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RESEARCH ARTICLE

A Cutting-Edge TOPSIS Approach for Navigating MCDM Challenges Under t-Intuitionistic Fuzzy Environments

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ABSTRACT The significance of t-intuitionistic fuzzy TOPSIS lies in its ability to address challenges related to uncertainty and ambiguity within the decision-making process. The incorporation of the "t" parameter as a t-norm and t-conorm operator offers a more comprehensive and precise approach, making it essential in scenarios where standard intuitionistic fuzzy TOPSIS approaches are insufficient. The implementation of this methodology across multiple domains enhances the dependability and robustness of decision-making procedures. The classification of the technique as an indispensable tool with a broad spectrum of applications is substantially bolstered by its fundamental qualities of versatility, adaptability, and flexibility. In this study, we introduce a novel distance measure called the lift-distance measure between t-intuitionistic fuzzy sets and examines its structural properties. Then, the superiority of this new distance measure is compared with some existing distance measures. To address circumstances with inherent ambiguity, we present a novel decision-making tool called the t-intuitionistic fuzzy TOPSIS technique based on the proposed distance measure. The integration of t-intuitionistic fuzzy terminology into this approach augments the versatility and inclusiveness of the TOPSIS methodology. The case analysis demonstrates that the developed strategy's effectiveness and precision are superior to those of established alternatives. By providing a flexible and all-encompassing tool for decision-making in conditional environments, the application of this methodology possesses the capacity to generate significant favorable results. In addition, a comparative analysis is undertaken, which includes established TOPSIS methods, to demonstrate the improved performance of the proposed method. The comparison results demonstrate the ability of the proposed method to effectively capture the imprecision or vague differences in t-intuitionistic fuzzy sets so as to obtain more accurate and reliable ranking results.

INDEX TERMS t-intuitionistic fuzzy set, distance measure, closeness coefficient, intuitionistic fuzzy TOPSIS technique, decision making, optimization.

I. INTRODUCTION

A. BACKGROUND

Operations research (OR) is an academic subject that uses advanced analytical methods to help make better decisions. It is a broader and more descriptive way to refer to this academic discipline, which is also known as management science. More generally, as business professionals, we can use OR to analyze and model a problem, gather objective data, and make a decision based on that data with solid scientific principles. It involves the examination of many alternatives based on specific and categorical criteria for

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arriving at an optimal or at least satisfactory solution to a given problem and involves the determination, by calculation, of an optimal solution from the set of all feasible solutions. Multi-criteria decision analysis is a process utilized in operations research wherein a large number of alternative options are compared and contrasted in order to ascertain the most optimal one. Multi-criteria decision-making (MCDM) methodology is a very important area of research on dealing with this type of complex decision-making problem involving a large number of criteria or objectives [1], [2]. As such, MCDM methodologies represent one of the most important tools that business analysts have to assess the competing alternatives a decision-maker may face when the problem involves a large number of complex and often inter-related objectives. In fact, techniques and methods for solving MCDM problems are very helpful for finding out how important different goals really are, for showing the trade-offs between goals that always come up during the selection process, and for making better, more complex decisions in the end. MCDM has enabled business analysts to generate more logically consistent recommendations for organizations when confronted with complex scenarios. Academics, engineers, environmental planners, regulators, and managers are just a few of the disciplines that have embraced MCDM techniques. Therefore, by utilizing MCDM, individuals in positions of authority are not only able to evaluate more critically but also attain superior performance and greater personal returns. Hence, it can serve as a tool for effectively managing decision-making processes when resources are limited, competing objectives are present, and the potential consequences of different courses of action are uncertain. Recent research investigations have concentrated on the utilization of the MCDM method to examine intricate decision-making challenges that encompass numerous objectives or criteria. So, many MCDM methods have been created to deal with these issues. These include the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), the Analytic Hierarchy Process (AHP), and the Analytic Network Process (ANP). The TOPSIS method is one of the most prevalent MCDM approaches. An exhaustive approach to identifying the optimal alternative is MCDM. Certainly, this approach makes decision-makers capable of perceiving the most suitable alternative by allowing them to compare many attributes over diverse criteria or objectives, as they can potentially lead to significant payoffs. Every available alternative is given a score based on the given criteria. The comparison of other options results from a mix of assessments, which shows its extreme power in both the comparison and the comprehensive value assigned to different alternatives. Consequently, to successfully compare other options, managers can use this generally accepted methodology to ensure that all criteria are considered without exclusion and bias. Hence, it is significant that this tool uses their own judgment in determining which of these methodologies is most appropriate for addressing a particular decision-making challenge. Conventional TOPSIS techniques in MCDM are insufficient for dealing with real-world situations because they heavily rely on exact numerical values and fail to account for the ambiguity and imprecision that are inherent in such problems. In order to address this lack of clarity, TOPSIS approaches have been augmented in the academic literature in uncertain contexts. Although the literature discusses many TOPSIS-based methods for decision-making difficulties, each technique has distinct strengths and weaknesses.

During the evaluation process, it is essential to determine the meaning of each attribute that represents the target. For example, when evaluating whether a teacher is fully prepared, a decision-maker may provide a binary opinion of "yes" or "no". However, there are many uncertainties and subjective terms such as "very poor", "good", or "excellent" that come up in practical decision-making scenarios. These terms do not always correspond to precise data. To address these issues, the fuzzy set theory ($\mathbb{FST}$) was introduced by Zadeh [3] in 1965. A mathematical framework designed to represent and analyze data that is ambiguous and imprecise. To address these uncertainties, the implementation of a "$\mathbb{FST}$" is feasible. The implementation of $\mathbb{FST}$ enables the expansion of evaluation values from the binary scale of 0,1 to a broader and more flexible range of [0, 1]. It defines $\mathbb{FS}$ in terms of membership degrees and this allow us to model gradual transitions between sets. The $\mathbb{FST}$ has been widely applied in engineering, business management, education, artificial intelligence and other domains. Fuzzy Logic has successfully managed multiple industrial systems, like air conditioners, washing machines, and elevators. Fuzzy numbers (FNs) are significant in $\mathbb{FST}$ as they provide a generalized platform for expressing imprecise, incomplete, and inconsistent information. FNs help solve MCDM problems and communicate and reflect evaluation information in multiple dimensions. However, some researchers have developed solutions for solving MCDM problems in classical fuzzy environments [4], [5], [6], [7], [8], [9]. The membership function in $\mathbb{FST}$ may not accurately represent complex information. With the progression of theoretical investigation and practical illustration, novel challenges have emerged. For instance, when a group of five individuals is asked to evaluate a teacher's patience with students, two may respond with agreement/yes, two may disagree/no, and one may feel uncertain or choose not to answer. It is essential to consider this hesitancy in the decision-making process. To tackle these problems, the intuitionistic fuzzy sets ($\mathbb{IFS}$) was initially developed by Atanassov [10] in 1986. The theory of $\mathbb{IFS}$ differs from $\mathbb{FS}$ in that they include a membership degree, non-membership degree, hesitancy degree, or intuitionistic index. These aspects align with humans who express decisions using negation, affirmation, or hesitation. Thus, $\mathbb{IFS}$s are more effective in describing and gathering decision-making information. In situations where decision-makers judgments are unclear, or knowledge about the problem is lacking, $\mathbb{IFS}$ is used to reduce uncertainty and better represent decision-makers preferences. This phenomenon presents more uncertainty than $\mathbb{FS}$, making it a more flexible framework. It can be helpful in circumstances where the uncertainty is more complex and traditional $\mathbb{FS}$ are unable to capture it.

B. LITERATURE REVIEW

The operations on $\mathbb{IFS}$ were defined by Zeng and Li [11]. A crucial aspect of the evaluation process is the aggregation of expert opinions towards reaching a collective decision. Researchers have studied $\mathbb{IFS}$ in the MCDM field due to this pioneering work and other significant contributions. Xu [12] proposed the intuitionistic fuzzy weighted averaging operator. Xu and Yager [13] introduced the intuitionistic fuzzy weighted geometric aggregation operator, the intuitionistic fuzzy ordered weighted geometric aggregation operator, and the intuitionistic fuzzy hybrid geometric aggregation operator. The intuitionistic fuzzy framework has been used to design and implement numerous MCDM methods.

The paper primarily focuses on the intuitionistic fuzzy TOPSIS ($\mathbb{IF}$-TOPSIS)method.

