IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received 16 February 2024, accepted 9 March 2024, date of publication 27 March 2024, date of current version 9 April 2024.
Digital Object Identifier 10.1109/ACCESS.2024.3381540

RESEARCH ARTICLE

Nonlinear Dynamic Observer Design for Uncertain Vehicle Systems With Disturbances

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This work was supported in part by the Innovation Capability Support Program of Shaanxi under Grant 2021TD-28 and Grant 2022KXJ-144, in part by the Inquiry Program of Guangzhou Gonglu Engineering Company under Grant 220238220503, and in part by Shaanxi Key Laboratory of New Transportation Energy and Automotive Energy Saving Program under Grant CHD300102221508.

ABSTRACT This paper investigates the dynamic observer (DO) design problem for a class of disturbed Lipschitz nonlinear vehicle systems with uncertainties, where the uncertainties are assumed to be in both system matrix and input matrix. The proposed DO has a unified form and the popularly used proportional observer and proportional integral observer can be considered as the particular cases of the proposed DO. Based on the algebraic constraints, the observer design issue is transformed into the asymptotically stability analysis problem. The sufficient criterion of the DO design is derived in terms of linear matrix equality (LMI), by constructing a Lyapunov function to guarantee the H-infinity performance. A numerical example is used to illustrate the performance of the proposed DO.

INDEX TERMS Dynamic observer, nonlinear system, uncertainty, LMI, H-infinity.

I. INTRODUCTION

The vehicle state information is important to the vehicle stability control system. However, the direct measurement of vehicle states requires expensive sensors which are not available for commercial vehicles but for scientific research only. Therefore, vehicle state observers, which can estimate the vehicle state, have attracted considerable attentions over the past decades. For example, in [1], a switched Takagi-Sugeno observer was proposed to estimate the sideslip angle, by using the nominal piecewise affine tire model and the tire dynamic load to establish the vehicle lateral dynamics model. In order to improve the control tracking performance, the authors of [2] proposed an angular velocity observer to estimate the steering angular of the front wheel. In [3], the authors solved the cornering stiffness estimation problem, for articulated vehicles. The lateral dynamics model of articulated vehicles was established by combining the bicycle model, linear tire model and modified Dugoff model, and the cornering stiffness was estimated by using a dual linear time-varying Kalman filter. For four wheel independent drive electric vehicle, the authors of [4] investigated the estimate problem of tire-road peak adhesion coefficient. Based on the tire longitudinal force obtained by using recursive-least-squares, an adaptive square root cubature Kalman filter and partitioned similarity principle were designed estimate the vehicle states and the tire road peak adhesion coefficient. In the application of electric vehicles [5], by using a seventh order polynomial fitting approach and two-RC equivalent circuit model to represent the lithium-ion batteries, a nonlinear observer was proposed to estimate the battery's offline parameters, which can be used to calculate the state of charge of lithium-ion batteries.

In the application of observers, the uncertainty has attracted considerable attention in the past years, which can deteriorate the performance of control system. In the observer design for uncertain systems, the difficulty is how to describe the uncertain term. There are some results about uncertainty representation. For the steer-by-wire system [6], a radial basis function neural network was used to model the uncertain

The associate editor coordinating the review of this manuscript and approving it for publication was Jorge Esteban Rodas Benítez.

nonlinearity of automated vehicles, which was caused by self-aligning torque and unknown friction torque. Then an observer was designed to estimate the angular velocity of the front wheel, by constructing Lyapunov function. In [7], the authors introduced a lumped vector to represent the uncertain term in practical vehicle model, which was caused by the modeling errors on longitudinal position and the potential communication delays, data packet losses. Then a second-order sliding mode observer was designed to estimate the uncertainty. The authors of [8] used a mathematical model of dual input and dual output system to represent the uncertain term in vehicle steering systems. Inserting the uncertainty into the vehicle system, the authors designed an H-infinity observer to solve the actuator fault diagnosis and reconfiguration problem. In [9], a Takagi-Sugeno fuzzy model was applied to represent the vehicle lateral system with parameters uncertain, which caused by side wind force and road adhesion coefficient variance. By treating the uncertainty as a unknown input, the authors proposed a unknown input observer to estimate the vehicle state and actuator faults, simultaneously. The authors of [10] developed a machine learning model to estimate the number of departing foreign visitors from Türkiye to gain a deeper understanding of their future behaviors. In [11], a distributed observer was designed as feed forward compensation to attenuate the uncertainty impact.

The external disturbance is another negative issue in the control system, which is need to pay more attention. One method to deal with the disturbance is to design an observer to estimate the disturbance and the system state simultaneously, and then take some measures to eliminate the negative influence. For example, in [12], by applying the Lyapunov theory, a nonlinear observer was designed for the anaerobic digestion model. In [13], in order to eliminate the effect of disturbance torque caused by the varying tire-road condition, a disturbance observer combined with the super-twisting algorithm was designed to compensate the disturbance torque acting on the front wheels. The authors of [14] proposed a sliding mode observer to estimate the self-aligning moment disturbance without required information about the tire parameters, by using Lyapunov stability theory. The authors of [15] proposed a disturbance observer for steer-by-wire system with unknown compound disturbances, which were caused by external disturbance, Euler approximation errors and neural network approximation error.

Another method to deal with the disturbance is H-infinity approach, which by minimizing the H-infinity norm from control variable to disturbance. In [16], an H-infinity observer was proposed, which can suppress the particle depletion during the execution of the standard particle observer after compromising the weights of particles. In [17], the authors proposed an observer-based control for path tracking problem of autonomous ground vehicles. The observer was designed to estimate both the sideslip angle and the vehicle yaw rate and the finite frequency H-infinity criteria was utilized to attenuate the disturbances.

The observers introduced previously are the kind of proportional observer (PO), which only has a proportional gain of estimation error. It is well known that there always exists static error in the estimation of PO. In order to reject the static error, the proportional integral observer (PIO) has been developed, which contains an extra integral gain of the estimation error. In [18], in order to improve the dynamic performance for output voltage tracking, a moving discretized control-set model predictive control with hybrid modulation was proposed for three-phase dual-active-bridge, by using a PIO to estimate the load current for the cost saving of the current sensor. The authors of [19] proposed a proportional-integral extend state observer for nuclear reactors, to estimate the reactor process variables and coolant thermal hydraulics. Moreover, a PIO, which can estimate both the measurement system variables and the actuator faults, was designed to diagnosis the fault for nonlinear systems in [20]. The H-infinity approach was applied to ensure robustness of PIO against sensor noise and disturbances for vehicle [21], [22]. In order to improve the tracking functional properties, the authors of [23] proposed a modified PIO to reconstruct the magnetic fluxes and the angular speed of induction motor for speed control system. Besides, in [24], a developed PIO design method was proposed for H-infinity state estimation problem for a class of discrete-time recurrent neural networks with time-varying delays, by considering the Bernoulli distributed random variables with certain probabilities to implement the observer. The authors of [25] and [26] investigated the application of PIO for the state of charge estimation of lithium-ion batteries.

