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RESEARCH ARTICLE

Linear Diophantine Fuzzy Clustering Algorithm Based on Correlation Coefficient and Analysis on Logistic Efficiency of Food Products

JEEVITHA KANNAN[1], VIMALA JAYAKUMAR[1], MUHAMMAD SAEED[2], TMADER ALBALLA[3], HAMIDEN ABD EL-WAHED KHALIFA[4,5], AND HEBA GHAREEB GOMAA[6]

[1]Department of Mathematics, Alagappa University, Karaikudi 630003, India
[2]Department of Mathematics, University of Management and Technology, Lahore, Punjab 54000, Pakistan
[3]Department of Mathematics, College of Sciences, Princess Nourah bint Abdulrahman University, P.O. Box 84428, Riyadh 11671, Saudi Arabia
[4]Department of Mathematics, College of Science, Qassim University, Buraydah 51452, Saudi Arabia
[5]Department of Operations and Management Research, Faculty of Graduate Studies for Statistical Research, Cairo University, Giza 12613, Egypt
[6]Institute for Management Information Systems, Suez 43332, Egypt

Corresponding author: Muhammad Saeed (muhammad.saeed@umt.edu.pk)

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ABSTRACT The significance of clustering algorithms lies in their ability to distinguish problems and devise customized solutions. In the broader context of clustering, fuzzy clustering is one of the crucial aspects. In response to the real-world clustering problems, this research suggests a new fuzzy cluster scheme of data under the linear diophantine fuzzy set(LDFS) framework. More precisely, LDF clustering is initiated with the aid of the correlation coefficient($\mathcal{CC}$) and weighted correlation coefficient($\mathcal{WCC}$) for LDFS. Due to their ability to quantify the degree of similarity between two elements, $\mathcal{CC}$ are valuable in clustering problems. The LDF- clustering algorithm comprises a well-integrated algorithm for managing uncertainty and $\mathcal{CC}$ among LDFS. Also, our approach to LDF clustering is compared to existing fuzzy clustering studies to assess its effectiveness. Since LDFS broadens the score space, the experimental evaluation of our proposed scheme enables Decision makers(DM) to freely select their score values. The theme of this study is to impart the commencement of LDF-clustering analysis and attempt to apply $\mathcal{CC}$ to the clustering problem. An interpretative example provides the analysis of the logistic efficiency of food products by employing an LDF-clustering algorithm.

INDEX TERMS LDFS, clustering algorithm, correlation coefficient, logistics, food products, optimization, decision making, algorithms.

I. INTRODUCTION

Clustering involves combining a set of objects into clusters that are associated with data characteristics, where the objects in clustered objects are more alike than those in other clusters. It is among the most commonly employed tools for analyzing data. Computational intelligence and pattern recognition research both heavily rely on clustering techniques. The applications of cluster analysis have been applied to various fields including pattern recognition, data mining, information retrieval, medicine, biology, and finance. Due to the vast amount of data that these fields handle, the techniques employed must be efficient in the usage of memory of the entire data set. The creation of specialized solutions to problems relies heavily on clustering algorithmic techniques. By clustering data, patterns, and order are made apparent by dividing it into related components.

Unlike classification, which involves class prediction on unlabeled data following supervised learning on pre-labeled data, clustering deals with unsupervised learning of unlabeled data. As a result, clustering algorithms can be used safely on a data set without much knowledge of it. The definition

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of a cluster can include both crisp and fuzzy features, with the former being defined as having definite boundaries and the latter being unclear. The process of clustering involves merging observed objects into clusters that satisfy the fundamental criteria listed below.

1. Objects that are part of the same group are considered similar. There are only homogeneous clusters.

2. Each cluster should be distinct from the others in such a way that the objects within each cluster are distinct from the objects found within the other clusters. Different clusters have heterogeneous compositions.

The variations among many fundamental items are frequently hazy in the actual world. When categorizing something, there will inevitably be a degree of uncertainty, which gives rise to fuzzy clustering analysis. Within the broader context of clustering, fuzzy clustering plays a very significant function. In 1965, Zadeh [53] created the fuzzy set(FS) theory. It is a group of objects with diverse levels of membership. Membership score(MS) is fuzzy numbers mappings that represent real values from 0 to 1 inclusive. Fuzzy clustering is a highly practical technology for data analysis. The field is still relatively new and expanding in numerous directions, with innovative and sophisticated new ideas, methodologies, and applications [32]. Later, in 1986, Atanassov [3] updated the FS and introduced an intuitionistic FS(IFS). These newer sets designate their participants to two distinct roles, membership and nonmembership score(NMS). Due to the fuzzy values used in these mappings, the sum of these mappings cannot be greater than 1I.

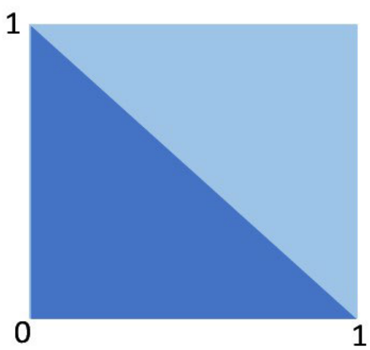

FIGURE 1. Codomain of IFS.

Numerous different clustering algorithms have been developed using IF data. The Pythagorean FS (PFS) was created by Yager [49], which makes it possible to reduce the IFS restrictions by retaining the sum of the squares of both scores within the unit intervalI. Later, Yager [50] developed q-rung orthopair fuzzy set(q-ROFS) as a generalization PFS, which relaxes the restrictions of PFS by the constraint that the sum of the q^{th} power of MS and NMS belongs to unit intervalI. But, certain constraints prevent decision-makers from selecting their own score. To alleviate these limitations, LDFS was designed.

The $\mathcal{CC}$ is a term that is used in statistical problems to describe the level of association between entities. One

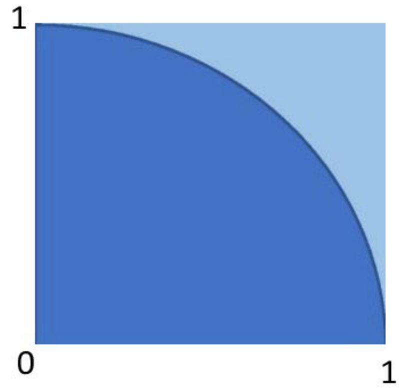

FIGURE 2. Codomain of PFS.

of the most often used metrics is correlation, which is also a crucial factor in procedures such as decision-making, pattern recognition, data analysis, and classification. Clustering problems can be tackled with the use of correlation coefficients, which indicate the level of similarity between two elements. In fuzzy sets, correlation measures outperform similarity measurements because they can determine the direction and strength of linkages in addition to determining the degree of resemblance. In addition to similarity metrics, correlations bring the advantage of both positive and negative relationships, offering a more complex interpretation of links across fuzzy sets. Correlation measures are better at recognizing broad patterns and trends within fuzzy sets because they can take covariance and variability into account. This leads to a more thorough study. Correlation measures provide a more comprehensive description of fuzzy set interactions by considering both similarity and dissimilarity factors. All things considered, their adaptability makes them the go-to option for a thorough study of fuzzy set relationships. A few $\mathcal{CC}$ were launched for various functional areas such as FS, IFS, PFS, and q-ROFS. However, due to their bounds limitations, they don't contain some kind of information. This motivates us to investigate the $\mathcal{CC}$ for LDFS since LDFS is the key aspect in eliminating the bounds on MS and NMS(I).

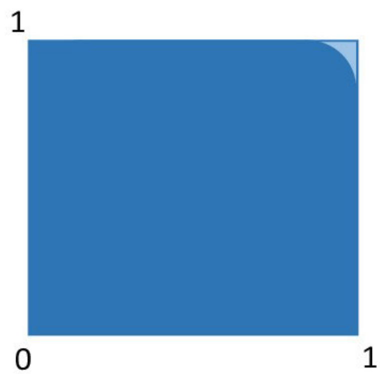

FIGURE 3. Codomain of q-ROFS.

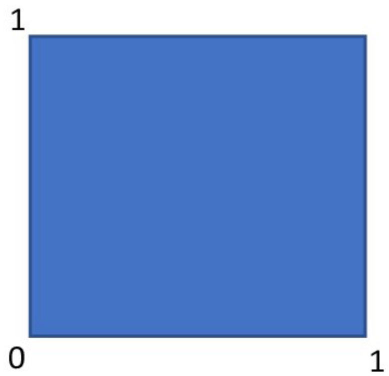

FIGURE 4. Codomain of LDFS.

A. LITERATURE REVIEW

Bonizzoni et al. [10] researched correlation clustering and consensus clustering. Akram et al. [4], [27] has accomplished some remarkable work and has suggested various clustering algorithms. According to Yang and Lin [51], they introduced type-2 FS similarity measures, which were used in clustering. The correlation coefficients of the hesitant FSs were established by Chen et al. [14] and applied to clustering analysis. The IFS clustering algorithm was explained by Xu et al. [47]. Chaudhuri [13] evaluated a probabilistic C means clustering algorithm under an IF environment. Lin et al. [28] suggests a novel distance measure that is based on the continuous optimal aggregation operator to address interval intuitionistic fuzzy clustering issues. Riaz et al., developed specialized fuzzy soft-max aggregation operators based on linear Diophantine sets for the development of an efficient algorithm for green supply chain issues [6].

Pearsons [36] has created a $\mathcal{CC}$ that finds application in various statistical analyses, including data analysis and classification patterns, clustering, medical diagnosis, and decision-making. The ineffectiveness of conventional correlation in managing data related to fuzzy scenarios has been acknowledged. To address these problems, several authors have broadened the definition of statistical correlation to fuzzy correlation. Chiang et al. [11] utilized mathematical statistics to determine the correlation coefficient between fuzzy data. Many Researchers have attempted to evaluate the impact of correlation between fuzzy sets [33], [52].

In 1991, Gerstenkorn and Manko [19] conducted the initial investigation into the $\mathcal{CC}$ of intuitionistic fuzzy sets. Bustince and Burillo [8] instigated the concept of a $\mathcal{CC}$ between the interval-valued intuitionistic fuzzy set(IVIFS). The $\mathcal{CC}$ and measure for IFSs in probabilistic spaces were investigated by Hong et Al. [20]. Many authors later researched and created correlations in various trends and other areas. Correlation studies carried out in the 1990s revealed that the coefficients of correlation are positioned between [0, 1]. There is a large number of $\mathcal{CC}$ introduced and analyzed between the interval 0 and 1 for the extensions of fuzzy sets. Later in the 21st century, scholars have been developing new methods for generating correlation coefficients for fuzzy sets within the range $[-1, 1]$ and it is reflected in the latest findings of some studies.

The $\mathcal{CC}$ between the IVIFSs are constructed by Nguyen and build a clustering analysis. Qu et al. [38] introduced a new clustering algorithm that utilizes $\mathcal{CC}$ for complex picture fuzzy sets to classify products by features. Park et al. [37] demonstrated the use of $\mathcal{CC}$ of IVIFS in MCDM. Garg [18] created a new $\mathcal{CC}$ among PFSs to assist individuals in making decisions. Ganie et al. [17] defined the PiFS correlation coordinators and applied them in medical diagnosis. Du [15] created the $\mathcal{CC}$ for the q-ROFS. Li and his team [30] utilized a clustering analysis, as an illustration, to showcase the superiority of the proposed method by examining two δ-$\mathcal{CC}$ for q-rung orthopair fuzzy conditions. Abbas et Al. [1] explores the clustering technique for q-ROF 2-tuple linguistic sets based on $\mathcal{CC}$. Through the use of counter-examples, Lin et [29] examines the inadequacies of the current $\mathcal{CC}$ and suggests a new directional correlation rate, as well as its weighted version for PFSs. Bashir [9] derived some improved $\mathcal{CC}$ for q-ROFS and suggested a cluster analysis in automobile classification.

Furthermore, the literature-based $\mathcal{CC}$ still exhibits certain restrictions on IFS, PFS, and q-ROFS. Hence, several academics in various systematic fields were interested in the practical benefits of LDFS, leading to the creation of several significant papers. Iampam et al. [21] explored it using different methods of Einstein aggregation for Multi-Criteria Decision Making(MCDM) problems, while Ayub et al. [5] determined the development of LDF relations and algebraic traits later on. In addition to describing the cosine similarity measure and its applications, Kamac [26] developed the highly complex LDFS. Riaz et al. [42] broadened the scope of the LDFS by including soft rough sets and their potential utilization in material handling equipment. Riaz et al. [41] created aggregation operators(AOs) that prioritized linear Diophantine fuzzy numbers (LDFNs) and utilized them to select third-party logistic service providers. Farid et al. [16] suggested the use of Einstein's prioritized linear Diophantine fuzzy AOs with applications. Frank AOs for linear Diophantine fuzzy numbers with interval values were recently developed by Riaz et al. [39]. By utilizing IVLDF data, Petchimuthu et al. [35] attempted to solve the supplier selection problem with its own AOs. Jeevitha et al. [25] discussed the problem of climate crisis by utilizing LDF DEMATEL approach. Subsequently, some enhancements of LDFS were addressed and discussed the application in the field of digital transformation [24], agri-drone [22], tender selection [23] medical diagnosis [46] Saeed et al. used hybrid fuzzy hypersoft structures for optimization and efficient decision-making purposes [43], [44], [45].

B. CONTRIBUTION OF THIS STUDY

The advancement in theory and the empirical use of clustering algorithms, LDFS, and $\mathcal{CC}$ led us to investigate these ideas.

