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RESEARCH ARTICLE

Comparative Analysis of Fuzzy Rule Interpolation Techniques Across Various Scenarios Using a Set of Benchmarks

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ABSTRACT This paper presents a set of benchmarks to evaluate the performance of Fuzzy Rule Interpolation (FRI) methods under various challenging conditions. FRI methods are widely used for handling sparse fuzzy rule bases and reducing decision complexity. Despite lacking overlap with the antecedents of any rule in the rule bases, FRI can still produce a conclusion. To unify the requirements of FRI methods, several conditions have been proposed. Among these, the convex and normal fuzzy set condition and the Piece-wise linearity condition are the most common. In this paper, we introduce new benchmark scenarios for testing FRI methods. These benchmarks aim to serve as a reference for evaluating and comparing the accuracy and effectiveness of FRI methods. By using these benchmarks, researchers can compare different FRI methods and identify areas for improvement in the field of fuzzy inference.

INDEX TERMS FRI benchmark, interpolation techniques, fuzzy rule interpolation, fuzzy interpolative, benchmark scenarios.

I. INTRODUCTION

Traditional fuzzy systems are based on a comprehensive fuzzy rule base that covers all conceivable input scenarios to generate meaningful results. However, it is often the case that incomplete sets of rules exist, regardless of whether they are based on input from human experts or on automated procedures. This fact implies that traditional fuzzy inference systems may not be well suited to situations with a limited number of fuzzy rules and where the input does not match any of the rule antecedents. To reduce this challenge, the Fuzzy Rule Interpolation (FRI) concept was developed.

FRI provides a pragmatic solution that provides reasonable results, even if the system contains a few fuzzy rules. FRI can detect the critical fuzzy rules required for inference and has the potential to streamline complex fuzzy models characterized by an abundance of rules.

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FRI is a valuable tool in situations where there is a lack of comprehensive expert knowledge or difficulty obtaining datasets. Using FRI is important to enhance the efficiency of complex classical fuzzy inference systems by identifying the important rules.

FRI was applied in different fields, such as control systems, prediction models, and decision-making. In [1], a creative method was presented to precisely target the detection of IoT-Botnet attacks within IoT smart environments. This method applied the adaptation of the LEast Squares-based FRI (LESFRI) technique [23].

Also, FRI methods were applied within the field of Intrusion Detection Systems (IDS) [3]. This investigation included the implementation of the FRI-IDS model as a means of detecting Distributed Denial of Service (DDoS) attacks.

Similarly, another strategy was proposed for the detection of different intrusions using FRI techniques. In [2], a system for recognizing facial expressions was designed, which classified and identified seven distinct facial expression types using the FRI concept. Also, in [4] the fuzzy decision model proposed for the selection of tourist hotel locations was created. This model proved the practicality and efficacy of the proposed B-FRI approach in the context of assessing and supporting decision-making for hotel location selection.

In [31], a new detection mechanism for multi-step attacks was introduced by the authors, utilizing FRI methods in a fuzzy automaton framework. The proposed mechanism demonstrated an impressive 97.836% detection rate. Notably, it not only identified multi-step attacks but also detected them in stages, particularly in cases where the planned attack was not fully developed, resulting in reduced potential harm. In [32], the authors presented an abnormality detection approach leveraging Fuzzy Rule Interpolation (FRI) along with Simple Network Management Protocol (SNMP) Management Information Base (MIB) parameters. This method streamlined the detection process by eliminating the time-consuming raw traffic processing component, which typically requires extensive computational resources.

In [33], the authors introduced a fuzzy inference-based anomaly-based intrusion detection system for detecting DDoS attacks. The utilization of a fuzzy inference system aimed to move beyond binary decisions, addressing issues associated with the limitations of IDS alert system awareness. In [37], a novel phishing website attack detection method was presented by the authors, mitigating problems linked to knowledge-based representation and binary decision issues. The proposed detection method, implemented using the Incircle-FRI method, was evaluated on an open-source benchmark phishing website dataset. The results demonstrated competitive accuracy, achieving a 97.58% detection rate, and effectively reducing false alerts.

Despite the many FRI methods proposed since 1991, the use of the FRI concept was not important due to the insufficiency of its practical application in different fields. The reason is the lack of comprehensive benchmarks and characteristics through which one can distinguish and compare interpolation methods.

The authors discussed in the literatures [7], [26], and [34] a general set of conditions for the FRI concept, where the comparison between interpolation methods was based on a set of specific examples. Our objective in this paper is to construct comprehensive benchmark scenarios (as a framework) that will be created to be a base for classifying and comparing the FRI methods.

The main contribution of this paper is summarized as follows:

- Introducing various benchmark scenarios that serve as a guide for evaluating FRI methods and identifying their strengths and weaknesses.
- Analyzing the proposed benchmark scenarios to standardize the evaluation process and help researchers assess the accuracy and effectiveness of FRI methods under various challenging conditions.
- These benchmark scenarios are valuable for advancing the field of fuzzy inference and improving the performance of FRI methods in real-world applications.
- These benchmark scenarios can serve as a standard reference for facilitating FRI methods classification and comparison.

This paper is organized as follows. Section (II) defines the notion of a complete - incomplete rule base and background related to fuzzy rule interpolation. Section (III) gives characteristics of the presented FRI conditions. Section (IV) presents the suggested set of benchmarks for evaluating the FRI methods. Experiments of the benchmarks will be provided in Section (V). Section (VI) presents conclusions.

II. PRELIMINARIES AND BACKGROUND RELATED TO FUZZY RULE INTERPOLATION

Fuzzy expert systems leverage imprecision and partial truth to mimic human reasoning, often employing approximate reasoning. Such systems involve linguistic variables, fuzzy rules, and a fuzzy inference method. Linguistic variables help interpret expressions, while fuzzy rules link input and output variables [6]. The fuzzy inference strategies use these rules to process new input (observation) data via approximate reasoning. The essential components include fuzzy rule bases, storing knowledge for inference, and a mechanism for computing outputs based on input and rules.

One popular approach is the Compositional Rule of Inference (CRI), seen in Mamdani's or Sugeno's fuzzy logic controllers. However, CRI is effective with dense rule bases covering the entire problem space, making sparse or incomplete rule bases challenging. Sparse rule bases may result from incomplete knowledge or the desire to reduce system complexity. CRI fails when input observations lack overlap with available rules, highlighting a limitation in handling sparse rule bases.

The Fuzzy Rule Interpolation (FRI) is an advanced inference mechanism for fuzzy rule-based systems, particularly when dealing with sparse rule bases. FRI addresses the limitations of the CRI by manipulating rules similar to unmatched observations, allowing for reasoning in situations where a complete rule base is impractical. FRI can interpolate outcomes with sparse knowledge, making two key contributions: facilitating reasoning on sparse rule bases and potentially simplifying dense rule bases through system computation reduction.

Fuzzy interpolative reasoning is an inference technique for sparse fuzzy rule-based systems. It is obvious that the number of fuzzy rules significantly affects the execution time of the fuzzy rule-based system, where the sparser the fuzzy rule bases of the system are, the faster the execution of the system is. In this situation, the input universe of discourse is covered completely by fuzzy rule bases through fuzzy interpolative reasoning methods. When an observation occurs, a consequence can be derived using fuzzy interpolative reasoning techniques.

The schema of the fuzzy rules represented in Fig. 1 is an example of complete rule bases, which are defined as follows:

Rule1: If $(Input_1$ *is* A_1 *and* $Input_2$ *is* $B_2)$ *Then (Output is* $C_1)$

Rule2: If $(Input_1$ *is* A_1 *and* $Input_2$ *is* $B_2)$ *Then (Output is* $C_1)$

Rule3: If (Input$_1$ *is* A_1*and* Input$_2$ *is* B_2) *Then* (Output *is* C_1)

$$\vdots$$

Rule8: If (Input$_1$ *is* A_1*and* Input$_2$ *is* B_2) *Then* (Output *is* C_1)

Rule9: If (Input$_1$ *is* A_1*and* Input$_2$ *is* B_2) *Then* (Output *is* C_1)

New Observation:

If (Input$_1$ *is* A_1*and* Input$_2$ *is* B_2) *Then* (Output *is* C_1)

Fig. 1 describes the relationship between inputs and outputs via complete rule bases in the fuzzy rule-based system as represented (Rule$_1$, Rule$_2$, …….., Rule$_9$). Thus, there are three rules (Rule$_2$, Rule$_4$, and Rule$_5$) that cover the new inputs, such as new observation 1 and new observation 2. Hence, one of the traditional inference systems (MAMDANI or SUGENOI) could be used to calculate the conclusion.

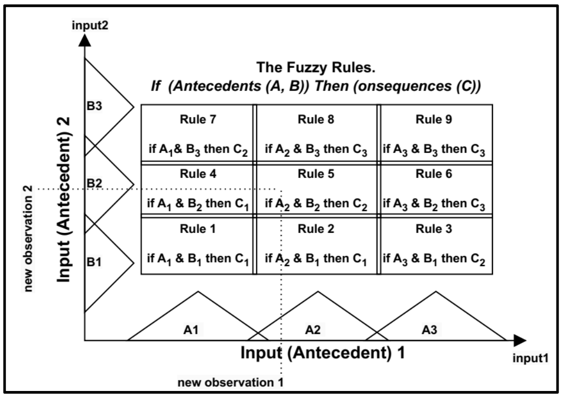


FIGURE 1. Dense fuzzy rule based system.

Fig. 2 describes the incomplete fuzzy rules in the fuzzy rule-based system: In this case, the classical compositional rule of inference cannot give a conclusion because there are no rules covering the new inputs, such as (new observation 1 and new observation 2).

The schema of fuzzy rules shown in the example in Fig. 2 is represented as follows:

Rule1: If (Input$_1$ *is* A_1*and* Input$_2$ *is* B_1) *Then* (Output *is* C_1)

Rule3: If (Input$_1$ *is* A_3*and* Input$_2$ *is* B_2) *Then* (Output *is* C_1)

Rule7: If (Input$_1$ *is* A_1*and* Input$_2$ *is* B_2) *Then* (Output *is* C_1)

Rule8: If (Input$_1$ *is* A_2*and* Input$_2$ *is* B_2) *Then* (Output *is* C_1)

Rule9: If (Input$_1$ *is* A_3*and* Input$_2$ *is* B_2) *Then* (Output *is* C_1)

New Observation:

If (Input$_1$ *is* A_1*and* Input$_2$ *is* B_2) *Then* (Output *is* C_1)

In this case, traditional inference methods cannot give any conclusion because the new observations1 or new observations2 are not covered by any of the current fuzzy rules in the system.

