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RESEARCH ARTICLE

Space Shift Keying (SSK) Transmission Over Rayleigh Fading Channels and Symmetric Alpha-Stable Noise

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ABSTRACT This paper presents a comprehensive analytical study of the average bit error probability (ABEP) performance of multiple-input multiple output (MIMO) communication systems employing Space Shift Keying (SSK) modulation operating over a mixture of Rayleigh fading and impulsive noise channels modelled by the symmetric alpha-stable distribution. By considering three key receiver structures, namely the genie-aided (GA), minimum distance (MD) and maximum likelihood (ML) type of receivers various analytical ABEP performance evaluation results are presented. Firstly, a novel analytical approach for evaluating the ABEP of SSK-MIMO systems, for the ideal benchmark GA receiver which has a-priori knowledge of both channel and noise coefficients, is introduced. The proposed methodology yields exact expressions for the ABEP performance of multiple-input single output (MISO) systems with two transmit antennas. For the general case of MIMO systems which employ an arbitrary number of transmit and/or receive antennas, accurate approximations and tight upper performance bounds are derived and their ABEP performance is analyzed. An asymptotic ABEP analysis is also carried out from which the diversity and the coding gains are derived. Secondly, a general class of minimum distance (MD) receivers, namely the L_p-norm receivers, is considered. An approximate analytical expression for the ABEP of the special case of L_2-norm (matched filter) receivers, whose performance is optimal for the additive white Gaussian noise channel (AWGN), is derived. For the general case of L_p-norm receivers, their ABEP performance is evaluated by means of Monte Carlo simulations, revealing that they significantly outperform the L_2-norm receivers. Thirdly, by considering the maximum-likelihood (ML) receiver, since its implementation complexity turns out to be prohibitively high, simple, suboptimal receiver configurations structures of the ML receiver are instead proposed. Analytical evaluation results verified by complementary computer simulations have shown that their ABEP performance is asymptotically, i.e., at high signal-to-noise ratios (SNR), optimal. For the GA, suboptimal ML and L_p-norm receiver structures, the impact of spatial correlation on their ABEP performance has also been analyzed and evaluated. The accuracy of the analytical approaches used in deriving the proposed receivers has been validated by equivalent numerical ABEP performance evaluation results accompanied by complementary Monte Carlo simulations.

INDEX TERMS Alpha stable distribution, average bit error probability, space shift keying (SSK) modulation, spatial Poisson process, multiple-input–multiple-output (MIMO) systems, network interference.

I. INTRODUCTION

Spatial modulation (SM) has emerged as an efficient and low-complexity transmission scheme for multiple-input–multiple-output (MIMO) wireless communication

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systems [2], [3], [4]. The working principle of SM is to use both conventional phase or amplitude modulation techniques, such as phase shift keying (PSK) and quadrature amplitude modulation (QAM), as well as a unique antenna index, selected from the set of transmitting antennas, to convey information. At every given time slot, only one transmitting antenna is active for data transmission whereas the remaining transmitting antennas are kept silent [3]. Furthermore, in [5], the so-called space shift keying (SSK) modulation, has been proposed as a low–complexity implementation of SM, so that the trade–off between receiver complexity and achievable data rate can be considered.

In the past, the performance of SM systems operating over fading and additive white Gaussian noise (AWGN) channels has been addressed in various research works, e.g. see [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17] and references therein. In these works, diversity techniques, such as maximum ratio combining (MRC), are employed at the receiver to increase the achievable diversity gain. It is well known that such schemes yield an optimal performance in the AWGN channel. However, such optimality is not valid for communication systems operating over impulsive noise channels [18], [19]. It is noted that impulsive noise which has non-Gaussian statistics occurs in various operating conditions, such as lightning discharges and network interference [20], [21].

Several probability distributions have been proposed in the open technical literature to model impulsive noise. Representative examples include the Middleton Class A (MCA) distribution [18] and the symmetric α-stable (SαS) distribution [22], [23]. The MCA distribution is a mixture of a large number of Gaussian random variables (RVs) with different variances and characterizes the sparsity of high-amplitude spikes in noise. This distribution has been used extensively in the past to model impulsive noise in communication systems, e.g., [19], [24], and [25].

On the other hand, the SαS distribution describes the statistical characteristics of the noise amplitude distribution. This distribution can accurately model aggregate interference in a multi-user network in which interfering nodes are spatially distributed according to a Poisson point process (PPP) [20], [26] as well as impulsive noise power line communication (PLC) systems [25], [27]. In recent years, the SαS distribution has emerged as an important alternative to the MCA distribution because it can more accurately model the heavy tail characteristics of the impulsive noise and its parameters can be estimated from the amplitude statistics in a consistent manner [28].

The performance of wireless systems operating in the presence of SαS noise has been addressed in several research works. For example, [21] and [29] have investigated the performance of diversity receivers and space-time block coding (STBC) systems, respectively, assuming Rayleigh fading channels. In [30] analytical expressions for the symbol error rate of M-ary modulation schemes of single-antenna systems operating over generalized fading channels, have been presented.

Furthermore, the problem of signal detection under SαS noise has been addressed in [31], [32], [33], [34], [35], and [36]. For example, in [31], the error performance of four different classes of receivers, namely the optimum maximum likelihood (ML), the minimum distance (Gaussian), the Cauchy and the limiter-plus-integrator, operating in the presence of SαS noise, has been addressed. In [32], the error performance of the ML, Gaussian and limiter-plus-integrator receiver has been analyzed in the presence of mixture Gaussian and stable noise. In [33], a max-min distributed detector for wireless sensor networks has been proposed. In [34], further performance evaluation results for suboptimal receivers, namely Gaussian, soft limiter, myriad and Cauchy, have been presented. In [35], the optimality of the myriad filter in the presence of alpha-stable noise has been investigated. In [36], a performance analysis of spectrum sensing schemes based on fractional lower order moments for cognitive radio has been presented. However, to the best of our knowledge, the design and performance analysis of SM systems operating in the presence of both fading and impulsive noise has not yet been considered in the open technical literature.

On the other hand, in order to analyze the error performance of SM systems in the presence of Gaussian noise, an optimum receiver based on ML detection is usually assumed, e.g, see [6]. Nevertheless, as it will become evident later on in our paper, the complexity of such a receiver in the presence of SαS noise is prohibitively high, as it involves the numerical computation of infinite integrals of special functions. In addition, the well known minimum distance (MD) receiver, which is optimal in the presence of Gaussian noise, performs poorly in a SαS noise environment.

Motivated by the above, in this paper we analyze in a systematic manner the performance of SSK systems operating over Rayleigh fading and SαS noise by considering the following generic receiver structures: *i*) Genie-aided receivers (GA) which assume complete a-priori knowledge of both channel and noise parameters and can be considered as a benchmark receiver; ii) MD receivers, which as well known have optimal performance in the AWGN channel; iii) ML receivers which although theoretically achieve the best performance, are not practical because of their high implementation complexity and thus, suboptimal reduced-complexity receiver structures are instead considered.

Within this framework, the main contributions of this paper can be summarized as follows:

- For the GA receiver structure, we present novel, single-integral expressions for the analytical evaluation of the average bit error Probability (ABEP) of SSK-MISO systems. The resulting analytical expressions are exact when the transmitter is equipped with two antennas. Both independent and identically distributed (i.i.d.) fading as well as correlated fading channels are considered.

TABLE 1. List of mathematical notations used in this paper.