The research seeks to propose innovative DM approaches that utilize correlation coefficients in practical scenarios, with the aim of establishing practical applications of these coefficient factors in FS theory. As a result, the fundamental components of this proposed clustering algorithm are established, such as the information energy of each LDFS, their relationship to each other, and the correlation between two LDFS. Following this, the equivalence matrix and matrix of $\mathcal{CC}$ are explained, and the associated composition of the correlation matrix is determined. The rules for λ-cutting classification are also discussed. The proposed definitions have a lot of intriguing properties and valuable outcomes. Additionally, the LDFS correlation coefficients are used to design the clustering algorithm. The presented algorithm is compared to the other methods discussed in the literature to verify the efficacy and suitability of LDF-Clustering.

In bringing out an innovative approach within the context of linear diophantine fuzzy sets (LDFS), this study significantly advances the field of clustering algorithms. In order to tackle real-world clustering problems, fuzzy clustering is essential. In this work, we present a novel fuzzy cluster scheme that makes use of the $\mathcal{CC}$ for LDFS. Adding CC to the LDF clustering algorithm improves its capacity to control uncertainty and measure how similar elements are to one another during the clustering process. Because of the enlarged score space of LDFS, the LDF-clustering algorithm described in this study not only offers a well-integrated solution to clustering problems but also gives decision-makers the freedom to choose score values.

The suggested LDF-clustering algorithm's applicability and efficacy are practically demonstrated through the interpretative example that centers on the logistic efficiency study of food products. This application demonstrates how the algorithm may be used to classify and examine logistics information pertaining to food items, offering valuable insights that can guide decision-making procedures. In the context of supply chain and logistics management, the larger consequence is that the suggested strategy helps to optimize logistical operations, which may result in increased effectiveness, lower costs, and better resource allocation. Overall, the study fills a practical need by providing a specialized tool for improving logistics efficiency through data-driven analysis and decision-making, in addition to furthering the subject of clustering algorithms. By using a range of features, LDF-Clustering is a versatile technology that can be used to build adaptable and flexible clusters, which will increase the logistic efficiency of food goods. This tactic enables more customised and effective logistical management of food goods, which raises customer satisfaction and efficiency.

Thus, this work lays the groundwork for the use of $\mathcal{CC}$ in clustering issue-solving and advances the field of LDF-clustering analysis.

C. HIGHLIGHTS AND EMPHASIS OF THIS STUDY

1. LDFS uses the reference parameter technique, which is becoming more and more used for in-depth analyses in various applications. This approach, which offers a broader space than existing sets, tries to address shortcomings in previous methods and provide a useful data selection.

2. The most obvious finding to emerge from this study is to extend the theory and applications of LDFS in terms of clustering analysis.

3. Another finding that stands out in this study is the formulation of $\mathcal{CC}$ for LDFS, which is employed for LDF-clustering.

4. An analysis of the literature review indicates that the present study is one of the pioneering effects in establishing the clustering algorithm in the LDFS environment. And this paper highlights this breakthrough.

5. The four different $\mathcal{CC}$ for LDFS has been identified in this work. The logistic effectiveness of food products has been analyzed with these $\mathcal{CC}$.

6. The comparative analysis with prior research has demonstrated and it exhibits the efficacy of this study.

7. The experimental section concludes with evaluations that highlight the precedence of the proposed method and the shortcomings of the foregoing methods.

D. RESEARCH GAP AND MOTIVATION

The lack of consideration for non-membership values in the literature that currently exists on fuzzy clustering is highlighted as the specific limitation that regulates the research gap. Non-membership values, which indicate the degree of dissimilarity or exclusion from a cluster, are important in fuzzy clustering. This gap is filled by the research, which gives decision-makers the freedom to select non-membership values based on personal preferences by incorporating non-membership grades into the suggested theory. This distinctive feature sets the suggested LDFS clustering algorithm apart from traditional fuzzy clustering techniques by improving its adaptability and applicability.

E. STRUCTURE OF THIS MANUSCRIPT

Section II comprises the basic definitions of existing research. The formulation of $\mathcal{CC}$ and $\mathcal{WCC}$ for LDFS has been assigned in Section III along with its characteristic theorems. In Section IV, the LDF-clustering algorithm is described and an example is analyzed. Also, it contains comparative analysis, shortcomings of prior research, precedence of the proposed method, and sensitive analysis. Section V concludes this study.

II. BACKGROUND OF THE STUDY

Some of the basic ideas underpinning existing systems are outlined in this section. $\mathfrak{P}$, the Universal set is used throughout the study.

Definition 1: A FS Δ on $\mathfrak{P}$ is interpreted as

$$\Delta = \{(\mathfrak{k}_i, \mu(\mathfrak{k}_i)) : \forall \mathfrak{k}_i \in \mathfrak{P}\}$$

where, $\mu(\mathfrak{k}_i) \in [0, 1]$ constituted as a MS for the member $\mathfrak{k}_i$ in $\mathfrak{P}$.

Definition 2: An IFS $\mathfrak{B}$ on $\mathfrak{P}$ is interpreted as

$$\mathfrak{B} = \{(\mathfrak{k}_i, \mu(\mathfrak{k}_i), \nu(\mathfrak{k}_i)) : \forall \mathfrak{k}_i \in \mathfrak{P}\}$$

where, $\mu(\mathfrak{k}_i), \nu(\mathfrak{k}_i) \in [0,1]$ *constituted as a MS and NMS contigent to the constraint* $0 \leq \mu(\mathfrak{k}_i) + \nu(\mathfrak{k}_i) \leq 1$ *for all the members* $\mathfrak{k}_i$ *in* $\mathfrak{P}$.

Definition 3: An LDFS $\mathfrak{L}$ *on* $\mathfrak{P}$ *is interpreted as*

$$\mathfrak{L} = \{(\mathfrak{k}_i, \langle \mu(\mathfrak{k}_i), \nu(\mathfrak{k}_i)\rangle, \langle \alpha(\mathfrak{k}_i), \beta(\mathfrak{k}_i)\rangle) : \forall \mathfrak{k}_i \in \mathfrak{P}\}$$

where, $\mu(\mathfrak{k}_i), \nu(\mathfrak{k}_i), \alpha(\mathfrak{k}_i), \beta(\mathfrak{k}_i) \in [0,1]$ *constituted as a MS and NMS and their reference parameters respectively. Also, they are contigent to the constraint* $0 \leq \alpha(\mathfrak{k}_i)\mu(\mathfrak{k}_i) + \beta(\mathfrak{k}_i)\nu(\mathfrak{k}_i) \leq 1$ *and* $0 \leq \alpha(\mathfrak{k}_i)+\beta(\mathfrak{k}_i) \leq 1$ *for all the members* $\mathfrak{k}_i$ *in* $\mathfrak{P}$.

III. CORRELATION COEFFICIENT FOR LINEAR DIOPHANTINE FUZZY SET

This section comprises four different $\mathcal{CC}$ for LDFS. Some of its theorems are utilized to explore its basic characteristics.

Definition 4: Let $\mathfrak{S}_1 = \{(\mathfrak{k}_i, \langle \mu_{\mathfrak{S}_1}(\mathfrak{k}_i), \nu_{\mathfrak{S}_1}(\mathfrak{k}_i)\rangle, \langle \alpha_{\mathfrak{S}_1}(\mathfrak{k}_i), \beta_{\mathfrak{S}_1}(\mathfrak{k}_i)\rangle) : \mathfrak{k}_i \in \mathfrak{P}\}$, $\mathfrak{S}_2 = \{(\mathfrak{k}_i, \langle \mu_{\mathfrak{S}_2}(\mathfrak{k}_i), \nu_{\mathfrak{S}_2}(\mathfrak{k}_i)\rangle, \langle \alpha_{\mathfrak{S}_2}(\mathfrak{k}_i), \beta_{\mathfrak{S}_2}(\mathfrak{k}_i)\rangle) : \mathfrak{k}_i \in \mathfrak{P}\}$ *be two* $\mathbb{LDFS}$ *over the set* $\mathfrak{P} = \{\mathfrak{k}_1, \mathfrak{k}_2, \mathfrak{k}_3, \ldots, \mathfrak{k}_\mathfrak{y}\}$. *Then the informational energies of* $\mathfrak{S}_1, \mathfrak{S}_2$ *are interpreted as*

$$\mathfrak{T}(\mathfrak{S}_1) = \sum_{i=1}^{\mathfrak{y}} \left((\mu_{\mathfrak{S}_1}(\mathfrak{k}_i))^2 + (\nu_{\mathfrak{S}_1}(\mathfrak{k}_i))^2 + (\alpha_{\mathfrak{S}_1}(\mathfrak{k}_i))^2 + (\beta_{\mathfrak{S}_1}(\mathfrak{k}_i))^2\right)$$

$$\mathfrak{T}(\mathfrak{S}_2) = \sum_{i=1}^{\mathfrak{y}} \left((\mu_{\mathfrak{S}_2}(\mathfrak{k}_i))^2 + (\nu_{\mathfrak{S}_2}(\mathfrak{k}_i))^2 + (\alpha_{\mathfrak{S}_2}(\mathfrak{k}_i))^2 + (\beta_{\mathfrak{S}_2}(\mathfrak{k}_i))^2\right)$$

Definition 5: Let $\mathfrak{S}_1$ *and* $\mathfrak{S}_2$ *be two* $\mathbb{LDFS}$. *Then the correlation between* $\mathfrak{S}_1$ *and* $\mathfrak{S}_2$ *is interpreted as*

$$\mathfrak{C}(\mathfrak{S}_1, \mathfrak{S}_2) = \left\{\begin{array}{l} \sum_{i=1}^{\mathfrak{y}} (\mu_{\mathfrak{S}_1}(\mathfrak{k}_i)\mu_{\mathfrak{S}_2}(\mathfrak{k}_i) + \nu_{\mathfrak{S}_1}(\mathfrak{k}_i)\nu_{\mathfrak{S}_2}(\mathfrak{k}_i) + \\ \alpha_{\mathfrak{S}_1}(\mathfrak{k}_i)\alpha_{\mathfrak{S}_2}(\mathfrak{k}_i) + \beta_{\mathfrak{S}_1}(\mathfrak{k}_i)\beta_{\mathfrak{S}_2}(\mathfrak{k}_i)) \end{array}\right\}$$

Remark:

(i)$\mathfrak{C}(\mathfrak{S}_1, \mathfrak{S}_1) = \mathfrak{T}(\mathfrak{S}_1)$

(ii)$\mathfrak{C}(\mathfrak{S}_1, \mathfrak{S}_2) = \mathfrak{C}(\mathfrak{S}_2, \mathfrak{S}_1)$.

Definition 6: Let $\mathfrak{S}_1$ *and* $\mathfrak{S}_2$ *be two* $\mathbb{LDFS}$ *over the set* $\mathfrak{P} = \{\mathfrak{k}_1, \mathfrak{k}_2, \mathfrak{k}_3, \ldots, \mathfrak{k}_\mathfrak{y}\}$. *Then the correlation coefficent*($\mathcal{CC}$) *of* $\mathfrak{S}_1, \mathfrak{S}_2$ *is interpreted as*

$$\mathfrak{Y}(\mathfrak{S}_1, \mathfrak{S}_2) = \frac{\mathfrak{C}(\mathfrak{S}_1, \mathfrak{S}_2)}{(\mathfrak{T}(\mathfrak{S}_1).\mathfrak{T}(\mathfrak{S}_2))^{\frac{1}{2}}}$$

Example 1: *Let* $\mathfrak{A} = \{ (\mathfrak{k}_1, \langle 0.7, 0.4\rangle, \langle 0.8, 0.1\rangle), (\mathfrak{k}_2, \langle 0.5, 0.7\rangle, \langle 0.6, 0.3\rangle), (\mathfrak{k}_3, \langle 0.8, 0.4\rangle, \langle 0.6, 0.4\rangle) \}$, $\mathfrak{B} = \{ (\mathfrak{k}_1, \langle 0.6, 0.4\rangle, \langle 0.5, 0.3\rangle), (\mathfrak{k}_2, \langle 0.7, 0.5\rangle, \langle 0.8, 0.2\rangle), (\mathfrak{k}_3, \langle 0.7, 0.6\rangle, \langle 0.6, 0.3\rangle)\}$ *be two LDFS.*

Then the informational energies of $\mathfrak{A}$ *and* $\mathfrak{B}$ *is calculated as*:

$$\begin{aligned}\mathfrak{T}(\mathfrak{A}) &= 0.7^2 + 0.4^2 + 0.8^2 + 0.1^2 + 0.5^2 + 0.7^2 + 0.6^2 \\ &\quad + 0.3^2 + 0.8^2 + 0.4^2 + 0.6^2 + 0.4^2 \\ &= 3.81\end{aligned}$$

$$\begin{aligned}\mathfrak{T}(\mathfrak{B}) &= 0.6^2 + 0.4^2 + 0.5^2 + 0.3^2 + 0.7^2 + 0.5^2 + 0.8^2 \\ &\quad + 0.2^2 + 0.7^2 + 0.6^2 + 0.6^2 + 0.3^2 \\ &= 3.58\end{aligned}$$

And the correlation between $\mathfrak{A}$ *and* $\mathfrak{B}$ *is calculated as*:

$$\begin{aligned}&\mathfrak{C}(\mathfrak{A}, \mathfrak{B}) \\ &= (0.7)(0.6) + (0.4)(0.4) + (0.8)(0.5) + (0.1)(0.3) \\ &\quad + (0.5)(0.7) + (0.7)(0.5) + (0.6)(0.8) + (0.3)(0.2) \\ &\quad + (0.8)(0.7) + (0.4)(0.6) + (0.6)(0.6) + (0.4)(0.3) \\ &= 3.53\end{aligned}$$

$$\begin{aligned}&\mathfrak{Y}(\mathfrak{S}_1, \mathfrak{S}_2) \\ &= \frac{\mathfrak{C}(\mathfrak{S}_1, \mathfrak{S}_2)}{(\mathfrak{T}(\mathfrak{S}_1).\mathfrak{T}(\mathfrak{S}_2))^{\frac{1}{2}}} \\ &= \frac{3.53}{((3.81)(3.58))^{\frac{1}{2}}} \\ &= \frac{3.53}{3.67} \\ &= 0.9619\end{aligned}$$

