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# **WE RESEARCH ARTICLE**

# Robust Tracking Control for Quadrotor UAV With External Disturbances and Uncertainties Using Neural Network Based MRAC

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**ABSTRACT** In this paper, an intelligent Model Reference Adaptive Control (MRAC) based on a neural network is proposed for robust tracking control of quadrotor UAV under external disturbances and parameter variations. First, the singularity-free dynamic model of the quadrotor is developed using Newton-Quaternion formalism. Then, conventional MRAC is designed to generate training data. With the generated data, the Feed Forward Neural Network (FFNN) and Recurrent Neural Network (RNN) are trained offline to get an initial set of network parameters for position controller parameters estimation and attitude control of the quadrotor, respectively, and an online learning algorithm is developed to update those network parameters in real-time. Finally, the performance of the designed Neural network-based MRAC has been evaluated using a numerical simulation in a nominal scenario and by introducing parametric variation and external disturbances as matched and unmatched uncertainties into the system. The simulation results show that the proposed controller has a better tracking performance and disturbance rejection capability compared with the Linear Quadratic Regulator (LQR) and conventional MRAC. Furthermore, the utilized control efforts are minimal and smooth proving functional safety and economical use of the controller. Therefore, the suggested controller is feasible for real-time implementation of the quadrotor UAV.

**INDEX TERMS** Feed forward neural network, LQR, model reference adaptive control, newton-quaternion formalism, online learning algorithm, recurrent neural network, quadrotor UAV.

#### **I. INTRODUCTION**

Unmanned Aerial Vehicle (UAV) is a kind of recent generation of autopilot aircraft deployed for various industry, civilian, and military applications. A quadrotor is a type of UAV that is driven by four rotors with the capability of Vertical takeoff and landing and other flight maneuverings. Due to this capability, low operation cost, and maneuverability at low altitudes and dangerous environments for high-risk applications, research on quadrotors has been growing enormously from time to time and currently become a research hotspot in the aviation field [\[1\],](#page-17-0) [\[2\]. Ci](#page-17-1)vilian use of quadrotor UAV includes aerial photography, inspection missions of railways

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<span id="page-0-1"></span>and power lines, surveillance for illegal imports and exports, road detection and tracking, fire detection and control, traffic management, border patrolling, crop monitoring and spraying, and search and rescue operation for missing persons and natural disasters [\[3\],](#page-17-2) [\[4\]. T](#page-17-3)he aforementioned applications attract control researchers to design robust control algorithms to achieve the assigned tasks precisely. Therefore, this research study follows the model-based highperformance closed-loop flight controller design for autopilot quadrotor UAV.

The Flight Control System (FCS) is the brain of a quadrotor UAV that is responsible for achieving the desired attitude and position while the quadrotor flies. Quadrotor UAV control researchers face many challenges in developing high-performance controllers for the FCS due to time-varying

<span id="page-1-0"></span>properties of the environment, nonlinear coupled dynamics of the system, uncertainties in modeling, and external disturbances from the wind [\[5\]. Th](#page-17-4)ese make the dynamics more challenging for control applications. To deal with such dynamics, adaptive control has been a promising technique that can improve the performance of control systems in the presence of uncertainty due to model degradation and uncertainty from the operating environment [\[6\]. Th](#page-17-5)e basic working principle of adaptive control laws is the Certainty Equivalence Principle (CEP), which is a principle that most current adaptive controllers are working on. Based on this principle, the parameters of the adaptive controller are derived from a set of uncertain plant parameters whose true values are unknown. According to CEP, the controller parameters are replaced by estimates and taken as the true parameters when the control laws are manipulated. However, this parameter estimation requires sufficiently fast parameter convergence for the adaptive controller to achieve the desired performance specification [\[7\],](#page-17-6) [\[8\].](#page-17-7)

<span id="page-1-2"></span>The adaptive control technique is an appropriate solution for dealing with parametric uncertainties, and it has better performance than robust control in dealing with uncertainties in constant or slow-varying parameters. However, robust control is advantageous in dealing with disturbances, quickly varying parameters, and unmodeled dynamics [\[9\]. H](#page-17-8)ere, the goal of robust control design is to maintain system performance with acceptable error within a certain range of model variations and inaccuracies, so it's well suited for applications where stability and reliability of the system are top priorities, where process dynamics are known and variation ranges for uncertainties can be estimated. Since robust control system design requires such a high level of expertise that can exactly estimate variation ranges for uncertainties, it has never been a popular solution for control problems in several industries [\[10\]. F](#page-17-9)or this reason, adaptive control gets more attention than robust control in industries among those, NASA has been performing a series of adaptive flight control studies since 70's [\[11\].](#page-17-10)

<span id="page-1-5"></span>Model Reference Adaptive Control (MRAC) is the type of adaptive control that aims to force the dynamic response of the system to asymptotically approach that of a desired reference model despite parametric uncertainties in the system. Nowadays, control engineers classify MRAC into direct and indirect MRAC [\[12\].](#page-17-11) In the case of direct MRAC, controller parameters are directly updated online without the need to estimate plant parameters [\[8\]. In](#page-17-7) this control topology, the purpose of MRAC is to make plant output behave like a reference model in the way that the adaptation rule works to estimate uncertain plant parameters whereas the controller works to achieve acceptable convergence to a desired output  $[13]$ . On the other hand, the indirect MRAC technique involves applying some system identification method to obtain a model of a system and its environment from input-output experiments, and the controller is redesigned online based on the estimated model.

Self-tuning regulator (STR) is an example of indirect MRAC. In this control topology, uncertain parameters of the plant model are estimated first, and then based on the estimated model controller parameters are computed separately [\[7\].](#page-17-6)

<span id="page-1-9"></span><span id="page-1-8"></span><span id="page-1-1"></span>The control systems that combine AI techniques such as neural networks, fuzzy logic, evolutionary competition, and Bayesian probability; with conventional adaptive control systems are classified under intelligent control systems due to their knowledge-based decision-making capabilities. From stated AI techniques, Artificial Neural Network (ANN) is considered as a complex adaptive system that can change its internal structure based on the information passing through it  $[14]$ . With these features, ANN is a widely used effective method to solve estimation and control problems, and its application areas in aviation fields have also increased  $[15]$ . Therefore, in this study intelligent flight controller is designed by combining the MRAC technique with the learning power of ANN, which are two equivalent topics under different umbrellas of research such as control engineering and machine learning, respectively [\[16\].](#page-17-15) Such systems can achieve sustained behavior in the presence of uncertainty in system dynamics, unpredictable environmental changes, unreliable sensor information, and actuator malfunction. Furthermore, can perform complex tasks as compared to conventional control techniques.

#### <span id="page-1-10"></span><span id="page-1-3"></span>A. RELATED WORKS

<span id="page-1-12"></span><span id="page-1-11"></span><span id="page-1-4"></span>Due to the significant increase in quadrotor UAV application areas, various control researchers have published studies on quadrotor UAV control. The Proportional Integral Derivative (PID) controller is the aviation industry's most widely used control algorithm. It is a linear controller with the advantages of being easy to tune and alter parameter gains, simple to construct, and strong resilience [\[17\].](#page-17-16) Authors in reference [\[18\]](#page-17-17) by linearizing the quadrotor model and decoupling the state, designed a PID controller to stabilize and control the quadrotor. The authors demonstrated that the PID controller is effective only within the range of  $\pm 10^0$  for roll and pitch angles, and the controller performance is highly affected by external disturbances. Using a PID controller for a quadrotor has the following problems: it's suitable only for linear systems, it loses control performance for wider angles of roll and pitch rotations, which limits quadrotor maneuverability near the specified range, and it can't tolerate external disturbances.

<span id="page-1-13"></span><span id="page-1-7"></span><span id="page-1-6"></span>Authors in reference [\[19\]](#page-17-18) did a comparative study between LQR and PID controller's performance on the quadrotor system. First, the authors drive a linear mathematical model of system dynamics and then simulate the two designed control techniques under the same initial condition. The authors verified that the LQR controller is more stable and robust than the PID controller. However, the LQR control mechanism has the following two drawbacks. First, it is

sensor-intensive. Second, LQR controllers can not handle unmodeled dynamics, which is the most common problem of implementing linear controllers in nonlinear systems. On the other hand, authors in [\[12\]](#page-17-11) and [\[20\]](#page-17-19) proposed a nonlinear robust MRAC approach for matched uncertainty in quadrotor UAV dynamics. The authors proved that this control approach loses its robustness for the unmatched uncertainty. Other solutions are proposed to solve the nonlinearity of the quadrotor dynamics and the effect of matched uncertainties.

<span id="page-2-2"></span>Intelligent control techniques have been emerged as promising control techniques in tolerating the harmful effects of unmatched uncertainty and complexity of the system dynamics. Authors in reference [\[21\]](#page-17-20) did a comparative analysis between neural network-based direct MRAC and conventional MRAC, and the author proved that neural network-based direct MRAC has a smaller rising time, steady-state error, and settling time than the conventional MRAC approach. Authors in [\[22\]](#page-17-21) also developed extended minimum resource allocating network (EMRAN) based MRAC for UAV control and compared controller performance with existing PID controller. The results prove that the controller has better performance in the presence of disturbances compared to the existing one. However, this study is only focused on offline trained neural networks that estimate the parameters of the controller but outdoor quadrotor applications require adaptive control, which can adapt its parameters to changing flight conditions of quadrotor dynamics. In this phenomenon, the idea of online training that continuously tunes network parameters with changing flight dynamics of the system with environmental conditions is crucial for quadrotor control.

<span id="page-2-4"></span>Authors in reference [\[23\]](#page-18-0) designed a neural network-based intelligent adaptive control for nonlinear systems. The authors developed an online trained neural network structure along with a conventional MRAC by obtaining training patterns for the network training from PI control. The authors found that the online tuned network compensates for the nonlinearity of the system that is not considered in conventional MRAC, and the proposed controller significantly improves the system performance by minimizing reference model tracking error, and it shows superior performance than PI-based MRAC and pure MRAC approaches. However, the nonlinearity of adaptation laws in conventional MRAC causes difficulty in tuning PI controller gains, so the author obtained PI controller gains manually by try and error method, and when the controller gains are large the generated training patterns are mostly corrupted by noise and the network unable to fit the patterns. These make the network training process tricky. Generally, in the literature survey, the recently proposed methods have been focused on offline-trained networks, and noisy sensitive data used to train the networks, which motivates us to develop a novel training and control approach for the quadrotor.

#### B. MAIN CONTRIBUTIONS

<span id="page-2-1"></span>In this study, the intelligent flight control technique, which combines ANN from the AI technique and MRAC from the conventional control technique, is developed for quadrotor control. In this approach, ANN is employed to learn and approximate the system dynamics meanwhile MRAC adjusts its parameters based on the error between the actual system output and the desired reference model output. The training pattern for the network is obtained from input-output patterns of the adaptation law of conventional MRAC, and the generalized Back Propagation Algorithm (BPA) is developed to update network parameters in real-time enabling the controller to adapt its parameters to changing flight conditions of the quadrotor dynamics. Furthermore, the proposed control technique is effective in tolerating matched and unmatched uncertainties, requires less processing time, and can perform its control action in all quadrotor maneuvering ranges. This implies that the proposed control technique is an effective control approach for real-time implementation on the quadrotor platform. Therefore, the objective of this study is to develop a high-performance ANN-based MRAC algorithm for quadrotor control with the following major contributions:

- <span id="page-2-3"></span>1) A novel neural network-based MRAC architecture for robust tracking control based on quaternion modeling of quadrotor UAV has been proposed,
- 2) To increase the performance of the designed controller, a more precise quaternion-based singularity-free quadrotor dynamic model is developed by applying Newton-Quaternion formalism and considering real-time scenarios of quadrotor maneuvering,
- 3) The proposed technique has been designed and tested under external disturbances and parameters uncertainties using simulation and proved to have a better performance than other similar controllers that are used for UAV control, and
- 4) An online learning algorithm has been introduced to enhance controller performance. Artificial neural networks are used to update weights and biases in real- time which in turn increases convergence rate and ensures the stability of the system.