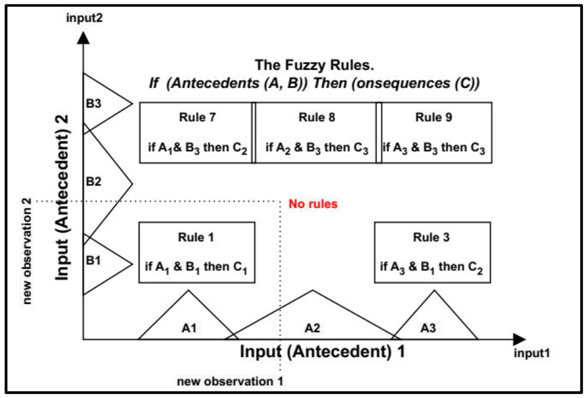


FIGURE 2. Sparse fuzzy rule-based system with overlapping between fuzzy sets.

In another scenario where traditional inference systems fail to yield conclusions, occurs when the system encompasses all fuzzy rules (complete fuzzy rules). Despite having complete fuzzy rules, the classical compositional rule of inference becomes incapable of concluding due to a gap between fuzzy sets, illustrated in Fig. 3. This gap derives from non-overlapping triangular fuzzy sets. Therefore, no rule adequately encompasses new inputs, such as new observation 1 and new observation 2.

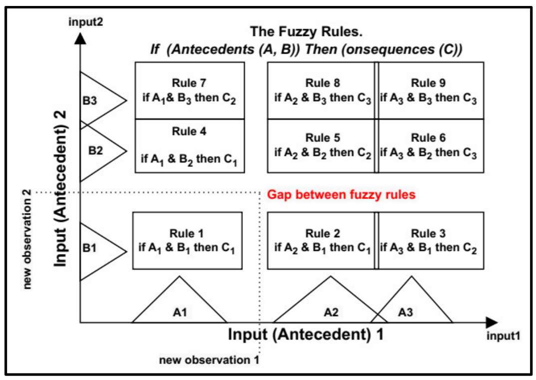


FIGURE 3. Gab between fuzzy sets (no-overlapping) with complete fuzzy rules.

Different FRI methods have been proposed in the literatures, including linear interpolation, extension KH [14], [15] central point-based interpolation, Conservation of the Relative Fuzziness interpolation (CRF) method [17], Improved Multidimensional modified α-Cut interpolation (IMUL) method [22], an enhanced α-Cut based fuzzy interpolative reasoning approach, which is based on the slopes of fuzzy triangular shape has been introduced. These methodologies enable direct deduction of conclusions from observations.

Additionally, several methods aim to improve the outputs by addressing potential issues such as abnormal conclusions in KH linear interpolation. The VKK method is a modification of the KH FRI, which was suggested by Vass et al. [20] The conclusion in this method can be computed based on the distance of the center points and the widths of the α-cuts, rather than the lower and upper distances.

The stabilized KH (KHstab) approach is proposed by Tikk et al. [8] to address and exclude the abnormality conclusion. This method has used the inverse of the distance between antecedents and observation, where all flanking of the observation will be used in computing the conclusion. Another modification of the original KH method is called The Modified α-Cut based Interpolation (MACI), which was presented by Tikk and Baranyi [19], MACI transforms fuzzy sets into vector descriptions, then calculates the conclusion, and finally, transforms back to the initial space.

On the other hand, some of the FRI methods require a two-step process to determine the conclusions. These approaches first generate an approximate fuzzy rule through specific similarity principles and then employ an approximate transformation to provide the conclusion.

For instance, there are several methodologies used for interpolation in the literature, including the Generalized Methodology (GM) [9], Polar Cuts (POC) [21], Least Squares (LES) [23], Vague Environment (VE) [25], and the Fuzzy Interpolation in the Vague Environment (FIVE) approach [24]. For Convex and Normal Fuzzy Sets (CNF), another method known as Incircle-FRI has been devised [18]. For all rules and observations, Incircle-FRI involves the production of CNF and PieceWise Linearity (PWL) outcomes [12], [16].

The author in [40] introduced Dynamic Fuzzy Interpolation based on Rule Assessment (RAD-FRI), which enhances the sparse rule base by incorporating high-quality interpolated rules. RAD-FRI improves the similarity function by considering the location of rules in a sparse rule base and filtering unused interpolated rules. The authors in [41] proposed a rough-fuzzy rule interpolation method to improve decision-making systems by including further uncertain information, enabling the implementation of fuzzy reasoning systems with incomplete rule bases. In [42], a novel FRI approach is described, which is based on mapping the structural patterns within a given fuzzy rule base onto a mathematically isomorphic data space such that the essential information embedded in the original rule base can be effectively captured, represented, and analyzed.

In [43], a new FRI approach is presented that uses density. The proposed method adaptively selects the closest rules that are within a certain range of the unmatched inputs, thus assuring the selected rules have high similarity to the inputs. In [44], an extended version of the Incircle FRI is introduced using a modified weight measure calculation and a shift technique. This weight measure estimation and shift ratio enabled the ability of extrapolation to be conducted implicitly, which also improved the performance results of the algorithm in the presence of multiple fuzzy rules and multidimensional priors.

III. CHARACTERISTICS OF FUZZY RULE INTERPOLATION CONDITIONS

FRI addresses the challenge of sparse fuzzy rule-based inference, but designing such systems requires careful consideration of multiple factors to ensure successful implementation. To evaluate and compare different techniques based on common principles, it is crucial to establish a set of criteria. Existing literatures (e.g., [8], [10], and [34]) has defined various criteria and properties from different perspectives when applying the FRI concept.

These criteria aim to ensure that FRI methods can produce different results as noted in the literatures [9], [11], and [13], where they produce normality and convexity conclusions, preserve multicollinearity, apply with different types of membership functions, deal with multidimensional environments, it reduces computational complexity.

Meeting most of these criteria with problem-specific parameters should result in a practical fuzzy rule interpolation technique. As a starting point for comparing and assessing FRI approaches, significant conditions have been presented to help unify FRI methods. Important FRI characteristics include the following:

A. AVOIDING THE INVALID CONCLUSION (NORMALITY)

FRI methods must, as a basic requirement, produce valid conclusion fuzzy sets, which implies that the resultant membership value must fall within the [1, 0] range and that only one membership function value should be associated with a single element of the conclusion. Based on the research presented in [19] and [30], this condition may be expressed as a set of limitations in FRI techniques that use α-Cut.

Suppose $X_j (j = 1,\ldots,n)$ is the input part and Y is the output part, the Cartesian product of the input part can be represented by $X = X_1 \times X_2 \times \ldots \times X_n$. A fuzzy rule (*IF-THEN*) described by R_i: *if* $A_{i1} \wedge A_{i2} \wedge \cdots \wedge A_{in}$, then B_i, where antecedents part $A_{ij} \in F(X_j)$, consequents part $B_i \in F(Y)$, and $F(Z)$ define all fuzzy subsets of Z. The Cartesian product of antecedents part (n-dimensional) $Aij, (j = 1,\ldots,n)$ of rule R_i can be indicated as $A_{(i)}$. The membership function of fuzzy set $A \in F(Z)$ is valid if it satisfies certain constraints based on α-cuts as shown in (1).

$$\begin{aligned}&\forall_{\alpha}, \alpha_1 < \alpha_2 \in (0, 1] : inf\{A_{\alpha}\} \leq sup\{A_{\alpha}\}\ and\\&inf\{A_{\alpha 1}\} \leq inf\{A_{\alpha 2}\}\ and\\&sup\{A_{\alpha 2}\} \leq sup\{A_{\alpha 1}\}\end{aligned} \tag{1}$$

where *"inf"* refers to the lower endpoints and *"sup"* refers to the upper endpoints of the real α-cuts of the fuzzy sets.

According to the conclusion of mapping I, indicated by $B^* = I(A^*) \in F(Y)$ will be a valid fuzzy set for any $A^* \in F(X)$, the correctness of the mapping may be ascertained. Further details and benchmarks related to this point can be found in [12], which demonstrates all notations of CNF condition.

Further details about the CNF benchmark examples related to this condition can be found in [12]. In which the authors proved all notations of the CNF condition and collected some cardinal rule-base and observation examples according to the first FRI method (KH FRI). Based on corollaries and equations that have also been set up to examine the normality of the fuzzy conclusion.

Fig.4 illustrates the goal of this condition, which is not to obtain a normality conclusion as shown by (B^*).

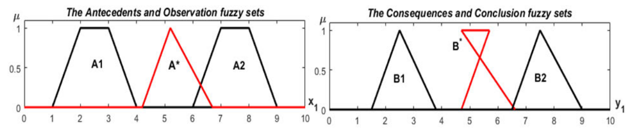


FIGURE 4. The condition of the invalid conclusion.

B. CONSERVATION ON THE LINEARITY SLOPES (PWL)

When the fuzzy sets used in the fuzzy rules are preserved on the linearity, the approximated sets must retain this characteristic. Consequently, inferences drawn from such rules and observations must adhere to a piecewise linear format as noted in the literatures [9], [11], and [13].

To ensure strict adherence to this condition, any further interpolation beyond calculation using odd points alone should be avoided (for more details see [28], [29]). To obtain a comprehensive understanding of this requirement, it is recommended that all notations of the piecewise linear condition be reviewed, along with examples, as demonstrated in [16].

