more than mere technical requirements. They are pivotal in bridging the gap between classical data formats and the quantum realm, thus unlocking the full spectrum of quantum computing's potential. In this section, we discuss those encoding patterns that are prominent and potentially more attractive for quantum system applications.

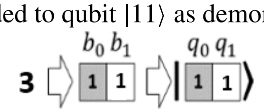
A. BASIS ENCODING



Basis encoding or computational basis encoding is the simplest quantum encoding technique as it involves the direct mapping of classical bits to qubits. In general, the mathematical form is:

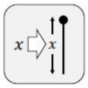
$$X \approx \sum_{i=-k}^{m} b_i 2^i \rightarrow |b_m \dots b_{-k}\rangle, \tag{1}$$

where numerical input data X is approximated by a binary bit string $(b_m \dots b_{-k})$ with a precision of k decimal places or $m+k$ significant digits [46]. In essence, an input number x is first approximated by a binary format $x := b_{n-1} \dots b_1 b_0$, which is then mapped directly to the corresponding quantum computational basis vector $|x\rangle := |b_{n-1} \dots b_1 b_0\rangle$. For example, a computational basis state of an n-qubit system such as $|3\rangle = |0011\rangle)$ is correlated with a classical n-bit-string (0011). In other words, the real number "3" in classical bits '11' is encoded to qubit $|11\rangle$ as demonstrated below:



This entails that for input numbers approximated by n-digits, n-qubits are required for their representation. To achieve this encoding, the initial $|0\rangle$ state of qubits that represent a '1' digit must be flipped into $|1\rangle$. For a single qubit, this transformation can be accomplished with a single operation, allowing this encoding to be prepared in linear time. In a sense, this represents the most direct form of computation, where each bit is essentially replaced by a qubit, enabling a 'computation' to operate in parallel on all bit sequences in a superposition [82]. This technique is categorized as digital encoding since it is beneficial for arithmetic computations [83]. However, this encoding, while simple, may be inefficient for large datasets since each classical bit requires a separate qubit, resulting in $n = m + k$, qubits required, as per equation (1).

B. AMPLITUDE ENCODING



Amplitude encoding is achieved by assigning different amplitude values to each classical bit. It encodes a classical input vector X of length N onto the amplitudes of an n-qubit state with $n = \log_2 (N)$. Typical mathematical notations are:

$$|\psi_x\rangle = \sum_{i=1}^{N} x_i |i\rangle, \quad or$$
$$X \rightarrow |\psi_x\rangle = \sum_{i=0}^{N-1} x_i |i\rangle, \tag{2}$$

where $|\psi_x\rangle$ denotes the n-qubit quantum state and corresponds to a normalized classical N-dimensional data point, x_i is the i^{th} item of x, and $|i\rangle$ is the i^{th} computational basis state in the Hilbert space. For instance, to encode the classical vector $x = (0, 0, 2, 3)$, first, the input vector needs to be normalized to a length of numeric '1' [3]. That is, according to the *Born rule* [5], the squared moduli of the N amplitudes of a quantum state must sum up to 1. It is similar to the basic concept of qubit states, where α and β are complex numbers and represent the amplitudes for 0 and 1 states as:

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle\,, \tag{3}$$

where $|\alpha|^2 + |\beta|^2 = 1$, i.e., the total of the squares of the amplitudes of all possible states in a superposition equals 1. Thus, the normalization factor:

$$x_{NF} = \sqrt{0^2 + 0^2 + 2^2 + 3^2} = \sqrt{13}, \tag{4}$$

and the resulting quantum mapping will be:

$$|\psi_{norm}\rangle = \frac{1}{\sqrt{13}}[0|00\rangle + 0|01\rangle + 2|10\rangle + 3|11\rangle)]. \tag{5}$$

Using the IBM and Qiskit SDK, the quantum circuit and other statistics of encoded data are illustrated in Fig. 5.

Here, it is pertinent to note that 4-dimensional data points are encoded into 2 qubits, i.e., multiple classical bits as input of N features can be encoded into $n = \log_2 N$ qubits. This is indeed a more efficient and compact representation (in terms of qubits) than basis, angle, or QRAM encodings. However, to correlate each amplitude with a component of the input vector, the dimension of the vector must be equal to a power of two; otherwise, shorter vectors must be padded with zeros to attain the dimension of 2^n.

Amplitude encoding is used in some quantum teleportation, and QKD protocols, e.g., the BB84, quantum prover authentication protocols, and many other QML algorithms, such as the Harrow, Hassidim, and Lloyd (HHL) algorithm for solving linear equations [84], and Quantum K-Nearest

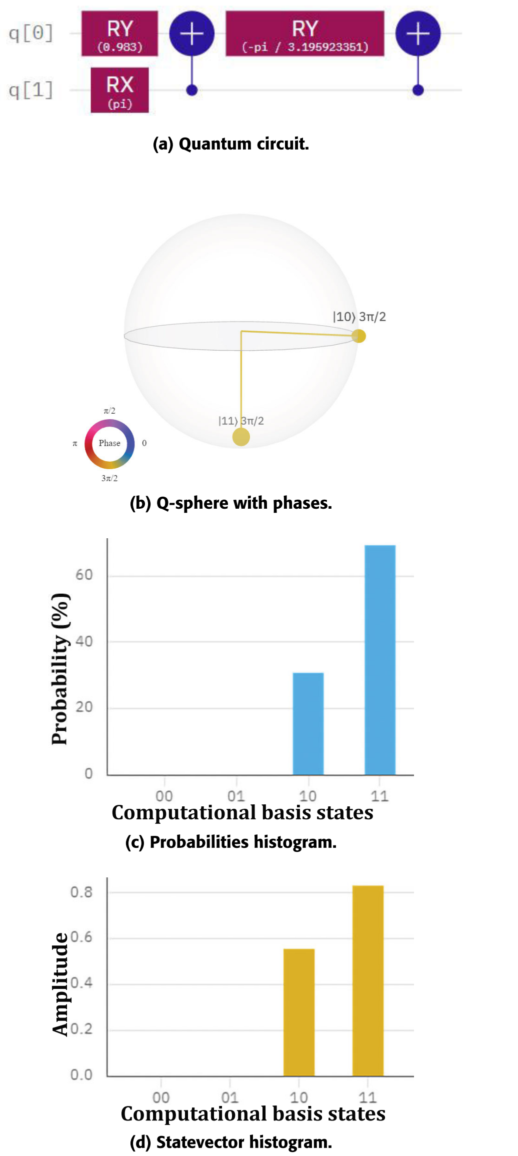


FIGURE 5. Quantum information mapping and analysis: classical data (0,0,2,3) mapping into qubits using amplitude encoding.

Neighbor (QKNN) algorithms [50], [85]. However, amplitude encoding also has some limitations; the number of measurement scales with the number of amplitudes as n-qubits contain 2^n amplitudes, and this is costly. Moreover, it is also vulnerable to certain types of noise, such as dephasing noise [3], [38].

