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RESEARCH ARTICLE

Decentralized Fuzzy Observer-Based Fault Estimation for Nonlinear Large-Scale Systems

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ABSTRACT This paper presents a fault estimation technique based on the decentralized fuzzy observer for nonlinear large-scale systems, which are considered to be consisted of fuzzy subsystems and uncertain interconnections. Based on a Takagi–Sugeno fuzzy model for the subsystems of the large-scale system and the decentralized fuzzy observer, the H_∞ performance of the fault estimation problem is established by using the estimation error model. By using H_∞ performance inequalities to address the fault estimation problem, the decentralized fuzzy observer design techniques are proposed to guarantee the fault estimation conditions. Also, sufficient conditions of observer design are converted into the linear matrix inequality formats. Finally, an example is provided to verify the effectiveness of the proposed decentralized fuzzy observer design techniques for fault estimation.

INDEX TERMS Decentralized control, estimation error, fault detection, fuzzy systems, large-scale systems, linear matrix inequalities, observers, stability analysis, sufficient conditions, Takagi–Sugeno model.

I. INTRODUCTION

Nowadays, most of the systems considered in various modern industries have very complex structures to represent the characteristics of the systems. Among the various characteristics, nonlinearities and interconnections cause greater difficulties in controlling and observing the systems, and thus these characteristics are recognized as problems to be solved in the control field [1], [2]. In particular, the interconnection problem is known as a very important problem in large-scale systems [3], [4], [5], [6], [7], which are composed of multiple subsystems, because the structural connection and the interaction of information between subsystems have a large effect on the entire large-scale system. Also, in the large-scale systems, the actuator faults or errors can pose many risks to control and operation of the system due to the large and complex system structure. Thus, detecting or estimating the fault signal is essential to ensure the safety and reliability of the large-scale systems [8], [9], [10], [11].

Apart from the large-scale system issue, the fault estimation is one of the technique for Fault Detection and Diagnosis (FDD), and has an advantage to be possible to directly estimate the fault signal. Accordingly, research on the fault estimation technique has received a lot of attention so far, and in particular, many studies on fault estimation techniques for nonlinear systems have been presented by using Takagi–Sugeno (T–S) fuzzy model approach [12]. In [13] and [14], the fault estimation techniques have been represented for nonlinear systems based on fuzzy model with continuous-time and discrete-time cases, respectively. As with [13] and [14], the fuzzy fault estimation studies have been developed to solve various problems, such as time-delay [15], [16], non-measurable premise variable [17], and quantization error [18], [19]. Also, recently, studies on fuzzy fault estimation have been presented: In [20] and [21], the fault estimation problems have been addressed for nonlinear fractional-order systems with unknown inputs

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and discrete-time fuzzy systems, respectively. The above studies have proposed improved fault estimation techniques than existing ones through novel observer structures and design approaches. In [22], the fault estimation observer has been developed for switched fuzzy stochastic systems, which have Brownian motion noise and switched subsystems. The fuzzy fault-tolerant control and fault detection based on the fault estimation have been proposed in [23] and [24], respectively.

One of the recent key research issues in FDD is to address the sampled-data problem. A robust sampled-data fault detecting filter has been developed for linear systems in [25], and a sampled-data fuzzy fault estimation technique has been developed for nonlinear systems in [26], respectively. Furthermore, expanding the research field to sampled-data observation techniques, an observer-based controller is designed for networked control systems to solve the network attack problem based on a interval type-2 fuzzy model in [27], and the sampled-data disturbance and fault diagnosis observers have been developed for interconnected semi-Markovian systems in [28].

Despite the many fuzzy fault estimation studies described above, research on fault estimation for large-scale systems is still very lacking. The reason is that fault estimation studies to date have been mostly applicable to cases where system information is accurately known, but in the large-scale system, it is difficult to know all system information accurately due to the unknown or uncertain interconnection problem. In fact, recent studies for fault estimation of large-scale systems have considered that all system information of interconnection is known, and have applied all system information about interconnection to the fault estimation observer [29], [30], [31], [32]. It means that the unknown or uncertain interconnection problem cannot be solved in [29], [30], [31], [32]. In addition, the above studies [29], [30], [31], [32] have the limitation of dealing the fault estimation for linear large-scale systems. In [33], the fuzzy fault estimation technique has been proposed for large-scale systems, but it has assumed that the system information of all interconnections is still known. Although the various approaches, such as fuzzy observer [34], fuzzy filter [35], [36], fuzzy isolation [37], and fuzzy detection [38], [39], have been presented for nonlinear large-scale systems with uncertain interconnection, the fault estimation technique has not been addressed yet. Especially, to the best of the author's knowledge, the decentralized fuzzy observer-based fault estimation has not been studied for nonlinear large-scale systems with uncertain interconnections so far.

Motivated by the above analysis for the previous techniques, the decentralized fuzzy observer design techniques are proposed for fault estimation of nonlinear large-scale systems with uncertain interconnections in this paper. The subsystems of nonlinear large-scale system are represented to T–S fuzzy model, and the decentralized fuzzy observer is considered based on the fuzzy subsystems. The fault estimation problem is addressed by using the estimation error model between the fuzzy subsystems of the large-scale system and the decentralized fuzzy observer. By using Lyapunov functional approach, some sufficient conditions are developed to design the decentralized fuzzy observer for fault estimation with satisfying both the stability condition and the H_∞ performance condition. In addition, in order to solve easily by a convex optimization tool, the proposed sufficient conditions are converted into various linear matrix inequality (LMI) formats. Finally, a simulation example is provided to prove the validity of the proposed ideas, techniques and procedures by result analysis and comparison.

This paper is organized as follows: Section II describes the fuzzy models of large-scale systems and the decentralized observer and fault estimation problem. The LMI conditions to design the decentralized fuzzy observer for fault estimation are proposed by using Lyapunov functional in Section III. The simulation example is given for illustration and comparison in Section IV. Finally, the conclusions are given in Section V.

Notation: The notations $(\cdot)^T$, He$\{\cdot\}$, and $*$ denote the transpose of the argument, the summation of the element and its transposed element, and the transposed element in symmetric positions, respectively. The subscripts k and l denote the subsystem indices, and subscripts i and j denote fuzzy rule indices. Also, $\mathcal{I}_N$ is a set of integers with $\{1, 2, \cdots, n\}$.

II. PRELIMINARIES

Consider a large-scale system composed of n nonlinear subsystems, which can be described as the T–S fuzzy model with the following fuzzy IF–THEN rules:

Fuzzy Rule i of kth subsystem: :

IF $z_{k1}(t)$ is $\Gamma_{ki1}, \cdots$, and $z_{kq}(t)$ is Γ_{kiq},

$$\text{THEN} \begin{cases} \dot{x}_k(t) = A_{ki}x_k(t) + B_{ki}\omega_k(t) + E_{ki}f_k(t) + h_k(x(t)) \\ y_k(t) = C_{ki}x_k(t) + D_{ki}\omega_k(t) \end{cases} \tag{1}$$

where $z_{kp}(t)$, $k \in \mathcal{I}_{kq}$, is the premise variable, Γ_{kip}, $(k, i, p) \in \mathcal{I}_n \times \mathcal{I}_r \times \mathcal{I}_q$, is a fuzzy set for $z_{kp}(t)$, $x_k(t) \in \mathbb{R}^{n_k}$, $\omega_k(t) \in \mathbb{R}^{m_k}$, and $y_k(t) \in \mathbb{R}^{l_k}$ are the state variable, disturbance, and measurement output of kth subsystem, respectively, $x(t) = \text{col}\{x_1(t), x_2(t), \ldots, x_n(t)\}$ is the whole state variable of the large-scale system, and $f_k(t) \in \mathbb{R}^{s_k}$ is the actuator fault input, which is assumed that the derivative of $f_k(t)$ is norm bounded. Also, A_{ki}, B_{ki}, E_{ki}, C_{ki} and D_{ki} denote nominal system matrices with appropriate dimensions for the ith fuzzy rule of the kth subsystem, and $h_k(x(t))$ is a nonlinear vector function for representing the interconnection of large-scale system and is assumed to satisfy the following Assumption:

Assumption 1: The vector function $h_k(x(t))$ is unknown, but satisfies the following quadratic inequality:

$$\left(h_k(x(t))\right)^T h_k(x(t)) \leq \alpha_k^2 x(t)^T H_k^T H_k x(t) \tag{2}$$

where $\alpha_k > 0$ is a bound scalar of the interconnection term, and H_k is a given constant matrix with appropriate dimension.