More recently, a kind of dynamic observer (DO) has been introduced, where there exists a dynamic variable in the observer gain [27]. There are some interesting results about DO. For example, in [28], a systematic constrained fuzzy integral sliding mode controller was designed for class of uncertain discrete-time nonlinear system, based on a DO along with H-infinity performance to attenuate disturbance. The authors of [29] proposed a robust anomaly detection DO for continuous linear roesser systems, by using the H-infinity performance indices to ensure the safety and the reliability of industrial monitoring systems. In [30], the DO was proposed for nonlinear systems for the first time. For a class of nonlinear systems where the nonlinearity was assumed to satisfy the Lipschitz condition, the authors derived the DO design approach from the solution of the linear matrix inequality (LMI). In [31], by using H-infinity criterion, a unified H-infinity DO was developed to estimate the damping force of an Electro-Rheological damper in an automotive suspension system, where the nonlinearity is bounded through a Lipschitz condition. Moreover, the authors of [32] and [33] investigated the DO design for semi-active suspension systems and automotive suspension systems, respectively. In [34], an H-infinity DO was designed for nonlinear systems with unknown inputs and disturbances.

Among the above published literatures, to the best of our knowledge, few results about DO design for nonlinear systems with both uncertainties and disturbances has been given. For instance, the authors of [28] used a nonlinear function to represent the matched uncertainty, without considering the state vector uncertainty. The authors of [29] mainly focused on the DO design for systems with disturbances. In [30], the DO design problem for nonlinear systems without disturbances and uncertainty has been solved. In [31] and [32], the author only solved the DO design problem for nonlinear systems with disturbances, where a nonlinear function was added in input matrix, and then they extended the results to nonlinear systems, where the state and input matrices both depend on the parameter vector in [33]. In [34], the authors proposed an H-infinity DO for nonlinear systems with disturbance and unknown inputs.

In summary, based on the analysis of the existing literature, there are still the following problems need to be solved:

(1) There are no research results about the application of DO for nonlinear systems with both uncertainties and disturbances;

(2) In the application of DO for vehicle systems, the uncertainty is not taken into account.

Motivated by the above observations, we will try to investigate the DO design problem for nonlinear vehicle systems in the presence of both uncertainties and disturbances. This study has the following main contributions:

(1) A unified form of DO is proposed for nonlinear systems in the presence of disturbances and uncertainties. Different from the previous research, the uncertainty and the nonlinearity are independent from each other, and the uncertainty is represented by using a function with bounded condition;

(2) The uncertainty is assumed to exist in both system matrix A and input matrix B, which is also different from existence results and more generalized, since the uncertainty can be bring in everywhere;

(3) By incorporating H-infinity disturbance rejection technique to attenuate the effect of disturbance and the Lyapunov function approach, a LMI is reconstructed to solve the DO design problem;

(4) The research results has been extended to vehicle systems, which are assumed to be nonlinear and in the presence of disturbances and uncertainties.

The paper is organized as follows: The nonlinear uncertain vehicle system model is constructed in Section II. Section III describes the observer design problem. Section IV presents parametrization results, of DO parameter matrices. Section V solves the DO design problem for uncertain vehicle systems with disturbances. Simulation example is presented in Section VI to illustrate the effectiveness and superiority of the proposed observer. Finally, the conclusion is given in Section VII.

Notation: $\mathbb{R}^n$ represents the n-dimensional Euclidean space; $\mathbb{R}^{n\times m}$ represents the set of all $n \times m$ real matrices; A^T denotes the transpose of matrix A; A is symmetric positive definite if and only if $A^T = A$ and $A > 0$; A^+ denotes the general inverse of matrix A which satisfies $AA^+A = A$; $\|\cdot\|_\infty$ is the H-infinity norm; I denotes the identity matrix of appropriated dimension and 0 denotes the zero matrix of appropriated dimension.

II. SYSTEM MODELING

To make the problem easy to analyze, the vehicle dynamic system is simplified to a two-degree-of-freedom model [8], which is shown in Figure 1. The two-degree-of-freedom model contains the yaw rate and side slip angle parameters that describe the vehicle handling stability and reflect the most basic characteristics of the required curve motion. The two-degree-of-freedom model quantitatively describes the key parameters that affect the lateral motion of vehicle, such as the position of vehicle centroid and the tire side deflection characteristics, which is the basis for the study of vehicle operating stability. Many theories and experiments have proved that the two-degree-of-freedom of a four-wheel vehicle model can reflect the actual physical process of the vehicle. In the figure, the vehicle is assumed to move to the right. The two front wheels are combined into one wheel and the two rear wheels are combined into one wheel. m is the center of gravity. v is the real speed.

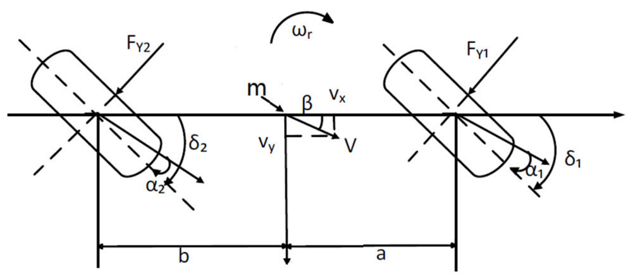

FIGURE 1. Two-degree-of-freedom model.

The major parameters of the two-degree-of-freedom vehicle model are given as follows:

F_{Y1} and F_{Y2} are the lateral forces of the front and rear wheels, respectively;

δ_1 and δ_2 are the front and rear steering angles, respectively;

α_1 and α_2 are the side angles of the front and rear wheels, respectively;

ω_r is the yaw rate of the vehicle;

β is the sideslip angle of the vehicle;

v_x is the longitudinal speed of the vehicle;

v_y is the lateral speed of the vehicle;

a and b are the distances from the center of gravity to the front and rear axles, respectively.