The rest of the paper is organized as follows. In section  $II$ , the detailed quaternion-based mathematical model of quadrotor flight dynamics is presented. In section [III,](#page-6-0) ANN-based MRAC design is described. In section  $IV$ , simulation results are presented. Finally, concluding remarks and possible further works are presented in section [V.](#page-17-22)

### <span id="page-2-0"></span>**II. QUATERNION-BASED MATHEMATICAL MODEL OF QUADROTOR UAV**

# A. PRELIMINARY NOTIONS OF QUADROTOR UAV

To model quadrotor dynamics, the coordinate frames of the quadrotor must be defined first. There are two coordinate frames, as shown in Figure [1,](#page-3-0) that the quadrotor will operate in: inertial frame  $(x_i, y_i, z_i)$  and body frame

<span id="page-3-0"></span>

**FIGURE 1.** Quadrotor free body diagram and its reference frames.

<span id="page-3-5"></span> $(x_h, y_h, z_h)$  [\[24\]. I](#page-18-1)nertial Frame (IF) is defined with respect to the ground with a positive *z*-axis pointing in the opposite direction of gravity, whereas Body Frame (BF) is defined with respect to the Center of Gravity (CoG) of the quadrotor with its axes fixed to the body. The necessity of defining those reference frames is because of

- 1) aerodynamics forces and torques are manipulated with respect to BF,
- 2) on-board builtin sensors measure quantities with their respective reference frame, for instance, Inertial Measurement Unit (IMU) gives readings with respect to BF whereas GPS gives readings with respect to IF, and
- 3) Newton laws are valid in IF only.

Therefore, projection matrices are required to transform quantities between the two frames and derive a system governing equations according to a single reference frame [\[25\].](#page-18-2)

Coordinate frames transformation is used to transform quantities between the two reference frames. To avoid the nonlinearity, geometric singularity, and computational cost usually associated with the Euler angles [\[26\],](#page-18-3) [\[27\],](#page-18-4) the quaternion approach is applied for coordinate frames transformation. A quaternion is a hyper-complex number with four dimensions that include a scalar part *qo*, and a three-dimensional vector part  $\vec{q} = [q_i, q_j, q_k]$  with orthonormal base  $[i, j, k]$   $[28]$ . Thus, the quaternion can be represented as follows:

<span id="page-3-8"></span>
$$
q = q_o + q_1 \hat{i} + q_2 \hat{j} + q_3 \hat{k} = [q_o, q_1, q_2, q_3] = (q_o, \vec{q}) \tag{1}
$$

To represent a valid 3D orientation or to compute valid vector rotations, the quaternion must be normalized. The normalized quaternion is ideally suitable for representing and transforming the orientation of coordinates between any two reference frames. Quaternion normalization can be done as follows:

$$
q = \frac{q_o + q_1\hat{i} + q_2\hat{j} + q_3\hat{k}}{\sqrt{q_o^2 + q_1^2 + q_2^2 + q_3^2}}
$$
 (2)

A unit quaternion provides a convenient mathematical notation for representing orientations in 3D space. Hence, to derive the quadrotor dynamic model, the unit quaternion notion is applied. The product between two quaternion  $q_a = (q_{oa}, \vec{q}_a)$  and  $q_b = (q_{ob}, \vec{q}_b)$ , is manipulated as follows:

$$
q_a \otimes q_b = (q_{oa}q_{ob} - \vec{q}_a \cdot \vec{q}_b, q_{oa}\vec{q}_b + q_{ob}\vec{q}_a + \vec{q}_a \times \vec{q}_b) \tag{3}
$$

<span id="page-3-9"></span>Coordinate transformation between two different reference frames by using quaternion rotation is computed as [\[29\]](#page-18-6)

<span id="page-3-2"></span><span id="page-3-1"></span>
$$
X = q \otimes X' \otimes q^{-1} \tag{4}
$$

where  $X$  is rotated vector and  $X'$  is vector to be rotated. Equation [4](#page-3-1) implies that multiplying two or more rotation quaternions produces another rotation quaternion that represents the total rotation obtained by performing each individual rotation for each quaternion. Vectors can be transformed from one axis system to another if first transformed into quaternions with a zero scalar value, also this is known as a pure quaternion. Thus, a quaternion-based rotation matrix that transforms quantities from IF to the BF is obtained by rewriting equation [4](#page-3-1) as follows:

<span id="page-3-10"></span><span id="page-3-3"></span>
$$
\begin{bmatrix} 0 \\ r_b \end{bmatrix} = q \otimes \begin{bmatrix} 0 \\ r_i \end{bmatrix} \otimes q^*
$$
 (5)

where  $r_b$  =  $[x_b, y_b, z_b]'$  and  $r_i$  =  $[x_i, y_i, z_i]'$  are position quantities with respect to BF and IF, respectively. By using the quaternion product in equation [3,](#page-3-2) a 3D rotation matrix  $R_I^B(q)$  [\[30\]](#page-18-7) that projects position quantities from BF into IF can be obtained by rearranging equation [5](#page-3-3) as follows:

<span id="page-3-6"></span>
$$
\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = \begin{bmatrix} 2(q_o^2 + q_1^2) - 1 & 2(q_1q_2 + q_oq_3) & 2(q_1q_3 - q_oq_2) \\ 2(q_1q_2 - q_oq_3) & 2(q_o^2 + q_2^2) - 1 & 2(q_2q_3 + q_oq_1) \\ 2(q_oq_2 + q_1q_3) & 2(q_2q_3 - q_oq_1) & 2(q_o^2 + q_3^2) - 1 \end{bmatrix} \begin{bmatrix} x_b \\ y_b \\ z_b \end{bmatrix}
$$
\n(6)

<span id="page-3-7"></span>Transformation matrix that projects BF rates  $\omega$  =  $[p, q, r]'$  into IF quaternion rates  $\dot{q}$  can be obtained as [\[31\],](#page-18-8) [\[32\]:](#page-18-9)

<span id="page-3-11"></span><span id="page-3-4"></span>
$$
\begin{bmatrix} \dot{q}_o \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \frac{1}{2} q \otimes \omega = \frac{1}{2} \begin{bmatrix} q_o & -q_1 & -q_2 & -q_3 \\ q_1 & q_o & -q_3 & q_2 \\ q_2 & q_3 & q_o & -q_1 \\ q_3 & -q_2 & q_1 & q_o \end{bmatrix} \begin{bmatrix} 0 \\ p \\ q \\ r \end{bmatrix} \tag{7}
$$

#### B. QUADROTOR FLIGHT DYNAMICS MODELING

Almost all systems in real life are nonlinear in nature, so control researchers design model-based either linear controllers by linearizing those nonlinear systems at some operating point, or design nonlinear controllers based on a simplified nonlinear model of the system by ignoring some features that are assumed to have less effect on the system. During simulation, the designed controller may show

promising results by effectively driving tracking error toward zero. However, during the implementation of the developed control algorithm in the real system, the error may not converge to zero. This is due to the designed control law, which is designed based on a linearized model not based on the real plant. So, unmodeled dynamics, which are neglected during modeling, may result in poor controller performance and may also cause the system unstable. To solve such harmful effects of unmodeled dynamics, in this section more realistic behavior of the quadrotor is enforced by considering every possible effect which has a significant change on the quadrotor dynamics. This point is raised from the general fact of control engineering: when the accuracy of the model improves, the controller performance will also improves though this increases the complexity of the model. Taking this general fact into consideration, the following assumptions have been made to model the quadrotor flight dynamics [\[33\],](#page-18-10) [\[34\]:](#page-18-11)

- <span id="page-4-5"></span>1) structure of the quadrotor is rigid with constant mass and uniform symmetry,
- 2) gravitational force doesn't change with altitude,
- 3) the resistance and inductance of Brushless Direct Current (BLDC) electric motor are negligible, and
- 4) at the hovering position, velocity of air and propellers are equal.

Parameter values of the system dynamics are presented in Table[.1.](#page-4-0)

<span id="page-4-0"></span>



# 1) QUADROTOR ROTATIONAL MOTION

Using Newton's 2*nd* law of rotational motion for fully actuated quadrotor rotational dynamics, the net moment derived from BF as follows:

$$
\sum \tau_i = J\alpha = [U_2, U_3, U_4]' - M_{gyro} - M_{aero}
$$
 (8)

$$
\dot{\omega} = J^{-1} \big[ [U_2, U_3, U_4]' - \omega \times (J\omega) - M_{\text{gyro}} - M_{\text{aero}} \big] \tag{9}
$$

where *J* is inertia matrix,  $\alpha = \dot{\omega} = [\dot{p}, \dot{q}, \dot{r}]'$  are angular accelerations in BF;  $U_2$ ,  $U_3$ , and  $U_4$  are moments on BF which are used to control roll, pitch, and yaw, respectively;  $M_{\text{evro}} = J_p \Omega \times \omega$  is a gyroscopic moment that the spinning propellers induce when the quadrotor rotates in the space [\[37\];](#page-18-14)  $M_{aero} = C_a \omega$  is aerodynamic frictional moment [\[38\];](#page-18-15)  $C_a$  is drag coefficient; and  $\Omega$  is net propeller angular speed. By rearranging equation [9,](#page-4-1) rotational dynamics of the quadrotor is obtained as

<span id="page-4-8"></span><span id="page-4-7"></span><span id="page-4-3"></span>
$$
\begin{cases}\n\dot{p} = \frac{1}{J_{xx}}U_2 + \frac{J_{yy} - J_{zz}}{J_{xx}}qr - \frac{J_p q}{J_{xx}}\Omega_r - \frac{C_{a_p}}{J_{xx}}p \\
\dot{q} = \frac{1}{J_{yy}}U_3 + \frac{J_{zz} - J_{xx}}{J_{yy}}pr + \frac{J_p p}{J_{yy}}\Omega_r - \frac{C_{a_q}}{J_{yy}}q \\
\dot{r} = \frac{1}{J_{zz}}U_4 + \frac{J_{xx} - J_{yy}}{J_{zz}}pq - \frac{C_{a_r}}{J_{zz}}rw\n\end{cases} (10)
$$

#### 2) QUADROTOR TRANSLATIONAL MOTION

The translational dynamics of the quadrotor is derived based on Newton's 2*nd* law of translational motion. Since Newton's laws are valid only in IF, the rotation matrix  $R_I^B$  is used to project the thrust force that is generated in BF onto IF. Thus, the translational dynamics of the quadrotor in IF is derived as

$$
\sum F_i = ma = R_I^B(q)U_1 - F_g - F_d \tag{11}
$$

<span id="page-4-9"></span><span id="page-4-2"></span>
$$
a = \frac{1}{m} [R_I^B(q)U_1 - F_g - F_d]
$$
 (12)