$\jmath^2 = -1$ denotes the imaginary unit
$\lvert z \rvert$ denotes the magnitude of the complex number z
z^* denotes the conjugate of the complex number z
$\Re\{z\}$ denotes the real part of the complex number z
$\Im\{z\}$ denotes the real part of the complex number z
$f(x) = o[g(x)]$ as $x \to x_0$ if $\lim_{x\to x_0} \frac{f(x)}{g(x)} = 0$
$\lVert \cdot \rVert_F^2$ denotes the square Frobenius norm
$(\cdot)^T$ denotes the matrix transpose
$(\cdot)^H$ denotes the hermitian transpose
$\odot$ denotes matrix Hadamard product
$\mathbb{C}$ denotes the set of complex numbers
$\mathbb{E}_X\langle\cdot\rangle$ denotes expectation with respect to the random variable (RV) X
$f_X(\cdot)$ denotes the Probability Density Function (PDF) of the RV X
$F_X(\cdot)$ denotes the Cumulative Distribution Function (CDF) of the RV X
$\mathcal{M}_X(s) = \mathbb{E}_X\langle\exp(-sX)\rangle$ denotes the Moment Generating Function (MGF) of the RV X
$\Phi_X(t) = \mathbb{E}_X\langle\exp(\jmath tX)\rangle$ denotes the Characteristic Function (CHF) of the RV X
$J_a(\cdot)$ is the Bessel function of the first kind and order a [1, eq. (8.402))]
$\Gamma(x) = \int_0^\infty \exp(-t)t^{x-1}\mathrm{d}t$ is the Gamma function [1, eq. (8.310/1)]
$\Gamma(a,x) = \int_x^\infty \exp(-t)t^{a-1}\mathrm{d}t$ is the upper incomplete Gamma function [1, eq. (8.350/2)]
$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty \exp(-t^2/2)\mathrm{d}t$ is the Gauss Q-function
$\mathrm{sign}(x)$ is the signum of the real number x
$S(x) = \int_0^x \sin(\pi t^2/2)\mathrm{d}t$ is the Fresnel sine integral [1, eq. (8.250/2)]
$C(x) = \int_0^x \cos(\pi t^2/2)\mathrm{d}t$ is the Fresnel cosine integral [1, eq. (8.250/3)]
$\mathrm{Si}(x) = \int_0^x \sin(t)/t\,\mathrm{d}t$ is the sine integral function [1, eq. (8.230/1)]
$\mathrm{Ci}(x) = -\int_x^\infty \cos(t)/t\,\mathrm{d}t$ is the cosine integral function [1, eq. (8.230/1)]
$\Pr\{\cdot\}$ denotes the probability operator
$\hat{\cdot}$ denotes estimated value at the receiver side
$\mathcal{F}\{f(x), x; t\} = \int_{-\infty}^{\infty} \exp(\jmath tx) f(x)\mathrm{d}x$ denotes the Fourier transform of the function $f(x)$

For SSK-MIMO systems equipped with two transmit antennas, approximate, yet accurate analytical expressions are further derived in terms of a single integral. An asymptotic analysis for high values of signal-to-noise ratio (SNR) is carried out, from which the diversity and coding gains are obtained. Upper bounds for the ABEP that become tight for high values of the SNR are deduced for SSK-MISO and SSK-MIMO systems equipped with arbitrary number of transmitting antennas. Furthermore, similar analytical expressions for the performance of SSK-MIMO systems are derived;

- We analyze the performance of SSK-MIMO systems assuming a generic class of minimum distance (MD) receiver by considering an alternative receiver structure that employs an L_p-norm distance metric. This receiver yields significant performance gains as compared to the traditional L_2-norm (matched filter) receiver;
- We introduce the optimum maximum likelihood (ML) detector and propose simpler, suboptimal receiver structures that yield a close-to-the-optimal performance at high SNR values;
- We analyze and evaluate the impact of spatial correlation on the ABEP performance of the GA, L_p-norm and suboptimal ML receiver structures.

The proposed analysis has been validated by numerically evaluated results and equivalent performance results obtained by means of Monte Carlo simulations.

The remainder of this paper is organized as follows. Section II outlines the system and noise models. In Section III the proposed receiver designs are presented and their ABEP performance in the presence of both fading and noise is evaluated. Performance evaluation results are presented in Section IV, whereas Section V concludes the paper.

Notations: A comprehensive list of all mathematical notations used in this paper appears in Table 1.

II. SYSTEM MODEL

Consider a $N_r \times N_t$ MIMO SSK system with N_r and N_t being the number of receiving and transmit antennas, respectively. Assuming a frequency-flat fading channel model the received complex signal vector, $\mathbf{y} \in \mathbb{C}^{N_r \times 1}$, can be expressed as [3]

$$\mathbf{y} = \sqrt{\rho}\mathbf{H}\mathbf{x} + \mathbf{n} \tag{1}$$

where ρ is the average transmitting power at each transmitting antenna; $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ is the complex channel matrix, $\mathbf{x} = [x_1, x_2, \ldots, x_{N_t}]^T$ is the $N_t \times 1$ transmitting vector whose i^{th} element, x_i, is given as

$$x_i = \begin{cases} 1 & \text{if the } i\text{th transmitting antenna is active} \\ 0 & \text{if the } i\text{th transmitting antenna is not active}. \end{cases}$$

and $\mathbf{n} \in \mathbb{C}^{N_r \times 1}$ is the additive noise vector at the receiver.

The elements of the channel matrix, h_{ij}, are complex Gaussian RVs, having zero mean and unit variance, i.e. $h_{ij} \sim \mathcal{CN}(0, 1)$. Hereafter and unless otherwise stated, it is assumed that h_{ij} are i.i.d. RVs. It is noted that this fading model has been used in the past for the channel modeling of practical multi-antenna systems if a half wavelength-spaced uniform linear array (ULA) is deployed in an isotropic scattering environment [37]. However, in practice, antenna correlation is also present and can degrade the performance of multi-antenna systems [38], [39]. Nevertheless, an exact performance evaluation of the system under consideration when correlated fading channels are assumed, is difficult - if not impossible, to be carried out. Therefore, the impact of channel correlation will be investigated analytically for specific antenna configurations only, namely for $N_r = 1$ and $N_t = 2$. For the general case of $N_r > 1$, the performance evaluation will be carried out by means of Monte Carlo simulations.

The i^{th} element of the noise vector, $[\mathbf{n}]_i$, is a complex RV having SαS distributed real and imaginary parts. Following [23, p. 117] and [21], here $[\mathbf{n}]_i$ is assumed to be a complex isometric RV, i.e. it can be written as

$$[\mathbf{n}]_i = \sqrt{A_i}\left(G_i^R + \jmath G_i^I\right) \tag{2}$$

where G_i^R and G_i^I are i.i.d. real valued Gaussian RVs with zero mean and unit variance and A_i follows an alpha-stable distribution. In general, a mathematically tractable closed-form expression for the PDF of A_i, suitable for the performance evaluation of multi-antenna systems, is not available in the open technical literature. Recently, generic closed-form expressions for the PDF of A_i in terms of the Fox's H-function have been presented in [30] and [36]. For specific values of α, these expressions can be further reduced to the PDFs of well known distributions, such as for $\alpha = 1$ the Cauchy and for $\alpha = 2$ the Gaussian. Therefore, in this paper we follow a CHF-based approach to transform the resulting average error-rate integral expressions into the Fourier transform domain, using the Parseval theorem. This approach is much more efficient than the conventional PDF-based approach, because a closed-form expression for the CHF of A_i requiring only elementary functions is readily available.

Specifically, the CHF of A_i is given in [22] and [23] as

$$\Phi_{A_i}(t) = \exp\left[-|\sigma t|^{\alpha/2}(1 - \jmath\, \mathrm{sign}(t))\omega(t, \alpha)\right] \tag{3}$$

where $\alpha \in (0, 2]$ is the characteristic exponent, $\sigma = [\cos(\pi\alpha/4)]^{2/\alpha}$ is the scale parameter and

$$\omega(t, \alpha) = \begin{cases} \tan\left(\frac{\pi\alpha}{4}\right) & \text{if } \alpha \neq 2 \\ \frac{2}{\pi}|\log(t)| & \text{if } \alpha = 2. \end{cases} \tag{4}$$

It is noted that for a SαS RV only the moments of order α or less exist [23, p. 22].

III. PERFORMANCE ANALYSIS AND RECEIVER DESIGN

In this section, several receiver structures for SSK systems operating in the presence of stable noise and Rayleigh fading will be presented. Hereafter, it is assumed that the channel matrix $\mathbf{H}$ is perfectly known at the receiver side. We first analyze the performance an ideal receiver which assumes perfect a-priori knowledge of the noise components A_i in (2). Next, the performance of practical receivers operating without knowledge of A_i will be presented.

