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RESEARCH ARTICLE

Optimizing Environmental Impact: MCDM-Based Approaches for Petrochemical Industry Emission Cuts

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ABSTRACT The petrochemical industry is a major contributor to carbon emissions, necessitating an urgent shift towards effective emission reduction techniques. However, a lack of essential data has hindered the development of strategies to address this issue, calling for a comprehensive approach. This study seeks to formulate effective approaches for mitigating carbon emissions in the petrochemical sector by assessing their impact and recognizing potential barriers to reduction. The primary objectives revolve around three key aspects: reducing energy intensity, optimizing CO2 emission reduction, and minimizing associated costs. To attain these objectives, we utilized a dataset represented as a Complex Multi-Fuzzy Hypersoft Set (CMFHSS), specifically designed to address data uncertainties through the incorporation of amplitude and phase terms (P-terms) of complex numbers (C-numbers). The research explores three decision-making techniques, namely Similarity Measures (SM), Entropy (ENT) and TOPSIS within CMFHSS. These techniques are applied to identify the most efficient carbon emission reduction strategy, with the goal of maximizing benefits while minimizing costs.

INDEX TERMS Decision-making, similarity measure, entropy, TOPSIS.

I. INTRODUCTION

The discharge of greenhouse gases into the atmosphere, known as carbon emissions, poses significant global challenges. This issue leads to a marked increase in the average temperature of Earth, disrupting the equilibrium of the world's climate. Consequently, it adversely affects the health of many individuals by contributing to air pollution. This problem has become a universally acknowledged and urgent issue among nations [1] due to factors such as uncontrolled population growth, industrialization, and rising energy demands, which are identified as primary contributors. To mitigate the growth of carbon emissions, countries are actively pursuing various strategies. Nations strongly support investments in renewable energy to reduce reliance on fossil fuels and cut down carbon emissions. Additionally, some countries emphasize the importance of energy conservation [2] to reduce carbon emissions by promoting reduced energy consumption.

Effectively addressing the challenge of carbon emissions requires a comprehensive understanding of its primary causes. Economic growth stands out as a significant influencer, as investments crucial to the gross domestic product drive economic growth. Increased energy consumption accompanies this growth, essential for industrial production [3]. Consequently, reliance on fossil fuels during economic expansion leads to increased carbon emissions. Globalization is also pivotal, granting large corporations access to new markets abroad. To succeed, these enterprises must meet diverse customer expectations across different

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countries, necessitating a significant increase in production capacities fueled by energy consumption [4]. Without appropriate measures, globalization will worsen carbon emissions. The contemporary era faces a significant global challenge highlighted by Alicja et al. [5]: climate change. The United Nations Intergovernmental Panel on Climate Change (IPCC), as pointed out by Alivia [6], attributes escalating temperatures since the mid-20th century to increased carbon emissions from heightened fossil fuel consumption. Despite the absence of a decline, global greenhouse gas emissions rose by 2.0 percent in both 2018 and 2019 [7].

The role of petrochemical usage and its derivatives is pivotal in driving urbanization through industrialization, albeit with significant environmental repercussions. This industry not only creates employment opportunities but also fosters integration with various industrial sectors, as highlighted by observations made by Donald [8]. Consequently, it has become a sought-after sector for numerous industrialized and industrializing nations, such as China, heavily relying on it for economic growth, as acknowledged by Couth and Trois [9] and Fan et al. [10]. In 2016, China's energy consumption accounted for over 23 percent of the global total, contributing nearly 30 percent of the world's overall carbon emissions, according to Ferella et al. [11]. An effective approach to reduce carbon emissions involves targeting key industrial sectors and implementing emission reduction objectives, as proposed by Fan et al. [12] and Glew et al. [13], with the petrochemical industry identified among these crucial sectors. Apart from emitting carbon during product combustion, the petrochemical industry consumes substantial energy, emerging as a significant source of both production and consumption-related carbon emissions, as cited by Hao et al. [14] and Hao et al. [15]. According to statistics from the National Development and Reform Commission, petrochemical enterprises constitute over a third of high-energy-consuming entities, with 340 out of 1000 falling under this category. In 2000, the petrochemical industry accounted for 28.3 percent of China's industrial energy consumption, utilizing 270.4 million tons of standard coal. This figure dramatically increased to 795.5 million tons of standard coal by 2017, constituting 27.0 percent of industrial energy consumption (NBS, 2000, 2019). Given its energy-intensive nature, significant carbon emissions, and high energy consumption, in-depth investigations into the petrochemical sector's carbon emissions are essential, as emphasized by Huang et al. [16].

Over the past century, numerous foundational theories have emerged, delving into the inherent vagueness of data and their potential to overcome the limitations of parametrization techniques. These theories have opened up new avenues of exploration in the field of fuzzy systems (FS) by incorporating probabilistic indicators for ambiguous events [17], [20], [21], [22], [23], [24]. Such contributions have been extensively discussed in combination frameworks outlined in a relevant work [29]. Within this context, De Luca and Termini [28] put forth a specific set of possibilities for fuzzy ENT, garnering increasing attention compared to SM [38], [39], [40], [41], [42]. SM holds significant importance in evaluating the similarity between two surfaces and has undergone extensive analysis by Pappis and colleagues in numerous studies [29], [30]. Building upon the concepts of ENT and SM, their application to internal and external determinants was presented in a study by Liu [31], and [32], showcasing their relevance in areas such as strategic planning, intelligence, and concerns related to accelerometers. In parallel, Al-Qudah and Hassan [18], [33] developed the theories of complex fuzzy set (CFS) and complex fuzzy soft set (CFSS), enabling the representation of complex two-dimensional characteristics. To enhance interpretability in practical applications, it is necessary to divide variables into data points. Smarandache addressed this necessity by introducing the Hypersoft set as an extension of the traditional soft set (SS) [36]. Later, this concept was expanded into a multi-attribute procedure, broadening the Hypersoft set [37], [62], [64], [65], [66], [67]. Saeed et al. also showcased practical applications of Hypersoft sets within a neutrosophic context [37], [62], [64], [65], [66], [67]. Additionally, various other theories have surfaced to discuss hybrid structures in fuzzy-like environments [61], [62], [63], contributing to a more comprehensive understanding of the domain. Nevertheless, the realm of energy technology has generally given limited attention to procurement management principles, particularly concerning multi-criteria strategic planning for RCET development. This emptiness serves as the catalyst for initiating this project, which seeks to explore numerical methods for making multi-criteria decisions in RCET. The project employs an innovative imprecise mathematical model that depends on wavelet estimations.

This study aims to develop effective strategies for mitigating carbon emissions within the petrochemical sector, examining their impact and identifying potential obstacles to reduction. The primary objectives encompass three aspects: decreasing energy intensity, optimizing CO2 emission reduction, and minimizing associated costs. To achieve these goals, we utilized a dataset represented as a complex multi-fuzzy hypersoft set (CMFHSS). This set is designed to address data uncertainties by incorporating amplitude and phase terms (P-terms) of complex numbers (C-numbers). The research investigates three decision-making techniques, namely SM, ENT and TOPSIS within CMFHSS. These techniques are employed to select the most efficient carbon emission reduction strategy, aiming for maximum benefits at minimal cost. The findings of this study can inform policy-making and guide petrochemical industry owners in selecting the most advantageous and cost-effective carbon emission reduction techniques.

A. MOTIVATION

Due to limitations in understanding and identifying specific approaches for reducing carbon emissions as discussed in previous research ([18], [19], [25], [26], [33]), this study aims to predict potential scenarios for carbon emission reduction strategies and accurately assess their outcomes. The strategies outlined in these references lack the depth required for a thorough data evaluation, hindering comprehensive understanding and informed decision-making. The assumptions presented in [19], [25], and [26] face challenges in handling two-dimensional (2D) data content, particularly in evaluating the magnitude of effects and the timeframes needed for impact, especially concerning specific criteria sub-values. Moreover, while [18] and [33] show their ability to organize relevant 2D data, they struggle with handling specifications involving sub-parameter types of characteristics. All the theories mentioned above encounter difficulties when dealing with multi-faceted, multi-phased data. To overcome these challenges, we have developed an extensive framework that integrates complex multi-fuzzy and hypersoft set methodologies. This approach offers three key advantages in terms of customization. The CMFHSS model introduces a diverse range of membership function quantities distributed across a complex framework along an imaginary axis. This expansion includes the incorporation of a new element called the P-term, specifically tailored to accommodate seasonal variability within the context. Furthermore, the CMFHSS attributes undergo further categorization into distinct elements, aiming to enhance accuracy in multiple directions or phases. This study seeks to create efficient strategies for cutting carbon emissions in the petrochemical industry, focusing on reducing energy use, optimizing CO2 reduction, and minimizing costs. Its objective is to pinpoint the most cost-effective strategies for accomplishing these objectives. The findings will serve as valuable insights for policymakers and business executives in shaping well-informed choices regarding the reduction of carbon emissions.