C. QSAMPLE ENCODING

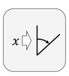
Qsample encoding is a hybrid approach as it combines features of both amplitude and basis encodings. This method establishes a connection between the probability distribution of a discrete random variable and the former quantum state form. In other words, it associates a classical discrete probability distribution $(p_1, \ldots, p_N)$ with the real amplitude vector

$v = (v_1, \ldots, v_{2^n})^T$, where $(v)^T$ is a transpose operation depicting the n-qubit quantum state [82], [86]. Essentially, in Qsample embedding, basis encoding resembles an empirical distribution function, with the amplitudes serving as data weights, akin to empirical probabilities. For example, in this encoding, the N features are encoded in the qubits while the information of interest is conveyed through the amplitudes. It combines a real amplitude vector with conventional binary probability distributions as:

$$|\psi\rangle = \sum_{i=1}^{2^n} \sqrt{p_i}|i\rangle, \tag{6}$$

where $|i\rangle$ denotes the computational basis state, and p_i represents the discrete probability associated with each state $|i\rangle$, which, when squared ($\sqrt{p_i} = v_i$), gives the amplitude for that state. This equation shows how a classical probability distribution can be mapped onto the amplitudes of a quantum state, with 2^n being the total number of computational basis states for an n-qubit system [87]. In this way, any discrete random variable can be represented solely by indexing its events. The hybrid scheme is advantageous because it allows data to be encoded both in qubits and amplitudes; the 'amplitudes' represent the information we are interested in, while the 'qubits' encode the N features. This dual approach makes Qsample encoding particularly useful in probabilistic QML models and quantum Boltzmann machines (QBMs) [88], [89]. In these applications, state preparation for a given probability distribution operates in the same manner, where a qubit-efficient algorithm is polynomial in the input, whereas an amplitude-efficient quantum algorithm is exponential in the input dimension N [82].

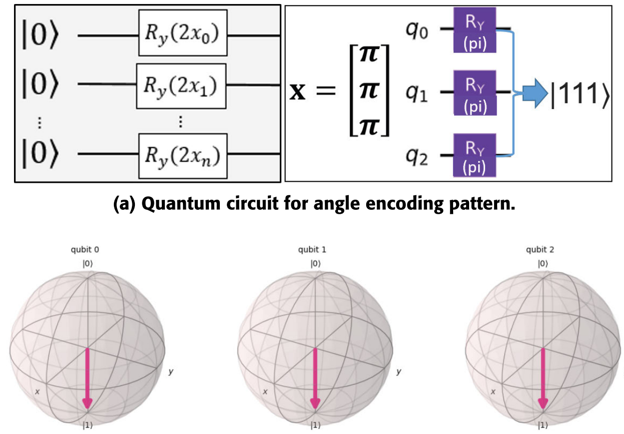
D. ANGLE OR ROTATION ENCODING



Angle encoding utilizes the phase or rotation property of a qubit to represent information. This encoding makes use of rotation gates to encode classical information. The general mathematical form is:

$$|x\rangle = \bigotimes_{i=1}^{n} R(x_i)\left|0^n\right\rangle, \text{ or}$$
$$|x\rangle = \bigotimes_{i=1}^{n} \left(\cos(x_i)|0\rangle + \sin(x_i)|1\rangle\right), \tag{7}$$

where $\bigotimes$ is the tensor product operation over n qubits, and R can be any Pauli gate (Rx, Ry, Rz) for x, y, and z-axes rotation, applied individually to each qubit to encode the feature x_i into the angle of rotation for that qubit. For example, in Fig. 6, the data point $\mathbf{x} = (\pi, \pi, \pi)$ can be encoded as $|111\rangle$. In this instance, we have also introduced an R_y gate, it is a single-qubit gate that rotates the qubit state around the y-axis of the Bloch sphere by a given angle. Consider a rotation operator gate $R_y(\theta)$: then θ-angle rotation around the y-axis



(a) Quantum circuit for angle encoding pattern.

(b) Bloch sphere of each qubit with its phase.

FIGURE 6. Angle or rotation embedding scheme.

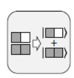
is expressed by:

$$\begin{aligned} &\mathrm{y} = \cos\left(\frac{\theta}{2}\right), \text{ or} \\ &\theta = 2\arccos(\mathrm{y}) = 2\cos^{-1}(0) = \pi \text{ radians}, \end{aligned} \tag{8}$$

where the R_y gate implements $\exp^{\left(-i\frac{\theta}{2}\mathrm{y}\right)}$ on the Bloch sphere, and causes the qubit state to be rotated by the specified angle around the y-axis, as illustrated in Fig. 6(b).

This technique encodes one data point at a time, rather than a whole dataset like basis or amplitude encoding. Thus, requires N qubits, i.e., 1 qubit/data point. However, another variant called *dense angle encoding* requires only half of the qubits to encode the same amount of data points. Angle or tensor product encoding is particularly useful for image processing (angle parameter of a qubit to store a color), i.e., to depict the color information of a pixel in the flexible representation for quantum images (FRQI), where the idea is to use different levels of angles for RGB information and the tensor product with location information (x-axis and y *or* z-axis) to represent an image [90]. This encoding method finds application in quantum neural networks (QNNs) as well as in the realm of data classification for QML models [50], [91]. Also, using its variant *arbitrary encoding* fosters the design of parameterized quantum circuits, where circuit parameters can be adjusted to optimize the performance or reduce the error rates of quantum circuits [43], [92].

E. QUANTUM ASSOCIATIVE MEMORY (QuAM) ENCODING



QuAM embedding utilizes superposition to encode a set of data points within a qubit register, aiming to prepare an equally weighted superposition of the basis-encoded values within that register. Specifically, the goal is to achieve a state where each data value, such as x_0, x_1, and x_2, is represented in both basis and amplitude encoding as a part of an equally weighted superposition, as exemplified in Fig. 7. The mathematical

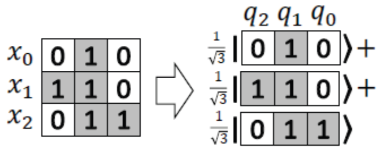


FIGURE 7. QuAM embedding scheme, each data value on the left is encoded using basis encoding with an amplitude of $(1/\sqrt{n})$.

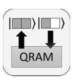
expression to represent this process is:

$$X \to \sum_{i=0}^{M-1} \frac{1}{\sqrt{M}} |x_i\rangle . \tag{9}$$

Equation (9) shows that, for an encoding of M data points, each state $|x_i\rangle$ is included in the superposition with an equal amplitude of $\frac{1}{\sqrt{M}}$ ensuring all states are equally likely when the quantum state is measured [46], [82].