Classical TOPSIS has thus been extended to fuzzy TOPSIS to account for the uncertainty in decision-maker evaluations; it has been implemented in numerous studies that address MCDM problems. Due to its reliable and effective ambiguity and imprecision, fuzzy TOPSIS is essential for decision-making. The robust fuzzy TOPSIS approach rapidly combines fuzzy sets to use subjective or ambiguous criteria in complicated real-world circumstances where correct data is difficult. Fuzzy TOPSIS is a method that can be used to select the best option among similar choices. It can also automate the selection process and overcome ambiguity and uncertainty. Its qualitative-quantitative integration improves environmental management, finance, and engineering. Fuzzy TOPSIS improves judgments and offers a systematic framework for evaluating and selecting solutions to complex issues with many criteria. Subjective opinions of benefits and adverse outcomes are considered during evaluation. The method's capacity to combine theoretical models into actual choice situations promotes well-informed, suitable, and efficient decision-making. Chen and Hwang [14] formulated the fuzzy TOPSIS method. Chu and Lin [15] developed a fuzzy TOPSIS technique for evaluating the selection process of robot selection. Several scholars have proposed various fuzzy MCDM techniques and their practical applications for portfolio selection problems [16], [17], [18]. It is widely acknowledged that assessing objects involves more than just positive or negative evaluations. Uncertainty can also be expressed through responses such as "no," "difficult to say," or "no opinion." When decision-makers want to convey uncertainty, there are better approaches than fuzzy TOPSIS. This is because it does not explicitly consider non-membership degrees. To address this issue and accurately incorporate the expert's level of uncertainty, it is essential to enhance the fuzzy TOPSIS methodology by integrating comprehensive techniques. The $\mathbb{IF}$-TOPSIS architecture represents a noteworthy improvement over conventional fuzzy TOPSIS methods. By incorporating resistance and non-membership levels, this comprehensive approach effectively manages uncertainty. Additionally, it accommodates varying levels of interest and motivation by thoroughly simulating decision scenarios. Applying this methodology results in more precise data analysis, incorporating a sense of hesitancy that accurately conveys the degree of ambiguity or belief expressed by decision-makers. The $\mathbb{IF}$-TOPSIS decision-making method also demonstrates superior discriminatory capability. Choosing between options, even in situations where the differences are minimal, can offer many advantages. Despite the complexity of such scenarios, this distinct quality allows individuals or organizations to function at their best. Many methods have been suggested to overcome the challenges inherent in the process of selecting a vendor. Specifically, specific models have been created to include provisions considering the rating process's imprecision. Boran et al. [19] proposed an integrated method based on the group decision- making (GDM) with the $\mathbb{IF}$-TOPSIS for supplier selection in a group decision-making scenario. This study employs many decision-makers and criteria to choose a supplier. The intuitionistic fuzzy multi-criteria group supplier selection technique study shows that TOPSIS may be successful. When the supplier selection process is meant to be more accurate, it should include different alternatives, their performance according to the decision maker's criteria, and the criteria' priority, which helps express uncertain knowledge more precisely. GDM professionals who understand and address problems may be preferable. The intuitionistic fuzzy weighted aggregate (IFWA) operator, as one of the multi-criteria aggregation operators, is able to utilize the DMs perspective for prioritizing the criteria and ranking of alternatives. Sen et al. [20] proposed several approaches, such as $\mathbb{IF}$-TOPSIS, $\mathbb{IF}$-MOORA, and $\mathbb{IF}$-GRA, to this empirical dataset to present a methodology for sustainable supplier selection. However, they also recognized the drawbacks of these methods. This area was also enhanced by Memari et al. [21] as they unveiled an $\mathbb{IF}$-TOPSIS technique that allows spare parts manufacturers to determine the eco-friendliest supplier. This method achieves reliable and accurate supplier rankings by considering thirty sub-criteria and nine primary criteria. This approach, empirically demonstrated through a case study, is a viable solution to debates about sustainable procurement. Secondly, it addresses identified theoretical gaps in a previous study [20]. Interval-valued intuitionistic fuzzy set (IVIFS) can enhance one's ability to handle uncertainties and ambiguities more efficiently. There are structured methods for addressing situations with inaccurate or ambiguous information. These methods can prove especially valuable when making decisions that rely on subjective criteria and personal preferences rather than objective facts. Due to the intricate and unpredictable nature of physical phenomena, researchers have focused on MCDM problems, necessitating the integration of interval-expressed criterion values. Ye [22] proposed and used an extended method for the TOPSIS method for GDM with IVIFS to address the partner selection issue. This technique is better suited for addressing the partner selection problem in an environment with incomplete and uncertain information. Boran [23] suggested combining 𝕀𝔽 preference relations that aim to get weights for criteria and the 𝕀𝔽-TOPSIS method that seeks to rank options when there is not enough information to decide on the best location for a facility. In GDM problems, it is essential to aggregate the opinions of decision-makers in order to evaluate the problem accurately. However, the existing aggregation operator has a limitation. Tan [24] developed a generalized interval-valued intuitionistic fuzzy geometric aggregation operator (IVIFGAO) to overcome these limitations. This operator can aggregate the opinions of individual decision-makers under an interval-valued intuitionistic fuzzy GDM environment. He also defined the Choquet integral-based Hamming distance between interval-valued intuitionistic fuzzy values. The operator is combined with TOPSIS on Choquet integral-based Hamming distance to investigate multi-criteria interval-valued intuitionistic fuzzy GDM. This approach considers the interaction phenomena among the decision-making problems. It has been observed that there has been no research on the topic of dynamic MAGDM problems in an intuitionistic fuzzy environment. To address this gap, Su et al. [25] studied dynamic intuitionistic fuzzy multiple attribute group decision-making problems. These problems involve using attribute values provided by multiple decision-makers or experts at different periods in the form of intuitionistic fuzzy numbers. They also provided a numerical example to explain their method. The evaluation of renewable energy technologies for electricity generation in Turkey using 𝕀𝔽-TOPSIS was done by Boran et al. [26]. Intepe et al. [27] proposed the interval-valued 𝕀𝔽-TOPSIS technique for solving the technological forecasting technique selection problem. When there are many alternatives, it can be difficult for decision-makers to evaluate them. In such situations, it may be necessary to categorize the options based on specific criteria. Preference relations commonly express decision-makers' preferences for alternatives or criteria. Xia and Xu [28] defined the intuitionistic fuzzy multiplicative preference relation and presented a methodology for GDM based on this notion. They provided an example of a developed technique to demonstrate the efficiency of this approach. Vahdani et al. [29] developed an extended elimination and choice translation reality (ELECTRE) technique for multi-criteria group decision-making (MCGDM) problems in the framework of 𝕀𝔽𝕊. They showcased the comparative analysis of the proposed ELECTRE approach and the existing 𝕀𝔽-TOPSIS methodology, utilizing an example application in flexible manufacturing systems. The study demonstrated the potential benefits of the ELECTRE approach and its superiority over the existing 𝕀𝔽-TOPSIS method. The Dombi operational parameter has a natural flexibility with variable resilience. It is significant in expressing experts' attitudes in decision-making. In [30], Hussain et al. proposed an intuitionistic fuzzy rough TOPSIS method based on Dombi operations. Furthermore, they demonstrate how to solve the MCGDM problem, which involves identifying the most critically ill COVID-19 patients using this approach. They conduct a comparative analysis with other existing methods to evaluate the effectiveness and superiority of the developed model. In [31], Kucukvar et al. described an intuitionistic fuzzy approach to representing the judgments of decision-makers used to evaluate the weights of various life cycle phases, sustainability indicators, and alternatives. Decision-makers require tools to evaluate the quality and reliability of information. Intuitionistic fuzzy entropy fulfills this need by offering a metric to assess the level of disorder or unpredictability in 𝕀𝔽𝕊. Joshi and Kumar [32] proposed a technique based on distance measures and 𝕀𝔽 entropy. Macias et al. [33] presented a TOPSIS decision-making model within an 𝕀𝔽 environment for evaluating the ergonomic compatibility of AMT. Yue [34] proposed a new methodology for GDM problems in an intuitionistic fuzzy environment. This model employs an advanced version of the TOPSIS technique to determine decision-makers' weights. The group decision for alternatives is formed by aggregating the decisions of individual decision-makers. The extended TOPSIS technique is utilized to establish a preference ranking of alternatives. Furthermore, a comparative analysis was conducted to showcase the significant technical advancements of this model compared to other methods. Chen [35] introduced a modified hybrid averaging method with an inclusion-based ordered weighted averaging operation to form a collective decision-making under intuitionistic fuzzy environment. Zhang and Xu [36] designed a soft computing method based on maximizing consensus and fuzzy TOPSIS to solve interval-valued intuitionistic fuzzy MAGDM problems involving both parts of the decision data. Tlig and Rebai [37] defined a fuzzy TOPSIS method based on intuitionistic fuzzy values to solve MCDM problems in which linguistic terms represent the performance rating values and the criteria weights. Using the 𝕀𝔽-TOPSIS, Rouyendegh et al. [38] investigated the problem of selecting green suppliers. Tiwari et al. [39] proposed an interval-valued 𝕀𝔽-TOPSIS method for the supplier selection problem. Wu et al. [40] presented a new 𝕀𝔽-TOPSIS method and demonstrated that it increases monotonically with these three linear phases. In the context of evidence theory, Ludmila et al. [41] presented the generalization of the TOPSIS method in an intuitionistic fuzzy environment. Baccour et al. [42] proposed semi-metric distance measures between 𝕀𝔽𝕊. Szmidt and Kacprzyk [43] defined the ideas of Haming and Euclidean distance measures on 𝕀𝔽𝕊. Distance measures using the 𝕀𝔽𝕊 were characterized by Wang and Xin [44]. In [45], Nagan et al. introduced the H-max distance measure of 𝕀𝔽𝕊. Yan and Chiclana [46] presented extended Hausdorff distance measure of 𝕀𝔽𝕊. In [47], Jin et al. defined a novel distance measure of 𝕀𝔽𝕊. Liu [48] devised a novel distance-based 𝕀𝔽-TOPSIS method for the instruction of physical education quality evaluation.

Shen et al. [49] extended 𝕀𝔽-TOPSIS method based on the proposed distance measure for MCDM. A practical application of credit risk assessment for prospective strategic partners was also discussed in the sane paper. Rouyendegh et al. [50] proposed a new 𝕀𝔽-TOPSIS for the wind farm site. Roszkowska et al. [51] investigated an 𝕀𝔽-TOPSIS to assess socioeconomic phenomena based on survey data. The theory of t-𝕀𝔽𝕊 can handle more complex and ambiguous decision-making problems where the decision-makers preferences may be more uncertain or vague or where the criterion are difficult to quantify. The t-𝕀𝔽𝕊 shows excellent promise and effectiveness in modelling human involvement in human-based intelligence to achieve modernization across multiple departments, including data analysis, data mining, image coding and interpretation, and intelligence systems. Sharma [52] presented the idea of t-intuitionistic fuzzy set (t-𝕀𝔽𝕊).

C. RESEARCH GAP

The process of selecting a smart phone is complex, as there are many subjective variables to consider. To make well-informed decisions, it is essential to use the MCDM approach, which helps balance vital factors with goals. Price, storage capacity, camera capabilities, battery life, features, and brand awareness should be considered when choosing a smartphone. However, the data might need to be more precise or clear, making the selection process challenging. One of the most effective methods of MCDM is the fuzzy TPSIS approach, which is widely recognized for its efficiency. In fuzzy TOPSIS, each smartphone selection factor's membership degree indicators are satisfied. However, the applicability of this approach is weak due to its reliance on membership (satisfaction) degree. In a real-world problem, a customer always pays equal attention to the consideration of the non-satisfactory degree during the selection of a smartphone, such as in situations that involve the respecting and consensus of the experts' opinions, significantly affecting the decision-making process. On the other hand, concerning the 𝕀𝔽-TOPSIS environment, the conflicting objectives are resolved considering each factor of smartphone selection's acceptance membership (satisfaction) and rejection membership (dissatisfaction). Consumers may assign varying degrees of importance to different attributes, such as pricing and features. Fuzzy TOPSIS makes it possible to depict these preferences by utilizing fuzzy numbers, which permit the simulation of compromises. For example, a customer may pay a premium for a smartphone with an exceptional camera. Additionally, they may need clarification regarding the significance of the most recent technological advancements in the camera's functionalities. However, fuzzy TOPSIS cannot handle situations where customers have doubts or reservations concerning particular criteria. In such cases, 𝕀𝔽-TOPSIS is incredibly beneficial. Utilizing non-membership degrees enables decision-makers to overtly articulate their uncertainties, offering a more accurate depiction of hesitancy within the decision-making procedure.The 𝕀𝔽-TOPSIS model has limitations in handling scenarios where there are varying priorities for both the price and storage capacity of specific devices. For instance, some may prioritize a 75% decrease in price, while others may prioritize an 80% increase in storage capacity. In these cases, it is necessary to design a mathematical mechanism that can handle this situation. In this way, the t-𝕀𝔽-TOPSIS emerges as a powerful methodology to counter such situations. The t-𝕀𝔽-TOPSIS is a practical and beneficial approach that utilizes distance measurements to rank and select externally established options. By incorporating satisfaction levels (acceptance), rejection, abstention, and priorities (parameter ''t'') linked to objectives, this methodology simplifies the decision-making process, improves the coherence of reasoning, and reduces subjectivity. This approach simplifies decision-making by enabling comprehensive evaluations, comparisons, and prioritization of alternatives. The strategy gains significance in the context of developed strategies largely because it offers decision-makers a framework that delivers optimal options based on a well-selected parameter value. It demonstrates efficacy and adaptability and is a valuable tool for those in positions of authority. Competitive advantages are derived from the capacity to navigate complicated decision-making scenarios efficiently. Implementing this methodology ensures a thorough and impartial assessment of the decision-making procedure, resulting in more advantageous outcomes.

D. MOTIVATIONS

A notable decision-making tool in multiple disciplines is the t-𝕀𝔽-TOPSIS technique, which is based on the t-norm and t-conorm operators. The corresponding motivation for initiating research is presented in the following discussion:

- The t-𝕀𝔽-TOPSIS methodology creates accurate decision-making standards by assessing uncertain data through t-norm and t-conorm operators. This technique proves to be exceptionally effective when dealing with intricate, mysterious, and elusive data. When dealing with complex situations that require informed assessments from multiple factors, having a cohesive framework to handle ambiguity, enhance flexibility, and reduce doubt can be highly beneficial.
- Employing a multi-criteria decision-making scheme, the t-𝕀𝔽-TOPSIS strategy is practical as well as effective. This paper consolidates a substantial portion of the intricate interconnections. Furthermore, it illuminates an extensive array of viewpoints regarding the issue and, in certain ways, expands the academic discipline in which these aspects are interrelated. Moreover, the utilization of decision makers linked to the 't' parameter can be advantageous in addressing specific challenges.
- By incorporating the 't' parameter into this methodology, it becomes more feasible to account for a diverse array of weights and criteria. This informed approach ultimately seeks to optimize and accelerate decision-making. By implementing this methodology, a precise

numerical framework can be established to facilitate the decision-making process. This innovative approach streamlines proximity coefficient calculations and criteria weighting distribution, ensuring impartial ranking and enhancing selection.

