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THEORY

Fixed-Time Stabilization for a Class of Stochastic Memristive Inertial Neural Networks With Time-Varying Delays

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ABSTRACT This article investigates the fixed-time stabilization (XTS) of a class of stochastic memristive inertial neural networks (MINNs) with time-varying delays using interval matrix method (IMM) within the framework of the Filipov solution. To streamline the analysis, the second-order differential system is converted into an ordinary first-order differential system through suitable variable transformations. Afterwards, three types of state feedback controllers were designed. It's worth noting that the third controller represents an improvement over the first two controllers. Consequently, we have derived several sufficient conditions for XTS. This approach results in a more conservative upper estimate for the settling time function (STF). In addition, the finite-time stabilization (FTS) criterion can be derived. Ultimately, the validity of the theoretical findings were confirmed by numerical simulation outcomes.

INDEX TERMS Feedback control, fixed-time stabilization, interval matrix method, inertial memristive neural networks.

I. INTRODUCTION

Nowadays, neural networks (NNs) have seen significant advancements in various engineering and scientific fields, including signal processing, image processing, pattern recognition and robotics [1], [2]. The stability of neural networks, as a prerequisite for ensuring task completion, has also emerged as a prominent research topic among scholars.

Meanwhile, memristive neural networks (MNNs) have become a research hotspot, replacing traditional resistors in artificial neural networks with memristors. The concept of memristors was initially proposed by scientist Chua in 1971, defining the resistance based on the amount of charge flowing through it [3]. However, the HP Lab team didn't confirm the physical existence of memristors until 2008 [4]. Due to their similarity to the memory characteristics of human brain synapses, memristors exhibit significant potential for a wide range of applications [5], [6], [7], [8]. Therefore, studying the dynamic behavior [9], [10], [11] of MNNs, which can simulate the human brain, is of significant importance. Furthermore, numerous studies have focused on the stability of MNNs. For example, in [12], Wen and Zeng analyzed the exponential stability of recurrent MNNs. Reference [13] investigated the global Mittag-Leffler stability and synchronization of fractional order MNNs. These studies contribute to a deeper understanding of the behavior and potential applications of MNNs.

As we know, traditional NNs are usually described by first-order differential equations. Nonetheless, in 1986, Babcock and Westervelt introduced the concept of INNs,

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as described in [14], by incorporating inductors into neural network circuits, which can be modeled using second-order differential equations. In addition, due to the physical significance and biological background of inertia and memristors, their combination can simulate more complex dynamic behaviors in nature. Consequently, a class of MINNs can be constructed by combining INNs with MNNs [15], [16], [17]. In recent years, there have been publications focusing on MINNs. For instance, the stability and existence of periodic solutions for delayed inertial BAM neural networks were studied [18]. In [19], Wang and Zeng studied FTS problem of MINNs with distributed time delays.

Undeniably, in the past few decades, theoretical and applied research on FTS and control has received considerable attention due to its qualitative transient characteristics within a given finite time [20], [21], [22]. Reference [23] revisited the concept of FTS using linear matrix inequality (LMI) theory and proposed fewer conditions to ensure the FTS of linear continuous time systems, and in [24], Meng and Shen investigated the FTS of linear systems. Additionally, Zhang et al. [25] explored the finite time control problem and derived certain sufficient conditions. References [26] and [27] examined finite time control for linear and nonlinear stochastic systems separately.

We observe that the literature on FTS often emphasizes the reliance of stability time on initial conditions, which may not always be available [28]. Consequently, XTS, which represents a novel approach to stability analysis, warrants further investigation. Notably, in [21], the fixed-time control problem of delayed recurrent MNNs ware examined through the use of Lyapunov functions and control algorithms. Similarly, [22] focused on the fixed time synchronization of impulsive NNs using differential inequalities. Moreover, [18] explored the concepts of FTS and XTS for high-order BAM. These studies make significant contributions towards comprehending and applying XTS in various research and engineering domains [18], [21], [29].

It is important to mention that in the field of engineering applications, models of time-delay stochastic neural networks are more common. Therefore, researchers have never stopped researching neural networks for time-delay stochastic systems. While some researches have been done on the XTS problem of stochastic MINNs with time delays, there is still much room for further exploration and development. Additionally, the predominant focus within current literature on MINNs revolves around FTS and exponential stability. For instance, [30] delved into the global exponential synchronization of MINNs. Furthermore, in [31], Chen and Li explored the notion of fixed-time projection synchronization for MNNs featuring discrete time delays. Recent investigations in [32] presented novel findings on global exponential non-divergence analysis for MINNs. Nonetheless, to the best of the author's knowledge, there remains a scarcity of research outcomes pertaining to the XTS of stochastic MINNs, which has been a driving force behind our research endeavors.

In particular, this article's main contributions can be summarized as follows: Firstly, efficient controllers are developed for the two-layer structure of MINNs, ensuring FTS and XTS of stochastic MINNs. Additionally, a time delay term is incorporated into the controller to effectively address delays, and new results on the XTS of MINNs are presented using LMI. Finally, the theoretical results are validated for correctness and effectiveness through simulation examples.

The rest of this paper is structured in the following manner. The Section II will outline the necessary preliminary work. Theoretical results will be presented in Section III. The effectiveness of these theoretical results will be demonstrated through numerical simulations in Section IV. Finally, the conclusion can be found in Section V.

II. PROBLEM STATEMENT AND PRELIMINARIES

A. MODE DESCRIPTIONS

Now, we present the mathematical model of stochastic MINNs, along with a set of assumptions, definitions, and lemmas. Contemplate a category of stochastic MINNs featuring time-varying delays

$$\begin{aligned} d\dot{m}_i(t) = [&-a_i^\star \dot{m}_i(t) - b_i^\star m_i(t) \\ &+ \sum_{j=1}^{n} c_{ij}\left(m_i(t)\right) f_j\left(m_j(t)\right) \\ &+ \sum_{j=1}^{n} d_{ij}\left(m_i(t) g_j\left(m_j(t-\imath(t))\right)\right)]dt \\ &+ \varpi_i\left(t, m_i(t), m_i(t-\imath(t))\right) dw(t), \end{aligned} \tag{1}$$

with the initial values

$$m_i(s) = \psi_i(s), \ \dot{m}_i(s) = \varphi_i(s), \ s \in [-\imath, 0]. \tag{2}$$

In the given context, $m_i(t)$ is identified as the state variable, where both $a_i^\star > 0$ and $b_i^\star > 0$. The second derivative of $m_i(t)$, also referred to as the inertial term, is pertinent to system (1). The nonlinear function, f_i, serves to exemplify the activation function of the neural network. Concurrently, $\imath(t)$ designates a time-varying delay, complying with $0 \le \imath(t) \le \imath$, $\dot{\imath}(t) \le X \le 1$, where $\imath$ and X are constants. $c_{ij}(m_i)$ and $d_{ij}(m_i)$ symbolize the memristive connection weights.

$$c_{ij}(m_i) = \begin{cases} \hat{c}_{ij}, |m_i| \le \Upsilon_i, \\ \check{c}_{ij}, |m_i| > \Upsilon_i, \end{cases} \tag{3}$$

$$d_{ij}(m_i) = \begin{cases} \hat{d}_{ij}, |m_i| \le \Upsilon_i, \\ \check{d}_{ij}, |m_i| > \Upsilon_i, \end{cases} \tag{4}$$

where $\hat{c}_{ij}, \check{c}_{ij}, \hat{d}_{ij}, \check{d}_{ij}$ are known constants with respect to memristors, $i, j = 1, 2, \ldots, n$.

To establish the primary results, we introduce some assumptions.

Assumption 1: For each i, the neuron activation function of neurons f_i and g_i are bounded and meet the Lipschitz condition with $l_i > 0, j_i > 0, i \in I_n$,

$$l_i^{\searrow} \leq \frac{f_i(m_1) - f_i(m_1)}{m_1 - m_2} \leq l_i^{\nwarrow}, l_i = max\{l_i^{\searrow}, l_i^{\nwarrow}\},$$
$$j_i^{\searrow} \leq \frac{g_i(m_1) - g_i(m_1)}{m_1 - m_2} \leq j_i^{\nwarrow}, j_i = max\{j_i^{\searrow}, j_i^{\nwarrow}\},$$
$$L^{*} = diag\{l_1^{\searrow} l_1^{\nwarrow}, l_2^{-} l_2^{\nwarrow}, \cdots l_n^{\searrow} l_n^{\nwarrow}\},$$
$$L^{**} = diag\{\frac{l_1^{\searrow} + l_1^{\nwarrow}}{2}, \frac{l_2^{\searrow} + l_2^{\nwarrow}}{2}, \cdots \frac{l_n^{\searrow} + l_n^{\nwarrow}}{2}\},$$

hold for all $m_1, m_2 \in R, m_1 \neq m_2$.

Assumption 2: There exist matrices $S_1 \geq 0, S_2 \geq 0$ and $\sigma(t, 0, 0) = 0$, such that

$$trace[\varpi^T(t, m(t, m(t - \imath(t))))\varpi(t, m(t, m(t - \imath(t))))] \leq m(t)^T S_1 m(t) + m^T(t - \imath(t)) S_2 m(t - \imath(t)).$$

Definition 1 [33]: If for any open set T that encompasses $\Omega(m_0)$, a set-valued map Ω with non-empty values is considered to be upper semi-continuous at $m_0 \in \Gamma \subset \mathbb{R}^n$, and there exists a neighborhood M of m_0 such that $\Omega(M) \subset T$, then $\Omega(m)$ is a closed (convex, compact) image for every $m_0 \in \Gamma$.