Consider a set of M, N-dimensional data points. The quantum algorithm for addressing and storing this data requires a set of $2N + 1$ qubits. Thereby, the algorithm requires $O(MN)$ steps to encode the patterns as a quantum superposition over N qubits. Thus, with $2N + 1$ qubits, the QuAM can store up to $M = 2^N$ patterns in $O(MN)$ steps and requires $O(\sqrt{M})$ time to associatively recall the entire pattern [93]. Such QuAM feature facilitates exponential quantum capacity and faster track pattern recognition in next-generation High Energy Physics (HEP) experiments [94], [95]. Additionally, this imperative encoding is entailed by Grover's algorithm for unstructured search to yield a quadratic speedup, a quantum variant of the Fourier transform, and the famous Shor's algorithm for the factorization of prime numbers [12], [15], [96].

F. QUANTUM RANDOM ACCESS MEMORY (QRAM) ENCODING



While basis and angle encoding schemes embed N classical features on N qubits, amplitude encoding maps N classical features on $\log_2 N$ qubits. However, these methods are relatively basic and do not account for the complexities of the dataset. A QRAM-based data loading approach could potentially overcome this shortcoming [97]. The QRAM encoding logic is based on the classical RAM concept. It has the same three basic components as CC-RAM: a memory array, an address register, and an output register. A CC-RAM that receives an address with a memory index, loads the data stored at this address into an output register. QRAM offers the same functionality, but the address and output registers are composed of qubits (quantum registers) rather than CC bits [98], [99], as depicted in Fig. 8.

The major advantage of a QRAM is that both the address and the output register can be in a superposition of multiple values, allowing for access to a superposition of data values simultaneously [100]. For instance, in Fig. 7, given an input address register that is in a superposition of addresses, $|\psi_a\rangle = \frac{1}{\sqrt{2}}(|\,|\mathbf{00}\rangle + |\mathbf{01}\rangle)$, the QRAM creates a superposition of addresses and their data values, i.e., $|\psi_{ax}\rangle = \frac{1}{\sqrt{2}}(|\mathbf{00}\rangle|010\rangle + \frac{1}{\sqrt{2}}(|\mathbf{01}\rangle|110\rangle)$.

In general, the mathematical form is:

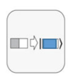
$$X \to \sum_{i=0}^{N-1} \frac{1}{\sqrt{N}} |i\rangle\, |x_i\rangle\, , or$$

$$\frac{1}{\sqrt{M}} \sum_{i=0}^{M-1} |a\rangle_i |0\rangle \xrightarrow{QRAM} \frac{1}{\sqrt{M}} \sum_{i=0}^{M-1} |a\rangle_i\, |x_a\rangle\, , \tag{10}$$

where M represents the superposition of all the addresses to be loaded in the address register, and N is the total number of data values. The term $|a\rangle_i$ represents the particular address of the i-th data value to be loaded, and $|x_a\rangle$ are the data values associated with that address. Thus, QRAM encoding requires $[\log_2 M]+ l$ qubits, i.e., $[\log_2 M]$ for the address register and additional l qubits to encode data values using basis encoding [46].

In other words, classical RAM uses N bits for the random addressing of $M = 2^N$ unique memory cells. Conversely, QRAM employs $\log_2 M$ qubits to address any quantum superposition of M memory cells, and crucially, it only requires $O(\log M)$ operations or switches/gates to be thrown to perform a memory call [98], [101]. This represents an exponential decrease in the power needed for addressing, thereby resulting in a significant improvement in efficiency and reducing the resources needed for memory access in QC systems. The computational characteristics of a QRAM closely resemble those of Basis and QuAM encoding schemes. As a result, it finds utility in comparable applications within the domain of QML and extends to other algorithms, e.g., quantum searching on a classical database, collision detection in cryptography, element distinctness, quantum oracle implementation, Quantum Support Vector Machine (QSVM), spin-photonic networks, telecommunications, etc. [97], [102], [103], [104], [105]. This is because the QRAM mechanism allows access to classically stored information in superposition by querying an index register, i.e., parallel processing of data (quantum parallelism) [52]. QRAM encoding is also suitable for solving linear equations in the HHL algorithm to process eigenvalues at the intermediate stage [106]. However, the practical limitations of producing larger QRAM products continue to be an open and difficult technological challenge for QC hardware manufacturers [58].

G. SUPERDENSE ENCODING



Qubits, as quantized units of quantum information, possess the remarkable ability to transmit and manipulate more information than classical bits [1], [107]. Superdense encoding addresses the pivotal question: *How much classical information can quantum states represent?* Contrary to encoding a single bit into a qubit, superdense encoding utilizes the principles of quantum entanglement and superposition to encode two classical bits across two entangled qubits [108], [109]. Specifically, the protocol uses the correlations between two qubits in a shared

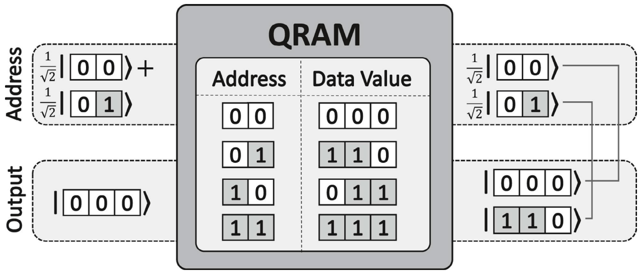


FIGURE 8. QRAM embedding functionality [46].

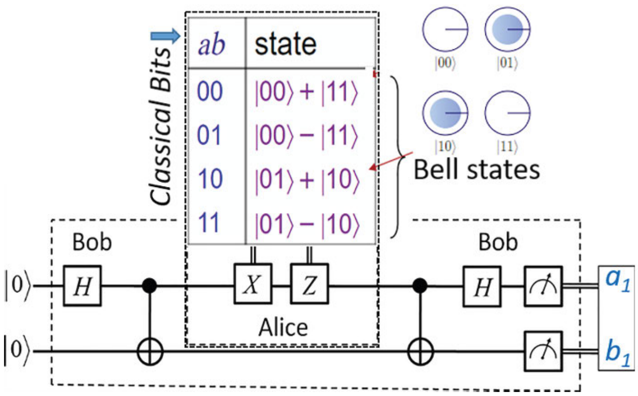


FIGURE 9. Superdense encoding principle.

entangled state to represent the information. In the superdense coding protocol, an entangled pair of qubits is prepared in one of the maximally entangled Bell states by a party, typically named Bob, as follows:

$$\text{(encoding: 00)} \Rightarrow |00\rangle \longrightarrow |\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle),$$
$$\text{(encoding: 01)} \Rightarrow |01\rangle \longrightarrow |\Phi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle),$$
$$\text{(encoding: 10)} \Rightarrow |10\rangle \longrightarrow |\Psi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle),$$
$$\text{(encoding: 11)} \Rightarrow |11\rangle \longrightarrow |\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle),$$

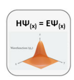
where $|\Phi^+\rangle$, $|\Phi^-\rangle$, $|\Psi^+\rangle$, and $|\Psi^-\rangle$ are the Bell states, which are maximally entangled states of two qubits.
For instance, Bob prepares an entangled pair of qubits and sends one to Alice while retaining the other. Alice encodes two classical bits, a and b, by performing specific quantum operations (Fig. 9) on her qubit based on the value of these bits:

If $(a, b) = (0, 0)$, Alice does nothing (identity operation).
If $(a, b) = (0, 1)$, Alice applies the Z gate.
If $(a, b) = (1, 0)$, Alice applies the X gate.
If $(a, b) = (1, 1)$, Alice applies both the X and Z gates.