E. NOVELTY OF THE CURRENT STUDY

The subsequent discussion highlights the novelty of the current research:

- The novel t-$\mathbb{IF}$-TOPSIS approach incorporates t − $\mathbb{IFS}$, which are superior to the conventional $\mathbb{IF}$-TOPSIS approach in their ability to represent uncertainty, hesitation, and ambiguity. It enables decision-makers to utilize data that may be perplexing effectively.
- The methodology under consideration presents an innovative strategy for understanding and contrasting intuitionistic fuzzy sets by integrating the t-norm and t-conorm operators. The parameter 't' facilitates flexibility in representing complex relationships between criteria and options across a range of decision scenarios.
- One of the noteworthy features of this approach is its intended incorporation of hesitancy into the decision-making process. The suggested framework recognizes that decision-makers may face inherent uncertainties during their evaluations and offers a structured way to deal with and incorporate these uncertainties effectively.
- A diverse range of t-norm and t-conorm operators are accessible to decision-makers to tailor the procedure to their specific criteria. This methodology has the capability to integrate multi-layered analysis, evaluate interrelationships with different levels of certainty, and accommodate a broad spectrum of domains and applications. This practical approach offers a mathematical framework to assess potential outcomes in circumstances where there is vagueness and inaccuracy.
- Through the custom of a novel methodology to determine the significance of criteria and compute the closeness coefficient, we enable unbiased decision-making. This trend represents a significant advancement in the field of decision-making, specifically in situations involving membership, non-membership, hesitation, and the 't' parameter.
- The capacity of this technique to capture complex and nonlinear relationships between criteria and judgments is well recognized. It exhibits significant advantages in many situations characterized by intricate and constantly evolving circumstances.

F. OBJECTIVE OF THE CURRENT RESEARCH WORK

This article aims to achieve specific objectives, which are as follows:

- Propose the idea of the lift distance measure between t-$\mathbb{IFS}$. This phenomenon holds great importance as it has the potential to aid decision-making in uncertain situations, where the classical $\mathbb{IFS}$ cannot deal with such situations. It is also relevant in tasks related to grouping and classification, feature selection, and algorithm design recommendations. The presented framework provides a quantitative methodology for assessing and comparing t-$\mathbb{IFS}$, leading to notable insights and informed decision-making in many fields of study.
- Explore and prove various mathematical properties of the newly defined lift distance measure.
- Propose the methodology of the t-$\mathbb{IF}$-TOPSIS. The t-$\mathbb{IF}$-TOPSIS is effective in addressing practical difficulties. When consumers are seeking to purchase a certain device and possess diverse requirements about cost and storage space, this approach proves to be quite beneficial. The t-$\mathbb{IF}$-TOPSIS methodology facilitates the production of different alternatives by selecting unique factors and identifying a viable solution that meets various preferences.
- The present research pertains to the practical implementation of the proposed approach aimed at discerning the most proficient individuals eligible for graduate scholarships. It will help in the identification and selection of the most meritorious candidates for these scholarships.
- To properly evaluate the effectiveness of the newly designed strategy, it is important to conduct a comparative analysis with established approaches. It will allow us to identify its strengths and weaknesses. We will carry out a thorough assessment using real-world data and scenarios to gauge how well the proposed strategy performs in comparison to other tactics.

The present content is organized into many sections to improve overall clarity and readability in the following ways: A succinct summary of the essential definitions of t-$\mathbb{IFS}$ that are crucial for the accomplishment of this study is discussed in Section II. A novel distance measure known as the lift-distance measure, which proceeds to examine its mathematical characteristics, is presented in Section III. This section also performs a comparative study to demonstrate its effectiveness. The t-$\mathbb{IF}$-TOPSIS approach is developed, and its efficacy is depicted in resolving the matter of selecting graduate scholarship students in Section IV. A comparison between the fuzzy TOPSIS, $\mathbb{IF}$-TOPSIS and t-$\mathbb{IF}$-TOPSIS techniques are discussed in Section V. This section also contains the limitations of the proposed techniques. Finally, the conclusions of the current study are presented in Section V.

II. PRELIMINARIES

This section provides an overview of a few fundamental concepts of t-$\mathbb{IFS}$ that are essential to comprehending the following research:

Definition 1 ([10]): Let $\mathfrak{U}$ be the universe of discourse. An intuitionistic fuzzy set ($\mathbb{IFS}$)$\mathcal{A}$ of $\mathfrak{U}$ is defined by the form $\mathcal{A} = \{(a_1, \mu_{\mathcal{A}}(a_1), \nu_{\mathcal{A}}(a_1)) : a_1 \in \mathcal{U}\}$, where $\mu_{\mathcal{A}}: \mathcal{U} \to [0,1]$ and $\nu_{\mathcal{A}}: \mathcal{U} \to [0,1]$ represents the membership and non-membership functions, respectively, such that: $0 \le \mu_{\mathcal{A}}(a_1) + \nu_{\mathcal{A}}(a_1) \le 1$. In addition, the hesitation degree of $\mathcal{A}$ is computed by the formula:

$$\pi_{\mathcal{A}}(a_1) = 1 - \mu_{\mathcal{A}}(a_1) - \nu_{\mathcal{A}}(a_1).$$

In the following, $\mathbb{IFS}(\mathcal{U})$ denotes the set of all $\mathbb{IFS}$ in $\mathcal{U}$

Definition 2 ([36]): A mapping $\mathbb{d}: \mathbb{IFS}(\mathcal{U}) \times \mathbb{IFS}(\mathcal{U})] \to [0, 1]$ is the distance measure of $\mathbb{IFS}$ of $\mathcal{U}$, if it satisfies the following conditions:

(DM1) $0 \le \mathbb{d}(\mathcal{A}, \mathcal{B}) \le 1$

(DM2) $\mathbb{d}(\mathcal{A}, \mathcal{B}) = \mathbb{d}(\mathcal{B}, \mathcal{A})$

(DM3) $\mathbb{d}(\mathcal{A}, \mathcal{B}) = 0 \Longleftrightarrow \mathcal{A} = \mathcal{B}$

(DM4) If $\mathcal{A} \subseteq \mathcal{B} \subseteq \mathcal{C}$, then $\mathbb{d}(\mathcal{A}, \mathcal{C}) \ge (\mathcal{A}, \mathcal{B})$ and $\mathbb{d}(\mathcal{A}, \mathcal{C}) \ge \mathbb{d}(\mathcal{B}, \mathcal{C})$, $\forall \mathcal{A}, \mathcal{B}, \mathcal{C} \in \mathbb{IFS}(\mathcal{U})$.

Definition 3 ([41]): Let $\mathcal{A}$ be an $\mathbb{IFS}$ of a universal set $\mathcal{U}$ and $t \in [0, 1]$. The $\mathbb{IFS}\mathcal{A}_t$ ofd $\mathcal{U}$ is called a t-$\mathbb{IFS}$ and is defined as:

$$\mu_{\mathcal{A}_t}(a_1) = min\{\mu_{\mathcal{A}}(a_1), t\} \text{and}$$
$$\nu_{\mathcal{A}_t}(a_1) = max\{\nu_{\mathcal{A}}(a_1), 1 - t\}, \forall a_1 \in \mathcal{U}.$$

The value of $\tau(a_1) = 1 - (\mu_{\mathcal{A}_t}(a_1) + \nu_{\mathcal{A}_t}(a_1))$ is called the degree of hesitancy. The t-$\mathbb{IFS}$ is of the form:

$$\mathcal{A}_t = \{(a_1, \mu_{\mathcal{A}_t}(a_1), \nu_{\mathcal{A}_t}(a_1)) : a_1 \in \mathcal{U}\}$$

where $\mu_{\mathcal{A}_t}$ and $\nu_{\mathcal{A}_t}$ are functions that assign degrees of membership and non-membership, respectively. Moreover, the functions $\mu_{\mathcal{A}_t}$ and $\nu_{\mathcal{A}_t}$ satisfy the condition:

$$0 \le \mu_{\mathcal{A}_t}(a_1) + \nu_{\mathcal{A}_t}(a_1) \le 1.$$

Definition 4: Let $\mathcal{A}_t = \{(a_1, \mu_{\mathcal{A}_t}(a_1), \nu_{\mathcal{A}_t}(a_1)) : a_1 \in \mathcal{U}\}$ and $\mathcal{B}_t = \{(a_1, \mu_{\mathcal{B}_t}(a_1), \nu_{\mathcal{B}_t}(a_1)) : a_1 \in \mathcal{U}\}$ be any two t-$\mathbb{IFS}$ on a universe $\mathcal{U}$.

1) $\mathcal{A}_t = \mathcal{B}_t \Longleftrightarrow \mu_{\mathcal{A}_t}(a_1) = \mu_{\mathcal{B}_t}(a_1)$ and $\nu_{\mathcal{A}_t}(a_1) = \nu_{\mathcal{B}_t}(a_1)$.
2) $\mathcal{A}_t \subseteq \mathcal{B}_t \Longleftrightarrow \mu_{\mathcal{A}_t}(a_1) \le \mu_{\mathcal{B}_t}(a_1)$ and $\nu_{\mathcal{A}_t}(a_1) \ge \nu_{\mathcal{B}_t}(a_1)$.

Definition 5 ([43]): The distance measure between $\mathbb{IFS}\mathcal{A}$ and $\mathcal{B}$ is defined as follows:

1) The Hamming distance measure $\mathbb{d}_H(\mathcal{A}, \mathcal{B})$ is defined as:

$$\mathbb{d}_H(\mathcal{A}, \mathcal{B}) = \frac{1}{2}\sum_{i=1}^{n}\left[\begin{array}{c}|\mu_{\mathcal{A}}(a_i) - \mu_{\mathcal{B}}(a_i)| + \\ |\nu_{\mathcal{A}}(a_i) - \nu_{\mathcal{B}}(a_i)| + |\pi_{\mathcal{A}}(a_i) - \pi_{\mathcal{B}}(a_i)|\end{array}\right]$$

2) The Euclidean distance measure $\mathbb{d}_E(\mathcal{A}, \mathcal{B})$ is defined as:

$$\mathbb{d}_E(\mathcal{A}, \mathcal{B}) = \sqrt{\frac{1}{2}\sum_{i=1}^{n}\left[\begin{array}{c}(\mu_{\mathcal{A}}(a_i) - \mu_{\mathcal{B}}(a_i))^2 + \\ (\nu_{\mathcal{A}}(a_i) - \nu_{\mathcal{B}}(a_i))^2 + (\pi_{\mathcal{A}}(a_i) - \pi_{\mathcal{B}}(a_i))^2\end{array}\right]}$$

Definition 6 ([44]): The distance measure $\mathbb{d}_1$ between $\mathbb{IFS}\mathcal{A}$ and $\mathcal{B}$ is defined as:

$$\mathbb{d}_1(\mathcal{A}, \mathcal{B}) = \frac{1}{n}\sum_{i=1}^{n}\left[\begin{array}{c}\frac{|\mu_{\mathcal{A}}(a_i) - \mu_{\mathcal{B}}(a_i)| + |\nu_{\mathcal{A}}(a_i) - \nu_{\mathcal{B}}(a_i)|}{4} \\ + \frac{max\{|\mu_{\mathcal{A}}(a_i) - \mu_{\mathcal{B}}(a_i)|, |\nu_{\mathcal{A}}(a_i) - \nu_{\mathcal{B}}(a_i)|\}}{2}\end{array}\right]$$

Definition 7 ([45]): The distance measure $\mathbb{d}_2$ between $\mathbb{IFS}\mathcal{A}$ and $\mathcal{B}$ is expressed as:

$$\mathbb{d}_2(\mathcal{A}, \mathcal{B}) = \frac{1}{n}\sum_{i=1}^{n}\left[\begin{array}{c}\frac{|\mu_{\mathcal{A}}(a_i) - \mu_{\mathcal{B}}(a_i)| + |\nu_{\mathcal{A}}(a_i) - \nu_{\mathcal{B}}(a_i)|}{4} \\ + \frac{|\max\{\mu_{\mathcal{A}}(a_i), \nu_{\mathcal{B}}(a_i)\} - \max\{\mu_{\mathcal{B}}(a_i), \nu_{\mathcal{A}}(a_i)\}|}{2}\end{array}\right]$$

Definition 8 ([46]): The distance measure $\mathbb{d}_3$ between $\mathbb{IFS}\mathcal{A}$ and $\mathcal{B}$ is expressed as:

$$\mathbb{d}_3(\mathcal{A}, \mathcal{B}) = \frac{1}{n}\sum_{i=1}^{n}[max\{|\mu_{\mathcal{A}}(a_i) - \mu_{\mathcal{B}}(a_i)|, |\nu_{\mathcal{A}}(a_i) - \nu_{\mathcal{B}}(a_i)|, |\pi_{\mathcal{A}}(a_i) - \pi_{\mathcal{B}}(a_i)|\}]$$

Definition 9 ([47]): The distance measure $\mathbb{d}_4$ between $\mathbb{IFS}\mathcal{A}$ and $\mathcal{B}$ is defined as:

$$\mathbb{d}_4(\mathcal{A}, \mathcal{B}) = \frac{1}{4n}\sum_{i=1}^{n}\left[\begin{array}{c}|\mu_{\mathcal{A}}(a_i) - \mu_{\mathcal{B}}(a_i)| + \\ |\nu_{\mathcal{A}}(a_i) - \nu_{\mathcal{B}}(a_i)| + |\pi_{\mathcal{A}}(a_i) - \pi_{\mathcal{B}}(a_i)| \\ +2max\left\{\begin{array}{c}|\mu_{\mathcal{A}}(a_i) - \mu_{\mathcal{B}}(a_i)|, |\nu_{\mathcal{A}}(a_i) - \nu_{\mathcal{B}}(a_i)|, \\ |\pi_{\mathcal{A}}(a_i) - \pi_{\mathcal{B}}(a_i)|\end{array}\right.\end{array}\right]$$

III. A NOVEL DISTANCE MEASURE FOR T-INTUITIONISTIC FUZZY SETS

A new distance measure, namely, the lift distance measure for t-$\mathbb{IFS}$, and its structural characteristics are discussed in the section that follows.