<span id="page-4-6"></span>where  $U_1$  is the total thrust force used to control the altitude,  $a = [\ddot{x}, \ddot{y}, \ddot{z}]'$  is the linear acceleration of quadrotor in IF,  $F_g$  = *mg* is a gravitational force along the *z* direction,  $F_d = C_d[x, \dot{y}, \dot{z}]'$  [\[39\]](#page-18-16) is air drag or friction force in IF which is proportional to linear velocity, *C<sup>d</sup>* is drag coefficient, *g* is gravity, and *m* is mass of quadrotor. Since the quadrotor translation is the underactuated system, only the altitude of the quadrotor is controlled directly, and any non-zero roll or pitch angle results in the quadrotor maneuvering along the *x* or *y* direction. Therefore, equation [12](#page-4-2) can be rewritten as

<span id="page-4-4"></span>
$$
\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \frac{1}{m} \begin{bmatrix} R_I^B(q) \begin{bmatrix} 0 \\ 0 \\ U_1 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} - \begin{bmatrix} C_{d_x} \dot{x} \\ C_{d_y} \dot{y} \\ C_{d_z} \dot{z} \end{bmatrix} \end{bmatrix}
$$
(13)  

$$
\begin{cases} \ddot{x} = 2(q_1q_3 - q_0q_2)\frac{U_1}{m} - \frac{C_{d_x}}{m}\dot{x} \\ \ddot{y} = 2(q_2q_3 + q_0q_1)\frac{U_1}{m} - \frac{C_{d_y}}{m}\dot{y} \\ \ddot{z} = \left[2(q_0^2 + q_3^2) - 1\right]\frac{U_1}{m} - g - \frac{C_{d_z}}{m}\dot{z} \end{cases}
$$
(14)

#### 3) AERODYNAMIC GROUND EFFECT

<span id="page-4-1"></span>Ground Effect (GE) is an aerodynamic phenomenon that the ground pushes the quadrotor up when the propeller

approaches the ground surface [\[40\]. F](#page-18-17)or the same transmitted power to the actuator, the propeller generates more thrust caused only by the presence of the ground as shown in Figure [2.](#page-5-0)

<span id="page-5-0"></span>

**FIGURE 2.** Variation of thrust generated at Mid Air maneuvering and Near-ground maneuvering due to GE in the quadrotor [\[33\].](#page-18-10)

Quadrotor has a stronger GE up to the altitude of  $z = 6R$ , so GE is considered for the altitude up to  $z = 6R$ . Hence, the region where  $z \ge 0.5R$  and  $z \le 6R$  is considered as In Ground Effect (IGE), and the region where  $z < 0.5R$  and  $z > 6R$ is considered as Out of Ground Effect (OGE) where GE is typically doesn't exist. Experiments show that at constant power, GE is mathematically expressed as [\[27\],](#page-18-4) [\[35\]](#page-18-12)

$$
f_{GE}(z) = \begin{cases} \frac{1}{1 - \gamma(\frac{R}{4z})^2} & 0.5R \le z \le 6R\\ 1 & otherwise \end{cases}
$$
(15)

where  $f_{GE}(z)$  is the ground effect factor which is decreased as altitude *z* increases.  $\gamma$  is the correction coefficient, which is estimated from the experiment, and it's used to adjust unpredictable airflow influence between the rotors of the quadrotor, and *R* is the radius of the propellers. To account the effect of GE, the thrust force model is modified [\[27\]](#page-18-4) as

$$
T_i = b\Omega^2 f_{GE}(z) \tag{16}
$$

The rate multiplication of the Coriolis term which is the product of quaternion rates is very small and has insignificant effects in the quadrotor governing equation. In addition to this, the net residues speed  $\Omega_r$  in equation [10,](#page-4-3) has a nonzero value for yaw rotation but yaw rotation doesn't have a gyroscopic effect. Therefore, both Coriolis term and net residues speed  $\Omega_r$  are ignored to simplify the rotational dynamics of the quadrotor. Hence, by combining equations [10](#page-4-3) and [14,](#page-4-4) <span id="page-5-3"></span><span id="page-5-1"></span>and considering the GE, the overall quadrotor flight dynamics is summarized as

$$
\tilde{x} = 2(q_1q_3 - q_0q_2)\frac{U_1}{m}f_{GE} - \frac{C_{d_x}}{m}\dot{x}
$$
 (17a)

$$
\ddot{y} = 2(q_2 q_3 + q_0 q_1) \frac{U_1}{m} f_{GE} - \frac{C_{d_y}}{m} \dot{y}
$$
(17b)

$$
\ddot{z} = \left[2(q_o^2 + q_3^2) - 1\right] \frac{U_1}{m} f_{GE} - g - \frac{C_{d_z}}{m} \dot{z} \tag{17c}
$$

$$
x = z(q_1q_3 - q_0q_2)\frac{U_1}{m}JGE - \frac{C_{d_y}}{m}x
$$
(17a)  

$$
\ddot{y} = 2(q_2q_3 + q_0q_1)\frac{U_1}{m}f_{GE} - \frac{C_{d_y}}{m}\dot{y}
$$
(17b)  

$$
\ddot{z} = \left[2(q_0^2 + q_3^2) - 1\right]\frac{U_1}{m}f_{GE} - g - \frac{C_{d_z}}{m}\dot{z}
$$
(17c)  

$$
\dot{p} = \frac{1}{J_{xx}}U_2f_{GE}(z) - \frac{C_{a_p}}{J_{xx}}p
$$
(17d)  

$$
\dot{q} = \frac{1}{J_{yy}}U_3f_{GE}(z) - \frac{C_{a_q}}{J_{yy}}q
$$
(17e)

$$
\dot{q} = \frac{1}{J_{yy}} U_{3} f_{GE}(z) - \frac{C_{a_q}}{J_{yy}} q \tag{17e}
$$

$$
\dot{r} = \frac{1}{J_{zz}} U_4 f_{GE}(z) - \frac{C_{a_r}}{J_{zz}} r \tag{17f}
$$

$$
\dot{q}_o = -\frac{1}{2}(pq_1 + qq_2 + rq_3) \tag{17g}
$$

$$
\dot{r} = \frac{1}{J_{zz}} U_4 f_{GE}(z) - \frac{C_{a_r}}{J_{zz}} r
$$
\n(17f)  
\n
$$
\dot{q}_o = -\frac{1}{2} (pq_1 + qq_2 + rq_3)
$$
\n(17g)  
\n
$$
\dot{q}_1 = \frac{1}{2} (pq_o + rq_2 - qq_3)
$$
\n(17h)  
\n
$$
\dot{q}_2 = \frac{1}{2} (qq_o + pq_3 - rq_1)
$$
\n(17i)  
\n
$$
\dot{q}_3 = \frac{1}{2} (rq_o + qq_1 - pq_2)
$$
\n(17j)

$$
\dot{q_2} = \frac{1}{2}(qq_o + pq_3 - rq_1) \tag{17i}
$$

$$
\dot{q_3} = \frac{1}{2}(rq_o + qq_1 - pq_2) \tag{17}
$$

Based on the free body diagram of the quadrotor in Figure [1](#page-3-0) and the relation between propeller angular speed  $\Omega$  and induced torque  $\tau$ , control allocation equations are obtained as follows

$$
\begin{cases}\n\Omega_1 = \sqrt{\frac{1}{f_{GE}(z)} \left( \frac{U_1}{4b} + \frac{U_2}{4b\ell} + \frac{U_3}{4b\ell} - \frac{U_4}{4d} \right)} \\
\Omega_2 = \sqrt{\frac{1}{f_{GE}(z)} \left( \frac{U_1}{4b} - \frac{U_2}{4b\ell} + \frac{U_3}{4b\ell} + \frac{U_4}{4d} \right)} \\
\Omega_3 = \sqrt{\frac{1}{f_{GE}(z)} \left( \frac{U_1}{4b} - \frac{U_2}{4b\ell} - \frac{U_3}{4b\ell} - \frac{U_4}{4d} \right)} \\
\Omega_4 = \sqrt{\frac{1}{f_{GE}(z)} \left( \frac{U_1}{4b} + \frac{U_2}{4b\ell} - \frac{U_3}{4b\ell} + \frac{U_4}{4d} \right)}\n\end{cases}
$$
\n(18)

and the overall residual or net propeller angular speed  $\Omega_r$  is obtained as

<span id="page-5-4"></span><span id="page-5-2"></span>
$$
\Omega_r = -\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4 \tag{19}
$$

Conversion from quaternion to Euler angle can be performed by using a canonical set of Euler angles as [\[41\]](#page-18-18)

$$
\begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} \arctan 2(2(q_o q_1 + q_2 q_3), q_o^2 - q_1^2 - q_2^2 + q_3^2) \\ \arcsin(2(q_o q_2 - q_3 q_1)) \\ \arctan 2(2(q_o q_3 + q_1 q_2), q_o^2 + q_1^2 - q_2^2 - q_3^2) \end{bmatrix}
$$
\n(20)

# <span id="page-6-0"></span>**III. INTELLIGENT FLIGHT CONTROL DESIGN OF QUADROTOR**

#### A. QUADROTOR CONTROLLER DESIGN STRATEGY

Due to the underactuation of the quadrotor translational dynamics, there is no way to directly control the lateral motion of the quadrotor. Therefore, a nested control approach has to be deployed in which desired attitude commands are generated by the outer position control loop based on the targeted response, and the inner attitude control loop stabilizes and controls the attitudes of the quadrotor based on the generated desired command. To ensure that every quadrotor output follows a specified trajectory, virtual control inputs are introduced to the system [\[42\].](#page-18-19) The physical interpretation of this control law is that control of the translation motion of the quadrotor depends on desired quaternion rotation  $q_d$  and total thrust force  $U_1$ . In the case of no external disturbance acts on the system, the virtual control inputs can be obtained as

$$
U = [U_x, U_y, U_z]' = R_I^B(q_d)U_1 f_{GE} - mg \qquad (21)
$$

<span id="page-6-1"></span>
$$
U_x = 2(q_{1_d}q_{3_d} - q_{0_d}q_{2_d})U_1f_{GE}
$$
 (22a)

$$
U_y = 2(q_{2_d}q_{3_d} + q_{0_d}q_{1_d})U_1f_{GE}
$$
 (22b)

$$
U_z = \left[2(q_{oa}^2 + q_{3a}^2) - 1\right]U_1f_{GE} - mg \qquad (22c)
$$

where  $U_x$ ,  $U_y$  and  $U_z$  are virtual position control inputs along *x*, *y* and *z* directions, respectively. This method of controlling a nonlinear system as if it is linear is called Nonlinear Dynamic Inversion (NDI). NDI is used to linearize the system, decouple the controlled variables, and separate the main model from the dynamic inversion model to compute a solution in closed form. By substituting equation  $22(a, b & c)$  $22(a, b & c)$  $22(a, b & c)$ into equation  $17(a,b\&c)$ , the translational quadrotor dynamics can be rewritten in terms of virtual control inputs as follows:

$$
\begin{cases}\n\ddot{x} = \frac{U_x}{m} - \frac{C_{d_x}}{m} \dot{x} \\
\ddot{y} = \frac{U_y}{m} - \frac{C_{d_y}}{m} \dot{y} \\
\ddot{z} = \frac{U_z}{m} - \frac{C_{d_z}}{m} \dot{z}\n\end{cases}
$$
\n(23)