A. GENIE AIDED (GA) RECEIVER

This receiver minimizes the probability of an erroneous symbol when the channel matrix $\mathbf{H}$ and the noise components A_i are a-priori known. In order to derive an analytical expression for the ABEP, the corresponding detection rule should first be derived. Letting $\mathbf{a} = [1/\sqrt{A_1}, 1/\sqrt{A_2}, \ldots, 1/\sqrt{A_{N_r}}]^T$ and $\mathbf{g} = [G_1, G_2, \ldots, G_{N_r}]^T$, (1) can be expressed as

$$\mathbf{y} \odot \mathbf{a} = \sqrt{\rho}(\mathbf{H}\,\mathbf{x}) \odot \mathbf{a} + \mathbf{g}. \tag{5}$$

Since the elements of $\mathbf{g}$ are $\mathcal{CN}(0, 2)$ RVs, the optimal detection rule, assuming that the symbol $\mathbf{x}_t$ has been transmitted, $\forall t \in \{1, 2, \ldots, N_t\}$, minimizes the following distance metric

$$\hat{\mathbf{x}}_t = \underset{t}{\mathrm{argmin}}\left\{\| \mathbf{y} \odot \mathbf{a} - \sqrt{\rho}(\mathbf{H}\,\mathbf{x}_t) \odot \mathbf{a} \|_F^2\right\}. \tag{6}$$

Using (6), the average pairwise error probability (APEP) that $\mathbf{x}_t$ was transmitted and $\mathbf{x}_m$ was received by the GA receiver, can be expressed as

$$Pr(\mathbf{x}_t \to \mathbf{x}_m) = \mathbb{E}_{\mathbf{a},\mathbf{H}}\left\langle Q\left(\sqrt{\frac{\rho \,\| [\mathbf{H}\,(\mathbf{x}_t - \mathbf{x}_m)] \odot \mathbf{a} \|_F^2}{4}}\right)\right\rangle. \tag{7}$$

In what follows, analytical expressions for the APEP will be derived, assuming three antenna configurations, namely 2×1, $N_t \times 1$ and $N_t \times N_r$.

1) MISO 2 × 1 SSK SYSTEMS

By considering a MISO system with $N_t = 2$ and $N_r = 1$, there are two possible transmitted symbols, $[1, 0]^T$ and $[0, 1]^T$. Moreover, the vector $\mathbf{a}$ becomes a scalar, i.e. $\mathbf{a} = 1/\sqrt{A}$, and the channel matrix $\mathbf{H} = [h_1\, h_2]$. Using (7), the APEP simplifies to

$$\mathrm{APEP} = \mathbb{E}_{A,h_1,h_2}\left\langle Q\left(\sqrt{\frac{\rho|h_1 - h_2|^2}{4A}}\right)\right\rangle. \tag{8}$$

This APEP can be evaluated using the following proposition.

Proposition 1: The APEP of a 2×1 SSK system employing a GA receiver operating in the presence of i.i.d. Rayleigh fading and SαS noise can be expressed in terms of a single integral as

$$\mathrm{APEP} = \frac{1}{2} - \frac{1}{2\pi}\int_0^\infty \Re\Big\{(F_1(t) + \jmath F_2(t)) \times \exp\left[-(ct)^{\alpha/2}\left(1 + \jmath \tan\left(\frac{\pi\alpha}{4}\right)\right)\right]\Big\}\, \mathrm{d}t \quad (9)$$

where $c = [\cos(\pi\alpha/4)]^{\frac{2}{\alpha}}$ and $F_1(t)$ and $F_2(t)$ are given by (10) and (11), respectively, as shown at the bottom of the next page.

Proof: Assuming i.i.d. Rayleigh fading, the RV $Z = |h_1 - h_2|^2$ follows an exponential distribution with $f_Z(z) = 0.5\exp(-0.5z)$ [5], [12], [40]. Thus, an analytical expression for the conditional APEP, given A, can be deduced by employing [41, eq. (14-3-7)] as

$$Pr(\mathbf{x}_t \to \mathbf{x}_m | A) = \frac{1}{2} - \frac{1}{2}\left(1 + \frac{4A}{\rho}\right)^{-1/2}. \quad (12)$$

A single integral representation for the APEP can be obtained as

$$\mathrm{APEP} = \frac{1}{2} - \frac{1}{2}\int_0^\infty \left(1 + \frac{4A}{\rho}\right)^{-1/2} f_A(A)\mathrm{d}A. \quad (13)$$

The integral in (13) is very difficult, if not impossible, to be solved, since a closed form for $f_A(A)$ is not available. Nevertheless, by observing that A is a positive RV and using the Parseval's theorem, this APEP can be expressed as

$$\mathrm{APEP} = \frac{1}{2} - \frac{1}{2\pi}\int_0^\infty \Re\{G(t)\Phi_A^*(t)\}\mathrm{d}t. \quad (14)$$

where $G(t) = \mathcal{F}\left\{\left(1 + \frac{4A}{\rho}\right)^{-1/2} u(A), A; t\right\}$, which can be conveniently expressed as $G(t) = F_1(t) + \jmath F_2(t)$, where

$$F_1(t) = \int_0^\infty \left(1 + \frac{4A}{\rho}\right)^{-1/2} \cos(A\,t)\mathrm{d}A \quad (15a)$$

$$F_2(t) = \int_0^\infty \left(1 + \frac{4A}{\rho}\right)^{-1/2} \sin(A\,t)\mathrm{d}A. \quad (15b)$$

By employing [1, 3.751/1] and [1, 3.751/2], (15a) and (15b) can be solved in closed-form yielding (10) and (11), respectively. Finally, using (3), (9) is readily obtained thus completing the proof. ■

In the following analysis, the impact of spatial correlation on the error probability of 2×1 SSK systems will be discussed. We assume that h_1 and h_2 are correlated complex Gaussian RVs with covariance matrix

$$\mathbb{C} = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12}^* & \sigma_2^2 \end{bmatrix} \quad (16)$$

Proposition 2: The APEP of a 2×1 SSK system employing a GA receiver operating in the presence of correlated Rayleigh fading and SαS noise can be expressed in terms of a single integral as

$$\mathrm{APEP} = \frac{1}{2} - \frac{1}{2\pi}\int_0^\infty \Re\Big\{\left(F_1'(t) + \jmath F_2'(t)\right) \times \exp\left[-(ct)^{\alpha/2}\left(1 + \jmath \tan\left(\frac{\pi\alpha}{4}\right)\right)\right]\Big\}\, \mathrm{d}t \quad (17)$$

where $c = [\cos(\pi\alpha/4)]^{\frac{2}{\alpha}}$ and $F_1'(t)$ and $F_2'(t)$ are given by (10) and (11), respectively, by replacing ρ with

$$\rho' = \rho(\sigma_1^2 + \sigma_2^2 - 2\Re\{\sigma_{12}\})/2. \quad (18)$$

Proof: Assuming correlated Rayleigh fading, the RV $Y = h_1 - h_2$ is complex Gaussian having zero mean and variance $\sigma^2 = \mathbb{E}\langle |Y|^2\rangle = \mathbb{E}\langle |h_1|^2\rangle + \mathbb{E}\langle |h_2|^2\rangle - 2\Re\{\mathbb{E}\langle h_1 h_2^*\rangle\} = \sigma_1^2 + \sigma_2^2 - 2\Re\{\sigma_{12}\}$. Therefore, an analytical expression for the APEP of 2×1 SSK operating in the presence of correlated Rayleigh fading can be readily deduced using Proposition 1 by scaling ρ with $\rho\sigma^2/2$, thus completing the proof. ■

2) MIMO $2 \times N_r$ SSK SYSTEMS

Let us now consider a MIMO system with $N_t = 2$ transmitting and N_r receiving antennas, for which, again, the two possible transmitted symbols are $[1, 0]^T$ and $[0, 1]^T$. Using (7), the APEP can be deduced as

$$\mathrm{APEP} = \mathbb{E}_{\mathbf{a},\mathbf{H}}\left\langle Q\left(\sqrt{\rho\sum_{r=1}^{N_r}\frac{|h_{1r} - h_{2r}|^2}{4A_r}}\right)\right\rangle. \quad (19)$$

Since for this case, an exact expression cannot be obtained, an accurate approximation for the APEP will be derived using the following proposition.

Proposition 3: An accurate approximation for the APEP of a $2 \times N_r$ SSK system employing a GA receiver operating in the presence of i.i.d. Rayleigh fading and SαS noise can be deduced as

$$\mathrm{APEP} \approx \frac{1}{12}\left[F_3\left(\frac{\rho}{8}\right)\right]^{N_r} + \frac{1}{4}\left[F_3\left(\frac{\rho}{6}\right)\right]^{N_r} \quad (20)$$

where $F_3(s)$ is given by (21), as shown at the bottom of the next page, and $c = [\cos(\pi\alpha/4)]^{\frac{2}{\alpha}}$.