Definition 2 [34]: For $\dot{m}(t) = f(m(t))$, where $m \in \mathbb{R}^n$ with right endpoints that exhibit discontinuities, the Filippov set-valued mapping can be delineated in the subsequent manner:

$$\Omega(m(t)) = \bigcap_{\delta>0} \bigcap_{\mu(T)=0} \overline{co}\left[f\left(B(m(t), \delta) \setminus T\right)\right].$$

B. INTERVAL MATRIX METHOD

let $\bar{c}_{ij} = \max\left\{c_{ij}^{*}, c_{ij}^{**}\right\}, \underline{c}_{ij} = \min\left\{c_{ij}^{*}, c_{ij}^{**}\right\}$. Meanwhile, we can get $\bar{d}_{ij}, \underline{d}_{ij}$ in the same way.

By using differential inclusions and set-valued mapping theory, this can be clearly seen that

$$d\dot{m}_i(t) \in [-a_i^{\sharp} m_i(t) - b_i^{\sharp} m_i(t) + \sum_{j=1}^{n} \overline{co}[c_{ij}(m_i(t))] f_j(m_j(t)) + \sum_{j=1}^{n} \overline{co}[d_{ij}(m_i(t)] g_j(m_j(t - \imath(t))))]dt + \varpi_i dw_i(t), \tag{5}$$

for $t \geq 0$, $i, j \in \Gamma_n$, there exist $\hat{c}_{ij}(m_i) \in [\bar{c}_{ij}, \underline{c}_{ij}] = \overline{co}[c_{ij}(m_i)]$, $\hat{d}_{ij}(m_i) \in [\bar{d}_{ij}, \underline{d}_{ij}] = \overline{co}[d_{ij}(m_i)]$ and $\varpi_i = \varpi_i(t, m_i(t), m_i(t - \imath(t)))$, thus

$$d\dot{m}_i(t) = [-a_i^{\sharp}\dot{m}_i(t) - b_i^{\sharp} m_i(t) + \sum_{j=1}^{n} c_{ij}(m_i(t)) f_j(m_j(t)) + \sum_{j=1}^{n} d_{ij}(m_i(t) g_j(m_j(t - \imath(t))))]dt + \varpi_i dw(t), \tag{6}$$

or equivalently,

$$\hat{c}_{ij}(m_i) = \lambda_{ij}^{\dagger}(t)\underline{c}_{ij} + (1 - \lambda_{ij}^{\dagger}(t))\bar{c}_{ij}, 0 \leq \lambda_{ij}^{\dagger}(t) \leq 1,$$
$$\hat{d}_{ij}(m_i) = \lambda_{ij}^{\ddagger}(t)\underline{d}_{ij} + (1 - \lambda_{ij}^{\ddagger}(t))\bar{d}_{ij}, 0 \leq \lambda_{ij}^{\ddagger}(t) \leq 1.$$

Choosing $\lambda_{ij}^{\dagger} = \frac{1 - o_{ij}^{\dagger}(t)}{2}$, $\lambda_{ij}^{\ddagger} = \frac{1 - o_{ij}^{\ddagger}(t)}{2}$, we have

$$\hat{c}_{ij}(m_i(t)) = \acute{c}_{ij} + o_{ij}^{\dagger}(t)\grave{c}_{ij},$$
$$\hat{d}_{ij}(m_i(t)) = \acute{d}_{ij} + o_{ij}^{\ddagger}(t)\grave{d}_{ij},$$

where $\acute{c}_{ij} = \frac{\bar{c}_{ij} + \underline{c}_{ij}}{2}$, $\acute{d}_{ij} = \frac{\bar{d}_{ij} + \underline{d}_{ij}}{2}$, $\grave{c}_{ij} = \frac{\bar{c}_{ij} - \underline{c}_{ij}}{2}$, $\grave{d}_{ij} = \frac{\bar{d}_{ij} - \underline{d}_{ij}}{2}$. The intervals $[\underline{c}_{ij}, \bar{c}_{ij}]$ can be characterized by their midpoints $\acute{c}_{ij}$ and $\acute{d}_{ij}$, as well as the half-lengths $\grave{c}_{ij}$ and $\grave{d}_{ij}$. Then, two measurable function $o_{ij}^{\dagger}(t)$ and $o_{ij}^{\ddagger}(t) \in \overline{co}[-1, 1]$ are selected

$$d\dot{m}_i = -[a_i^{\sharp}\dot{m}_i - b_i^{\sharp} m_i \sum_{j=1}^{n}(\acute{c}_{ij}(m_i) + o_{ij}^{\dagger}\grave{c}_{ij}) f_j(m_j) + \sum_{j=1}^{n}(\acute{d}_{ij}(m_i) + o_{ij}^{\ddagger}\grave{d}_{ij}) g_j(m_j)]dt + \varpi_i dw_i. \tag{7}$$

Let

$$\hat{C}(m) = \hat{c}_{ij}(m_i), \hat{D}(m) = \hat{d}_{ij}(m_i),$$
$$O^{\dagger}(t) = o_{ij}^{\dagger}(t), O^{\ddagger}(t) = o_{ij}^{\ddagger}(t),$$
$$A^{\star} = diag(a_1^{\star}, a_2^{\star}, \cdots, a_n^{\star})^T,$$
$$f(m) = (f_1(m_1), f_2(m_2), \cdots, f_n(m_n))^T,$$
$$g(m) = (g_1(m_1), g_2(m_2), \cdots, g_n(m_n))^T,$$
$$\varpi = (\varpi_1, \varpi_2, \cdots, \varpi_n), w(t) = (w_1(t), w_2, \cdots, w_n(t)).$$

Then, we set

$$\acute{C} = (\acute{c}_{ij})_{n\times n}, \Diamond C(m(t)) = o_{ij}^{\dagger}(t)\grave{c}_{ij} = H_1 F G_1,$$
$$\acute{D} = (\acute{d}_{ij})_{n\times n}, \Diamond D(m(t)) = o_{ij}^{\ddagger}(t)\grave{d}_{ij} = H_2 F G_2,$$
$$H_1 = (\sqrt{\grave{c}_{11}} i_1, \cdots, \sqrt{\grave{c}_{1n}} i_1, \cdots, \sqrt{\grave{c}_{n1}} i_n, \cdots, \sqrt{\grave{c}_{nn}} i_n),$$
$$G_1 = (\sqrt{\grave{c}_{11}} i_1, \cdots, \sqrt{\grave{c}_{1n}} i_1, \cdots, \sqrt{\grave{c}_{n1}} i_n, \cdots, \sqrt{\grave{c}_{nn}} i_n)^T,$$

$i_p (p = 1, \cdots, n)$ is the recognition matrix, provided $\mathbf{F} = \{f \mid diag\{o_{11}, \cdots, o_{1n}, \cdots, o_{n1}, \cdots, o_{nn}\}\}$, where $\| o_{ij} \| \leq 1$, and $\mathbf{F}^T\mathbf{F} \leq I$. Obviously, MINNs (7) are transformed into

$$d\dot{m}(t) = [-A^{\star}\dot{m}(t) - B^{\star} m(t) + (\acute{C} + H_1 F G_1) f(m(t)) + (\acute{D} + H_2 F G_2) g(m(t - \imath(t)))]dt + \varpi dw(t). \tag{8}$$

C. PRELIMINARIES

Then, the linear variables is transformed

$$\hbar(t) = \Theta m(t) + \dot{m}(t),$$

To formulate criteria for XTS, we devise the following feedback controller

$$\begin{cases} u_1 = -K_1 m(t) - K_2 sign(m(t)) - K_3 sign(m(t))|m(t)|^q \\ v_1 = -Z_1 \hbar(t) - Z_2 sign(\hbar(t)) - Z_3 sign(\hbar(t))|\hbar(t)|^q, \end{cases} \tag{9}$$

is the designed feedback control law, $K_1, K_2, K_3, Z_1, Z_2, Z_3$ are control strength matrices, $q > 1$.

Therefore, the controlled close-loop system is obtain as

$$\begin{cases} \dot{m} = -\Theta m(t) + \hbar(t) + u_1(t) \\ \dot{\hbar} = -Am(t) - B\hbar(t) + (\acute{C} + H_1FG_1)f(x(t)) \\ \quad +(\acute{D} + H_2FG_2)g(m(t-\imath(t))) + \varpi dw(t) + v_1(t). \end{cases} \tag{10}$$

where $\Theta > 0$, $B = A^\star - \Theta$, $A = B^\star - A^\star\Theta + \Theta^2$. Generally, Θ is given, which makes $B > 0$.

Definition 3 [35]: The system (1) is globally FTS at the origin if it exhibits Lyapunov stability and finite-time convergence (FTC). FTC implies that, for any initial state $m_0 \in \mathbb{R}^{r+n}$, there exists a function $T : \mathbb{R}^{n+m} \setminus \{0\} \to (0, +\infty)$, known as the STF, such that $\lim_{t\to T(m_0)} m(t, m_0) = 0$ and $m(t, m_0) = 0$ for all $t \geq T(m_0)$.