The authors in [16] presented benchmark scenarios that were constructed to assess the KH FRI's compliance with PWL requirements, serving as a reference for evaluation and comparison with other FRI methods. These scenarios were created based on various equations and notations related to the PWL property, highlighting the problematic aspects of the KH FRI method, and constructing benchmark examples for testing other FRI methods against situations that do not meet the linearity condition for KH FRI. In the study, necessary and sufficient notations and equations demonstrating the PWL property for the KH FRI method were determined, discussing the relationship between linear approximation and real function conclusions. The paper compared several FRI methods (KHstab, VKK, FRIPOC, and VEIN) based on PWL benchmark examples. The results indicate that the KH FRI does not satisfy the PWL property. Among the methods compared, KHstab and FRIPOC suffer in preserving PWL, while the VKK and VEIN methods generally succeed, except for some benchmark examples with minor deviations in linearity.

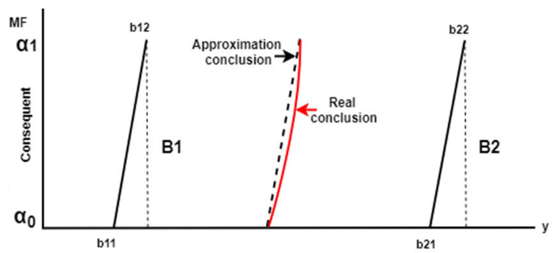


FIGURE 5. The condition of the linearity slopes of the fuzzy sets.

Fig. 5 shows the difference between the approximation conclusion and the real conclusion.

C. THE RELATIONSHIP BETWEEN THE ANTECEDENT AND CONSEQUENT FUZZY SETS (CONTINUITY)

According to the membership functions, the most common and simple ones used with the FRI concept are singleton, triangular, and trapezoidal fuzzy sets. Suppose that we have the fuzzy rules:

R_1: $A_1 \rightarrow B_1$, R_2: $A_2 \rightarrow B_2$, and R_3: $A_3 \rightarrow B_3$

As described in Fig. 6, the antecedent fuzzy sets are represented by triangular fuzzy sets, the consequences are represented by singleton fuzzy sets, and there is a new observation represented by triangular fuzzy set; therefore, based on the definition and continuity property, if the observation fuzzy set is similar (as the same type) to the antecedent fuzzy sets, then the conclusion fuzzy set should be similar to the consequence fuzzy sets.

The definition related to the continuity property between fuzzy sets of (antecedent and observation) and (consequences and conclusion), as mentioned in [26] and [30], is described as follows: The fuzzy set Z, it will be by FSZ: F(Z) × F(Z) → (R). Then, for any (A_1, A_2, A^*) $\in F(X)$, if $FS_x(A*, A_{i1}) \geq FSx(A^*, A_{i2})$, then $FS_y(I(A^*), B_{i1}) \geq FS_y(A^*, B_{i2})$, where R_{ij}: $A_{ij} \rightarrow B_{ij}$ $(j=1,2)$ refer to the two-rules from the rule-base (R).

Many scientists often focus solely on the scenario of this principle, which is when the observation exactly matches a rule antecedent. This case is generally referred to as rule base compatibility. In the field of logic, the principle of similarity is equivalent to Modus Ponens (MP) and is considered a hallmark of model continuity represented by the fuzzy-relation of the rule-base [27]. Fig. 6 shows the similar degree between observation and conclusion.

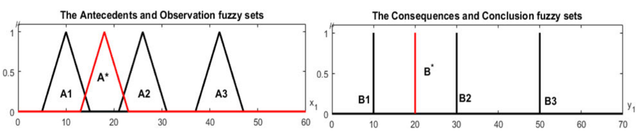


FIGURE 6. The condition of the mapping between antecedent and consequent.

D. THE FUZZINESS OF THE APPROXIMATED CONCLUSION (FUZZINESS)

Suppose the knowledge base provides certain information from fuzzy input data. In that case, a precise conclusion can be expected if all the consequences of the rules considered during interpolation are from a singleton fuzzy set. Regarding this condition, as demonstrated in the literature [26], there are two opposing viewpoints on this issue.

Case 1: If (A^*) is a singleton fuzzy set, then the deduced conclusion I(A^*) should be a singleton fuzzy set, as shown in Fig. 7.

Case 2: If all fuzzy rules (antecedents, consequents, and observation) (I) contributing to the calculation of the conclusion I(A^*) and the observation (A^*) are singletons, then I(A^*) should also be a singleton, as shown in Fig. 8.

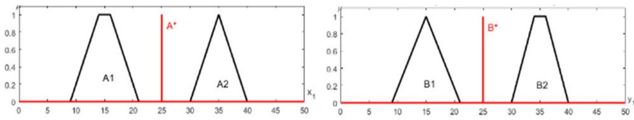


FIGURE 7. The condition of the fuzziness of the approximated result for (case1).

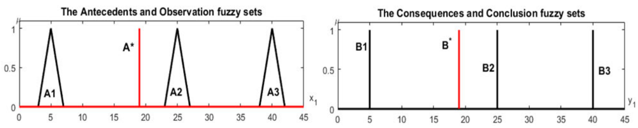


FIGURE 8. The condition of the fuzziness of the approximated result for (case2).

E. THE STABILITY OF APPROXIMATION (STABILITY)

In this property, the goal is to approximate the relationship between the antecedent and consequent universes to the highest possible degree. This approximation should be stable and separate from the position of the measure points, with confluence towards the compared function as the number of measure points approaches infinity. This condition ensures that the shape of the observation is identical to that of the conclusion, resulting in a stable and accurate approximation as illustrated by Fig. 9.

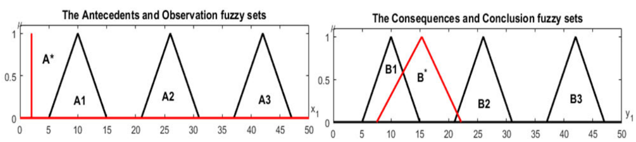


FIGURE 9. The condition of the stability of approximation.

F. THE FUZZY SETS OVERLAPPING (OVERLAPPING)

A competent fuzzy rule interpolation approach must be able to account for rules in which antecedents and consequences have overlapped. This implies that the method works in problem domains where neighboring fuzzy rules have certain common constituents. Fig. 10 describes the overlapping property and the location of the observation between the antecedents and consequences.

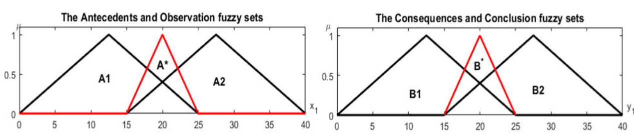


FIGURE 10. The condition of the overlapping between fuzzy sets.

G. EXTRAPOLATION CAPABILITY (FUZZINESS)

A method employing mapping I is considered suitable for extrapolation when it can generate a conclusion for an observation located in an extrapolative position. If observation A^* located so that A_{i1} and A_{i2} exist such that ($A_{i1} \prec A^* \prec A_{i2}$), then FRI is applied based on rules R_{i1} and R_{i2} to obtain the conclusion. Otherwise, when all rule antecedents A_i ($i = 1, \ldots, r$) either precede or are preceded by A^*:

$$\forall_i < \in [1, r] : If A^* \prec A_1 \, or A_1 \prec A^* \qquad (2)$$

On the other side, if every rule antecedent either precedes or follows A^* in the observed sequence, signifying the absence of A^* positioned between any antecedent pairs, the prerequisites for applying FRI based on extrapolation are not fulfilled as shown in Fig. 11.

This difference in positioning plays an important role in determining the method's capability to extrapolate conclusions for observations situated between antecedents or if its applicability is restricted to scenarios, where observations exactly align with specific antecedents.

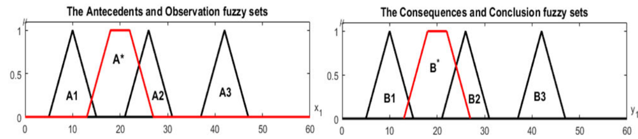


FIGURE 11. The condition of the extrapolation of the observation.

IV. THE SUGGESTED SET OF BENCHMARK SCENARIOS FOR EVALUATING THE FRI METHODS

Many FRI methods have been developed over the past decade, but most of them have weaknesses. Researchers in fuzzy interpolation attempted to address these deficiencies and issues in these methods to identify and improve recurring problems in evaluating FRI methods. The authors highlight several shortcomings in prior research. These include:

Lack of Comprehensive Conditions in FRI Analysis: Approach-oriented papers, such as those referenced in [10], [19], and [26], are criticized for introducing new FRI concepts without providing comprehensive conditions in the Fuzzy Rule Interpolation (FRI) analysis. The conditions they present are deemed insufficient to fully justify the proposed approaches or aspects.

Limited Scope in Summarization-Oriented Papers: Summarization-oriented papers, exemplified by [11], [34], [38], and [39], are faulted for offering brief summaries that predominantly focus on specific aspects of FRI. These summaries are considered lacking in coverage and depth, making them susceptible to criticism on similar grounds. Insufficient Evaluation Criteria: The evaluation criteria used in prior research are mentioned as lacking in some cases. For instance, [19] is noted for its analysis and comparison of MACI's general applicability, complexity, approximative power, and fuzziness of conclusion, indicating that these criteria might not be comprehensive enough.

Limited Methodological Exploration: Some summarization-oriented papers, including those by Jenei [26] and Baranyi et al. [10], are criticized for a lack of methodological exploration. The focus on applicability, consistency, and shape-preservation is noted, but the authors argue that these aspects are part of a summarization approach rather than a comprehensive exploration of methodologies.

The authors in this paper introduce new benchmarks and conditions for testing FRI methods. These benchmarks aim to serve as a reference for evaluating and comparing the accuracy and effectiveness of FRI methods; thus, researchers can identify areas for improvement in the field of fuzzy inference.

We selected specific fuzzy rules and fuzzy set configurations based on extensive analysis of the inherent challenges in FRI methods. The construction of benchmark scenarios is a crucial aspect of our evaluation framework. Moreover, during our analysis to generate the scenarios, the following properties will be considered to create the benchmarks:

- **Positioning of Observations:**

Examines where observations lie concerning rule bases, considering possibilities such as in-between, partial overlap, or alignment with existing rules.

- **Range Values of Antecedent and Consequent Components:**

Considers the range values of Antecedent (ANT) and Consequent (CON) components, providing insights into the scope and variability of these components within the FRI system.