Theorem 1: Let $\mathfrak{S}_1, \mathfrak{S}_2$ *be two* $\mathbb{LDFS}$ *and* $\mathfrak{Y}(\mathfrak{S}_1, \mathfrak{S}_2)$ *be the* $\mathscr{CC}$ *of* $\mathfrak{S}_1$ *and* $\mathfrak{S}_2$. *Then*

(C1) $0 \leq \mathfrak{Y}(\mathfrak{S}_1, \mathfrak{S}_2) \leq 1$

(C2) $\mathfrak{Y}(\mathfrak{S}_1, \mathfrak{S}_2) = \mathfrak{Y}(\mathfrak{S}_2, \mathfrak{S}_1)$

(C3) $\mathfrak{Y}(\mathfrak{S}_1, \mathfrak{S}_2) = 1$ *if* $\mathfrak{S}_1 = \mathfrak{S}_2$

Proof: (C1) Since $\mu_{\mathfrak{S}_1}(\mathfrak{k}_i)$, $\nu_{\mathfrak{S}_1}(\mathfrak{k}_i)$, $\alpha_{\mathfrak{S}_1}(\mathfrak{k}_i)$, $\beta_{\mathfrak{S}_1}(\mathfrak{k}_i)$, $\mu_{\mathfrak{S}_2}(\mathfrak{k}_i)$,$\nu_{\mathfrak{S}_2}(\mathfrak{k}_i)$, $\alpha_{\mathfrak{S}_2}(\mathfrak{k}_i)$, $\beta_{\mathfrak{S}_2}(\mathfrak{k}_i) \in [0,1]$ for all $\mathfrak{k}_i \in \mathfrak{P}$, we have $\mathfrak{C}(\mathfrak{S}_1, \mathfrak{S}_2) \geq 0$, $\mathfrak{T}(\mathfrak{S}_1) \geq 0$, $\mathfrak{T}(\mathfrak{S}_2) \geq 0$, hence we have that

$$\mathfrak{Y}(\mathfrak{S}_1, \mathfrak{S}_2) \geq 0$$

Next, to prove that $\mathfrak{Y}(\mathfrak{S}_1, \mathfrak{S}_2) \leq 1$,

$$\begin{aligned}&\mathfrak{Y}(\mathfrak{S}_1, \mathfrak{S}_2) \\ &= \frac{\mathfrak{C}(\mathfrak{S}_1, \mathfrak{S}_2)}{(\mathfrak{T}(\mathfrak{S}_1).\mathfrak{T}(\mathfrak{S}_2))^{\frac{1}{2}}} \\ &= \frac{\left\{\begin{array}{l}\sum_{i=1}^{\mathfrak{y}}(\mu_{\mathfrak{S}_1}(\mathfrak{k}_i)\mu_{\mathfrak{S}_2}(\mathfrak{k}_i) + \nu_{\mathfrak{S}_1}(\mathfrak{k}_i)\nu_{\mathfrak{S}_2}(\mathfrak{k}_i) \\ \quad + \alpha_{\mathfrak{S}_1}(\mathfrak{k}_i)\alpha_{\mathfrak{S}_2}(\mathfrak{k}_i) + \beta_{\mathfrak{S}_1(\mathfrak{k}_i)\beta_{\mathfrak{S}_2}(\mathfrak{k}_i))}\end{array}\right\}}{(\mathfrak{T}(\mathfrak{S}_1).\mathfrak{T}(\mathfrak{S}_2))^{\frac{1}{2}}} \\ &= \frac{\left\{\begin{array}{l}\sum_{i=1}^{\mathfrak{y}}(\mu_{\mathfrak{S}_1}(\mathfrak{k}_i)\mu_{\mathfrak{S}_2}(\mathfrak{k}_i) + \nu_{\mathfrak{S}_1}(\mathfrak{k}_i)\nu_{\mathfrak{S}_2}(\mathfrak{k}_i) \\ \quad + \alpha_{\mathfrak{S}_1}(\mathfrak{k}_i)\alpha_{\mathfrak{S}_2}(\mathfrak{k}_i) + \beta_{\mathfrak{S}_1(\mathfrak{k}_i)\beta_{\mathfrak{S}_2}(\mathfrak{k}_i))}\end{array}\right\}}{\left\{\begin{array}{l}\sum_{i=1}^{\mathfrak{y}}((\mu_{\mathfrak{S}_1}(\mathfrak{k}_i))^2 + (\nu_{\mathfrak{S}_1}(\mathfrak{k}_i))^2 + (\alpha_{\mathfrak{S}_1}(\mathfrak{k}_i))^2 \\ \quad +(\beta_{\mathfrak{S}_1}(\mathfrak{k}_i))^2) \sum_{i=1}^{\mathfrak{y}}((\mu_{\mathfrak{S}_2}(\mathfrak{k}_i))^2 + \\ \quad (\nu_{\mathfrak{S}_2}(\mathfrak{k}_i))^2 + (\alpha_{\mathfrak{S}_2}(\mathfrak{k}_i))^2 + (\beta_{\mathfrak{S}_2}(\mathfrak{k}_i))^2)\end{array}\right\}^{\frac{1}{2}}}\end{aligned}$$

Also, we have

$$\begin{aligned}&\mathfrak{C}(\mathfrak{S}_1, \mathfrak{S}_2) \\ &= \left\{\begin{array}{l} \mu_{\mathfrak{S}_1}(\mathfrak{k}_1)\mu_{\mathfrak{S}_2}(\mathfrak{k}_1) + \nu_{\mathfrak{S}_1}(\mathfrak{k}_1)\nu_{\mathfrak{S}_2}(\mathfrak{k}_1) + \\ \alpha_{\mathfrak{S}_1}(\mathfrak{k}_1)\alpha_{\mathfrak{S}_2}(\mathfrak{k}_1) + \beta_{\mathfrak{S}_1}(\mathfrak{k}_1)\beta_{\mathfrak{S}_2}(\mathfrak{k}_1) \\ \mu_{\mathfrak{S}_1}(\mathfrak{k}_2)\mu_{\mathfrak{S}_2}(\mathfrak{k}_2) + \nu_{\mathfrak{S}_1}(\mathfrak{k}_2)\nu_{\mathfrak{S}_2}(\mathfrak{k}_2) + \\ \alpha_{\mathfrak{S}_1}(\mathfrak{k}_2)\alpha_{\mathfrak{S}_2}(\mathfrak{k}_2) + \beta_{\mathfrak{S}_1}(\mathfrak{k}_2)\beta_{\mathfrak{S}_2}(\mathfrak{k}_2) \\ + \ldots + \\ \mu_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{y})\mu_{\mathfrak{S}_2}(\mathfrak{k}_\mathfrak{y}) + \nu_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{y})\nu_{\mathfrak{S}_2}(\mathfrak{k}_\mathfrak{y}) + \\ \alpha_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{y})\alpha_{\mathfrak{S}_2}(\mathfrak{k}_\mathfrak{y}) + \beta_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{y})\beta_{\mathfrak{S}_2}(\mathfrak{k}_\mathfrak{y}) \end{array}\right\}\end{aligned}$$

$$\mathfrak{C}(\mathfrak{S}_1,\mathfrak{S}_2)^2 = \begin{Bmatrix} \mu_{\mathfrak{S}_1}(\mathfrak{k}_1)\mu_{\mathfrak{S}_2}(\mathfrak{k}_1)+\nu_{\mathfrak{S}_1}(\mathfrak{k}_1)\nu_{\mathfrak{S}_2}(\mathfrak{k}_1)+ \\ \alpha_{\mathfrak{S}_1}(\mathfrak{k}_1)\alpha_{\mathfrak{S}_2}(\mathfrak{k}_1)+\beta_{\mathfrak{S}_1}(\mathfrak{k}_1)\beta_{\mathfrak{S}_2}(\mathfrak{k}_1) \\ \mu_{\mathfrak{S}_1}(\mathfrak{k}_2)\mu_{\mathfrak{S}_2}(\mathfrak{k}_2)+\nu_{\mathfrak{S}_1}(\mathfrak{k}_2)\nu_{\mathfrak{S}_2}(\mathfrak{k}_2)+ \\ \alpha_{\mathfrak{S}_1}(\mathfrak{k}_2)\alpha_{\mathfrak{S}_2}(\mathfrak{k}_2)+\beta_{\mathfrak{S}_1}(\mathfrak{k}_2)\beta_{\mathfrak{S}_2}(\mathfrak{k}_2) \\ +\ldots+ \\ \mu_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{y})\mu_{\mathfrak{S}_2}(\mathfrak{k}_\mathfrak{y})+\nu_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{y})\nu_{\mathfrak{S}_2}(\mathfrak{k}_\mathfrak{y})+ \\ \alpha_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{y})\alpha_{\mathfrak{S}_2}(\mathfrak{k}_\mathfrak{y})+\beta_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{y})\beta_{\mathfrak{S}_2}(\mathfrak{k}_\mathfrak{y}) \end{Bmatrix}^2$$

Using cauchy schwatrz inequality, $(\sum u_i v_i)^2 \leq \sum u_i^2 \sum v_i^2$, we have

$$\mathfrak{C}(\mathfrak{S}_1,\mathfrak{S}_2)^2 \leq \begin{Bmatrix} (\mu_{\mathfrak{S}_1}(\mathfrak{k}_1))^2+(\nu_{\mathfrak{S}_1}(\mathfrak{k}_1))^2+(\alpha_{\mathfrak{S}_1}(\mathfrak{k}_1))^2+ \\ (\beta_{\mathfrak{S}_1}(\mathfrak{k}_1))^2+(\mu_{\mathfrak{S}_1}(\mathfrak{k}_2))^2+\ldots+ \\ (\mu_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{y}))^2+(\nu_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{y}))^2+(\alpha_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{y}))^2+ \\ (\beta_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{y}))^2\}\{(\mu_{\mathfrak{S}_2}(\mathfrak{k}_1))^2+(\nu_{\mathfrak{S}_1}(\mathfrak{k}_1))^2+ \\ (\alpha_{\mathfrak{S}_1}(\mathfrak{k}_1))^2+(\beta_{\mathfrak{S}_1}(\mathfrak{k}_1))^2+(\mu_{\mathfrak{S}_1}(\mathfrak{k}_2))^2 \\ +\ldots+(\mu_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{y}))^2+(\nu_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{y}))^2+ \\ (\alpha_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{y}))^2+(\beta_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{y}))^2\} \end{Bmatrix}$$

$$= \begin{Bmatrix} \left\{\sum_{\mathfrak{i}=1}^{\mathfrak{y}}\left((\mu_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{i}))^2+(\nu_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{i}))^2+\right.\right. \\ \left.\left.(\alpha_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{i}))^2+(\beta_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{i}))^2\right)\right\} \\ \left\{\sum_{\mathfrak{i}=1}^{\mathfrak{y}}\left((\mu_{\mathfrak{S}_2}(\mathfrak{k}_\mathfrak{i}))^2+(\nu_{\mathfrak{S}_2}(\mathfrak{k}_\mathfrak{i}))^2+\right.\right. \\ \left.\left.(\alpha_{\mathfrak{S}_2}(\mathfrak{k}_\mathfrak{i}))^2+(\beta_{\mathfrak{S}_2}(\mathfrak{k}_\mathfrak{i}))^2\right)\right\} \end{Bmatrix}$$

$$= \mathfrak{T}(\mathfrak{S}_1)\mathfrak{T}(\mathfrak{S}_2)$$

$$\mathfrak{C}(\mathfrak{S}_1,\mathfrak{S}_2) \leq (\mathfrak{T}(\mathfrak{S}_1).\mathfrak{T}(\mathfrak{S}_2))^{\frac{1}{2}}$$

Hence, $\mathfrak{Y}(\mathfrak{S}_1,\mathfrak{S}_2) \leq 1$.

(C2) The proof is straight forward.

(C3) If $\mathfrak{S}_1 = \mathfrak{S}_2$, then

$$\mathfrak{Y}(\mathfrak{S}_1,\mathfrak{S}_2) = \frac{\mathfrak{C}(\mathfrak{S}_1,\mathfrak{S}_2)}{\sqrt{\mathfrak{T}(\mathfrak{S}_1).\mathfrak{T}(\mathfrak{S}_2)}} = \frac{\mathfrak{C}(\mathfrak{S}_1,\mathfrak{S}_1)}{\sqrt{\mathfrak{T}(\mathfrak{S}_1).\mathfrak{T}(\mathfrak{S}_1)}} = \frac{\mathfrak{T}(\mathfrak{S}_1)}{\mathfrak{T}(\mathfrak{S}_1)}$$

■

Definition 7: Let $\mathfrak{w}_\mathfrak{i} = \{\mathfrak{w}_1, \mathfrak{w}_2, \ldots, \mathfrak{w}_\mathfrak{y}\}$ *be the weight vector for the elements in the universe set such that* $\sum_i \mathfrak{w}_\mathfrak{i} = 0$ *for all* $\mathfrak{w}_\mathfrak{i} > 0$. *Let* $\mathfrak{S}_1$ *and* $\mathfrak{S}_2$ *be two* $\mathbb{LDFS}$ *over the set* $\mathfrak{P} = \{\mathfrak{k}_1, \mathfrak{k}_2, \mathfrak{k}_3, \ldots, \mathfrak{k}_\mathfrak{y}\}$. *Then the weighted correlation coefficient(WCC) of* $\mathfrak{S}_1, \mathfrak{S}_2$ *is interpreted as*

$$\mathfrak{Y}_\mathfrak{w}(\mathfrak{S}_1,\mathfrak{S}_2) = \frac{\mathfrak{C}(\mathfrak{S}_1,\mathfrak{S}_2)}{(\mathfrak{T}(\mathfrak{S}_1).\mathfrak{T}(\mathfrak{S}_2))^{\frac{1}{2}}}$$

The expression can be written as shown in the equation at the bottom of the next page.