Definition 10: Let $\mathcal{A}_t$ and $\mathcal{B}_t$ be any two t-$\mathbb{IFS}$ on the universe of discourse $\mathcal{U} = \{a_1, a_2, a_3, \ldots, a_n\}$. The lift distance $d_{\mathcal{LD}}(\mathcal{A}_t, \mathcal{B}_t)$ between $\mathcal{A}_t$ and $\mathcal{B}_t$ is defined as follows:

If $\nu_{\mathcal{A}_t}(a_i) = \nu_{\mathcal{B}_t}(a_i)$ then

$$d_{\mathcal{LD}}(\mathcal{A}_t, \mathcal{B}_t) = \frac{1}{2n}\sum_{i=1}^{n}\left[\begin{array}{c}|\mu_{\mathcal{A}_t}(a_i) - \mu_{\mathcal{B}_t}(a_i)| + \\ |\pi_{\mathcal{A}_t}(a_i) - \pi_{\mathcal{B}_t}(a_i)|\end{array}\right]$$

If $\nu_{\mathcal{A}_t}(a_i) \neq \nu_{\mathcal{B}_t}(a_i)$ then

$$d_{\mathcal{LD}}(\mathcal{A}_t, \mathcal{B}_t) = \frac{1}{2n}\sum_{i=1}^{n}\left[\begin{array}{c}|\mu_{\mathcal{A}_t}(a_i)| + |\mu_{\mathcal{B}_t}(a_i)| + \\ |\nu_{\mathcal{A}_t}(a_i) - \nu_{\mathcal{B}_t}(a_i)| + |\pi_{\mathcal{A}_t}(a_i) - \pi_{\mathcal{B}_t}(a_i)|\end{array}\right].$$

Theorem 1: Let $\mathcal{A}_t$, $\mathcal{B}_t$ and $\mathcal{C}_t$ be any three t-$\mathbb{IFS}$ defined in $\mathcal{U}$. The lift distance $d_{\mathcal{LD}}$ admits all the structural properties of a distance measure.

Proof. Consider $\mathcal{A}_t = \{(a, \mu_{\mathcal{A}_t}(a), \nu_{\mathcal{A}_t}(a)) : a \in \mathcal{U}\}$, $\mathcal{B}_t = \{(a, \mu_{\mathcal{B}_t}(a), \nu_{\mathcal{B}_t}(a)) : a \in \mathcal{U}\}$ and

$\mathcal{C}_t = \{(a, \mu_{\mathcal{C}_t}(a), \nu_{\mathcal{C}_t}(a)) : a \in \mathcal{U}\}$ be any three t$-\mathbb{IFS}$ on $\mathcal{U}$.

For (DM1):

Case I: This property is first solved for $\nu_{\mathcal{A}_t}(a) = \nu_{\mathcal{B}_t}(a)$.

Consider $\mu_{\mathcal{A}_t}(a), \mu_{\mathcal{B}_t}(a), \pi_{\mathcal{A}_t}(a), \pi_{\mathcal{B}_t}(a) \in [0, 1]$ such that $\mu_{\mathcal{A}_t}(a) - \mu_{\mathcal{B}_t}(a),\ \pi_{\mathcal{A}_t}(a) - \pi_{\mathcal{B}_t}(a) \in [0, 1]$.

Now, consider the following inequality:

$$0 \le \frac{1}{2}\left[\left|\mu_{\mathcal{A}_t}(a) - \mu_{\mathcal{B}_t}(a)\right| + \left|\pi_{\mathcal{A}_t}(a) - \pi_{\mathcal{B}_t}(a)\right|\right] \le 1.$$

By taking the summation over all $i = 1, 2, \ldots, n$ in the above inequality, we obtain the following relation:

$$0 \le \frac{1}{2n}\sum_{i=1}^{n}\left[\begin{array}{c}\left|\mu_{\mathcal{A}_t}(a_i) - \mu_{\mathcal{B}_t}(a_i)\right| + \\ \left|\pi_{\mathcal{A}_t}(a_i) - \pi_{\mathcal{B}_t}(a_i)\right|\end{array}\right] \le 1.$$

Thus, $0 \le d_{\mathcal{LD}}(\mathcal{A}_t, \mathcal{B}_t) \le 1$.

Case II: For the case $\nu_{\mathcal{A}_t}(a) \ne \nu_{\mathcal{B}_t}(a)$, the following inequality can be proven: $0 \le d_{\mathcal{LD}}(\mathcal{A}_t, \mathcal{B}_t) \le 1$.

For (DM2):

The symmetry of measure $d_{\mathcal{LD}}(\mathcal{A}_t, \mathcal{B}_t)$ with respect to their arguments is obvious, so $d_{\mathcal{LD}}(\mathcal{A}_t, \mathcal{B}_t) = d_{\mathcal{LD}}(\mathcal{B}_t, \mathcal{A}_t)$.

For (DM3):

Consider $d_{\mathcal{LD}}(\mathcal{A}_t, \mathcal{B}_t) = 0$

$$\Longleftrightarrow \frac{1}{2n}\sum_{i=1}^{n}\left[\begin{array}{c}\left|\mu_{\mathcal{A}_t}(a_i) - \mu_{\mathcal{B}_t}(a_i)\right| + \\ \left|\pi_{\mathcal{A}_t}(a_i) - \pi_{\mathcal{B}_t}(a_i)\right|\end{array}\right] = 0.$$

$$\Longleftrightarrow \left|\mu_{\mathcal{A}_t}(a) - \mu_{\mathcal{B}_t}(a)\right| + \left|\pi_{\mathcal{A}_t}(a) - \pi_{\mathcal{B}_t}(a)\right| = 0$$

$$\Longleftrightarrow \mu_{\mathcal{A}_t}(a) - \mu_{\mathcal{B}_t}(a) = 0,\ \pi_{\mathcal{A}_t}(a) - \pi_{\mathcal{B}_t}(a) = 0.$$

$$\Longleftrightarrow \mu_{\mathcal{A}_t}(a) = \mu_{\mathcal{B}_t}(a),\ \pi_{\mathcal{A}_t}(a_i) = \pi_{\mathcal{B}_t}(a), \forall a \in \mathcal{U}.$$

$$\Longleftrightarrow \mathcal{A}_t = \mathcal{B}_t.$$

For (DM4):

Case I: This property is first solved for $\nu_{\mathcal{A}_t}(a) = \nu_{\mathcal{B}_t}(a)$.

Let $\mathcal{A}_t \subseteq \mathcal{B}_t \subseteq \mathcal{C}_t$, then

$$\mu_{\mathcal{A}_t}(a) \le \mu_{\mathcal{B}_t}(a) \le \mu_{\mathcal{C}_t}(a)$$

and

$$\nu_{\mathcal{A}_t}(a) \ge \nu_{\mathcal{B}_t}(a) \ge \nu_{\mathcal{C}_t}(a).$$

Then, we have the following inequalities:

$$\left|\mu_{\mathcal{A}_t}(a) - \mu_{\mathcal{B}_t}(a)\right| \le \left|\mu_{\mathcal{A}_t}(a) - \mu_{\mathcal{C}_t}(a)\right| \quad (1)$$

$$\left|\nu_{\mathcal{A}_t}(a) - \nu_{\mathcal{B}_t}(a)\right| \le \left|\nu_{\mathcal{A}_t}(a) - \nu_{\mathcal{C}_t}(a)\right| \quad (2)$$

$$\left|\mu_{\mathcal{B}_t}(a) - \mu_{\mathcal{C}_t}(a)\right| \le \left|\mu_{\mathcal{A}_t}(a) - \mu_{\mathcal{C}_t}(a)\right| \quad (3)$$

$$\left|\nu_{\mathcal{B}_t}(a) - \nu_{\mathcal{C}_t}(a)\right| \le \left|\nu_{\mathcal{A}_t}(a) - \nu_{\mathcal{C}_t}(a)\right| \quad (4)$$

$$\left|\pi_{\mathcal{A}_t}(a) - \pi_{\mathcal{B}_t}(a)\right| \le \left|\pi_{\mathcal{A}_t}(a) - \pi_{\mathcal{C}_t}(a)\right| \quad (5)$$

$$\left|\pi_{\mathcal{B}_t}(a) - \pi_{\mathcal{C}_t}(a)\right| \le \left|\pi_{\mathcal{A}_t}(a) - \pi_{\mathcal{C}_t}(a)\right| \quad (6)$$

Consider

$$d_{\mathcal{LD}}(\mathcal{A}_t, \mathcal{C}_t) = \frac{1}{2n}\sum_{i=1}^{n}\left[\begin{array}{c}\left|\mu_{\mathcal{A}_t}(a_i) - \mu_{\mathcal{C}_t}(a_i)\right| + \\ \left|\pi_{\mathcal{A}_t}(a_i) - \pi_{\mathcal{C}_t}(a_i)\right|\end{array}\right]$$

The following result is obtained by applying (1) and (5) to the equation above:

$$d_{\mathcal{LD}}(\mathcal{A}_t, \mathcal{C}_t) \ge \frac{1}{2n}\sum_{i=1}^{n}\left[\begin{array}{c}\left|\mu_{\mathcal{A}_t}(a_i) - \mu_{\mathcal{B}_t}(a_i)\right| + \\ \left|\pi_{\mathcal{A}_t}(a_i) - \pi_{\mathcal{B}_t}(a_i)\right|\end{array}\right]$$

Consequently, $d_{\mathcal{LD}}(\mathcal{A}_t, \mathcal{C}_t) \ge d_{\mathcal{LD}}(\mathcal{A}_t, \mathcal{B}_t)$.

In the same way, we obtained that

$$d_{\mathcal{LD}}(\mathcal{A}_t, \mathcal{C}_t) \ge d_{\mathcal{LD}}(\mathcal{B}_t, \mathcal{C}_t).$$

Case II: Now, the property is solved for $\nu_{\mathcal{A}_t}(a) \ne \nu_{\mathcal{B}_t}(a)$. Consider

$$d_{\mathcal{LD}}(\mathcal{A}_t, \mathcal{C}_t) = \frac{1}{2n}\sum_{i=1}^{n}\left[\begin{array}{c}\left|\mu_{\mathcal{A}_t}(a_i)\right| + \left|\mu_{\mathcal{C}_t}(a_i)\right| + \\ \left|\nu_{\mathcal{A}_t}(a_i) - \nu_{\mathcal{C}_t}(a_i)\right| + \left|\pi_{\mathcal{A}_t}(a_i) - \pi_{\mathcal{C}_t}(a_i)\right|\end{array}\right]$$

When equations (1) and (5) are applied to the above equation, we get:

$$d_{\mathcal{LD}}(\mathcal{A}_t, \mathcal{C}_t) \ge \frac{1}{2n}\sum_{i=1}^{n}\left[\begin{array}{c}\left|\mu_{\mathcal{A}_t}(a_i)\right| + \left|\mu_{\mathcal{B}_t}(a_i)\right| + \\ \left|\nu_{\mathcal{A}_t}(a_i) - \nu_{\mathcal{B}_t}(a_i)\right| + \left|\pi_{\mathcal{A}_t}(a_i) - \pi_{\mathcal{B}_t}(a_i)\right|\end{array}\right].$$

Then, we get the inequality $d_{\mathcal{LD}}(\mathcal{A}_t, \mathcal{C}_t) \ge d_{\mathcal{LD}}(\mathcal{A}_t, \mathcal{B}_t)$.