Definition 4 [36]: The origin of system (1) is globally XTS if it exhibits both global FTS characteristics and if $T(m_0)$ is bounded. In simpler terms, there is a positive constant $T_{\max}$ for which $T(m_0) \leq T_{\max}$ holds for all $m_0 \in \mathbb{R}^{r+s}$.

Lemma 1 (Schur Complement [37]): It's generally accepted that there is a given symmetric matrix $\Lambda = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ * & \Lambda_{22} \end{bmatrix}$, where $\Lambda_{11} \in \mathbb{R}^{n\times n}$, which is equivalently written as follows:

(1) $\Lambda < 0$;

(2)$\Lambda_{11} < 0$, $\Lambda_{22} - \Lambda_{12}^T\Lambda_{11}^{-1}\Lambda_{12} < 0$;

(3)$\Lambda_{22} <, \Lambda_{12}\Lambda_{22}^{-1}\Lambda_{12}^T < 0$.

Lemma 2 [38]: If we have $\Xi^T = \Xi$, Γ and Z with suitable dimensions,

$$\Xi + \Gamma FZ + Z^T F^T \Gamma^T < 0,$$

satisfies $F^T F \leq I$, and there exists $\varepsilon > 0$,

$$\Xi + \epsilon^{-1}\Gamma\Gamma^T + \epsilon Z^T Z < 0.$$

Lemma 3 [39]: We can make the assumption that $V(\cdot) : \mathbb{R}^n \to \mathbb{R}_+ \cup \{0\}$ is C-regular, and the vector function $m(t) \in \mathbb{R}^n$ is absolutely continuous within any compact subinterval of $[0, +\infty)$. If $V(m(t)) \leq -\eth V^n(m(t))$ for all $t \geq 0$, where $\eth > 0$ and $0 < \eta < 1$, then it follows that $V(m(t)) \equiv 0$, and the STF can be calculated as $T^* = \frac{V^{1-\eta}(m(0))}{\eth(1-\eta)}$.

Lemma 4 [40]: If $m_1, m_2, \cdots, m_n > 0$, $0 < p \leq 1$, $q > 1$, then

$$\sum_{i=1}^{n} m_i^p \geq \left(\sum_{i=1}^{n} m_i\right)^p, \sum_{i=1}^{n} m_i^q \geq n^{1-q}\left(\sum_{i=1}^{n} m_i\right)^q.$$

Lemma 5 [41]: If there exists a continuous and radically unbounded function $V : \mathbb{R}^n \to \mathbb{R}_+ \cup \{0\}$ such that $\mathbb{L}V(m) \leq -\gamma V^p(m) - \tau V^q(m)$, where $\gamma > 0$, $\tau > 0$, $0 < p < 1$, and $q > 1$, then the zero solution of systems (11) is deemed FTS, and the corresponding STF can be estimated as $T_{max} = \frac{1}{\gamma(1-p)} + \frac{1}{\tau(q-1)}$.

Lemma 6 [42]: Suppose $V(\cdot) : \mathbb{R}^n \to \mathbb{R}_+ \cup \{0\}$ is a continuously and radically unbounded function that satisfies the inequality $\mathbb{L}V(m) \leq -\gamma V^p(m) - \tau V^q(m)$, where $\gamma > 0$, $\tau > 0$, $p = 1 - \frac{1}{2\zeta}$ and $q = 1 + \frac{1}{2\zeta}$ for $\zeta > 1$. Then, the origin of the systems (11) is XTS, and $T_{max} = \frac{\pi\zeta}{\sqrt{\gamma\tau}}$. Furthermore, the STF can be estimated as $T_{max} = \frac{\pi\zeta}{\sqrt{\gamma\tau}}$.

III. MAIN RESULTS

Theorem 1: If Assumption 1 is satisfied. The MINNs (10) can achieve stochastic XTF if there exist several positive definite matrices X_1, X_2, positive diagonal matrices M_1, M_2, two general matrices W_1, W_2, and some scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6 > 0$, $P < \beta I$, such that

$$\begin{pmatrix} \psi_1 & 0 & 0 & L^{**}\check{M}_1 & 0 & \psi_2 \\ * & \check{\beta}S_2 - L^{**}\check{M}_2 & 0 & 0 & L^{**}\check{M}_2 & \psi_3 \\ * & * & \psi_4 & 0 & 0 & \psi_5 \\ * & * & * & -\check{M}_1 & 0 & 0 \\ * & * & * & * & -\check{M}_2 & 0 \\ * & * & * & * & * & \psi_6 \end{pmatrix} < 0, \tag{11}$$

$$\begin{aligned} \psi_1 &= -2\Gamma X_1 + \varepsilon_1 I - 2W_1 + \check{\beta}S_1 - L^*\check{M}_1, \\ \psi_2 &= (X_1, X_1L^T, X_1L^TG_1^T, 0, 0, 0), \\ \psi_3 &= (0, 0, 0, X_2J^T, X_2J^TG_2^T, 0), \\ \psi_4 &= -2BX_2 - 2W_2 + \varepsilon_2AA^T + \varepsilon_3\acute{C}\acute{C}^T + \varepsilon_4H_1H_1^T \\ &\quad + \varepsilon_5\acute{D}\acute{D}^T + \varepsilon_6H_2H_2^T, \\ \psi_5 &= (0, 0, 0, 0, 0, X_2), \\ \psi_6 &= diag(-\varepsilon_2I, -\varepsilon_3I, -\varepsilon_4I, -\varepsilon_5I, -\varepsilon_6I, -\varepsilon_1I), \\ \check{S}_1 &= X_1S_1X_1^T, \\ \check{S}_2 &= X_2S_2X_2^T, \\ \check{\beta} &= X_1\beta IX_1^T, \\ \check{M}_1 &= X_1M_1X_1^T, \\ \check{M}_2 &= X_2M_2X_2^T. \end{aligned}$$

The controlled (10) systems are XTS. Meanwhile, $K_1 = W_1X_1^{-1}$, $Z_1 = W_2X_2^{-1}$, furthermore,

$$T_{max} = \frac{2}{\lambda} + \frac{2}{\rho(q-1)},$$

where

$$\begin{aligned} \lambda &= \frac{2min\{\lambda_{min}(P_1K_2), \lambda_{min}(P_2Z_2)\}}{\sqrt{max\{\lambda_{max}(P_1), \lambda_{max}(P_2)\}}}, \\ \rho &= \frac{2min\{\lambda_{min}(P_1K_3), \lambda_{min}P_2Z_3\} \cdot n^{\frac{1-q}{2}}}{\sqrt{max\{\lambda_{max}(P_1), \lambda_{max}(P_2)\}^{p+1}}}, \end{aligned} \tag{12}$$

that is to say,

$$\begin{aligned} \lambda &= \lambda_1\lambda_2^{-\frac{1}{2}}, \rho = \lambda_3\lambda_2^{-\frac{p+1}{2}}, \\ \lambda_1 &= 2min\{\lambda_{min}(P_1K_2, P_2Z_2)\}, \\ \lambda_2 &= max\{\lambda_{max}(P_1), \lambda_{max}(P_2)\}, \\ \lambda_3 &= 2n^{\frac{1-q}{2}}min\{\lambda_{min}(P_1K_3), \lambda_{min}(P_2Z_3)\}. \end{aligned}$$

Proof: The Lyapunov functional is constructed as

$$V(t) = m(t)^T P_1 m(t) + \hbar(t)^T P_2 \hbar(t), \tag{13}$$

then,

$$\begin{aligned} &dV(t) \\ &= \mathbb{L}V(t)dt + 2\hbar(t)P_2\varpi dw(t), \end{aligned} \tag{14}$$

$$\begin{aligned} &\mathbb{L}V(t) \\ &= trace(\varpi^T P_2 \varpi) - 2m(t)^T P_1 \Gamma m(t) + 2m(t)^T P_1 \hbar(t) \\ &\quad + 2m(t)^T P_1 u_1(t) - 2n(t)^T P_2 A m(t) - 2\hbar(t)^T P_2 B \hbar(t) \\ &\quad + 2\hbar(t)^T P_2 (\acute{C} + H_1 F G_1) f(m(t)) \\ &\quad + 2n(t)^T P_2 (\acute{D} + H_2 F G_2) g(m(t - \iota(t))) + 2\hbar(t)^T P_2 v_1(t), \end{aligned} \tag{15}$$

$$\begin{aligned} &2m(t)^T P_1 \hbar(t) - 2\hbar(t)^T P_2 A m(t) \\ &\leq \varepsilon_1 m(t)^T P_1 P_1 m(t) + \varepsilon_1^{-1} \hbar(t)^T \hbar(t) + \varepsilon_2 \hbar(t)^T P_2 A A^T P_2 \hbar(t) \\ &\quad + \varepsilon_2^{-1} m(t)^T m(t), \end{aligned} \tag{16}$$

$$\begin{aligned} &2\hbar(t)^T P_2 (\acute{C} + H_1 F G_1) f(m(t)) \\ &\leq \varepsilon_3 \hbar(t)^T P_2 \acute{C} \acute{C}^T P_2 \hbar(t) + \varepsilon_3^{-1} m(t)^T L^T L m(t) \\ &\quad + \varepsilon_4 \hbar(t)^T P_2 H_1 H_1^T P_2 \hbar(t) + \varepsilon_4^{-1} m(t)^T L^T G_1^T G_1 L m(t), \end{aligned} \tag{17}$$

$$\begin{aligned} &2\hbar(t)^T P_2 (\acute{D} + H_2 F G_2) G(m(t - \iota(t))) \\ &\leq \varepsilon_5 \hbar(t)^T P_2 \acute{D} \acute{D}^T P_2 \hbar(t) + \varepsilon_5^{-1} m(t - \iota(t))^T J^T J m(t - \iota(t)) \\ &\quad + \varepsilon_6 \hbar(t)^T P_2 H_2 H_2^T P_2 \hbar(t) \end{aligned} \tag{18}$$

$$\begin{aligned} &trace(\varpi^T P_2 \varpi) \\ &\leq \beta(m(t)^T S_1 m(t) + m(t - \iota(t))^T S_2 m(t - \iota(t))). \end{aligned} \tag{19}$$