Theorem 2: Let $\mathfrak{S}_1$, $\mathfrak{S}_2$ *be two* $\mathbb{LDFS}$ *and* $\mathfrak{Y}_\mathfrak{w}(\mathfrak{S}_1,\mathfrak{S}_2)$ *be the* $\mathscr{WCC}$ *of* $\mathfrak{S}_1$ *and* $\mathfrak{S}_2$. *Then*

(C1) $0 \leq \mathfrak{Y}_\mathfrak{w}(\mathfrak{S}_1,\mathfrak{S}_2) \leq 1$

(C2) $\mathfrak{Y}_\mathfrak{w}(\mathfrak{S}_1,\mathfrak{S}_2) = \mathfrak{Y}(\mathfrak{S}_2,\mathfrak{S}_1)$

(C3) $\mathfrak{Y}_\mathfrak{w}(\mathfrak{S}_1,\mathfrak{S}_2) = 1$ *if* $\mathfrak{S}_1 = \mathfrak{S}_2$

Proof: (C1) Since $\mu_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{i})$,$\nu_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{i})$,$\alpha_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{i})$,$\beta_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{i})$, $\mu_{\mathfrak{S}_2}(\mathfrak{k}_\mathfrak{i})$, $\nu_{\mathfrak{S}_2}(\mathfrak{k}_\mathfrak{i})$,$\alpha_{\mathfrak{S}_2}(\mathfrak{k}_\mathfrak{i})$,$\beta_{\mathfrak{S}_2}(\mathfrak{k}_\mathfrak{i}) \in [0, 1]$ for all $\mathfrak{k}_\mathfrak{i} \in \mathfrak{P}$, we have $\mathfrak{C}(\mathfrak{S}_1,\mathfrak{S}_2) \geq 0$, $\mathfrak{T}(\mathfrak{S}_1) \geq 0$, $\mathfrak{T}(\mathfrak{S}_2) \geq 0$, hence we have that

$$\mathfrak{Y}_\mathfrak{w}(\mathfrak{S}_1,\mathfrak{S}_2) \geq 0$$

Next, to prove that $\mathfrak{Y}_\mathfrak{w}(\mathfrak{S}_1,\mathfrak{S}_2) \leq 1$,

$$\mathfrak{Y}_\mathfrak{w}(\mathfrak{S}_1,\mathfrak{S}_2) = \frac{\mathfrak{C}(\mathfrak{S}_1,\mathfrak{S}_2)}{(\mathfrak{T}(\mathfrak{S}_1).\mathfrak{T}(\mathfrak{S}_2))^{\frac{1}{2}}} = \frac{\left\{\begin{array}{r}\sum_{\mathfrak{i}=1}^{\mathfrak{y}}\mathfrak{w}_\mathfrak{i}(\mu_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{i})\mu_{\mathfrak{S}_2}(\mathfrak{k}_\mathfrak{i})+\nu_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{i})\nu_{\mathfrak{S}_2}(\mathfrak{k}_\mathfrak{i})+ \\ \alpha_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{i})\alpha_{\mathfrak{S}_2}(\mathfrak{k}_\mathfrak{i})+\beta_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{i})\beta_{\mathfrak{S}_2}(\mathfrak{k}_\mathfrak{i}))\end{array}\right\}}{\left\{\begin{array}{r}\sum_{\mathfrak{i}=1}^{\mathfrak{y}}\mathfrak{w}_\mathfrak{i}((\mu_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{i}))^2+(\nu_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{i}))^2+\alpha_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{i}))^2 \\ +(\beta_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{i}))^2)\sum_{\mathfrak{i}=1}^{\mathfrak{y}}\mathfrak{w}_\mathfrak{i}((\mu_{\mathfrak{S}_2}(\mathfrak{k}_\mathfrak{i}))^2+ \\ (\nu_{\mathfrak{S}_2}(\mathfrak{k}_\mathfrak{i}))^2+(\alpha_{\mathfrak{S}_2}(\mathfrak{k}_\mathfrak{i}))^2+(\beta_{\mathfrak{S}_2}(\mathfrak{k}_\mathfrak{i}))^2)\end{array}\right\}^{\frac{1}{2}}}$$

Also, we have, as shown in the equation at the bottom of the next page.

Using cauchy schwatrz inequality, $(\sum u_i v_i)^2 \leq \sum u_i^2 \sum v_i^2$, we have

$$\mathfrak{C}(\mathfrak{S}_1,\mathfrak{S}_2)^2 \leq \begin{Bmatrix} [(\sqrt{\mathfrak{w}_1}\mu_{\mathfrak{S}_1}(\mathfrak{k}_1))^2+(\sqrt{\mathfrak{w}_1}\nu_{\mathfrak{S}_1}(\mathfrak{k}_1))^2+ \\ (\sqrt{\mathfrak{w}_1}\alpha_{\mathfrak{S}_1}(\mathfrak{k}_1))^2+(\sqrt{\mathfrak{w}_1}\beta_{\mathfrak{S}_1}(\mathfrak{k}_1))^2+ \\ (\sqrt{\mathfrak{w}_2}\mu_{\mathfrak{S}_1}(\mathfrak{k}_2))^2+\ldots++(\sqrt{\mathfrak{w}_\mathfrak{y}}\alpha_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{y}))^2+ \\ (\sqrt{\mathfrak{w}_\mathfrak{y}}\beta_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{y}))^2][(\sqrt{\mathfrak{w}_1}\mu_{\mathfrak{S}_2}(\mathfrak{k}_1))^2+ \\ (\sqrt{\mathfrak{w}_1}\nu_{\mathfrak{S}_1}(\mathfrak{k}_1))^2+(\sqrt{\mathfrak{w}_1}\alpha_{\mathfrak{S}_1}(\mathfrak{k}_1))^2+ \\ (\sqrt{\mathfrak{w}_1}\beta_{\mathfrak{S}_1}(\mathfrak{k}_1))^2+(\sqrt{\mathfrak{w}_2}\mu_{\mathfrak{S}_1}(\mathfrak{k}_2))^2 \\ +\ldots+(\sqrt{\mathfrak{w}_\mathfrak{y}}\alpha_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{y}))^2+(\sqrt{\mathfrak{w}_\mathfrak{y}}\beta_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{y}))^2] \end{Bmatrix}$$

$$\leq \begin{Bmatrix} \left\{\sum_{\mathfrak{i}=1}^{\mathfrak{y}}\mathfrak{w}_\mathfrak{i}\left((\mu_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{i}))^2+(\nu_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{i}))^2+\right.\right. \\ \left.\left.(\alpha_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{i}))^2+(\beta_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{i}))^2\right)\right\}\times \\ \left\{\sum_{\mathfrak{i}=1}^{\mathfrak{y}}\mathfrak{w}_\mathfrak{i}\left((\mu_{\mathfrak{S}_2}(\mathfrak{k}_\mathfrak{i}))^2+(\nu_{\mathfrak{S}_2}(\mathfrak{k}_\mathfrak{i}))^2+\right.\right. \\ \left.\left.(\alpha_{\mathfrak{S}_2}(\mathfrak{k}_\mathfrak{i}))^2+(\beta_{\mathfrak{S}_2}(\mathfrak{k}_\mathfrak{i}))^2\right)\right\} \end{Bmatrix}$$

$$= \mathfrak{T}(\mathfrak{S}_1)\mathfrak{T}(\mathfrak{S}_2)$$

$$\mathfrak{C}(\mathfrak{S}_1,\mathfrak{S}_2) \leq (\mathfrak{T}(\mathfrak{S}_1).\mathfrak{T}(\mathfrak{S}_2))^{\frac{1}{2}}$$

Hence, $\mathfrak{Y}_\mathfrak{w}(\mathfrak{S}_1,\mathfrak{S}_2) \leq 1$.

(C2) The proof is straight forward.

(C3) If $\mathfrak{S}_1 = \mathfrak{S}_2$, then

$$\mathfrak{Y}_\mathfrak{w}(\mathfrak{S}_1,\mathfrak{S}_2) = \frac{\mathfrak{C}(\mathfrak{S}_1,\mathfrak{S}_2)}{\sqrt{\mathfrak{T}(\mathfrak{S}_1).\mathfrak{T}(\mathfrak{S}_2)}}$$

$$= \frac{\mathfrak{C}(\mathfrak{S}_1, \mathfrak{S}_1)}{\sqrt{\mathfrak{T}(\mathfrak{S}_1).\mathfrak{T}(\mathfrak{S}_1)}}$$
$$= \frac{\mathfrak{T}(\mathfrak{S}_1)}{\mathfrak{T}(\mathfrak{S}_1)}$$

■

Definition 8: Let $\mathfrak{S}_1$ and $\mathfrak{S}_2$ be two $\mathbb{LDFS}$ over the set $\mathfrak{P} = \{\mathfrak{k}_1, \mathfrak{k}_2, \mathfrak{k}_3, \ldots, \mathfrak{k}_\mathfrak{y}\}$. Then the correlation coefficent($\mathscr{CC}$) of $\mathfrak{S}_1, \mathfrak{S}_2$ is interpreted as

$$\mathfrak{Y}'(\mathfrak{S}_1, \mathfrak{S}_2) = \frac{\mathfrak{C}(\mathfrak{S}_1, \mathfrak{S}_2)}{\max(\mathfrak{T}(\mathfrak{S}_1), \mathfrak{T}(\mathfrak{S}_2))}$$

Theorem 3: Let $\mathfrak{S}_1, \mathfrak{S}_2$ be two $\mathbb{LDFS}$ and $\mathfrak{Y}'(\mathfrak{S}_1, \mathfrak{S}_2)$ be the $\mathscr{CC}$ of $\mathfrak{S}_1$ and $\mathfrak{S}_2$. Then

(C1) $0 \leq \mathfrak{Y}'(\mathfrak{S}_1, \mathfrak{S}_2) \leq 1$

(C2) $\mathfrak{Y}'(\mathfrak{S}_1, \mathfrak{S}_2) = \mathfrak{Y}'(\mathfrak{S}_2, \mathfrak{S}_1)$

(C3) $\mathfrak{Y}'(\mathfrak{S}_1, \mathfrak{S}_2) = 1$ if $\mathfrak{S}_1 = \mathfrak{S}_2$

Proof: (C1) Since $\mu_{\mathfrak{S}_1}(\mathfrak{k}_i)$, $\nu_{\mathfrak{S}_1}(\mathfrak{k}_i)$, $\alpha_{\mathfrak{S}_1}(\mathfrak{k}_i)$, $\beta_{\mathfrak{S}_1}(\mathfrak{k}_i)$, $\mu_{\mathfrak{S}_2}(\mathfrak{k}_i)$,$\nu_{\mathfrak{S}_2}(\mathfrak{k}_i)$, $\alpha_{\mathfrak{S}_2}(\mathfrak{k}_i)$, $\beta_{\mathfrak{S}_2}(\mathfrak{k}_i) \in [0, 1]$ for all $\mathfrak{k}_i \in \mathfrak{P}$, we have $\mathfrak{C}(\mathfrak{S}_1, \mathfrak{S}_2) \geq 0$, $\mathfrak{T}(\mathfrak{S}_1) \geq 0$, $\mathfrak{T}(\mathfrak{S}_2) \geq 0$, hence we have that

$$\mathfrak{Y}'(\mathfrak{S}_1, \mathfrak{S}_2) \geq 0$$

Next, to prove that $\mathfrak{Y}'(\mathfrak{S}_1, \mathfrak{S}_2) \leq 1$,

$$\mathfrak{Y}'(\mathfrak{S}_1, \mathfrak{S}_2) = \frac{\left\{\begin{array}{r} \sum_{i=1}^{\mathfrak{y}} (\mu_{\mathfrak{S}_1}(\mathfrak{k}_i)\mu_{\mathfrak{S}_2}(\mathfrak{k}_i) + \nu_{\mathfrak{S}_1}(\mathfrak{k}_i)\nu_{\mathfrak{S}_2}(\mathfrak{k}_i) \\ + \alpha_{\mathfrak{S}_1}(\mathfrak{k}_i)\alpha_{\mathfrak{S}_2}(\mathfrak{k}_i) + \beta_{\mathfrak{S}_1(\mathfrak{k}_i)\beta_{\mathfrak{S}_2}(\mathfrak{k}_i))} \end{array}\right\}}{\max(\mathfrak{T}(\mathfrak{S}_1), \mathfrak{T}(\mathfrak{S}_2))}$$

Also, from the previous theorem we have

$$\mathfrak{C}(\mathfrak{S}_1, \mathfrak{S}_2) \leq \max(\mathfrak{T}(\mathfrak{S}_1), \mathfrak{T}(\mathfrak{S}_2))$$
$$\implies \mathfrak{Y}'(\mathfrak{S}_1, \mathfrak{S}_2) \leq 1.$$

(C2) The proof is straight forward.