B. EXPOSITION OF A STUDY

Section II outlines the methodology grounded in the CMFHSS framework. Transitioning to Section III, the focus shifts to presenting the results. Lastly, Section IV provides discussions and conclusive remarks derived from the findings of the article. For a deeper grasp of the algorithm's operation, please consult Figure 1, depicting a frame diagram.

II. METHODOLOGY

In this section, we explore fundamental concepts such as FS, SS, FSS, FHSS, MFS, MFSS, CMFSS, SM, ENT, CFH-set, and CFH-subset.

Definition 1 [19]: Definition 1: The concept of fuzzy set, denoted as

$$R = \{(y, I(y))|, y \in Y\}, \tag{1}$$

is characterized by a mapping $I : Y \rightarrow [0, 1]$, where Y represents a collection of objects, and $I(y)$ signifies the membership grade of $y \in Y$.

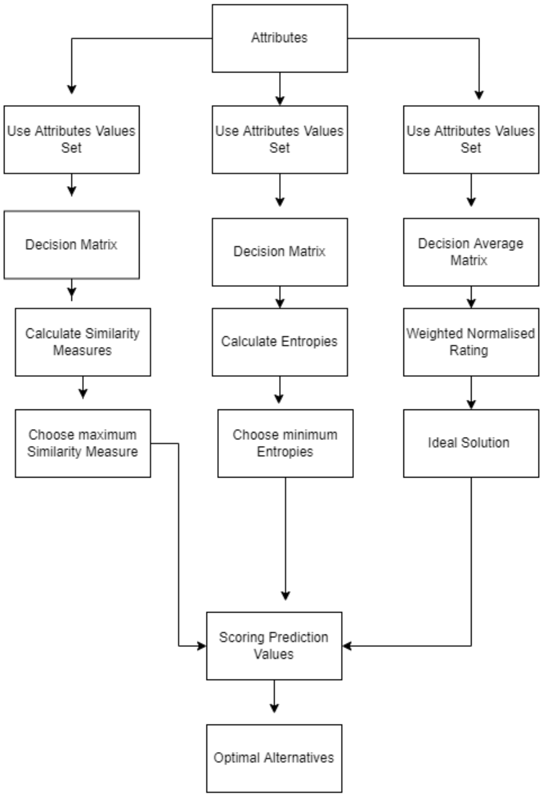

FIGURE 1. Diagram outlining the structure of proposed algorithms.

Definition 2: According to another perspective provided by [25], a pair (I, Q) is identified as soft set over the universe Y. Here, I is a mapping defined as

$$I : Q \rightarrow P(Y), \tag{2}$$

where for any $\epsilon \in Q$, $I(\epsilon)$ can be interpreted as the ϵ approximate elements of the SS (I, Q).

Definition 3: As stated by [26], considering Y as the initial universe and Q as the set of parameters, a pair (I, Q) is recognized as fuzzy soft set over Y. Here, $P(Y)$ denotes the power set of all fuzzy subsets of Y, and

$$I : Q \rightarrow P(Y), . \tag{3}$$

Definition 4: In the context elaborated by [37], where Y denotes the universal set and $I(Y)$ represents all fuzzy subsets of Y, the fuzzy hypersoft set is defined for distinct attributes $m_1, m_2, m_3, \cdots, m_n$ with attribute values belonging to sets $M_1, M_2, M_3, \ldots, M_n$ respectively. The FHSS is represented as the pair (Σ_L, L) over Y, characterized by a map

$$\Sigma_L : L \rightarrow I(Y), \tag{4}$$

where $L = F1 \times F2 \times F3 \times \ldots \times Fn$.

Definition 5 [55]: Let k be a non-zero, non-negative integer, and $Y \neq \Phi$. An multi fuzzy set Q in Y is an ordered sequence

$$Q = \{\langle y, \lambda_1(y), \ldots, \lambda_k(y)\rangle : y \in Y\}, \tag{5}$$

where

$$\lambda_i : Y \rightarrow O_i = [0, 1], i = 1, 2, \ldots, k. \tag{6}$$

The multi-membership map of multi fuzzy sets Q, denoted as

$$\lambda_Q(y) = (\lambda_1(y), \ldots, \lambda_k(y)), \tag{7}$$

represents the collection of these sets in Y as $M^k FS(Y)$.

Definition 6: [27] A pair (I, Q) is considered an multi fuzzy soft set with dimension k if

$$I : Q \to M^k FS(Y), \tag{8}$$

and $I(e)$, where $e \in Q$, is its collection of e-approximate members.

Definition 7 [18]: A pair (I, Q) is a complex multi fuzzy soft set of dimension k over Y if

$$I : Q \to CM^k(Y) \tag{9}$$

and is represented as

$$(I, Q) = \{\langle \epsilon, I(\epsilon) \rangle : \epsilon \in Q, I(\epsilon) \in CM^k(Y)\}. \tag{10}$$

Here, $I(\epsilon)$ is defined as

$$\{\langle y, \lambda^s I(\epsilon)(y) = \rho^s I(\epsilon)(y).\epsilon^{i\omega^s I(\epsilon)(y)} \rangle : \epsilon \in Q, y \in Y, \} \tag{11}$$

, where $\lambda^s I(\epsilon)(y) s \in k$ is a complex multi-membership function for $y \in X$ with real-valued functions A part

$$= (\rho^s I(\epsilon)(y)) s \in k \in [0, 1] \tag{12}$$

and P-part

$$= (\{\omega^s I(\epsilon)\}(y))_{s \in k}. \tag{13}$$

The collection of all such sets is denoted as $CM^k FSS$. Here, s = 1, 2, …, k.

Definition 8: [35] A function S from $FS(Y, E) \times FS(Y, E)$ to $[0, 1]$ is called a SM for fuzzy soft set if it satisfies the following points:

1) $S(X_Q, \Phi_Q) = 0$ for any $Q \in E$, and $S((I, Q), (I, Q)) = 1$ for any $(I, Q) \in FS(Y, E)$,
2) $S((I, Q), (J, C)) = S((J, C), (I, Q))$ for any $(I, Q), (J, C) \in FS(Y, E)$,
3) For any $(I, Q), (J, C), (H, O) \in FS(Y, E)$, if $(I, Q) \subseteq (J, C) \subseteq (H, O)$, then $S((H, O), (I, Q)) = \min(S((H, O), (J, C)), S((J, C), (I, Q)))$.

Definition 9 [35]: A real-valued function E from $FS(Y, E)$ to $[0, \infty]$ for fuzzy soft set is called an ENT if it satisfies the given conditions:

1) $E(I, Q) = 0$ if (I, Q) is an SS.
2) $E(I, Q) = 1$ if $I(e) = 0.5$ for any $e \in Q$, where [0.5] is the FS with membership function $0.5 = 0.5$ for every $y \in Y$,
3) If (I, Q) is a crisp set compared to (J, C), i.e., for $e \in Q$ and $y \in Y$, $I(e)(y) \leq J(e)(y)$ if $J(e)(y) \leq 0.5$, and $I(e)(y) \geq J(e)(y)$ if $J(e)(y) \geq 0.5$, then $E(I, Q) \leq E(J, C)$,
4) $E(I, Q) = E(I^c, Q)$, where (I^c, Q) is the complement of fuzzy soft set (I, Q), written as $I^c(e) = (I(e))^c$ for every $e \in Q$.

Definition 10: [62] Let $M_1, M_2, M_3, \ldots, M_n$ be disjoint sets with attribute values corresponding to n distinct attributes $m_1, m_2, m_3, \ldots, m_n$, where $n \geq 1$. Define $G = M_1 \times M_2 \times M_3 \times \ldots \times M_n$, and let $\xi(\underline{y})$ be a CF-set over Y for all $\underline{\epsilon} = (c_1, c_2, c_3, \ldots, c_n) \in G$. The *complex fuzzy hypersoft set* (CFH-set) ϖ_G over Y is then defined as:

$$\varpi_G = \{(\underline{\epsilon}, \xi(\underline{\epsilon})) : \underline{\epsilon} \in G, \xi(\underline{\epsilon}) \in C(Y)\} \tag{14}$$

where

$$\xi : G \to C(Y), \quad \xi(\underline{\epsilon}) = \emptyset \text{ if } \underline{\epsilon} timesinG. \tag{15}$$

Here, $\xi(\underline{\epsilon})$ serves as a CF-approximate function of ϖ_G, and its value is referred to as the $\underline{\epsilon}$-member of the CFH-set for all $\underline{\epsilon} \in G$.