- **Number of Rule Bases:**

The rationale for evaluating the number of rule bases in fuzzy rule interpolation is to comprehend the complexity inherent in both the antecedent and consequent parts of the system. The quantity of rule bases serves as a metric that guides the allocation of resources, helping to allocate computational capability and memory efficiently. Additionally, the insights derived from assessing the number of rule bases play a key role in informing modeling decisions and contribute to achieving optimal performance and accuracy in the interpolation process.

- **Dimensionality of Input and Output Variables:**

Examines the dimensionality of input and output variables, offering insights into the complexity and multidimensional nature of the FRI scenarios under evaluation.

- **Degree of Fuzziness for Fuzzy Sets:**

Evaluates the degree of fuzziness associated with the LeftSide and RightSide fuzzy sets, providing a measure of the uncertainty or imprecision incorporated into the FRI method.

- **The type of Fuzzy Sets:**

The selection of types of fuzzy sets relies on the nature of the issue, the available information, and the desired level of granularity and flexibility in modeling uncertainty. These shapes (triangular, trapezoidal, or singleton) offer a balance between interpretability, simplicity, and significance in catching and describing imprecise information, allowing fuzzy logic systems to effectively capture and process uncertain information in various applications, such as control systems, decision-making, and pattern recognition.

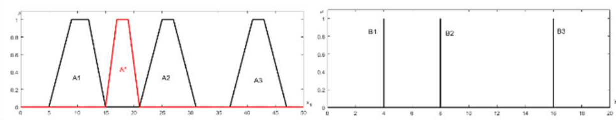
In summary, these properties encompass various aspects crucial for constructing comprehensive benchmark scenarios, offering a nuanced evaluation of FRI methods under diverse conditions. In the following, we present the proposed benchmarks based on all the above-mentioned properties as follows:

A. BENCHMARK SCENARIOS OF THE "CONTINUITY" CONDITION

This condition (Condition (IV-A)) encompasses four standardized scenarios (Scenario (1) - Scenario (4)) formulated and carefully constructed to serve as a benchmark for comparison, mainly focusing on the continuity condition. These scenarios contained variations in the dimensionality of the inputs and outputs and used different membership functions for each part of the inputs and outputs. The specific characteristics for each benchmark example are represented by input values, output values, and the number of rules for each scenario as of the condition (IV-A):

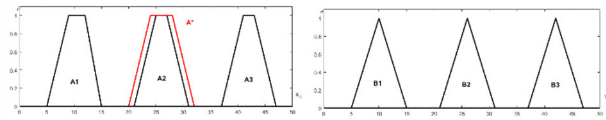
Condition (IV-A) - Scenario (1)

A_1=[5 9 12 15], A_2=[21 25 27 31], A_3=[37 41 43 47], B_1=[4], B_2=[8], B_3=[16], A^*=[15 17 19 21] , and Fuzzy Rules are $A_1 \rightarrow B_1$, $A_3 \rightarrow B_3$



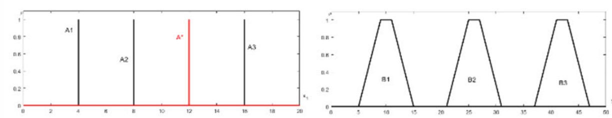
Condition (IV-A) - Scenario (2)

A_1=[5 9 12 15], A_2=[21 25 27 31], A_3=[37 41 43 47], B_1=[5 10 15], B_2=[21 26 31], B_3=[37 42 47], A^*=[20 24 28 32] , and Fuzzy Rules are $A_1 \rightarrow B_1$, $A_3 \rightarrow B_3$



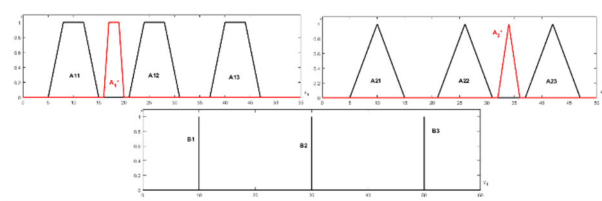
Condition (IV-A) - Scenario (3)

A_1=[4], A_2=[8], A_3=[16], B_1=[5 9 12 15], B_2=[21 25 27 31], B_3=[37 41 43 47], A^*=[12] , and Fuzzy Rules are $A_1 \rightarrow B_1$, $A_3 \rightarrow B_3$



Condition (IV-A) – Scenario (4)

A_{11}=[5 8 12 15], A_{12}=[21 24 28 31], A_{13}=[37 40 44 47], A_{21}=[5 10 15], A_{22}=[21 26 31], A_{23}=[37 42 47], B_1=[10], B_2=[30], B_3=[50], A^*_1=[16 17 19 20] , A^*_2=[32 34 36] , and Fuzzy Rules are A_{11} and $A_{21} \rightarrow B_1$, A_{12} and $A_{22} \rightarrow B_2$, A_{13} and $A_{23} \rightarrow B_3$



B. BENCHMARK SCENARIOS OF THE "FUZZINESS" CONDITION

The fuzziness condition comprises several standard scenarios used for evaluating the preservation of fuzziness in FRI methods. These scenarios are intended to investigate the behavior of FRI techniques in maintaining the fuzziness of the input and output fuzzy sets. The specific details of each scenario, including the properties of the input and output components and the number of rules, are provided below:

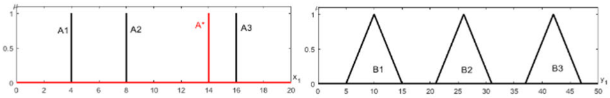
The first approach (IV-B1) of the fuzziness condition is related to when the observation is a singleton fuzzy set.

The second approach (IV-B2) of the fuzziness condition is related when the observation and consequent are singleton fuzzy sets. Both approaches (IV-B1) and (IV-B2) consist of four scenarios, each featuring distinct fuzzy sets and shapes.

The First Approach (IV-B1) of the Fuzziness Condition

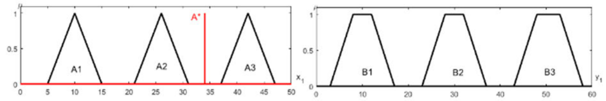
Condition (IV-B1) – Scenario (1)

A_1=[4], A_2=[8], A_3=[16], B_1=[5 10 15], B_2=[21 26 31], B_3=[37 42 47], A^*=[14] , and Fuzzy Rules are $A_1 \to B_1$, $A_3 \to B_3$



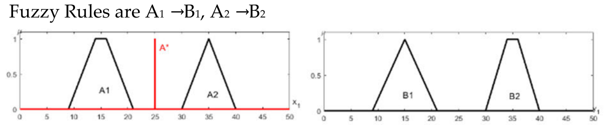
Condition (4.2-A) – Scenario (2)

A_1=[5 10 15], A_2=[21 26 31], A_3=[37 42 47], B_1=[3 8 12 17], B_2=[23 28 32 37], B_3=[43 48 53 58], A^*=[34] , and Fuzzy Rules are $A_1 \to B_1$, $A_3 \to B_3$



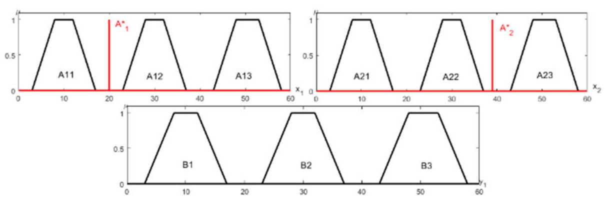
Condition (IV-B1) – Scenario (3)

A_1=[9 14 16 21], A_2=[30 35 40], B_1=[9 15 21], B_2=[30 34 36 40], A^*=[25] , and Fuzzy Rules are $A_1 \to B_1$, $A_2 \to B_2$



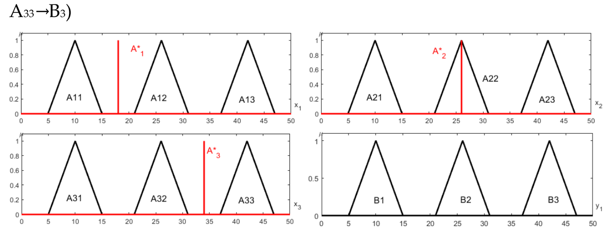
Condition (IV-B1) – Scenario (4)

A_{11}=[3 8 12 17], A_{12}=[23 28 32 37], A_{13}=[43 48 53 58], A_{21}=[3 8 12 17], A_{22}=[23 28 32 37], A_{23}=[43 48 53 58], B_1=[3 8 12 17], B_2=[23 28 32 37], B_3=[43 48 53 58] , A^*_1=[20] , A^*_2=[39] , and Fuzzy Rules are: (A_{11} and $A_{21} \to B_1$), (A_{12} and $A_{22} \to B_2$), (A_{13} and $A_{23} \to B_3$).