To design a controller for 2*nd* order system, a small quaternion approach is implemented to rewrite quadrotor rotational dynamics in quaternion. This assumption is true for stabilizing the quadrotor near to hovering state. Since the unit quaternion approach is utilized to represent the orientation of the quadrotor, the following conditions are always true for near hovering state maneuverings such as  $q_o \approx 1$ , and  $q_1 = q_2 = q_3 \approx 0$ . In this scenario, the relationship between BF rates and quaternion rates, which is expressed

in equation [7,](#page-3-4) becomes

$$
\begin{bmatrix} \dot{q}_o \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ p \\ q \\ r \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 \\ p \\ q \\ r \end{bmatrix}
$$
 (24)

Hence,

<span id="page-6-2"></span>
$$
\begin{bmatrix} p \\ q \\ r \end{bmatrix} = 2 \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix}
$$
 (25)

<span id="page-6-6"></span>and then taking time derivative on both sides result in

<span id="page-6-3"></span>
$$
\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = 2 \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix}
$$
 (26)

Then, substitute equations [25](#page-6-2) and [26](#page-6-3) into equation  $17(d, e \& f)$  $17(d, e \& f)$ . Finally,  $2^{nd}$  order rotational dynamics of the quadrotor in terms of a quaternion can be obtained as

<span id="page-6-5"></span>
$$
\begin{cases}\n\ddot{q}_1 = \frac{1}{J_{xx}} \left( \frac{1}{2} U_2 - C_{a_p} \dot{q}_1 \right) \\
\ddot{q}_2 = \frac{1}{J_{yy}} \left( \frac{1}{2} U_3 - C_{a_q} \dot{q}_2 \right) \\
\ddot{q}_3 = \frac{1}{J_{zz}} \left( \frac{1}{2} U_4 - C_{a_r} \dot{q}_3 \right)\n\end{cases} (27)
$$

In the suggested control strategy, the attitude controller takes control action based on the deviation of the actual quaternion from the desired quaternion. Even though the attitude controller is designed to track only the vector part of quaternion, from the property of unit quaternion, the actual scalar part of quaternion *q<sup>o</sup>* will be converged to its desired *qo<sup>d</sup>* by the following relation:

$$
q_o = \sqrt{1 - (q_1^2 + q_2^2 + q_3^2)}
$$
 (28)

By assigning those virtual control inputs, now quadrotor dynamics become fully actuated since there are six control inputs such as  $U_x$ ,  $U_y$ ,  $U_z$ ,  $U_2$ ,  $U_3$ , and  $U_4$  for controlling six degrees of freedom quadrotor maneuvering along *x*, *y*,*z*, *q*1, *q*2, &*q*<sup>3</sup> direction, respectively. Therefore, each trajectory of the quadrotor can be controlled separately by corresponding control input.

<span id="page-6-4"></span>To design flight controller, state variables can be defined as follows,  $x = [x, \dot{x}, y, \dot{y}, z, \dot{z}, q_1, \dot{q}_1, q_2, \dot{q}_2, q_3, \dot{q}_3] =$  $[x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}]'$  where *x*, *y*, and *z* are linear position and  $\dot{x}$ ,  $\dot{y}$  and  $\dot{z}$  are speeds along  $x$ ,  $y$ , and  $z$  axis, respectively; whereas  $q_1$ ,  $q_2$ , and  $q_3$  are orientation in quaternion and  $\dot{q}_1$ ,  $\dot{q}_2$ , and  $\dot{q}_3$  are quaternion rates along roll, pitch, and yaw direction, respectively. Thus, this state vector has two parts namely the position control represented by the vector  $[x, \dot{x}, y, \dot{y}, z, \dot{z}]$ , and orientation control represented by vector  $[q_1, \dot{q}_1, q_2, \dot{q}_2, q_3, \dot{q}_3]$ . By rewriting equation  $23$  and  $27$  in state space equation form, the final

quadrotor dynamics model, which is defined from fixed IF, is obtained as

$$
\dot{x} = f(x, U) = f(x) + g(x)U
$$
(29)  

$$
\begin{cases}\n\dot{x}_1 = x_2 \\
\dot{x}_2 = \frac{1}{m} (U_x - C_{d_x}x_2) \\
\dot{x}_3 = x_4\n\end{cases}
$$
  

$$
\dot{x}_4 = \frac{1}{m} (U_y - C_{d_y}x_4)
$$
  

$$
\dot{x}_5 = x_6
$$
  

$$
\dot{x}_6 = \frac{1}{m} (U_z - C_{d_z}x_6)
$$
(30)  

$$
\dot{x}_7 = x_8
$$
  

$$
\dot{x}_8 = a_1 U_2 - d_1 x_8
$$
  

$$
\dot{x}_9 = x_{10}
$$
  

$$
\dot{x}_{10} = a_2 U_3 - d_2 x_{10}
$$
  

$$
\dot{x}_{11} = x_{12}
$$
  

$$
\dot{x}_{12} = a_3 U_4 - d_3 x_{12}
$$
(31)

where

\n- \n
$$
a_1 = \frac{1}{2J_{xx}}, \text{ and } d_1 = \frac{C_{ap}}{J_{xx}},
$$
\n
\n- \n $a_2 = \frac{1}{2J_{yy}}, \text{ and } d_2 = \frac{C_{aq}}{J_{yy}},$ \n
\n- \n $a_3 = \frac{1}{2J_{yy}}, \text{ and } d_3 = \frac{C_{ap}}{J_{zz}}.$ \n
\n

Based on this setup, the novel quadrotor UAV control approach that this research study contributed is shown in Figure [3.](#page-7-0)

<span id="page-7-0"></span>

**FIGURE 3.** Hierarchical hybrid control architecture of the quadrotor.

To tackle the aforementioned problem of the absence of direct actuation control for the position of the quadrotor, the linearized model is established in equation [23](#page-6-4) by using the NDI method. Thus, Feedforward ANN-based MRAC (NNMRAC) is designed for position control of the quadrotor as a control architecture in Figure [4.](#page-7-1) To obtain input-output data for ANN training, conventional MRAC for linearized systems is designed, first. Then, by exporting input-output data of adaptation law of MRAC system to MATLAB<sup>(R)</sup> workspace, FFNN is trained to estimate position controller parameters.

<span id="page-7-1"></span>

**FIGURE 4.** NN-based MRAC of quadrotor position control architecture.

On the other hand, the attitude controller has one to one relationship between control inputs and corresponding outputs i.e., there are direct actuation controls for the attitude that drive the quadrotor in the desired direction. Hence, RNNbased STR (RNNSTR) is designed to control the attitude of the quadrotor as shown in Figure [5.](#page-8-0) In this control architecture, the RNN plant model acts as a Black-box identifier which is trained first to generate one step ahead prediction of plant output, and the process is known as system identification. Then, by inserting trained plant model network parameters into the appropriate locations in the overall control architecture, the controller network will be trained. Here, the controller network is trained to generate control action that will drive estimated plant output toward reference input by following the desired reference model response [\[43\].](#page-18-20) When the controller network is trained, the plant model network is considered as a fixed network. To stabilize the system, control input must be one step delayed to separate the two processes since system identification must be accomplished before the controller takes control action without overlapping.

<span id="page-7-2"></span>Since both direct and indirect adaptive control approaches are implemented to design closed-loop controllers for the quadrotor, the overall proposed quadrotor control architecture becomes a hybrid control system as shown in Figure [3.](#page-7-0)

#### B. QUADROTOR POSITION CONTROLLER DESIGN

To design a Neural Network-based MRAC, the following two steps have been followed. The first step is designing conventional MRAC for second-order systems, which is described in equation [23.](#page-6-4) The second step is training FFNN

<span id="page-8-0"></span>

**FIGURE 5.** RNNSTR of quadrotor attitude control architecture.

offline first to estimate the controller parameters and then implementing online learning algorithm to update network parameters in real-time.

#### 1) CONVENTIONAL MRAC DESIGN OF 2*ND* ORDER SYSTEM

The implicit assumption in the design of the MRAC system is that the control designer is sufficiently familiar with the plant and its operating environment under consideration so that the designer can specify the desired behavior of the plant in terms of the output of a reference model. In the MRAC technique, the reference model describes system performance; and the adaptive controller forces the system to behave like the reference model. Here, the parameters of the adaptive controller are adjusted based on the deviation of actual plant output from the desired reference model output. As shown in Figure [4,](#page-7-1) MRAC system has two loops. The first loop is a normal or inner loop, which is an ordinary loop consisting of a plant and nominal nonadaptive controller, that is used to track the set point and stabilize the system [\[44\]. T](#page-18-21)he second loop is parameter adjustment or outer loop that adjusts controller parameters in such a way that driving reference model response tracking error toward zero.

<span id="page-8-3"></span><span id="page-8-1"></span>Mathematical techniques like MIT rule and Lyapunov direct method can be used to develop adaptation rules. Even though MIT rule is a widely used straightforward technique to design MRAC to achieve satisfactory results during controller performance evaluation, it is very sensitive to the changes in the amplitude of the reference input. Hence, it can't guarantee global stability in MRAC system [\[45\]. O](#page-18-22)n the other hand, Lyapunov direct method is a well-known method that can be applied to ensure the overall stability in the MRAC system [\[46\]. I](#page-18-23)t is based on establishing some positive definite or Lyapunov function of tracking and estimation error, first, and then adaptation laws are derived in a way that makes the gradient of the candidate Lyapunov function negative. In this way, the candidate Lyapunov function will decrease as time goes forward and derived adaptation laws give bounded asymptotic convergence of control parameters so that the controller can ensure global stability of the overall system.

Since the translational dynamics of the quadrotor in equation [23](#page-6-4) is a 2*nd* order system, MRAC is designed for 2<sup>nd</sup> order system. To design MRAC, the plant dynamics with matched uncertainty is given as

$$
\dot{x} = Ax + B(U - \theta^{*T} \phi(x))
$$
\n(31)

where  $f(x) = \theta^*^T \phi(x)$  is matched uncertainty,  $\theta =$  $[\theta_1, \theta_2, \ldots]$  are unknown constant vector, and  $[\phi_1(x), \phi_2(x), \ldots]$ are vector of known bounded basis functions. The reference model for 2*nd* order system is given as

$$
\dot{x}_m = A_m x_m + B_m r \tag{32}
$$

The ideal control law is defined as

$$
U^* = k_r^* r - k_x^* x + \theta^{*T} \phi(x)
$$
 (33)

where  $U^*$  is the ideal control law, and  $k_r^*$ ,  $k_x^*$ , and  $\theta^*$  are final constant estimated values of controller gains when the plant model output perfectly tracks the reference model response. The actual control law that perfectly cancels out the matched uncertainty is defined as

$$
U = k_r r(t) - k_x x(t) + \theta^T \phi(x)
$$
 (34)

where  $k_r$ ,  $k_x$ , and  $\theta$  are actual adaptation parameters. Estimation errors is defined as:  $\tilde{k_x} = k_x - k_x^*$ ,  $\tilde{k_r} = k_r - k_r^*$ and  $\tilde{\theta} = \theta - \theta^*$ . To drive the adaptation law by using the Lyapunov direct method, let's choose the positive definite Lyapunov candidate function of tracking and estimation error as [\[9\]](#page-17-8)

$$
V(e, \tilde{k}_r, \tilde{k}_x, \tilde{\theta}) = e^T P e + |b| \left( \frac{\tilde{k}_r^2}{\gamma_r} + \tilde{k}_x \gamma_x^{-1} \tilde{k}_x^T + \tilde{\theta}^T \gamma_{\theta}^{-1} \tilde{\theta} \right) \tag{35}
$$