In order to obtain the vehicle system, based on Newton's second law, we can obtain the following equations:

$$m_g v_x(\dot{\beta} + \omega_r) = F_{Y1} + F_{Y2}, \tag{1a}$$

$$I_z\omega_r = aF_{Y1} - bF_{Y2}, \tag{1b}$$

where m_g is the vehicle mass and I_z is the yaw moment of inertia around the center of mass.

Furthermore, the lateral forces of the front and rear wheels F_{Y1} and F_{Y2} can be expressed by using the following equations:

$$F_{Y1} = [k_1 + \sigma(t)Nk_1]\alpha_1, \tag{2a}$$
$$F_{Y2} = [k_2 + \sigma(t)Nk_2]\alpha_2, \tag{2b}$$

where k_1 and k_2 are the given lateral stiffness values of the front and rear wheels, respectively. N represents the deviation from the lateral stiffness amplitude.

The term $\sigma(t)$ is a time varying function, which is used to represent the uncertainty in vehicle systems such as structure errors or time-delays. It is assumed to satisfy the following condition:

$$\sigma(t) \leq 1. \tag{3}$$

Furthermore, the side angles α_1 and α_2 can be expressed by:

$$\alpha_1 = \beta + a\omega_r / v_x - \delta_1, \tag{4a}$$
$$\alpha_2 = \beta - b\omega_r / v_x - \delta_2, \tag{4b}$$

Subsequently, by substituting the representations (3) and (4) into vehicle system (1), we can obtain the following new representation of vehicle system:

$$\dot{x} = (A + \Delta A)x + (B + \Delta B)u, \tag{5a}$$
$$y = Cx, \tag{5b}$$

where vectors $x = \begin{bmatrix} \omega_r \\ \beta \end{bmatrix}, u = \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix}$, matrices $A = \begin{bmatrix} \frac{a^2k_1+b^2k_2}{\upsilon_x I_z} & \frac{ak_1-bk_2}{I_z} \\ \frac{ak_1-bk_2}{m_g \upsilon_x^2} - 1 & \frac{k_1+k_2}{m_g \upsilon_x} \end{bmatrix}, B = \begin{bmatrix} -\frac{ak_1}{I_z} & \frac{bk_2}{I_z} \\ -\frac{k_1}{m_g \upsilon_x} & -\frac{k_2}{m_g \upsilon_x} \end{bmatrix}$ and $C = I$.

The uncertain terms ΔA and ΔB can be represented by

$$\Delta A = M_1 g(t) N_1, \tag{6}$$
$$\Delta B = M_2 g(t) N_2, \tag{7}$$

where matrices $M_1 = \begin{bmatrix} \frac{a^2Nk_1+b^2Nk_2}{\upsilon_x I_z} & \frac{aNk_1-bNk_2}{I_z} \\ \frac{aNk_1-bNk_2}{m_g \upsilon_x^2} & \frac{Nk_1+Nk_2}{m_g \upsilon_x} \end{bmatrix}$, $M_2 = \begin{bmatrix} -\frac{aNk_1}{I_z} & \frac{bNk_2}{I_z} \\ -\frac{Nk_1}{m_g \upsilon_x} & -\frac{Nk_2}{m_g \upsilon_x} \end{bmatrix}, N_1 = N_2 = I, g(t) = \begin{bmatrix} \sigma(t) & 0 \\ 0 & \sigma(t) \end{bmatrix}$.

On the other hand, the vehicle system is a nonlinear complex system, and there always exist disturbances in the actual driving [8]. Considering the above factors, we add the nonlinear and disturbance terms in (5). Then the final vehicle system model is described as follows:

$$\dot{x} = (A + \Delta A)x + (B + \Delta B)u + f(x) + Dw, \tag{8a}$$
$$y = Cx + Kw, \tag{8b}$$

where vector w represents the disturbance of bounded energy. D and K are known constant matrices of appropriate dimensions.

The nonlinear function $f(x)$ is assumed to satisfy the following Lipschitz condition:

$$\left\| f(x) - f(\hat{x}) \right\| \leq \gamma \left\| x - \hat{x} \right\|, \tag{9}$$

with Lipschitz constant $\gamma > 0$.

III. PROBLEM FORMULATION

For system (8), we proposed the following DO:

$$\dot{z} = Fz + Jy + Hv + Eu + Tf(\hat{x}), \tag{10a}$$
$$\dot{v} = Pz + Qy + Gv, \tag{10b}$$
$$\hat{x} = Rz + Sy, \tag{10c}$$

where $z \in \mathbb{R}^q$ is the state variable of the observer, with $n - p \leq q \leq n$, n is the dimension of system state x, $v \in \mathbb{R}^r$ is an auxiliary variable and $\hat{x} \in \mathbb{R}^n$ is the estimate of x. Matrices F,J,H,E,T,P,Q,G,R and S are unknown and of appropriate dimensions to be determined.

Remark 1: The observer design problem is to determine all the parameter matrices of observer (10) such that:

- *For $w = 0$, the estimation error $e \to 0$ for $t \to \infty$;*
- *For $w \neq 0$, $\|T_{we}\|_\infty < \lambda$,*

where $e = \hat{x} - x$, $\|T_{we}\|_\infty$ represents the transfer function from w to e, and λ is a positive scalar.

In order to solve the observer design problem, let us introduce a new error variable ε:

$$\varepsilon = z - Tx, \tag{11}$$

Then the derivative of ε is given as follows:

$$\begin{aligned} \dot{\varepsilon} &= \dot{z} - T\dot{x} \\ &= F\varepsilon + Hv + (FT + JC - TA)x + (E - TB)u \\ &\quad - T\Delta Ax - T\Delta Bu + JKw - TDw + T[f(\hat{x}) - f(x)]. \end{aligned} \tag{12}$$

Furthermore, we can obtain the following results about the derivative of v and system estimate $\hat{x}$:

$$\dot{v} = Pz + Qy + Gv = P\varepsilon + (PT + QC)x + Gv + QKw \tag{13}$$

and

$$\hat{x} = Rz + Sy = R\varepsilon + (RT + SC)x + SKw. \tag{14}$$

From (12), (13) and (14), one can see that if the following algebraic constraints are satisfied:

$$\begin{aligned} FT + JC - TA &= 0, \\ E - TB &= 0, \\ PT + QC &= 0, \\ RT + SC &= I, \end{aligned} \tag{15}$$

we can obtain the following equations:

$$\begin{aligned} \dot{\varepsilon} &= Fz + Hv - T\Delta Ax - T\Delta Bu \\ &\quad + (JK - TD)w + T[f(\hat{x}) - f(x)], \end{aligned} \tag{16}$$

$$\dot{v} = P\varepsilon + Gv + QKw, \tag{17}$$
$$e = R\varepsilon + SKw, \tag{18}$$