Remark 1: In Assumption 1, it is assumed that the unknown interconnection function consists of α_k and H_k. Here, α_k means the maximum bound of the interconnections, and H_k represents the structure of the interconnection. In other words, the higher the value of α_k is, the stronger the degree of interconnections is. Also, when the value of α_k is determined, it indicates that Assumption 1 is always satisfied in the smaller interconnection bound than the value of α_k. In general, there is a need to obtain the maximum interconnection bound α_k with guaranteeing the stability condition or observing performance. However, in the fault estimation case, since the purpose is to optimize the fault estimation performance, it is hard to optimize the maximum interconnection bound α_k. Thus, in this paper, we assume that the maximum interconnection bound α_k is a given scalar, which is often assumed in the decentralized filtering techniques [36], [40].

Using center-average defuzzification, product inference and singleton fuzzifier, IT–THEN rule (1) can be inferred as the following fuzzy subsystem:

$$\begin{aligned}\dot{x}_k(t) &= \sum_{i=1}^{r} \mu_{ki}(z_k(t))\big(A_{ki}x_k(t) + B_{ki}\omega_k(t) + E_{ki}f_k(t)\big)\\ &\quad + h_k(x(t))\\ y_k(t) &= \sum_{i=1}^{r} \mu_{ki}(z_k(t))\big(C_{ki}x(t) + D_{ki}\omega_k(t)\big)\end{aligned} \tag{3}$$

where

$$\mu_{ki}(z_k(t)) = \eta_{ki}(z_k(t)) \Big/ \sum_{i=1}^{r} \eta_{ki}(z_k(t)),$$

$$\eta_{ki}(z_k(t)) = \prod_{p=1}^{q} \Gamma_{kip}(z_{kp}(t))$$

in which $\Gamma_{kip} = \mathcal{U}_{z_{kp}(t)} \subset \mathbb{R} \rightarrow \mathbb{R}_{[0,1]}$ is the membership function of $z_{kp}(t)$ on compact set $\mathcal{U}_{z_{kp}(t)}$.

To design the fuzzy observer for fault estimation, some mathematical assumptions have to be considered as follows:

Assumption 2: In kth subsystem, the state variable $x_k(t)$ is not measurable, but the premise variable $z_k(t)$ and the output variable $y_k(t)$ are measurable.

Assumption 3: Each pair of (A_{ki}, C_{ki}) is observable for $(k, i) \in \mathcal{I}_n \times \mathcal{I}_r$. Also, matrices E_{ki} are a full column rank, and matrices C_{ki} are a full row rank for $(k, i) \in \mathcal{I}_n \times \mathcal{I}_r$.

Based on the fuzzy subsystem (3) and assumptions, we suppose a decentralized fuzzy observer model as follows:

$$\begin{aligned}\dot{\hat{x}}_k(t) &= \sum_{i=1}^{r} \mu_{ki}(z_k(t))\big(A_{ki}\hat{x}_k(t) + E_{ki}\hat{f}_k(t)\\ &\quad + L_{ki}(y_k(t) - \hat{y}_k(t))\big)\\ \hat{y}_k(t) &= \sum_{i=1}^{r} \mu_{ki}(z_k(t))C_{ki}\hat{x}_k(t)\\ \dot{\hat{f}}_k(t) &= \sum_{i=1}^{r} \mu_{ki}(z_k(t))F_{ki}(y_k(t) - \hat{y}_k(t))\end{aligned} \tag{4}$$

where $\hat{x}(t) \in \mathbb{R}^{n_k}$, $\hat{y}_k(t) \in \mathbb{R}^{l_k}$ and $\hat{f}(t) \in \mathbb{R}^{s_k}$ are the state, output and fault to be estimated, respectively, and L_{ki} and F_{ki} are the decentralized fuzzy observer gain matrices with appropriate dimensions.

Remark 2: The dynamic equation of the fault estimation observer (4) for the actuator fault input is presented only using the error of the output, because the output variable is only available for the system (3) and the information about the actuator fault input can be not obtained at all. The structural limitations of the fault estimation observer impose some constraints and conservatism, such as the derivative of $f_k(t)$ has to be norm bounded and the fault estimation error is minimized relative to the derivative of the actuator fault input [13], [15], [16], [29].

Then, substituting (4) into (3) and defining the state observer error $e_{x_k}(t) = x_k(t) - \hat{x}_k(t)$ and the fault estimation error $e_{f_k}(t) = f_k(t) - \hat{f}_k(t)$, the error model is obtained as follows:

$$\begin{aligned}\dot{e}_k(t) &= \big(\tilde{A}_k(t) - \tilde{L}_k(t)\tilde{C}_k(t)\big)e_k(t)\\ &\quad + \big(\tilde{B}_k(t) - \tilde{L}_k(t)\tilde{D}_k(t)\big)v_k(t) + \tilde{h}_k(x(t))\end{aligned} \tag{5}$$

where

$$\tilde{A}_k(t) = \sum_{i=1}^{r} \mu_{ki}(z_k(t)) \begin{bmatrix} A_{ki} & E_{ki} \\ 0 & 0 \end{bmatrix},$$

$$\tilde{L}_k(t) = \sum_{i=1}^{r} \mu_{ki}(z_k(t)) \begin{bmatrix} L_{ki} \\ F_{ki} \end{bmatrix},$$

$$\tilde{C}_k(t) = \sum_{i=1}^{r} \mu_{ki}(z_k(t)) \begin{bmatrix} C_{ki} & 0 \end{bmatrix},$$

$$\tilde{B}_k(t) = \sum_{i=1}^{r} \mu_{ki}(z_k(t)) \begin{bmatrix} B_{ki} & 0 \\ 0 & I \end{bmatrix},$$

$$\tilde{D}_k(t) = \sum_{i=1}^{r} \mu_{ki}(z_k(t)) \begin{bmatrix} D_{ki} & 0 \end{bmatrix},$$

and $e_k(t) = \mathrm{col}\{e_{x_k}(t), e_{f_k}(t)\}$, $v_k(t) = \mathrm{col}\{\omega_k(t), \dot{f}_k(t)\}$, and $\tilde{h}_k(x(t)) = \mathrm{col}\{h_k(x(t)), 0\}$.

From the error model (5), the objective of fault estimation problem can be stated as follows:

Problem 1: Find the observer gain matrices L_{ki} and F_{ki} and minimize a scalar $\gamma > 0$ such that the following H_∞ performance is guaranteed:

1) The equilibrium point of the large-scale system based on error subsystem (5) is asymptotically stable when $v_k(t) = 0$ and $h_k(x(t)) = 0$.
2) The following inequality is satisfied for a scalar γ under zero initial condition:

$$\begin{aligned}&\sum_{k=1}^{n} \int_0^\infty \|e_k(t)\|^2 dt\\ &\quad \le \gamma^2 \sum_{k=1}^{n} \int_0^\infty \Big(\|v_k(t)\|^2 + \|x_k(t)\|^2\Big) dt.\end{aligned} \tag{6}$$

Remark 3: Unlike the previous fault estimation studies, the term of $x_k(t)$ is included in the inequality for H_∞ performance of fault estimation in the second condition of Problem 1. The term of $x_k(t)$ is required, because the information of the uncertain interconnection is not possible to be considered in the decentralized fuzzy observer (4), so the error model (5) still has an uncertain interconnection function $h_k(x(t))$. In papers [34], [36] for designing the fuzzy observer or a filter for large-scale systems with uncertain interconnections, an inequality for H_∞ performance as the form of the inequality (6) has been used. Thus, it is reasonable to use inequality (6) for fault estimation technique. In addition, in this paper, a decentralized fuzzy observer design technique for fault estimation based on an inequality without the term of $x_k(t)$ is also introduced in Corollary 1, and the fault estimation performance of the proposed decentralized fuzzy observers based two inequalities are compared and verified in Section IV.