Proof: It can be observed that an exact analytical expression for the expectation in (19) is very difficult, if not impossible, to be obtained. An accurate approximate solution of (19) can be deduced by employing the tight exponential approximation for the Q-function proposed in [42, eq. (14)], i.e., $Q(x) \approx \frac{1}{12}\exp(-x^2/2) + \frac{1}{4}\exp(-2x^2/3)$. Since the entries of $\mathbf{H}$ and $\mathbf{a}$ are i.i.d. and using the definition of the MGF, (19) can be approximated as

$$\mathrm{APEP} \approx \frac{1}{12}\left[\mathcal{M}_{Z_r}\left(\frac{\rho}{8}\right)\right]^{N_r} + \frac{1}{4}\left[\mathcal{M}_{Z_r}\left(\frac{\rho}{6}\right)\right]^{N_r} \quad (22)$$

where $Z_r = |h_{1r} - h_{2r}|^2/A_r$, $\forall r \in \{1, 2, \ldots, N_r\}$. Conditioning on A_r, Z_r follows an exponential distribution with the following MGF

$$\mathcal{M}_{Z_r}(s) = \frac{A_r}{A_r + 2s} = 1 - \frac{2s}{A_r + 2s}. \quad (23)$$

Thus, an analytical expression for the APEP can be deduced by averaging $\mathcal{M}_{Z_r}(s)$ over A_r. In doing so, the evaluation of the following integral is required

$$F_3(s) = 1 - \int_0^\infty \frac{2s}{A + 2s} f_{A_r}(A)\mathrm{d}A. \quad (24)$$

By employing the Parseval's theorem, $F_3(s)$ can be expressed as

$$F_3(s) = 1 - \frac{1}{\pi}\int_0^{\infty} H(t)\Phi^*_{A_r}(t)\}\mathrm{d}t, \tag{25}$$

where

$$H(t) = \mathcal{F}\left\{\frac{2s}{A+2s}u(A), A; t\right\}. \tag{26}$$

The real and imaginary parts of $H(t)$ can be evaluated, respectively, as

$$\Re\{H(t)\} = \int_0^{\infty} \frac{2s\cos(A\,t)}{A+2s}\mathrm{d}A \tag{27a}$$

$$\Im\{H(t)\} = \int_0^{\infty} \frac{2s\sin(A\,t)}{A+2s}\mathrm{d}A. \tag{27b}$$

Using [1, Eq. (3.722/1)], [1, Eq. (3.722/3)] and after performing some algebraic manipulations, (21) is obtained, thus completing the proof. ■

Next, we present a simple closed-form expression for the APEP that becomes asymptotically tight at high SNR values. Based on this expression the diversity and coding gains of the GA can be deduced. The following result holds.

Proposition 4: For $\rho \to \infty$, the APEP of $2 \times N_r$ SSK system employing a GA receiver is given as

$$\text{APEP} = \frac{1}{2\sqrt{\pi}}\frac{\Gamma\left(\frac{N_r\alpha+1}{2}\right)}{\Gamma\left(\frac{N_r\alpha}{2}+1\right)}\left[\Gamma\left(1+\frac{\alpha}{2}\right)\right]^{N_r}\left(\frac{\rho}{8}\right)^{-N_r\alpha/2} + o\left(\rho^{-(N_r+1)\alpha/2}\right) \tag{28}$$

Proof: Using the well known Graig's representation of the Gauss Q-function [38],

$$Q(x) = \frac{1}{\pi}\int_0^{\pi/2}\exp\left(-\frac{x^2}{2\sin^2(\theta)}\right)\mathrm{d}\theta \tag{29}$$

(19) can be written as

$$\text{APEP} = \frac{1}{\pi}\int_0^{\pi/2}\mathbb{E}_{\mathbf{a},\mathbf{H}}\left\langle\exp\left(-\frac{\rho}{2\sin^2(\theta)}\sum_{r=1}^{N_r}\frac{|h_{1r}-h_{2r}|^2}{4A_r}\right)\right\rangle\mathrm{d}\theta \tag{30}$$

By observing that RVs A_r are i.i.d., each product term in (30) has the same expected value. Therefore, (30) can be written as

$$\text{APEP} = \frac{1}{\pi}\int_0^{\pi/2}\mathbb{E}_{A,\mathbf{H}}\left\langle\exp\left(-\frac{\rho}{8A\sin^2(\theta)}\sum_{r=1}^{N_r}|h_{1r}-h_{2r}|^2\right)\right\rangle\mathrm{d}\theta \tag{31}$$

where A is any of the RVs A_r. Observe that the RVs $W_r = |h_{1r} - h_{2r}|^2$ follow an exponential distribution with PDF $f_{W_r}(w) = 0.5\exp(-0.5w)$ and MGF $\mathcal{M}_{W_r}(s) = (1+2s)^{-1}$. By taking the expectation of (31) with respect to $\mathbf{H}$ and using the definition of the MGF, (31) can be expressed as

$$\text{APEP} = \frac{1}{\pi}\int_0^{\pi/2}\mathbb{E}_A\left\langle\left(1+\frac{\rho}{4A\sin^2(\theta)}\right)^{-N_r}\right\rangle\mathrm{d}\theta. \tag{32}$$

The expectation with respect to A can be evaluated as

$$\mathbb{E}_A\left\langle\left(1+\frac{\rho}{4A\sin^2(\theta)}\right)^{-N_r}\right\rangle = \int_0^{\infty}\left(1+\frac{\rho}{4A\sin^2(\theta)}\right)^{-N_r} f_A(A)\mathrm{d}A. \tag{33}$$

For large values of A, an asymptotic analytical expression for the PDF of A, $f_A(A)$, is given as [22]

$$f_A(A) = \frac{\alpha}{2\pi}\sin\left(\frac{\pi\alpha}{2}\right)\Gamma(\alpha/2)A^{-\alpha/2-1} + o(A^{-1-\alpha}) \tag{34}$$

By substituting (34) into (33) and employing [1, eq. (3.241/4)] as well as the well-known identities $\Gamma(1+x) = x\Gamma(x)$ [1, eq. (8.331/1)] and $\Gamma(x)\Gamma(1-x) = \pi/\sin(\pi x)$ [1, eq. (8.334/3)], (33) can be evaluated in closed form as

$$\mathbb{E}_A\left\langle\left(1+\frac{\rho}{4A\sin^2(\theta)}\right)^{-N_r}\right\rangle = \frac{\Gamma(1+\alpha/2)\Gamma(N_r-\alpha/2)}{\Gamma(N_r)\Gamma(1-\alpha/2)}\left(\frac{\rho}{4\sin^2(\theta)}\right)^{-\alpha/2}. \tag{35}$$

Finally, by substituting (35) into (33) and using [1, eq. (3.621/1)], [1, eq. (8.338/1)] and [1, eq. (8.384/1)], (28) is readily obtained, thus completing the proof. ■

$$F_1(t) = \frac{1}{2}\sqrt{\frac{\pi\rho}{2t}}\left\{\left[1-2C\left(\sqrt{\frac{\rho t}{2\pi}}\right)\right]\cos\left(\frac{\rho t}{4}\right)+\left[1-2S\left(\sqrt{\frac{\rho t}{2\pi}}\right)\right]\sin\left(\frac{\rho t}{4}\right)\right\} \tag{10}$$

$$F_2(t) = \frac{1}{2}\sqrt{\frac{\pi\rho}{2t}}\left\{\left[1-2S\left(\sqrt{\frac{\rho t}{2\pi}}\right)\right]\cos\left(\frac{\rho t}{4}\right)+\left[2C\left(\sqrt{\frac{\rho t}{2\pi}}\right)-1\right]\sin\left(\frac{\rho t}{4}\right)\right\} \tag{11}$$

$$F_3(s) = 1 - \frac{1}{\pi}\int_0^{\infty}\Re\left\{\frac{\jmath}{t}+2\,s\exp\left(-\jmath 2\,s\,t\right)\left(\frac{\jmath\pi}{2}-\mathrm{Ci}(2\,s\,t)-\jmath\,\mathrm{Si}(2\,s\,t)\right)\exp\left[-(ct)^{\alpha/2}\left(1+\jmath\tan\left(\frac{\pi\alpha}{4}\right)\right)\right]\right\}\mathrm{d}t \tag{21}$$

As it can be observed, the diversity gain of the proposed system configuration depends on both the number of the receiving antennas, N_r, as well the parameter α. Moreover, the resulting coding gain is also a function of N_r and α. By differentiating the natural logarithm of the coding gain defined in (10) with respect to α and N_r separately, it can be seen that the coding gain is a monotonically decreasing function of both parameters.