In this case, (16), (17) and (18) can be rewritten as the following systems:

$$\dot{\zeta} = \mathbb{A}\zeta + \mathbb{B}\Delta Ax + \mathbb{B}\Delta Bu + \mathbb{D}w + \mathbb{F}[f(\hat{x}) - f(x)], \tag{19a}$$
$$e = \mathbb{C}\zeta + \mathbb{E}w, \tag{19b}$$

where vector $\zeta = \begin{bmatrix} \varepsilon \\ v \end{bmatrix}$, matrices $\mathbb{A} = \begin{bmatrix} F & H \\ P & G \end{bmatrix}$, $\mathbb{B} = \begin{bmatrix} -T \\ 0 \end{bmatrix}$, $\mathbb{D} = \begin{bmatrix} JK - TD \\ QK \end{bmatrix}$, $\mathbb{F} = \begin{bmatrix} T \\ 0 \end{bmatrix}$, $\mathbb{C} = \begin{bmatrix} R & 0 \end{bmatrix}$ and $\mathbb{E} = SK$.

*Remark 2: Based on the above analysis, the observer design problem in **Remark 1** is reduced to the asymptotically stability analysis of system (19), in other words, to determine matrices $\mathbb{A}, \mathbb{B}, \mathbb{C}, \mathbb{D}, \mathbb{E}$ and $\mathbb{F}$ such that:*

- *Conditions (15) are satisfied,*
- *For $w = 0$, the estimation error $e \to 0$for $t \to \infty$;*
- *For $w \neq 0$, $\|T_{we}\|_\infty < \lambda$,*

Next, we give the following lemma and assumption that will be used in the sequel of the paper:

Lemma 1 ([35]): Let $\mathcal{C}$, $\mathcal{D}$ and $\mathcal{G}$ be real matrices of appropriate dimensions and $\mathcal{D}$ satisfies $\mathcal{D}^T\mathcal{D} \leq I$. Then for any scalar $\rho > 0$ and vectors $a, b \in R^n$, we have

$$2a^T\mathcal{C}\mathcal{D}\mathcal{G}b \leq \rho^{-1}a^T\mathcal{C}\mathcal{C}^Ta + \rho b^T\mathcal{G}^T\mathcal{G}b. \tag{20}$$

Assumption 1: System 8 is asymptotically stable against the internal and external disturbances.

IV. PARAMETRIZATION

From observer (10) one can see that there are many parameter matrices need to be determined. In order to simplify the calculation, some parametrization results will be presented based on the algebraic conditions (15). Firstly, from conditions (15), we have the following equation:

$$\begin{bmatrix} F & J \\ P & Q \\ R & S \end{bmatrix} \begin{bmatrix} T \\ C \end{bmatrix} = \begin{bmatrix} TA \\ 0 \\ I \end{bmatrix}. \tag{21}$$

The condition for (21) to have a solution is the following rank condition

$$rank \begin{bmatrix} T \\ C \end{bmatrix} = n \tag{22}$$

must be satisfied.

Assume that we choose a matrix T to satisfy the condition (22), then by following the procedure in [30], we can obtain the following results:

$$F = TA\alpha_1 - Z_1\beta_1, \tag{23}$$
$$J = TA\alpha_2 - Z_1\beta_2, \tag{24}$$
$$P = -Z_2\beta_1, \tag{25}$$
$$Q = -Z_2\beta_2, \tag{26}$$
$$R = a_1 - Z_3\beta_1, \tag{27}$$
$$S = a_2 - Z_3\beta_2. \tag{28}$$

where

$$\alpha_1 = \Sigma^+ \begin{bmatrix} I \\ 0 \end{bmatrix}, \beta_1 = (I - \Sigma\Sigma^+) \begin{bmatrix} I \\ 0 \end{bmatrix}, \tag{29}$$
$$\alpha_2 = \Sigma^+ \begin{bmatrix} 0 \\ I \end{bmatrix}, \beta_2 = (I - \Sigma\Sigma^+) \begin{bmatrix} 0 \\ I \end{bmatrix}, \tag{30}$$
$$\Sigma = \begin{bmatrix} T \\ C \end{bmatrix}, \tag{31}$$
$$Z_1 = \begin{bmatrix} I & 0 & 0 \end{bmatrix} Z, \tag{32}$$
$$Z_1 = \begin{bmatrix} I & 0 & 0 \end{bmatrix} Z, \tag{33}$$
$$Z_1 = \begin{bmatrix} I & 0 & 0 \end{bmatrix} Z \tag{34}$$

and Z is an arbitrary matrix of appropriate dimension.

From the above analysis, one can see that once the matrix T is determined, matrices $\alpha_1, \alpha_2, \beta_1$ and β_2 can be deduced. Moreover, once matrices Z_1, Z_2 and Z_3 are determined, all the matrices F, J, P, Q, R and S can be deduced. In this case, the observer design problem is transferred to determine matrices T, H, E, G, Z_1, Z_2 and Z_3.

Remark 3: One can see from (19b) that estimation error $e \to 0$ when vector $\zeta \to 0$,i.e., e is independent of matric $\mathbb{C}$. In this case, without loss of generality, we can fix matrix $Z_3 = 0$ and obtain

$$R = a_1, S = a_2.$$

Remark 4: According to the above results, we can rewrite matrices $\mathbb{A}$, $\mathbb{C}$ and $\mathbb{D}$ of system (19) as follows:

$$\mathbb{A} = \begin{bmatrix} F & H \\ P & G \end{bmatrix} = \begin{bmatrix} TA\alpha_1 - Z_1\beta_1 & H \\ -Z_2\beta_1 & G \end{bmatrix} = \mathbb{A}_1 + \mathbb{Z}\mathbb{A}_2, \tag{35}$$

with $\mathbb{A}_1 = \begin{bmatrix} TA\alpha_1 & 0 \\ 0 & 0 \end{bmatrix}$, $\mathbb{Z} = \begin{bmatrix} Z_1 & H \\ Z_2 & G \end{bmatrix}$, $\mathbb{A}_2 = \begin{bmatrix} -\beta_1 & 0 \\ 0 & I \end{bmatrix}$ and