III. MAIN RESULTS

In this section, the decentralized fuzzy observer design techniques for fault estimation have been proposed for nonlinear large-scale system (3) based on error model (5) with considering Problem 1. Before presenting the main results, following lemma has to be considered for the proof of main results.

Lemma 1 ([41]): Consider the following inequality:

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(z(t))\mu_j(z(t))\Lambda_{ij} < 0$$

where $\mu_i(z(t)) \geq 0$ and $\sum_{i=1}^{r}\mu_i(z(t)) = 1$ and Λ_{ij} is a real symmetric matrix for $(i,j) \in \mathcal{I}_r \times \mathcal{I}_r$. Then, the above inequality is fulfilled by the following conditions:

$$\begin{aligned} &\Lambda_{ii} < 0, && i \in \mathcal{I}_r,\\ &\frac{1}{r-1}\Lambda_{ii} + \frac{1}{2}\big(\Lambda_{ij} + \Lambda_{ji}\big) < 0, && (i,j) \in \mathcal{I}_r \times \mathcal{I}_w, \end{aligned}$$

where $\mathcal{I}_r \times \mathcal{I}_w$ denotes for all pairs $(i,j) \in \mathcal{I}_r \times \mathcal{I}_r$ such that $1 \leq i \neq j \leq r$.

Now, based on the above lemma, the sufficient condition of the decentralized fuzzy observer design for fault estimation of the error model (5) is addressed in the following theorem.

Theorem 1: If there exist some matrices $P_k = P_k^T > 0$ and N_{ki} and some scalars $\rho > 0$, $\sigma_k > 0$ and $\hat{\gamma}$ such that the following LMIs are satisfied:

$$\min \hat{\gamma} \quad \text{subject to}$$

$$\Psi_{kii} < 0, \qquad (k,i) \in \mathcal{I}_n \times \mathcal{I}_r \tag{7}$$

$$\frac{1}{r-1}\Psi_{kii} + \frac{1}{2}\big(\Psi_{kij} + \Psi_{kji}\big) < 0, \qquad (k,i,j) \in \mathcal{I}_n \times \mathcal{I}_r \times \mathcal{I}_w \tag{8}$$

$$-\hat{\gamma}I + n\sigma_l\alpha_l^2 H_{lk}^T H_{lk} < 0 \qquad (k,l) \in \mathcal{I}_n \times \mathcal{I}_n \tag{9}$$

where

$$\Psi_{kij} = \begin{bmatrix} \mathrm{He}\{P_k\tilde{A}_{ki} - N_{ki}\tilde{C}_{kj}\} + \rho I & P_k\tilde{B}_{ki} - N_{ki}\tilde{D}_{kj} & P_k \\ * & -\hat{\gamma}I & 0 \\ * & * & -\sigma_k I \end{bmatrix},$$

$$\tilde{A}_{ki} = \begin{bmatrix} A_{ki} & E_{ki} \\ 0 & 0 \end{bmatrix}, \qquad \tilde{B}_{ki} = \begin{bmatrix} B_{ki} & 0 \\ 0 & I \end{bmatrix},$$

$$\tilde{C}_{ki} = \begin{bmatrix} C_{ki} & 0 \end{bmatrix}, \qquad \tilde{D}_{ki} = \begin{bmatrix} D_{ki} & 0 \end{bmatrix}$$

and α_l is a given positive scaler for interconnection bound, and H_{lk} is the submatrix having n_k columns of H_l from the ν_kth column vector for $\nu_k = n_1 + n_2 + \cdots + n_{k-1} + 1$, then the decentralized fuzzy observer (4) guarantees the H_∞ fault estimation performance for a large-scale system based on the fuzzy subsystem (3), and γ is a minimum H_∞ performance value of fault estimation. In addition, the decentralized fuzzy filter gain matrices are given by $\mathrm{col}\{L_{ki}, F_{ki}\} = P_k^{-1}N_{ki}$, and $\gamma = \sqrt{\hat{\gamma}/\rho}$ is a minimum H_∞ performance value of fault estimation.

Proof: First, to obtain the sufficient condition for satisfying the first condition of Problem 1, we consider a Lyapunov function candidate with $v_k(t) = 0$ and $h_k(x(t)) = 0$ as follows:

$$V(t) = \sum_{k=1}^{n} V_k(e_k(t)) = \sum_{k=1}^{n} e_k(t)^T P_k e_k(t)$$

where $P_k = P_k^T > 0$. Then, the first time derivative of the Lyapunov functional candidate becomes

$$\dot{V}(t) = \sum_{k=1}^{n}\big(\dot{e}_k(t)^T P_k e_k(t) + e_k(t)^T P_k \dot{e}_k(t)\big). \tag{10}$$

Due to $v_k(t) = 0$ and $h_k(x(t)) = 0$, the following equation can be obtained by substituting (3) and (5) into (10):

$$\begin{aligned}\dot{V}(t) = \sum_{k=1}^{n}\Big(\Big(\big(\tilde{A}_k(t) - \tilde{L}_k(t)\tilde{C}_k(t)\big)e_k(t)\Big)^T P_k e_k(t) \\ + e_k(t)^T P_k\Big(\big(\tilde{A}_k(t) - \tilde{L}_k(t)\tilde{C}_k(t)\big)e_k(t)\Big)\Big). \end{aligned} \tag{11}$$

Thus, if the inequality $\mathrm{He}\{P_k(\tilde{A}_k(t) - \tilde{L}_k(t)\tilde{C}_k(t))\} < 0$, which is guaranteed by LMIs (7) and (8), is satisfied, then the first time derivative of $V(t)$ is majorized by $\dot{V}(t) < 0$.

Now, we establish the H_∞ performance criteria to solve the second condition of Problem 1 based on the error system (5) with a zero initial condition by using the following inequality:

$$\begin{aligned}\mathcal{J} = \dot{V}(t) + \rho\sum_{k=1}^{n}\big(e_k(t)^T e_k(t) - \gamma^2 v_k(t)^T v_k(t) \\ - \gamma^2 x_k(t)^T x_k(t)\big) \end{aligned} \tag{12}$$

where ρ is a positive scalar.

Then, by substituting (5) into (12), we have

$$\begin{aligned}\mathcal{J} = \sum_{k=1}^{n}\Big(\Big(\big(\tilde{A}_k(t) - \tilde{L}_k(t)\tilde{C}_k(t)\big)e_k(t) \\ + \big(\tilde{B}_k(t) - \tilde{L}_k(t)\tilde{D}_k(t)\big)v_k(t) + \tilde{h}_k(x(t))\Big)^T P_k e_k(t)\end{aligned}$$

$$
\begin{aligned}
&+ e_k(t)^T P_k\Big(\big(\tilde{A}_k(t)-\tilde{L}_k(t)\tilde{C}_k(t)\big)e_k(t)\\
&+ \big(\tilde{B}_k(t)-\tilde{L}_k(t)\tilde{D}_k(t)\big)v_k(t)+\tilde{h}_k(x(t))\Big)\Big)\\
&+ \rho\sum_{k=1}^{n}\big(e_k(t)^T e_k(t)-\gamma^2 v_k(t)^T v_k(t)-\gamma^2 x_k(t)^T x_k(t)\big)\\
\le& \sum_{k=1}^{n}\bigg(\Big(\big(\tilde{A}_k(t)-\tilde{L}_k(t)\tilde{C}_k(t)\big)e_k(t)\\
&+ \big(\tilde{B}_k(t)-\tilde{L}_k(t)\tilde{D}_k(t)\big)v_k(t)\Big)^T P_k e_k(t)\\
&+ e_k(t)^T P_k\Big(\big(\tilde{A}_k(t)-\tilde{L}_k(t)\tilde{C}_k(t)\big)e_k(t)\\
&+ \big(\tilde{B}_k(t)-\tilde{L}_k(t)\tilde{D}_k(t)\big)v_k(t)\Big)\Big)\\
&+ \tilde{h}_k(x(t))^T P_k e_k(t)+e_k(t)^T P_k\tilde{h}_k(x(t))\Big)\\
&+ \rho\sum_{k=1}^{n}\big(e_k(t)^T e_k(t)-\gamma^2 v_k(t)^T v_k(t)-\gamma^2 x_k(t)^T x_k(t)\big)\\
\le& \sum_{k=1}^{n}\bigg(\Big(\big(\tilde{A}_k(t)-\tilde{L}_k(t)\tilde{C}_k(t)\big)e_k(t)\\
&+ \big(\tilde{B}_k(t)-\tilde{L}_k(t)\tilde{D}_k(t)\big)v_k(t)\Big)^T P_k e_k(t)\\
&+ e_k(t)^T P_k\Big(\big(\tilde{A}_k(t)-\tilde{L}_k(t)\tilde{C}_k(t)\big)e_k(t)\\
&+ \big(\tilde{B}_k(t)-\tilde{L}_k(t)\tilde{D}_k(t)\big)v_k(t)\Big)\\
&+ \sigma_k^{-1}e_k(t)^T P_k^2 e_k(t)+\rho e_k(t)^T e_k(t)-\rho\gamma^2 v_k(t)^T v_k(t)\bigg)\\
&+ \sum_{k=1}^{n}\Big(-\rho\gamma^2 x_k(t)^T x_k(t)+\sigma_k\tilde{h}_k(x(t))^T\tilde{h}_k(x(t))\Big)
\end{aligned}
\tag{13}
$$