Similarly, $d_{\mathcal{LD}}(\mathcal{A}_t, \mathcal{C}_t) \ge d_{\mathcal{LD}}(\mathcal{B}_t, \mathcal{C}_t)$.

Thus $d_{\mathcal{LD}}(\mathcal{A}_t, \mathcal{C}_t) \ge d_{\mathcal{LD}}(\mathcal{A}_t, \mathcal{B}_t)$ and $d_{\mathcal{LD}}(\mathcal{A}_t, \mathcal{C}_t) \ge d_{\mathcal{LD}}(\mathcal{B}_t, \mathcal{C}_t)$. Consequently, $d_{\mathcal{LD}}$ is a distance measure.

An analysis of the subsequent illustration will assist in the comprehension of the notion of $d_{\mathcal{LD}}$:

Example 3.3. Consider

$$\mathcal{A}_t = \{(a_1, 0.4, 0.5), (a_2, 0.3, 0.4), (a_3, 0.4, 0.5\}$$

and

$$\mathcal{B}_t = \{(a_1, 0.6, 0.4), (a_2, 0.5, 0.4), (a_3, 0.7, 0.3\}$$

be any two t-$\mathbb{IFS}$ on $\mathcal{U} = \{a_1, a_2, a_3\}$.

The lift distance measure $d_{\mathcal{LD}}(\mathcal{A}_t, \mathcal{B}_t)$ between $\mathcal{A}_t$ and $\mathcal{B}_t$ is:

$$d_{\mathcal{LD}}(\mathcal{A}_t, \mathcal{B}_t) = \frac{1}{2(3)}\left[\begin{array}{c}(0.6 + 0.4 + 0.1 + 0.1) \\ + (0.2 + 0.2) + (0.7 + 0.4 + 0.2 + 0.1)\end{array}\right]$$

Hence, $d_{\mathcal{LD}}(\mathcal{A}_t, \mathcal{B}_t) = 0.5$.

Theorem 2: The lift distance measure $d_{\mathcal{LD}}$ satisfies the following conditions:

$$d_{\mathcal{LD}}(\mathcal{A}_t, \mathcal{C}_t) \le d_{\mathcal{LD}}(\mathcal{A}_t, \mathcal{B}_t) + d_{\mathcal{LD}}(\mathcal{B}_t, \mathcal{C}_t)$$

Proof. **Case I:** First, we solve this property for $\nu_{\mathcal{A}_t}(a) = \nu_{\mathcal{B}_t}(a)$.

In view of Definition (3.1), we have

$$d_{\mathcal{LD}}(\mathcal{A}_t, \mathcal{C}_t) = \frac{1}{2n}\sum_{i=1}^{n}\left[\begin{array}{c}\left|\mu_{\mathcal{A}_t}(a_i)\right| + \left|\mu_{\mathcal{C}_t}(a_i)\right| + \left|\nu_{\mathcal{A}_t}(a_i) - \nu_{\mathcal{C}_t}(a_i)\right| \\ + \left|\pi_{\mathcal{A}_t}(a_i) - \pi_{\mathcal{C}_t}(a_i)\right|\end{array}\right]$$

$$d_{\mathcal{LD}}(\mathcal{A}_t, \mathcal{C}_t) = \frac{1}{2n}\sum_{i=1}^{n}\left[\begin{array}{c}\left|\mu_{\mathcal{A}_t}(a_i) - \mu_{\mathcal{B}_t}(a_i) + \mu_{\mathcal{B}_t}(a_i) - \mu_{\mathcal{C}_t}(a_i)\right| \\ + \left|\pi_{\mathcal{A}_t}(a_i) - \pi_{\mathcal{B}_t}(a_i) + \pi_{\mathcal{B}_t}(a_i) - \pi_{\mathcal{C}_t}(a_i)\right|\end{array}\right]$$

$$\le \frac{1}{2n}\sum_{i=1}^{n}\left[\begin{array}{c}\left|\mu_{\mathcal{A}_t}(a_i) - \mu_{\mathcal{B}_t}(a_i)\right| + \left|\mu_{\mathcal{B}_t}(a_i) - \mu_{\mathcal{C}_t}(a_i)\right| + \\ \left|\pi_{\mathcal{A}_t}(a_i) - \pi_{\mathcal{B}_t}(a_i)\right| + \left|\pi_{\mathcal{B}_t}(a_i) - \pi_{\mathcal{C}_t}(a_i)\right|\end{array}\right]$$

Thus, $d_{\mathcal{LD}}(\mathcal{A}_t, \mathcal{C}_t) \le d_{\mathcal{LD}}(\mathcal{A}_t, \mathcal{B}_t) + d_{\mathcal{LD}}(\mathcal{B}_t, \mathcal{C}_t)$.

TABLE 1. Comparative study of proposed lift distance measure with existing distance measures.

	Case-I	Case-II	Case-III	Case-IV
$\mathcal{A}^i_{0.8}$	⟨0.3,0.2,0.5⟩	⟨0.3,0.2,0.5⟩	⟨0.4,0.4,0.2⟩	⟨0.6,0.3,0.1⟩
$\mathcal{B}^i_{0.8}$	⟨0.15,0.25,0.6⟩	⟨0.16,0.26,0.58⟩	⟨0.5,0.4,0.1⟩	⟨0.5,0.4,0.1⟩
$\mathbb{d}_H$[43]	0.15	0.14	0.1	0.1
$\mathbb{d}_E$[43]	0.13	0.12	0.1	0.1
$\mathbb{d}_1$[44]	0.12	0.12	0.075	0.1
$\mathbb{d}_2$[45]	0.5	0.5	0.01	0.05
$\mathbb{d}_3$[46]	0.15	0.14	0.1	0.1
$\mathbb{d}_4$[47]	0.12	0.12	0.075	0.1
$d_{\mathcal{LD}}$	0.30	0.32	0.1	0.6

Case II: Now, we solve this property for $\nu_{\mathcal{A}_t}(a)\neq\nu_{\mathcal{B}_t}(a)$. According to Definition (3.1), we have

$$d_{\mathfrak{LD}}(\mathcal{A}_t, \mathfrak{C}_t)$$

$$= \frac{1}{2n}\sum_{i=1}^{n}\begin{bmatrix}\left|\mu_{\mathcal{A}_t}(a_i)\right|+\left|\mu_{\mathfrak{C}_t}(a_i)\right|+\left|\nu_{\mathcal{A}_t}(a_i)-\nu_{\mathfrak{C}_t}(a_i)\right| \\ +\left|\pi_{\mathcal{A}_t}(a_i)-\pi_{\mathfrak{C}_t}(a_i)\right|\end{bmatrix}$$

$$= v\frac{1}{2n}\sum_{i=1}^{n}\begin{bmatrix}\left|\mu_{\mathcal{A}_t}(a_i)\right|+\left|-\mu_{\mathcal{B}_t}(a_i)+\mu_{\mathcal{B}_t}(a_i)-\mu_{\mathfrak{C}_t}(a_i)\right|+ \\ \left|\nu_{\mathcal{A}_t}(a_i)-\nu_{\mathcal{B}_t}(a_i)+\nu_{\mathcal{B}_t}(a_i)-\nu_{\mathfrak{C}_t}(a_i)\right|+ \\ \left|\pi_{\mathcal{A}_t}(a_i)-\pi_{\mathcal{B}_t}(a_i)+\pi_{\mathcal{B}_t}(a_i)-\pi_{\mathfrak{C}_t}(a_i)\right|\end{bmatrix}$$

$$\leq \frac{1}{2n}\sum_{i=1}^{n}\left[\left|\mu_{\mathcal{A}_t}(a_i)\right|+\left|\mu_{\mathcal{B}_t}(a_i)\right|+\left|\nu_{\mathcal{A}_t}(a_i)-\nu_{\mathcal{B}_t}(a_i)\right|\right.$$
$$+\left|\pi_{\mathcal{A}_t}(a_i)-\pi_{\mathcal{B}_t}(a_i)\right|+\left|\mu_{\mathcal{B}_t}(a_i)-\mu_{\mathfrak{C}_t}(a_i)\right|$$
$$\left.+\left|\nu_{\mathcal{B}_t}(a_i)-\nu_{\mathfrak{C}_t}(a_i)\right|+\left|\pi_{\mathcal{B}_t}(a_i)-\pi_{\mathfrak{C}_t}(a_i)\right|\right]$$

This shows that $d_{\mathfrak{LD}}(\mathcal{A}_t, \mathfrak{C}_t) \leq d_{\mathfrak{LD}}(\mathcal{A}_t, \mathcal{B}_t) + d_{\mathfrak{LD}}(\mathcal{B}_t, \mathfrak{C}_t)$, as shown at the bottom of the next page.

A. EFFICIENCY OF THE NEW PROPOSED LIFT DISTANCE MEASURE

We highlight the efficiency of the proposed lift distance measure by comparing it with existing distance measures. The following table illustrates the findings of our analysis, providing valuable insights into the efficacy of the proposed distance measure in the t-$\mathbb{IF}$ domain.

From Cases I and II of the above Table, it is quite evident that the distance measures developed in [44], [45], and [47] have some limitations. Specifically, they do not comply with the following relation $\mathbb{d}\left(\mathcal{A}^1_{0.8}, \mathcal{B}^1_{0.8}\right) = \mathbb{d}\left(\mathcal{A}^2_{0.8}, \mathcal{B}^2_{0.8}\right)$, when $\mathcal{A}^1_{0.8} = \mathcal{A}^2_{0.8}$ and $\mathcal{B}^1_{0.8} \neq \mathcal{B}^2_{0.8}$. These measures may not be suitable for certain situations, which exposes their lack of reasonability.

Moreover, in Cases III and IV of the above Table, the distance measure developed in [43] and [46] does not satisfy the relation $\mathbb{d}\left(\mathcal{A}^3_{0.8}, \mathcal{B}^3_{0.8}\right) = \mathbb{d}\left(\mathcal{A}^4_{0.8}, \mathcal{B}^4_{0.8}\right)$, where $\mathcal{A}^3_{0.8} \neq \mathcal{A}^4_{0.8}$ and $\mathcal{B}^3_{0.8} = \mathcal{B}^4_{0.8}$. This contradiction serves to highlight the insufficiency of the existing distance measures.

It is important to note that the proposed lift distance measure is a valuable tool in distinguishing between alternatives, making it a necessary component of the decision-making process, particularly in the TOPSIS method. This distance measure is also essential for quantitatively comparing two t-$\mathbb{IFS}$s and analyzing incomplete or unclear data. The lift distance measure is a flexible MCDM problem-solving tool that helps in pattern identification, decision-making, and clustering.

IV. INNOVATIVE t-INTUITIONISTIC FUZZY TOPSIS METHODOLOGY

The following section introduces a novel approach called t-$\mathbb{IF}$-TOPSIS, which is implemented in t-$\mathbb{IF}$ environments and serves as a resolution for MCDM issues. Experimental execution will be utilized to illustrate the efficacy and viability of the proposed methodology in practical contexts.

The t-$\mathbb{IF}$-TOPSIS is among the most recognized decision-making techniques, and it's widely used due to its mathematical foundation, simplicity, and easy applicability. The t-$\mathbb{IF}$-TOPSIS technique is employed when a situation has many options and a number of criteria that must be assessed with comprehensive analyses. The architectural design of this system is designed and executed with the capability of handling very complex and unforeseen situations in the environment. This framework demonstrates a significant ability to efficiently handle and overcome obstacles that emerge from unanticipated and unclear decision-making processes. Manipulating the 't' parameter in uncertainty modeling to align with particular characteristics and prerequisites of the field allows for a more accurate depiction of decision-makers' inclinations and uncertainties. Many recognize the methodology's effectiveness in addressing intricate and bewildering matters, thereby enabling a more accurate and robust approach to the decision-making process.