3) MIMO $N_t \times N_r$ SSK SYSTEMS

It is first noted that for $N_t > 2$, an exact expression for the APEP is even more difficult to be obtained. Nevertheless, by employing the well known union bound technique, a tight upper bound on the APEP of the considered system can be deduced as [7], [8]

$$\text{APEP} \leq \frac{N_t^{-1}}{\log_2(N_t)} \times \sum_{t_1=1}^{N_t} \sum_{t_2 \neq t_1 = 1}^{N_t} N_b(t_1, t_2) \text{APEP}(t_1 \rightarrow t_2) \quad (36)$$

where $\text{APEP}(t_1 \rightarrow t_2)$ denotes the APEP related to the pair of transmitting symbols t_1 and t_2, which can be evaluated using the results of Propositions I and II, and $N_b(t_1, t_2)$ is the Hamming distance between t_1 and t_2.

B. L_p-NORM RECEIVER

For this generic class of minimum distance (MD) receivers, the optimal detection rule, assuming that the symbol $\mathbf{x}_t$ has been transmitted, $\forall t \in \{1, 2, \ldots, N_t\}$, minimizes the following distance metric [36], [43], [44]

$$\hat{\mathbf{x}}_t = \underset{t}{\text{argmin}} \left\{ \| \, \mathbf{y} - \sqrt{\rho} \mathbf{H} \, \mathbf{x}_t \, \|^p \right\}. \quad (37)$$

In (37), we select $p < \alpha$ because only in this case the moments of the SαS distribution are finite and the so-called fractional low order statistics (FLOS) are regarded as a useful signal processing approach for designing detectors operating over stable noise [36]. For the special case of $p = 2$, the well-known matched filter receiver is obtained, i.e.

$$\hat{\mathbf{x}}_t = \underset{t}{\text{argmin}} \left\{ \| \, \mathbf{y} - \sqrt{\rho} \mathbf{H} \mathbf{x}_t \, \|_F^2 \right\}. \quad (38)$$

However, the L_2-norm receivers, which can be optimized for the additive white Gaussian noise (AWGN) channel, performs poorly in SαS noise channels [43]. Furthermore, it has been shown in [21] that, as N_r increases, no diversity gain is achieved, while there exists a threshold of the α parameter of the stable distribution below which system performance degrades. These observations have been also verified by the performance evaluation results which will be presented later on in Section IV.

Nevertheless, although the L_2-norm receiver is optimal for only the AWGN channel, it has been widely used in practical applications because of its maximal ratio combining (MRC) property and thus, its performance over SαS noise is still of interest and it will be also considered here.

In the following, approximate analytical expressions for the APEP of L_2-norm receivers will be presented. It is noted that, since obtaining an analytical APEP expression for the general L_p-norm receiver it is very difficult, if not impossible, only computer simulated ABEP performance results will be presented later on in Section IV.

Using (38), the APEP for the L_2-norm receivers can be deduced as

$$\text{APEP} = \mathbb{E}_{\{A_r\}_{r=1}^{N_r}, \mathbf{H}} \left\langle Q \left[\sqrt{\frac{\rho \left(\sum_{r=1}^{N_r} Z_r \right)^2}{4 \sum_{r=1}^{N_r} Z_r A_r}} \right] \right\rangle, \quad (39)$$

where $Z_r = |h_{1r} - h_{2r}|^2$.

As the expectation in (39) is very difficult, if not impossible, to be evaluated analytically, an accurate analytical approximation for (39) that is tight over the entire SNR region, will be proposed. Specifically, the following result holds.

Proposition 5: The APEP of $2 \times N_r$ SSK system employing an L_2-norm receiver can be accurately approximated as

$$\text{APEP} \approx \frac{1}{2} - \frac{1}{2} \sum_{k=0}^{N_r - 1} \sum_{j=0}^{k} \frac{(-1)^{k-j} (2k)! S_{kj}}{2^{2k} k! j! (k-j)!} \left(\frac{\rho N_r}{4} \right)^{\frac{1}{2}+k-j} \quad (40)$$

where S_{kj} is given by (41), as shown at the bottom of the next page.

Proof: See Appendix V. ■

Finally, it is noted that for the more general case of a $N_t \times N_r$ MIMO system, an upper bound for the APEP can be obtained in a straightforward way, by employing (36), (40) and (41).

C. ML RECEIVER

For the ML receiver, its detection rule can be mathematically expressed as

$$\begin{aligned} \hat{\mathbf{x}}_t &= \underset{t}{\text{argmax}} \left\{ \prod_{r=1}^{N_r} f_\alpha(||[\mathbf{y}]_r - \sqrt{\rho}[\mathbf{H}\mathbf{x}_t]_r||) \right\} \\ &= \underset{t}{\text{argmax}} \left\{ \sum_{r=1}^{N_r} \log f_\alpha(||[\mathbf{y}]_r - \sqrt{\rho}[\mathbf{H}\mathbf{x}_t]_r||) \right\} \end{aligned} \quad (42)$$

where $f_\alpha(||x||)$ is the PDF of the magnitude $r = ||x||$ of a N-dimensional SαS distributed random vector given by [22, eq. (7.5.5)]

$$f_\alpha(r) = \frac{2^{1-N/2}}{\Gamma(N/2)} \int_0^\infty \exp(-\sigma^\alpha t^\alpha) J_{N/2-1}(rt)^{N/2} \mathrm{d}t. \quad (43)$$

Clearly, the implementation complexity of this ML receiver is prohibitive, since it involves the numerical evaluation of a Hankel transform and also depends on the noise parameters σ and α. Nevertheless, asymptotically optimal receivers with reduced implementation complexity, as compared to the ML receivers, can be obtained for high SNR values. Specifically, by employing [23, p. 118], the tail PDF of r can simplifies to

$$f_\alpha(r) = \alpha 2^\alpha \frac{\sin\left(\frac{\pi\alpha}{2}\right)}{\pi\alpha/2} \frac{\Gamma\left(\frac{\alpha+2}{2}\right)\Gamma\left(\frac{\alpha+N}{2}\right)}{\Gamma(N/2)} r^{-\alpha-1} + o\left(r^{-2\alpha-1}\right). \quad (44)$$

Using the dominant term of (44) in (42) and after some straightforward mathematical simplifications, the detection rule of the resulting suboptimal receiver can be deduced as

$$\hat{\mathbf{x}}_t = \underset{t}{\operatorname{argmin}} \left\{ \sum_{r=1}^{N_r} \log ||[\mathbf{y}]_r - \sqrt{\rho}[\mathbf{H}\mathbf{x}_t]_r|| \right\}. \quad (45)$$

This receiver performs asymptotically optimal at high SNR values and its performance does not depend on specific values of the impulsive noise parameters while its implementation complexity is significantly lower than that of its ML counterpart. Furthermore, using the well known relationship

$$\log(z) = 2\tanh^{-1}\left(\frac{z-1}{z+1}\right), \quad (46)$$

(45) can be conveniently expressed as

$$\hat{\mathbf{x}}_t = \underset{t}{\operatorname{argmin}} \left\{ \sum_{r=1}^{N_r} \tanh^{-1}\left(\frac{||[\mathbf{y}]_r - \sqrt{\rho}[\mathbf{H}\mathbf{x}_t]_r|| - 1}{||[\mathbf{y}]_r - \sqrt{\rho}[\mathbf{H}\mathbf{x}_t]_r|| + 1} \right) \right\}. \quad (47)$$

The advantage of (47) over (45), is that *the function* $\tanh^{-1}(z)$ *converges faster than the logarithm around* $z = 0$. Specifically, it turns out that only 12 odd powers of z are necessary for achieving double precision accuracy. On the other hand, $\tanh^{-1}(z)$ is bounded, while the logarithm function in (45) is not. Therefore, (47) is more appropriate for numerical evaluation, especially when lookup tables are employed for the numerical evaluation of $\tanh^{-1}(z)$.