where σ_k is a positive scalar. Also, from (2) of Assumption 1, the following inequality is satisfied:

$$
\begin{aligned}
\sum_{k=1}^{n}\tilde{h}_k(x(t))^T\tilde{h}_k(x(t)) &\le \sum_{k=1}^{n}\alpha_k^2 x(t)^T H_k^T H_k x(t)\\
&= \sum_{k=1}^{n}\sum_{l=1}^{n}\alpha_k^2 x_l(t)^T H_{kl}^T H_{kl} x_l(t)\\
&= \sum_{k=1}^{n}\sum_{l=1}^{n}\alpha_l^2 x_k(t)^T H_{lk}^T H_{lk} x_k(t)
\end{aligned}
\tag{14}
$$

where $H_k = [H_{k1}\;\; H_{k2}\;\; \cdots\;\; H_{kn}]$ and H_{kl} has n_l columns.

Applying the inequality (14) into (13) yields

$$
\begin{aligned}
\mathcal{J} \le& \sum_{k=1}^{n}\bigg(\Big(\big(\tilde{A}_k(t)-\tilde{L}_k(t)\tilde{C}_k(t)\big)e_k(t)\\
&+ \big(\tilde{B}_k(t)-\tilde{L}_k(t)\tilde{D}_k(t)\big)v_k(t)\Big)^T P_k e_k(t)\\
&+ e_k(t)^T P_k\Big(\big(\tilde{A}_k(t)-\tilde{L}_k(t)\tilde{C}_k(t)\big)e_k(t)\\
&+ \big(\tilde{B}_k(t)-\tilde{L}_k(t)\tilde{D}_k(t)\big)v_k(t)\Big)\\
&+ \sigma_k^{-1}e_k(t)^T P_k^2 e_k(t)+\rho e_k(t)^T e_k(t)-\rho\gamma^2 v_k(t)^T v_k(t)\bigg)\\
&+ \sum_{k=1}^{n}\sum_{l=1}^{n}\Big(-\frac{1}{n}\rho\gamma^2 x_k(t)^T x_k(t)\\
&+ \sigma_l\alpha_l^2 x_k(t)^T H_{lk}^T H_{lk} x_k(t)\Big)\\
=& \sum_{k=1}^{n}\begin{bmatrix} e_k(t)\\ v_k(t)\end{bmatrix}^T\begin{bmatrix}\Psi_k^{11}(t) & \Psi_k^{12}(t)\\ * & -\rho\gamma^2 I\end{bmatrix}\begin{bmatrix} e_k(t)\\ v_k(t)\end{bmatrix}\\
&+ \frac{1}{n}\sum_{k=1}^{n}\sum_{l=1}^{n}x_k(t)^T\big(-\rho\gamma^2 I+n\sigma_l\alpha_l^2 H_{lk}^T H_{lk}\big)x_k(t)
\end{aligned}
\tag{15}
$$

where $\Psi_k^{11}(t) = \mathrm{He}\{P_k(\tilde{A}_k(t)-\tilde{L}_k(t)\tilde{C}_k(t))\}+\sigma_k^{-1}P_k^2+\rho I$ and $\Psi_k^{12}(t) = P_k(\tilde{B}_k(t)-\tilde{L}_k(t)\tilde{D}_k(t))$.

Thus, if the following inequalities are satisfied

$$
\begin{bmatrix}\Psi_k^{11}(t) & \Psi_k^{12}(t)\\ * & -\rho\gamma^2 I\end{bmatrix} < 0, \tag{16}
$$

$$
-\rho\gamma^2 I + n\sigma_l\alpha_l^2 H_{lk}^T H_{lk} < 0, \tag{17}
$$

then $\mathcal{J} < 0$ is satisfied. Furthermore, by using Schur complement to inequality (16), applying Lemma 1 and denoting $P_k\times\mathrm{col}\{L_{ki}, F_{ki}\} = N_{ki}$ and $\rho\gamma^2 = \hat{\gamma}$, the condition of $\mathcal{J} < 0$ is majorized as LMIs (7), (8) and (9).

Also, by integrating (12) from 0 to ∞, we obtain

$$
\begin{aligned}
&\int_0^\infty \mathcal{J}dt\\
&= V(\infty)-V(0)+\rho\bigg(\sum_{k=1}^{n}\int_0^\infty e_k^T(t)e_k(t)dt\\
&\quad -\gamma^2\sum_{k=1}^{n}\int_0^\infty\big(v_k(t)^T(t)v_k(t)+x_k(t)^T x_k(t)\big)\bigg)dt\\
&< 0.
\end{aligned}
\tag{18}
$$

Finally, from $V(0) = 0$ by zero initial condition and $V(\infty) \ge 0$, if LMIs (7), (8) and (9) are satisfied, then H_∞ performance (6) is guaranteed from the result of (18). Thus, by inequalities (7), (8) and (9), we can guarantee both the stability condition of large-scale system without fault, disturbance and interconnection and H_∞ performance of the estimation error model based on (5).

Remark 4: In Theorem 1, the LMIs have to be satisfied with a given maximum interconnection bound α_k. It means that the proposed decentralized fuzzy observer designed by Theorem 1 guarantees the H_∞ fault estimation performance for (5), even if the interconnection bound has any value below the given maximum interconnection bound α_k.

Remark 5: In Problem 1, γ has to be minimized for H_∞ performance of the fault estimation, but the objective of LMIs of Theorem 1 is that $\hat{\gamma}$ is minimized. Here, $\hat{\gamma} = \rho\gamma^2$ is defined by two variable γ and ρ, Thus, minimizing $\hat{\gamma}$ in LMIs (7), (8) and (9) may not minimize γ, because the value of ρ can be also changed by LMIs. To solve this problem, a value of ρ is given in advance. Then, γ is possible to be minimized by minimizing $\hat{\gamma}$.

Theorem 1 shows the decentralized fuzzy observer design technique for fault estimation of large-scale systems. Also, the LMIs of Theorem 1 are converged the sufficient conditions of fault estimation fuzzy observer design for the fuzzy system without considering the interconnections. This convergence is shown in the following theorem:

Theorem 2: The LMIs (7), (8) and (9) of Theorem 1 can be the sufficient conditions for fault estimation fuzzy observer design of the general fuzzy system such as followings:

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(z(t))\big(A_i x(t) + B_i\omega(t) + E_i f(t)\big)$$
$$y(t) = \sum_{i=1}^{r} \mu_i(z(t))\big(C_i x(t) + D_i\omega(t)\big) \tag{19}$$

Proof: When $h_k(t) = 0$, we can suppose $\alpha_k = 0$ in Assumption 1. Thus, the LMI (9) is always satisfied for any $\hat{\gamma} > 0$. Next, considering $\rho = 1$ and a sufficiently large value of $\sigma_k > 0$, the LMIs (7) and (8) are fulfilled by the following inequality:

$$\sum_{i=1}^{r}\sum_{j=1}^{r} \mu_{ki}(z_k(t))\mu_{kj}(z_k(t)) \times \begin{bmatrix} \mathrm{He}\{P_k\tilde{A}_{ki} + P_k L_{ki}\tilde{C}_{kj}\} + \rho I & P_k\tilde{D}_{ki} \\ * & -\gamma^2 I \end{bmatrix} < 0 \tag{20}$$

Here, with ignoring the subscripts k and l, the inequality (20) is the sufficient condition to guarantee the H_∞ fault estimation of the fuzzy system (19) via Lyapunov functional $V(e(t)) = e(t)^T P e(t)$ and $\mathcal{J} = \dot{V}(e(t)) + e(t)^T e(t) - \gamma^2 v(t)^T v(t))$. Thus, when considering $h_k(t) = 0$, LMIs (7), (8) and (9) can be converged the sufficient conditions of the fault estimation fuzzy observer design for general fuzzy systems.