After encoding, Alice sends her qubit back to Bob. Bob, now in possession of both qubits, can decode the two classical bits by performing a Bell state measurement [110]. While Alice encodes information by acting on a single qubit, it is essential to understand that both qubits are needed to retrieve the result, illustrating that quantum superdense coding enables a sender to encode a two-classical-bit message using pre-shared entanglement [111]. This process allows Bob to recover the exact state prepared by Alice, thereby retrieving the two classical bits.

Superdense encoding, leveraging the Bell states, has several applications in quantum secure communication and quantum cryptography [112], [113], [114]. In a quantum communication protocol, utilizing pre-shared quantum entanglement enhances the data transmission rate via a quantum channel, enabling the transmission of two classical bits using two entangled qubits, as opposed to classical transmissions [115], [116]. In other words, superdense encoding increases the maximum information rate to two bits per qubit, compared to one bit in classical communications, provided that Alice and Bob have access to entangled qubits. This means that the superdense encoding scheme uses the entangled state of the two qubits to represent four possible combinations (Fig. 9), thereby encoding two bits of information across the pair. When one of the entangled qubits is manipulated (by Alice, in the standard protocol), and then both qubits are measured together (by Bob), the result is the transmission of two classical bits. Furthermore, it is important to note that this is a distinct concept from quantum teleportation; while both rely on entanglement, superdense encoding transmits two classical bits using entangled qubits, whereas teleportation sends one qubit's state using two classical bits for communication [3], [117].

H. HAMILTONIAN EVOLUTION ANSATZ ENCODING



In previous embedding approaches, we encoded features explicitly into quantum states, but the Hamiltonian encoding strategy differentiates itself by encoding data into the dynamics of a quantum system, which is why it is also called dynamic encoding [82]. More specifically, rather than preparing a quantum state that contains features or a distribution in its mathematical description, the Hamiltonian scheme implicitly encodes the feature's information by allowing them to define the evolution of the quantum system. The data are used to construct a Hamiltonian (energy) operator, and then an initial state evolves under this Hamiltonian for a given time [3].

The time evolution of a quantum mechanical system is delineated by the Schrödinger equation: $i\hbar\frac{d}{dt}|\psi\rangle = H|\psi\rangle$, which describes how the quantum state $|\psi\rangle$ evolves over time. Here, i is the imaginary unit, $\hbar$ is the reduced Planck's constant, and H is the Hamiltonian operator [118]. The Hamiltonian represents the total energy of the system (including kinetic and potential energies) and plays a central role in determining the time evolution of the system. For time-independent Hamiltonians, the Schrödinger equation's solutions, given an initial condition $|\psi(t = 0)\rangle = |\psi_0\rangle$ can be expressed as:

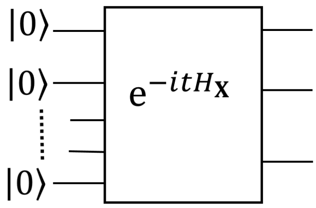
$$|\psi(t)\rangle = U(t)|\psi_0\rangle,$$
$$\text{where } U(t) = e^{-i\frac{t}{\hbar}H}, \tag{11}$$

is the unitary time-evolution operator and corresponds to the unitary matrix [119].

Hamiltonian encoding associates a system's Hamiltonian with a matrix representing the data transformation, such as the design matrix X that holds feature vectors as rows, or the Gram matrix $X^T X$ of the data [5], [87]. Consider the Hamiltonian encoding that maps the Hamiltonian H of a quantum circuit as:

$$\left|\psi'\right\rangle = \mathrm{e}^{-iH_{\mathbf{X}}t}|\psi\rangle, \tag{12}$$

illustrated through:



where a matrix $\mathbf{X}$ embodies a dataset, exemplified by an $M \times N$ dimensional data matrix with rows consisting of feature vectors. However, preprocessing tricks might be required to transform the data matrix into a Hermitian matrix [82]. For Hamiltonian encoding to be applied, we need to be able to implement an evolution on a quantum computer as:

$$\left|\psi'\right\rangle = e^{-iH_A t}|\psi\rangle, \tag{13}$$

where $|\psi\rangle$ is the initial quantum state characterizing a system composed of n qubits. The Hamiltonian H_A serves to encode a Hermitian matrix A of the same dimensions, signifying that the matrix representation of H_A is entry-wise equivalent to A. The state $|\psi'\rangle$ represents the final state or evolved quantum state that now contains the information encoded into the Hamiltonian, e.g., eigenvalues of H in the phase of the amplitudes. This implies that Hamiltonian encoding gives the algorithm the ability to extract the eigenvalues of the feature-matrices or multiply these eigenvalues to an amplitude vector. This functionality, involving the encoding of matrices into the Hamiltonian for time evolution, can be advantageous, as demonstrated by the renowned HHL algorithm for matrix inversion [84].

Furthermore, this encoding scheme is used in quantum annealing and adiabatic QC approaches to solve optimization problems [120]. Therefore, this embedding technique finds applications in various algorithms, including quantum optical neural networks (QONNs), continuous-variable (CV-QNNs), hybrid factoring algorithms where information is encoded in an Ising Hamiltonian, Variational Quantum Eigensolver (VQE) for molecular energy calculations, Quantum tomography for Quantum Approximate Optimization Algorithm (QAOA), and quantum error correction (QEC) [19], [121], [122], [123]. Nonetheless, Hamiltonian encoding faces key demerits such as the complexity of simulating complex Hamiltonians digitally (by decomposing the time-evolution operator into a sequence of quantum gates), accumulation of errors over longer evolution time, significant resource demands exceeding current quantum hardware capabilities, and challenges in scalability and precision control [124].

In summary, Table 2 provides a comparative analysis of quantum encoding techniques in terms of mathematical forms, number of qubits, runtime complexity, and applications. However, note that there are several other encoding patterns that exist, e.g., space encoding [65], matrix or dynamic encoding [82], Schmidt decomposition [129], instantaneous quantum polynomial (IQP) style encoding [130], Schrödinger's cat code encoding [131], QAOA ansatz encoding [132], time-bin encoding [133], parity encoding [134], arbitrary continuous-variable encoding [135], Fock encoding and coherent-state encoding schemes [136], etc.