Definition 11 [62]: Let $\varpi_{W_1} = (\xi_1, W_1)$ and $\varpi_{W_2} = (\xi_2, W_2)$ be two CFH-sets over the same Y. The set $\varpi_{W_1} = (\xi_1, W_1)$ is considered the CFH-subset of $\varpi_{W_2} = (\xi_2, W_2)$ if:

1) $W_1 \subseteq W_2$,
2) For all $\underline{y} \in W_1$, $\xi_1(\underline{y}) \subseteq \xi_2(\underline{y})$, i.e., $r_{W_1}(\underline{y}) \leq r_{W_2}(\underline{y})$ and $\omega_{W_1}(\underline{y}) \leq \omega_{W_2}(\underline{y})$, where $r_{W_1}(\underline{y})$ and $\omega_{W_1}(\underline{y})$ are amplitude and phase terms of $\xi_1(\underline{y})$, and $r_{W_2}(\underline{y})$ and $\omega_{W_2}(\underline{y})$ are amplitude and phase terms of $\xi_2(\underline{y})$

III. RESULTS

Throughout this section, the following data is considered: $\mathcal{D} = A_1 \times A_2 \times A_3 \times \ldots \times A_n$, $\mathcal{E} = B_1 \times B_2 \times B_3 \times \ldots \times B_n$, $\mathcal{R} = C_1 \times C_2 \times C_3 \times \ldots \times C_n$, $e = (e_1, e_2, e_3, \ldots e_n)$, $\aleph = N_1 \times N_2 \times N_3 \times \ldots \times N_n$.

Definition 12: Let $m_1, m_2, m_3, \ldots, m_n$ denote distinct attributes with corresponding attribute values belonging to the sets $M_1, M_2, M_3, \ldots, M_n$ respectively, where $M_i \cap M_j = \emptyset$ for $i \neq j$. A pair $(\mathcal{J}, \mathcal{D})$ is termed as an MFHSS of dimension k over Y, where $\mathcal{J}$ is a function defined as

$$\mathcal{J} : \mathcal{D} \to M^k FHS(Y). \tag{16}$$

For $e \in \mathcal{D}$, $\mathcal{J}(e)$ can be interpreted as the set of approximate elements of the MFHSS $(\mathcal{J}, \mathcal{D})$.

Definition 13: A pair $(\mathcal{J}, \mathcal{D})$ is termed as a CMFHSS of dimension k over Y, where $\mathcal{J}$ is a mapping given by

$$\mathcal{J} : \mathcal{D} \to CM^k(Y). \tag{17}$$

A complex multi-fuzzy hypersoft set of dimension $k(CM^k FHSS(Y))$ is a mapping from parameters to $CM^k(Y)$. It is a parameterized family of complex multi-fuzzy subsets of Y, and it can be expressed as:

$$(\mathcal{J}, \mathcal{D}) = \{\langle e, \mathcal{J}(e) \rangle : e \in \mathcal{D}, \mathcal{J}(e) \in CM^k(Y)\}, \tag{18}$$

where

$$\mathcal{J}(e) = \{\langle y, \lambda^s \mathcal{J}(e)(y) = \rho^s \mathcal{J}(e)(y).e^{i\omega^s \mathcal{J}(e)(y)} \rangle : e \in \mathcal{D}, y \in Y\} \tag{19}$$

$s = 1, 2, \ldots k$, where $\mu^s \mathcal{J}(e)(y) s \in k$ represents a complex-valued grade of multi-membership function $y \in Y$. By definition, the values of $\lambda^s \mathcal{J}(e)(y) s \in k$ may all lie in the complex plane within the unit circle, and are thus of the form

$$[\lambda^s \mathcal{J}(e)(y) = \rho^s \mathcal{J}(e)(y).e^{i\omega^s \mathcal{J}(e)(y)}] s \in k, \tag{20}$$

where ($i^2 = -1$), each of the A-terms

$$(\rho^s \mathcal{J}(e)(y))s \in k \tag{21}$$

and the P-terms

$$(\omega^s \mathcal{J}(e)(y))s \in k \tag{22}$$

are both real-valued, and

$$(\rho^s \mathcal{J}(e)(y))_{s \in k} \in [0, 1], \tag{23}$$

The set of all $CM^k FHSS$ in Y is denoted by $CM^k FHSS(Y)$.

Example 1: Consider a scenario where an individual seeks a loan from one of several banks for a specific duration. Let $Y = \{y_1 =$ JP Morgan, $y_2 =$ Wells Fargo, $y_3 =$ Goldman Sachs $\}$ represent the set of three banks in the USA. Assuming a year consists of four periods with varying interest rates in each period, let $a_1 =$ Repayment tenor, $a_2 =$ Interest rate, $a_3 =$ Documentation, representing distinct attributes whose attribute values belong to the sets E_1, E_2, E_3. Define $E_1 = \{f_1 =$ Flexible, $f_2 =$ Difficult $\}$, $E_2 = \{f_3 =$High, $f_4 = Low\}$, $E_3 = \{f_5 =$Easy$\}$. We construct CMFHSS having three dimensions.

In this example, the A-terms signify the degrees of association with the arrangement of interest rates, and the P-terms signify the degrees of association with the period of seasons corresponding to the attribute values. In the CMF value $y_1/(0.8e^{i2\pi(2/4)}, y_2/0.2e^{i2\pi(4/4)}, y_3/0.3e^{i2\pi(3/4)}$, the first value $(0.8e^{i2\pi(2/4)}$ indicates that the interest rate of the loan is high in the late spring, since the A-term 0.8 is close to one and the P-term (2/4) indicates the year (the late spring season), which is the second period with respect to the attribute values (f_1, f_3, f_5). Similarly, the subsequent membership value $0.2e^{i2\pi(4/4)}$ indicates that the interest rate is low in the winter, as the P-term 0.2 is close to zero, and the P-term (4/4) corresponds to the fourth season of the year (the winter season) with respect to the attribute values (f_1, f_3, f_5).

Now, we will outline the basic concepts and operations of CMFHSS.

Definition 14: Let $(\mathcal{J}, \mathcal{D})$ and $(\varpi, \mathcal{E})$ be two $CM^k FHSS$ defined over Y. We say that $(\mathcal{J}, \mathcal{D})$ is a CMFHSS subset of $(\varpi, \mathcal{E})$ if the following conditions hold:

1) $\mathcal{D} \subseteq \mathcal{E}$, and
2) For all $e \in \mathcal{D}$, $\mathcal{J}(e) \sqsubseteq \varpi(e)$.

In this situation, we can denote $\mathcal{J}(e) \sqsubseteq \varpi(e)$.

A. BASIC OPERATIONS ON CMFHSS-SETS

This section delves into foundational theoretical operations and principles concerning CMFHSS sets, including discussions on union, intersection, complement, De Morgan's law, and associativity.

Definition 15: Consider a CM^{k}FHSS over Y denoted as $(\mathcal{J}, \mathcal{D})$, where $(\mathcal{J}, \mathcal{D})^c$ represents its complement. The complement is defined as

$$(\mathcal{J}, \mathcal{D})^c = (\mathcal{J}^c, \neg \mathcal{D}), \tag{24}$$

where

$$\mathcal{J}^c : \mathcal{D} \to CM^k(Y) \tag{25}$$

is a mapping given by $\mathcal{J}^c(e) =$

$$\{\langle y, \lambda^s_{\mathcal{J}^c(e)}(y) = \rho^s_{\mathcal{J}^c(e)}(y).e^{i\omega^s_{\mathcal{J}^c(e)}(y)}\rangle : e \in \neg \mathcal{D},\ y \in Y\}, \tag{26}$$

where

$$\rho^s F^c(e)(y) = 1 - \rho^s \mathcal{J}(e)(y) \tag{27}$$

represents the complement of the A-term and

$$\omega^s \mathcal{J}^c(e)(y) = 2\pi - i\omega^s \mathcal{J}(e)(y) \tag{28}$$

denotes the complement of the P-term.