Let $\{\mathbb{A}_1, \mathbb{A}_2, \mathbb{A}_3, \ldots, \mathbb{A}_m\}$ be the set of distinct alternatives, and $\{\mathbb{C}_1, \mathbb{C}_2, \ldots, \mathbb{C}_n\}$ be the set of attributes. Let $W_t = \{w_1, w_2, w_3, \ldots, w_n\}$ denotes the t-$\mathbb{IF}$ associated weight vector corresponding to these criterions such that w_j is a t-$\mathbb{IF}$ values. Each alternative $\mathbb{A}_\mathrm{i}(1 \leq i \leq m)$ is evaluated on the set of attributes $\mathbb{C}_j\,(1 \leq j \leq n)$. This evaluation is expressed having entries t-$\mathbb{IF}$ values $\beta_{ij} = \langle \mu_{ij}, \nu_{ij}, \pi_{ij} \rangle$, where μ_{ij} and ν_{ij} indicates the level of the satisfaction and dissatisfaction of each alternative $\mathbb{A}_i$ corresponding to attribute $\mathbb{C}_j$ satisfying the conditions:

$$0 \leq \mu_{ij} + \nu_{ij} \leq 1 \text{ and } 0 \leq \pi_{ij} \leq 1.$$

The algorithm for solving the MCDM problem in a t-intuitionistic fuzzy environment is designed as follows:

Step I: The information of an expert is summarized in a t-$\mathbb{IF}$ decision matrix

$$\mathcal{D}_t = [\beta_{ij}]_{m\times n} = [\langle \mu_{ij}, \nu_{ij}, \pi_{ij} \rangle]_{m\times n}.$$

Step II: Obtain a t-$\mathbb{IF}$ normalized decision matrix $\mathcal{N}_t = [r_{ij}]_{m\times n}$(if necessary), where r_{ij} is computed by the following formula:

$$r_{ij} = \begin{cases} \dfrac{\beta_{ij}}{\underset{1\leq i\leq m}{max}\left\{\beta_{ij}^2\right\}} & \text{for benefit type attribute} \\ 1 - \dfrac{\beta_{ij}}{\underset{1\leq i\leq m}{max}\left\{\beta_{ij}^2\right\}} & \text{for cost type attribute} \end{cases} \tag{7}$$

Step III: The t-$\mathbb{IF}$ weighted normalized matrix $\mathcal{W}_{ij} = [\gamma_{ij}]_{m\times n}$, $(1 \leq i \leq m, 1 \leq j \leq n)$ with respect to w_j and $\mathcal{N}_t$ is computed as follows:

$$\begin{aligned} \gamma_{ij} &= w_j \bullet r_{ij} = \langle \mu_{\gamma_{ij}}, \nu_{\gamma_{ij}}, \pi_{\gamma_{ij}} \rangle \\ &= \left\langle \begin{matrix} \frac{\mu_{w_j}\times\mu_{r_{ij}}}{\mu_{w_j}+\mu_{r_{ij}}-\mu_{w_j}\times\mu_{r_{ij}}}, \frac{\nu_{w_j}+\nu_{r_{ij}}-2\nu_{w_j}\times\nu_{r_{ij}}}{1-\nu_{w_j}\times\nu_{r_{ij}}}, \\ 1-\mu_{\gamma_{ij}}-\nu_{\gamma_{ij}} \end{matrix} \right\rangle \end{aligned} \tag{8}$$

Step IV: Obtain the t-intuitionistic fuzzy positive-ideal solution (t$-\mathbb{IFPIS}$)$\mathbb{C}_j^+ = \left(\mu_{\mathbb{C}_j^+}, \nu_{\mathbb{C}_j^+}, \pi_{\mathbb{C}_j^+}\right)$ and the t-intuitionistic fuzzy negative-ideal solution (t$-\mathbb{IFNIS}$)$\mathbb{C}_j^- = \left(\mu_{\mathbb{C}_j^-}, \nu_{\mathbb{C}_j^-}, \pi_{\mathbb{C}_j^-}\right)$ from $\mathcal{W}_{ij}$ by the following formulae:

$$\mathbb{C}_j^+ = \begin{cases} \left(\begin{matrix} (\max_i \mu_{\gamma_{ij}}), (\min_i \nu_{\gamma_{ij}}), \\ \nu \\ 1-(\max_i \mu_{\gamma_{ij}}) - \left(\min_i \nu_{\gamma_{ij}}\right) \end{matrix}\right) & \text{for benefit criteria} \\ \left(\begin{matrix} (\min_i \mu_{\gamma_{ij}}), (\max_i \nu_{\gamma_{ij}}), \\ 1-(\min_i \mu_{\gamma_{ij}}) - \left(\max_i \nu_{\gamma_{ij}}\right) \end{matrix}\right) & \text{for cost criteria} \end{cases} \tag{9}$$

$$\mathbb{C}_j^- = \begin{cases} \left(\begin{matrix} (\min_i \mu_{\gamma_{ij}}), (\max_i \nu_{\gamma_{ij}}), \\ 1-(\min_i \mu_{\gamma_{ij}}) - \left(\max_i \nu_{\gamma_{ij}}\right) \end{matrix}\right) & \text{for benefit criteria} \\ \left(\begin{matrix} (\max_i \mu_{\gamma_{ij}}), (\min_i \nu_{\gamma_{ij}}), \\ 1-(\max_i \mu_{\gamma_{ij}}) - \left(\min_i \nu_{\gamma_{ij}}\right) \end{matrix}\right) & \text{for cost criteria} \end{cases} \tag{10}$$

Step V: Compute the lift distance measures for each alternative $\mathbb{A}_i$ as follows:

$$d_{\mathbb{A}_i}^+\left(\mathcal{W}_{ij}, \mathbb{C}_j^+\right) = \begin{cases} \frac{1}{2n}\left\{\sum_{i=1}^n \left[\begin{matrix} \left|\mu_{\gamma_{ij}} - \mu_{\mathbb{C}_j^+}\right| + \\ \left|\pi_{\gamma_{ij}} - \pi_{\mathbb{C}_j^+}\right| \end{matrix}\right]\right\} & \text{if} \nu_{\gamma_{ij}} = \nu_{\mathbb{C}_j^+} \\ \frac{1}{2n}\left\{\sum_{i=1}^n \left[\begin{matrix} \left|\mu_{\gamma_{ij}}\right| + \left|\mu_{\mathbb{C}_j^+}\right| + \\ \left|\nu_{\gamma_{ij}} - \nu_{\mathbb{C}_j^+}\right| + \left|\pi_{\gamma_{ij}} - \pi_{\mathbb{C}_j^+}\right| \end{matrix}\right]\right\} & \text{if} \nu_{\gamma_{ij}} \neq \nu_{\mathbb{C}_j^+} \end{cases} \tag{11}$$

$$d_{\mathbb{A}_i}^-\left(\mathcal{W}_{ij}, \mathbb{C}_j^-\right) = \begin{cases} \frac{1}{2n}\left\{\sum_{i=1}^n \left[\begin{matrix} \left|\mu_{\gamma_{ij}} - \mu_{\mathbb{C}_j^-}\right| + \\ \left|\pi_{\gamma_{ij}} - \pi_{\mathbb{C}_j^-}\right| \end{matrix}\right]\right\} & \text{if} \nu_{\gamma_{ij}} = \nu_{\mathbb{C}_j^-} \\ \frac{1}{2n}\left\{\sum_{i=1}^n \left[\begin{matrix} \left|\mu_{\gamma_{ij}}\right| + \left|\mu_{\mathbb{C}_j^-}\right| + \\ \left|\nu_{\gamma_{ij}} - \nu_{\mathbb{C}_j^-}\right| + \left|\pi_{\gamma_{ij}} - \pi_{\mathbb{C}_j^-}\right| \end{matrix}\right]\right\} & \text{if} \nu_{\gamma_{ij}} \neq \nu_{\mathbb{C}_j^-} \end{cases} \tag{12}$$

Step VI: Determine the closeness coefficient $\mathcal{X}_{\mathbb{A}_i}$ for each alternative $\mathbb{A}_i$ as follows:

$$\mathcal{X}_{\mathbb{A}_i} = \frac{\min\left(d_{\mathbb{A}_i}^+\left(\mathcal{W}_{ij}, \mathbb{C}_j^+\right), d_{\mathbb{A}_i}^-\left(\mathcal{W}_{ij}, \mathbb{C}_j^-\right)\right)}{\max\left(d_{\mathbb{A}_i}^+\left(\mathcal{W}_{ij}, \mathbb{C}_j^+\right), d_{\mathbb{A}_i}^-\left(\mathcal{W}_{ij}, \mathbb{C}_j^-\right)\right)}, \quad 0 \leq \mathcal{X}_{\mathbb{A}_i} \leq 1 \tag{13}$$

Step VII: Rank the alternatives according to $\mathcal{X}_{\mathbb{A}_i}$ and select the most desirable alternatives.

$$d_{\mathcal{LD}}(\mathcal{A}_t, \mathcal{C}_t) = \begin{cases} \frac{1}{2n}\sum_{i=1}^n \left\{\begin{matrix} \left|\mu_{\mathcal{A}_t}(a_i)\right| + \left|\mu_{\mathcal{B}_t}(a_i)\right| + \left|\nu_{\mathcal{A}_t}(a_i) - \nu_{\mathcal{B}_t}(a_i)\right| + \left|\pi_{\mathcal{A}_t}(a_i) - \pi_{\mathcal{B}_t}(a_i)\right| + \\ \left|\mu_{\mathcal{B}_t}(a_i) - \mu_{\mathcal{C}_t}(a_i)\right| + \left|\pi_{\mathcal{B}_t}(a_i) - \pi_{\mathcal{C}_t}(a_i)\right| \end{matrix}\right\} & \text{if } \nu_{\mathcal{B}_t}(a_i) = \nu_{\mathcal{C}_t}(a_i) \\ \frac{1}{2n}\sum_{i=1}^n \left\{\begin{matrix} \left|\mu_{\mathcal{A}_t}(a_i)\right| + \left|\mu_{\mathcal{B}_t}(a_i)\right| + \left|\nu_{\mathcal{A}_t}(a_i) - \nu_{\mathcal{B}_t}(a_i)\right| + \left|\pi_{\mathcal{A}_t}(a_i) - \pi_{\mathcal{B}_t}(a_i)\right| + \\ \left|\mu_{\mathcal{B}_t}(a_i) - \mu_{\mathcal{C}_t}(a_i)\right| + \left|\nu_{\mathcal{B}_t}(a_i) - \nu_{\mathcal{C}_t}(a_i)\right| + \left|\pi_{\mathcal{B}_t}(a_i) - \pi_{\mathcal{C}_t}(a_i)\right| \end{matrix}\right\} & \text{if} \nu_{\mathcal{B}_t}(a_i) \neq \nu_{\mathcal{C}_t}(a_i) \end{cases}$$

$$d_{\mathcal{LD}}(\mathcal{A}_t, \mathcal{C}_t) \leq \begin{cases} \frac{1}{2n}\sum_{i=1}^n \left\{\begin{matrix} \left|\mu_{\mathcal{A}_t}(a_i)\right| + \left|\mu_{\mathcal{B}_t}(a_i)\right| + \left|\nu_{\mathcal{A}_t}(a_i) - \nu_{\mathcal{B}_t}(a_i)\right| + \left|\pi_{\mathcal{A}_t}(a_i) - \pi_{\mathcal{B}_t}(a_i)\right| + \\ \left|\mu_{\mathcal{B}_t}(a_i) - \mu_{\mathcal{C}_t}(a_i)\right| + \left|\pi_{\mathcal{B}_t}(a_i) - \pi_{\mathcal{C}_t}(a_i)\right| \end{matrix}\right\} & \text{if } \nu_{\mathcal{B}_t}(a_i) = \nu_{\mathcal{C}_t}(a_i) \\ \frac{1}{2n}\sum_{i=1}^n \left\{\begin{matrix} \left|\mu_{\mathcal{A}_t}(a_i)\right| + \left|\mu_{\mathcal{B}_t}(a_i)\right| + \left|\nu_{\mathcal{A}_t}(a_i) - \nu_{\mathcal{B}_t}(a_i)\right| + \left|\pi_{\mathcal{A}_t}(a_i) - \pi_{\mathcal{B}_t}(a_i)\right| + \\ \left|\mu_{\mathcal{B}_t}(a_i)\right| + \left|\mu_{\mathcal{C}_t}(a_i)\right| + \left|\nu_{\mathcal{B}_t}(a_i) - \nu_{\mathcal{C}_t}(a_i)\right| + \left|\pi_{\mathcal{B}_t}(a_i) - \pi_{\mathcal{C}_t}(a_i)\right| \end{matrix}\right\} & \text{if} \nu_{\mathcal{B}_t}(a_i) \neq \nu_{\mathcal{C}_t}(a_i) \end{cases}$$

TABLE 2. Intuitionistic fuzzy decision matrix $\mathcal{D}$.