Condition (IV-B1) – Scenario (5)

A_{11}=[5 10 15], A_{12}=[21 26 31], A_{13}=[37 42 47], A_{21}=[5 10 15], A_{22}=[21 26 31], A_{23}=[37 42 47], A_{31}=[5 10 15], A_{32}=[21 26 31], A_{33}=[37 42 47], B_1=[5 10 15], B_2=[21 26 31], B_3=[37 42 47], A^*_1=[18] , A^*_2=[26] , A^*_3=[34], and Fuzzy Rules are: (A_{11} and A_{21} and $A_{31} \to B_1$), (A_{12} and A_{22} and $A_{32} \to B_2$), (A_{13} and A_{32} and $A_{33} \to B_3$)



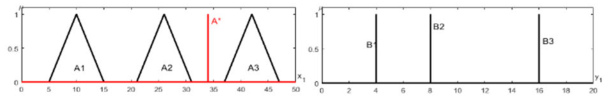
C. BENCHMARK SCENARIOS OF THE "STABILITY" CONDITION

This requirement comprises four standard scenarios designed for the stability condition, which simulate the stability of the observation with a conclusion. Three scenarios are represented by (1-D) input and (1-D) output, and one scenario is described by (2-D) input and (1-D) output. The specific details of each scenario, including the properties of the input and output components and the number of rules, are provided in condition (IV-C):

The Second Approach (IV-B2) of the Fuzziness Condition

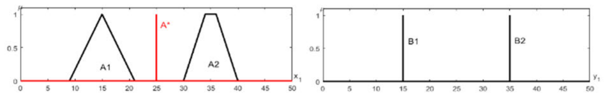
Condition (IV-B2) – Scenario (1)

A_1=[5 10 15], A_2=[21 26 31], A_3=[37 42 47], B_1=[4], B_2=[8], B_3=[16], A^*=[34] , and Fuzzy Rules are $A_1 \to B_1$, $A_3 \to B_3$



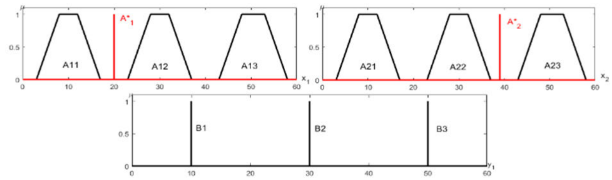
Condition (IV-B2) – Scenario (2)

A_1=[9 15 21], A_2=[30 34 36 40], B_1=[15], B_2=[35], A^*=[25] , and Fuzzy Rules are $A_1 \to B_1$, $A_2 \to B_2$



Condition (IV-B2) – Scenario (3)

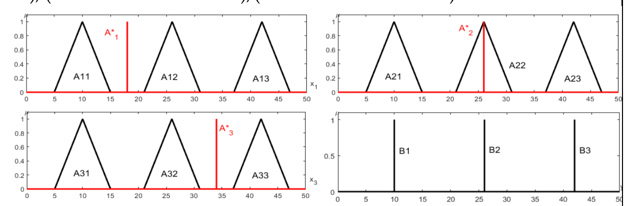
A_{11}=[3 8 12 17], A_{12}=[23 28 32 37], A_{13}=[43 48 53 58], A_{21}=[3 8 12 17], A_{22}=[23 28 32 37], A_{23}=[43 48 53 58], B_1=[10], B_2=[26], B_3=[42], A^*_1=[20] , A^*_2=[39] , and Fuzzy Rules are: (A_{11} and $A_{21} \to B_1$), (A_{12} and $A_{22} \to B_2$), (A_{13} and $A_{23} \to B_3$).



Condition (IV-B2) – Scenario (4)

A_{11}=[5 10 15], A_{12}=[21 26 31], A_{13}=[37 42 47], A_{21}=[5 10 15], A_{22}=[21 26 31], A_{23}=[37 42 47], A_{31}=[5 10 15], A_{32}=[21 26 31], A_{33}=[37 42 47], B_1=[10], B_2=[26], B_3=[42],

A^*_1=[18] , A^*_2=[26] , A^*_3=[34], and Fuzzy Rules are: (A_{11} and A_{21} and $A_{31} \to B_1$), (A_{12} and A_{22} and $A_{32} \to B_2$), (A_{13} and A_{32} and $A_{33} \to B_3$)



D. BENCHMARK SCENARIOS OF THE "OVERLAPPING" CONDITION

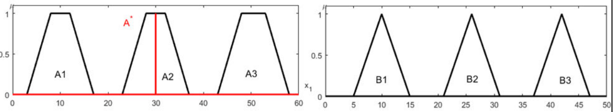
In cases where the input and output fuzzy sets are overlapping, this condition is suitable to check the validity of the FRI methods. Four scenarios are designed to check this condition; three of them are described in case (1-D) input and (1-D) output, and one scenario is described by (2-D) input and (1-D) output. The specific details of each scenario, including the properties of the input and output and the number of rules, are provided in condition (IV-D):

E. BENCHMARK SCENARIOS OF THE "EXTRAPOLATION" CONDITION

In cases where the observation is outside the range of the input part (antecedent), it is called the extrapolation condition. This condition (Condition (IV-E)) contains five standardized scenarios (Scenario (1)–Scenario (5)) developed and carefully created to perform as a benchmark for comparison. These scenarios contained variations in the dimensionality of the inputs and outputs and used different membership functions for each part of the inputs and outputs. The specific attributes for each scenario are represented by input values, output

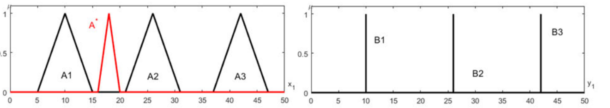
Condition (IV-C) – Scenario (1)

A_1=[3 8 12 17], A_2=[23 28 32 37], A_3=[43 48 53 58], B_1=[5 10 15], B_2=[21 26 31], B_3=[37 42 47], A^*=[30] , and Fuzzy Rules are $A_1 \rightarrow B_1$, $A_3 \rightarrow B_3$



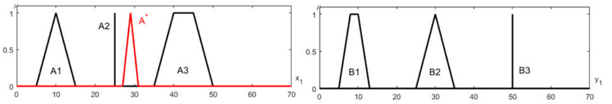
Condition (IV-C) – Scenario (2)

A_1=[5 10 15], A_2=[21 26 31], A_3=[37 42 47], B_1=[10], B_2=[26], B_3=[42], A^*=[16 18 20] , and Fuzzy Rules are $A_1 \rightarrow B_1$, $A_2 \rightarrow B_2$, $A_3 \rightarrow B_3$



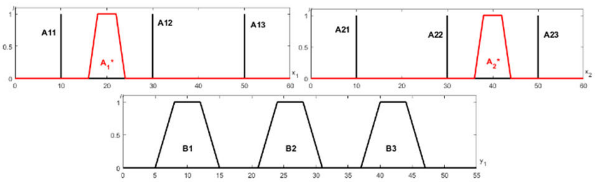
Condition (IV-C) – Scenario (3)

A_1=[5 10 15], A_2=[25], A_3=[35 40 45 50], B_1=[5 8 10 13], B_2=[25 30 35], B_3=[50], A^*=[27 29 31] , and Fuzzy Rules are $A_1 \rightarrow B_1$, $A_2 \rightarrow B_2$, $A_3 \rightarrow B_3$



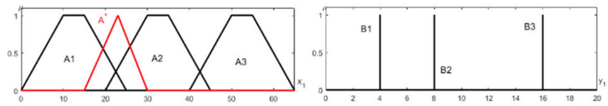
Condition (IV-C) – Scenario (4)

A_{11}=[10], A_{12}=[30], A_{13}=[50], A_{21}=[10], A_{22}=[30], A_{23}=[50], B_1=[5 8 12 15], B_2=[21 24 28 31], B_3=[37 40 44 47], A^*_1=[16 18 22 24] , A^*_2=[36 38 42 44], and Fuzzy Rules are: (A_{11} and $A_{21} \rightarrow B_1$), (A_{12} and $A_{22} \rightarrow B_2$), (A_{13} and $A_{23} \rightarrow B_3$).



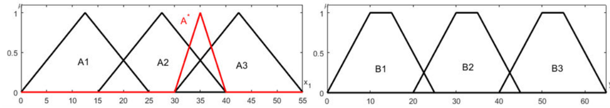
Condition (IV-D) – Scenario (1)

A_1=[0 10 15 25], A_2=[20 30 35 45], A_3=[40 50 55 65], B_1=[4], B_2=[8], B_3=[16], A^*=[15 23 30] , and Fuzzy Rules are $A_1 \rightarrow B_1$, $A_2 \rightarrow B_2$, $A_3 \rightarrow B_3$



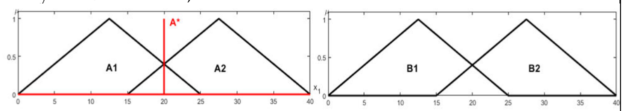
Condition (IV-D) – Scenario (2)

A_1=[0 12.5 25], A_2=[15 27.5 40], A_3=[30 42.5 55], B_1=[0 10 15 25], B_2=[20 30 35 45], B_3=[40 50 55 65], A^*=[30 35 40] , and Fuzzy Rules are $A_1 \rightarrow B_1$, $A_2 \rightarrow B_2$, $A_3 \rightarrow B_3$



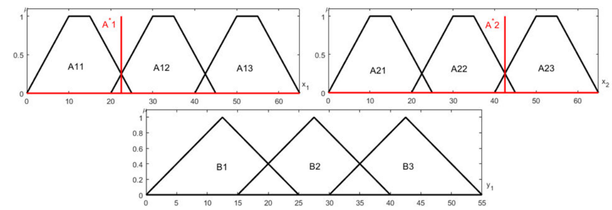
Condition (IV-D) – Scenario (3)

A_1=[0 12.5 25], A_2=[15 27.5 40], B_1=[0 12.5 25], B_2=[15 27.5 40], A^*=[20] , and Fuzzy Rules are $A_1 \rightarrow B_1$, $A_2 \rightarrow B_2$



Condition (IV-D) – Scenario (4)

A_{11}=[0 10 15 25], A_{12}=[20 30 35 45], A_{13}=[40 50 55 65], A_{21}=[0 10 15 25], A_{22}=[20 30 35 45], A_{23}=[40 50 55 65], B_1=[0 12.5 25], B_2=[15 27.5 40], B_3=[30 42.5 55], A^*_1=[22.5] , A^*_2=[42.5] , and Fuzzy Rules are: (A_{11} and $A_{21} \rightarrow B_1$), (A_{12} and $A_{22} \rightarrow B_2$), (A_{13} and $A_{23} \rightarrow B_3$).



values, and the number of rules for each scenario of the condition (IV-E):

V. EXPERIMENTS OF THE BENCHMARK SCENARIOS

In this section, we present a comprehensive evaluation of multiple FRI methods utilizing the suggested benchmark scenarios. The evaluated methods, including KH [14], [15], KHSTB [8], MACI [19], IMUL [22], CRF [17], FIVE [24], VKK [20], GM [9], POC [21], LES [23], VEIN [25], and INCIRCLE [18], were subjected to strict checks based on specific benchmark scenarios. Each method's performance was analyzed in terms of its adherence to the benchmark scenarios, allowing for a detailed comparison. These benchmarks aimed to provide a thorough understanding of each method's strengths and weaknesses in handling diverse FRI scenarios.