Example 2: Extending from example 1, let's consider

$$\begin{aligned}\mathcal{J}(f_1, f_3, f_5) = {} & y_1/(0.1e^{i2\pi(1/4)}, 0.3e^{i2\pi(2/4)}, 0.4e^{i2\pi(2/4)}),\\ & y_2/(0.1e^{i2\pi(3/4)}, 0.5e^{i2\pi(2/4)}, 0.2e^{i2\pi(6/4)}),\\ & y_3/(0.5e^{i2\pi(1/4)}, 0.4e^{i2\pi(4/4)}, 0.1e^{i2\pi(2/4)}),\end{aligned}$$

Utilizing definition 19, the complement is derived as follows:

$$\begin{aligned}\mathcal{J}^c(f_1, f_3, f_5) = {} & y_1/(0.1e^{i2\pi(2/4)}, 0.8e^{i2\pi(0/4)}, 0.1e^{i2\pi(2/4)}),\\ & y_2/(0.1e^{i2\pi(2/4)}, 0.5e^{i2\pi(3/4)}, 0.1e^{i2\pi(4/4)}),\\ & y_3/(0.7e^{i2\pi(2/4)}, 0.8e^{i2\pi(1/4)}, 0.4e^{i2\pi(2/4)}).\end{aligned}$$

B. ENT ON CMFHS-SETS

Fuzzy ENT stands as a fundamental attribute of f-sets, specifically addressing the primary query in f-set handling - the extent of fuzziness. ENT functions as a pivotal tool for quantifying the degree of fuzziness within Fuzzy Sets (FS). This section introduces the concept of ENT within the context of Carbon Mitigation Frameworks for the Petrochemical Industry (CMFHSS). A series of interconnected theorems and practical applications have been devised to implement the newly established ENT-based CMFHSS. These developments underscore its significance and validation in optimizing techniques for reducing carbon emissions. The central aim revolves around three core facets: the reduction of energy intensity, the maximal mitigation of CO2 emissions, and the minimization of associated costs.

Definition 15: A function $E : CM^k FHSS(Y) \to [0, 1]$ is considered ENT on $CM^k FHSS$ if it satisfies the following conditions:

1) For any $\mathfrak{Z}$ and $\mathfrak{K}$, $E(\mathfrak{Z}, \mathfrak{K}) = 0$ if and only if $\rho^s F(e)(y) = 1$ and $\omega^s F(e)(e)(y) = 2\pi$ for all $e \in \mathfrak{K}$ and $y \in Y$, where $s = 1, 2, \ldots, k$.
2) For any $\mathfrak{Z}$ and $\mathfrak{K}$, $E(\mathfrak{Z}, \mathfrak{K}) = 1$ if and only if $\rho^s \mathfrak{Z}(e)(y) = 0.5$ and $\omega^j \mathfrak{Z}(e)(y) = \pi$ for all $e \in \mathfrak{K}$ and $y \in Y$, where $s = 1, 2, \ldots, k$.
3) $E(\mathfrak{Z}, \mathfrak{K}) = E(\mathfrak{Z}, \mathfrak{K})^c$.
4) If $(\mathfrak{Z}, \mathfrak{K}) \subseteq (\varkappa, \mathfrak{K})$, meaning $\rho^s \mathfrak{Z}(e)(y) \leq \rho^s \varkappa(e)(y)$ and $\omega^s \mathfrak{Z}(e)(y) \leq \omega^s \varkappa(e)(y)$ for all $e \in E$ and $y \in Y$, where $s = 1, 2, \ldots, k$, then $E(\mathfrak{Z}, \mathfrak{K}) \geq E(\varkappa, \mathfrak{K})$.

1) THE IMPLEMENTATION OF ENT-BASED CMFHSS FOR REDUCING CARBON EMISSIONS TAKES INTO ACCOUNT PARAMETRIC UNCERTAINTIES

Addressing the urgent issue of carbon emissions necessitates swift resolution. Several studies have identified key factors contributing to this challenge [1]. Some researchers argue that globalization and economic growth directly fuel the increase in carbon emissions [68], [69]. Moreover, other studies emphasize factors such as environmental consciousness and financial considerations in this regard [70], [71]. However, attempting to enhance all these factors simultaneously seems impractical due to the associated high costs. What's needed is a new study aimed at identifying the most precise approach for effectively managing carbon emissions reduction techniques. Prioritizing among various criteria can enable efficient resource utilization and pinpoint the most suitable methods to mitigate carbon emissions [72]. This study delves into the intricate task of singling out and prioritizing the primary contributors to carbon emissions specifically within the petrochemical sector. It also proposes effective methodologies to curtail these emissions. Traditional decision-making processes within industries have grappled with complex variables and interconnected elements. To tackle this challenge, the adoption of an ENT-based CMFHSS as a decision-making tool presents a systematic and efficient approach to assess and rank the influential factors driving carbon emissions. Here, ENTserves as a robust mathematical concept that quantifies uncertainty and disorder within a system. The use of ENT-based decision-making aims to streamline the identification of carbon emission reduction techniques that wield the most significant impact on emissions in the petrochemical industry. Additionally, it suggests optimal techniques that yield maximum benefits while minimizing costs. The detailed steps of this model are outlined in Fig. 2. This model operates through three distinct phases. Initially, experts from the petrochemical industry aid in determining the linguistic parameters for information. Subsequently, data collected from this industry is structured into CMFHSS format, aligning with the established expert parameters. Finally, a prescribed algorithm is applied to select the most pivotal emission reduction techniques that promise substantial benefits at minimal costs for the petrochemical industry.

Let C represent a non-empty universal set, where $C \subset P$ denotes the set of procedures under consideration, outlined as $C = \{c_1, c_2, \ldots, c_m\}$. Let $\mathfrak{E} = P_1 \times P_2 \times \ldots \times P_n$, with $n \geq 1$, and P_i signifies the array of all emission reduction attributes associated with the emission factor p_i, $i = 1, 2, 3, \ldots, n$. The procedural stages for the envisioned ENT using CMFHSS are outlined as follows:

1) Input each of the CMFHSS techniques.
2) Calculate ENT for each CMFHSS technique using the formula

$$E(\mathfrak{Z}, \mathfrak{E}) = \frac{1}{2m}\Sigma_{l=1}^{m}[E_l^r(\mathfrak{Z}, \mathfrak{E}) + E_l^{\omega}\frac{(\mathfrak{Z}, \mathfrak{E})}{2\pi}], \tag{29}$$

3) Identify the CMFHSS technique with the minimum ENT and select it as the optimal solution.
4) In case of multiple optimal choices, choose any one.

Example 1: In a scenario where a petrochemical company executive seeks to identify potential methods for reducing emissions for specific clients, they aim to enlist expertise to assess carbon emission techniques. Consider the set $X = \{a, b, c\}$ representing experts who offer their opinions on emission reduction techniques based on CMFHSS attributes. Let $a_1 =$ Efficiency, $a_2 =$ Cost-effectiveness, and $a_3 =$ Impact, signifying unique attributes linked to sets $\delta_1, \delta_2, \delta_3$. Here, $\delta_1 = \{\delta_1 =$ Renewable energy usage, $\delta_2 =$ Process optimization$\}$, $\delta_2 = \{\delta_3 =$ Low$\}$, and $\delta_3 = \{\delta_4 =$ Sustainable, $\delta_5 =$ Recycling$\}$. The assessment of each emission reduction technique's appeal is represented in CMFHSS as $(\mathfrak{Z}, \mathfrak{E})$, $(\varkappa, \mathfrak{E})$, and $(\square, \mathfrak{E})$, respectively.