D	$\mathbb{C}_1$	$\mathbb{C}_2$	$\mathbb{C}_3$	$\mathbb{C}_4$	$\mathbb{C}_5$	$\mathbb{C}_6$
$\mathbb{A}_1$	⟨0.6,0.3,0.1⟩	⟨0.7,0.3,0⟩	⟨0.4,0.4,0.2⟩	⟨0.3,0.7,0⟩	⟨0.8,0.2,0⟩	⟨0.4,0.6,0⟩
$\mathbb{A}_2$	⟨0.3,0.5,0.2⟩	⟨0.4,0.5,0.1⟩	⟨0.6,0.3,0.1⟩	⟨1,0,0⟩	⟨0.4,0.4,0.2⟩	⟨0.6,0.4,0⟩
$\mathbb{A}_3$	⟨0.5,0.4,0.1⟩	⟨0.8,0.2,0⟩	⟨0.5,0.4,0.1⟩	⟨0.8,0.1,0.1⟩	⟨0.7,0.3,0⟩	⟨0.7,0.3,0⟩
$\mathbb{A}_4$	⟨0.7,0.2,0.1⟩	⟨0.5,0.4,0.1⟩	⟨0.4,0.5,0.1⟩	⟨0.9,0.1,0⟩	⟨0.3,0.5,0.2⟩	⟨0.3,0.5,0.2⟩
$\mathbb{A}_5$	⟨0.8,0.2,0⟩	⟨0.6,0.3,0.1⟩	⟨0.8,0.2,0⟩	⟨0.2,0.7,0.1⟩	⟨0.5,0.3,0.2⟩	⟨0.5,0.4,0.1⟩
$\mathbb{A}_6$	⟨0.4,0.5,0.1⟩	⟨0.7,0.2,0.1⟩	⟨0.7,0.2,0.1⟩	⟨0.4,0.4,0.2⟩	⟨0.4,0.5,0.1⟩	⟨1,0,0⟩
$\mathbb{A}_7$	⟨0.5,0.5,0⟩	⟨1,0,0⟩	⟨0.3,0.6,0.1⟩	⟨0.4,0.5,0.1⟩	⟨0.6,0.4,0⟩	⟨0.6,0.3,0.1⟩

TABLE 3. 0.9-intuitionistic fuzzy decision matrix $\mathcal{D}_{0.90}$.

$\mathcal{D}_{0.90}$	$\mathbb{C}_1$	$\mathbb{C}_2$	$\mathbb{C}_3$	$\mathbb{C}_4$	$\mathbb{C}_5$	$\mathbb{C}_6$
$\mathbb{A}_1$	⟨0.6,0.3,0.1⟩	⟨0.7,0.3,0⟩	⟨0.4,0.4,0.2⟩	⟨0.3,0.7,0⟩	⟨0.8,0.2,0⟩	⟨0.4,0.6,0⟩
$\mathbb{A}_2$	⟨0.3,0.5,0.2⟩	⟨0.4,0.5,0.1⟩	⟨0.6,0.3,0.1⟩	⟨0.9,0.1,0⟩	⟨0.4,0.4,0.2⟩	⟨0.6,0.4,0⟩
$\mathbb{A}_3$	⟨0.5,0.4,0.1⟩	⟨0.8,0.2,0⟩	⟨0.5,0.4,0.1⟩	⟨0.8,0.1,0.1⟩	⟨0.7,0.3,0⟩	⟨0.7,0.3,0⟩
$\mathbb{A}_4$	⟨0.7,0.2,0.1⟩	⟨0.5,0.4,0.1⟩	⟨0.4,0.5,0.1⟩	⟨0.9,0.1,0⟩	⟨0.3,0.5,0.2⟩	⟨0.3,0.5,0.2⟩
$\mathbb{A}_5$	⟨0.8,0.2,0⟩	⟨0.6,0.3,0.1⟩	⟨0.8,0.2,0⟩	⟨0.2,0.7,0.1⟩	⟨0.5,0.3,0.2⟩	⟨0.5,0.4,0.1⟩
$\mathbb{A}_6$	⟨0.4,0.5,0.1⟩	⟨0.7,0.2,0.1⟩	⟨0.7,0.2,0.1⟩	⟨0.4,0.4,0.2⟩	⟨0.4,0.5,0.1⟩	⟨0.9,0.1,0⟩
$\mathbb{A}_7$	⟨0.5,0.5,0⟩	⟨0.9,0.1,0⟩	⟨0.3,0.6,0.1⟩	⟨0.4,0.5,0.1⟩	⟨0.6,0.4,0⟩	⟨0.6,0.3,0.1⟩

A. APPLICATION OF t-INTUITIONISTIC FUZZY TOPSIS IN REAL-WORLD SCENARIO

In the following section, we provide a numerical example to demonstrates the effectiveness of this approach.

The mathematics department of a certain university wants to award three merit scholarships to the three most intelligent researchers from a list of many applicants. After an initial screening, only seven scholars $\{\mathbb{A}_1, \mathbb{A}_2, \mathbb{A}_3, \mathbb{A}_4, \mathbb{A}_5, \mathbb{A}_6, \mathbb{A}_7\}$ were chosen for further investigation. A committee was constituted to examine the credibility of these scholars on the basis of a set criterion $\{\mathbb{C}_1, \mathbb{C}_2, \mathbb{C}_3, \mathbb{C}_4, \mathbb{C}_5, \mathbb{C}_6\}$, where $\mathbb{C}_1$: CGPA; A good CGPA is crucial for scholarship applications, it reflects academic performance and potential to excel. It positions a competitive candidate and can open doors to fully funded scholarships.

$\mathbb{C}_2$: Number of research papers; Publishing research papers in reputable journals shows critical thinking abilities and endurance. It requires independent analysis and evaluation of the subject matter. Having 2-3 published papers in peer-reviewed journals is recommended for securing competitive scholarships.

$\mathbb{C}_3$: Quality of research; Quality of research provides robust evidence, is ethical, stands up to scrutiny, and can be used to inform policy making. It should adhere to principles of professionalism, transparency, accountability, and auditability.

$\mathbb{C}_4$: Research proposal; A research proposal outlines a project's research method, summarizing its material. Researchers and university students use research proposals to complete dissertations, obtain funding, and meet course requirements.

$\mathbb{C}_5$: Personal statement; A personal statement of purpose for a scholarship should include an introduction, passion for the course, skills, future goals, experiences, and motivation for applying.

$\mathbb{C}_6$: Interview; During scholarship interviews, students are evaluated on their strengths, weaknesses, goals, personalities, and academics to provide a more personalized understanding of the candidate beyond their application or resume, respectively.

The findings of the committee about the performance of seven scholars are arranged as an $\mathbb{IF}$ decision matrix $\mathfrak{D}$ having intuitionistic fuzzy values as its entries. These details are summarized in the following Table 2. It has been decided that those students who achieve 90% of each criterion will be selected for this scholarship.

Step I: The $0.90 - \mathbb{IF}$ decision matrix $\mathcal{D}_{0.90}$ for the value of the parameter $t = 0.90$ is obtained by applying Definition (2.3) on Table 2 and is presented in the following Table 3.

Step II: The intuitionistic fuzzy weighted vector set W corresponding to the set of criterions designated by the committee is given below:

$$W = \left\{ \begin{array}{l} \langle 0.7, 0.3, 0\rangle, \langle 0.6, 0.2, 0.2\rangle, \langle 0.3, 0.4, 0.3\rangle, \\ \langle 0.7, 0.3, 0\rangle, \langle 0.3, 0.3, 0.4\rangle, \langle 0.7, 0.1, 0.2\rangle \end{array} \right\}.$$

The 0.9- intuitionistic fuzzy associated weight vector $W_{0.90}$ corresponding to set of criterions is determined by applying

TABLE 4. 0.9-intuitionistic fuzzy Weighted decision matrix $\mathcal{W}_{ij}$.

Alternative $(\mathbb{A}_i)$	*Criteria* $(\mathbb{C}_j)$	$\mathcal{W}_{ij}$
$\mathbb{A}_1$	$\mathbb{C}_1$	⟨0.4773,0.4615,0.0612⟩
	$\mathbb{C}_2$	⟨0.4773,0.4043,0.1184⟩
	$\mathbb{C}_3$	⟨0.2069,0.5774,0.2217⟩
	$\mathbb{C}_4$	⟨0.2658,0.7342,0⟩
	$\mathbb{C}_5$	⟨0.2791,0.4043,0.3166⟩
	$\mathbb{C}_6$	⟨0.3415,0.6170,0.0415⟩
$\mathbb{A}_2$	$\mathbb{C}_1$	⟨0.2658,0.5882,0.1460⟩
	$\mathbb{C}_2$	⟨0.3158,0.5556,0.1286⟩
	$\mathbb{C}_3$	⟨0.25,0.5227,0.2273⟩
	$\mathbb{C}_4$	⟨0.6495,0.3505,0⟩
	$\mathbb{C}_5$	⟨0.2069,0.5227,0.2704⟩
	$\mathbb{C}_6$	⟨0.4773,0.4375,0.0852⟩
$\mathbb{A}_3$	$\mathbb{C}_1$	⟨0.4118,0.5227,0.0655⟩
	$\mathbb{C}_2$	⟨0.5217,0.3333,0.1450⟩
	$\mathbb{C}_3$	⟨0.2308,0.5714,0.1978⟩
	$\mathbb{C}_4$	⟨0.5957,0.3505,0.0538⟩
	$\mathbb{C}_5$	⟨0.2658,0.5227,0.2115⟩
	$\mathbb{C}_6$	⟨0.5385,0.3505,0.1110⟩
$\mathbb{A}_4$	$\mathbb{C}_1$	⟨0.5385,0.4043,0⟩
	$\mathbb{C}_2$	⟨0.4286,0.4043,0.1671⟩
	$\mathbb{C}_3$	⟨0.2069,0.,0.1681⟩
	$\mathbb{C}_4$	⟨0.6495,0.3505,0⟩
	$\mathbb{C}_5$	⟨0.1765,0.5882,0.2353⟩
	$\mathbb{C}_6$	⟨0.2653,05263,0.2079⟩
$\mathbb{A}_5$	$\mathbb{C}_1$	⟨0.5957,0.4043,0⟩
	$\mathbb{C}_2$	⟨0.4286,0.4043,0.1671⟩
	$\mathbb{C}_3$	⟨0.2791,0.4783,0.2426⟩
	$\mathbb{C}_4$	⟨0.1842,0.7342,0.0816⟩
	$\mathbb{C}_5$	⟨0.2308,0.4615,0.3077⟩
	$\mathbb{C}_6$	⟨0.4118,0.4375,0.1507⟩
$\mathbb{A}_6$	$\mathbb{C}_1$	⟨0.3415,0.5882,0.0703⟩
	$\mathbb{C}_2$	⟨0.4773,0.3333,0.1894⟩
	$\mathbb{C}_3$	⟨0.2658,0.4783,0.2559⟩
	$\mathbb{C}_4$	⟨0.3415,0.5227,0.1358⟩
	$\mathbb{C}_5$	⟨0.2069,0.5882,0.2049⟩
	$\mathbb{C}_6$	⟨0.6495,0.1818,0.1687⟩
$\mathbb{A}_7$	$\mathbb{C}_1$	⟨0.4118,0.5882,0⟩
	$\mathbb{C}_2$	⟨0.5625,0.2653,0.1722⟩
	$\mathbb{C}_3$	⟨0.1765,0.6842,0.1393⟩
	$\mathbb{C}_4$	⟨0.3415,0.5882,0.0703⟩
	$\mathbb{C}_5$	⟨0.25,0.5227,0.2273⟩
	$\mathbb{C}_6$	⟨0.4773,0.3505,0.1722⟩

Definition (2.3) on W and is express as follows:

$$W_{0.90} = \left\{ \begin{array}{c} \langle 0.7, 0.3, 0\rangle, \langle 0.6, 0.2, 0.2\rangle, \langle 0.3, 0.4, 0.3\rangle, \\ \langle 0.7, 0.3, 0\rangle, \langle 0.3, 0.4, 0.3\rangle, \langle 0.7, 0.1, 0.2\rangle \end{array} \right\}.$$

Step III: The $0.90 - \mathbb{IF}$ weighted decision matrix is determined by applying equation (8) on Table 3 and $W_{0.90}$.