(C3) If $\mathfrak{S}_1 = \mathfrak{S}_2$, then

$$\mathfrak{Y}'(\mathfrak{S}_1, \mathfrak{S}_2) = \frac{\mathfrak{C}(\mathfrak{S}_1, \mathfrak{S}_2)}{\max\{\mathfrak{T}(\mathfrak{S}_1), \mathfrak{T}(\mathfrak{S}_2)\}}$$
$$= \frac{\mathfrak{C}(\mathfrak{S}_1, \mathfrak{S}_1)}{\max\{\mathfrak{T}(\mathfrak{S}_1), \mathfrak{T}(\mathfrak{S}_1)\}}$$
$$= \frac{\mathfrak{T}(\mathfrak{S}_1)}{\mathfrak{T}(\mathfrak{S}_1)}$$

■

Definition 9: Let $\mathfrak{S}_1$ and $\mathfrak{S}_2$ be two $\mathbb{LDFS}$ over the set $\mathfrak{P} = \{\mathfrak{k}_1, \mathfrak{k}_2, \mathfrak{k}_3, \ldots, \mathfrak{k}_\mathfrak{y}\}$. Then the weighted correlation coefficent($\mathscr{CC}$) of $\mathfrak{S}_1, \mathfrak{S}_2$ is interpreted as shown in the equation at the bottom of the next page.

Theorem 4: Let $\mathfrak{S}_1$, $\mathfrak{S}_2$ be two $\mathbb{LDFS}$ and $\mathfrak{Y}'_\mathfrak{w}(\mathfrak{S}_1, \mathfrak{S}_2)$ be the $\mathscr{CC}$ of $\mathfrak{S}_1$ and $\mathfrak{S}_2$. Then

(C1) $0 \leq \mathfrak{Y}'_\mathfrak{w}(\mathfrak{S}_1, \mathfrak{S}_2) \leq 1$

(C2) $\mathfrak{Y}'_\mathfrak{w}(\mathfrak{S}_1, \mathfrak{S}_2) = \mathfrak{Y}'(\mathfrak{S}_2, \mathfrak{S}_1)$

(C3) $\mathfrak{Y}'_\mathfrak{w}(\mathfrak{S}_1, \mathfrak{S}_2) = 1$ if $\mathfrak{S}_1 = \mathfrak{S}_2$

Proof: Similar to the previous one. ■

IV. LDF-CLUSTERING ALGORITHM

In this section, the LDF-clustering algorithm is established by employing the definitions of CC, Correlation matrix, and Equivalance matrix. An illustration of the logistic efficiency of food products is experimented with to represent the applicability and reliability of the suggested clustering algorithm.

Definition 10: Let $\mathfrak{S}_\mathfrak{r}$ be the set of $\mathbb{LDFS}$ and $\mathcal{C} = (\zeta_{ij})_{r\times r}$ be a correlation matrix($\mathscr{CM}$), where $\zeta_{ij} = \mathfrak{Y}(\mathfrak{S}_1, \mathfrak{S}_2)$ and it satisfies,

1) $0 \leq \zeta_{ij} \leq 1$, $i, j = 1, 2 \ldots, r$.
2) $\zeta_{ij} = 1$, when $i = j$, $i = 1, 2, \ldots, r$.
3) $\zeta_{ij} = \zeta_{ji}$, $i, j = 1, 2 \ldots, r$.

$$\mathfrak{Y}_\mathfrak{w}(\mathfrak{S}_1, \mathfrak{S}_2) = \left\{ \frac{\sum_{i=1}^{\mathfrak{y}} \mathfrak{w}_i(\mu_{\mathfrak{S}_1}(\mathfrak{k}_i)\mu_{\mathfrak{S}_2}(\mathfrak{k}_i) + \nu_{\mathfrak{S}_1}(\mathfrak{k}_i)\nu_{\mathfrak{S}_2}(\mathfrak{k}_i) + \alpha_{\mathfrak{S}_1}(\mathfrak{k}_i)\alpha_{\mathfrak{S}_2}(\mathfrak{k}_i) + \beta_{\mathfrak{S}_1}(\mathfrak{k}_i)\beta_{\mathfrak{S}_2}(\mathfrak{k}_i))}{\left\{\begin{array}{c} \sum_{i=1}^{\mathfrak{y}} \mathfrak{w}_i((\mu_{\mathfrak{S}_1}(\mathfrak{k}_i))^2 + (\nu_{\mathfrak{S}_1}(\mathfrak{k}_i))^2 + (\alpha_{\mathfrak{S}_1}(\mathfrak{k}_i))^2 + (\beta_{\mathfrak{S}_1}(\mathfrak{k}_i))^2) \\ \sum_{i=1}^{\mathfrak{y}} \mathfrak{w}_i((\mu_{\mathfrak{S}_2}(\mathfrak{k}_i))^2 + (\nu_{\mathfrak{S}_2}(\mathfrak{k}_i))^2 + (\alpha_{\mathfrak{S}_2}(\mathfrak{k}_i))^2 + (\beta_{\mathfrak{S}_2}(\mathfrak{k}_i))^2) \end{array}\right\}^{\frac{1}{2}}} \right\}$$

$$\mathfrak{C}(\mathfrak{S}_1, \mathfrak{S}_2) = \left\{\begin{array}{l} \sqrt{\mathfrak{w}_1}\mu_{\mathfrak{S}_1}(\mathfrak{k}_1)\sqrt{\mathfrak{w}_1}\mu_{\mathfrak{S}_2}(\mathfrak{k}_1) + \sqrt{\mathfrak{w}_1}\nu_{\mathfrak{S}_1}(\mathfrak{k}_1)\sqrt{\mathfrak{w}_1}\nu_{\mathfrak{S}_2}(\mathfrak{k}_1) + \\ \sqrt{\mathfrak{w}_1}\alpha_{\mathfrak{S}_1}(\mathfrak{k}_1)\sqrt{\mathfrak{w}_1}\alpha_{\mathfrak{S}_2}(\mathfrak{k}_1) + \sqrt{\mathfrak{w}_1}\beta_{\mathfrak{S}_1}(\mathfrak{k}_1)\sqrt{\mathfrak{w}_1}\beta_{\mathfrak{S}_2}(\mathfrak{k}_1) \\ \sqrt{\mathfrak{w}_2}\mu_{\mathfrak{S}_1}(\mathfrak{k}_2)\sqrt{\mathfrak{w}_2}\mu_{\mathfrak{S}_2}(\mathfrak{k}_2) + \sqrt{\mathfrak{w}_2}\nu_{\mathfrak{S}_1}(\mathfrak{k}_2)\sqrt{\mathfrak{w}_2}\nu_{\mathfrak{S}_2}(\mathfrak{k}_2) + \\ \sqrt{\mathfrak{w}_2}\alpha_{\mathfrak{S}_1}(\mathfrak{k}_2)\sqrt{\mathfrak{w}_2}\alpha_{\mathfrak{S}_2}(\mathfrak{k}_2) + \sqrt{\mathfrak{w}_2}\beta_{\mathfrak{S}_1}(\mathfrak{k}_2)\sqrt{\mathfrak{w}_2}\beta_{\mathfrak{S}_2}(\mathfrak{k}_2) \\ + \ldots + \\ \sqrt{\mathfrak{w}_\mathfrak{y}}\alpha_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{y})\sqrt{\mathfrak{w}_\mathfrak{y}}\alpha_{\mathfrak{S}_2}(\mathfrak{k}_\mathfrak{y}) + \sqrt{\mathfrak{w}_\mathfrak{y}}\beta_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{y})\sqrt{\mathfrak{w}_\mathfrak{y}}\beta_{\mathfrak{S}_2}(\mathfrak{k}_\mathfrak{y}) \end{array}\right\}$$

$$\mathfrak{C}(\mathfrak{S}_1, \mathfrak{S}_2)^2 = \left\{\begin{array}{l} \sqrt{\mathfrak{w}_1}\mu_{\mathfrak{S}_1}(\mathfrak{k}_1)\sqrt{\mathfrak{w}_1}\mu_{\mathfrak{S}_2}(\mathfrak{k}_1) + \sqrt{\mathfrak{w}_1}\nu_{\mathfrak{S}_1}(\mathfrak{k}_1)\sqrt{\mathfrak{w}_1}\nu_{\mathfrak{S}_2}(\mathfrak{k}_1) + \\ \sqrt{\mathfrak{w}_1}\alpha_{\mathfrak{S}_1}(\mathfrak{k}_1)\sqrt{\mathfrak{w}_1}\alpha_{\mathfrak{S}_2}(\mathfrak{k}_1) + \sqrt{\mathfrak{w}_1}\beta_{\mathfrak{S}_1}(\mathfrak{k}_1)\sqrt{\mathfrak{w}_1}\beta_{\mathfrak{S}_2}(\mathfrak{k}_1) \\ \sqrt{\mathfrak{w}_2}\mu_{\mathfrak{S}_1}(\mathfrak{k}_2)\sqrt{\mathfrak{w}_2}\mu_{\mathfrak{S}_2}(\mathfrak{k}_2) + \sqrt{\mathfrak{w}_2}\nu_{\mathfrak{S}_1}(\mathfrak{k}_2)\sqrt{\mathfrak{w}_2}\nu_{\mathfrak{S}_2}(\mathfrak{k}_2) + \\ \sqrt{\mathfrak{w}_2}\alpha_{\mathfrak{S}_1}(\mathfrak{k}_2)\sqrt{\mathfrak{w}_2}\alpha_{\mathfrak{S}_2}(\mathfrak{k}_2) + \sqrt{\mathfrak{w}_2}\beta_{\mathfrak{S}_1}(\mathfrak{k}_2)\sqrt{\mathfrak{w}_2}\beta_{\mathfrak{S}_2}(\mathfrak{k}_2) \\ + \ldots + \\ \sqrt{\mathfrak{w}_\mathfrak{y}}\alpha_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{y})\sqrt{\mathfrak{w}_\mathfrak{y}}\alpha_{\mathfrak{S}_2}(\mathfrak{k}_\mathfrak{y}) + \sqrt{\mathfrak{w}_\mathfrak{y}}\beta_{\mathfrak{S}_1}(\mathfrak{k}_\mathfrak{y})\sqrt{\mathfrak{w}_\mathfrak{y}}\beta_{\mathfrak{S}_2}(\mathfrak{k}_\mathfrak{y}) \end{array}\right\}^2$$

Definition 11: *Let $\mathcal{C} = (\zeta_{ij})_{r\times r}$ be the $\mathscr{CM}$, if $\mathcal{C}^2 = \mathcal{C} \odot \mathcal{C} = (\bar{\zeta_{ij}})_{r\times r}$ then $\mathcal{C}^2$ is called a composition matrix of $\mathcal{C}$, where $\bar{\zeta_{ij}} = max_k\{min\zeta_{ik}, \zeta_{kj}\}, i, j = 1, 2, \ldots, r$*

Theorem 5: *Let $\mathcal{C} = (\zeta_{ij})_{r\times r}$ be the $\mathscr{CM}$, then the composition matrix $\mathcal{C}^2 = \mathcal{C} \odot \mathcal{C} = (\bar{\zeta_{ij}})_{r\times r}$ is a $\mathscr{CM}$.*

Theorem 6: *Let $\mathcal{C} = (\zeta_{ij})_{r\times r}$ be the $\mathscr{CM}$, then for any non-negative integers $\mathfrak{a}_1, \mathfrak{a}_2$ the composition matrix $\mathcal{C}^{\mathfrak{a}_1+\mathfrak{a}_2} = \mathcal{C}^{\mathfrak{a}_1} \odot \mathcal{C}^{\mathfrak{a}_2}$ is still a correlation matrix.*

Definition 12: *Let $\mathcal{C} = (\zeta_{ij})_{r\times r}$ be the $\mathscr{CM}$, then $\mathcal{C}$ is said to be equivalent correlation matrix if*

$$\mathcal{C}^2 \subseteq \mathcal{C} \implies max_k\{min\zeta_{ik}, \zeta_{kj}\} \leq \zeta_{ij}$$

Definition 13: *Let $\mathcal{C} = (\zeta_{ij})_{r\times r}$ be the $\mathscr{CM}$ then after the finite terms of compositions: $\mathcal{C} \to \mathcal{C}^2 \to \mathcal{C}^4 \to \mathcal{C}^8 \to \ldots \mathcal{C}^{2^k} \to$ there must exist a positive integer k such that $\mathcal{C}^{2^k} = \mathcal{C}^{2^{(k+1)}}$ and $\mathcal{C}^{2^k}$ is also an equivalent correlation matrix.*

Definition 14: *Let $\mathcal{C} = (\zeta_{ij})_{r\times r}$ be an equivalent correlation matrix. Then we call $\mathcal{C}_\lambda = ({}_\lambda\zeta_{ij})_{r\times r}$, the λ - cutting matrix of $\mathcal{C}$ where,*

$${}_\lambda\zeta_{ij} = \begin{cases} 0 & if\, \zeta_{ij} \leq \lambda \\ 1 & if\, \zeta_{ij} \geq \lambda \end{cases}$$

and λ is the confidence level with $\lambda \in [0, 1]$.

A. MATHEMATICAL MODELLING

To illustrate LDF-clustering's usefulness in high-dimensional situations and uncertainty handling, this subsection presents the main lines of research on algorithmic and computational augmentations of LDF-clustering.

Step 1: Let $\mathfrak{S}_\mathfrak{r}$ be the set of $\mathbb{LDFS}$ in $\mathfrak{P} = \{\mathfrak{k}_1, \mathfrak{k}_2, \ldots, \mathfrak{k}_\mathfrak{y}\}$. Using the formula, CC between $\mathbb{LDFS}$ can be determined and then put it as correlation matrix $\mathcal{C} = (\zeta_{ij})_{r\times r}$

$$\mathcal{C} = \begin{bmatrix} \zeta_{11} & \zeta_{12} & \cdots & \zeta_{1r} \\ \zeta_{21} & \zeta_{22} & \cdots & \zeta_{2r} \\ \vdots & \vdots & \vdots & \vdots \\ \zeta_{r1} & \zeta_{r2} & \cdots & \zeta_{rr} \end{bmatrix}$$

where $\zeta_{ij} = \mathfrak{Y}(\mathfrak{S}_\mathrm{i}, \mathfrak{S}_\mathrm{j})$

Step 2: Check whether $\mathcal{C} = (\zeta_{ij})_{r\times r}$ is an equivalent matrix. If it does not, then determine the corresponding $\mathscr{CM}$ $\mathcal{C}^{2^k}$ which satisfies $\mathcal{C}^{2^k} = \mathcal{C}^{2^{k+1}}$.