Therefore, we have

$$\begin{aligned} &2m(t)^T P_1 u_1(t) + 2n(t)^T P_2 v_1(t) \\ &= -2m(t)^T P_1 K_1 m(t) - 2m(t)^T P_1 K_2 sign(m(t)) \\ &\quad - 2m(t)^T P_1 K_3 sign(m(t))|m(t)|^q - 2\hbar(t)^T P_2 Z_1 \hbar(t) \\ &\quad - 2\hbar(t)^T P_2 Z_2 sign(\hbar(t)) - 2\hbar(t)^T P_2 Z_3 sign(\hbar(t))|\hbar(t)|^q. \end{aligned} \tag{20}$$

Therefore, according to Assumption 1, for positive diagonal matrices M_1 and M_2, we can satisfy the following inequalities:

$$\begin{aligned} &-\begin{pmatrix} m(t) \\ f(m(t)) \end{pmatrix}^T \begin{pmatrix} L^* M_1 & -L^{**} M_1 \\ -L^{**} M_1 & M_1 \end{pmatrix} \begin{pmatrix} m(t) \\ f(m(t)) \end{pmatrix} \\ &-\begin{pmatrix} m(t-\iota(t)) \\ f(m(t-\iota(t))) \end{pmatrix}^T \begin{pmatrix} L^* M_2 & -L^{**} M_2 \\ -L^{**} M_2 & M_2 \end{pmatrix} \\ &\times \begin{pmatrix} m(t-\iota(t)) \\ f(m(t-\iota(t))) \end{pmatrix} \\ &\geq 0. \end{aligned} \tag{21}$$

Combining the above equations, one gets

$$\begin{aligned} \mathbb{L}V(t) &\leq m(t)^T \Phi_{11} m(t) + m(t - \iota(t))^T \Phi_{22} m(t - \iota(t)) \\ &\quad + \hbar(t)^T \Phi_{33} \hbar(t) + f(m(t))^T (-M_1) f(m(t)) \\ &\quad + f(m(t - \iota(t)))^T (-M_2) f(m(t - \iota(t))) \\ &\quad + f(m(t))^T L^{**} M_1 m(t) + m(t)^T L^{**} M_1 f(m(t)) \\ &\quad + f(m(t - \iota(t)))^T L^{**} M_2 m(t) \\ &\quad + m(t - \iota(t))^T L^{**} M_2 f(m(t - \iota(t))) \\ &\quad - 2m(t)^T P_1 K_2 sign(m(t)) \\ &\quad - 2m(t)^T P_1 K_3 sign(m(t))|m(t)|^q \\ &\quad - 2\hbar(t)^T P_2 Z_2 sign(\hbar(t)) \\ &\quad - 2\hbar(t)^T P_2 Z_3 sign(\hbar(t))|\hbar(t)|^q, \end{aligned} \tag{22}$$

where

$$\begin{aligned} \Phi_{11} &= -2P_1\Theta + \varepsilon_1 P_1 P_1 + \varepsilon_2^{-1} I + \varepsilon_3^{-1} L^T L \\ &\quad + \varepsilon_4^{-1} L^T G_1^T G_1 L - 2P_1 K_1 + \beta S_1 - L^* M_1 \\ \Phi_{22} &= \varepsilon_5^{-1} J^T J + \varepsilon_6^{-1} J^T G_2^T G_2 J + \beta S_2 \\ \Phi_{33} &= -2P_2 B + \varepsilon_1^{-1}\Theta + \varepsilon_2 P_2 A A^T P_2 + \varepsilon_3 P_2 \acute{C}\acute{C}^T P_2 \\ &\quad + \varepsilon_4 P_2 H_1 H_1^T P_2 + \varepsilon_5 P_2 \acute{D}\acute{D}^T P_2 + \varepsilon_6 P_2 H_2 H_2^T P_2 \\ &\quad - 2P_2 Z_1 \end{aligned}$$

Then, it follows that

$$\begin{aligned} \mathbb{L}V(t) &\leq -\lambda_{min}(P_1 K_2 + K_2^T P_1) \sum_{i=1}^{n} |m_i(t)| \\ &\quad - \lambda_{min}(P_2 Z_2 + Z_2^T P_2) \sum_{i=1}^{n} |\hbar_i(t)| \\ &\quad - \lambda_{min}(P_1 K_3 + K_3^T P_1) \sum_{i=1}^{n} |m_i(t)|^{q+1} \\ &\quad - \lambda_{min}(P_2 Z_3 + Z_3^T P_2) \sum_{i=1}^{n} |\hbar_i(t)|^{q+1}. \end{aligned} \tag{23}$$

From lemma 4 and $q > 1$,

$$\sum_{i=1}^{n} |m_i(t)|^{q+1} \geq n^{\frac{1-q}{2}} (m(t)^T m(t))^{\frac{q+1}{2}},$$

letting

$$\begin{aligned} \lambda_1 &= 2min\left\{\lambda_{min}(P_1 K_2), \lambda_{min}(P_2 Z_2)\right\}, \\ \lambda_2 &= max\left\{\lambda_{max}(P_1), \lambda_{max}(P_2)\right\}, \\ \lambda_3 &= 2n^{\frac{1-q}{2}} min\left\{\lambda_{min}(P_1 K_3), \lambda_{min}(P_2 Z_3)\right\}, \end{aligned}$$

we can get

$$\begin{aligned} \mathbb{E}\left\{dV(t)\right\} &\leq -\frac{2\lambda_{min}(P_1 K_2, P_2 Z_2)}{\sqrt{\lambda_{max}(P_1, \lambda_{max}(P_2))}} \mathbb{E}(V(t))^{\frac{1}{2}} \\ &\quad - \frac{2\lambda_{min}(P_1 K_3, P_2 Z_3) \cdot n^{\frac{1-q}{2}}}{(\sqrt{\lambda_{max}(P_1), \lambda_{max}(P_2)})^{\rho+1}} \mathbb{E}(V(t))^{\frac{q+1}{2}} \\ &= -\lambda \mathbb{E}(V(t))^{\frac{1}{2}} - \rho \mathbb{E}(V(t))^{\frac{q+1}{2}}, \end{aligned} \tag{24}$$

where $\lambda = \lambda_1 \lambda_2^{-\frac{1}{2}}$, $\rho = \lambda_3 \lambda_2^{-\frac{q+1}{2}}$. Then, from Lemma 3, the system (10) can achieve XTS,

$$T_{max} = \frac{1}{\lambda(1 - \frac{1}{2})} + \frac{1}{\rho(\frac{q+1}{2} - 1)} = \frac{2}{\lambda} + \frac{2}{\rho(q-1)}.$$

Remark 1: Now, let's examine a specific case of Theorem 1. Assuming the conditions of Theorem 1 remain valid, if $q = 2$, then the MINNs (10) can achieve XTS under controller (9), the gain matrices of the controller are $K_1 = W_1X_1^{-1}, Z_1 = W_2X_2^{-1}$, furthermore, $T_{max} = \frac{\pi}{\sqrt{\lambda\rho}}$, where
$\lambda = \lambda_1\lambda_2^{-1}$,
$\rho = \lambda_3\lambda_2^{-\frac{3}{2}}, \lambda_1 = 2min\{\lambda_{min}(P_1K_2), \lambda_{min}(P_2Z_2)\}$,
$\lambda_2 = max\{\lambda_{max}(P_1), \lambda_{max}(P_2)\}$,
$\lambda_3 = 2n^{-\frac{1}{2}}min\{\lambda_{min}(P_1K_3), \lambda_{min}(P_2Z_3)\}$.

In order to compare with XTS, we also studied the FTS of the system, the following feedback controller has been designed:

$$\begin{cases} \tilde{u}_1 = -K_1m(t) - K_2sign(m(t)), \\ \tilde{v}_1 = -Z_1\hbar(t) - Z_2sign(\hbar(t)), \end{cases} \tag{25}$$

where K_1, Z_1 are control gains to be determined later, therefore, the following corollary can be easily derived from lemma 3.