The outcomes of this procedure are summarized in the following Table 4.

TABLE 5. 0.9-Intuitionistic fuzzy positive and negative ideal solutions.

	$\mathbb{C}_j^+$	$\mathbb{C}_j^-$
$\mathbb{C}_1$	⟨0.5957,0.4043,0⟩	⟨0.2658,0.5882,0.1460⟩
$\mathbb{C}_2$	⟨0.5625,0.2653,0.1722⟩	⟨0.3158,0.5556,0.1286⟩
$\mathbb{C}_3$	⟨0.2791,0.4783,0.2426⟩	⟨0.1765,0.6842,0,1393⟩
$\mathbb{C}_4$	⟨0.6495,0.3505,0⟩	⟨0.1842,0.7342,0.0816⟩
$\mathbb{C}_5$	⟨0.2791,0.4043,0.3166⟩	⟨0.1765,0.5882,0.2353⟩
$\mathbb{C}_6$	⟨0.6495,0.1818,0.1687⟩	⟨0.2658,0.6170,0.1172⟩

TABLE 6. Lift Distance measures $d^+_{\mathbb{A}_i}$ and $d^-_{\mathbb{A}_i}$.

	$d^+_{\mathbb{A}_i}(\mathcal{W}_{ij}, \mathbb{C}_j^+)$	$d^-_{\mathbb{A}_i}(\mathcal{W}_{ij}, \mathbb{C}_j^-)$
$\mathbb{A}_1$	0.4198	0.2517
$\mathbb{A}_2$	0.3650	0.2934
$\mathbb{A}_3$	0.3792	0.3771
$\mathbb{A}_4$	0.2870	0.1500
$\mathbb{A}_5$	0.3104	0.3134
$\mathbb{A}_6$	0.3160	0.3033
$\mathbb{A}_7$	0.4231	0.2680

TABLE 7. Closeness coefficient $\mathcal{X}_{\mathbb{A}_i}$.

$\mathbb{A}_i$	$\mathcal{X}_{\mathbb{A}_i}$
$\mathbb{A}_1$	0.5996
$\mathbb{A}_2$	0.8038
$\mathbb{A}_3$	0.9945
$\mathbb{A}_4$	0.5226
$\mathbb{A}_5$	0.9904
$\mathbb{A}_6$	0.9598
$\mathbb{A}_7$	0.6334

Step IV: The 0.90-$\mathbb{IFPIS}$ $\mathbb{C}_j^+$ and $0.90 - \mathbb{IFNIS}$ $\mathbb{C}_j^-$ are derived by applying the equations (9) and (10) respectively on Table 4. The calculated values of these solutions are arranged in the following Table 5.

Step V: The lift distance measures $d^+_{\mathbb{A}_i}$ and $d^-_{\mathbb{A}_i}$ of each alternative $\mathbb{A}_i$ are obtained by applying equation (11) and equation (12) on Table 3 and Table 5, respectively. The computed values are listed in the following Table 6.

Step VI: Calculate closeness coefficient $\mathcal{X}_{\mathbb{A}_i}$ of each alternative $\mathbb{A}_i$ by applying equation (13) on Table 6. The outcomes of this calculation are summarized in the following Table 7.

TABLE 8. Comparative evaluation of proposed and existing TOPSIS techniques.

Techniques	$\mathbb{A}_1$	$\mathbb{A}_2$	$\mathbb{A}_3$	$\mathbb{A}_4$	$\mathbb{A}_5$	$\mathbb{A}_6$	$\mathbb{A}_7$	Ranking
Fuzzy TOPSIS	0.7614	0.9394	0.4796	0.9032	0.5539	0.6654	0.6029	$\mathbb{A}_2 \succ \mathbb{A}_4 \succ \mathbb{A}_1 \succ \mathbb{A}_6 \succ \mathbb{A}_7 \succ \mathbb{A}_5 \succ \mathbb{A}_3$
$\mathbb{IF}$-TOPSIS	0.7350	0.6505	0.7881	0.6330	0.9114	0.8571	0.6246	$\mathbb{A}_5 \succ \mathbb{A}_6 \succ \mathbb{A}_3 \succ \mathbb{A}_1 \succ \mathbb{A}_2 \succ \mathbb{A}_4 \succ \mathbb{A}_7$
0.6- $\mathbb{IF}$-TOPSIS	0.5138	0.6660	0.0334	0.5598	0.2095	0.5004	0.8064	$\mathbb{A}_7 \succ \mathbb{A}_2 \succ \mathbb{A}_4 \succ \mathbb{A}_1 \succ \mathbb{A}_6 \succ \mathbb{A}_5 \succ \mathbb{A}_3$
0.7- $\mathbb{IF}$-TOPSIS	0.6893	0.8513	0.3203	0.8859	0.5716	0.7582	0.8556	$\mathbb{A}_4 \succ \mathbb{A}_7 \succ \mathbb{A}_2 \succ \mathbb{A}_6 \succ \mathbb{A}_1 \succ \mathbb{A}_5 \succ \mathbb{A}_3$
0.8- $\mathbb{IF}$-TOPSIS	0.5028	0.7109	0.6700	0.8673	0.9806	0.7843	0.6988	$\mathbb{A}_5 \succ \mathbb{A}_4 \succ \mathbb{A}_6 \succ \mathbb{A}_2 \succ \mathbb{A}_7 \succ \mathbb{A}_3 \succ \mathbb{A}_1$

TABLE 9. Comparative analysis of proposed startegy with existing technique.

Aspects	Techniques		
	Fuzzy TOPSIS	$\mathbb{IF}$-TOPSIS	t-$\mathbb{IF}$-TOPSIS
Information Granularity	Fuzzy linguistic terms are employed to represent subjective assessments.	By incorporating non-membership degrees, it offers a more comprehensive and elaborate representation.	Through the utilization of the t parameter, it enables the adjustment of the granularity of the aggregation function.
Handling Uncertainty	It tackles uncertainty by using fuzzy sets and linguistic terms.	Incorporating non-membership degrees helps capture hesitation and uncertainty more accurately.	Introducing a parameterized approach takes effects even further, significantly enhancing flexibility in handling uncertainty. It has more control and customization when dealing with uncertain situations.
Customization	It offers less flexibility in terms of customization when compared to the other two techniques.	Incorporating the non-membership degree makes the aggregation function more flexible and adaptable to different scenarios.	Its parameterized approach offers a higher level of customization, enabling users to adjust the aggregation levels according to their needs.
Parametrization	There is no parameter involved.	There is no parameter involved.	It includes the "t" parameter.

From the Table 7, we note that

$$\mathbb{A}_3 \succ \mathbb{A}_5 \succ \mathbb{A}_6 \succ \mathbb{A}_2 \succ \mathbb{A}_7 \succ \mathbb{A}_1 \succ \mathbb{A}_4.$$

Consequently, the scholar $\mathbb{A}_3$, $\mathbb{A}_5$ and $\mathbb{A}_6$ are selected to award the three merit scholarships.

V. COMPARATIVE ANALYSIS OF THE PROPOSED TECHNIQUE

In this section, we establish a comprehensive comparative analysis to show case the validity and efficiency of the proposed technique with existing strategies.

The following Table 8 describes the solutions of the above-mentioned problem corresponding to various values of the parameter t. From the following Table 8, it is quite evident that $\mathbb{IF}$-TOPSIS offers only one solution to a problem. On the other hand, t-$\mathbb{IF}$-TOPSIS provides the class of solutions based on the value of the parameter's 't'.

It means that the selection of a parameters gives flexible choices about a particular decision. For instance, in view of the above decision-making problem one can select different scholarship awardees with respect to the values of the parameter t, whereas there is only one choice of selecting the scholarship awardees in $\mathbb{IF}$ environment. The t-$\mathbb{IF}$-TOPSIS offers the ability to precisely modify $\mathbb{IF}$-TOPSIS within various choice frameworks, improving its coherence and flexibility. This strategy emerges as a helpful tool for addressing the complexities linked to uncertainty in the decision-making approach. Fuzzy TOPSIS takes into account the degree of membership but does not explicitly consider reluctance or non-membership during the assessment process. Moreover, it is important to note that the Fuzzy TOPSIS strategy developed in [53], [54], and [55] becomes a special case of the recently defined strategy by considering the non-membership degree is equal to zero. By incorporating t-norm and t-conorm parameters into t-$\mathbb{IF}$-TOPSIS, decision-makers can represent their preferences more flexibly, as well as handle uncertainty better, because these criteria can be adjusted to reflect different levels of moderation or expansiveness in the decision-making process. Table 9 presents a comparative analysis of the suggested methodology with the existing strategies. The evaluation incorporates various benefits, including information granularity, uncertainty handling, customization, and parameterization. The study aims to assess the efficacy of the new approach concerning the mentioned parameters.

A. LIMITATIONS OF THE CURRENT TECHNIQUES

In this subsection, we discuss a limitation of the proposed technique.

The following are the limitations of the proposed techniques:

- The t-$\mathbb{IF}$ setting may not be suitable for scenarios where the sum of the degree of membership and non-membership of an element exceeds 1. To overcome this limitation, we recommend exploring this technique in more adaptable contexts, such as Pythagorean fuzzy, fermentation fuzzy, picture fuzzy, and spherical fuzzy environments.
- The existence of uncertainty about assigning the weight to the set of criteria is another limitation of this method. The t-$\mathbb{IF}$-TOPSIS may not counter this challenge. Therefore, we recommend t-$\mathbb{IF}$-AHP to address this issue

VI. CONCLUSION

The utilization of MCDM techniques is crucial for decision-makers facing complex and uncertain environments. It allows them to make informed choices that align with their goals and objectives. By incorporating the parameter "t" into the t-$\mathbb{IF}$-TOPSIS methodology, the flexibility and customization of decision-making processes are enhanced. It is achieved by enabling the manipulation of t-norm and t-conorm procedures. The incorporation of the parameter "t" provides decision-makers with a more intricate portrayal of their convictions and reservations in assessments. In this article, we have defined the idea of lift distance measures and established their structural properties. We have also proposed t-$\mathbb{IF}$-TOPSIS methodology to solve MCDM problems more effectively. We have effectively used this method to address the MCDM issue of merit scholarship awards at a particular university.

In the future, we will concentrate on applying the proposed lift distance measure to solve MCDM problems in various physical phenomena, specifically environmental protection, and the engineering sector in t-$\mathbb{IF}$ setting. In addition, we will develop this methodology to a more generalized environments like Pythagorean fuzzy, picture fuzzy, spherical fuzzy and fermentation fuzzy environments to counter the limitations of this methodology in t-$\mathbb{IF}$ environment. We will also design t-$\mathbb{IF}$-VIKOR mechanism to give more effective solutions of MCDM problems under t-$\mathbb{IF}$ knowledge.

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