$$\mathbb{D} = \begin{bmatrix} TA\alpha_2 - Z_1\beta_2 - TD \\ -Z_2\beta_2 \end{bmatrix} = \mathbb{D}_1 + \mathbb{Z}\mathbb{D}_2, \tag{36}$$

with $\mathbb{D}_1 = \begin{bmatrix} TA\alpha_2 - TD \\ 0 \end{bmatrix}$, $\mathbb{D}_2 = \begin{bmatrix} -\beta_2 \\ 0 \end{bmatrix}$ and

$$\mathbb{C} = \begin{bmatrix} \alpha_1 & 0 \end{bmatrix}. \tag{37}$$

*Remark 5: Based on all the above results, one can see that the unknown parameters H, G, Z_1 and Z_2 are contained in one matrix $\mathbb{Z}$. If matrix $\mathbb{Z}$ is determined, matrices H and G can be obtained. Besides, by using the matrices Z_1 and Z_2 , matrices F, J, P and Q can be deduced according to (23), (24), (25) and (26). Then, all the parameters matrices of DO (10) can be determined. In this case, the observer design problem in **Remark 2** is reduced to determine matrix $\mathbb{Z}$ such that*

- *Conditions (18) are satisfied,*
- *For $w = 0$, the estimation error $e \to 0$ for $t \to \infty$;*
- *For $w \neq 0$, $\|T_{we}\|_\infty < \lambda$,*

V. MAIN RESULTS

In this section, the previous results will be used to solve the DO design problem for uncertain vehicle systems in the presence of disturbances. For system (8), we give the following theorem to determine matrix $\mathbb{Z}$:

Theorem 1: For a given positive constant scalar λ, if there exist a symmetric positive definite matrix P_1, a matrix $\mathbb{P}$, and a given positive scalar γ, such that the following LMI holds

$$\begin{bmatrix} \Omega_1 & P_1\mathbb{B}M_1 & P_1\mathbb{B}M_2 & P_1\mathbb{F} & 0 & 0 & \Omega_2 \\ * & -I & 0 & 0 & 0 & 0 & 0 \\ * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & -\frac{1}{\gamma}I & 0 & 0 & 0 \\ * & * & * & * & N_1^TN_1 & 0 & 0 \\ * & * & * & * & * & N_2^TN_2 & 0 \\ * & * & * & * & * & * & \Omega_3 \end{bmatrix} < 0, \quad (38)$$

where

$$\Omega_1 = \mathbb{A}_1^TP_1 + P_1\mathbb{A}_1^T + \mathbb{A}_2^T\mathbb{P}^T + \mathbb{P}\mathbb{A}_2 + (1+\gamma)\mathbb{C}^T\mathbb{C}, \quad (39a)$$

$$\Omega_2 = P_1\mathbb{D}_1 + \mathbb{P}\mathbb{D}_2 + (\gamma+1)\mathbb{C}^T\mathbb{E}, \quad (39b)$$

$$\Omega_3 = (\gamma+1)\mathbb{E}^T\mathbb{E} - \lambda^2 I. \quad (39c)$$

Then system (8) is asymptotically stable for $w = 0$, and $\|T_{we}\|_\infty < \lambda$ for $w \neq 0$.

In this case, matrix $\mathbb{Z}$ can be deduced by

$$\mathbb{Z} = P_1^{-1}\mathbb{P}. \quad (40)$$

Proof: According to the literature [36], the problem presented in **Remark 5** can be summarized as to construct the Lyapunov function V such that the following inequality must holds:

$$\dot{V} + e^Te - \lambda^2 w^Tw < 0. \quad (41)$$

Let us choose the following Lyapunov function:

$$V = \zeta^TP_1\zeta. \quad (42)$$

where P_1 is a symmetric positive definite matrix.

Then the derivative of V is given by

$$\begin{aligned}\dot{V} &= \dot{\zeta}^TP_1\zeta + \zeta^TP_1\dot{\zeta} \\ &= \left\{\mathbb{A}\zeta + \mathbb{B}\Delta Ax + \mathbb{B}\Delta Bu + \mathbb{D}w + \mathbb{F}[f(\hat{x}) - f(x)]\right\}^T P_1\zeta \\ &\quad + \zeta^TP_1\left\{\mathbb{A}\zeta + \mathbb{B}\Delta Ax + \mathbb{B}\Delta Bu + \mathbb{D}w + \mathbb{F}[f(\hat{x}) - f(x)]\right\}^T \\ &= \zeta^T\mathbb{A}^TP_1\zeta + \zeta^TP_1\mathbb{A}\zeta + \zeta^TP_1\mathbb{D}w + w^T\mathbb{D}^TP_1\zeta \\ &\quad + 2\zeta^TP_1\mathbb{B}\Delta Ax + 2\zeta^TP_1\mathbb{B}\Delta Bu + 2\zeta^TP_1\mathbb{F}[f(\hat{x}) - f(x)].\end{aligned} \quad (43)$$

By inserting (35) and (36) into (43), we can obtain the following equation

$$\begin{aligned}\dot{V} &= \zeta^T[(\mathbb{A}_1 + \mathbb{Z}\mathbb{A}_2)^TP_1 + P_1(\mathbb{A}_1 + \mathbb{Z}\mathbb{A}_2)]\zeta \\ &\quad + \zeta^TP_1(\mathbb{D}_1 + \mathbb{Z}\mathbb{D}_2)w + w^T(\mathbb{D}_1 + \mathbb{Z}\mathbb{D}_2)^TP_1\zeta \\ &\quad + 2\zeta^TP_1\mathbb{B}\Delta Ax + 2\zeta^TP_1\mathbb{B}\Delta Bu + 2\zeta^TP_1\mathbb{F}[f(\hat{x}) - f(x)].\end{aligned} \quad (44)$$

Furthermore, define the matrix $\mathbb{P} = P_1\mathbb{Z}$ yields

$$\begin{aligned}\dot{V} &= \zeta^T[\mathbb{A}_1^TP_1 + P_1\mathbb{A}_1 + \mathbb{A}_2^T\mathbb{P}^T + \mathbb{P}\mathbb{A}_2]\zeta \\ &\quad + \zeta^T(P_1\mathbb{D}_1 + \mathbb{P}\mathbb{D}_2)w + w^T(P_1\mathbb{D}_1 + \mathbb{P}\mathbb{D}_2)^T\zeta \\ &\quad + 2\zeta^TP_1\mathbb{B}\Delta Ax + 2\zeta^TP_1\mathbb{B}\Delta Bu + 2\zeta^TP_1\mathbb{F}[f(\hat{x}) - f(x)].\end{aligned} \quad (45)$$