In the domain of QIP, the evaluation of the runtime for data embedding or encoding algorithms often hinges on a central metric: the asymptotic complexity (Big-O notation, as referenced in Fig. 4 and Table 2). This metric reflects the increase in the number of elementary operations required relative to the input size, thereby indicating the runtime's growth rate with increasing input [75], [76]. However, the concept of 'input' varies across different contexts. For instance, in QML, an algorithm's input is the data itself. An efficient algorithm in this context operates in polynomial time relative to the data input's dimension (N) and the number of data points (M). In QC, efficiency is defined by an algorithm's polynomial runtime in relation to the number of qubits. Given that data can be encoded into either qubits or amplitudes, the term "efficient" assumes different implications in QML, often leading to confusion. To clarify, we suggest categorizing algorithms as either 'qubit-efficient' or 'amplitude-efficient', based on the considered input type [5], [82]. If the data is encoded into the amplitudes or operators of a quantum system (e.g., in amplitude and Hamiltonian encoding), then amplitude-efficient state preparation routines are also efficient in terms of dataset size. Conversely, when data is encoded into qubits, qubit-efficient state preparation aligns with dataset size efficiency. For example, state preparation for basis encoding is qubit-efficient as it requires at most n-gates [119], though this approach demands a significant number of qubits, particularly for high-precision data representation. It's noteworthy that if a QML algorithm shows polynomial behavior in n (qubit-efficient), it implies a logarithmic runtime dependence on the dataset size [80]. The notion of exponential speedups from qubit-efficient QML algorithms can be perplexing to machine learning practitioners. This is because the time required to load the MN features from memory hardware is inherently linear in MN. Exponential speedups are only feasible if state preparation is also achieved in a qubit-efficient manner [81], [137].

Various quantum encoding techniques, such as basis encoding, amplitude encoding, Qsample encoding, and others, convert classical data into quantum states with different runtime complexity, as outlined in Table 2. For instance, we can observe that a key advantage of amplitude encoding

TABLE 2. Comparative analysis of popular encoding techniques.

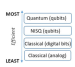
Encoding Methods	Mathematical Form	# Qubits	Runtime	Applications
Basis Encoding	$X \approx \sum_{i=-k}^{m} b_i 2^i \rightarrow \lvert b_m \ldots b_{-k}\rangle$	N	$\mathcal{O}(MN)$ [82]	Arithmetic operation [83].
Amplitude Encoding	$X \rightarrow \lvert\psi_x\rangle = \sum_{i=0}^{N-1} x_i \lvert i\rangle$	$\log N$	$\mathcal{O}(MN)$ / $\mathcal{O}(\log(MN))^*$ [82]	Harrow, Hassidim and Lloyd (HHL) [84]. Quantum K-Nearest Neighbor (QKNN) [83]. Satellite image data compression [125].
Qsample Encoding	$X \rightarrow \lvert\psi\rangle = \sum_{i=1}^{2^n} \sqrt{p_i} \lvert i\rangle$	N	$\mathcal{O}(2^N)$ / $\mathcal{O}(N)^*$ [82]	Probabilistic models [126]. Bayesian networks [86]. Quantum Boltzmann machines (QBMs) [88].
Angle Encoding	$X \rightarrow \lvert x\rangle = \bigotimes_i^n R(x_i) \lvert 0^n\rangle$ $\lvert x\rangle = \bigotimes_{i=1}^n (\cos(x_i) \lvert 0\rangle + \sin(x_i) \lvert 1\rangle)$	N	$\mathcal{O}(N)$ [199]	Quantum error handling [91], [92]. Quantum image processing (QIMP) [90]. Quantum neural network (QNN) [91].
QuAM Encoding	$X \rightarrow \sum_{i=0}^{M-1} \frac{1}{\sqrt{M}} \lvert x_i\rangle$	$2N + 1$	$\mathcal{O}(MN)$ [93]	Shor's algorithm [12]. Grover's algorithm [15]. Quantum holography [127]
QRAM Encoding	$X \rightarrow \sum_{i=0}^{N-1} \frac{1}{\sqrt{N}} \lvert i\rangle \lvert x_i\rangle$	$[\log M] + l$	$\mathcal{O}(\log M)$ [98]	Quantum database search [97], [102]. Quantum cryptography [103]. Eigenvalues processing in HHL algorithm [106]. Telecommunications [104], [105].
Superdense Encoding	$X \rightarrow \lvert\psi_s\rangle = \frac{1}{\sqrt{2}}(\lvert 00\rangle \pm \lvert 11\rangle)$ $= \frac{1}{\sqrt{2}}(\lvert 01\rangle \pm \lvert 10\rangle)$ ψ_s can be $\lvert\Phi^+\rangle, \lvert\Phi^-\rangle, \lvert\Psi^+\rangle, \lvert\Psi^-\rangle$	2 qubits (entangled pair) per 2 classical bits	$\mathcal{O}(1)$ [109]	Quantum security [112], [128]. Quantum communication networks [114]. Quantum secure direct communication (QSDC) [114].
Hamiltonian Encoding	$X \rightarrow \lvert\psi'\rangle = e^{-iH\mathbf{x}t} \lvert\psi\rangle$	$\log N$	$\mathcal{O}(MN)$ / $\mathcal{O}(\log(MN))^*$ [82]	Hybrid factoring algorithm [19]. Adiabatic quantum computation (AQC) [120]. Quantum tomography [122].

In Runtime column: M is the number of inputs or data points, N the number of features of each data point.
* Only applies for certain models with dataset of M inputs each with N features [82].

is its efficiency, requiring only $n = \log MN$ qubits or $O(\log(MN))$ to encode a dataset with M inputs and N features each [138]. Unlike basis, amplitude, and Hamiltonian encoding, which represent an entire dataset within a quantum system, Qsample encoding uniquely represents a probability distribution over random variables [82]. This implies that the amplitude vector characterizes the distribution of a classical discrete random variable, independent of the number of inputs M. Viewing a quantum state as a Qsample akin to a probabilistic model offers a promising intersection between machine learning/QML and QC, particularly with Boltzmann or Gibbs distributions [32]. Moreover, for arithmetic computations, a digital encoding like basis encoding might be preferable. Conversely, to maximize data storage within a limited number of qubits, compact encoding such as amplitude encoding could be optimal. Yet, it's important to consider that state preparation for amplitude encoding is operationally costly. This reflects the fact that there is no universally best encoding for quantum computation that addresses different problems on current devices. It is, therefore, an important and nontrivial open question to determine which encoding or embedding scheme is best suited for machine learning, or QC.

Overall, the objective of QML is to reduce the complexity, whether in terms of the number of operations (time complexity) or the amount of data required (sample complexity), for tasks such as model training, test vector classification, or concept generation. QML applications with inherently quantum inputs can sometimes offer exponential advantages over their classical counterparts in terms of either sample or time complexity. For example, Shor's quantum algorithm can factor an integer P (product of two prime numbers) with a complexity of $O\left((\log P)^3\right)$, indicating polynomial time complexity [12], [56]. Also, classical approaches to searching an unsorted database require $O(N)$ time for a linear search, N typically represents the number of entries in the database. On the other hand, Grover's quantum algorithm performs the same task in $O(\sqrt{N})$ time, which represents a quadratic speedup over the classical approach [57].