1) This task can be accomplished with the assistance of an expert. Carbon-Neutral or Low-Carbon Initiatives =

$$(\mathfrak{Z}, \mathfrak{K}) = \Bigg\{\mathfrak{Z}(\delta_1, \delta_3, \delta_4) = \Bigg\{\frac{(0.5\exp^{i0.2\pi}, 0.8\exp^{i0.1\pi}, 0.1\exp^{i0.2\pi})}{a}, \frac{(0.3\exp^{i0.1\pi}, 0.2\exp^{i0.5\pi}, 0.2\exp^{i0.4\pi})}{b}, \frac{(0.3\exp^{i0.5\pi}, 0.4\exp^{i0.2\pi}, 0.7\exp^{i0.1\pi})}{c}\Bigg\},$$

$$\Bigg\{\mathfrak{Z}(\delta_1, \delta_3, \delta_5) = \Bigg\{\frac{(0.4\exp^{i0.2\pi}, 0.2\exp^{i0.3\pi}, 0.7\exp^{i0.7\pi})}{a}, \frac{(0.5\exp^{i0.1\pi}, 0.8\exp^{i0.2\pi}, 0.6\exp^{i0.3\pi})}{b}, \frac{(0.1\exp^{i0.6\pi}, 0.5\exp^{i0.8\pi}, 0.1\exp^{i0.4\pi})}{c}\Bigg\},$$

$$\Bigg\{\mathfrak{Z}(\delta_2, \delta_3, \delta_4) = \Bigg\{\frac{(0.6\exp^{i0.7\pi}, 0.1\exp^{i0.5\pi}, 0.5\exp^{i0.1\pi})}{a}, \frac{(0.7\exp^{i0.6\pi}, 0.2\exp^{i0.9\pi}, 0.1\exp^{i0.4\pi})}{b}, \frac{(0.7\exp^{i0.2\pi}, 0.4\exp^{i0.9\pi}, 0.3\exp^{i0.7\pi})}{c}\Bigg\}$$

$$\Bigg\{\mathfrak{Z}(\delta_2, \delta_3, \delta_5) = \Bigg\{\frac{(0.7\exp^{i0.6\pi}, 0.2\exp^{i0.9\pi}, 0.1\exp^{i0.2\pi})}{a}, \frac{(0.1\exp^{i0.5\pi}, 0.7\exp^{i0.2\pi}, 0.4\exp^{i0.8\pi})}{b}, \frac{(0.2\exp^{i0.5\pi}, 0.7\exp^{i0.7\pi}, 0.8\exp^{i0.4\pi})}{c}\Bigg\}\Bigg\},$$

Optimization of Manufacturing Processes = $(\varkappa, \mathfrak{K})$

$$= \Bigg\{\mathfrak{Z}(\delta_1, \delta_3, \delta_4) = \Bigg\{\frac{(0.2\exp^{i0.7\pi}, 0.2\exp^{i0.8\pi}, 0.9\exp^{i0.2\pi})}{a}, \frac{(0.7\exp^{i0.5\pi}, 0.2\exp^{i0.6\pi}, 0.3\exp^{i0.8\pi})}{b}, \frac{(0.4\exp^{i0.3\pi}, 0.8\exp^{i0.9\pi}, 0.5\exp^{i0.3\pi})}{c}\Bigg\},$$

$$\Bigg\{\mathfrak{Z}(\delta_1, \delta_3, \delta_5) = \Bigg\{\frac{(0.6\exp^{i0.9\pi}, 0.3\exp^{i0.3\pi}, 0.2\exp^{i0.9\pi})}{a}, \frac{(0.2\exp^{i0.8\pi}, 0.6\exp^{i0.2\pi}, 0.9\exp^{i0.4\pi})}{b}, \frac{(0.2\exp^{i0.8\pi}, 0.1\exp^{i0.6\pi}, 0.6\exp^{i0.2\pi})}{c}\Bigg\},$$

$$\Bigg\{\mathfrak{Z}(\delta_2, \delta_3, \delta_4) = \Bigg\{\frac{(0.3\exp^{i0.3\pi}, 0.1\exp^{i0.7\pi}, 0.8\exp^{i0.4\pi})}{a}, \frac{(0.6\exp^{i0.2\pi}, 0.7\exp^{i0.2\pi}, 0.7\exp^{i0.1\pi})}{b},$$

$$\frac{(0.2\exp^{i0.7\pi},0.1\exp^{i0.9\pi},0.1\exp^{i0.8\pi})}{c}\Bigg\}$$

$$\Bigg\{\mathfrak{Z}(\delta_2,\delta_3,\delta_5)=\Bigg\{\frac{(0.7\exp^{i0.2\pi},0.8\exp^{i0.2\pi},0.5\exp^{i0.1\pi})}{a},$$
$$\frac{(0.8\exp^{i0.2\pi},0.6\exp^{i0.7\pi},0.2\exp^{i0.4\pi})}{b},$$
$$\frac{(0.2\exp^{i0.5\pi},0.7\exp^{i0.2\pi},0.8\exp^{i0.4\pi})}{c}\Bigg\}\Bigg\},$$

Use of Renewable Energy Sources= $(\Box,\mathfrak{K})$ =

$$\Bigg\{\mathfrak{Z}(\delta_1,\delta_3,\delta_4)=\Bigg\{\frac{(0.2\exp^{i0.6\pi},0.2\exp^{i0.8\pi},0.4\exp^{i0.7\pi})}{a},$$
$$\frac{(0.5\exp^{i0.2\pi},0.9\exp^{i0.4\pi},0.1\exp^{i0.9\pi})}{b},$$
$$\frac{(0.6\exp^{i0.2\pi},0.8\exp^{i0.2\pi},0.8\exp^{i0.2\pi})}{c}\Bigg\},$$

$$\Bigg\{\mathfrak{Z}(\delta_1,\delta_3,\delta_5)=\Bigg\{\frac{(0.8\exp^{i0.6\pi},0.9\exp^{i0.1\pi},0.3\exp^{i0.6\pi})}{a},$$
$$\frac{(0.5\exp^{i0.2\pi},0.3\exp^{i0.8\pi},0.2\exp^{i0.6\pi})}{b},$$
$$\frac{(0.4\exp^{i0.8\pi},0.5\exp^{i0.2\pi},0.4\exp^{i0.3\pi})}{c}\Bigg\},$$

$$\Bigg\{\mathfrak{Z}(\delta_2,\delta_3,\delta_4)=\Bigg\{\frac{(0.3\exp^{i0.7\pi},0.3\exp^{i0.4\pi},0.9\exp^{i0.4\pi})}{a},$$
$$\frac{(0.3\exp^{i0.6\pi},0.8\exp^{i0.4\pi},0.7\exp^{i0.5\pi})}{b},$$
$$\frac{(0.1\exp^{i0.9\pi},0.2\exp^{i0.1\pi},0.5\exp^{i0.6\pi})}{c}\Bigg\}$$

$$\Bigg\{\mathfrak{Z}(\delta_2,\delta_3,\delta_5)=\Bigg\{\frac{(0.1\exp^{i0.5\pi},0.5\exp^{i0.1\pi},0.2\exp^{i0.7\pi})}{a},$$
$$\frac{(0.4\exp^{i0.8\pi},0.7\exp^{i0.2\pi},0.4\exp^{i0.6\pi})}{b},$$
$$\frac{(0.1\exp^{i0.4\pi},0.8\exp^{i0.1\pi},0.7\exp^{i0.4\pi})}{c}\Bigg\}\Bigg\}.$$

2) Using the formula outlined in the algorithm (see Table 1), the Entropies of $(\mathfrak{Z},\mathfrak{K})$, $(\varkappa,\mathfrak{K})$, and $(\Box,\mathfrak{K})$ can be calculated as follows:
   The entropies as given below $E(\mathfrak{Z},\mathfrak{K}) = 0.59$, $E(\varkappa,\mathfrak{K}) = 0.43$, $E(\Box,\mathfrak{K}) = 0.41$ respectively.
3) The most efficient choice is to select $(\Box,\mathfrak{K})$ since it possesses the lowest ENT value among the options available.
4) Harnessing renewable energy sources stands as the most efficient approach to curbing carbon emissions.

TABLE 1. **Entropies.**

$E^r_1(\mathfrak{Z},\mathfrak{K})$	0.6
$E^r_2(\mathfrak{Z},\mathfrak{K})$	0.1
$E^r_3(\mathfrak{Z},\mathfrak{K})$	0.7
$E^r_4(\mathfrak{Z},\mathfrak{K})$	0.6
$E^\omega_1(\mathfrak{Z},\mathfrak{K})$	2.2
$E^\omega_2(\mathfrak{Z},\mathfrak{K})$	1.74
$E^\omega_3(\mathfrak{Z},\mathfrak{K})$	2.81
$E^\omega_4(\mathfrak{Z},\mathfrak{K})$	0.722
$E^r_1(\varkappa,\mathfrak{K})$	4.012
$E^r_2(\varkappa,\mathfrak{K})$	0.122
$E^r_3(\varkappa,\mathfrak{K})$	0.7
$E^r_4(\varkappa,\mathfrak{K})$	0.6
$E^\omega_1(\varkappa,\mathfrak{K})$	2.612
$E^\omega_2(\varkappa,\mathfrak{K})$	4.75
$E^\omega_3(\varkappa,\mathfrak{K})$	8.71
$E^\omega_4(\varkappa,\mathfrak{K})$	1.62
$E^r_1(\Box,\mathfrak{K})$	0.61
$E^r_2(\Box,\mathfrak{K})$	0.71
$E^r_3(\Box,\mathfrak{K})$	0.9
$E^r_4(\Box,\mathfrak{K})$	0.76
$E^\omega_1(\Box,\mathfrak{K})$	1.8
$E^\omega_2(\Box,\mathfrak{K})$	3.5
$E^\omega_3(\Box,\mathfrak{K})$	2.1
$E^\omega_4(\Box,\mathfrak{K})$	3.1