Remark 6: Theorem 2 shows that the proposed decentralized fuzzy observer for fault estimation converges the fault estimation fuzzy observer for general fuzzy systems. It means that the H_∞ fault estimation performance of Theorem 1 can be improved as the value of the interconnection bound α_k is decreased. The change of the H_∞ fault estimation performance according to the value of α_k is verified in Section IV.

To conquer the limitation of including the term of $x_k(t)$ in the inequality (6) for H_∞ performance of Problem 1, a novel decentralized fuzzy observer design technique for fault estimation is introduced in the following corollary:

Corollary 1: If there exist some matrices $P_k = P_k^T > 0$, $Q_k = Q_k^T > 0$ and N_{ki} and some scalars $\rho > 0$ and $\hat{\gamma}$ such that the following inequalities are satisfied:

$$\min \hat{\gamma} \quad \text{subject to}$$
$$\Phi_{klii} < 0, \qquad (k,l,i) \in \mathcal{I}_n \times \mathcal{I}_n \times \mathcal{I}_r, \tag{21}$$
$$\frac{1}{r-1}\Phi_{klii} + \frac{1}{2}\big(\Phi_{klij} + \Phi_{klji}\big) < 0, \qquad (k,l,i,j) \in \mathcal{I}_n \times \mathcal{I}_n \times \mathcal{I}_r \times \mathcal{I}_q, \tag{22}$$

where

$$\Phi_{klij} = \begin{bmatrix} \Phi_{kij}^{11} & \Phi_{kij}^{12} & \mathcal{P}_k & \tilde{H}_{lk}^T \\ * & -\hat{\gamma} I & 0 & 0 \\ * & * & -\sigma_k I & 0 \\ * & * & * & -\frac{1}{2n\alpha_l^2}\sigma_k^{-1} I \end{bmatrix},$$
$$\Phi_{kij}^{11} = \mathrm{diag}\{\mathrm{He}\{Q_k A_{ki}\}, \mathrm{He}\{P_k\tilde{A}_{ki} - N_{ki}\tilde{C}_{kj}\} + \rho I\},$$
$$\Phi_{kij}^{12} = \begin{bmatrix} Q_k\hat{B}_{ki} & Q_k E_{ki} \\ P_k\tilde{B}_{ki} - N_{ki}\tilde{C}_{kj} & 0 \end{bmatrix},$$
$$\mathcal{P}_k = \mathrm{diag}\{Q_k, P_k\},$$
$$\tilde{H}_{lk}^T = \mathrm{col}\{H_{lk}^T, 0\},$$
$$\hat{B}_{ki} = \begin{bmatrix} B_{ki} & 0 \end{bmatrix}$$

and σ_k and α_l are given positive scalars, then the decentralized fuzzy observer (4) guarantees the H_∞ fault estimation performance for a large-scale system based on the fuzzy subsystem (3). In addition, the decentralized fuzzy observer gain matrices are given by $\mathrm{col}\{L_{ki}, F_{ki}\} = P_k^{-1}N_{ki}$, and $\gamma = \sqrt{\hat{\gamma}/\rho}$ is a minimum H_∞ performance value of fault estimation.

Proof: From fuzzy systems based on (3) and error model (5), we consider

$$\dot{\chi}_k(t) = \Xi_k(t)\chi_k(t) + M_k(t)\upsilon_k(t) + \hat{h}_k(x(t)) \tag{23}$$

where

$$\Xi_k(t) = \begin{bmatrix} A_k(t) & 0 \\ 0 & \tilde{A}_k(t) - \tilde{L}_k(t)\tilde{C}_k(t) \end{bmatrix},$$
$$M_k(t) = \begin{bmatrix} \hat{B}_k(t) & E_k(t) \\ \tilde{B}_k(t) - \tilde{L}_k(t)\tilde{D}_k(t) & 0 \end{bmatrix},$$
$$A_k(t) = \sum_{i=1}^{r} \mu_{ki}(z_k(t))A_{ki},$$
$$\hat{B}_k(t) = \sum_{i=1}^{r} \mu_{ki}(z_k(t))\hat{B}_{ki},$$
$$E_k(t) = \sum_{i=1}^{r} \mu_{ki}(z_k(t))E_{ki}$$

and $\chi_k(t) = \mathrm{col}\{x_k(t), e_k(t)\}$, $\upsilon_k(t) = \mathrm{col}\{v_k(t), f_k(t)\}$, and $\hat{h}_k(x(t)) = \mathrm{col}\{h_k(x(t)), \tilde{h}_k(x(t))\}$.

Then, based on the model (23), we firstly consider a Lyapunov function candidate with $\upsilon_k(t) = 0$ and $h_k(x(t)) = 0$ as follows:

$$V(t) = \sum_{k=1}^{n} V_k(\chi_k(t)) = \sum_{k=1}^{n} \chi_k(t)^T \mathcal{P}_k \chi_k(t)$$

where $\mathcal{P}_k = \mathcal{P}_k^T > 0$. Then, the first time derivative of the Lyapunov functional candidate becomes

$$\begin{aligned}\dot{V}(t) &= \sum_{k=1}^{n}\left(\dot{\chi}_k(t)^T\mathcal{P}_k\chi_k(t) + \chi_k(t)^T\mathcal{P}_k\dot{\chi}_k(t)\right)\\ &= \sum_{k=1}^{n}\left(\left(\Xi_k(t)\chi_k(t)\right)^T\mathcal{P}_k\chi_k(t) + \chi_k(t)^T\mathcal{P}_k\left(\Xi_k(t)\chi_k(t)\right)\right)\end{aligned} \tag{24}$$

Thus, if the inequality $\mathrm{He}\{\mathcal{P}_k\Xi_k(t)\} < 0$, which is guaranteed by LMIs (21) and (22) by defining $\mathcal{P}_k = \mathrm{diag}\{Q_k, P_k\}$ with $Q_k = Q_k^T > 0$ and $P_k = P_k^T > 0$ without the loss of generality, is satisfied, then the first time derivative of $V(t)$ is majorized by $\dot{V}(t) < 0$.

Also, we establish the H_∞ performance criteria with a zero initial condition by using the following inequality:

$$\mathcal{J} = \dot{V}(t) + \rho\sum_{k=1}^{n}\left(e_k(t)^Te_k(t) - \gamma^2\upsilon_k(t)^T\upsilon_k(t)\right) \tag{25}$$

where ρ is a positive scalar. Now, through a similar procedure of Theorem 1, we obtain the following inequality:

$$\begin{aligned}\mathcal{J} \le \sum_{k=1}^{n}&\Big(\left(\Xi_k(t)\chi_k(t) + M_k(t)\upsilon_k(t)\right)^T\mathcal{P}_k\chi_k(t)\\ &+ \chi_k(t)^T\mathcal{P}_k\left(\Xi_k(t)\chi_k(t) + M_k(t)\upsilon_k(t)\right)\\ &+ \sigma_k^{-1}\chi_k(t)^TP_k^2\chi_k(t) + \rho e_k(t)^Te_k(t) - \rho\gamma^2 v_k(t)^Tv_k(t)\Big)\\ &+ \sum_{k=1}^{n}\sum_{l=1}^{n}2\sigma_l\alpha_l^2\chi_k(t)^T\hat{H}_{lk}^T\hat{H}_{lk}\chi_k(t)\\ &= \frac{1}{n}\sum_{k=1}^{n}\sum_{l=1}^{n}\begin{bmatrix}\chi_k(t)\\ \upsilon_k(t)\end{bmatrix}^T\begin{bmatrix}\Phi_{kl}(t) & P_kM_k(t)\\ * & -\rho\gamma^2I\end{bmatrix}\begin{bmatrix}\chi_k(t)\\ \upsilon_k(t)\end{bmatrix}\end{aligned}$$

where $\Phi_{kl}(t) = \mathrm{He}\{P_k\Xi_k(t)\} + \sigma_k^{-1}P_k^2 + \rho\hat{I} + 2n\sigma_l\alpha_l^2\hat{H}_{lk}^T\hat{H}_{lk}$ and $\hat{H}_{lk} = \begin{bmatrix}H_{lk} & 0\end{bmatrix}$ and $\hat{I} = \mathrm{diag}\{0, I\}$.