IV. CHALLENGES AND FUTURE RESEARCH PROSPECTS



Modern data systems have an ever-growing gap between the available information storage and the bit-per-second rates [139], quantum information technology is expected to be applied to the sixth generation (6G) of cellular networks to achieve performance gains [140]. In future network communication systems, quantum information technology will not fully replace classical information communication technology. In fact, classical information communication technology and quantum information technology will complement each other and evolve together to enable hybrid classical-quantum

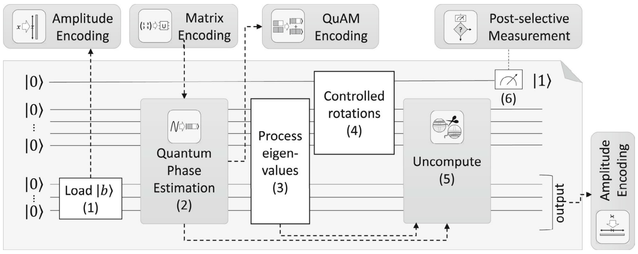


FIGURE 10. Applied embedding strategies and patterns for the HHL algorithm [84].

systems with better performance in diverse sectors to solve real-life issues [141], [142]. Nevertheless, currently, quantum computers are facing obstacles in the field of quantum encoding/embedding schemes; a few of them are below:

1) **Selection of Encoding Technique:** One of the primary challenges in quantum encoding schemes is the intricate task of selecting an appropriate encoding technique. With a plethora of encoding methods available, ranging from basis and amplitude encoding to more sophisticated strategies (Table 2), determining the most suitable approach for a given QC task becomes non-trivial. The efficiency and performance of quantum algorithms heavily depend on the encoding, making the selection process a critical factor in optimizing quantum computations. This is because the goal is usually either greater accuracy, greater speed, or greater opportunity for analytical insight into the system. What distinguishes embedding techniques is usually the proportion and frequency of interchange between quantum and classical techniques. The lack of a one-size-fits-all approach necessitates careful consideration of the task requirements and the efficiency of the encoding method [143].

   To address that challenge, further research is needed to investigate the potential of hybrid quantum algorithms for data fitting, for instance, in [39], [40], [144], and [128]. "Hybrid" encoding patterns aim to leverage the advantages of the best appropriate approaches and enhance the representation and manipulation of classical data in quantum states. For example, a combination of varied encoding patterns has been applied in the case of the HHL algorithm (Fig. 10), which is a well-known quantum algorithm designed to solve systems of linear equations exponentially faster than the best-known classical algorithms under certain conditions [84], [145]. Fig. 10 illustrates the possible ways to pass from one encoding pattern to another one and the data conversions that are used throughout the algorithm. The HHL algorithm uses a combination of amplitude encoding and matrix encoding in conjunction with quantum phase estimation (QPE), followed by QuAM encoding schemes. Essentially, it can find the solution vector $\mathbf{x}$ for a given linear equation $A\mathbf{x} = \mathbf{b}$, where A is a Hermitian matrix and $\mathbf{b}$ is a given vector. If A is invertible and Hermitian, and $\mathbf{b}$ is known, the HHL algorithm can be used to find the solution vector $\mathbf{x}$, i.e., $|x\rangle = A^{-1}|b\rangle$. Due to the spectral theorem [146], $|b\rangle$ can be written in terms of the eigenvectors $\{u_i\}$ of A as $|b\rangle = \sum_{j=0}^{n-1} \beta_j |u_j\rangle$, where coefficients β_j represent the amplitudes for each eigenvector state. That equation can be reformulated for $|x\rangle$ using the estimated eigenvalues λ_j as follows: $|x\rangle = \sum_{j=0}^{n-1} \lambda_j^{-1} b_j |u_j\rangle$, which is actually the desired output of the HHL algorithm. A more detailed description of conversions is in [147] and [45].

   To compute the $|x\rangle$, in the first step of the HHL quantum circuit (Fig. 10), amplitude encoding is applied where vector $|b\rangle$ is loaded, while the matrix A is encoded using matrix encoding. In step 2, the QPE routine is introduced, whose goal is to estimate the phases that correspond to the eigenvalues of the operation. From Fig.10's quantum circuit, it can be observed that the QPE pattern utilizes amplitude encoding and matrix encoding as input and produces output in basis encoding. After the QPE, QuAM encoding is employed to refine the probability amplitudes of the desired quantum states. As discussed in the QuAM encoding section, it facilitates the preparation of an equally weighted superposition of basis-encoded values to enhance the probability amplitudes of the desired quantum states. In steps 3–4, the algorithm proceeds to process eigenvalues obtained from QPE and applies controlled rotations to a set of qubits. These rotations are proportional to the inverses of the eigenvalues, as the angle of rotation is dependent on the inverse of the estimated eigenvalues. To reverse the QPE process, step 5 involves uncomputing to disentangle the auxiliary qubits used during QPE from the system, leaving the quantum state that encodes the solution. Given that the inversion of the eigenvalues is carried out through a probabilistic process, a post-selective measurement is subsequently applied. Finally, the algorithm converts the digital encoding (QuAM) of the inverted eigenvalues into an analogous (amplitude) encoding. This encoding will represent the solution $\mathbf{x}$ to the linear system; this is also referred to as the "quantum analog-digital conversion" process [106]. However, there are certain issues with the HHL algorithm that compromise the exponential speed-up [148], e.g., loading $|b\rangle$ in amplitude encoding may require an exponential number of operations—the 'input problem'. Also, the algorithm produces the solution vector $|x\rangle$ in amplitude encoding, which possibly requires multiple repetitions to estimate the amplitudes—the 'output problem' [149].

   Furthermore, data compression is one of the most fundamental questions in information theory [139]. It is necessary to opt for the best suitable quantum compression techniques (QCT) based on their compression performance with the given dataset. For example, for a satellite image dataset, if angle embedding is selected, then the minimum of n qubits encodes n classical features. Currently, NISQ technology is constrained by a limited number of qubits and faces challenges in preserving the coherence of multiple qubits. Therefore, basis and angle embedding schemes may not be optimal for satellite images due to the substantial qubit requirement for encoding classical data. Amplitude embedding, on the other hand, allows the encoding of 2^n classical data features using only n qubits. This exponential reduction in the number of required qubits makes amplitude embedding a more suitable choice for compressing satellite image data [125]. QML also helps to enhance the various classical machine learning methods for better analysis and prediction using complex measurements [150]. Additionally, novel QIP methods need to be explored for information processing, such as one-dimensional time series and two-dimensional images, in either the space or frequency domain to improve the fidelity of quantum coding [31], [151].