C. LEVERAGING CMFHS-SET SIMILARITY MEASURES FOR OPTIMIZING CARBON EMISSION REDUCTION STRATEGIES IN THE PETROCHEMICAL INDUSTRY TO IMPROVE SUSTAINABILITY PRACTICES

One of the primary focal points in environmental research revolves around diminishing carbon emissions. Addressing and reducing carbon emissions stands as a pivotal element in the battle against climate change and its adverse impacts. Rather than concentrating solely on Renewable Energy (RE), it is crucial to explore a diverse range of methods designed to limit carbon emissions across different sectors. These methods encompass a broad spectrum of strategies, spanning from adopting energy-efficient practices in households, industries, and organizations to embracing innovative technological advancements that promote cleaner and more sustainable energy sources. To confront this complex challenge, a comprehensive approach is recommended, utilizing a combined fuzzy Multiple Criteria Decision-Making (MCDM) framework based on similarity measures. This framework aims to assess and prioritize techniques for reducing carbon emissions suitable for practical implementation. The evaluation involves a thorough analysis that takes into account various factors, including social, institutional, technological, financial, and environmental aspects. For further elaboration on the eight distinct categories of carbon emission reduction techniques explored.

Similarity measures assess the resemblance among different patterns, images, or sets and find extensive application in the domain of fs-sets. In this context, we introduce a definition of a SM for *CMFHSS* as follows.

Definition 2: A function symbolized as

$$R : CM^kFHSS(Z) \times CM^kFHSS(Z) \rightarrow [0,1] \tag{30}$$

is regarded as possessing SM characteristics between two CM^kFHSS structures, specifically represented by $(\mathcal{Y},\mathcal{J})$ and $(\kappa,\mathcal{J})$, if it complies with certain fundamental principles. These criteria are articulated as follows:

1) $R((\mathcal{Y},\mathcal{J}),(\kappa,\mathcal{J})) = R((\kappa,\mathcal{J}),(\mathcal{Y},\mathcal{J}))$,
2) $R((\mathcal{Y},\mathcal{J}),(\kappa,\mathcal{J})) = 1 \Leftrightarrow (\mathcal{Y},\mathcal{J}) = (\kappa,\mathcal{J})$, $R((\mathcal{Y},\mathcal{J}),(\kappa,\mathcal{J})) = 0 \Leftrightarrow$ for all $e \in \mathcal{J}$,
3) $x \in Z$, and $s = 1, 2, 3\ldots, K$, the following conditions are satisfied: $\xi^s\mathcal{Y}(e) = 1, \xi^s\kappa(e) = 0$ or $\xi^s\mathcal{Y}(e) = 0, \xi^s\kappa(e) = 1$ and $\delta^s\mathcal{Y}(e) = 2\pi, \delta^s\kappa(e) = 0$ or $\delta^s\mathcal{Y}(e) = 0, \delta^s\kappa(e) = 2\pi$,

4) For any $(\mathcal{Y}, \mathcal{J}), (\kappa, \mathcal{J})$, and $(\Box, \mathcal{J})$ in CM^kFHSS, if $(\mathcal{Y}, \mathcal{J}) \subseteq (\kappa, \mathcal{J}) \subseteq (\Box, \mathcal{J})$, then $R((\mathcal{Y}, \mathcal{J}), (\Box, \mathcal{J})) \leq R((\mathcal{Y}, \mathcal{J}), (\kappa, \mathcal{J}))$ and $R((\mathcal{Y}, \mathcal{J}), (\Box, \mathcal{J})) \leq R((\kappa, \mathcal{J}), (\Box, \mathcal{J}))$. This leads us to the formulation of the equation employed in assessing the SM between two CM^kFHSS structures.

1) CARBON EMISSION REDUCTION TECHNIQUES IN PETROCHEMICAL INDUSTRY

This section introduces novel strategies for reducing carbon emissions in the petrochemical industry by employing advanced techniques. The aim is to develop innovative carbon emission reduction methods tailored specifically for this industry's operations. These techniques focus on minimizing carbon footprint while maintaining operational efficiency.

2) UTILIZATION OF THE SUGGESTED SIMILARITY-DEPENDENT CMFHSS

Let $Z \neq \Omega$ represent the comprehensive set, and assume $Z \subset B$, symbolizing alternative elements indicated by $X = z_1, z_2, \ldots, z_m$. Consider $\mathcal{J} = B_1 \times B_2 \times \ldots \times B_r$, where $r \geq 1$, and B_i represents the collection of all attribute values related to attribute b_i for $i = 1, 2, 3, \ldots, r$.

The sequence for developing the proposed similarity based on CMFHSS is as follows:

1) Enter each CMFHSS.
2) Compute the similarity gauge for each CMFHSS using the equation: $R((\mathcal{Y}, \mathcal{J}), (\kappa, \mathcal{J}))$

$$\frac{1}{2m}\sum_{l=1}^{m}\left[R_l^q((\mathcal{Y}, \mathcal{J}), (\kappa, \mathcal{J})) + \frac{R_l^\delta((\mathcal{Y}, \mathcal{J}), (\kappa, \mathcal{J}))}{2\pi}\right], \tag{31}$$

where $R_l^q((\mathcal{Y}, \mathcal{J}), (\kappa, \mathcal{J})) =$

$$1 - \frac{1}{n}\sum_{l=1}^{n}\max\{|\xi^s\mathcal{Y}(e)(zp) - \tau^s\kappa(e)(zp)| : s \in k\}, \tag{32}$$

and $R_l^\delta((\mathcal{Y}, \mathcal{J}), (\kappa, \mathcal{J})) =$

$$2\pi - \frac{1}{n}\sum_{l=1}^{n}\max|\delta^s\mathcal{Y}(e)(zp) - \delta^s\kappa(e)(zp)| : s \in k. \tag{33}$$

3) Identify the CMFHSS demonstrating the highest similarity and designate it as the most optimal.
4) If multiple optimal CMFHSS options are identified, any one of them can be chosen.

Example 2: The petrochemical industry faces a challenge in reducing carbon emissions, necessitating intervention by an administration experiencing a downward trend. To address this, the authorities have established four independent panels and an evaluation board. Each panel has proposed four distinct initiatives, which they have subsequently submitted to the administration.

Let $X = \{a, b, c\}$ be represent three experts. Define a_1 = Emission Reduction Targets, a_2 = Technology Deployment, a_3 = Policy Instruments, as distinct attributes with corresponding values belonging to sets $\delta_1, \delta_2, \delta_3$. Here, $\delta_1 = \{\delta_1$ = Percentage Reduction, δ_2 = Time frame$\}$, $\delta_2 = \{\delta_3$ = Process Optimization$\}$, $\delta_3 = \{\delta_4$ = Emission Trading Schemes, δ_5 = Environmental Impact Assessment$\}$.

1) The aim is to determine the most efficient carbon reduction methods according to the specified parameters Structure for CMFHSS is represented in the following tables.