Corollary 1: Suppose Assumption 1 holds. The MINNs will achieve stochastic FTS if there exist several positive definite matrices X_1, X_2, positive diagonal matrices M_1, M_2, two general matrices W_1, W_2, and some scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6 > 0$. $P < \beta I$, such that

$$\begin{pmatrix} \psi_1 & 0 & 0 & L^{**}\check{M}_1 & 0 & \psi_2 \\ * & \check{\beta}S_2 - L^{**}\check{M}_2 & 0 & 0 & L^{**}\check{M}_2 & \psi_3 \\ * & * & \psi_4 & 0 & 0 & \psi_5 \\ * & * & * & -\check{M}_1 & 0 & 0 \\ * & * & * & * & -\check{M}_2 & 0 \\ * & * & * & * & * & \psi_6 \end{pmatrix} < 0,$$

where

$$\begin{aligned} \psi_1 &= -2\Theta X_1 + \varepsilon_1 I - 2W_1 + \check{\beta}S_1 - L^*\check{M}_1, \\ \psi_2 &= (X_1, X_1L^T, X_1L^TG_1^T, 0, 0, 0), \\ \psi_3 &= (0, 0, 0, X_2J^T, X_2J^TG_2^T, 0), \\ \psi_4 &= -2P_2X_2 - 2W_2 + \varepsilon_2AA^T + \varepsilon_3\acute{C}\acute{C}^T + \varepsilon_4H_1H_1^T \\ &\quad + \varepsilon_5\acute{D}\acute{D}^T + \varepsilon_6H_2H_2^T, \\ \psi_5 &= (0, 0, 0, 0, 0, X_2), \\ \psi_6 &= diag(-\varepsilon_2 I, -\varepsilon_3 I, -\varepsilon_4 I, -\varepsilon_5 I, -\varepsilon_6 I, -\varepsilon_1 I), \\ \check{S}_1 &= X_1S_1X_1^T, \\ \check{S}_2 &= X_2S_2X_2^T, \\ \check{\beta} &= X_1\beta IX_1^T, \\ \check{M}_1 &= X_1M_1X_1^T, \\ \check{M}_2 &= X_2M_2X_2^T. \end{aligned}$$

What is more, the upper bound of the STF for stabilization can be estimated as $T_{max} = \frac{\lambda_2(E||x(0)||^{\frac{1}{2}} + E||y(0)||^{\frac{1}{2}})}{\alpha}, \alpha = \lambda_1\lambda_2^{-\frac{1}{2}}$.

Now, we consider another feedback controller, which is a more common case, where $0 < p < 1, q > 1$.

$$\begin{cases} u_2 = -K_1m(t) - K_2sign(m(t))|m(t)|^p - K_3sign(m(t))|m(t)|^q \\ v_2 = -Z_1\hbar(t) - Z_2sign(\hbar(t))|\hbar(t)|^p - Z_3sign(\hbar(t))|\hbar(t)|^q. \end{cases} \tag{26}$$

Theorem 2: If Assumption 1 holds. The MINNs (10) can achieve stochastic XTS if there exist several positive definite matrices X_1, X_2, positive diagonal matrices M_1, M_2, two general matrices W_1, W_2, and some scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6 > 0, P < \beta I$, such that

$$\begin{pmatrix} \psi_1 & 0 & 0 & L^{**}\check{M}_1 & 0 & \psi_2 \\ * & \check{\beta}S_2 - L^{**}\check{M}_2 & 0 & 0 & L^{**}\check{M}_2 & \psi_3 \\ * & * & \psi_4 & 0 & 0 & \psi_5 \\ * & * & * & -\check{M}_1 & 0 & 0 \\ * & * & * & * & -\check{M}_2 & 0 \\ * & * & * & * & * & \psi_6 \end{pmatrix} < 0, \tag{27}$$

where

$$\begin{aligned} \psi_1 &= -2\Theta X_1 + \varepsilon_1 I - 2W_1 + \check{\beta}S_1 - L^*\check{M}_1, \\ \psi_2 &= (X_1, X_1L^T, X_1L^TG_1^T, 0, 0, 0), \\ \psi_3 &= (0, 0, 0, X_2J^T, X_2J^TG_2^T, 0), \\ \psi_4 &= -2BX_2 - 2W_2 + \varepsilon_2AA^T + \varepsilon_3\acute{C}\acute{C}^T + \varepsilon_4H_1H_1^T \\ &\quad + \varepsilon_5\acute{D}\acute{D}^T + \varepsilon_6H_2H_2^T, \\ \psi_5 &= (0, 0, 0, 0, 0, X_2), \\ \psi_6 &= diag(-\varepsilon_2 I, -\varepsilon_3 I, -\varepsilon_4 I, -\varepsilon_5 I, -\varepsilon_6 I, -\varepsilon_1 I), \\ \check{\beta} &= X_1\beta IX_1^T, \\ \check{M}_1 &= X_1M_1X_1^T, \\ \check{M}_2 &= X_2M_2X_2^T. \end{aligned}$$

Then, the controlled system (10) is XTS with $K_1 = W_1X_1^{-1}, Z_1 = W_2X_2^{-1}$. What is more,

$$T_{max} = \frac{2}{\alpha(1-p)} + \frac{2}{\beta(q-1)}.$$

where $\alpha = \lambda_1\lambda_2^{-\frac{p+1}{2}}$, $\beta = \lambda_3\lambda_2^{-\frac{q+1}{2}}$, $\lambda_1 = 2min\{\lambda_{min}(P_1K_2), \lambda_{min}(P_2Z_2)\}$, $\lambda_2 = min\{\lambda_{max}(P_1), \lambda_{max}(P_2)\}$, $\lambda_3 = 2min\{\lambda_{min}(P_1K_3)\cdot n^{\frac{1-q}{2}}, \lambda_{min}(P_2Z_3)\cdot n^{\frac{1-q}{2}}\}$.

Proof:

$$V(t) = m(t)^TP_1m(t) + \hbar(t)^TP_2\hbar(t), \tag{28}$$

Similarly, we have

$$\begin{aligned} \mathbb{L}V(t) \le\ & m(t)^T\Phi_{11}m(t) + m(t-\iota(t))^T\Phi_{22}m(t-\iota(t)) \\ & + \hbar(t)^T\Phi_{33}\hbar(t) + f(m(t))^T(-M_1)f(m(t)) \\ & + f(m(t-\iota(t)))^T(-M_2)f(m(t-\iota(t))) \\ & + f(m(t))^TL^{**}M_1m(t) + m(t)^TL^{**}M_1f(m(t)) \\ & + f(m(t-\iota(t)))^TL^{**}M_2m(t) \\ & + m(t-\iota(t))^TL^{**}M_2f(m(t-\iota(t))) \end{aligned}$$

$$
\begin{aligned}
&- 2m(t)^T P_1 K_2 sign(m(t))|m(t)|^p \\
&- 2m(t)^T P_1 K_3 sign(m(t))|m(t)|^q \\
&- 2\hbar(t)^T P_2 Z_2 sign(\hbar(t))|\hbar(t)|^p \\
&- 2\hbar(t)^T P_2 Z_3 sign(\hbar(t))|\hbar(t)|^q, \quad (29)
\end{aligned}
$$

where

$$
\begin{aligned}
\Phi_{11} &= -2P_1\Theta + \varepsilon_1 P_1 P_1 + \varepsilon_2^{-1} I + \varepsilon_3^{-1} L^T L \\
&\quad + \varepsilon_4^{-1} L^T L G_1^T G_1 L - 2P_1 K_1 + \beta S_1 - L^* M_1 \\
\Phi_{22} &= \varepsilon_5^{-1} J^T J + \varepsilon_6^{-1} J^T G_2^T G_2 J + \beta S_2 \\
\Phi_{33} &= -2P_2 B + \varepsilon_1^{-1} I + \varepsilon_2 P_2 A A^T P_2 + \varepsilon_3 P_2 \acute{C}\acute{C}^T P_2 \\
&\quad + \varepsilon_4 P_2 H_1 H_1^T P_2 + \varepsilon_5 P_2 \acute{D}\acute{D}^T P_2 + \varepsilon_6 P_2 H_2 H_2^T P_2 \\
&\quad - 2P_2 Z_1.
\end{aligned}
$$

By Lemmas 1, in conjunction with inequality (11) with $P_1 = X_1^{-1}$ and $P_2 = X_2^{-1}$

$$
\begin{aligned}
\mathbb{L}V(t) \le &-\lambda_{min}(P_1 K_2 + K_2^T P_1)|\sum_{i=1}^{n} m_i(t)|^{p+1} \\
&- \lambda_{min}(P_2 Z_2 + Z_2^T P_2)|\sum_{i=1}^{\hbar} \hbar_i(t)|^{p+1} \\
&- \lambda_{min}(P_1 K_3 + K_3^T P_1)|\sum_{i=1}^{n} m_i(t)|^{q+1} \\
&- \lambda_{min}(P_2 Z_3 + Z_3^T P_2)|\sum_{i=1}^{n} \hbar_i(t)|^{q+1}. \quad (30)
\end{aligned}
$$

Due to $0 < p < 1$, together with lemma 4,one gets

$$
\sum_{i=1}^{n} |m_i(t)|^{p+1} \ge (m(t)^T m(t))^{\frac{p+1}{2}},
$$

Letting $\alpha = \lambda_1 \lambda_2^{-\frac{p+1}{2}}$, $\beta = \lambda_1 \lambda_3^{-\frac{q+1}{2}}$, we can get

$$
\mathbb{E}|dV(t)| \le -\alpha[V(t)]^{\frac{p+1}{2}} - \beta[V(t)]^{\frac{q+1}{2}}.
$$

Moreover,

$$
T_{max} = \frac{2}{\alpha(1-p)} + \frac{2}{\beta(q-1)}. \quad (31)
$$

Remark 2: Now, let's consider a special case of Theorem 1. Assuming that the conditions in Theorem 2 still hold, if $p = 1 - \frac{1}{\zeta}$, $q = 1 + \frac{1}{\zeta}$ and $\zeta > 1$, then we have $\frac{p+1}{2} = 1 - \frac{1}{2\zeta}$, $\frac{q+1}{2} = 1 + \frac{1}{2\zeta}$. The system (11) can achieve XTS under the controller (12). Moreover, a more precise estimation of the STF can be derived using Lemma 8: $T_{max} = \frac{\pi\mu}{\sqrt{\alpha\beta}}$.