The gradient of the Lyapunov candidate function can be obtained as

$$
\dot{V}(e, \tilde{k_r}, \tilde{k_x}, \tilde{\theta}) = -e^T Q e + 2|b|\tilde{k_r} \left( -re^T \bar{P} sgn(b) + \gamma_r^{-1} \dot{\tilde{k_r}} \right) \n+ 2|b|\tilde{k_x} \left( xe^T \bar{P} sgn(b) + \gamma_x^{-1} \dot{\tilde{k_x}} \right) \n+ 2|b|\tilde{\theta}^T \left( -\phi e^T \bar{P} sgn(b) + \gamma_{\theta}^{-1} \dot{\tilde{\theta}} \right)
$$
\n(36)

<span id="page-8-2"></span>The adaptation laws, which make closed-loop error dynamics uniformly stable, for  $2^{nd}$  system can be obtained by making the gradient of the Lyapunov function negative as follows:

$$
\begin{cases}\n\dot{k}_r = \gamma_r r e^T \bar{P} s g n(b) \\
\dot{k}_x = -\gamma_x x e^T \bar{P} s g n(b) \\
\dot{\theta} = \gamma_\theta \phi e^T \bar{P} s g n(b)\n\end{cases}
$$
\n(37)

In most cases, if *A* and *B* are known, then *k<sup>x</sup>* and/or  $k_r$  can be designed by any standard non-adaptive control techniques to stabilize the closed-loop system and enable plant output to follow a reference input [\[9\]. Sin](#page-17-8)ce all the states of the quadrotor can be obtained with the help of sensors onboard platforms, all the states of the quadrotor dynamics can be assumed to be available. Therefore, by considering the adaptation parameter  $k_x$  as a full-state feedback gain,  $k_x$  will be designed from robust non-adaptive servo LQR control as a control architecture in Figure [4.](#page-7-1)

• Reference model for  $2^{nd}$  order system is designed based on desired transient response of  $t_s = 3$  second settling time and 5% maximum overshoot:

$$
x_m = \frac{4}{s^2 + 2.76s + 4}r\tag{38}
$$

• Translation quadrotor dynamics in state space form rewritten as

$$
\begin{cases} \n\dot{x}_1 = x_2 \\ \n\dot{x}_2 = \frac{1}{m} \left( U_x - C_{d_x} x_2 \right) \n\end{cases} \n\tag{39}
$$

• Since the objective control design is minimizing tracking error, error dynamics to design LQR becomes

$$
\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = -\frac{C_{d_x}}{m} z_2 - \frac{U_x}{m} \end{cases}
$$
(40)

$$
U_x = U_{LQR} + U_{adaptive}
$$
 (41)

where  $U_{LQR}$  is non adaptive controller with fixed gain  $k_x$ , and *Uadaptive* is adaptive control with adaptive parameters  $k_r$  and  $\theta$ . By using a similar approach,  $U_y$  and  $U_z$  are designed to control the position of the quadrotor along *y* and *z* directions.

# 2) FFNN TRAINING TO ESTIMATE POSITION CONTROLLER PARAMETERS

<span id="page-9-3"></span>ANN with a sufficient number of network parameters has well-proved properties to universally approximate any smooth arbitrary functions to an acceptable level of accuracy. This is the core property of ANN for its superiority over other approximation methods. Furthermore, once trained, ANN requires less computation time and memory storage to perform its tasks [\[47\]. H](#page-18-24)owever, choosing the right number of ANN parameters is the main problem in training ANN since there is no defined rule to select it. Therefore, most of the time ANN parameters are chosen by trial and error, keeping in mind that the smaller the numbers are, the better the network is in terms of memory storage and processing time. To estimate position controller parameters, a three-layer ANN with 10 neurons in the hidden layer is trained and tested with 5000 samples of simulation data, and its performance is shown in Figure [6.](#page-9-0)

<span id="page-9-0"></span>

**FIGURE 6.** Trained FFNN training, validation and test performances.

 $\sqrt{ }$ 

 $\overline{\phantom{a}}$ 

C. DESIRED ALTITUDE AND QUATERNION COMPUTATION [D](#page-6-1)esired  $U_1$  and  $q_d$  are derived by rewriting equation [22](#page-6-1)  $(a,b&c)$  and using unit quaternion relation, as follows:

<span id="page-9-1"></span>
$$
\frac{U_x}{U_1 f_{GE}} = 2(q_{1_d} q_{3_d} - q_{0_d} q_{2_d})
$$
\n(42a)

$$
\frac{U_y}{U_1 f_{GE}} = 2(q_{2_d} q_{3_d} + q_{o_d} q_{1_d})
$$
\n(42b)

$$
\frac{U_z + mg}{U_1 f_{GE}} = q_{oa}^2 - q_{1d}^2 - q_{2d}^2 + q_{3d}^2 \qquad (42c)
$$

$$
\begin{bmatrix} \frac{U_z + mg}{U_1 f_{GE}} = q_{o_d}^2 - q_{1_d}^2 - q_{2_d}^2 + q_{3_d}^2 & (42c) \\ 1 = q_{o_d}^2 + q_{1_d}^2 + q_{2_d}^2 + q_{3_d}^2 & (42d) \end{bmatrix}
$$

To derive  $U_1$ , square both side of equations  $42(a-c)$ , and then take sum as follows:

$$
\begin{aligned} & \left[ \frac{U_x}{U_1 f_{GE}} \right]^2 + \left[ \frac{U_y}{U_1 f_{GE}} \right]^2 + \left[ \frac{U_z + mg}{U_1 f_{GE}} \right]^2 \\ &= [2(q_{1_d} q_{3_d} - q_{0_d} q_{2_d})]^2 + [2(q_{2_d} q_{3_d} + q_{0_d} q_{1_d})]^2 \\ &+ [q_{0_d}^2 - q_{1_d}^2 - q_{2_d}^2 + q_{3_d}^2]^2 \\ &= \left[ q_{0_d}^2 + q_{1_d}^2 + q_{2_d}^2 + q_{3_d}^2 \right]^2 \end{aligned}
$$

For unit quaternion rotation,  $q_{o_d}^2 + q_{1_d}^2 + q_{2_d}^2 + q_{3_d}^2 = 1$ . Hence,

<span id="page-9-2"></span>
$$
\[U_x^2 + U_y^2 + (U_z + mg)^2\] \frac{1}{U_1^2 f_{GE}^2}
$$
  
= 1 (43)

$$
U_1 = \frac{1}{f_{GE}} \sqrt{U_x^2 + U_y^2 + (U_z + mg)^2}
$$
 (44)

From desired rotation around *z*-axis  $\psi_d$ , corresponding desired quaternion is obtained by using axis-angle relationship in [\[29\]](#page-18-6) as follows:

$$
q_{3_d} = \sin(\frac{\psi_d}{2})\tag{45}
$$

To obtain the desired scalar quaternion  $q_{o_d}$ , take the sum of equation  $42(c \& d)$  $42(c \& d)$ , first as

$$
\frac{U_z + mg}{U_1 f_{GE}} + 1 = 2q_{o_d}^2 + 2q_{3_d}^2 \tag{46}
$$

$$
2q_{o_d}^2 = \frac{U_z + mg}{U_1 f_{GE}} - 2q_{3_d}^2 + 1 \tag{47}
$$

Then, substitute equation  $44 \& 45$  $44 \& 45$  $44 \& 45$  into equation  $47$ . Finally, *qo<sup>d</sup>* can be obtained as

$$
q_{o_d} = \frac{1}{\sqrt{2}} \sqrt{\frac{U_z + mg}{\sqrt{U_x^2 + U_y^2 + (U_z + mg)^2}}} - 2\sin^2(\frac{\psi_d}{2}) + 1
$$
 (48)

To derive  $q_{1d}$  and  $q_{2d}$ , first, rewrite equation  $42(a \& b)$  $42(a \& b)$  $42(a \& b)$  as follows:

$$
\frac{1}{U_1 f_{GE}} \begin{bmatrix} U_x \\ U_y \end{bmatrix} = 2 \begin{bmatrix} q_{3_d} & -q_{o_d} \\ q_{o_d} & q_{3_d} \end{bmatrix} \begin{bmatrix} q_{1_d} \\ q_{2_d} \end{bmatrix}
$$
(49)

From the  $2\times 2$  matrix inverse property, the inverse is obtained, first as

$$
\begin{bmatrix} q_{1_d} \\ q_{2_d} \end{bmatrix} = \frac{1}{2U_1 f_{GE}} * \frac{1}{q_{o_d}^2 + q_{3_d}^2} \begin{bmatrix} q_{3_d} & q_{o_d} \\ -q_{o_d} & q_{3_d} \end{bmatrix} \begin{bmatrix} U_x \\ U_y \end{bmatrix}
$$
(50)

Then, substitute equations  $44$ ,  $45 \& 48$  $45 \& 48$  $45 \& 48$  into equa-tion [50.](#page-10-3) Finally,  $q_{1d}$  and  $q_{2d}$  can be obtained as follows:

$$
q_{1_d} = \frac{\sin(\frac{\psi_d}{2})U_x + \frac{1}{\sqrt{2}}U_y \sqrt{\frac{U_z + mg}{\sqrt{U_x^2 + U_y^2 + (U_z + mg)^2}}} - 2\sin^2(\frac{\psi_d}{2}) + 1}{U_z + mg + \sqrt{U_x^2 + U_y^2 + (U_z + mg)^2}}
$$
(51)

and

$$
q_{2_d} = \frac{\sin(\frac{\psi_d}{2})U_y - \frac{1}{\sqrt{2}}U_x\sqrt{\frac{U_z + mg}{\sqrt{U_x^2 + U_y^2 + (U_z + mg)^2}} - 2\sin^2(\frac{\psi_d}{2}) + 1}}{U_z + mg + \sqrt{U_x^2 + U_y^2 + (U_z + mg)^2}}
$$
(52)

Equations [44,](#page-9-2) [45,](#page-10-0) [48,](#page-10-2) [51,](#page-10-4) and [52](#page-10-5) are dealt with inside the conversion block as shown in Figure [3.](#page-7-0)

#### D. QUADROTOR ATTITUDE CONTROLLER DESIGN

To make the response of the attitude controller fast relative to the position controller response, a reference model with a settling time of 2 seconds and zero overshoot is selected as

$$
y_m = \frac{4}{s^2 + 4s + 4}r\tag{53}
$$

As expressed earlier in Figure [5,](#page-8-0) the overall RNNSTR architecture consists of two separate networks, which are

<span id="page-10-1"></span><span id="page-10-0"></span>trained separately to perform different tasks. One is trained to estimate the model, and the other is trained to generate control actions that drive the system toward reference input. In this scenario, the first step is estimating the plant model by generating input and output data of the system model in equation [27.](#page-6-5) To train the plant model network, first, the maximum-minimum input of the system is obtained from equation [18](#page-5-2) by considering 75% of the maximum speed of the BLDC motor running at 10000 RPM that yields 1.1/−1.1 Nm torque; and the desired output is seated to lay in the range of  $[-1 \ 1]$  quaternion unit. Then, 5000 samples of input-output data are generated with a sampling time of 0.05 seconds. Finally, a three-layer ANN with 10 neurons in the hidden layer is configured and trained. The performance of the trained plant model network on training, validation, and testing datasets is shown in Figure [7,](#page-10-6) [8,](#page-11-0) and [9.](#page-11-1) As can be seen from these figures, the training, validation, and testing data errors are minimal i.e., the plant model network estimates the quadrotor rotation model precisely.