Renewable Energy Integration$= (\varkappa, \mathfrak{K})$

$$= \left\{\varkappa(\delta_1, \delta_3, \delta_4) = \left\{\frac{(0.1\exp^{i0.5\pi}, 0.1\exp^{i0.7\pi}, 0.2\exp^{i0.1\pi})}{a},\right.\right.$$

$$\frac{(0.6\exp^{i0.9\pi}, 0.1\exp^{i0.7\pi}, 0.1\exp^{i0.7\pi})}{b},$$

$$\left.\frac{(0.3\exp^{i0.6\pi}, 0.9\exp^{i0.1\pi}, 0.3\exp^{i0.7\pi})}{c}\right\},$$

$$\varkappa(\delta_1, \delta_3, \delta_5) = \left\{\frac{(0.7\exp^{i0.2\pi}, 0.3\exp^{i0.7\pi}, 0.1\exp^{i0.2\pi})}{a},\right.$$

$$\frac{(0.1\exp^{i0.2\pi}, 0.1\exp^{i0.2\pi}, 0.3\exp^{i0.1\pi})}{b},$$

$$\left.\frac{(0.4exp^{i0.7\pi}, 0.5\exp^{i0.6\pi}, 0.5\exp^{i0.2\pi})}{c}\right\},$$

$$\varkappa(\delta_2, \delta_3, \delta_4) = \left\{\frac{(0.1\exp^{i0.2\pi}, 0.3\exp^{i0.7\pi}, 0.7\exp^{i0.3\pi})}{a},\right.$$

$$\frac{(0.8\exp^{i0.2\pi}, 0.8\exp^{i0.2\pi}, 0.7\exp^{i0.3\pi})}{b},$$

$$\left.\frac{(0.3\exp^{i0.7\pi}, 0.9\exp^{i0.4\pi}, 0.1\exp^{i0.4\pi})}{c}\right\},$$

$$\varkappa(\delta_2, \delta_3, \delta_5) = \left\{\frac{(0.6\exp^{i0.9\pi}, 0.9\exp^{i0.1\pi}, 0.2\exp^{i0.2\pi})}{a},\right.$$

$$\frac{(0.7\exp^{i0.4\pi}, 0.1\exp^{i0.2\pi}, 0.7\exp^{i0.2\pi})}{b},$$

$$\left.\left.\frac{(0.2\exp^{i0.6\pi}, 0.2\exp^{i0.5\pi}, 0.2\exp^{i0.1\pi})}{c}\right\},\right\},$$

Energy Efficiency Improvements$= (\Box, \mathfrak{K})$

$$= \left\{\Box(\delta_1, \delta_3, \delta_4) = \left\{\frac{(0.1\exp^{i0.2\pi}, 0.4\exp^{i0.2\pi}, 0.4\exp^{i0.5\pi})}{a},\right.\right.$$

$$\frac{(0.6\exp^{i0.1\pi}, 0.2\exp^{i0.6\pi}, 0.2\exp^{i0.7\pi})}{b},$$

$$\left.\frac{(0.2\exp^{i0.6\pi}, 0.1\exp^{i0.7\pi}, 0.3\exp^{i0.6\pi})}{c}\right\},$$

$$\Box(\delta_1, \delta_3, \delta_5) = \left\{\frac{(0.1\exp^{i0.7\pi}, 0.6\exp^{i0.3\pi}, 0.4\exp^{i0.9\pi})}{a},\right.$$

$$\frac{(0.2\exp^{i0.7\pi}, 0.6\exp^{i0.3\pi}, 0.6\exp^{i0.3\pi})}{b},$$

$$\left.\frac{(0.1\exp^{i0.6\pi}, 0.3\exp^{i0.7\pi}, 0.8\exp^{i0.9\pi})}{c}\right\},$$

$$\Box(\delta_2, \delta_3, \delta_4) = \left\{\frac{(0.8\exp^{i0.9\pi}, 0.7\exp^{i0.2\pi}, 0.1\exp^{i0.4\pi})}{a},\right.$$

$$\frac{(0.9\exp^{i0.3\pi}, 0.4\exp^{i0.2\pi}, 0.4\exp^{i0.1\pi})}{b},$$

$$\left.\frac{(0.6\exp^{i0.7\pi}, 0.1\exp^{i0.7\pi}, 0.2\exp^{i0.1\pi})}{c}\right\},$$

$$\Box(\delta_2, \delta_3, \delta_5) = \left\{\frac{(0.6\exp^{i0.9\pi}, 0.2\exp^{i0.8\pi}, 0.4\exp^{i0.6\pi})}{a},\right.$$

$$\frac{(0.7\exp^{i0.8\pi}, 0.1\exp^{i0.2\pi}, 0.7\exp^{i0.7\pi})}{b},$$

$$\frac{(0.3 \exp^{i0.6\pi}, 0.2 \exp^{i0.7\pi}, 0.7 \exp^{i0.9\pi})}{c}\Bigg\}, \Bigg\},$$

Switching to Renewable Energy Sources$= (\square, \mathfrak{K})$

$$= \Bigg\{\square(\delta_1, \delta_3, \delta_4) = \Bigg\{ \frac{(0.2 \exp^{i0.7\pi}, 0.5 \exp^{i0.7\pi}, 0.2 \exp^{i0.4\pi})}{a},$$
$$\frac{(0.2 \exp^{i0.6\pi}, 0.5 \exp^{i0.8\pi}, 0.3 \exp^{i0.6\pi})}{b},$$
$$\frac{(0.7 \exp^{i0.6\pi}, 0.3 \exp^{i0.7\pi}, 0.2 \exp^{i0.9\pi})}{c}\Bigg\},$$

$$\square(\delta_1, \delta_3, \delta_5) = \Bigg\{ \frac{(0.6 \exp^{i0.9\pi}, 0.2 \exp^{i0.8\pi}, 0.2 \exp^{i0.9\pi})}{a},$$
$$\frac{(0.2 \exp^{i0.8\pi}, 0.6 \exp^{i0.1\pi}, 0.6 \exp^{i0.9\pi})}{b},$$
$$\frac{(0.2 \exp^{i0.6\pi}, 0.2 \exp^{i0.7\pi}, 0.9 \exp^{i0.7\pi})}{c}\Bigg\},$$

$$\square(\delta_2, \delta_3, \delta_4) = \Bigg\{ \frac{(0.6 \exp^{i0.9\pi}, 0.2 \exp^{i0.8\pi}, 0.3 \exp^{i0.3\pi})}{a},$$
$$\frac{(0.2 \exp^{i0.8\pi}, 0.9 \exp^{i0.1\pi}, 0.7 \exp^{i0.8\pi})}{b},$$
$$\frac{(0.3 \exp^{i0.6\pi}, 0.2 \exp^{i0.7\pi}, 0.6 \exp^{i0.3\pi})}{c}\Bigg\},$$

$$\square(\delta_2, \delta_3, \delta_5) = \Bigg\{ \frac{(0.3 \exp^{i0.4\pi}, 0.2 \exp^{i0.8\pi}, 0.2 \exp^{i0.9\pi})}{a},$$
$$\frac{(0.2 \exp^{i0.8\pi}, 0.6 \exp^{i0.1\pi}, 0.7 \exp^{i0.2\pi})}{b},$$
$$\frac{(0.3 \exp^{i0.6\pi}, 0.1 \exp^{i0.9\pi}, 0.5 \exp^{i0.6\pi})}{c}\Bigg\}, \Bigg\},$$

Framework and perfect CMFHSS are $(\mathfrak{Z}, \mathfrak{K}) =$

$$\Bigg\{\mathfrak{Z}(\delta_1, \delta_3, \delta_4) = \Bigg\{ \frac{(0.4 \exp^{i0.1\pi}, 0.2 \exp^{i0.7\pi}, 0.2 \exp^{i0.5\pi})}{a},$$
$$\frac{(0.9 \exp^{i0.1\pi}, 0.7 \exp^{i0.2\pi}, 0.4 \exp^{i0.9\pi})}{b},$$
$$\frac{(0.6 \exp^{i0.4\pi}, 0.6 \exp^{i0.3\pi}, 0.7 \exp^{i0.2\pi})}{c}\Bigg\},$$

$$\mathfrak{Z}(\delta_1, \delta_3, \delta_5) = \Bigg\{ \frac{(0.1 \exp^{i0.8\pi}, 0.2 \exp^{i0.1\pi}, 0.5 \exp^{i0.4\pi})}{a},$$
$$\frac{(0.9 \exp^{i0.1\pi}, 0.7 \exp^{i0.2\pi}, 0.1 \exp^{i0.3\pi})}{b},$$
$$\frac{(0.1 \exp^{i0.6\pi}, 0.1 \exp^{i0.4\pi}, 0.4 \exp^{i0.3\pi})}{c}\Bigg\},$$

$$\mathfrak{Z}(\delta_2, \delta_3, \delta_4) = \Bigg\{ \frac{(0.4 \exp^{i0.8\pi}, 0.1 \exp^{i0.7\pi}, 0.1 \exp^{i0.3\pi})}{a},$$
$$\frac{(0.1 \exp^{i0.7\pi}, 0.4 \exp^{i0.2\pi}, 0.9 \exp^{i0.1\pi})}{b},$$
$$\frac{(0.7 \exp^{i0.3\pi}, 0.1 \exp^{i0.3\pi}, 0.6 \exp^{i0.2\pi})}{c}\Bigg\},$$

$$\mathfrak{Z}(\delta_2, \delta_3, \delta_5) = \Bigg\{ \frac{(0.3 \exp^{i0.3\pi}, 0.2 \exp^{i0.6\pi}, 0.1 \exp^{i0.2\pi})}{a},$$
$$\frac{(0.5 \exp^{i0.1\pi}, 0.7 \exp^{i0.2\pi}, 0.5 \exp^{i0.9\pi})}{b},$$
$$\frac{(0.7 \exp^{i0.3\pi}, 0.8 \exp^{i0.4\pi}, 0.7 \exp^{i0.8\pi})}{c}\Bigg\}, \Bigg\},$$