The FRI toolbox was developed in a MATLAB environment by Johanyák et. al. in [35] and in an OCTAVE environment by Alzubi et. al. in [5]. The main purpose of the FRI toolbox is to unify various FRI methods, which played a crucial role in facilitating the evaluation methods. The current version of the FRI toolbox is available to download in [36]. It includes the following methods: KH, KH Stabilized, MACI, IMUL, CRF, VKK, GM, FRIPOC, LESFRI, and INCIRCLE. The package of the FRI toolbox contains software with a graphical user interface, providing easy-to-use access [7].

A. EXPERIMENTS OF FRI METHODS BASED ON THE BENCHMARK SCENARIOS OF THE "CONTINUITY" CONDITION

Table 1 provides a comprehensive overview of how different FRI methods fare in meeting the continuity mapping between rule-base parts condition. This helps researchers and practitioners to make informed decisions when selecting appropriate methods for specific applications.

The table indicates that the MACI method successfully met all conditions for all four benchmark scenarios, demonstrating its robustness. On the other hand, the LES method fulfilled all conditions except for scenario (IV-A.2), showing a minor limitation in this specific case. As for the IMUL method, it failed to meet two benchmark scenarios, namely scenario (IV-A.1) and scenario (IV-A.4), displaying areas where improvement is needed. Similarly, the CRF method struggled to meet the conditions for all benchmark scenarios, suggesting significant shortcomings. In contrast, several other methods, such as KH, KHSTB, FIVE, VKK, GM, POC, VEIN, and INCIRCLE, showed mixed results in fulfilling the conditions for the benchmark scenarios, highlighting the variability in their performance.

TABLE 1. **Summary of the evaluation FRI techniques according to benchmark scenarios of the continuity condition (IV-A).**

Scenarios of the Continuity Condition (IV-A)				
Methods	Scenario (IV-A.1)	Scenario (IV-A.2)	Scenario (IV-A.3)	Scenario (IV-A.4)
KH[14][15]	¢	ӿ	√	¢
KHSTB[8]	¢	ӿ	√	¢
MACI[19]	√	√	√	√
IMUL[22]	ӿ	√	√	ӿ
CRF[17]	ӿ	ӿ	ӿ	ӿ
FIVE[24]	√	¢	ӿ	¢
VKK[20]	¢	ӿ	√	√
GM[9]	√	¢	¢	√
POC[21]	√	¢	√	√
LES[23]	√	√	√	√
VEIN[25]	ӿ	¢	ӿ	¢
INCIRCLE[18]	√	√	ӿ	√

A sign (√) indicates that the condition is fulfilled, a sign (◎) indicates that the condition failed, and a sign (◎) indicates that there is no result or there a problem with conclusion.

B. EXPERIMENTS OF FRI METHODS BASED ON THE BENCHMARK SCENARIOS OF THE "FUZZINESS" CONDITION

Table 2 shows the evaluation summary of the current interpolations' methods according to the fuzziness benchmark, where the conclusion must preserve the same ratio of fuzziness on the left and right sides.

The conclusion based on Condition (IV-B1) can be expected if all the consequences of the rules taken into consideration during the interpolation are singleton-shaped, i.e., the knowledge base produces certain information from fuzzy input data. The table shows that MACI, KH, KHSTB, and VEIN failed to preserve the degree of support for fuzzy rules in all benchmark scenarios. Table 2 describes some FRI methods that passed in some scenarios and failed in others to apply this benchmark, such as the IMUL, POC, LES, and VKK methods. Regarding the FIVE method is only successful in scenario (IV-B1.4). CRF, GM, and INCIRCLE methods were able to preserve this Condition (IV-B1) in all benchmark scenarios.

Table 3 shows the evaluation summary of the current FRI methods based on the benchmarks for condition (IV-B2), where the conclusion must preserve the same ratio of fuzziness on the left and right sides.

Based on the evaluation results for all scenarios (IV-B2.1)–(IV-B2.4)., all methods except for KH, KHSTB, and VEIN fulfill the preservation of fuzziness conclusion condition (IV-B2). KH and KHSTB failed to fulfill the condition, especially in those scenarios that contain multi-dimensional inputs and different shapes, such as triangles and trapezoids. The FIVE method has succeeded in one scenario (IV-B2.3); other scenarios have no results, and the LES method has no result in one scenario (IV-B2.2). According to the IMUL, CRF, VKK, GM, POC, and Incircle methods have been successful in all scenarios, as shown in Table 3.

TABLE 2. **Summary of the evaluation FRI techniques according to the benchmark scenarios of the fuzziness condition (IV-B1).**

Scenarios of the Fuzziness Condition (IV-B1)					
Methods	Scenario (IV-B1.1)	Scenario (IV-B1.2)	Scenario (IV-B1.3)	Scenario (IV-B1.4)	Scenario (IV-B1.5)
KH[14][15]	ӿ	¢	¢	ӿ	ӿ
KHSTB[8]	ӿ	¢	¢	ӿ	ӿ
MACI[19]	ӿ	ӿ	ӿ	ӿ	ӿ
IMUL[22]	√	ӿ	ӿ	ӿ	√
CRF[17]	√	√	√	√	√
FIVE[24]	¢	¢	¢	√	¢
VKK[20]	ӿ	¢	¢	√	√
GM[9]	¢	√	√	√	√
POC[21]	ӿ	ӿ	ӿ	√	√
LES[23]	ӿ	ӿ	¢	√	√
VEIN[25]	¢	¢	¢	ӿ	¢
INCIRCLE[18]	√	√	√	√	√

A sign (√) indicates that the condition is fulfilled, a sign (◎) indicates that the condition failed, and a sign (◎) indicates that there is no result or there a problem with conclusion.

TABLE 3. **Summary of the evaluation FRI techniques according to the benchmark scenarios of the fuzziness condition (IV-B2).**

Scenarios of the Fuzziness Condition (IV-B2)				
Methods	Scenario (IV-B2.1)	Scenario (IV-B2.2)	Scenario (IV-B2.3)	Scenario (IV-B2.4)
KH[14][15]	¢	¢	¢	¢
KHSTB[8]	¢	¢	¢	¢
MACI[19]	√	√	√	√
IMUL[22]	√	√	√	√
CRF[17]	√	√	√	√
FIVE[24]	¢	¢	√	¢
VKK[20]	√	√	√	√
GM[9]	√	√	√	√
POC[21]	√	√	√	√
LES[23]	√	¢	√	√
VEIN[25]	¢	¢	ӿ	¢
INCIRCLE[18]	√	√	√	√

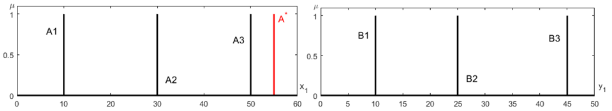
A sign (√) indicates that the condition is fulfilled, a sign (◎) indicates that the condition failed, and a sign (◎) indicates that there is no result or there a problem with conclusion.

C. EXPERIMENTS OF FRI METHODS BASED ON THE BENCHMARK SCENARIOS OF THE "STABILITY" CONDITION

Table 4 explains the evaluation summary of the current FRI methods according to the stability condition (IV-C), where the conclusions must be stable even using different membership functions in both input and output parts.

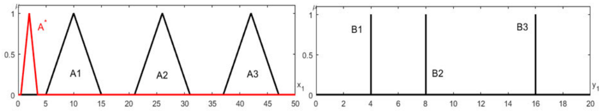
Condition (IV-E) – Scenario (1)

A_1=[10], A_2=[30], A_3=[50], B_1=[10], B_2=[25], B_3=[45], A^*=[55] , and Fuzzy Rules are $A_1 \rightarrow B_1$, $A_2 \rightarrow B_2$, $A_3 \rightarrow B_3$



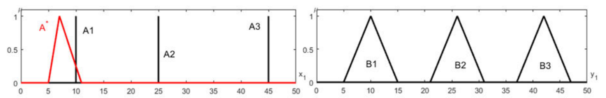
Condition (IV-E) – Scenario (2)

A_1=[5 10 15], A_2=[21 26 31], A_3=[37 42 47], B_1=[4], B_2=[8], B_3=[16], A^*=[0.5 2 3.5] , and Fuzzy Rules are $A_1 \rightarrow B_1$, $A_3 \rightarrow B_3$



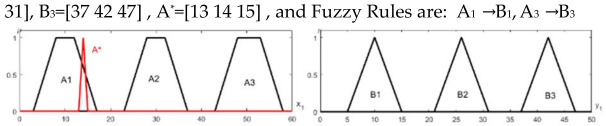
Condition (IV-E) – Scenario (3)

A_1=[10], A_2=[25], B_1=[5 10 15], B_2=[21 26 31], B_3=[37 42 47], A^*=[5 7 11] , and Fuzzy Rules are $A_1 \rightarrow B_1$, $A_2 \rightarrow B_2$, $A_3 \rightarrow B_3$



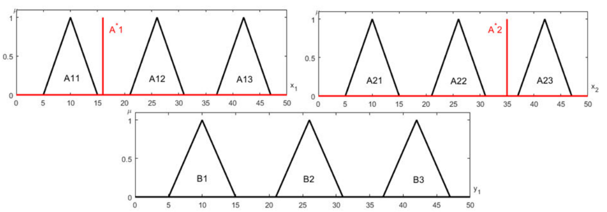
Condition (IV-E) – Scenario (4)

A_1=[3 8 12 17], A_2=[23 28 32 37], A_3=[43 48 53 58], B_1=[5 10 15], B_2=[21 26 31], B_3=[37 42 47] , A^*=[13 14 15] , and Fuzzy Rules are: $A_1 \rightarrow B_1$, $A_3 \rightarrow B_3$



Condition (IV-E) – Scenario (5)

A_{11}=[5 10 15], A_{12}=[21 26 31], A_{13}=[37 42 47], A_{21}=[5 10 15], A_{22}=[21 26 31], A_{23}=[37 42 47], A_{31}=[5 10 15], A_{32}=[21 26 31], A_{33}=[37 42 47], B_1=[5 10 15], B_2=[21 26 31], B_3=[37 42 47], A^*_1=[16] , A^*_2=[35] , and Fuzzy Rules are: (A_{11} and $A_{23} \rightarrow B_1$), (A_{12} and $A_{22} \rightarrow B_2$), (A_{13} and $A_{23} \rightarrow B_3$).