Thus, if the following inequalities are satisfied

$$\begin{bmatrix}\mathrm{He}\{P_k\Xi_k(t)\} + \sigma_k^{-1}P_k^2 + \rho\hat{I} + 2n\sigma_l\alpha_l^2\hat{H}_{lk}^T\hat{H}_{lk} & P_kM_k(t)\\ * & -\rho\gamma^2I\end{bmatrix} < 0 \tag{26}$$

then $\mathcal{J} < 0$ is satisfied. In addition, by defining $\mathcal{P}_k = \mathrm{diag}\{Q_k, P_k\}$, using Schur complement to inequality (26), applying Lemma 1 and denoting $P_k \times \mathrm{col}\{L_{ki}, F_{ki}\} = N_{ki}$ and $\rho\gamma^2 = \hat{\gamma}$, the condition of $\mathcal{J} < 0$ is majorized as inequalities (7) and (8).

Finally, by integrating (25) from 0 to ∞, we obtain the following condition from zero initial condition

$$\sum_{k=1}^{n}\int_0^\infty e_k^T(t)e_k(t)dt \le \gamma^2\sum_{k=1}^{n}\int_0^\infty \upsilon_k(t)^T\upsilon_k(t)dt. \tag{27}$$

Thus, we can guarantee both the stability condition of large-scale system without fault, disturbance and interconnection and H_∞ performance for fault estimation.

Remark 7: In Corollary 1, the decentralized fuzzy observer for fault estimation is designed through the inequality $\sum_{k=1}^{n}\int_0^\infty e_k^T(t)e_k(t)dt \le \gamma^2\sum_{k=1}^{n}\int_0^\infty \upsilon_k(t)^T\upsilon_k(t)dt$, not the inequality (6). However, in order to satisfy the LMIs (21) and (22) of Corollary 1, the satisfaction of inequality $A_{ki}Q_k + Q_kA_{ki}$ is essential, which means that all subsystems of the large-scale system has to be asymptotically stable. Also, LMIs of Corollary 1 are more conservative than LMIs of Theorem 1, because the value of σ_k has to be given in Corollary 1. Thus, Theorem 1 is possible to have better performance than Corollary 1. The performance difference is confirmed in Section IV through a comparison of two techniques.

Remark 8: The major contribution of this paper is addressed as follows:

- The decentralized fuzzy observer is designed for fault estimation of nonlinear large-scale systems with uncertain interconnections, which has not been studied so far, to the best of the author's knowledge.
- The H_∞ performance for fault estimation is guaranteed by using two different inequalities, and two approaches have been compared theoretically.
- It is shown that the proposed decentralized fuzzy observer design technique converges to the fault estimation fuzzy observer design of a general fuzzy system as the maximum interconnection bound α_k becomes smaller.

IV. NUMERICAL EXAMPLE

In this section, we provide simulation results to demonstrate the effectiveness of the proposed decentralized fuzzy observer design techniques for fault estimation. We consider the simulation example [35] as an interconnected mass-spring-damper mechanical system composed of two subsystems and connected by a spring. Also, the nonlinear large-scale system dynamic equation is addressed as follows:

$$\begin{aligned}&m_k\ddot{\theta}_k(t) + d_k(\dot{\theta}_k(t))\dot{\theta}_k(t) + \kappa_k\theta_k(t) + h_{kl}(\theta(t))\\ &\quad = \omega_k(t) + f_k(t)\\ &y_k(t) = \dot{\theta}_k(t) + \omega_k(t)\end{aligned}$$

where $(k, l) \in \mathcal{I}_2 \times \mathcal{I}_2$. Also, $\theta_k(t)$ and $\dot{\theta}_k(t)$, which are the state variables of mass-spring-damper mechanical system, are the relative position and velocity of the mass in the kth subsystem, respectively, $\theta(t) = [\theta_1(t)^T \;\; \theta_2(t)^T]^T$ and $y_k(t)$ is the measured output. The parameter m_k is the masses of kth subsystem with $m_1 = m_2 = 1kg$, the parameter κ_k is the stiffness of the springs of kth subsystem with $\kappa_1 = 0.2\ N/m$ and $\kappa_1 = 0.3\ N/m$, the function $d_k(\dot{\theta}_k(t))$ means the damping coefficients of the nonlinear dampers as $d_k(\dot{\theta}_k(t)) = d_{k_1} + d_{k_2}\dot{\theta}_k(t)^2$ with $d_{1_1} = 0.6\ N\cdot s/m$, $d_{1_2} = 0.8\ N\cdot s/m$, $d_{2_1} = 0.5\ N\cdot s/m$ and $d_{2_2} = 0.7\ N\cdot s/m$, the function $h_{kl}(\theta(t)) = \kappa(\theta_k(t) - \theta_l(t))$, which is the interconnection function for two subsystems connecting by uncertain spring constant κ. and the functions $\omega_k(t)$ and $f_k(t)$ means disturbance and actuator fault input of kth subsystem, respectively.

Then, by considering $\dot{\theta}_k(t) \in [-\Omega_k \quad \Omega_k]$ with $\Omega_1 = \Omega_2 = 1$ and defining $\theta_k(t) = x_{k_1}(t)$ and $\dot{\theta}_k(t) = x_{k_2}(t)$, the large-scale fuzzy system composed of two subsystems can be represented as follows:

$$\begin{aligned}\dot{x}_k(t) &= \sum_{i=1}^{2} \mu_{ki}(\dot{\theta}_k(t))\big(A_{ki}x_k(t) + B_{ki}\omega_k(t) + E_{ki}f_k(t)\big)\\ &\quad + \kappa H_k x(t)\\ y_k(t) &= \sum_{i=1}^{2} \mu_{ki}(\dot{\theta}_k(t))\big(C_{ki}x_k(t) + D_{ki}w_k(t)\big)\end{aligned}$$

where

$$\begin{aligned}A_{k1} &= \begin{bmatrix} 0 & 1 \\ -\kappa_k/m_k & -d_{k1}/m_k \end{bmatrix},\\ A_{k2} &= \begin{bmatrix} 0 & 1 \\ -\kappa_k/m_k & -d_{k1}/m_k - d_{k2}(\Omega_k)^2 \end{bmatrix},\\ B_{ki} &= \begin{bmatrix} 0 \\ 1/m_k \end{bmatrix}, \quad E_{ki} = \begin{bmatrix} 0 \\ 1/m_k \end{bmatrix},\\ C_{ki} &= \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad D_{ki} = 1,\\ H_1 &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1/m_1 & 0 & 1/m_1 \end{bmatrix},\\ H_2 &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1/m_2 & 0 & -1/m_2 \end{bmatrix},\\ \mu_{k1}(\dot{\theta}_k(t)) &= 1 - \frac{(\dot{\theta}_k(t))^2}{(\Omega_k)^2},\\ \mu_{k2}(\dot{\theta}_k(t)) &= 1 - \mu_{k1}(\dot{\theta}_k(t))\end{aligned}$$

for $(k, i) \in \mathcal{I}_2 \times \mathcal{I}_2$ and the maximum interconnection bound is considered as $\alpha_k = \kappa = 0.02$, which means that the decentralized fuzzy observer satisfies the H_∞ performance for fault estimation, even if the interconnection bound has any value below $0.02\ N/m$.