Remark 3: For any initial condition, the stability time for a fixed time is constrained by a predetermined constant, which can be pre-computed using systems and controller parameters.

Remark 4: In this article, it is evident that the $sign(m_i(t))$ plays a pivotal role in the controller design. However, it's difficult to handle. Therefore, we can use the continuous term $\frac{m_i(t)}{|m_i(t)| + \delta_i}$ as an approximation of $sign(m_i(t))$, where the constant δ_i is small enough.

To investigate the FTS of MINNs (10), we have designed the following feedback controller

$$
\begin{cases}
\tilde{u}_2 = -K_1 m(t) - K_2 sign(m(t))|m(t)|^p, \\
\tilde{v}_2 = -Z_1 \hbar(t) - Z_2 sign(\hbar(t))|\hbar(t)|^p,
\end{cases} \quad (32)
$$

Utilizing Lemma 3 as a foundation, we can easily deduce the subsequent corollary.

Corollary 2: We continue to assume that all the requirements specified in Theorem 2 remain satisfied, based on lemma 9, MINNs (10) can attain FTS with the controller (12), where $E\{T_0\} \le \frac{\lambda_2(E||x(0)||^{\frac{1-p}{2}} + E||y(0)||^{\frac{1-p}{2}})}{\chi}$ with $\chi = 2\lambda_1\lambda_2^{-\frac{1-p}{2}}$.

Remark 5: From inequality (31), it is evident that the STF of FTS is influenced by both the initial values and coefficients. In other words, once the initial values are set, the upper time bound is solely determined by the coefficients, which can be manipulated through the gain matrices.

The first few controllers are relatively common and simple, and a new controller is as follows. In our approach, we not only introduce a delay term but also incorporate both the $sign(m_i(t))$ function and absolute value function to more effectively address time-varying delays in the system (11).

$$
\begin{cases}
u_3 = K_1 m(t) - K_2 sign(m(t))|m(t)|^p - K_3 sign(m(t))|m(t)|^q, \\
v_3 = Z_1 \hbar(t) - Z_2 sign(\hbar(t))|\hbar(t)|^p - Z_3 sign(\hbar(t))|\hbar(t)|^q \\
\quad + Z_4 sign(\hbar(t))|\hbar(t - \iota(t))| + \tilde{M} sign(\hbar(t))(\frac{|\hbar(t)|}{\|\hbar(t)\|^2}).
\end{cases} \quad (33)
$$

Theorem 3: If Assumptions 1 and 2 are satisfied, then the stochastic MINNs (10) can attain XTS provided there exist several positive definite matrices X_1, X_2, positive diagonal matrices M_1, M_2, two general matrices W_1, W_2, W_3, W_4, and some scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6 > 0$, $P < \beta I$, such that

$$
\begin{pmatrix}
\psi_1 & 0 & 0 & L^{**}\check{M}_1 & 0 & \psi_2 \\
* & \beta\check{S}_2 - L^{**}\check{M}_2 & 0 & 0 & L^{**}\check{M}_2 & \psi_3 \\
* & * & \psi_4 & 0 & 0 & \psi_5 \\
* & * & * & -\check{M}_1 & 0 & 0 \\
* & * & * & * & -\check{M}_2 & 0 \\
* & * & * & * & * & \psi_6
\end{pmatrix} < 0, \quad (34)
$$

$$
\begin{pmatrix}
2W_3 & X_2 N^T G_2^T \\
* & -\varepsilon_5 I
\end{pmatrix} < 0, \quad (35)
$$

$$
\acute{D} J X_2 + W_4 < 0, \quad (36)
$$

where

$$
\psi_1 = -2\Theta X_1 + \varepsilon_1 I - 2W_1 + \check{\beta} S_1 - L^* \check{M}_1,
$$

$$\psi_2 = (X_1, X_1 L^T, X_1 L^T G_1^T, 0),$$
$$\psi_3 = (0, 0, 0, 0),$$
$$\psi_4 = -2BX_2 - 2W_2 + \varepsilon_2 AA^T + \varepsilon_3 \acute{C}\acute{C}^T + \varepsilon_4 H_1 H_1^T + \varepsilon_5 H_2 H_2^T,$$
$$\psi_5 = (0, 0, 0, X_2),$$
$$\psi_6 = diag(-\varepsilon_2 I, -\varepsilon_3 I, -\varepsilon_4 I, -\varepsilon_1 I),$$
$$\check{\beta} = X_1 \beta I X_1^T,$$
$$\check{M}_1 = X_1 M_1 X_1^T,$$
$$\check{M}_2 = X_2 M_2 X_2^T.$$

Then the controlled systems are XTS, and $K_1 = W_1 X_1^{-1}, Z_1 = W_2 X_2^{-1}, Z_4 = W_4 X_2^{-1}, \tilde{M} = W_3 X_2^{-1}$, furthermore,

$$T_{max} = \frac{2}{\jmath(1-p)} + \frac{1}{\Bbbk(q-1)},$$

where $\jmath = 2\lambda_1 \lambda_2^{-\frac{p+1}{2}}, \Bbbk = 2\lambda_1 \lambda_3^{-\frac{q+1}{2}}$, $\lambda_1 = min\{\lambda_{min}(P_1 K_2), \lambda_{min}(P_2 Z_2)\}$, $\lambda_2 = min\{\lambda_{min}(P_1 K_3) \cdot n^{\frac{1-q}{2}}, \lambda_{min}(P_2 Z_3) \cdot n^{\frac{1-q}{2}}\}$, $\lambda_3 = max\{\lambda_{max}(P_1), \lambda_{max}(P_2)\}$.

Proof:

$$V(t) = m(t)^T P_1 m(t) + \hbar(t)^T P_2 \hbar(t). \tag{37}$$

Therefore,

$$\begin{aligned} &2\hbar(t)^T P_2(\acute{D} + H_2 F G_2) g(m(t-\imath(t))) \\ &\le 2\hbar(t)^T P_2 \acute{D} J |(m(t-\imath(t))| + \varepsilon_5^{-1} M^T G_2^T G_2 M \\ &\quad + \varepsilon_5 \hbar(t)^T P_2 H_2 H_2^T P_2 \hbar(t). \end{aligned} \tag{38}$$

Similarly, we can prove that

$$\begin{aligned} \mathbb{L}V(t) \le\; & m(t)^T \Phi_{11} m(t) + m(t-\imath(t))^T \Phi_{22} m(t-\imath(t)) \\ & + \hbar(t)^T \Phi_{33} \hbar(t) + f(m(t))^T(-M_1) f(m(t)) \\ & + f(m(t-\imath(t)))^T (-M_2) f(m(t-\imath(t))) \\ & + f(m(t))^T L^{**} M_1 m(t) + m(t)^T L^{**} M_1 f(m(t)) \\ & + f(m(t-\imath(t)))^T L^{**} M_2 m(t) \\ & + m(t-\imath(t))^T L^{**} M_2 f(m(t-\imath(t))) \\ & - 2m(t)^T P_1 K_2 sign(m(t)) |m(t)|^p \\ & - 2m(t)^T P_1 K_3 sign(m(t)) |m(t)|^q \\ & - 2\hbar(t)^T P_2 Z_2 sign(\hbar(t)) |\hbar(t)|^p \\ & - 2\hbar(t)^T P_2 Z_3 sign(\hbar(t)) |\hbar(t)|^q \\ & + 2\hbar(t) P_2 Z_4 sign(\hbar(t)) |m((t-\imath(t)))| + +2P_2 \tilde{M}. \end{aligned} \tag{39}$$

where

$$\Phi_{11} = -2P_1\Theta + \varepsilon_1 P_1 P_1 + \varepsilon_2^{-1} I + \varepsilon_3^{-1} L^T L + \varepsilon_4^{-1} L^T L G_1^T G_1 L - 2P_1 K_1 + \beta S_1 - L^* M_1$$
$$\Phi_{22} = \varepsilon_5^{-1} J^T J + \varepsilon_6^{-1} J^T G_2^T G_2 J + \beta S_2$$
$$\Phi_{33} = -2P_2 B + \varepsilon_1^{-1} \Gamma + \varepsilon_2 P_2 A A^T P_2 + \varepsilon_3 P_2 \acute{C}\acute{C}^T P_2 + \varepsilon_4 P_2 H_1 H_1^T P_2 + \varepsilon_5 P_2 \acute{D}\acute{D}^T P_2 + \varepsilon_6 P_2 H_2 H_2^T P_2 - 2P_2 Z_1.$$

The remaining steps of the proof follow the same procedure.