Step 3: For an assurance level λ, we construct λ- clustering matrix $\mathcal{C}_\lambda$(using the definition). If all the elements of i^{th} line are same as the corresponding elements of j^{th} column in $\mathcal{C}_\lambda$. Then the $\mathbb{LDFS}$ $\mathfrak{S}_1$ and $\mathfrak{S}_2$ are of the same type. We can allocate all $\mathfrak{S}_\mathrm{i}$ by using this principle. The systematic work-flow of our proposed method is demonstrated in figure 5.

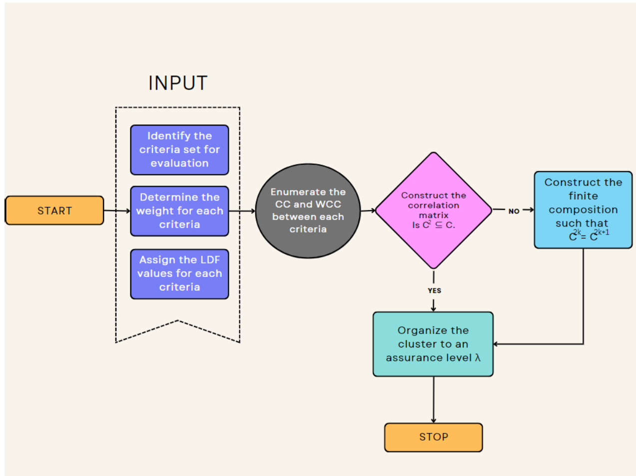

FIGURE 5. Work-Flow of proposed LDF-clustering.

B. AN EVALUATION OF LOGISTICS EFFICIENCY OF FOOD PRODUCTS

Logistic efficiency: The ratio of logistics output to input is used to define its efficiency. Utilizing fewer resources to provide quicker and better logistics output is the aim of logistical efficiency. The application of pertinent performance indicators, standards, and methodologies, as well as the evaluation system created by pertinent institutions, and the collection of assessment findings are all considered to be components of the definition of logistics efficiency evaluation. Logistics efficiency evaluation is described as assessing the effectiveness and effects of prior activities by gathering, processing, categorizing, analyzing, entering, and publishing pertinent data, making proper judgments, and acting appropriately.

Considering the set $\mathscr{S}$ ={Fresh Quality($\bar{Q}_1$), The Price($\bar{Q}_2$), Appearance($\bar{Q}_3$), Taste($\bar{Q}_4$), Place of Origin($\bar{Q}_5$), Brand($\bar{Q}_6$)} of criteria that determine the logistic efficiency of food products(Shown in figure 6).

1) DESCRIPTION OF PARAMETERS

1) **Fresh Quality:** It shows the level of freshness and general food product quality.
2) **Cost:** It reflects the price of the food item.
3) bf Appearance: It explains the food product's physical features.
4) bf Taste: It captures the characteristics of the food product's flavour and taste.
5) **Place of Origin:** It shows the location of the food product's production or sourcing.

$$\mathfrak{Y}'_\mathfrak{w}(\mathfrak{S}_1, \mathfrak{S}_2) = \frac{\mathfrak{C}(\mathfrak{S}_1, \mathfrak{S}_2)}{\max(\mathfrak{T}(\mathfrak{S}_1), \mathfrak{T}(\mathfrak{S}_2))}$$

$$\mathfrak{Y}'_\mathfrak{w}(\mathfrak{S}_1, \mathfrak{S}_2) = \left\{ \frac{\sum_{\mathrm{i}=1}^{\mathfrak{y}} \mathfrak{w}_\mathrm{i}(\mu_{\mathfrak{S}_1}(\mathfrak{k}_\mathrm{i})\mu_{\mathfrak{S}_2}(\mathfrak{k}_\mathrm{i}) + \nu_{\mathfrak{S}_1}(\mathfrak{k}_\mathrm{i})\nu_{\mathfrak{S}_2}(\mathfrak{k}_\mathrm{i}) + \alpha_{\mathfrak{S}_1}(\mathfrak{k}_\mathrm{i})\alpha_{\mathfrak{S}_2}(\mathfrak{k}_\mathrm{i}) + \beta_{\mathfrak{S}_1}(\mathfrak{k}_\mathrm{i})\beta_{\mathfrak{S}_2}(\mathfrak{k}_\mathrm{i}))}{\max\left\{ \begin{array}{c} \sum_{\mathrm{i}=1}^{\mathfrak{y}} \mathfrak{w}_\mathrm{i}((\mu_{\mathfrak{S}_1}(\mathfrak{k}_\mathrm{i}))^2 + (\nu_{\mathfrak{S}_1}(\mathfrak{k}_\mathrm{i}))^2 + (\alpha_{\mathfrak{S}_1}(\mathfrak{k}_\mathrm{i}))^2 + (\beta_{\mathfrak{S}_1}(\mathfrak{k}_\mathrm{i}))^2), \\ \sum_{\mathrm{i}=1}^{\mathfrak{y}} \mathfrak{w}_\mathrm{i}((\mu_{\mathfrak{S}_2}(\mathfrak{k}_\mathrm{i}))^2 + (\nu_{\mathfrak{S}_2}(\mathfrak{k}_\mathrm{i}))^2 + (\alpha_{\mathfrak{S}_2}(\mathfrak{k}_\mathrm{i}))^2 + (\beta_{\mathfrak{S}_2}(\mathfrak{k}_\mathrm{i}))^2) \end{array} \right\}} \right\}$$

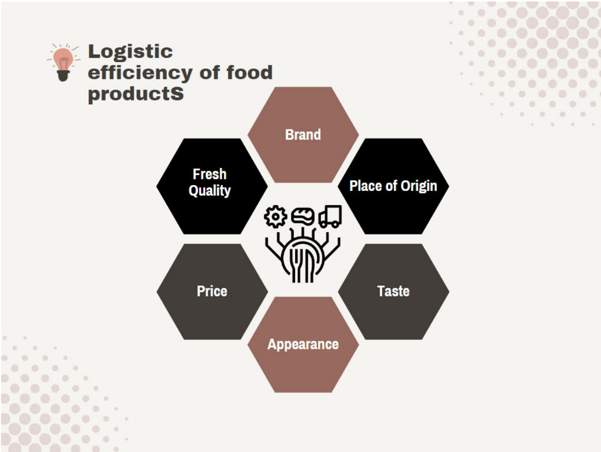

FIGURE 6. Criteria for food products.

TABLE 1. LDF values for each attributes.

Attributes	LDF value
$\mathcal{Q}_1$	$(\mathfrak{a}_1, \langle 0.73, 0.54\rangle, \langle 0.66, 0.32\rangle)$, $(\mathfrak{a}_2, \langle 0.92, 0.21\rangle, \langle 0.70, 0.29\rangle)$ $(\mathfrak{a}_3, \langle 0.82, 0.24\rangle, \langle 0.79, 0.18\rangle)$
$\mathcal{Q}_2$	$(\mathfrak{a}_1, \langle 0.68, 0.34\rangle, \langle 0.73, 0.16\rangle)$, $(\mathfrak{a}_2, \langle 0.54, 0.49\rangle, \langle 0.82, 0.11\rangle)$ $(\mathfrak{a}_3, \langle 0.73, 0.26\rangle, \langle 0.64, 0.23\rangle)$
$\mathcal{Q}_3$	$(\mathfrak{a}_1, \langle 0.56, 0.36\rangle, \langle 0.70, 0.24\rangle)$, $(\mathfrak{a}_2, \langle 0.69, 0.31\rangle, \langle 0.79, 0.12\rangle)$ $(\mathfrak{a}_3, \langle 0.72, 0.26\rangle, \langle 0.85, 0.12\rangle)$
$\mathcal{Q}_4$	$(\mathfrak{a}_1, \langle 0.82, 0.16\rangle, \langle 0.92, 0.07\rangle)$, $(\mathfrak{a}_2, \langle 0.79, 0.11\rangle, \langle 0.85, 0.21\rangle)$ $(\mathfrak{a}_3, \langle 0.65, 0.47\rangle, \langle 0.72, 0.21\rangle)$
$\mathcal{Q}_5$	$(\mathfrak{a}_1, \langle 0.71, 0.32\rangle, \langle 0.85, 0.13\rangle)$, $(\mathfrak{a}_2, \langle 0.82, 0.19\rangle, \langle 0.87, 0.11\rangle)$ $(\mathfrak{a}_3, \langle 0.92, 0.29\rangle, \langle 0.89, 0.09\rangle)$
$\mathcal{Q}_6$	$(\mathfrak{a}_1, \langle 0.62, 0.34\rangle, \langle 0.75, 0.21\rangle)$, $(\mathfrak{a}_2, \langle 0.73, 0.12\rangle, \langle 0.80, 0.17\rangle)$ $(\mathfrak{a}_3, \langle 0.49, 0.54\rangle, \langle 0.76, 0.21\rangle)$

Let $\{\mathfrak{a}_1, \mathfrak{a}_2, \mathfrak{a}_3\}$ be three set of decision-makers.

Step 1: To begin this process, LDF values has been assinged to each of the criteria and it is shown in the table 1.

Step 2: Next, the Correlation matrix $\mathcal{C}$ is computed with the help of the definitions. And then, the equivalance matrix is determined at after certain number of steps.

$$\mathcal{C} = \begin{bmatrix} 1 & 0.9559 & 0.9798 & 0.9491 & 0.9753 & 0.9600 \\ 0.9559 & 1 & 0.9808 & 0.9556 & 0.9715 & 0.9515 \\ 0.9798 & 0.9808 & 1 & 0.9616 & 0.9908 & 0.9721 \\ 0.9491 & 0.9556 & 0.9616 & 1 & 0.9766 & 0.9829 \\ 0.9753 & 0.9715 & 0.9908 & 0.9766 & 1 & 0.9683 \\ 0.9600 & 0.9515 & 0.9721 & 0.9829 & 0.9683 & 1 \end{bmatrix}$$

$$\mathcal{C}^2 = \mathcal{C} \odot \mathcal{C}$$

$$\mathcal{C}^2 = \begin{bmatrix} 1 & 0.9798 & 0.9798 & 0.9753 & 0.9798 & 0.9721 \\ 0.9798 & 1 & 0.9808 & 0.9766 & 0.9808 & 0.9721 \\ 0.9798 & 0.9808 & 1 & 0.9766 & 0.9908 & 0.9721 \\ 0.9753 & 0.9766 & 0.9766 & 1 & 0.9766 & 0.9829 \\ 0.9798 & 0.9808 & 0.9908 & 0.9766 & 1 & 0.9766 \\ 0.9721 & 0.9721 & 0.9721 & 0.9829 & 0.9766 & 1 \end{bmatrix}$$

TABLE 2. Assurance level of λ for CC.

Class	Assurance level for λ	Clusters
(6)	$0.9908 < \lambda \leq 1$	$\{\mathcal{Q}_1\}$,$\{\mathcal{Q}_2\}$,$\{\mathcal{Q}_3\}$, $\{\mathcal{Q}_5\}$,$\{\mathcal{Q}_4\}$, $\{\mathcal{Q}_6\}$
(5)	$0.9808 < \lambda \leq 0.9908$	$\{\mathcal{Q}_1\}$,$\{\mathcal{Q}_2\}$,$\{\mathcal{Q}_3, \mathcal{Q}_5\}$, $\{\mathcal{Q}_4\}$, $\{\mathcal{Q}_6\}$
(4)	$0.9829 < \lambda \leq 0.9808$	$\{\mathcal{Q}_1, \mathcal{Q}_2, \mathcal{Q}_3, \mathcal{Q}_5\}$,$\{\mathcal{Q}_4\}$, $\{\mathcal{Q}_6\}$
(3)	$0.9798 < \lambda \leq 0.9829$	$\{\mathcal{Q}_1\}$, $\{\mathcal{Q}_2, \mathcal{Q}_3, \mathcal{Q}_5\}$, $\{\mathcal{Q}_4, \mathcal{Q}_6\}$
(2)	$0.9766 < \lambda \leq 0.9798$	$\{\mathcal{Q}_1, \mathcal{Q}_2, \mathcal{Q}_3, \mathcal{Q}_5\}$, $\{\mathcal{Q}_4, \mathcal{Q}_6\}$
(1)	$0 < \lambda \leq 0.9766$	$\{\mathcal{Q}_1, \mathcal{Q}_2, \mathcal{Q}_3, \mathcal{Q}_4, \mathcal{Q}_5, \mathcal{Q}_6\}$

$$\mathcal{C}^4 = \begin{bmatrix} 1 & 0.9798 & 0.9798 & 0.9766 & 0.9798 & 0.9766 \\ 0.9798 & 1 & 0.9808 & 0.9766 & 0.9808 & 0.9766 \\ 0.9798 & 0.9808 & 1 & 0.9766 & 0.9908 & 0.9766 \\ 0.9766 & 0.9766 & 0.9766 & 1 & 0.9766 & 0.9829 \\ 0.9798 & 0.9808 & 0.9908 & 0.9766 & 1 & 0.9766 \\ 0.9766 & 0.9766 & 0.9766 & 0.9829 & 0.9766 & 1 \end{bmatrix}$$

$$\mathcal{C}^8 = \begin{bmatrix} 1 & 0.9798 & 0.9798 & 0.9766 & 0.9798 & 0.9766 \\ 0.9798 & 1 & 0.9808 & 0.9766 & 0.9808 & 0.9766 \\ 0.9798 & 0.9808 & 1 & 0.9766 & 0.9908 & 0.9766 \\ 0.9766 & 0.9766 & 0.9766 & 1 & 0.9766 & 0.9829 \\ 0.9798 & 0.9808 & 0.9908 & 0.9766 & 1 & 0.9766 \\ 0.9766 & 0.9766 & 0.9766 & 0.9829 & 0.9766 & 1 \end{bmatrix}$$

Step 3: Since, $\mathcal{C}^8 = \mathcal{C}^4$, $\mathcal{C}^4$ is a equivalence matrix.