Now, by using the MATLAB LMI Control Toolbox with FEASP and RK4 algorithm with sampling time $T = 1 \times 10^{-4}s$, the decentralized fuzzy observer gains are obtained from (7), (8) and (9) of Theorem 1 with supposing $\rho_k = 0.001$ as follows:

$$\begin{aligned}L_{11}^{Thm.1} &= \begin{bmatrix} 7.3725 \\ 17.2448 \end{bmatrix}, & L_{12}^{Thm.1} &= \begin{bmatrix} 7.3458 \\ 17.1516 \end{bmatrix},\\ L_{21}^{Thm.1} &= \begin{bmatrix} 7.5078 \\ 17.9646 \end{bmatrix}, & L_{22}^{Thm.1} &= \begin{bmatrix} 7.4871 \\ 17.8866 \end{bmatrix},\\ F_{11}^{Thm.1} &= 65.1269, & F_{12}^{Thm.1} &= 64.8237,\\ F_{21}^{Thm.1} &= 65.6381, & F_{22}^{Thm.1} &= 65.4073\end{aligned}$$

and the minimum value of γ for H_∞ performance is 0.002. Next, from LMIs (21) and (22) of Corollary 1, the decentralized fuzzy observer gains of (4) is also obtained as follows:

$$\begin{aligned}L_{11}^{Cor.1} &= \begin{bmatrix} 1.1959 \\ 10.1241 \end{bmatrix}, & L_{12}^{Cor.1} &= \begin{bmatrix} 2.0342 \\ 10.1789 \end{bmatrix},\\ L_{21}^{Cor.1} &= \begin{bmatrix} 11.4999 \\ 35.7307 \end{bmatrix}, & L_{22}^{Cor.1} &= \begin{bmatrix} 23.6586 \\ 62.5225 \end{bmatrix},\end{aligned}$$

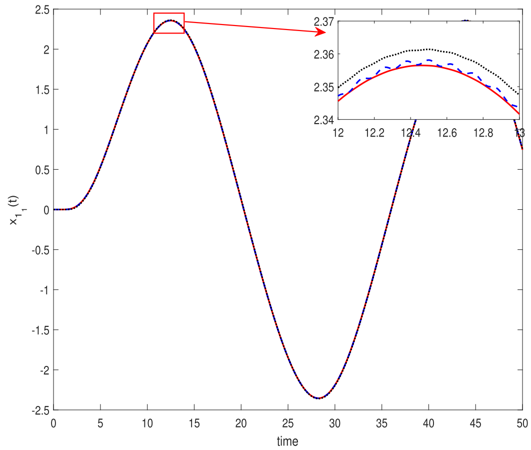

FIGURE 1. The time responses of first state variable for first subsystem: $x_{1_1}(t)$ (solid), $\hat{x}_{1_1}(t)$ of Theorem 1 (dashed) and $\hat{x}_{1_1}(t)$ of Corollary 1 (dotted).

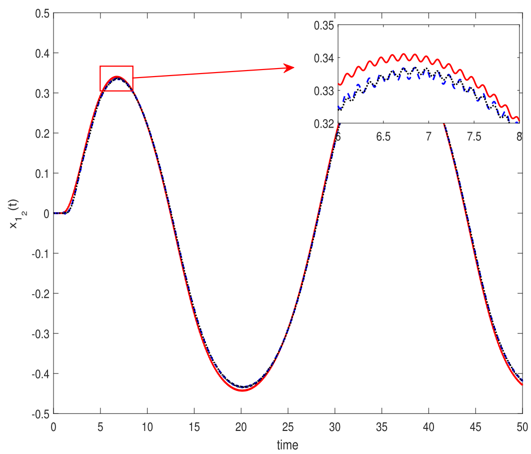

FIGURE 2. The time responses of second state variable for first subsystem: $x_{1_2}(t)$ (solid), $\hat{x}_{1_2}(t)$ of Theorem 1 (dashed) and $\hat{x}_{1_2}(t)$ of Corollary 1 (dotted).

$$\begin{aligned}F_{11}^{Cor.1} &= 10.5221, & F_{12}^{Cor.1} &= 17.7412,\\ F_{21}^{Cor.1} &= 61.1944, & F_{22}^{Cor.1} &= 125.4574.\end{aligned}$$

and the minimum value of γ for H_∞ performance is 0.12.

To show the performance of proposed fault estimation observers, we consider the disturbance and the actuator fault input, respectively:

$$\begin{aligned}\omega_k(t) &= 0.05\sin(50t),\\ f_k(t) &= \begin{cases} 0, & t \le 1 \\ 0.5\sin(0.2(t-1)), & 1 < t \end{cases}\end{aligned}$$

Then, the time responses of each state variable of subsystems and estimated state variable of decentralized fuzzy observers are shown in Figs. 1, 2, 3 and 4, and the time responses of each actuator fault input and estimated fault are shown in Figs. 5 and 6, respectively. As shown in figures, it can be seen that both proposed decentralized fuzzy observers have well estimated the state variables and fault inputs. In particular,

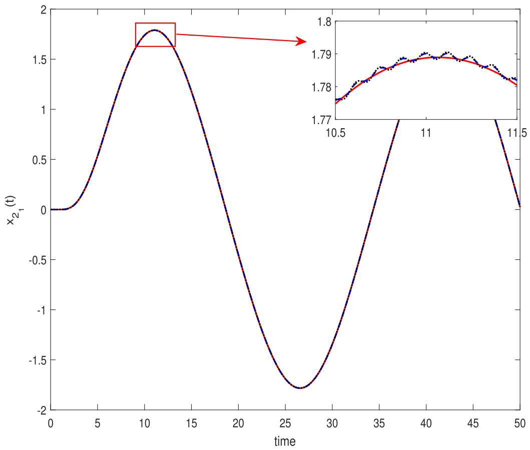

FIGURE 3. The time responses of first state variable for second subsystem: $x_{2_1}(t)$ (solid), $\hat{x}_{2_1}(t)$ of Theorem 1 (dashed) and $\hat{x}_{2_1}(t)$ of Corollary 1 (dotted).

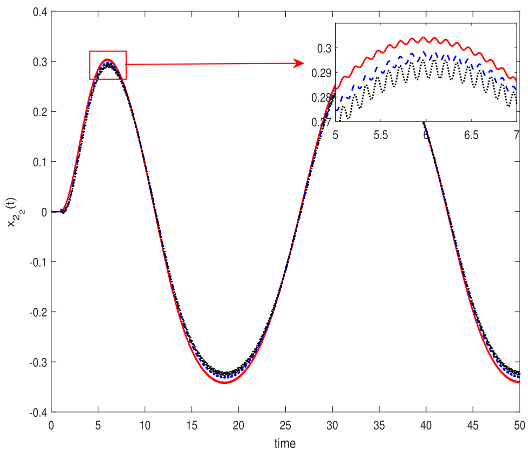

FIGURE 4. The time responses of second state variable for second subsystem: $x_{2_2}(t)$ (solid), $\hat{x}_{2_2}(t)$ of Theorem 1 (dashed) and $\hat{x}_{2_2}(t)$ of Corollary 1 (dotted).

it can be confirmed that the result of Theorem 1 is better than the result of Corollary 1 in terms of the fault estimation performance. The performance difference for fault estimation of two techniques can be shown in more detail in Figs. 7 and 8. Also, to identify the performance differences according to the maximum interconnection bound α_k, we consider the performance measure function as follows:

$$\mathcal{P} = \sqrt{\sum_{k=1}^{2} \int_0^{50} e_k(t)^T e_k(t)dt \Big/ \sum_{k=1}^{2} \int_0^{50} v_k(t)^T v_k(t)dt}$$

and the results of the performance measure function are compared in Table 1. In addition, to emphasize the superiority of the proposed techniques, the results of the performance measure function for general fault estimation fuzzy observer without considering the interconnection based on [13] is added in Table 1.