Remark 6: This study's innovation stems from the simplification of parameter constraints and the utilization of a controller grounded in interval matrix methods, thus presenting an effective and viable control strategy. In Theorems 1 and 2, controllers are simple and commonly used, making it easy to verify the obtained system stabilization criteria. In Theorem 3, we can observe that controller (33) handle the time delay term differently by incorporating proportional terms of PID control. Meanwhile, some articles provide integral terms for PID control. The different forms lead to different effects of controllers, and these forms are widely used in engineering.

IV. NUMERICAL SIMULATIONS

Within this section, we substantiate the theoretical findings with numerical simulations.

Consider the following stochastic MINNs

$$\begin{cases} \dot{m}_i = -\xi_i m_i + \hbar_i \\ \dot{\hbar}_i = -a_i m_i - b_i \hbar_i + \sum_{j=1}^n c_{ij}(m_i) f_i(m_j(t)) \\ \quad + \sum_{j=1}^n d_{ij}(m_i) g_i(m_j(t-\imath(t))) + \varpi_i dw_i. \end{cases} \tag{40}$$

$a_i = \xi_i^2 - a_i^\star \xi_i + b_i^\star, b_i = a_i^\star - \xi_i$, $i = 1, 2$, $f_j(x) = g_j(x) = tanh(x)$, and the delay with time change is $\imath_j(t) = 0.75 + 0.25 sin(2t)$. Correspondingly, the closed-loop system is as follows:

$$\begin{cases} \dot{m}_i = -\xi_i m_i + \hbar_i + u_i \\ \dot{\hbar}_i = -a_i m_i - b_i \hbar_i + \sum_{j=1}^n c_{ij}(m_i) f_i(m_j(t)) \\ \quad + \sum_{j=1}^n d_{ij}(m_i) g_i(m_j(t-\imath(t))) + \varpi_i dw_i + v_i. \end{cases} \tag{41}$$

The system parameters are taken as

$$A = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.07 \end{pmatrix}, \quad B = \begin{pmatrix} 5 & 0 \\ 0 & 10 \end{pmatrix},$$
$$\Theta = \begin{pmatrix} 0.05 & 0 \\ 0 & 0.05 \end{pmatrix},$$
$$L^* = \begin{pmatrix} 0.08 & 0 \\ 0 & 0.08 \end{pmatrix}, \; L^{**} = \begin{pmatrix} 0.3 & 0 \\ 0 & 0.3 \end{pmatrix},$$
$$C(m_i) = \begin{pmatrix} c_{11}(m_{i1}) & c_{12}(m_{i2}) \\ c_{21}(m_{i1}) & c_{22}(m_{i2}) \end{pmatrix},$$
$$D(m_i) = \begin{pmatrix} d_{11}(m_{i1}) & d_{12}(m_{i2}) \\ d_{21}(m_{i1}) & d_{22}(m_{i2}) \end{pmatrix},$$

Select the memristor connection weights:

$$c_{11}(m_1(t)) = \begin{cases} -0.2, & |m_1| \le 0.1. \\ 0.2, & |m_1| > 0.1. \end{cases}$$

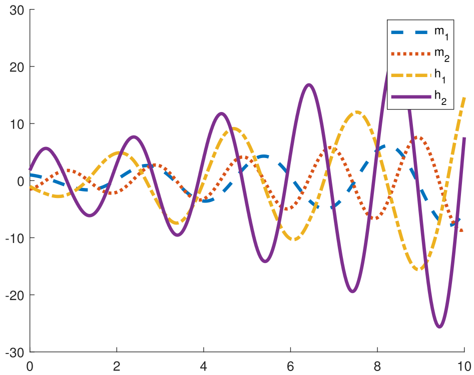

FIGURE 1. State trajectories without controller.

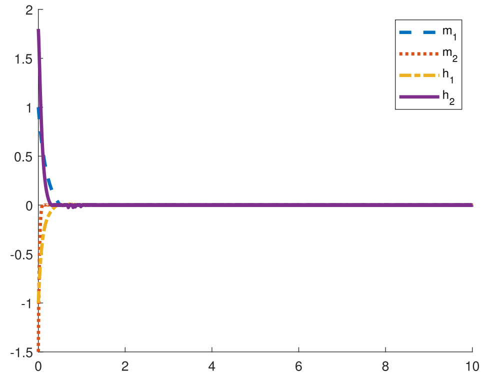

FIGURE 2. State trajectories under the controller (42).

$$c_{12}(m_1(t)) = \begin{cases} -1.5, |m_1| \le 0.1. \\ 1.5, |m_1| > 0.1. \end{cases}$$

$$c_{21}(m_2(t)) = \begin{cases} -0.2, |m_2| \le 0.1. \\ 0.2, |m_2| > 0.1. \end{cases}$$

$$c_{22}(m_2(t)) = \begin{cases} -0.25, |m_2| \le 0.1. \\ 0.25, |m_2| > 0.1. \end{cases}$$

$$d_{11}(m_1(t)) = \begin{cases} -1.4, |m_1| \le 0.1. \\ 1.4, |m_1| > 0.1. \end{cases}$$

$$d_{12}(m_1(t)) = \begin{cases} -0.5, |m_1| \le 0.1. \\ 0.5, |m_1| > 0.1. \end{cases}$$

$$d_{21}(m_2(t)) = \begin{cases} -0.3, |m_2| \le 0.1. \\ 0.3, |m_2| > 0.1. \end{cases}$$

$$d_{22}(m_2(t)) = \begin{cases} -2.5, |m_2| \le 0.1. \\ 2.5, |m_2| > 0.1. \end{cases}$$

$\sigma_1 = \sigma_2 = 0.3m_1(t) + 0.1m_1(t - \imath(t))$, therefore

$$S_1 = S_2 = \begin{pmatrix} 0.12 & 0 \\ 0 & 0.04 \end{pmatrix},$$

The state trajectories of MINNs (40) are illustrated in Fig. 1, without the implementation of any controller. From Fig. 1, the initial values of MINNs (40) are $\psi_1(s) = 1, \psi_2(s) = -1.5, \varphi_1(s) = -1, \varphi_2(s) = 1.8, s \in [-2, 0]$. We analyze the behavior of states in the time domain, which leads to the conclusion that the system (39) is unstable. Hence, our objective is to design suitable controller capable of stabilizing the unstable systems within a fixed time.

Next, we choose $q = 3.25$. Then, the delayed control law (12) is

$$\begin{cases} u_1 &= K_1 m(t) - K_2 sign(m(t))|m(t)| - K_3 sign(m(t))|m(t)|^q. \\ v_1 &= Z_1 \hbar(t) - Z_2 sign(\hbar(t))|\hbar(t)| - Z_3 sign(\hbar(t))|\hbar(t)|^q. \end{cases} \tag{42}$$

According to Theorem 1, we get

$$X_1 = \begin{pmatrix} 0.1638 & 0 \\ 0 & 0.0195 \end{pmatrix}, \ X_2 = \begin{pmatrix} 0.1192 & 0 \\ 0 & 0.0990 \end{pmatrix},$$

$$W_1 = \begin{pmatrix} 0.6141 & 0 \\ 0 & 0.52500 \end{pmatrix}, \ W_2 = \begin{pmatrix} 1.9479 & 0 \\ 0 & 1.2421 \end{pmatrix},$$

$$K_1 = \begin{pmatrix} 3.7496 & 0 \\ 0 & 26.9746 \end{pmatrix}, \ Z_1 = \begin{pmatrix} 16.3400 & 1.3996 \\ 0.6433 & 12.5510 \end{pmatrix},$$

$$Z_3 = Z_4 = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}, \ K_3 = K_4 = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix},$$

$$\check{M}_1 = \begin{pmatrix} 0.8890 & 0 \\ 0 & 0.8890 \end{pmatrix}, \ \check{M}_2 = \begin{pmatrix} 0.8890 & 0 \\ 0 & 0.8890 \end{pmatrix},$$

$$\check{\beta} = \begin{pmatrix} -6.5195 & 0 \\ 0 & -19.5585 \end{pmatrix}.$$

$\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = \varepsilon_5 = \varepsilon_6 = 1.0490.$

From Theorem 1, it can be concluded that the MINNs (41) can achieve stabilization within a fixed time using the delayed state feedback control law (42) with $\psi_1(s) = 1, \psi_2(s) = -1.5, \varphi_1(s) = -1, \varphi_2(s) = 1.8, s \in [-2, 0]$. Fig. 2 illustrates the state trajectories of system (41) under the controller (42). Moreover, it is evident that the states approach zero in fixed-time. This leads to the inference that MINNs (41) accomplish XTS via the utilization of the control laws (42).