According to **Lemma 1** and the representations (6) and (7), by choosing $\rho = 1$, we can obtain the following inequalities:

$$\begin{aligned}2\zeta^TP_1\mathbb{B}\Delta Ax &= 2\zeta^TP_1\mathbb{B}M_1g(t)N_1x \\ &\le \zeta^TP_1\mathbb{B}M_1(P_1\mathbb{B}M_1)^T\zeta + x^TN_1^TN_1x, \quad (46) \\ 2\zeta^TP_1\mathbb{B}\Delta Bu &= 2\zeta^TP_1\mathbb{B}M_2g(t)N_2u \\ &\le \zeta^TP_1\mathbb{B}M_2(P_1\mathbb{B}M_2)^T\zeta + u^TN_2^TN_2u. \quad (47)\end{aligned}$$

Subsequently, by considering the Lipshitz condition (9), we have the following inequality:

$$\begin{aligned}&2\zeta^TP_1\mathbb{F}[f(\hat{x}) - f(x)] \\ &\le 2\left\|\zeta^TP_1\mathbb{F}\right\| \times \|f(\hat{x}) - f(x)\| \\ &\le 2\left\|\zeta^TP_1\mathbb{F}\right\| \times \gamma \times \|\hat{x} - x\| \\ &= \gamma \times 2\left\|\zeta^TP_1\mathbb{F}\right\| \times \|e\| \\ &\le \gamma \times (\left\|\zeta^TP_1\mathbb{F}\right\|^2 + \|e\|^2) \\ &= \gamma\zeta^TP_1\mathbb{F}(P_1\mathbb{F})^T\zeta + \gamma e^Te.\end{aligned} \quad (48)$$

Inserting inequalities (46), (47) and (48) into (45) yields

$$\begin{aligned}\dot{V} &\le \zeta^T[\mathbb{A}_1^TP_1 + P_1\mathbb{A}_1 + \mathbb{A}_2^T\mathbb{P}^T + \mathbb{P}\mathbb{A}_2]\zeta \\ &\quad + \zeta^T(P_1\mathbb{D}_1 + \mathbb{P}\mathbb{D}_2)w + w^T(P_1\mathbb{D}_1 + \mathbb{P}\mathbb{D}_2)^T\zeta \\ &\quad + \zeta^TP_1\mathbb{B}M_1(P_1\mathbb{B}M_1)^T\zeta + x^TN_1^TN_1x \\ &\quad + \zeta^TP_1\mathbb{B}M_2(P_1\mathbb{B}M_2)^T\zeta + u^TN_2^TN_2u \\ &\quad + \gamma\zeta^TP_1\mathbb{F}(P_1\mathbb{F})^T\zeta + \gamma e^Te.\end{aligned} \quad (49)$$

On the other hand, from (19b), we have

$$e^Te = \zeta^T\mathbb{C}^T\mathbb{C}\zeta + \zeta^T\mathbb{C}^T\mathbb{E}w + w^T\mathbb{E}^T\mathbb{C}\zeta + w^T\mathbb{E}^T\mathbb{E}w. \quad (50)$$

By inserting (50) into inequality (49), we have

$$\begin{aligned}&\dot{V} + e^Te - \lambda^2w^Tw \\ &\le \zeta^T[\mathbb{A}_1^TP_1 + P_1\mathbb{A}_1 + \mathbb{A}_2^T\mathbb{P}^T + \mathbb{P}\mathbb{A}_2]\zeta \\ &\quad + \zeta^T(P_1\mathbb{D}_1 + \mathbb{P}\mathbb{D}_2)w + w^T(P_1\mathbb{D}_1 + \mathbb{P}\mathbb{D}_2)^T\zeta \\ &\quad + \zeta^TP_1\mathbb{B}M_1(P_1\mathbb{B}M_1)^T\zeta + x^TN_1^TN_1x \\ &\quad + \zeta^TP_1\mathbb{B}M_2(P_1\mathbb{B}M_2)^T\zeta + u^TN_2^TN_2u + \gamma\zeta^TP_1\mathbb{F}(P_1\mathbb{F})^T\zeta \\ &\quad + (\gamma+1)(\zeta^T\mathbb{C}^T\mathbb{C}\zeta + \zeta^T\mathbb{C}^T\mathbb{E}w + w^T\mathbb{E}^T\mathbb{C}\zeta + w^T\mathbb{E}^T\mathbb{E}w) \\ &\quad - \lambda^2w^Tw.\end{aligned} \quad (51)$$

Or equivalently,

$$\dot{V} + e^Te - \lambda^2w^Tw$$

$$\leq \begin{bmatrix} \zeta \\ x \\ u \\ w \end{bmatrix}^T \begin{bmatrix} \Omega' & 0 & 0 & \Omega_2 \\ * & N_1^T N_1 & 0 & 0 \\ * & * & N_2^T N_2 & 0 \\ * & * & * & \Omega_3 \end{bmatrix} \begin{bmatrix} \zeta \\ x \\ u \\ w \end{bmatrix} \tag{52}$$

where

$$\begin{aligned}\Omega' = &\mathbb{A}_1^T P_1 + P_1 \mathbb{A}_1 + \mathbb{A}_2^T \mathbb{P}^T + \mathbb{P}\mathbb{A}_2 \\ &+ P_1 \mathbb{B} M_1 (P_1 \mathbb{B} M_1)^T + P_1 \mathbb{B} M_2 (P_1 \mathbb{B} M_2)^T \\ &+ \gamma P_1 \mathbb{F}(P_1 \mathbb{F})^T + (\gamma + 1)\mathbb{C}^T \mathbb{C}.\end{aligned} \tag{53}$$

It is obvious that a sufficient for (41) to be satisfied is the following LMI must holds

$$\begin{bmatrix} \Omega' & 0 & 0 & \Omega_2 \\ * & N_1^T N_1 & 0 & 0 \\ * & * & N_2^T N_2 & 0 \\ * & * & * & \Omega_3 \end{bmatrix} < 0. \tag{54}$$

In this case, by using Schur complement, LMI (54) becomes (38). The proof is completed