2) **Limitation of Error-free Fully Quantum (Gate-based) Computers:** Another significant challenge involves the accurate assessment of the hardware resources required to implement quantum-based algorithms. The reason is that quantum systems are inherently susceptible to errors and decoherence. Implementing effective error-correction mechanisms within encoding schemes is a critical challenge to ensure the integrity of encoded information. This includes the precise evaluation of factors such as the execution of utilized gates and the necessary number of qubits. These existing limitations in fully quantum hardware, characterized by noise and errors, hinder the robust implementation of quantum encoding schemes [152], [153]. Quantum encoding schemes need to dynamically adapt to the constraints and limitations of quantum hardware. The evolution of quantum processors (e.g., IBM Quantum's systems: 27-qubit Falcon processors, 65-qubit Hummingbird processors, and 127-qubit Eagle R3 processors [154], [155] or higher-level), introduces new challenges, such as varying qubit connectivity and gate fidelities. Ensuring the adaptability of encoding schemes to evolving hardware configurations is a key research challenge. In this regard, research efforts are directed toward developing robust error-correction techniques tailored for various quantum encoding methods. For instance, IBM scientists introduced an end-to-end quantum error correction (QEC) protocol that implements fault-tolerant memory based on a family of low-density parity check (LDPC) codes with a high encoding rate that achieves an error threshold of 0.8% for the standard circuit-based noise model [156]. In addition, some applications, such as QEC firmware and "Surface Code" techniques are designed to mitigate gate errors and improve overall accuracy [43], [157], [158]. Also, recently, the Fluxonium-Transmon-Fluxonium (FTF) architecture has showcased a single-qubit gate fidelity of 99.99% and a two-qubit gate fidelity of 99.90% [159]. However, for large sets, gate accuracy remains a challenge in large-scale QC, and ongoing research and development aim to enhance the quality of quantum gates.
In addition, IBM's work on error mitigation techniques, particularly zero noise extrapolation (ZNE) and probabilistic error cancellation (PEC), is a significant step toward making quantum computations more accurate and reliable, even with the current limitations of quantum hardware [160], [161]. These approaches have been tested on IBM's 127-qubit quantum Eagle processors, showing how current QC technology can be pushed to its limits and still yield useful results for the realization of near-term quantum applications [155].
Moreover, while NISQ technology is on the horizon, quantum computers with 50–100 qubits may surpass today's classical digital computers in certain tasks [35], [162]. However, quantum gate noise will constrain the size of reliably executable quantum circuits [155]. NISQ devices will be valuable for exploring many-body quantum physics and other applications, but a 100- or 127-qubit QC won't immediately revolutionize the world — it should be seen as a significant step toward more potent quantum technologies in the future. Quantum technologists should continue to strive to enhance the accuracy of quantum gates with fully fault-tolerant QC. Consequently, overcoming these limitations, particularly realizing fault-tolerant quantum computation, is vital for the practical implementation of quantum encoding schemes.

3) **QRAM Unavailability**: QC is a proposed solution for computationally intensive problems, particularly in machine learning, where processing large datasets is essential [98], [163], [164]. Currently, the absence of a QRAM capable of efficiently encoding and reliably storing such information as a quantum state poses a significant hardware challenge in QC. Thereby, researchers are tasked with devising alternative strategies or anticipating advancements in quantum hardware to fully harness the potential of QRAM.
To tackle that challenge, the practical implementation of QRAM can be explored for real-world QC systems; for instance, relevant approaches are presented in [99], [156], [165], [166], and [167]. The bucket brigade (BB) and circuit-based flip-flop (FF) models for QRAM have been proposed in [52], [98], and [101]. These approaches will not only streamline the process of designing efficient NISQ-based computers but also bring out complete/full quantum (qubits-based) systems for solving today's real-life practical problems [92], [162]. Moreover, as early analog classical computers were replaced by digital electronic computers, we expect that eventually, in the near future, all NISQ-type algorithms will be replaced by gate-based algorithms.

4) **Integration of Quantum Computing with Machine Learning for Encoding:** Machine learning on classical models is well established, but it demands significant computational resources, especially when dealing with complex and high-volume data processing. To mitigate that, QC can exploit the principle of quantum parallelism, allowing them to process a large number of possibilities simultaneously. This could be beneficial in tasks like optimization problems or searching large solution spaces [12], [164]. However, the challenge mentioned pertains to the difficulties and complexities introduced when trying to merge these two intricate domains — QC principles and machine learning methodologies.
In this context, different research groups introduced the concepts of supervised machine learning modeling using QC, which deals with feature selection, parameter encoding, and parameterized circuit formation [163] and [168]. For instance, in [38] and [169], the team showcased the practicality of their suggested quantum embedding method through simulations and tests on standard datasets such as Iris and Breast Cancer. Their findings suggest that the quantum embedding search approach for supervised QML, i.e., the QES architecture, surpasses manual methods in terms of predictive performance. Also, they explored manipulating the entanglement level to manage and constrain the search space to a feasible size for practical implementations. Additionally, [67] examines the effect of data encoding on the expressive power of variational models in QML.Moreover, the concept of "quantum geometric deep learning" strives to establish a framework for crafting neural network architectures capable of efficiently handling quantum datasets by encoding relevant symmetries and physical principles [170]. This involves integrating both QC principles and geometric deep learning techniques to tackle the distinctive challenges and opportunities inherent in QIP.
Furthermore, the current momentum in the development of near-term quantum devices, coupled with the pursuit of fault-tolerant systems, has spurred researchers to explore the implications of substituting quantum circuits for traditional or supervised machine learning models [22], [154], [155], [159]. These inquiries often pertain to constructs referred to as quantum neural networks (QNNs) [91], [171]. Leading entities like IBM, Rigetti, and Xanadu have applied QC to enhance machine learning, particularly neural networks. Various notable case studies involve mapping neural networks to quantum processors and using quantum circuits to accelerate the inference phase of a trained neural network [72], [172], [173], [174], [175], [176], [177], [178]. These efforts have demonstrated the practical application of QC in the fields of artificial intelligence and machine learning, showcasing the potential of quantum processors to handle complex computational tasks.
For instance, a noteworthy advancement in this area is the development of continuous-variable quantum neural networks (CV-QNNs) [179], offering a versatile approach to designing neural networks on quantum computers. These networks (QNNs) utilize a variational quantum circuit within the CV framework, enabling the encoding of quantum information in continuous degrees of freedom, like electromagnetic field amplitudes. Featuring a layered structure of continuously parameterized gates, this quantum circuit is universal for CV quantum computation. This model incorporates both affine transformations and nonlinear activation functions by utilizing Gaussian and non-Gaussian gates, respectively [180]. The non-Gaussian gates are key to introducing both the essential nonlinearity and universality of the model. The architecture of the CV model enables the QNN to perform complex, non-linear transformations while maintaining a unitary nature.
Similarly, quantum kernel embedding utilizes qumodes (also referred to as CV states, which offer a viable alternative to discrete or digital qubits) to map classical data into a quantum feature space [73], [135]. This effectively prepares the data for quantum processing in a manner that leverages the continuous nature of CV systems. These experiments and case studies demonstrated the integration of classical networks into quantum frameworks, suggesting quantum variants for specialized network models, including recurrent, convolutional, and residual networks. In essence, qumodes are a key component of both quantum kernel embeddings (for data representation) and CV-QNNs (for quantum computation and learning), reflecting their versatility and critical role in the realm of CV quantum computing and QML.
In addition, the emergence of NISQ computers has opened up exciting prospects for achieving quantum speedups in machine learning tasks. For instance, matrix product states pre-training for QML benchmark on the novel image dataset, i.e., the Fashion-MNIST dataset [150], [181]. Thus, while the potential for enhancing encoding through machine learning is significant, navigating the intricacies of combining quantum and classical computational paradigms demands careful consideration. Achieving a seamless integration that maximizes encoding efficiency is an active area of research.
5) **Decoding Method (From Quantum to Classical Data)**: Efficient hybrid communication between classical and quantum components poses challenges not only in encoding but also in quantum decoding as well. Bridging the gap between quantum and classical information processing introduces complexities related to data transfer, synchronization, and minimizing information