Finally, it also interesting to consider the so-called generalized Cauchy (GR) receiver, whose detection rule is given as [31]

$$\hat{\mathbf{x}}_t = \underset{t}{\operatorname{argmin}} \left\{ \sum_{r=1}^{N_r} \log\left(c^2 + ||[\mathbf{y}]_r - \sqrt{\rho}[\mathbf{H}\mathbf{x}_t]_r||\right) \right\}. \quad (48)$$

In (48), c is an arbitrary parameter which is chosen to minimize the resulting ABEP. It can be observed that for $c = 0$, (48) reduces to (45) whereas for $c = 1$, (48) yields the optimal ML receiver operating in the presence of Cauchy noise, namely for $\alpha = 1$ (see eq.(3)). Note that the implementation complexity of the GC receiver can be further reduced by employing (46).

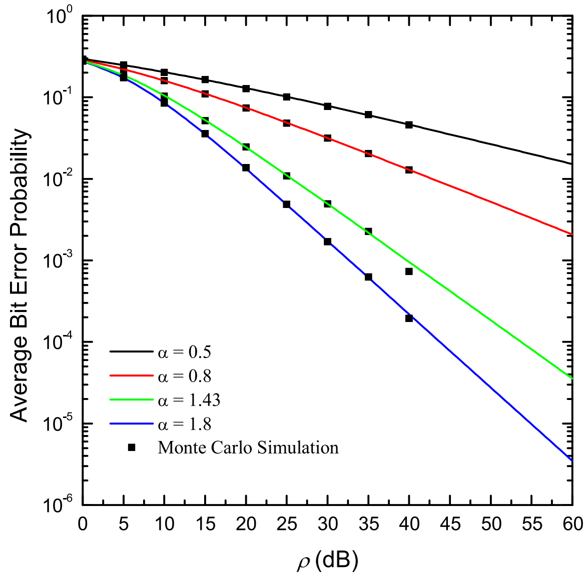

FIGURE 1. ABEP of 2 × 1 SSK systems with GA receiver as a function of the SNR, ρ, for various values of α.

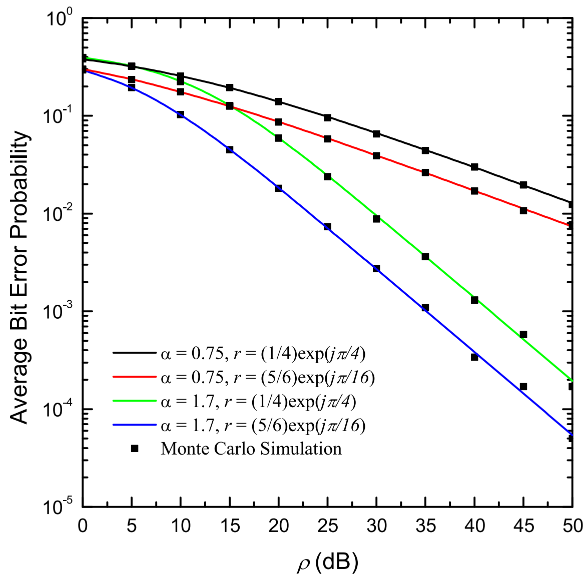

FIGURE 2. ABEP of 2 × 1 SSK systems with GA receiver operating in the presence of correlated Rayleigh fading channels, as a function of the SNR, ρ, for various values of α and correlation coefficient, r.

IV. PERFORMANCE EVALUATION RESULTS AND DISCUSSION

The performance of the previously presented receiver structures operating in the presence of impulsive noise will be assessed by means of numerical evaluation and complementary computer simulation results. To ensure the accuracy of

$$S_{kj} = \frac{1}{\pi}\int_0^\infty \Re\left\{ \exp\left[-N_r(ct)^{\alpha/2}\left(1 + \jmath\tan\left(\frac{\pi\alpha}{4}\right)\right) - \frac{\jmath\rho N_r t}{4}\right] \Gamma\left(\frac{1}{2} - k + j, -\frac{\jmath\rho N_r t}{4}\right)(-\jmath t)^{-\frac{1}{2}+k-j} \right\} \mathrm{d}t \quad (41)$$

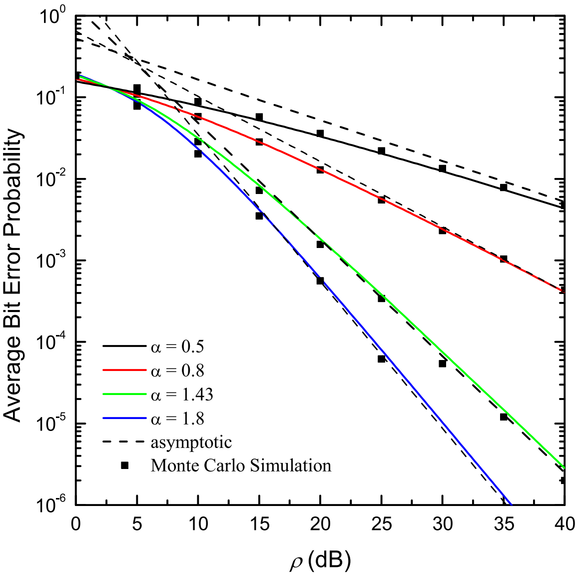

FIGURE 3. ABEP of 2 × 2 SSK systems with GA receiver as a function of the SNR, ρ, for various values of α.

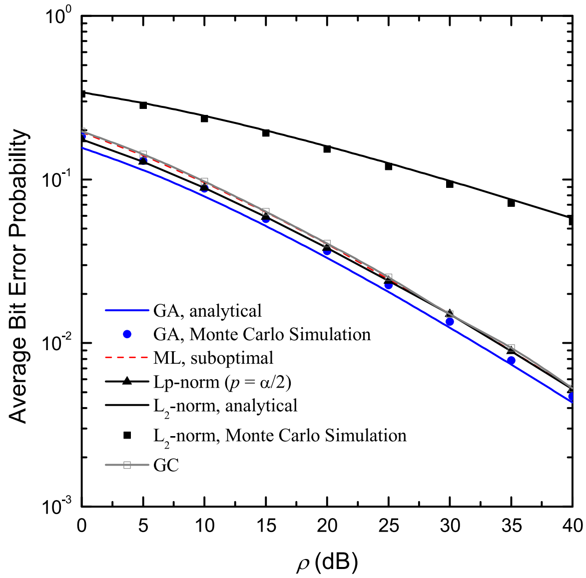

FIGURE 5. ABEP of 2 × 2 SSK systems with various type of receivers in the presence of severe impulsive noise ($\alpha = 0.5$).

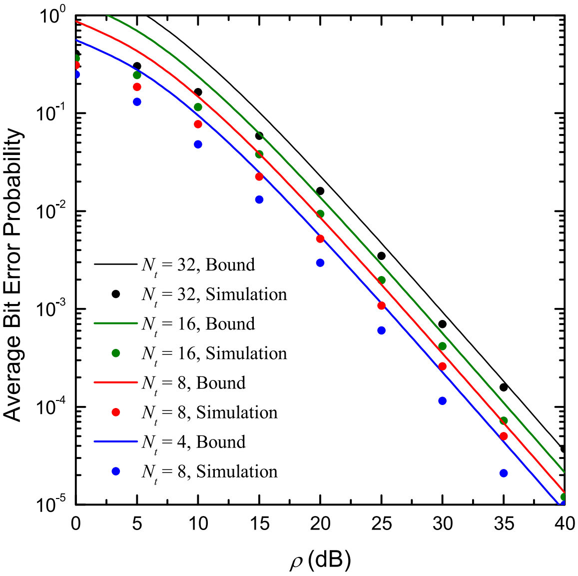

FIGURE 4. ABEP of $N_t \times 2$ SSK systems with GA receiver as a function of the SNR, ρ, for various values of N_t in the presence of moderately impulsive noise ($\alpha = 1.43$).