Remark 6: Based on the previous results, the observer design can be summarized in the following design procedure:

1) *Choose the matrix T according to (22);*
2) *Compute the matrix Σ according to (31);*
3) *Compute the matrices $\alpha_1, \alpha_2, \beta_1$ and β_2 according to (29) and (30);*
4) *Compute the matrices $\mathbb{A}_1, \mathbb{A}_2, \mathbb{D}_1, \mathbb{D}_2$ and $\mathbb{C}$ according to* ***Remark 5***;
5) *Compute the matrix $\mathbb{Z}$ according to the* ***Theorem 1***;
6) *Deduce matrices F, J, P and Q, according to (23), (24), (25) and (26).*
7) *Deduce matrix E according (18).*

VI. NUMERICAL EXAMPLE

In this section, a numerical example [8] is presented to show the performance of the proposed DO. The initial system states are $x_1(0) = 1$ and $x_2(2) = 0.4$. The nonlinear function $f(x) = [0.1 \sin x_1 \; 0.8 \sin x_2]^T$. The parameters of system uncertainty are $\sigma = 0.25$ and $N = 0.25$. By following the design procedure in Remark 7, we can obtain that $\lambda = 3.64$ and $\gamma = 2.86$.

The simulation results are shown in the following figures. **Figure 2** represents the disturbance and the control input, which are added in the simulation. The external disturbance $w = \sin 0.3\pi t$ is a sine wave signal from time 12s to 24s. The control input u is a step signal from time 15s to 35s.

Figure 3represents the original system state x_1 and the estimate $\hat{x}_1$ obtained from DO. **Figure 5** represents the original system state x_2 and the estimate $\hat{x}_2$ obtained from DO. The solid line represents the original system state. The dashed line represents the estimates obtained from DO. **Figure 4** represents the estimation error e_1 obtained from DO. **Figure 6** represents the estimation error e_2 obtained from DO.

From the above figures, one can see the performance of the proposed DO. Take **Figure 3** for example. Firstly, let us see the case that the control input is absence and the external disturbance is appearance (from time 12s to 15s). Due to the

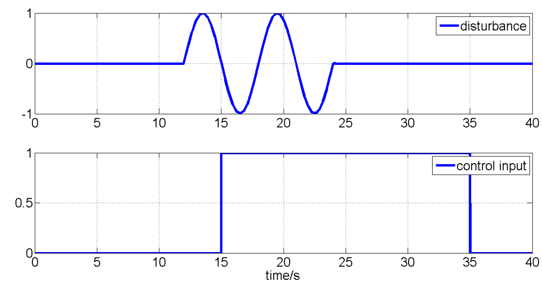

FIGURE 2. The disturbance w and the control input u.

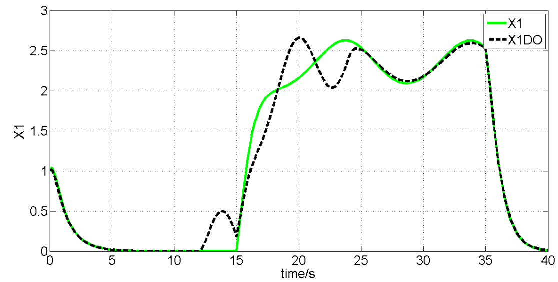

FIGURE 3. The state x_1 and its estimate $\hat{x}_1$ (solid line: original state; dashed line: DO).

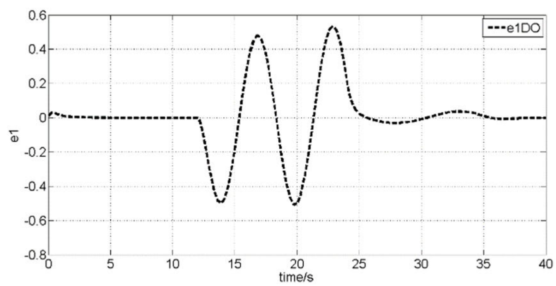

FIGURE 4. The estimation error e_1.

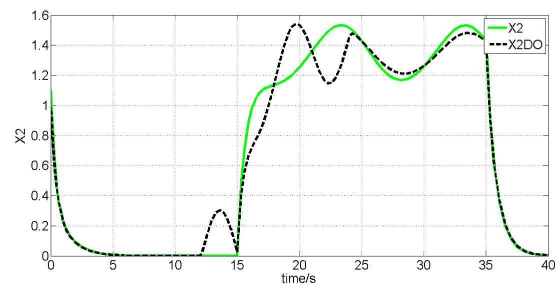

FIGURE 5. The state x_2 and its estimate $\hat{x}_2$ (solid line: original state; dashed line: DO).

appearance of disturbance signal, the estimate is not zero. By using the H-infinity approach, the negative influence of external disturbance on the estimation is limited.

Secondly, let us turn to the case that the control input and the external disturbance are both appearance (from 15s to 24s). In this case, similarly to case above, by using the H-infinity approach, the negative effect on estimate is limited.

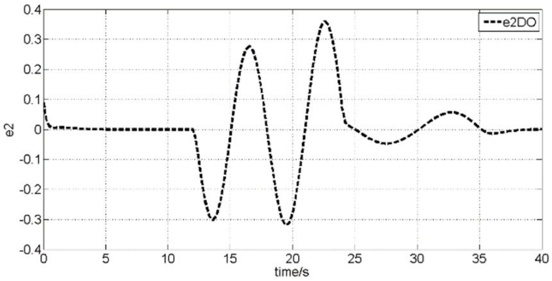

FIGURE 6. The estimation error e_2.

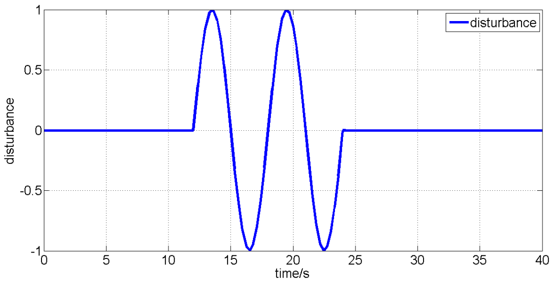

FIGURE 7. The disturbance w.