<span id="page-10-6"></span><span id="page-10-3"></span><span id="page-10-2"></span>

<span id="page-10-5"></span><span id="page-10-4"></span>**FIGURE 7.** Plant model network training performance.

<span id="page-10-7"></span>The second step is training the controller network by generating input-out data of the reference model in equation [53.](#page-10-7) To train the controller network, 4000 data are generated by considering the maximum/minimum size of the desired quaternion unit 1/−1. Here, before training the controller network, plant network parameters are set into appropriate places of the overall network configuration. In this step, only controller network parameters are updated whereas plant network parameters remain fixed. The desired reference

<span id="page-11-0"></span>

**FIGURE 8.** Plant model network validation performance.

<span id="page-11-1"></span>

**FIGURE 9.** Plant model network testing performance.

model response and tracking performance of RNNSTR for uniform random reference input signal on the training dataset is shown in Figure [10.](#page-11-2) The result shows that the plant network output(green) precisely tracks the reference model output(blue).

<span id="page-11-2"></span>

**FIGURE 10.** Tracking performance of RNNSTR on generated dataset.

These results, which are shown in Figures [7](#page-10-6) to [10,](#page-11-2) are RNNSTR attitude controller training to control *q*<sup>1</sup> rotation only. By applying the same procedure for *q*<sup>2</sup> and *q*3, RNNSTR attitude controllers are designed as well.

#### E. ONLINE LEARNING ALGORITHM

<span id="page-11-4"></span>Offline learning is not always applicable for real-time implementation. In some scenarios, online learning is needed to update the parameters of the network in real-time. However, using the initial set of network parameters from offline training increases the convergence rate and ensures the stability of the system [\[48\].](#page-18-25) Therefore, to increase the performance of the trained network in position and attitude control architectures in Figure [4](#page-7-1) and [5,](#page-8-0) both learning methodologies are applied in this study. Here, offline training is deployed to get the initial set of weights and biases of the network and then, the online learning algorithm is developed to update those weights and biases in real-time. These weights and biases are updated only when the output error exceeds a certain predefined threshold value. To update weight and biases online, a generalized BPA is utilized as follows:

• Once the network output  $y_i$  is obtained, output layer delta δ*j* is computed, first. Then, output layer weights *wji* and biases  $b_i$  are updated as

<span id="page-11-3"></span>
$$
\begin{cases}\n\delta_j = y_i(1 - y_i)e_o \\
w_{ji}(k + 1) = w_{ji}(k) + \mu \delta_j(k)x_i(k) \\
b_j(k + 1) = b_j(k) + \mu \delta_j(k)\n\end{cases}
$$
\n(54)

where delta  $\delta$  in ANN represents the rate at which the error changes with respect to the output of a particular neuron.

• Then, the error at the hidden layer is obtained as

$$
e_h = \sum_{j=1}^{N_o} \sum_{h=1}^{M_o} w_{kh} \delta_j
$$
 (55)

where  $\mu$  is the learning rate,  $N<sub>o</sub>$  is the maximum number of output nodes, *M<sup>o</sup>* is the maximum number of hidden layer neurons, *wkh* is the weights of hidden to the output layer, and *e<sup>h</sup>* is the error at each node in the hidden layer. Once hidden layer error and delta are computed, the delta rule in equation [54](#page-11-3) is applied similarly to update the hidden layer weights and biases. In this study, the network is deployed to retrain online by BPA when the error exceeds  $\pm 0.0005$ , and the flow chart for the online learning algorithm is shown in Figure [11.](#page-12-0)

<span id="page-12-0"></span>



# F. LQR CONTROLLER DESIGN FOR QUADROTOR UAV

LQR is an optimal linear control algorithm that operates in a dynamic system by minimizing a suitable cost function. It is often used when information about the

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system is completely known and undesirable disturbances like wind that can affect the system are absent. LQR computes the maximum gains to minimize the cost function, which enhances the performance of the system. Generally, high control effort is required to achieve maximum state regulation so that the system states are driven strongly to a stable state [\[37\]. T](#page-18-14)o design LQR, Jacobian approximation is utilized to linearize the nonlinear quadrotor dynamics that can be written as

<span id="page-12-5"></span><span id="page-12-1"></span>
$$
\dot{Z} = AZ + BU \tag{56}
$$

where  $Z \in R^6$  and  $U \in R$  are error state variable and input, respectively and the cost function *J* for which the controller seeks to minimize is given as [\[49\]](#page-18-26)

$$
J = \int_0^\infty (Z^T Q Z + U^T R U) \tag{57}
$$

where  $Q = Q^T \ge 0$  and  $R = R^T > 0$  are positive definite symmetric matrices that weigh the relative importance of the existing error as well as the energy expenditure to stabilize the system [\[50\]. B](#page-18-27)y applying the optimality principle to the optimal control problem, the goal of LQR is to minimize the cost function given in equation [57,](#page-12-1) and the equation is reduced to a simple Riccatti equation as

<span id="page-12-6"></span>
$$
PA + ATP - PBR-1BTP + Q = 0
$$
 (58)

where  $P = P^T \ge 0$  is a positive definite symmetric matrix obtained from equation [58.](#page-12-2) The optimal state feedback gain matrix is obtained as

<span id="page-12-2"></span>
$$
k = R^{-1}B^T P \tag{59}
$$

and the closed-loop LQR controller that drives the cost function towards zero as time goes to infinity is manipulated as

<span id="page-12-4"></span>
$$
U = r - kX \tag{60}
$$

where  $r$  is the reference input, and  $\overline{X}$  is state vector of the system. The closed-loop control architecture that is deployed to design the LQR controller is shown in Figure [12](#page-12-3)

<span id="page-12-3"></span>

**FIGURE 12.** LQR control architecture of the quadrotor UAV.

To design LQR, the quadrotor dynamics is linearized at a hovering position. Thus, the linearized model of the quadrotor translation dynamics from equation [23](#page-6-4) is obtained as

$$
\begin{cases}\n\ddot{x} = \frac{U_x}{\mu} \\
\ddot{y} = \frac{\mu_y}{\mu} \\
\ddot{z} = \frac{\mu_z}{m}\n\end{cases}
$$
\n(61)

Position error dynamics in state space form are obtained as

$$
\dot{Z}_p = A_p Z_p + B_p U_p \qquad (62)
$$
\n
$$
\begin{bmatrix}\n\dot{e}_x \\
\dot{e}_y \\
\dot{e}_y \\
\dot{e}_z \\
\dot{e}_z\n\end{bmatrix} = \begin{bmatrix}\n0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0\n\end{bmatrix} \begin{bmatrix}\ne_x \\
e_x \\
e_y \\
e_z \\
\dot{e}_z\n\end{bmatrix}
$$
\n
$$
+ \begin{bmatrix}\n0 & 0 & 0 & 0 \\
-2.1368 & 0 & 0 & 0 \\
0 & -2.1368 & 0 & 0 \\
0 & 0 & -2.1368 & 0 \\
0 & 0 & 0 & -2.1368\n\end{bmatrix} \begin{bmatrix}\n0 \\
U_x \\
U_y \\
U_y \\
U_z\n\end{bmatrix}
$$
\n
$$
B_p \qquad (63)
$$

where  $Z_p$  is a position error vector, which is the difference between the desired and actual state of the system. By using try and error finally, the weighting matrix *Q* and *R* are selected for position controller as

$$
Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$
(64)

The optimal state feedback gain matrix for position controller can be obtained as

$$
k_p = \begin{bmatrix} 1 & 1.3914 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1.3914 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1.3914 \end{bmatrix}
$$
 (65)

Similarly, the attitude controller is designed by linearizing attitude dynamics, which is given in equation [66.](#page-13-1) At the hovering position, the linear model for rotational dynamics can be obtained as

$$
\begin{cases}\n\ddot{q}_1 = \frac{1}{2J_{xx}} U_2 \\
\ddot{q}_2 = \frac{1}{2J_{yy}} U_3 \\
\ddot{q}_3 = \frac{1}{2J_{zz}} U_4\n\end{cases}
$$
\n(66)

Attitude error dynamics in state space form are obtained as

$$
\dot{Z}_a = A_a Z_a + B_a U_a \tag{67}
$$
\n
$$
\begin{bmatrix}\n\dot{e}_{q_1} \\
\dot{e}_{q_1} \\
\dot{e}_{q_2} \\
\dot{e}_{q_3} \\
\dot{e}_{q_3}\n\end{bmatrix} = \begin{bmatrix}\n0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0\n\end{bmatrix} \begin{bmatrix}\ne_{q_1} \\
\dot{e}_{q_1} \\
\dot{e}_{q_2} \\
\dot{e}_{q_3} \\
\dot{e}_{q_4} \\
\dot{e}_{q_5}\n\end{bmatrix}
$$
\n
$$
\dot{Z}_a
$$
\n
$$
+ \begin{bmatrix}\n0 & 0 & 0 & 0 & 0 \\
-102.9654 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -56.8117\n\end{bmatrix} \begin{bmatrix}\n0 \\
U_2 \\
U_3 \\
U_4\n\end{bmatrix}
$$
\n
$$
B_a
$$
\n(68)

where  $Z_a$  is the attitude error vector. The Weighting matrix *Q* and *R* for attitude control design are selected as the same as that of position controller, which is described in equation [64.](#page-13-2) The optimal state feedback gain matrix for the attitude controller can be obtained as

<span id="page-13-4"></span>
$$
k_a = \begin{bmatrix} 1 & 1.0097 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1.0097 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1.0174 \end{bmatrix}
$$
 (69)

By using the values of the optimal state feedback gain matrix in the equation  $65$  and  $69$ , LQR in equation  $60$  is designed to stabilize and control the position and attitude of the quadrotor.

# <span id="page-13-0"></span>**IV. SIMULATION RESULTS AND DISCUSSION**

# A. HELICAL TRAJECTORY TRACKING

<span id="page-13-2"></span>A helical trajectory is obtained by using the following desired trajectories:  $x_d = \cos t$ ,  $y_d = \sin t$ ,  $z_d = t$ , and  $\psi_d =$ −0.5 rad. The tracking performance of the controller is shown as

<span id="page-13-5"></span><span id="page-13-3"></span>

<span id="page-13-1"></span>**FIGURE 13.** 3D helical trajectory tracking performance.

<span id="page-14-0"></span>

<span id="page-14-1"></span>**FIGURE 14.** Translational trajectory tracking for helical path.



<span id="page-14-2"></span>**FIGURE 15.** Desired quaternion tracking for helical path.



<span id="page-14-3"></span>**FIGURE 16.** Required control efforts to track helical path.