According to the results, IMUL is the only successful method in all four scenarios of the benchmarks of the stability condition (IV-C). Meanwhile, the INCIRCLE method succeeded in three scenarios but failed in one scenario (IV-C.2). In contrast, MACI, POC, and LES succeeded in fulfilling two scenarios for each one. While CRF, VKK, KH, KHSTB, and GM failed to fulfill the condition, which failed in three scenarios and succeeded in one scenario for each one, according to the FIVE and VEIN methods, there was no result to fulfill the condition for all scenarios.

D. EXPERIMENTS OF FRI METHODS BASED ON THE BENCHMARK SCENARIOS OF THE "OVERLAPPING" CONDITION

Table 5 shows the evaluation summary of the current FRI methods according to the benchmark scenarios of the overlapping antecedent and consequent rule bases. The methods are evaluated on benchmarks of the overlapping condition (IV-D) for all scenarios (IV-D.1) - (IV-D.4). Therefore, we can conclude that for all FRI methods, only KHSTB has no conclusions for all scenarios. While INCIRCLE succeeded with all scenarios. Other FRI methods have succeeded in some of the scenarios and failed in others.

TABLE 4. Summary of the evaluation FRI techniques according to the benchmark scenarios of the stability condition (IV-C).

Methods	Scenario (IV-C.1)	Scenario (IV-C.2)	Scenario (IV-C.3)	Scenario (IV-C.4)
KH[14][15]	¢	¢	¢	√
KHSTB[8]	¢	¢	¢	√
MACI[19]	x	x	√	√
IMUL[22]	√	√	√	√
CRF[17]	√	x	¢	¢
FIVE[24]	¢	¢	¢	x
VKK[20]	√	x	¢	¢
GM[9]	√	x	¢	¢
POC[21]	√	x	√	¢
LES[23]	√	x	¢	√
VEIN[25]	¢	¢	¢	¢
INCIRCLE[18]	√	x	√	√

A sign (√) indicates that the condition is fulfilled, a sign (⊚) indicates that the condition failed, and a sign (⊚) indicates that there is no result or there a problem with conclusion.

E. EXPERIMENTS OF FRI METHODS BASED ON THE BENCHMARK SCENARIOS OF THE "EXTRAPOLATION" CONDITION

Table 4 shows the evaluation summary of the current FRI methods used, which can be expected if they are working with extrapolation issue or not. The number of rules is important for this condition, especially in the case of the multidimensional antecedent, where the conclusion of the FRI methods can be calculated based on the adjacent two rules to the observation.

Table 4 describes some FRI methods that passed in some scenarios and failed in others to apply this benchmark; only the LES method was successful in all scenarios; next, the KHstab and Incircle methods were successful in four scenarios and failed in scenario (IV-E.4). While the POC method is successful in two scenarios (IV-E.2) and (IV-E.5). Regarding the rest of the FRI methods, they failed to apply the extrapolation benchmark for all scenarios.

Table 7 and Fig. 12 present the performance metrics for different methods across varying conditions (A to E) and offer an overall evaluation via the "Total_Average" column. Here is a detailed investigation:

For KH method, an average of 29% is achieved across all conditions, with the highest score observed only in Condition E. For KHSTB method showcases an average performance of 25%, with the highest score observed only in Condition E. For MACI method exhibits robust performance

TABLE 5. Summary of the evaluation FRI techniques according to the benchmark scenarios of the overlapping condition (IV-D).

Scenarios of the Overlapping Condition (IV-D)				
Methods	Scenario (IV-D.1)	Scenario (IV-D.2)	Scenario (IV-D.3)	Scenario (IV-D.4)
KH[14][15]	¢	√	¢	¢
KHSTB[8]	¢	¢	¢	¢
MACI[19]	√	√	¢	¢
IMUL[22]	¢	√	¢	¢
CRF[17]	√	√	¢	¢
FIVE[24]	√	¢	¢	¢
VKK[20]	¢	√	√	√
GM[9]	√	¢	√	√
POC[21]	√	¢	√	√
LES[23]	√	√	√	¢
VEIN[25]	√	¢	¢	¢
INCIRCLE[18]	√	√	√	√

A sign (√) indicates that the condition is fulfilled, a sign (◎) indicates that the condition failed, and a sign (◎) indicates that there is no result or there a problem with conclusion.

TABLE 6. Summary of the evaluation FRI techniques according to the benchmark scenarios of the extrapolation condition (IV-E).

Scenarios of the Extrapolation Condition (IV-E)					
Methods	Scenario (IV-E.1)	Scenario (IV-E.2)	Scenario (IV-E.3)	Scenario (IV-E.4)	Scenario (IV-E.5)
KH[14][15]	x	x	x	x	x
KHSTB[8]	√	√	√	x	√
MACI[19]	x	x	x	√	x
IMUL[22]	x	x	x	x	x
CRF[17]	x	x	x	x	x
FIVE[24]	¢	¢	¢	¢	¢
VKK[20]	x	x	x	x	x
GM[9]	x	x	x	x	x
POC[21]	¢	√	¢	x	√
LES[23]	√	√	√	√	√
VEIN[25]	¢	¢	¢	¢	¢
INCIRCLE[18]	√	√	√	x	√

A sign (√) indicates that the condition is fulfilled, a sign (◎) indicates that the condition failed, and a sign (◎) indicates that there is no result or there a problem with conclusion.

across conditions, boasting an impressive overall average of 73%, excelling notably in Conditions A, B2, and E. For IMUL method delivers a high overall average of 79%, demonstrating strong performance in all conditions except Condition A and D. For CRF method achieves a 36% overall average, with notable strength in Condition B2. For FIVE method) attains a 16% overall average, with moderate performance in Conditions A, B1, B2, and D. For VKK method achieves a 65% overall average, excelling particularly in Conditions B2, and E. For GM method scores a 55% overall average, with notable strengths in Conditions B1, B2, and D. For POC method attains a 57% overall average, demonstrating consistent performance across various conditions. For LES method also achieves a 57% overall average, performing well in Conditions A, B2, and D. For VEIN method registers a low 4% overall average, indicating limited effectiveness across all conditions. For INCIRCLE method stands out with an impressive 92% overall average, showcasing strong performance in all conditions.

These performance metrics offer valuable insights into the relative strengths and weaknesses of each method across diverse conditions, facilitating the evaluation and selection of the most appropriate method for specific scenarios.

TABLE 7. summary of benchmark-based evaluation results for the fri methods.

Methods	Condition A	Condition B1	Condition B2	Condition C	Condition D	Condition E
KH	25%	0%	0%	25%	25%	100%
KHSTB	25%	0%	0%	25%	0%	100%
MACI	100%	40%	100%	50%	50%	100%
IMUL	50%	100%	100%	100%	25%	100%
CRF	0%	40%	100%	25%	50%	0%
FIVE	25%	20%	25%	0%	25%	0%
VKK	50%	40%	100%	25%	75%	100%
GM	50%	80%	100%	25%	75%	0%
POC	75%	40%	100%	50%	75%	0%
LES	100%	40%	75%	50%	75%	0%
VEIN	0%	0%	0%	0%	25%	0%
INCIRCLE	75%	100%	100%	75%	100%	100%

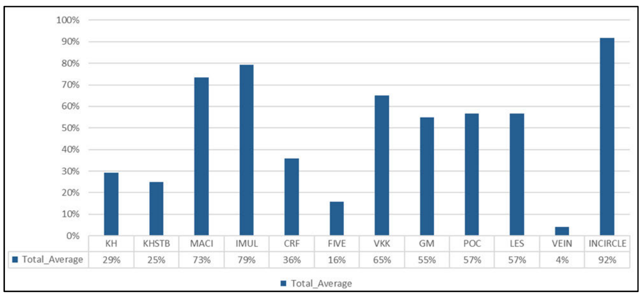


FIGURE 12. The Average ratios of the evaluation FRI methods according to benchmarks.

VI. CONCLUSION

Several conditions have been presented to demonstrate a unified framework for the requisites of Fuzzy Rule Interpolation (FRI) methods. This paper contributes by presenting a series of benchmarks created for the evaluation of FRI methods, clarifying the essential conditions they ought to perform. These benchmarks a useful reference points, offering recommendations for the classification and comparison of different

FRI methodologies. Using these benchmarks not only enables the comparison and evaluation of FRI methods but also performs as a guide to assess their strengths and weaknesses. The overarching purpose is to formalize the evaluation process, enabling researchers to measure the accuracy and efficacy of FRI methods under various and challenging conditions. These benchmarks have significant possibilities for advancing the field of fuzzy inference and improving the practical applicability of FRI methods in real-world scenarios.

From the results gathered in this study, it is concluded that MACI, IMUL, VKK, GM, POC, LES, and INCIRCLE consistently meet the specified benchmarks, demonstrating high success across various evaluation benchmark scenarios. Particularly, the Incircle FRI method appears to be especially proficient, producing meaningful results, as detailed in the accompanying Tables (I - VI). In contrast, several other FRI methods either experienced failure or produced inconclusive results across a majority of benchmarks. This discernment highlights the efficacy of the Incircle FRI method and highlights the differences in performance among different FRI approaches based on the applied benchmarks.

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REFERENCES

[1] M. Al-Kasassbeh, M. Almseidin, K. Alrfou, and S. Kovacs, "Detection of IoT-botnet attacks using fuzzy rule interpolation," *J. Intell. Fuzzy Syst.*, vol. 39, no. 1, pp. 421–431, Jul. 2020.

[2] W. D. Ofor, N. D. Nwiabu, and D. Matthias, "Face detection and expression recognition using fuzzy rule interpolation," *Int. J. Comput. Sci. Eng.*, vol. 7, no. 5, pp. 1683–1689, May 2019.

[3] M. Almseidin and S. Kovacs, "Intrusion detection mechanism using fuzzy rule interpolation," *J. Theor. Applied Inf. Technol.*, vol. 96, no. 16, pp. 5473–5488, 2018.

[4] Y. Jiang, S. Jin, and J. Peng, "Hierarchical fuzzy rule interpolation and its application for hotels location selection," *Int. J. Cognit. Informat. Natural Intell.*, vol. 10, no. 1, pp. 55–79, Jan. 2016.