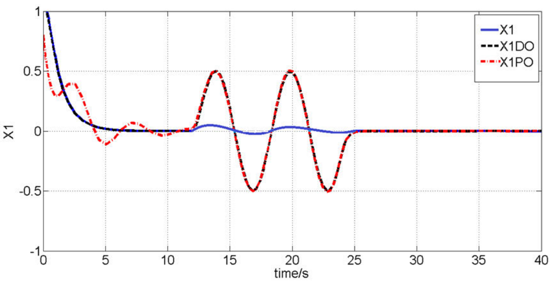

FIGURE 8. The state x_1 and its estimates (solid line: original state; dashed line: DO; dash-dotted line: PO).

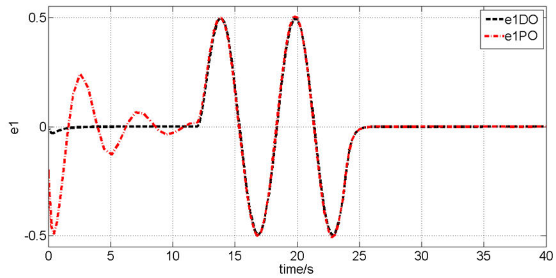

FIGURE 9. The estimation errors e_1 (dashed line: DO; dashed-dotted line: PO).

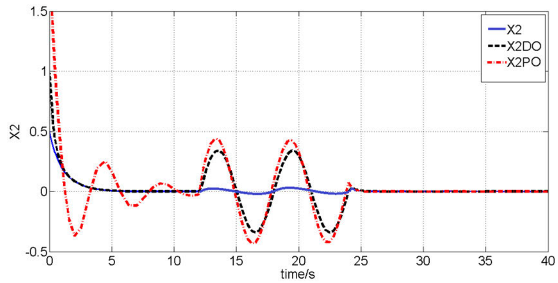

FIGURE 10. The state x_2 and its estimates (solid line: original state; dashed line: DO; dash-dotted line: PO).

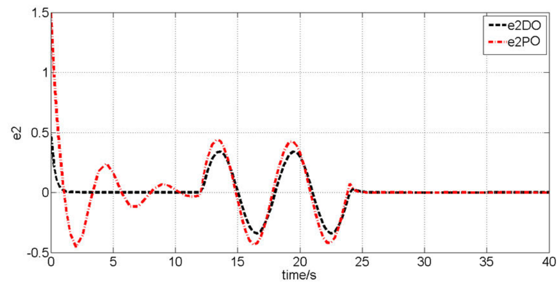

FIGURE 11. The estimation errors e_2 (dashed line: DO; dashed-dotted line: PO).

Lastly, in the case that the control input is appearance and the external disturbance is absence (from 24s to 35s). The estimate obtained from DO can track the original system state exactly.

Corresponding to the estimation of system state, the estimation error shows more clearly the performance of the proposed DO. Take **Figure 4** for example. In the case that the disturbance appears (from time 12s to 24s), by using the H-infinity approach, the influence of disturbance on estimation error is limited.

The comparison between the proposed DO and the PO has also been done. The simulation results are shown in the following figures. The external disturbance $w = \sin 0.3\pi t$ is a sine wave signal from time 12s to 24s, which is shown in **Figure 7**. The control input u is 0.

Figure 8 and **Figure 10** represent the original system states and their estimates obtained from DO and PO. The solid line represents the original system state. The dashed line represents the estimates obtained from DO and the dash-dotted line represents the estimates obtained from PO.

Figure 9 and **Figure 11** represent the estimation errors obtained from DO and PO. The dashed line represents the estimation error obtained from DO and the dash-dotted line represents the estimation error obtained from PO.

From the above figures, one can see the performances of the proposed DO, compared with PO. For state x_1, let us turn to Figure 8. At the beginning of the simulation, there are some oscillations in the estimation of PO, while there are no oscillations in the estimation of the proposed observer. This is because the existence of extra vector v in the proposed observer, which can restrain the oscillation. In the case that the external disturbance appears in the simulation, the proposed DO has the same estimation error as PO for state x_1.

For state x_2, let us turn to Figure 10. At the beginning of the simulation, there are some oscillations in the estimation of PO, while there are no oscillations in the estimation of the proposed observer. Furthermore, in the case that the external disturbance appears in the simulation, the proposed DO has also small estimation error for state x_2.

VII. CONCLUSION

This paper has investigated the DO design problem for nonlinear vehicle systems in the presence of uncertainties and disturbances. The nonlinearity of vehicle systems is assumed to satisfy Lipschitz condition, and the unknown uncertainty is assumed to be dependent in both system state and control input matrices. The H-infinity approach is utilized to mitigate the disturbance effect on the estimation performance.

The vehicle system is constructed, based on the two-degree-of-freedom model, which contains the nonlinearity, the uncertainty and the disturbance.

Then the algebraic constraints of observer parameter matrices are derived, based on the analysis of estimation error. Based on the analysis of the algebraic constraints, we can reduce the number of unknown parameter matrices. Firstly, we obtain the expression of parameter matrices F, J, P, Q, R and S. Then we found that if matrices Z_1 and Z_2 are determined, matrices F, J, P and Q can be deduced. Then the unknown parameter matrices are reduced to matrices Z_1, Z_2, H and G. Furthermore, we put the four matrices into one matrix $\mathbb{Z}$, which can be obtained by solving the optimization problem.

Next, the observer design problem is transferred into an asymptotically stability analysis problem, by rebuilding the state representation of estimation error. Sufficient conditions for the existence of the DO is given in terms of LMI, based on the Lyapunov function.

In the end, a numerical example is presented to illustrate the performance of the proposed DO. From the simulation results, it is proved the performance of the proposed DO. By using the H-infinity method, the proposed DO can deal with the influence of the disturbances. In terms of the uncertainty, the proposed DO can keep robust. Furthermore, compared with PO, the proposed DO also has good performance. The results proves the reliability and the availability of the proposed DO.

In the future, we will extend the research to the following parts:

a. Observer design for systems with time-delay. Time-delay is another inherent characteristic of practical system, which can be caused by the system structure and information transformation, and can deteriorate the performance of control system. We will try to address the observer design problem for time-delay systems, by finding a suitable representation of time-delay.

b. Observer-based control. The proposed DO will be extended to observer-based control for practical systems, especially for vehicle systems. By using the proposed DO, we can improve the estimation performance of the unmeasured variables of vehicle systems, such as sideslip angel, yaw rate and tire-road peak adhesion coefficient.

c. Fractional calculus theory. The fractional calculus theory and concepts is a useful tool to solve control problem for nonlinear systems [37], [38]. It can be one potential research direction in the future. We will try to solve the fractional observer design for non-linear systems, based on the research results.

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