<span id="page-14-4"></span>

**FIGURE 18.** Translational main trajectory tracking in nominal scenario.

<span id="page-14-5"></span>

**FIGURE 19.** Quaternion-based rotational trajectory tracking in nominal scenario.

<span id="page-14-6"></span>

**FIGURE 20.** Control efforts in nominal scenario.

a desired reference model response, so the suggested controller effectively stabilize the quadrotor system with a high tracking precision. Furthermore, the control efforts that enabled the quadrotor to achieve this task successfully are shown in Figure [16.](#page-14-2) By manipulating equation [18](#page-5-2) with the rated speed of the BLDC motor, the maximum

Figure [13,](#page-13-5) [14](#page-14-0)  $\&$  [15](#page-14-1) illustrate that within a finite time, both translational and rotational trajectories converged to

<span id="page-15-1"></span>

**FIGURE 21.** 3D main trajectory tracking in the presence of Matched uncertainty.



**FIGURE 22.** Translation tracking in the presence of matched uncertainties.

control effort for  $U_1$ ,  $U_2$ ,  $U_3$ , and  $U_4$  can be obtained as 13.07 N, 1.47 Nm, 1.47 Nm, and 0.25 Nm, respectively. Compared with these, the utilized control efforts to take control action are minimal and smooth.<sup>[1](#page-15-0)</sup> These prove the feasibility of the derived control algorithm. Here, minimal control effort is explicitly stated, in comparison with the control effort that can be obtained when the actuator of the quadrotor rotates at its rated speed. Hence, the proposed control algorithm guarantees accurate desired reference model trajectory tracking by stabilizing the quadrotor system at the expense of appropriate control efforts.

#### B. MAIN TRAJECTORY TRACKING

The main trajectory is developed specifically for aerial photography from the quadrotor by considering altitude  $= 9$  m, quadrotor speed  $= 2$  m/s, and lateral overlap i.e., the area of one image includes the area already captured in another image = 20%. The generated 3D trajectory and controller tracking performance are shown in Figure [17.](#page-14-3)

<span id="page-15-2"></span>

**FIGURE 23.** Rotation tracking in the presence of matched uncertainties.

<span id="page-15-3"></span>

**FIGURE 24.** Control efforts in the presence of matched uncertainties.

As can be depicted in Figure [18](#page-14-4) & [19,](#page-14-5) the rotational and translational quadrotor trajectory precisely track the desired reference model response. Moreover, 3D trajectory tracking in Figure [17](#page-14-3) proves that the controller guarantees accurate desired reference model response tracking by perfectly stabilizing the quadrotor when it flies on the top of the field as a desired stated path. Furthermore, the control inputs results in Figure [20](#page-14-6) show that the control efforts required to drive the quadrotor are smooth and minimal which proves the feasibility of the proposed control algorithm.

# C. MAIN TRAJECTORY TRACKING PERFORMANCE IN THE PRESENCE OF MATCHED UNCERTAINTY

Aerodynamic drag coefficients and inertia depend on the speed of the quadrotor, the density of air, and the altitude of flight, so it is crucial to design a controller that can tolerate these parametric uncertainties in the quadrotor model. Therefore, to validate the robustness of the proposed control algorithm in the presence of parametric variations that can be considered as a matched uncertainty, parametric variations with time are conducted as follows:  $J_{xx} = 2J_{xx_0}$ ,  $J_{yy} = 2J_{yy_o}, J_{zz} = 2J_{zz_o}, c_{d_x} = c_{d_y} = c_{d_z} = (1 + 0.1t)c_{d_o},$ and  $c_{a_p} = c_{a_q} = c_{a_r} = (1 + 0.01t)c_{a_o}$  where subscript '*o*' is included to indicate parameters at the nominal scenarios.

<span id="page-15-4"></span>In Figure [21-](#page-15-1)[23,](#page-15-2) the tracking errors are acceptable, this implies that the proposed controller tolerates parametric

<span id="page-15-0"></span><sup>&</sup>lt;sup>1</sup>Minimal and smooth control effort indicate better functional safety and economical use of the control system [\[51\],](#page-18-28) [\[52\]. R](#page-18-29)elatively large control effort requires large power from an electronic speed controller (ESC) that limits flight time since the quadrotor has a limited energy source, this makes the control system more expensive related to battery size; and oscillation rate in the control signal determines the intensity of wear of moving mechanical parts that probably damage and reduce aging of mechanical parts of the actuator.

<span id="page-16-0"></span>

<span id="page-16-1"></span>**FIGURE 25.** Tracking performance for quickly varying disturbance for case-1.



Step input tracking performance in the presence of slowly varying external disturbance

**FIGURE 26.** Tracking performance for slowly varying disturbance for case-2.

variations in the system by guaranteeing accurate desired trajectory tracking in the presence of parametric variations. This proves the robustness of the proposed control technique since the controller keeps the tracking error within the acceptable range. However, to track the given trajectory, the pose of the quadrotor in Figure [23](#page-15-2) is different than that of the nominal scenario in Figure [19.](#page-14-5) Moreover, as time elapse increases, parametric variations become large and large also

demanding larger control efforts at the turning point as shown in Figure [24.](#page-15-3) Hence, the proposed controller guarantees accurate desired trajectory tracking with the expense of additional control effort relative to the nominal operation in Figure [20.](#page-14-6)

# D. PERFORMANCE ANALYSIS IN THE PRESENCE UNMATCHED UNCERTAINTY

Quadrotor maneuvering in the field is exposed to external disturbance from the wind, so performance analysis is done by applying unknown external disturbances as unmatched uncertainties for case-1:  $d = \sin(3t)$  and case-2:  $d =$ sin(0.03*t*). Comparison is done based on the tracking performance of online ANN-based MRAC, offline ANN-based MRAC, conventional MRAC, and LQR control approaches. Notice that the disturbances that have been considered here are arbitrary functions since the uncertainty is assumed as unmatched, the controller doesn't have knowledge about it.

In Figure [25,](#page-16-0) for quickly varying disturbance in case-1, the error is very large for LQR and conventional MRAC; and in the case of online ANN-based MRAC and offline ANN-based MRAC the tracking errors are within a specified range i.e.,  $|e| \leq 0.0005$ , which shows the robustness of the ANN-based control system. Furthermore, from Figure [26,](#page-16-1) for slowly varying disturbance in case-2, the proposed controller with online tuned ANN parameters, outperforms other presented control strategies. Moreover, in both scenarios, LQR has poor external disturbance rejection capability compared to the presented adaptive control strategies. Therefore, these results prove that online learning algorithms enhance the external disturbance rejection capability of the control system.

#### <span id="page-17-22"></span>**V. CONCLUSION AND FUTURE WORKS**

A neural network-based MRAC is proposed in this article for the position and attitude control of the quadrotor in the presence of external disturbances and uncertainties. To design the suggested controller, first, quadrotor flight dynamics is modeled by considering every phenomenon that has a significant effect on the quadrotor UAV maneuvering. Then, the proposed controller is designed, and for performance comparison, the LQR controller is designed as well. Finally, to validate the effectiveness of the suggested control approach, numerical simulations have been carried out in nominal scenarios and the presence of matched and unmatched uncertainties. The simulation results validate the effectiveness and superiority of the proposed control technique in achieving high tracking precision and disturbance rejection capability in the nominal scenarios and the presence of matched and unmatched uncertainties. Furthermore, the utilized control efforts to achieve the desired tasks are minimal and smooth. These prove functional safety and economical use of the overall control architecture, so the proposed controller is feasible for real-time implementation of the quadrotor platform. Therefore, the future works of this study will be implementing a fully autonomous quadrotor prototype using the proposed intelligent flight control technique.