[5] M. Alzubi, M. Almseidin, M. A. Lone, and S. Kovacs, "Fuzzy rule interpolation toolbox for the GNU open-source OCTAVE," in *Proc. 17th Int. Conf. Emerg. eLearning Technol. Appl. (ICETA)*, Nov. 2019, pp. 16–22.

[6] L. T. Kóczy, J. Botzheim, and T. D. Gedeon, "Fuzzy models and interpolation," in *Forging New Frontiers: Fuzzy Pioneers I*, 2007, pp. 111–131.

[7] M. Alzubi, Z. C. Johanyák, and S. Kovács, "Fuzzy rule interpolation methods and FRI toolbox," 2019, *arXiv:1904.12178*.

[8] D. Tikk, I. Joó, L. Kóczy, P. Várlaki, B. Moser, and T. D. Gedeon, "Stability of interpolative fuzzy KH controllers," *Fuzzy Sets Syst.*, vol. 125, no. 1, pp. 105–119, Jan. 2002.

[9] C. Z. Johanyák and S. Kovács, "Survey on various interpolation based fuzzy reasoning methods," *Prod. Syst. Inf. Eng.*, vol. 3, no. 1, pp. 39–56, 2020.

[10] P. Baranyi, L. T. Koczy, and T. D. Gedeon, "A generalized concept for fuzzy rule interpolation," *IEEE Trans. Fuzzy Syst.*, vol. 12, no. 6, pp. 820–837, Dec. 2004.

[11] Z. C. Johanyák and S. Kovács, "A brief survey and comparison on various interpolation based fuzzy reasoning methods," *Acta Polytechnica Hungarica*, vol. 3, no. 1, pp. 91–105, 2006.

[12] M. Alzubi and S. Kovacs, "Some considerations and a benchmark related to the CNF property of the koczy-hirota fuzzy rule interpolation," 2019, *arXiv:1911.05041*.

[13] Z. C. Johanyák and A. Berecz, "Survey on practical applications of fuzzy rule interpolation," in *Proc. 1st Int. Sci. Exp. Conf. TEAM*, 2009, pp. 205–213.

[14] L. Kóczy and K. Hirota, "Interpolative reasoning with insufficient evidence in sparse fuzzy rule bases," *Inf. Sci.*, vol. 71, nos. 1–2, pp. 169–201, Jun. 1993.

[15] L. T. Koczy and K. Hirota, "Size reduction by interpolation in fuzzy rule bases," *IEEE Trans. Syst., Man, Cybern., B Cybern.*, vol. 27, no. 1, pp. 14–25, Feb. 1997.

[16] M. Alzubi and S. Kovács, "Investigating the piece-wise linearity and benchmark related to koczy-hirota fuzzy linear interpolation," *J. Theor. Appl. Inf. Technol.*, vol. 97, no. 11, pp. 3098–3111, 2019.

[17] L. Kóczy, K. Hirota, and T. D. Gedeon, "Fuzzy rule interpolation by the conservation of relative fuzziness," *J. Adv. Comput. Intell. Intell. Inform.*, vol. 4, no. 1, pp. 95–101, 2000.

[18] M. Alzubi and S. Kovacs, "Interpolative fuzzy reasoning method based on the incircle of a generalized triangular fuzzy number," *J. Intell. Fuzzy Syst.*, vol. 39, no. 1, pp. 709–729, Jul. 2020.

[19] D. Tikk and P. Baranyi, "Comprehensive analysis of a new fuzzy rule interpolation method," *IEEE Trans. Fuzzy Syst.*, vol. 8, no. 3, pp. 281–296, Jun. 2000.

[20] G. Vass, L. Kalmar, and L. Koczy, "Extension of the fuzzy rule interpolation method," in *Proc. Int. Conf. Fuzzy Sets Theory Appl.*, 1992, pp. 1–6.

[21] Z. C. Johanyák and S. Kovács, "Fuzzy rule interpolation based on polar cuts," in *Computational Intelligence, Theory and Applications*. Berlin, Germany: Springer, 2006, pp. 18–20.

[22] K. W. Wong, T. Gedeon, and D. Tikk, "An improved multidimensional alpha-cut based fuzzy interpolation technique," in *Proc. Int. Conf. Artif. Intell. Sci. Technol. (AISAT)*. Hobart, TAS, Australia: University of Tasmania, 2000, pp. 29–32.

[23] Z. C. Johanyák and S. Kovács, "Fuzzy rule interpolation by the least squares method," in *Proc. 7th Int. Symp. Hung. Researchers Comput. Intell. (HUCI)*, vol. 2425, Nov. 2006, pp. 495–506.

[24] S. Kovács and L. T. Kóczy, "Application of an approximate fuzzy logic controller in an AGV steering system, path tracking and collision avoidance strategy," in *Fuzzy Set Theory and Applications, in Tatra Mountains Mathematical Publications*, vol. 16. Bratislava, Slovakia: Mathematical Institute Slovak Academy of Sciences, 1999, pp. 456–467.

[25] Z. C. Johanyák and S. Kovács, "Vague environment-based two-step fuzzy rule interpolation method," in *Proc. 5th Slovakian-Hungarian Joint Symp. Appl. Mach. Intell. Inform. (SAMI)*, Jan. 2007, pp. 189–200.

[26] S. Jenei, "Interpolation and extrapolation of fuzzy quantities revisited—An axiomatic approach," *Soft Comput.*, vol. 5, no. 3, pp. 179–193, Jun. 2001.

[27] I. Perfilieva and S. Lehmke, "Correct models of fuzzy IF–THEN rules are continuous," *Fuzzy Sets Syst.*, vol. 157, no. 24, pp. 3188–3197, Dec. 2006.

[28] Q. Shen and Z. Huang, "Fuzzy interpolation with generalized representative values," in *Proc. U.K. Workshop Comput. Intell.*, 2004, pp. 161–171.

[29] Z. Huang and Q. Shen, "Scale and move transformation-based fuzzy interpolative reasoning: A revisit," in *Proc. IEEE Int. Conf. Fuzzy Syst.*, Jul. 2004, pp. 623–628.

[30] D. Dubois and H. Prade, "Gradual inference rules in approximate reasoning," *Inf. Sci.*, vol. 61, nos. 1–2, pp. 103–122, Apr. 1992.

[31] M. Almseidin, I. Piller, M. Al-Kasassbeh, and S. Kovacs, "Fuzzy automaton as a detection mechanism for the multi-step attack," *Int. J. Adv. Sci., Eng. Inf. Technol.*, vol. 9, no. 2, pp. 575–586, Mar. 2019.

[32] M. Almseidin, M. Al-kasassbeh, and S. Kovacs, "Fuzzy rule interpolation and SNMP-MIB for emerging network abnormality," *Int. J. Adv. Sci., Eng. Inf. Technol.*, vol. 9, no. 3, pp. 735–744, May 2019.

[33] M. Almseidin, J. Al-Sawwa, M. Alkasassbeh, and M. Alweshah, "On detecting distributed denial of service attacks using fuzzy inference system," *Cluster Comput.*, vol. 26, no. 2, pp. 1337–1351, Apr. 2023.

[34] D. Tikk, Z. C. Johanyák, S. Kovács, and K. W. Wong, "Fuzzy rule interpolation and extrapolation techniques: Criteria and evaluation guidelines," *J. Adv. Comput. Intell. Intell. Informat.*, vol. 15, no. 3, pp. 254–263, May 2011.

[35] Z. C. Johanyak, D. Tikk, S. Kovacs, and K. Wai Wong, "Fuzzy rule interpolation MATLAB toolbox—FRI toolbox," in *Proc. IEEE Int. Conf. Fuzzy Syst.*, Vancouver, BC, Canada, Jul. 2006, pp. 1427–1433.

[36] FRI—SFMI. *Fuzzy Rule Interpolation Toolbox—Sparse Fuzzy Model Identification Toolbox*. [Online]. Available: http://fri.uni-miskolc.hu/

[37] M. Almseidin, M. Alkasassbeh, M. Alzubi, and J. Al-Sawwa, "Cyber-phishing website detection using fuzzy rule interpolation," *Cryptography*, vol. 6, no. 2, p. 24, May 2022.

[38] L. T. Kóczy and D. Tikk, "A survey of fuzzy interpolation techniques," in *Proc. 1st Int. Symp. Hungarian Researchers Comput. Intell. (HUCI)*, Budapest, Hungary, 2000, pp. 5–13.

[39] L. T. Koczy and L. Muresan, "Fuzzy systems with interpolation. An overview," in *Proc. Joint 9th IFSA World Congr. 20th NAFIPS Int. Conf.*, Vancouver, BC, Canada, 2001, pp. 2494–2498.

[40] R. Xu, C. Shang, J. Lin, and Q. Shen, "Similarity function-assisted dynamic fuzzy rule interpolation: An improved approach," in *Proc. IEEE Int. Conf. Fuzzy Syst. (FUZZ)*, Aug. 2023, pp. 1–6.

[41] C. Chen and Q. Shen, "Rough-fuzzy rule interpolation for data-driven decision making," in *Proc. 20th U. K. Workshop Comput. Intell.*, Aberystwyth, Wales, U.K., Cham, Switzerland: Springer, 2022, pp. 27–37.

[42] C. Jiang, S. Jin, C. Shang, and Q. Shen, "Towards utilization of rule base structure to support fuzzy rule interpolation," *Expert Syst.*, vol. 40, no. 5, Jun. 2023, Art. no. e13097.

[43] F. Gao, "Density-based approach for fuzzy rule interpolation," *Appl. Soft Comput.*, vol. 143, Aug. 2023, Art. no. 110402.

[44] M. Alzubi, M. Almseidin, S. Kovacs, J. Al-Sawwa, and M. Alkasassbeh, "EI-FRI: Extended incircle fuzzy rule interpolation for multidimensional antecedents, multiple fuzzy rules, and extrapolation using total weight measurement and shift ratio," *J. Robot. Control (JRC)*, vol. 5, no. 1, pp. 217–227, 2024.

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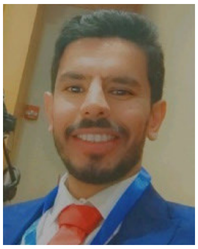

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