According to Theorem 2, we design the following controller:

$$\begin{cases} u_2 = K_1 m(t) - K_2 sign(m(t))|m(t)|^p - K_3 sign(m(t))|m(t)|^q. \\ v_2 = Z_1 \hbar(t) - Z_2 sign(\hbar(t))|\hbar(t)|^p - Z_3 sign(\hbar(t))|\hbar(t)|^q. \end{cases} \tag{43}$$

Similarly, we choose $p = 0.3$, $q = 3.25$, and the feasible solutions can be obtained from LMI.

$$X_1 = \begin{pmatrix} 2.8956 & 0 \\ 0 & 4.7489 \end{pmatrix}, \ X_2 = \begin{pmatrix} 2.1154 & 0 \\ 0 & 1.7562 \end{pmatrix},$$

$$W_1 = \begin{pmatrix} 11.9117 & 0 \\ 0 & 12.0391 \end{pmatrix}, \ W_2 = \begin{pmatrix} 34.5657 & 0 \\ 0 & 22.0418 \end{pmatrix},$$

$$K_1 = \begin{pmatrix} 4.1136 & 0 \\ 0 & 2.5352 \end{pmatrix}, \ Z_1 = \begin{pmatrix} 16.3400 & 0 \\ 0 & 12.5510 \end{pmatrix},$$

$$Z_3 = Z_4 = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}, \ K_3 = K_4 = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix},$$

$$\check{M}_1 = \begin{pmatrix} 14.1027 & 0 \\ 0 & 14.1027 \end{pmatrix}, \ \check{M}_2 = \begin{pmatrix} 14.1027 & 0 \\ 0 & 14.1027 \end{pmatrix},$$

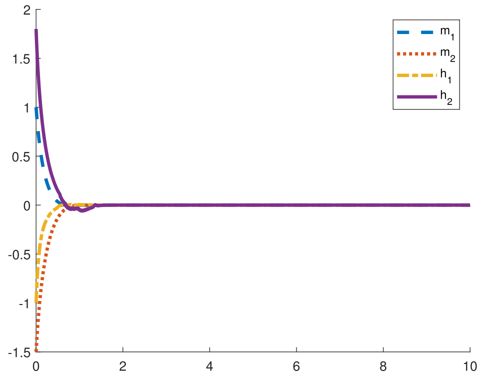

FIGURE 3. State trajectories under the controller (43).

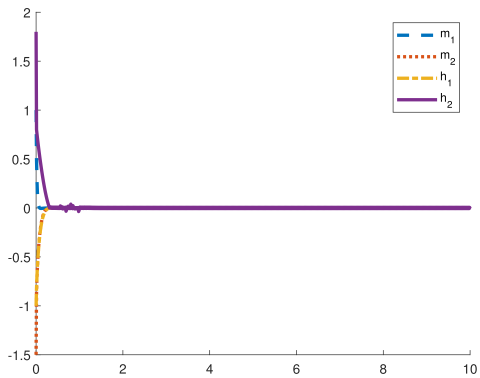

FIGURE 4. State trajectories under the controller (44).

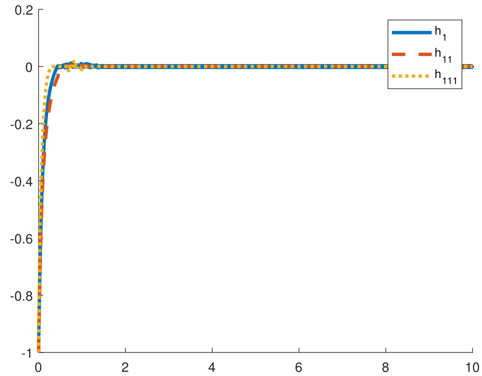

FIGURE 5. State trajectories of $\hbar_1$ under three controllers.

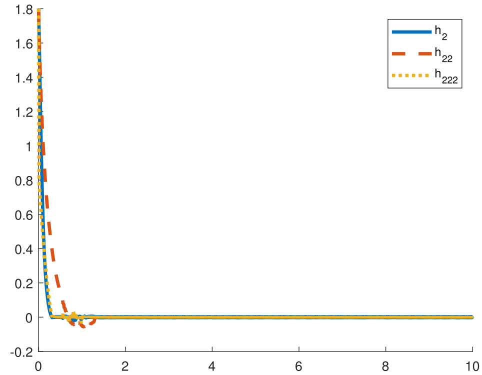

FIGURE 6. State trajectories of $\hbar_2$ under three controllers.

$$\check{\beta} = \begin{pmatrix} -108.1205 & 0 \\ 0 & -324.3616 \end{pmatrix}.$$

$\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = \varepsilon_5 = \varepsilon_6 = 18.6155$.

It can be concluded that system (10) is XTS. In Fig.3, it is clear that the state trajectories converge to zero for $\psi_1(s) = 1, \psi_2(s) = -1.5, \varphi_1(s) = -1, \varphi_2(s) = 1.8, s \in [-2, 0]$.

By comparing Fig.2 and Fig.3, it can be seen that among the two universal state feedback controllers of Theorem 1 and Theorem 2, the STF from Theorem 1 is less than that of Theorem 2, while the state trajectories of Theorem 2 are smoother.

In Theorem 3, we choose $p = 0.9$, $q = 4.25$. Then, adding a time delay term to the controllers to handle time delay in different ways, the controller (44) is as follows

$$\begin{cases} u_3 = K_1 m(t) - K_2 sign(m(t))|m(t)|^p - K_3 sign(m(t))|m(t)|^q. \\ v_3 = Z_1\hbar(t) - Z_2 sign(\hbar(t))|\hbar(t)|^p - Z_3 sign(\hbar(t))|\hbar(t)|^q \\ \quad +Z_4 sign(\hbar(t))|\hbar(t-\iota(t))| + \tilde{M} sign(\hbar(t))(\dfrac{|\hbar(t)|}{\| \hbar(t) \|^2}). \end{cases} \tag{44}$$

Correspondingly, the system parameters are taken as

$$A = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.07 \end{pmatrix}, \ B = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}, \ \Theta = \begin{pmatrix} 10 & 0 \\ 0 & 2 \end{pmatrix},$$

Then, one gets

$$X_1 = \begin{pmatrix} 0.0207 & 0 \\ 0 & 0.2500 \end{pmatrix}, \ X_2 = \begin{pmatrix} 0.5062 & 0 \\ 0 & 0.4737 \end{pmatrix},$$
$$W_1 = \begin{pmatrix} 0.4662 & 0 \\ 0 & 0.2909 \end{pmatrix}, \ W_2 = \begin{pmatrix} 0.9816 & 0 \\ 0 & -1.1420 \end{pmatrix},$$
$$W_3 = \begin{pmatrix} -0.5024 & 0 \\ 0 & -0.5288 \end{pmatrix}, \ W_4 = \begin{pmatrix} -0.9924 & 0 \\ 0 & -0.9924 \end{pmatrix},$$
$$K_1 = \begin{pmatrix} 22.4908 & 0 \\ 0 & 1.1636 \end{pmatrix}, \ K_2 = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix},$$
$$K_3 = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}, \ \check{M}_1 = \begin{pmatrix} 0.9007 & 0 \\ 0 & 0.8802 \end{pmatrix},$$
$$\check{M}_2 = \begin{pmatrix} 0.9025 & 0 \\ 0 & 0.9025 \end{pmatrix}, \ Z_1 = \begin{pmatrix} 1.9392 & 0 \\ 0 & -2.4107 \end{pmatrix},$$
$$Z_2 = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}, \ Z_3 = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix},$$
$$Z_4 = \begin{pmatrix} -1.9605 & 0 \\ 0 & -2.0948 \end{pmatrix}, \ \tilde{M} = \begin{pmatrix} -0.9925 & 0 \\ 0 & -1.1163 \end{pmatrix},$$
$$\check{\beta} = \begin{pmatrix} -6.6180 & 0 \\ 0 & -19.8540 \end{pmatrix},$$

$\varepsilon_1 = 1.1215$, $\varepsilon_2 = 1.0075$, $\varepsilon_3 = 1.0612$, $\varepsilon_4 = 0.9939$, $\varepsilon_5 = 1.0073$.

Thus, it is obvious in Fig.3 that state trajectories converge to zero, which verify the effectiveness of Theorem 3.

Based on Theorem 1 and Theorem 2, MINNs (41) can achieve XTS under the controllers (42) and (43). In fact, according to Theorem 3, MINNs (40) can also achieve XTS under controller (44). Numerical examples indicate that, under the same controller parameters, the settling time under controller (44) is smaller than the settling time under controllers (42) and (43). Therefore, compared to Theorem 1 and Theorem 2, Theorem 3 can provide a smaller upper estimate for the settling time.

Observing Figs. 5 and 6, it becomes apparent that the STF specified in Theorem 3 is shorter than that in Theorem 2. In other words, Theorem 3 better deals with the delay term of the systems by adding a delay term to the controllers and combining the $sign(m_i(t))$ function and absolute value function, thus providing a smaller upper estimate for the STF.

Furthermore, adopting a controller structured in matrix form not only simplifies its complexities but also optimizes computations in higher dimensions, thereby significantly enhancing the computational efficiency of the control system. This method enables us to meet various stability requirements within a unified control framework, simultaneously strengthening system stability and reducing control expenses.

V. CONCLUSION

This article has addressed the XTS of a type of stochastic MINNs. First, the approach begins with transforming the neural networks into systems with interval coefficients using the IMM. Simultaneously, by applying Filipov discontinuity theory and employing variable transformation, the issue of second-order equations is converted into a system of first-order equations. Through the design of Lyapunov-Krasovskii functional, Ito formula and LMI method, three different controllers are formulated to achieve FTS for any initial value. Subsequently, the acquired outcomes are represented in the format of LMI and can be validated through MATLAB. Additionally, the upper limit of FTS is estimated, which remains unaffected by the system's initial conditions. Furthermore, to emphasize the difference between FTS and XTS, this article extends some implications to guarantee FTS of the system. These results contribute to the advancement and enhancement of previous work on MINNs. Finally, the theoretical findings are validated through numerical simulations.

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