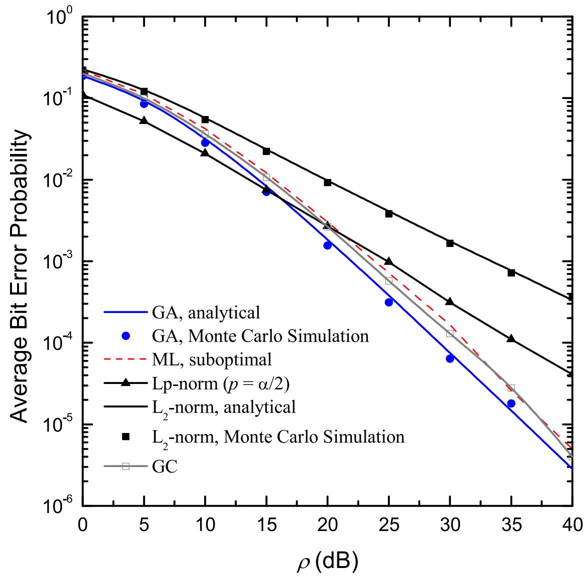

FIGURE 6. ABEP of 2 × 2 SSK systems with various type of receivers in the presence of moderately impulsive noise ($\alpha = 1.43$).

the ABEP performance evaluation results obtained by means of computer simulations using Monte Carlo error counting techniques, at least 5×10^6 random samples have been used.

Firstly, using (9)-(11), the ABEP performance of a 2×1 SSK system employing a GA receiver has been evaluated and the results are presented in Fig. 1, for various values of α. As it can be observed, analytical results are in perfect agreement with Monte Carlo simulations thus validating the accuracy of the theoretical analysis. Furthermore, it should be emphasized that the main advantage of (9) is that it yields accurate results with significantly lower implementation complexity as compared to performing long-running and time consuming Monte-Carlo simulations.

Fig. 2 illustrates the impact of antenna correlation on the ABEP performance of a 2×1 SSK system employing a GA receiver for $\alpha = 0.75$ and $\alpha = 1.7$. The elements of the covariance matrix are $\sigma_{12} \in \{1/4 \exp(\imath\pi/4), 5/6 \exp(\imath\pi/6)\}$, $\sigma_1^2 = \sigma_2^2 = 1$. The performance results clearly show that the presence of antenna correlation significantly degrades ABEP performance. Specifically, for $\alpha = 1.7$, there is a 6 dB performance degradation as the magnitude of the correlation coefficient increases from 1/4 to 5/6. Similar findings can be observed for lower values of α, i.e. for $\alpha = 0.75$.

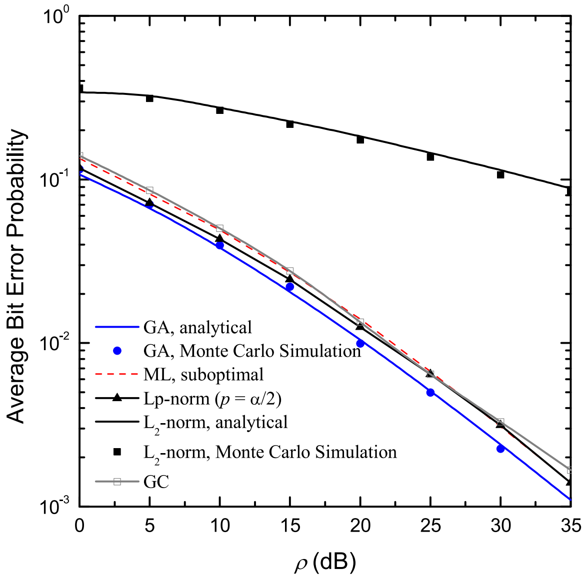

FIGURE 7. ABEP of 2 × 3 SSK systems with various type of receivers in the presence of severe impulsive noise ($\alpha = 0.5$).

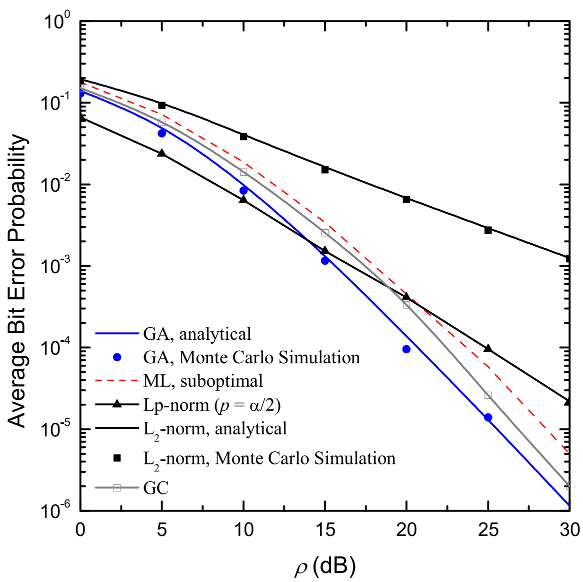

FIGURE 8. ABEP of 2 × 3 SSK systems with various type of receivers in the presence of moderately impulsive noise ($\alpha = 1.43$).

The analytical and asymptotic ABEP performance of 2×2 SSK systems employing GA receivers for various values of α, i.e. $\alpha \in \{0.5, 0.8, 1.43, 1.8\}$ is illustrated in Fig. 3. As it can be observed, the asymptotic results, obtained using Proposition 4 predict well the diversity and coding gains for all considered values of α.

Fig. 4 depicts the ABEP performance of $N_t \times 2$ SSK systems employing GA, assuming $\alpha = 1.43$ and various values of N_t. Analytical ABEP results have been obtained by employing the union bound in (36). The exact ABEP performance of the considered system has been evaluated by means of Monte Carlo simulations. As it can be observed, the

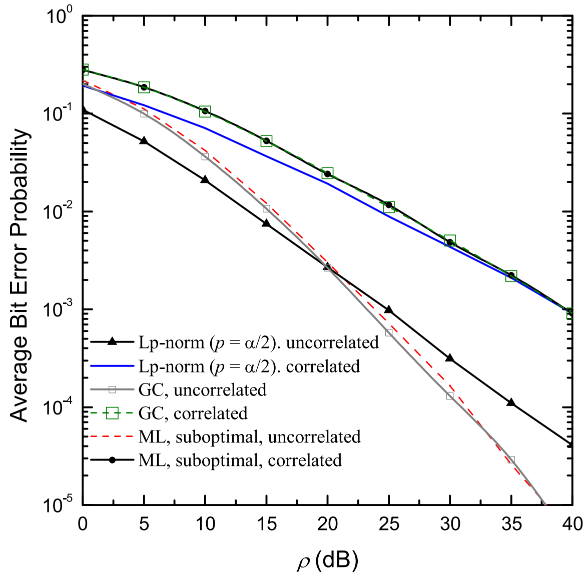

FIGURE 9. ABEP of 2 × 2 SSK systems with various type of receivers in the presence of correlated fading channels and moderately impulsive noise ($\alpha = 1.43$).

proposed upper bound for the ABEP is quite tight for high values of ρ, especially for large N_t.

Next, the ABEP performance of 2×2 and 2×3 SSK systems employing GA, L_2-norm, L_p-norm, suboptimal ML and GC receivers for severe, i.e., $\alpha = 0.5$, and moderate, i.e., $\alpha = 1.43$ impulsive noise channels will be presented. For the GA and L_2-norm receivers analytical performance evaluation results have been obtained using (9), (19), (20), and (39), respectively. On the other hand, for the L_p-norm, suboptimal ML and GC receivers performance evaluations results using Monte-Carlo computer simulations have been obtained using (37), (45) and (48), respectively.

For the severe impulsive noise channel the value $\alpha = 0.5$ has been chosen because such channel occurs in an interference environment caused by interfering nodes distributed according to a PPP on a two-dimensional plane [20]. For the moderately impulsive noise channel, $\alpha = 1.43$ has been selected because it corresponds to the interference caused by laptop computers [45]. For the GC receiver, a value of $c = \sqrt{\alpha/(2-\alpha)}$ has been used [21], [35], whereas for the L_p-norm receiver, $p = \alpha/2$ was used.

Fig. 5 depicts the ABEP performance of 2×2 SSK systems for all the receiver structures operating over the severe impulsive noise channel. It is evident that the analytical ABEP performance evaluation results obtained for the L_2-norm receivers using (39), are very tight over the entire SNR region. It is also noted that the L_2-norm receiver yields the worst performance, followed by the GC receiver, the suboptimal ML receiver, the L_p-norm and the GA. Furthermore, the performance of the GC receiver is almost identical to that of the suboptimal ML receiver for all SNR values. On the other hand, the L_p-norm receiver performs slightly better than the GC and the suboptimal ML receiver at low and medium SNR values, i.e. when SNR < 15 dB. Nevertheless, the L_p-norm, the suboptimal ML and the GC receivers have similar performances for high SNR values, i.e. when SNR > 25 dB, suffering from a 2 dB degradation as compared to the performance of the ideal GA receiver.