#### **REFERENCES**

- <span id="page-17-0"></span>[\[1\] K](#page-0-0). Chen, C. Ye, C. Wu, H. Wang, L. Jin, F. Zhu, and H. Hong, ''A novel open-closed-loop control strategy for quadrotor trajectory tracking on realtime control and acquisition platform,'' *Appl. Sci.*, vol. 13, no. 5, p. 3251, Mar. 2023.
- <span id="page-17-1"></span>[\[2\] E](#page-0-0). Okyere, A. Bousbaine, G. T. Poyi, A. K. Joseph, and J. M. Andrade, ''LQR controller design for quad-rotor helicopters,'' *J. Eng.*, vol. 2019, no. 17, pp. 4003–4007, Jun. 2019.
- <span id="page-17-2"></span>[\[3\] A](#page-0-1). Eltayeb, M. F. Rahmat, M. A. M. Basri, M. A. M. Eltoum, and M. S. Mahmoud, ''Integral adaptive sliding mode control for quadcopter UAV under variable payload and disturbance,'' *IEEE Access*, vol. 10, pp. 94754–94764, 2022.
- <span id="page-17-3"></span>[\[4\] P](#page-0-1). K. Reddy Maddikunta, S. Hakak, M. Alazab, S. Bhattacharya, T. R. Gadekallu, W. Z. Khan, and Q.-V. Pham, ''Unmanned aerial vehicles in smart agriculture: Applications, requirements, and challenges,'' *IEEE Sensors J.*, vol. 21, no. 16, pp. 17608–17619, Aug. 2021.
- <span id="page-17-4"></span>[\[5\] N](#page-1-0). P. Nguyen, N. X. Mung, H. L. N. N. Thanh, T. T. Huynh, N. T. Lam, and S. K. Hong, ''Adaptive sliding mode control for attitude and altitude system of a quadcopter UAV via neural network,'' *IEEE Access*, vol. 9, pp. 40076–40085, 2021.
- <span id="page-17-5"></span>[\[6\] V](#page-1-1). R. Puttige, ''Neural network based adaptive control for autonomous flight of fixed wing unmanned aerial vehicles,'' Ph.D. dissertation, UNSW Sydney, 2009.
- <span id="page-17-6"></span>[\[7\] S](#page-1-2). Bhattacharyya, D. Cofer, D. J. Musliner, J. Mueller, and E. Engstrom, ''Certification considerations for adaptive systems: Technical report,'' NASA, Washington, DC, USA, Tech. Rep., 218702, 2015.
- <span id="page-17-7"></span>[\[8\] G](#page-1-2). Pin, Y. Wang, A. Serrani, and T. Parisini, ''Dynamic certainty equivalence adaptive control by nonlinear parameter filtering,'' in *Proc. 59th IEEE Conf. Decis. Control (CDC)*, Dec. 2020, pp. 1454–1459.
- <span id="page-17-8"></span>[\[9\] N](#page-1-3). T. Nguyen and N. T. Nguyen, *Model-Reference Adaptive Control*. Berlin, Germany: Springer, 2018.
- <span id="page-17-9"></span>[\[10\]](#page-1-4) G. Cheng, D. Li, and M. He, ''Model-free coking furnace adaptive control,'' *Hydrocarbon Process.*, vol. 78, no. 12, p. 73, 1999.
- <span id="page-17-10"></span>[\[11\]](#page-1-5) S. S. Haider, "Simplified neural networks algorithms for function approximation and regression boosting on discrete input spaces,'' Ph.D. dissertation, Univ. Manchester, 2011.
- <span id="page-17-11"></span>[\[12\]](#page-1-6) B. Whitehead and S. Bieniawski, ''Model reference adaptive control of a quadrotor UAV,'' in *Proc. AIAA Guid., Navigat., Control Conf.*, Aug. 2010, p. 8148.
- <span id="page-17-12"></span>[\[13\]](#page-1-7) J. C. Lopez-Hoyos, J. S. Cervantes-Rojas, P. Ordaz, and O. Sandre-Hernandez, ''Model reference adaptive control for an unmanned aerial vehicle with variable-mass payloads,'' in *Proc. 18th Int. Conf. Electr. Eng., Comput. Sci. Autom. Control (CCE)*, 2021, pp. 1–6.
- <span id="page-17-13"></span>[\[14\]](#page-1-8) R. Burns, *Advanced Control Engineering*. Amsterdam, The Netherlands: Elsevier, 2001.
- <span id="page-17-14"></span>[\[15\]](#page-1-9) T. Oktay, S. Arik, I. Turkmen, M. Uzun, and H. Celik, ''Neural network based redesign of morphing UAV for simultaneous improvement of roll stability and maximum lift/drag ratio,'' *Aircr. Eng. Aerosp. Technol.*, vol. 90, no. 8, pp. 1203–1212, Nov. 2018.
- <span id="page-17-15"></span>[\[16\]](#page-1-10) N. A. Bakshi, ''Model reference adaptive control of quadrotor UAVs: A neural network perspective,'' *Adapt. Robust Control Syst.*, p. 135, Mar. 2018.
- <span id="page-17-16"></span>[\[17\]](#page-1-11) P. Priya and S. S. Kamlu, "Robust control algorithm for drones," in *Aeronautics-New Advances*. IntechOpen, 2022.
- <span id="page-17-17"></span>[\[18\]](#page-1-12) L. E. Romero, D. F. Pozo, and J. A. Rosales, "Quadcopter stabilization by using PID controllers,'' *Maskana*, vol. 5, pp. 175–186, Jan. 2014.
- <span id="page-17-18"></span>[\[19\]](#page-1-13) P. Saraf, M. Gupta, and A. M. Parimi, "A comparative study between a classical and optimal controller for a quadrotor,'' in *Proc. IEEE 17th India Council Int. Conf. (INDICON)*, Dec. 2020, pp. 1–6.
- <span id="page-17-19"></span>[\[20\]](#page-2-1) T. Bierling, ''Comparative analysis of adaptive control techniques for improved robust performance,'' Ph.D. dissertation, Technische Univ. Munchen, 2014.
- <span id="page-17-20"></span>[\[21\]](#page-2-2) A. Assefa, ''Neural network based direct MRAC tech-nique for improving tracking performance for nonlinear pendulum system,'' *J. Informat. Electr. Electron. Eng. (JIEEE)*, vol. 1, no. 2, pp. 1–15, Nov. 2020.
- <span id="page-17-21"></span>[\[22\]](#page-2-3) D. Noble and S. Bhandari, "Neural network based nonlinear model reference adaptive controller for an unmanned aerial vehicle,'' in *Proc. Int. Conf. Unmanned Aircr. Syst. (ICUAS)*, Jun. 2017, pp. 94–103.
- <span id="page-18-0"></span>[\[23\]](#page-2-4) R. Prakash and R. Anita, ''Neuro- PI controller based model reference adaptive control for nonlinear systems,'' *Int. J. Eng., Sci. Technol.*, vol. 3, no. 6, pp. 44–60, Jan. 1970.
- <span id="page-18-1"></span>[\[24\]](#page-3-5) Y. Y. Lv, W. Huang, J. Liu, and Z. F. Peng, "A sliding mode controller of quadrotor based on unit quaternion,'' *Appl. Mech. Mater.*, vols. 536–537, pp. 1087–1092, Apr. 2014.
- <span id="page-18-2"></span>[\[25\]](#page-3-6) A. Hussein and R. Abdallah, "Autopilot design for a quadcopter," Univ. Khartoum, Khartoum, Sudan, Tech. Rep., 2017.
- <span id="page-18-3"></span>[\[26\]](#page-3-7) B. J. Emran and H. Najjaran, ''A review of quadrotor: An underactuated mechanical system,'' *Annu. Rev. Control*, vol. 46, pp. 165–180, Jan. 2018.
- <span id="page-18-4"></span>[\[27\]](#page-3-7) F. Ahmadinejad, J. Bahrami, M. B. Menhaj, and S. S. Ghidary, ''Autonomous flight of quadcopters in the presence of ground effect,'' in *Proc. 4th Iranian Conf. Signal Process. Intell. Syst. (ICSPIS)*, 2018, pp. 217–223.
- <span id="page-18-5"></span>[\[28\]](#page-3-8) H. A. Gonzalez, ''Robust tracking of dynamic targets with aerial vehicles using quaternion-based techniques,'' Ph.D. dissertation, Univ. de Technologie de Compiegne, 2019.
- <span id="page-18-6"></span>[\[29\]](#page-3-9) M. E. Guerrero-Sánchez, H. Abaunza, P. Castillo, R. Lozano, and C. D. García-Beltrán, ''Quadrotor energy-based control laws: A unitquaternion approach,'' *J. Intell. Robotic Syst.*, vol. 88, nos. 2–4, pp. 347–377, Dec. 2017.
- <span id="page-18-7"></span>[\[30\]](#page-3-10) J. Sanwale, P. Trivedi, M. Kothari, and A. Malagaudanavar, ''Quaternionbased position control of a quadrotor unmanned aerial vehicle using robust nonlinear third-order sliding mode control with disturbance cancellation,'' *Proc. Inst. Mech. Eng., G, J. Aerosp. Eng.*, vol. 234, no. 4, pp. 997–1013, Mar. 2020.
- <span id="page-18-8"></span>[\[31\]](#page-3-11) J. Cariño, H. Abaunza, and P. Castillo, ''A fully-actuated quadcopter representation using quaternions,'' *Int. J. Control*, vol. 96, no. 12, pp. 3132–3154, Dec. 2023.
- <span id="page-18-9"></span>[\[32\]](#page-3-11) S. R. Nekoo, J. Á. Acosta, and A. Ollero, "Quaternion-based statedependent differential Riccati equation for quadrotor drones: Regulation control problem in aerobatic flight,'' *Robotica*, vol. 40, no. 9, pp. 3120–3135, Sep. 2022.
- <span id="page-18-10"></span>[\[33\]](#page-4-5) L. Baoying, L. Mingqiu, and Y. Junwei, "Research on pose control of quadrotor UAV based on sliding mode active disturbance rejection,'' in *Proc. 4th Int. Conf. Electron. Inf. Technol. Comput. Eng.*, Nov. 2020, pp. 663–668.
- <span id="page-18-11"></span>[\[34\]](#page-4-5) O. Doukhi, A. R. Fayjie, and D. J. Lee, "Intelligent controller design for quad-rotor stabilization in presence of parameter variations,'' *J. Adv. Transp.*, vol. 2017, pp. 1–10, Jan. 2017.
- <span id="page-18-12"></span>[\[35\]](#page-4-6) D. D. C. Bernard, F. Riccardi, M. Giurato, and M. Lovera, "A dynamic analysis of ground effect for a quadrotor platform,'' *IFAC-PapersOnLine*, vol. 50, no. 1, pp. 10311–10316, Jul. 2017.
- <span id="page-18-13"></span>[\[36\]](#page-4-6) N. Ahmed and M. Chen, "Sliding mode control for quadrotor with disturbance observer,'' *Adv. Mech. Eng.*, vol. 10, no. 7, Jul. 2018, Art. no. 168781401878233.
- <span id="page-18-14"></span>[\[37\]](#page-4-7) J. M. Selfridge and G. Tao, "A multivariable adaptive controller for a quadrotor with guaranteed matching conditions,'' *Syst. Sci. Control Eng.*, vol. 2, no. 1, pp. 24–33, Dec. 2014.
- <span id="page-18-15"></span>[\[38\]](#page-4-8) C. Bensalah, N. K. M'Sirdi, and A. Naamane, ''Full modelling and sliding mode control for a quadrotor UAV in visual servoing task,'' in *Proc. IMAACA*, 2019, pp. 1–11.
- <span id="page-18-16"></span>[\[39\]](#page-4-9) R. Beard, "Quadrotor dynamics and control rev 0.1," Tech. Rep., 2008.
- <span id="page-18-17"></span>[\[40\]](#page-5-3) P. Sanchez-Cuevas, G. Heredia, and A. Ollero, ''Characterization of the aerodynamic ground effect and its influence in multirotor control,'' *Int. J. Aerosp. Eng.*, vol. 2017, pp. 1–17, Aug. 2017.
- <span id="page-18-18"></span>[\[41\]](#page-5-4) P. J. Sanchez-Cuevas, V. Martin, G. Heredia, and A. Ollero, ''Aerodynamic effects in multirotors flying close to obstacles: Modelling and mapping,'' in *Proc. 4th Iberian Robot. Conf.*, vol. 1. Springer, 2020, pp. 63–74.
- <span id="page-18-19"></span>[\[42\]](#page-6-6) Z. Li, X. Ma, and Y. Li, "Model-free control of a quadrotor using adaptive proportional derivative-sliding mode control and robust integral of the signum of the error,'' *Int. J. Adv. Robotic Syst.*, vol. 15, no. 5, Sep. 2018, Art. no. 172988141880088.
- <span id="page-18-20"></span>[\[43\]](#page-7-2) M. T. Matthews and S. Yi, "Model reference adaptive control and neural network based control of altitude of unmanned aerial vehicles,'' in *Proc. SoutheastCon*, 2019, pp. 1–8.
- <span id="page-18-21"></span>[\[44\]](#page-8-1) H. Alimohammadi, B. B. Alagoz, A. Tepljakov, K. Vassiljeva, and E. Petlenkov, ''A NARX model reference adaptive control scheme: Improved disturbance rejection fractional-order PID control of an experimental magnetic levitation system,'' *Algorithms*, vol. 13, no. 8, p. 201, Aug. 2020.
- <span id="page-18-22"></span>[\[45\]](#page-8-2) S. Pankaj, J. S. Kumar, and R. K. Nema, "Comparative analysis of MIT rule and Lyapunov rule in model reference adaptive control scheme,'' *Innov. Syst. Des. Eng.*, vol. 2, no. 4, pp. 154–162, 2011.
- <span id="page-18-23"></span>[\[46\]](#page-8-3) P. Jain and M. J. Nigam, "Design of a model reference adaptive controller using modified MIT rule for a second order system,'' *Advance Electron. Electric Eng.*, vol. 3, no. 4, pp. 477–484, 2013.
- <span id="page-18-24"></span>[\[47\]](#page-9-3) Y. Shin, ''Neural network based adaptive control for nonlinear dynamic regimes,'' Ph.D. dissertation, Georgia Inst. Technol., Atlanta, Georgia, 2005.
- <span id="page-18-25"></span>[\[48\]](#page-11-4) W. R. De Mel, "Artificial neural network based adaptive controller for DC motors,'' Tech. Rep., 2004.
- <span id="page-18-26"></span>[\[49\]](#page-12-5) H. Purnawan and E. B. Purwanto, ''Design of linear quadratic regulator (LQR) control system for flight stability of LSU-05,'' *J. Phys., Conf.*, vol. 890, Sep. 2017, Art. no. 012056.
- <span id="page-18-27"></span>[\[50\]](#page-12-6) K. Ogata, *Modern Control Engineering*, 5th ed. 2010.
- <span id="page-18-28"></span>[\[51\]](#page-15-4) M. R. Naseh and M. Haeri, ''Robust synchronization of chaotic systems using active sliding mode control with minimum control effort,'' *Int. J. Modern Phys. B*, vol. 25, no. 17, pp. 2271–2288, Jul. 2011.
- <span id="page-18-29"></span>[\[52\]](#page-15-4) S. Skogestad, "Tuning for smooth PID control with acceptable disturbance rejection,'' *Ind. Eng. Chem. Res.*, vol. 45, no. 23, pp. 7817–7822, Nov. 2006.



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