loss. This is because decoding quantum-to-classical (Q2C) data typically involves substantial overhead as the quantum circuit must be sampled repeatedly to obtain meaningful data readout. The decoding problem has been studied quite a lot in communication settings, based mostly on coherent state encodings. One of the issues is that optimal decoding requires a collective operation on the quantum data [182], [183].

To overcome such challenges, different strategies have been adopted. For instance, [184] proposed optimized quantum wavelet transform (QWT) and quantum Haar transform (QHT) techniques. Similar to the classical Haar transform, quantum circuits can be developed to perform the so-called QHT. The QHT-based Q2C method demonstrated a superior 15-fold higher space efficiency compared to the quantum Fourier transform (QFT)-based Q2C method. While not as space-efficient as QHT, QFT-based decoding is invaluable for problems where the periodicity of quantum data plays a pivotal role. The decoding method based on QFT is particularly appealing for applications in image or audio processing, where data attributes like spectral bandwidth are crucial for output analysis [185]. Despite its comparative inefficiency in space utilization, QFT excels in applications that demand high precision and complex data manipulation [186].

Additionally, the proposed zero-depth (i.e., quantum circuits with minimal depth) QWT method exhibited remarkable enhancements in execution time, showing up to a 14% improvement over conventional Q2C and a substantial 78% improvement over QFT-based Q2C. Likewise, classical Slepian-Wolf coding involves quantum side information, where two correlated classical components are compressed and their quantum counterparts serve as side information during the decoding process [187]. The use of quantum side information can significantly enhance the efficiency of classical data compression and decoding, particularly in scenarios where classical and quantum data are intricately linked. This method demonstrates the potential for synergistic integration between classical and quantum information theories, offering novel pathways for data compression and encryption [188].

Although the aforementioned techniques offer promising solutions to Q2C decoding, several challenges remain. These encompass the necessity of reducing redundant sampling to minimize overhead in total execution time and implementing error correction strategies to counteract quantum noise and decoherence [42], [189]. Therefore, it is also essential to consider these decoding challenges for seamless integration into practical QC workflows. Moreover, a comparative analysis of these techniques reveals trade-offs between space efficiency, execution time, and applicability to different QC. For instance, while QHT and QWT excel in efficiency and speed, their application might be limited by the specific requirements of the quantum algorithm in use. Conversely, QFT-based decoding and Slepian-Wolf coding with quantum side information offer broader applicability at the expense of space or time efficiency.

6) **Quantum State Characterization and Measurement:** The delicate nature of quantum states makes their precise determination challenging, introducing uncertainties that impact the reliability of quantum encoding schemes. Developing improved techniques for state characterization and measurement is crucial for advancing quantum encoding capabilities.

In this perspective, Avagyan [190] and Notarnicola and Olivares [191] have introduced techniques such as the optical setup of a local oscillator on a beam splitter and a hybrid feed-forward receiver (HFFRE) for characterizing quantum states. These methods employ measurement configurations inspired by homodyne detection. Similarly, other researchers have employed strategies for quantum measurements that involve the calibration of coherent-state receivers [192], [193]. Specifically, they have demonstrated a quantum receiver for coherent communication capable of unconditionally discriminating among nonorthogonal coherent states [194], [195], [196]. Moreover, Lin and colleagues [197] proposed a protocol that addresses state preparation and measurement (SPAM) errors independently. Additionally, a quantum-state tomography technique by employing conditional generative adversarial networks (QST-CGAN) was presented in [198], showcasing an ability to adapt to noise and reconstruct the underlying state with up to two orders of magnitude fewer iterative steps than maximum-likelihood estimation (MLE) methods. These innovative approaches open avenues for applying state-of-the-art quantum state tomography (QST) and deep learning techniques in the classification and reconstruction of quantum states, even in the presence of various forms of noise.

In outlook, the realization of quantum computational supremacy through encoding schemes hinges on the imperative demonstration of their efficacy. This necessitates a comprehensive quantitative evaluation, considering key factors such as spatial complexity (total gate count), temporal complexity (circuit depth and execution time), and accuracy (fidelity/similarity). The proposed techniques should be rigorously benchmarked against existing embedding methods to establish their prowess in advancing QC capabilities [14], [199], [200].

V. CONCLUSION

This paper presented a comprehensive review of quantum embedding techniques, offering a thorough exploration of this intricate subject matter. It provided a framework for representing classical data as quantum states in a Hilbert space, which enables the use of quantum algorithms to perform more efficient computations on the data. We discussed quantum encoding schemes with a focus on quantum information processing to allow more complex information to be represented in the quantum state of a qubit. Basis, amplitude, angle, Qsample, QuAM, QRAM, Hamiltonian, and superdense are just a few examples of quantum encoding techniques, each with their strengths and potential applications. While classical bits and logic gates are limited in their ability to represent and process complex information, quantum encoding techniques enable exponential speedup and secure communication in quantum networks. Thus, data encoding strategies are critical for preprocessing data, deciding the number of qubits, compiling/correlating quantum data, designing quantum circuits, and efficiently executing quantum algorithms. To summarize, this article has strived to offer a panoramic overview of quantum embedding techniques and their multifaceted applications across a wide range of fields.

In conclusion, quantum computing encoding techniques offer a powerful new tool for encoding information that could have significant implications for a wide range of applications. By taking advantage of the unique properties of qubits, quantum encoding techniques can offer significant improvements over classical encoding techniques in terms of efficiency and information density. This research has important implications for the development of novel practical quantum information processing systems.

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