As shown Table 1, the superiority of the performance of the proposed decentralized fuzzy observer for fault estimation

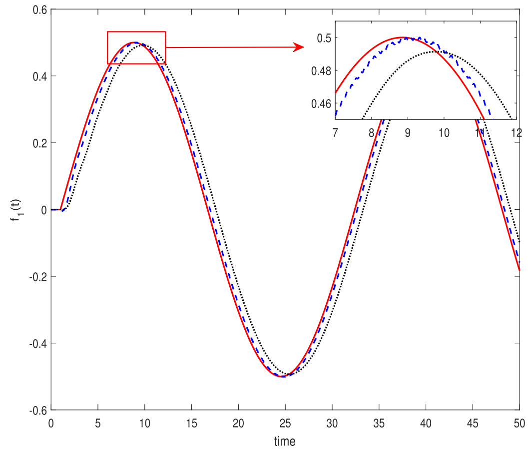

FIGURE 5. The time responses of actuator fault input for first subsystem: $f_1(t)$ (solid), $\hat{f}_1(t)$ of Theorem 1 (dashed) and $\hat{f}_1(t)$ of Corollary 1 (dotted).

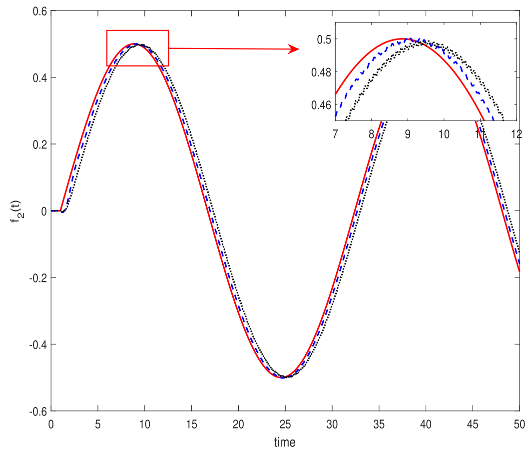

FIGURE 6. The time responses of actuator fault input for second subsystem: $f_2(t)$ (solid), $\hat{f}_2(t)$ of Theorem 1 (dashed) and $\hat{f}_2(t)$ of Corollary 1 (dotted).

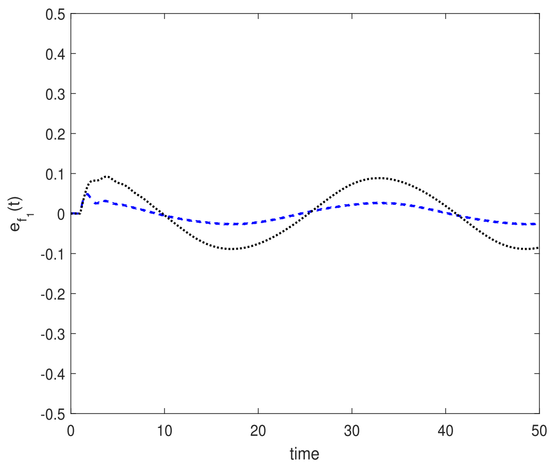

FIGURE 7. The time responses of estimation error of the actuator fault input for first subsystem: $e_{f_1}(t)$ of Theorem 1 (dashed) and $e_{f_1}(t)$ of Corollary 1 (dotted).

can be confirmed once again. Also, by the results of Table 1, [13] has not presented the relationship between the

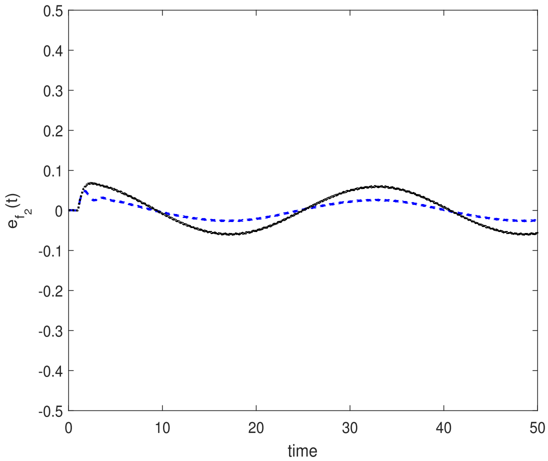

FIGURE 8. **The time responses of estimation error of the actuator fault input for second subsystem: $e_{f_2}(t)$ of Theorem 1 (dashed) and $e_{f_2}(t)$ of Corollary 1 (dotted).**

TABLE 1. **Performance comparison of the fuzzy fault estimation observer.**

Maximum interconnection bound	0.02	0.05	0.1
Theorem 1	0.475	0.646	0.841
Corollary 1	1.402	1.408	Infeasible
[13]	1.954	1.582	1.141

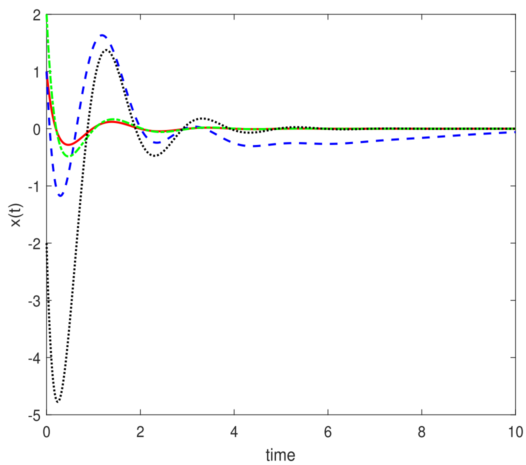

FIGURE 9. **The time responses of estimation error of the state variables by Theorem 1: $e_{x_{1_1}}(t)$ (solid), $e_{x_{1_2}}(t)$ (dashed), $e_{x_{2_1}}(t)$ (dash-dotted) and $e_{x_{2_2}}(t)$ (dotted).**

maximum interconnection bound and the fault estimation performance, because the interconnection problem is not considered in [13]. However, it can be known that Theorem 1 presents the better fault estimation performance when the smaller maximum interconnection bound is considering. Thus, we can guaranteed that the proposed technique is effective for large-scale systems with uncertain interconnections.

Finally, to show the asymptotic stability of error system (5) with $v_k(t) = 0$ and $h_k(x(t)) = 0$, we consider initial condition as $x_1(0) = [1 \quad -1]^T$ and $x_2(0) = [2 \quad -2]^T$, and its simulation results of Theorem 1 and Corollary 1 are shown in Figs 9 and 10, respectively. As shown in the figures,

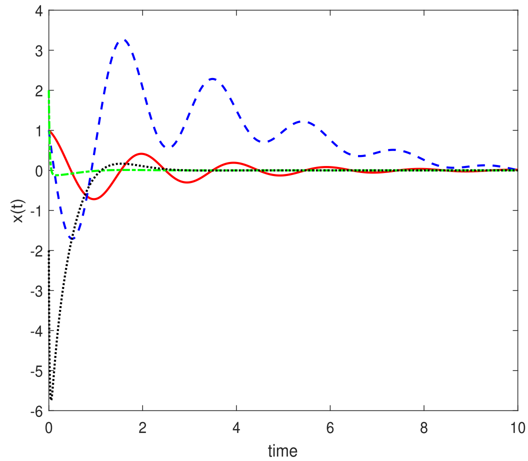

FIGURE 10. **The time responses of estimation error of the state variables by Corollary 1: $e_{x_{1_1}}(t)$ (solid), $e_{x_{1_2}}(t)$ (dashed), $e_{x_{2_1}}(t)$ (dash-dotted) and $e_{x_{2_2}}(t)$ (dotted).**

all estimation errors converge to zero when $\omega_k(t) = 0$, $f_k(t) = 0$ and $h_k(x(t)) = 0$. Thus, we know that the proposed decentralized fuzzy fault estimation observers satisfy the objective of fault estimation problem of Problem 1.

V. CONCLUSION

This paper has established the decentralized fuzzy observer design techniques for fault estimation of nonlinear large-scale systems. The nonlinear large-scale system has been considered by fuzzy subsystems with uncertain interconnections. By using the fuzzy subsystems and the decentralized fuzzy observer, the estimation error model has been represented and the fault estimation problem has been addressed. Based on the Lyapunov functional and H_∞ performance inequalities, the decentralized fuzzy observer design techniques for fault estimation have been derived into the sufficient conditions to satisfy the H_∞ performance, and its constructive design conditions have been presented into LMI formats. Finally, an example has been provided to demonstrate the effectiveness of the proposed decentralized fuzzy observer design techniques.

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