Step 4: The λ-cutting matrix is formed to determine the classifications 2 Let $\mathfrak{w} = \{0.3,\ 0.5,\ 0.2\}$ be the weight for three decision-makers $\{\mathfrak{a}_1, \mathfrak{a}_2, \mathfrak{a}_3\}$. Same computational work flow is carried out by utilizing $\mathcal{WCC}$ and obatined the cluster as follows.

Step 2:

$$\mathcal{C} = \begin{bmatrix} 1 & 0.9413 & 0.9759 & 0.9539 & 0.9733 & 0.9673 \\ 0.9413 & 1 & 0.9803 & 0.9463 & 0.9642 & 0.9483 \\ 0.9759 & 0.9803 & 1 & 0.9646 & 0.9903 & 0.9778 \\ 0.9539 & 0.9463 & 0.9649 & 1 & 0.9823 & 0.9798 \\ 0.9733 & 0.9642 & 0.9903 & 0.9823 & 1 & 0.9792 \\ 0.9673 & 0.9483 & 0.9798 & 0.9798 & 0.9792 & 1 \end{bmatrix}$$

Step: 3

$$\mathcal{C}^2 = \begin{bmatrix} 1 & 0.9759 & 0.9759 & 0.9733 & 0.9759 & 0.9759 \\ 0.9759 & 1 & 0.9803 & 0.9649 & 0.9803 & 0.9778 \\ 0.9759 & 0.9803 & 1 & 0.9823 & 0.9903 & 0.9778 \\ 0.9733 & 0.9649 & 0.9823 & 1 & 0.9823 & 0.9778 \\ 0.9759 & 0.9803 & 0.9903 & 0.9823 & 1 & 0.9798 \\ 0.9759 & 0.9778 & 0.9778 & 0.9778 & 0.9798 & 1 \end{bmatrix}$$

$$\mathcal{C}^4 = \begin{bmatrix} 1 & 0.9759 & 0.9759 & 0.9759 & 0.9759 & 0.9759 \\ 0.9759 & 1 & 0.9803 & 0.9803 & 0.9803 & 0.9798 \\ 0.9759 & 0.9803 & 1 & 0.9823 & 0.9903 & 0.9798 \\ 0.9759 & 0.9803 & 0.9823 & 1 & 0.9823 & 0.9798 \\ 0.9759 & 0.9803 & 0.9903 & 0.9823 & 1 & 0.9798 \\ 0.9759 & 0.9798 & 0.9798 & 0.9798 & 0.9798 & 1 \end{bmatrix}$$

$$\mathcal{C}^8 = \begin{bmatrix} 1 & 0.9759 & 0.9759 & 0.9759 & 0.9759 & 0.9759 \\ 0.9759 & 1 & 0.9803 & 0.9803 & 0.9803 & 0.9798 \\ 0.9759 & 0.9803 & 1 & 0.9823 & 0.9903 & 0.9798 \\ 0.9759 & 0.9803 & 0.9823 & 1 & 0.9823 & 0.9798 \\ 0.9759 & 0.9803 & 0.9903 & 0.9823 & 1 & 0.9798 \\ 0.9759 & 0.9798 & 0.9798 & 0.9798 & 0.9798 & 1 \end{bmatrix}$$

Step 4: Since, $\mathcal{C}^8 = \mathcal{C}^4$, $\mathcal{C}^4$ is a equivalence matrix.

TABLE 3. Assurance level of λ for WCC.

Class	Assurance level for λ	Clusters
(6)	$0.9903 < \lambda \leq 1$	$\{Q_1\}$,$\{Q_2\}$,$\{Q_3\}$, $\{Q_5\}$,$\{Q_4\}$, $\{Q_6\}$
(5)	$0.9823 < \lambda \leq 0.9903$	$\{Q_1\}$,$\{Q_2\}$,$\{Q_3, Q_5\}$, $\{Q_4\}$, $\{Q_6\}$
(4)	$0.9803 < \lambda \leq 0.9823$	$\{Q_1\}$,$\{Q_2\}$,$\{Q_3, Q_4, Q_5\}$, $\{Q_6\}$
(3)	$0.9798 < \lambda \leq 0.9803$	$\{Q_1\}$, $\{Q_2, Q_3, Q_5, Q_4\}$, $\{Q_6\}$
(2)	$0.9759 < \lambda \leq 0.9798$	$\{Q_1\}$, $\{Q_2, Q_3, Q_4, Q_5, Q_6\}$
(1)	$0 < \lambda \leq 0.9759$	$\{Q_1, Q_2, Q_3, Q_4, Q_5, Q_6$

C. COMPARATIVE ANALYSIS

The closer inspection of table 4 shows the comparison among the various clustering algorithms. The deficiencies of previous cluster analysis are clearly mentioned. Some of the data have bounds on their grade values, making them unrepresentable by previously developed theories. This amply demonstrates the superiority of our suggested method. Our proposed approach has unique contributions that we will highlight, including its capacity to handle uncertainty, the weighted correlation coefficient and correlation coefficient for LDFS that it incorporates, and the flexibility it provides to decision-makers in terms of score value selection because of the expanded score space. Hence, our proposed approach can be used in any type of complex situation. The table also shows how our suggested algorithm is superior to others.

D. SHORTCOMINGS OF PRIOR APPROACH AND PRECEDENCE OF CURRENT APPROACH

Shortcomings:

1) The inability of fuzzy sets to describe non-membership scores limits their ability to simulate numerous real-life scenarios.
2) Even with dual scores, some problems cannot be modeled in IFS and PFS. The strong restrictions of the characteristic function force the DM to restrict their choice values.
3) While q-ROFS does allow for some flexibility in their characteristic function, the DM is restricted to particular domains.
4) Inadequately $\mathcal{CC}$ of foregoing structures (IFS, PFS, and q-ROFS) has bounds on their domain, these constraints render them incompatible with some information that is required for certain problems within MCDM.

Precedence:

1) The premise of LDFS is the generalization of structures like IFS, PFS, and q-ROFS, which denotes that it can handle all types of information.
2) Because they model four different functions, including membership, non-membership, and reference parameters, they are superior to all existing theories when viewed as a structure.
3) In the aforementioned environment, setting some functions to zero still resolves the given issue.
4) The proposed $\mathcal{CC}$ also serves as a generalization of the $\mathcal{CC}$ for FS, IFS, PFS, and q-ROFS. This enables the resolution of problems in other frameworks.

E. SENSITIVITY ANALYSIS

We perform sensitivity analysis on the current section by adjusting various weights to ensure the stability of the new approach. This means that we can adjust the weight values of any two criteria without changing the other criteria's values. There are three potential cases when three criteria are present: $\mathfrak{a}_1 - \mathfrak{a}_2$; $\mathfrak{a}_1 - \mathfrak{a}_3$; $\mathfrak{a}_2 - \mathfrak{a}_3$.

Case (i): $\mathfrak{a}_1 - \mathfrak{a}_2$. Consider $\mathfrak{w} = \{0.5, 0.3, 0.2\}$.

Step 1: Construct the Correlation matrix $\mathcal{C}$ by employing the needed definitions.

$$\mathcal{C} = \begin{bmatrix} 1 & 0.9559 & 0.9788 & 0.9382 & 0.9694 & 0.9666 \\ 0.9559 & 1 & 0.9828 & 0.9577 & 0.9751 & 0.9623 \\ 0.9788 & 0.9828 & 1 & 0.9581 & 0.9895 & 0.8190 \\ 0.9382 & 0.9577 & 0.9581 & 1 & 0.9800 & 0.9798 \\ 0.9694 & 0.9751 & 0.9895 & 0.9800 & 1 & 0.9785 \\ 0.9666 & 0.9623 & 0.8190 & 0.9793 & 0.9785 & 1 \end{bmatrix}$$

Step 2: Then the respective equivalence matrix is computed.

$$\mathcal{C}^2 = \begin{bmatrix} 1 & 0.9788 & 0.9788 & 0.9694 & 0.9788 & 0.9788 \\ 0.9788 & 1 & 0.9828 & 0.9751 & 0.9828 & 0.9751 \\ 0.9788 & 0.9828 & 1 & 0.9800 & 0.9895 & 0.9785 \\ 0.9694 & 0.9751 & 0.9800 & 1 & 0.9800 & 0.9793 \\ 0.9788 & 0.9828 & 0.9895 & 0.9800 & 1 & 0.9793 \\ 0.9788 & 0.9751 & 0.9785 & 0.9793 & 0.9793 & 1 \end{bmatrix}$$

$$\mathcal{C}^4 = \begin{bmatrix} 1 & 0.9788 & 0.9788 & 0.9788 & 0.9788 & 0.9788 \\ 0.9788 & 1 & 0.9828 & 0.9800 & 0.9828 & 0.9793 \\ 0.9788 & 0.9828 & 1 & 0.9800 & 0.9895 & 0.9793 \\ 0.9788 & 0.9800 & 0.9800 & 1 & 0.9800 & 0.9793 \\ 0.9788 & 0.9828 & 0.9895 & 0.9800 & 1 & 0.9793 \\ 0.9788 & 0.9793 & 0.9793 & 0.9793 & 0.9793 & 1 \end{bmatrix}$$

$$\mathcal{C}^8 = \begin{bmatrix} 1 & 0.9788 & 0.9788 & 0.9788 & 0.9788 & 0.9788 \\ 0.9788 & 1 & 0.9828 & 0.9800 & 0.9828 & 0.9793 \\ 0.9788 & 0.9828 & 1 & 0.9800 & 0.9895 & 0.9793 \\ 0.9788 & 0.9800 & 0.9800 & 1 & 0.9800 & 0.9793 \\ 0.9788 & 0.9828 & 0.9895 & 0.9800 & 1 & 0.9793 \\ 0.9788 & 0.9793 & 0.9793 & 0.9793 & 0.9793 & 1 \end{bmatrix}$$

Step 3: Since, $\mathcal{C}^8 = \mathcal{C}^4$, $\mathcal{C}^4$ is a equivalence matrix.

Step 4: The λ-cutting matrix is formed to determine the classifications (Refer Table:5)

Case (ii): $\mathfrak{a}_1 - \mathfrak{a}_3$, Comsider $\mathfrak{w} = \{0.2, 0.5, 0.3\}$

Step 1: the Correlation matrix $\mathcal{C}$ is computed.

$$\mathcal{C} = \begin{bmatrix} 1 & 0.9431 & 0.9771 & 0.9605 & 0.9772 & 0.9623 \\ 0.9431 & 1 & 0.9793 & 0.9460 & 0.9229 & 0.9412 \\ 0.9771 & 0.9793 & 1 & 0.9672 & 0.9911 & 0.9723 \\ 0.9605 & 0.9460 & 0.9672 & 1 & 0.9798 & 0.9882 \\ 0.9772 & 0.9229 & 0.9911 & 0.9798 & 1 & 0.9715 \\ 0.9623 & 0.9421 & 0.9723 & 0.9882 & 0.9715 & 1 \end{bmatrix}$$

$$\mathcal{C}^2 = \begin{bmatrix} 1 & 0.9771 & 0.9771 & 0.9772 & 0.9772 & 0.9723 \\ 0.9771 & 1 & 0.9793 & 0.9672 & 0.9793 & 0.9723 \\ 0.9772 & 0.9793 & 1 & 0.9798 & 0.9911 & 0.9723 \\ 0.9772 & 0.9672 & 0.9798 & 1 & 0.9798 & 0.9882 \\ 0.9772 & 0.9793 & 0.9911 & 0.9798 & 1 & 0.9798 \\ 0.9723 & 0.9723 & 0.9723 & 0.9882 & 0.9798 & 1 \end{bmatrix}$$

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TABLE 4. An examination of preceding algorithms for clustering.

Reference	Data information	MCDM Approach	Application Area	Charateristics
Chen et.al [14]	HFS	Clustering Analysis based on HF cc	Cars Classification	Fails to describe NMS
Chaudhuri [13]	IFS	Possibilistic C-means Clustering Algorithm	Investment Portfolio	The range of data limits the DM to certain bound
Nyugen et.al [48]	IFS	Clustering Algorithm based on IF CC	Pattern Recognition	The range of data limits the DM to certain bound
Chaira [12]	IFS	Clustering Analysis by Distance function	Abnormal region in Mammogram images	The range of data limits the DM to certain bound
Lin et.al [28]	IVIFS	Clustering symmetric information entropy	Soil Classification	The range of data limits the DM to certain bound
Asim [2]	IHFS	Clustering Algorithm based on IF CC	University classification	The range of data limits the DM to certain bound
lin et.al [29]	PyFS	Clustering Algorithm based on PF directional CC	Medical diagnosis	The range of data limits the DM to certain bound
Qu et.al [38]	CPFS	Clustering Algorithm based on CPF CC	Laptop Classification	The range of data limits the DM to certain bound
Bashir et.al [9]	q-ROFS	Clustering Algorithm based on q-ROF CC	Vehicle Classification	The range of data limits the DM to certain bound
Abbas et.al [1]	q-ROF 2-tuple	Clustering Algorithm based on q-ROF CC	CIM software classification	The range of data limits the DM to certain bound
Proposed	LDFS	LDF-Clustering algorithm based on LDF-CC	Logistic efficiency of Food products classification	Enlarges the range of data and enables the DM to choose their own choice.