Fig. 6 depicts the performance of the same 2×2 SSK systems operating in the presence of moderately impulsive noise channels ($\alpha = 1.43$). These results show that the GC receiver yields a slightly better performance than the suboptimal ML receiver for all SNR values. Moreover, it can be observed that the L_p-norm receiver outperforms the GA receiver for SNR < 15 dB. Nevertheless the GA yields better performance for higher SNR values. It is also noted that the suboptimal ML and the GC receiver outperform the L_p-norm receiver for SNR > 20 dB. The reason behind the improved performance of the L_p-norm receiver for this range of SNR values is that the proposed L_p-norm receiver structure, with $p = \alpha/2$, closely resembles the optimal linear Rake receiver for the detection of binary signals contaminated by SαS noise proposed in [46]. This receiver, has been optimized in [46] for a specific range of values of α, namely for $1 \leq \alpha \leq 2$, and has been shown to yield better performance as compared to traditional diversity receivers, such as the maximal ratio combining (MRC). This observation further motivates the design and optimization of sophisticated L_p-norm receivers to increase the SNR range for which they outperform receiver configurations such as those considered in this study. Nevertheless, such an analysis is beyond the scope of the current work and is left for future research.

Figs. 7 and 8 depict the performance of 2×3 SSK systems with all considered receiver types for severe and moderately impulsive noise channels, respectively. It can be observed that the relative performance of the receivers under consideration for the given values of α is almost similar to the case of $N_r = 2$ receiving antennas. Nevertheless, the performance gap of all receivers slightly increases as the number of receiving antennas, N_r, increases from 2 to 3. It is noted that, the numerical results have shown that the L_2-norm receiver does not provide any diversity gain, as it is expected. In other words, there are no notable performance enhancements as N_r increases from 2 to 3 for a constant α. Moreover, for the severe impulse noise channel, the performance of the L_p-norm receiver is slightly worse than the performance of the GA receiver for low-to-medium SNR values. For the case of moderately impulsive noise channels, the L_p-norm receiver outperforms the GA receiver for SNR values of up to 15 dB.

Finally, Fig. 9 illustrates the impact of antenna correlation on a 2×2 SSK system operating in the presence of moderately impulsive noise channels and employing L_p-norm, suboptimal ML and GC receivers. The covariance matrix is given by

$$\mathbf{C} = \begin{bmatrix} 1 & \sigma_{12} & \sigma_{13} & \sigma_{14} \\ \sigma_{12}^* & 1 & \sigma_{23} & \sigma_{24} \\ \sigma_{13}^* & \sigma_{23}^* & 1 & \sigma_{34} \\ \sigma_{14}^* & \sigma_{24}^* & \sigma_{34}^* & 1 \end{bmatrix} \tag{49}$$

with $\sigma_{12} = 1/2\exp(\jmath\pi/2)$, $\sigma_{13} = 1/3\exp(\jmath\pi/3)$, $\sigma_{14} = 1/4\exp(\jmath\pi/4)$, $\sigma_{23} = 1/3\exp(\jmath\pi/6)$, $\sigma_{24} = 1/4\exp(\jmath\pi/4)$ and $\sigma_{34} = 1/8\exp(\jmath\pi/8)$. As it is evident from the obtained results, antenna correlation severely degrades the bit error rate performance for all considered receiver structures. Specifically, when correlated fading is considered, the performance loss for the L_p-norm receiver is approximately 15 dB (at ABEP $= 10^{-3}$). For the suboptimal ML and GC receivers, similar performance losses have been observed. Also, when correlated fading channels are assumed, the ABEP performance of suboptimal ML and GC receivers is almost identical. The performance of the L_p-norm receiver exhibits similar behavior as in the uncorrelated fading case, i.e., it outperforms the GC and suboptimal ML receivers for SNR values of up to 35 dB, whereas for larger SNR values the suboptimal ML and GC receivers slightly outperform the L_p-norm receiver.

To summarize, for the performance of the GA receiver, the obtained ABEP results predict well the diversity and coding gains, derived in Proposition 4. When being compared to other receiver structures, the L_2-norm receiver does not perform well under highly impulsive noise conditions and thus, it should not be the preferable choice. The L_p-norm receiver with $p = \alpha/2$, provides an improved performance as compared to the L_2-norm receiver and it even outperforms the GA receiver for moderate impulsive noise and low SNR values. Nevertheless, the suboptimal ML and the GC receivers outperform the L_p-norm receivers at high SNR values, as they achieve a higher diversity gain. Finally, it has been shown that spatial correlation severely degrades the performance of all considered receiver types, even under mildly impulsive noise scenarios.

V. CONCLUSION

In this paper, we have provided an extensive ABEP performance evaluation of MIMO SSK systems operating over Rayleigh fading channels and SαS noise. Three key receiver structures have been considered, namely the GA, MD and ML. For the GA receiver, analytical ABEP expressions for MISO and MIMO SSK have been obtained in terms of single integrals that can be efficiently evaluated numerically. The diversity order of this receiver has also been evaluated. The performance of a generic class of MD receivers, namely the L_p-norm distance-based receivers, has also been investigated. For the special case of the L_2 receiver, accurate approximate analytical ABEP expressions have also been obtained that are tight in the entire SNR regime. The optimal ML receiver has also been deduced and its performance has been evaluated. Because of its high implementation complexity and the fact that it depends on noise parameters, alternative suboptimal receiver structures have been investigated, whose performance is asymptotically optimal at high SNR values. Future work includes the extension of our results to more generic index modulation systems, the performance evaluation in the presence of generalized fading channels, the derivation of exact and asymptotic ABEP results for the L_p-norm receiver, as well as the investigation of improved receiver structures, suitable for signal detection in noise channels modelled as a mixture of Gaussian and alpha-stable noise.

APPENDIX
PROOF OF PROPOSITION 4

In order to provide a mathematically tractable approximation for (39), the sum $S_1 = \sum_{r=1}^{N_r} Z_r A_r$ is first approximated with another sum $S_2 = {N_r}^{-1} \sum_{r=1}^{N_r} Z_r \sum_{r=1}^{N_r} A_r$. The advantage of this approach is that, according to the sum Chebyshev and rearrangement inequalities [47], S_2 is bounded (either upper or lower) by S_1. Therefore, the resulting APEP can be approximated as

$$\text{APEP} \approx \mathbb{E}_{\{A_r\}_{r=1}^{N_r}, \{Z_r\}_{r=1}^{N_r}} \left\langle Q\left(\sqrt{\frac{\rho N_r \sum_{r=1}^{N_r} Z_r}{4 \sum_{r=1}^{N_r} A_r}}\right)\right\rangle. \quad \text{(A-50)}$$

Using [41, eq. (14-4-15)] along with the binomial theorem yields

$$\text{APEP} \approx \frac{1}{2} - \frac{1}{2} \sum_{k=0}^{N_r-1} \sum_{j=0}^{k} \binom{2k}{k} \binom{k}{j} \frac{(-1)^{k-j}}{2^{2k}} \times \mathbb{E}_{\{A_r\}_{r=1}^{N_r}} \left\langle \mu^{k-j+\frac{1}{2}} \right\rangle, \quad \text{(A-51)}$$

where $\mu = x/(A+x)$, $x = \rho N_r/4$ and $A = \sum_{r=1}^{N_r} A_r$. By employing the Parseval's theorem, $\mathbb{E}_{\{A_r\}_{r=1}^{N_r}} \langle \mu^q \rangle$, where $q = k - j + \frac{1}{2}$, can be deduced as

$$\mathbb{E}_{\{A_r\}_{r=1}^{N_r}} \langle \mu^q \rangle = \frac{1}{\pi} \int_0^\infty \Re \left\{ G(t) \times \exp\left[-N_r (ct)^{\alpha/2} \left(1 + \jmath \tan\left(\frac{\pi \alpha}{4}\right)\right)\right]\right\} \mathrm{d}t \quad \text{(A-52)}$$

where

$$G(t) = \int_0^\infty \left(\frac{x}{A+x}\right)^q \exp(\jmath A t) \mathrm{d}A. \quad \text{(A-53)}$$

Using [48, eq. (2.1.2/1)] and after performing some straightforward algebraic manipulations, (A-53) can be evaluated in closed-form yielding (21), and thus the proof is completed.

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