TABLE 5. Case:1 Cluster Analysis.

Class	Assurance Level	Clusters
6	$0.9895 \leq \lambda \leq 1$	$\{Q_1\}, \{Q_2\}, \{Q_3\}, \{Q_4\}, \{Q_5\}, \{Q_6\}$
5	$0.9828 \leq \lambda \leq 0.9895$	$\{Q_1\}, \{Q_2\}, \{Q_3, Q_5\}\ \{Q_4\}, \{Q_6\}$
4	$0.9800 \leq \lambda \leq 0.9828$	$\{Q_1\}, \{Q_2, Q_3, Q_5\}\ \{Q_4\}, \{Q_6\}$
3	$0.9793 \leq \lambda \leq 0.9800$	$\{Q_1\}, \{Q_2, Q_3, Q_4, Q_5\}\ \{Q_6\}$
2	$0.9788 \leq \lambda \leq 0.9793$	$\{Q_1\}, \{Q_2, Q_3, Q_4, Q_5, Q_6\}$
1	$0 \leq \lambda \leq 0.9788$	$\{Q_1, Q_2, Q_3, Q_4, Q_5, Q_6\}$

TABLE 6. Case:2 Cluster Analysis.

Class	Assurance Level	Clusters
6	$0.9911 \leq \lambda \leq 1$	$\{Q_1\}, \{Q_2\}, \{Q_3\}, \{Q_4\}, \{Q_5\}, \{Q_6\}$
5	$0.9882 \leq \lambda \leq 0.9911$	$\{Q_1\}, \{Q_2\}, \{Q_3, Q_5\}\ \{Q_4\}, Q_6\}$
4	$0.9798 \leq \lambda \leq 0.9882$	$\{Q_1\}, \{Q_2\}, \{Q_3, Q_5\}\ \{Q_4, Q_6\}$
3	$0.9793 \leq \lambda \leq 0.9798$	$\{Q_1\},\{Q_2\}, \{Q_3, Q_4, Q_5, Q_5\}$
2	$0.9772 \leq \lambda \leq 0.9793$	$\{Q_1\}, \{Q_2, Q_3, Q_4, Q_5, Q_6\}$
1	$0 \leq \lambda \leq 0.9772$	$\{Q_1, Q_2, Q_3, Q_4, Q_5, Q_6\}$

$$\mathcal{C}^4 = \begin{bmatrix} 1 & 0.9772 & 0.9771 & 0.9772 & 0.9772 & 0.9772 \\ 0.9771 & 1 & 0.9793 & 0.9793 & 0.9793 & 0.9793 \\ 0.9772 & 0.9793 & 1 & 0.9798 & 0.9911 & 0.9798 \\ 0.9772 & 0.9672 & 0.9798 & 1 & 0.9798 & 0.9882 \\ 0.9772 & 0.9793 & 0.9911 & 0.9798 & 1 & 0.9798 \\ 0.9772 & 0.9793 & 0.9798 & 0.9882 & 0.9798 & 1 \end{bmatrix}$$

$$\mathcal{C}^8 = \begin{bmatrix} 1 & 0.9772 & 0.9771 & 0.9772 & 0.9772 & 0.9772 \\ 0.9771 & 1 & 0.9793 & 0.9793 & 0.9793 & 0.9793 \\ 0.9772 & 0.9793 & 1 & 0.9798 & 0.9911 & 0.9798 \\ 0.9772 & 0.9672 & 0.9798 & 1 & 0.9798 & 0.9882 \\ 0.9772 & 0.9793 & 0.9911 & 0.9798 & 1 & 0.9798 \\ 0.9772 & 0.9793 & 0.9798 & 0.9882 & 0.9798 & 1 \end{bmatrix}$$

Step 3: Since, $\mathcal{C}^8 = \mathcal{C}^4$, $\mathcal{C}^4$ is a equivalence matrix.

Step 4: The λ-cutting matrix is formed to determine the classifications (Refer Table:6)

Case (iii): $\mathfrak{a}_2 - \mathfrak{a}_3$, Consider $\mathfrak{w} = \{0.3, 0.2, 0.5\}$

Step 1: the Correlation matrix $\mathcal{C}$ is generated.

$$\mathcal{C} = \begin{bmatrix} 1 & 0.9691 & 0.9837 & 0.9492 & 0.9793 & 0.9510 \\ 0.9691 & 1 & 0.9808 & 0.9633 & 0.9769 & 0.9504 \\ 0.9837 & 0.9808 & 1 & 0.9605 & 0.9917 & 0.9645 \\ 0.9492 & 0.9633 & 0.9605 & 1 & 0.9712 & 0.9815 \\ 0.9793 & 0.9769 & 0.9917 & 0.9712 & 1 & 0.9555 \\ 0.9510 & 0.9504 & 0.9645 & 0.9815 & 0.9555 & 1 \end{bmatrix}$$

Step 2: Enumerate the equivalence matrix.

$$\mathcal{C}^2 = \begin{bmatrix} 1 & 0.9808 & 0.9831 & 0.9712 & 0.9831 & 0.9645 \\ 0.9808 & 1 & 0.9808 & 0.9712 & 0.9808 & 0.9645 \\ 0.9831 & 0.9808 & 1 & 0.9712 & 0.9917 & 0.9645 \\ 0.9712 & 0.9712 & 0.9712 & 1 & 0.9712 & 0.9815 \\ 0.9831 & 0.9808 & 0.9917 & 0.9712 & 1 & 0.9712 \\ 0.9645 & 0.9645 & 0.9645 & 0.9815 & 0.9712 & 1 \end{bmatrix}$$

$$\mathcal{C}^4 = \begin{bmatrix} 1 & 0.9808 & 0.9831 & 0.9712 & 0.9831 & 0.9712 \\ 0.9808 & 1 & 0.9808 & 0.9712 & 0.9808 & 0.9712 \\ 0.9831 & 0.9808 & 1 & 0.9712 & 0.9917 & 0.9712 \\ 0.9712 & 0.9712 & 0.9712 & 1 & 0.9712 & 0.9815 \\ 0.9831 & 0.9808 & 0.9917 & 0.9712 & 1 & 0.9712 \\ 0.9645 & 0.9712 & 0.9712 & 0.9815 & 0.9712 & 1 \end{bmatrix}$$

$$\mathcal{C}^8 = \begin{bmatrix} 1 & 0.9808 & 0.9831 & 0.9712 & 0.9831 & 0.9712 \\ 0.9808 & 1 & 0.9808 & 0.9712 & 0.9808 & 0.9712 \\ 0.9831 & 0.9808 & 1 & 0.9712 & 0.9917 & 0.9712 \\ 0.9712 & 0.9712 & 0.9712 & 1 & 0.9712 & 0.9815 \\ 0.9831 & 0.9808 & 0.9917 & 0.9712 & 1 & 0.9712 \\ 0.9645 & 0.9712 & 0.9712 & 0.9815 & 0.9712 & 1 \end{bmatrix}$$

Step 3: Since, $\mathcal{C}^8 = \mathcal{C}^4$, $\mathcal{C}^4$ is a equivalence matrix.

Step 4: The λ-cutting matrix is formed to determine the classifications (Refer Table:7)

The results of Case 1, 2, 3 as shown in the table 5,6,7 respectively, evident that all the criteria are of same group

TABLE 7. Case:3 Cluster Analysis.

Class	Assurance Level	Clusters
6	$0.9895 \leq \lambda \leq 1$	$\{Q_1\}, \{Q_2\}, \{Q_3\}, \{Q_4\}, \{Q_5\}, \{Q_6\}$
5	$0.9828 \leq \lambda \leq 0.9895$	$\{Q_1\}, \{Q_2\}, \{Q_3, Q_5\} \{Q_4\}, \{Q_6\}$
4	$0.9895 \leq \lambda \leq 1$	$\{Q_1\}, \{Q_2\}, \{Q_3, Q_5\} \{Q_4, Q_6\}$
3	$0.9895 \leq \lambda \leq 1$	$\{Q_2\}, \{Q_1, Q_3, Q_5\} \{Q_4, Q_6\}$
2	$0.9712 \leq \lambda \leq 0.9808$	$\{Q_1, Q_2, Q_3, Q_5,\}, \{Q_4, Q_6\}$
1	$0 \leq \lambda \leq 0.9712$	$\{Q_1, Q_2, Q_3, Q_4, Q_5, Q_6\}$

for the values $0 \leq \lambda \leq 0.9788$, $0 \leq \lambda \leq 0.9772$, and $0 \leq \lambda \leq 0.9712$. This makes it clear that the upper limits of the intervals are rather near to one another, showing that the presented procedure is stable with regard to changes in the weights of the DM.

F. LIMITATIONS AND CHALLENGES

1) Computational Complexity: As the number of data points and clusters rises, fuzzy clustering techniques may become computationally complex. Their scalability for big datasets could be limited as a result.
2) Lack of Standardisation: Fuzzy clustering lacks a standardized assessment framework, which makes it difficult to compare different algorithms, in contrast to other classic clustering techniques that have well-established standards and benchmarks.

V. CONCLUSION

This study set out to develop a new approach to advance the field of fuzzy clustering in the context of linear diophantine fuzzy sets (LDFS) information. Concurrently, it investigated the influence of $\mathcal{CC}$ in the LDF-Clustering Algorithm. The findings of this study are given in two contexts using $\mathcal{WCC}$ and $\mathcal{CC}$. These contexts were applied, and two unique clusters were formed that shed light on the logistical effectiveness of food products. The comparative analysis carried out in this work emphasizes the importance of the LDF-clustering algorithm we have presented. This study adds to the current discussion on fuzzy clustering by comparing its performance to other approaches. It also highlights the usefulness and efficiency of the suggested algorithm in clarifying patterns and relationships found in logistic efficiency data. Thus, the study's findings support the LDF-Clustering algorithm's encouraging potential as a useful tool for improving logistics comprehension and management in the context of food product analysis.

Future research is advised to explore the application of decision-making methods, namely TOPSIS, MARCOS, and CODAS, in the particular setting of linear diophantine fuzzy (LDF) environments. Researchers would probably investigate modifications and adaptations that are consistent with the intrinsic properties of LDF data in order to determine the effectiveness of these techniques in LDF settings. The goal of this further research is to improve the suitability of these decision-making strategies in situations where linear diophantine fuzzy environments pose particular difficulties, hence advancing the general comprehension and use of these approaches in real-world environments.

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JEEVITHA KANNAN received the M.Sc. degree from the Department of Mathematics, Alagappa University, Karaikudi, Tamilnadu, India, where she is currently pursuing the Ph.D. degree in mathematics. She has presented many papers at several international conferences. Her research areas are fuzzy algebra, fuzzy lattice ordered structures, soft computing, and decision-making theory. She is a university topper in her post-graduate.

VIMALA JAYAKUMAR received the Ph.D. degree in mathematics from Alagappa University, Karaikudi, Tamil Nadu, India, in 2007. She has 19 years of teaching experience. She has presented many papers in various international conferences. She has published three books. She has completed eight Ph.D.'s and currently five ongoing under her guidance. She has published more than 50 research articles in various reputed journals. Her research interests include algebra, lattice theory, fuzzy mathematics, algebraic hyperstructures, fuzzy algebraic hyperstructures, group theory, soft computing, neutrosophic sets, and multicriteria decision-making. She received the Best Researcher Award in mathematics by the International Multidisciplinary Research Foundation (IMRF), in 2015 and 2016; and the Distinguished Women in Science 2017 by VIWA. She has professionally visited more than five countries.

MUHAMMAD SAEED was born in Pakistan, in July 1970. He received the Ph.D. degree in mathematics from Quaid-i-Azam University, Islamabad, Pakistan, in 2012. He taught mathematics at intermediate and degree level with exceptional results. He was involved as a teacher trainer for professional development for more than five years. He was the Chairperson of the Department of Mathematics, UMT, Lahore, from 2014 to 2021. Under his dynamic leadership, the Mathematics Department has produced more than 30 Ph.D.'s. He has supervised 30 M.S. students and seven Ph.D. students. He has published more than 150 articles in recognized journals. His research interests include fuzzy mathematics, rough sets, soft set theory, hypersoft set, neutrosophic sets, algebraic and hybrid structures of soft sets and hypersoft sets, multicriteria decision making, optimizations, artificial intelligence, pattern recognition and optimization under convex environments, graph theory in fuzzy-like, soft-like, and hypersoft-like environments, similarly, distance measures and their relevant operators in multipolar hybrid structures. He was awarded as the Best Teacher, in 1999, 2000, 2001, and 2002.

TMADER ALBALLA received the Ph.D. degree in system modeling and analysis from the Department of Statistical Sciences and Operations Research, Virginia Commonwealth University, Richmond, VA, USA. She is currently an Assistant Professor with the Mathematics and Statistics Department, Sciences College, Princess Nourah bint Abdulrahman University, Riyadh, Saudi Arabia.

HAMIDEN ABD EL-WAHED KHALIFA received the Ph.D. degree from the Faculty of Science, Tanta University, Tanta, Gharbia Governorate, Egypt. She is a full professor of operations research (mathematical programming). She is recently attached with the Operations Research Department, Faculty of Graduate Studies for Statistical Research, Cairo University, Giza, Egypt, as a Professor. She is also attached with the Department of Mathematics, College of Science and Arts, Qassim University, Buraydah, Saudi Arabia, for research projects. She has published more than 100 publications in SCI journals. Her research interests include game theory, multi-objective programming, fuzzy mathematics, rough sets, and decision making.

HEBA GHAREEB GOMAA is with the Institute for Management Information Systems, Egypt, and specializes in optimization and algorithmic design.