2) Calculate the SM for $(\mathfrak{Z}, \mathfrak{K})$, $(\varkappa, \mathfrak{K})$, and $(\square, \mathfrak{K})$ by applying the algorithm detailed in Step (2) using the formula provided in Table 2.
   Therefore, the level of resemblance between $(\mathfrak{Z}, \mathfrak{K})$ and $(\varkappa, \mathfrak{K})$, $(\square, \mathfrak{K})$, $(\square, \mathfrak{K})$ respectively is given by
   $S_1 = S((\mathfrak{Z}, \mathfrak{K}), (\varkappa, \mathfrak{K})) = 0.661$,
   $S_2 = S((\mathfrak{Z}, \mathfrak{K}), (\square, \mathfrak{K})) = 0.602$,
   $S_3 = S((\mathfrak{Z}, \mathfrak{K}), (\square, \mathfrak{K})) = 0.642$.
3) As a result, $(\varkappa, \mathfrak{K})$ demonstrates the highest SM, suggesting that integrating Renewable Energy is the most effective strategy for reduction of carbon emission technique.

Example 5: For example, 4, in a situation where we have one-dimensional details akin to $(\omega, \mathfrak{K})$

$$= \Bigg\{\varkappa(\delta_1, \delta_3, \delta_4) = \Bigg\{ \frac{(0.5 \exp^{i2\pi(zero)}, 0.4 \exp^{i2\pi(zero)}, 0.1 \exp^{i2\pi(zero)}}{a},$$
$$\frac{(0.7 \exp^{i2\pi(zero)}, 0.4 \exp^{i2\pi(zero)}, 0.2 \exp^{i2\pi(zero)})}{b},$$
$$\frac{(0.7 \exp^{i2\pi(zero)}, 0.6 \exp^{i2\pi(zero)}, 0.4 \exp^{i2\pi(zero)})}{c}\Bigg\},$$

$$\varkappa(\delta_1, \delta_3, \delta_5) = \Bigg\{ \frac{(0.6 \exp^{i2\pi(zero)}, 0.5 \exp^{i2\pi(zero)}, 0.1 \exp^{i2\pi(zero)})}{a},$$
$$\frac{(0.2 \exp^{i2\pi(zero)}, 0.7 \exp^{i2\pi(zero)}, 0.9 \exp^{i2\pi(zero)})}{b},$$
$$\frac{(0.2 \exp^{i2\pi(zero)}, 0.5 \exp^{i2\pi(zero)}, 0.8 \exp^{i2\pi(zero)})}{c}\Bigg\},$$

$$\varkappa(\delta_2, \delta_3, \delta_4) = \Bigg\{ \frac{(0.4 \exp^{i2\pi(zero)}, 0.7 \exp^{i2\pi(zero)}, 0.5 \exp^{i2\pi(zero)})}{a},$$
$$\frac{(0.1 \exp^{i2\pi(zero)}, 0.2 \exp^{i2\pi(zero)}, 0.1 \exp^{i2\pi(zero)})}{b},$$
$$\frac{(0.4 \exp^{i2\pi(zero)}, 0.9 \exp^{i2\pi(zero)}, 0.6 \exp^{i2\pi(zero)})}{c}\Bigg\},$$

$$\varkappa(\delta_2, \delta_3, \delta_5) = \Bigg\{ \frac{(0.6 \exp^{i2\pi(zero)}, 0.7 \exp^{i2\pi(zero)}, 0.2 \exp^{i2\pi(zero)})}{a},$$
$$\frac{(0.5 \exp^{i2\pi(zero)}, 0.7 \exp^{i2\pi(zero)}, 0.4^{i2\pi(zero)})}{b},$$
$$\frac{(0.8 \exp^{i2\pi(zero)}, 0.1 \exp^{i2\pi(zero)}, 0.8 \exp^{i2\pi(zero)})}{c}\Bigg\}, \Bigg\},$$

$(\square, \mathfrak{K})$

$$= \Bigg\{\square(\delta_1, \delta_3, \delta_4) = \Bigg\{ \frac{(0.7 \exp^{i2\pi(zero)}, 0.8 \exp^{i2\pi(zero)}, 0.7 \exp^{i2\pi(zero)})}{a},$$
$$\frac{(0.5 \exp^{i2\pi(zero)}, 0.5 \exp^{i2\pi(zero)}, 0.2 \exp^{i2\pi(zero)})}{b},$$
$$\frac{(0.7 \exp^{i2\pi(zero)}, 0.5 \exp^{i2\pi(zero)}, 0.7 \exp^{i2\pi(zero)})}{c}\Bigg\},$$

$$\square(\delta_1, \delta_3, \delta_5) = \Bigg\{ \frac{(0.5 \exp^{i2\pi(zero)}, 0.1 \exp^{i2\pi(zero)}, 0.5 \exp^{i2\pi(zero)})}{a},$$
$$\frac{(0.5 \exp^{i2\pi(zero)}, 0.2 \exp^{i2\pi(zero)}, 0.9 \exp^{i2\pi(zero)})}{b},$$
$$\frac{(0.4 \exp^{i2\pi(zero)}, 0.3 \exp^{i2\pi(zero)}, 0.7 \exp^{i2\pi(zero)})}{c}\Bigg\},$$

$$\square(\delta_2, \delta_3, \delta_4) = \Bigg\{ \frac{(0.8 \exp^{i2\pi(zero)}, 0.4 \exp^{i2\pi(zero)}, 0.3 \exp^{i2\pi(zero)})}{a},$$
$$\frac{(0.6 \exp^{i2\pi(zero)}, 0.2 \exp^{i2\pi(zero)}, 0.4 \exp^{i2\pi(zero)})}{b},$$
$$\frac{(0.5 \exp^{i2\pi(zero)}, 0.8 \exp^{i2\pi(zero)}, 0.1 \exp^{i2\pi(zero)})}{c}\Bigg\},$$

$$\square(\delta_2, \delta_3, \delta_5) = \Bigg\{ \frac{(0.8 \exp^{i2\pi(zero)}, 0.4 \exp^{i2\pi(zero)}, 0.2 \exp^{i2\pi(zero)})}{a},$$
$$\frac{(0.7 \exp^{i2\pi(zero)}, 0.7 \exp^{i2\pi(zero)}, 0.2 \exp^{i2\pi(zero)})}{b},$$
$$\frac{(0.1 \exp^{i2\pi(zero)}, 0.8 \exp^{i2\pi(zero)}, 0.5 \exp^{i2\pi(zero)})}{c}\Bigg\}, \Bigg\},$$

$(\square, \mathfrak{K})$

$$= \Bigg\{\square(\delta_1, \delta_3, \delta_4) = \Bigg\{ \frac{(0.2 \exp^{i2\pi(zero)}, 0.4 \exp^{i2\pi(zero)}, 0.5 \exp^{i2\pi(zero)})}{a},$$
$$\frac{(0.7 \exp^{i2\pi(zero)}, 0.1 \exp^{i2\pi(zero)}, 0.2 \exp^{i2\pi(zero)})}{b},$$
$$\frac{(0.8 \exp^{i2\pi(zero)}, 0.2 \exp^{i2\pi(zero)}, 0.6 \exp^{i2\pi(zero)})}{c}\Bigg\},$$

$$\Box(\delta_1,\delta_3,\delta_5)=\left\{\frac{(0.1\exp^{i2\pi(zero)},0.1\exp^{i2\pi(zero)},0.6\exp^{i2\pi(zero)})}{a},\frac{(0.2\exp^{i2\pi(zero)},0.2\exp^{i2\pi(zero)},0.1\exp^{i2\pi(zero)})}{b},\frac{(0.5\exp^{i2\pi(zero)},0.2\exp^{i2\pi(zero)},0.1\exp^{i2\pi(zero)})}{c}\right\},$$

$$\Box(\delta_2,\delta_3,\delta_4)=\left\{\frac{(0.6\exp^{i2\pi(zero)},0.1\exp^{i2\pi(zero)},0.5\exp^{i2\pi(zero)})}{a},\frac{(0.6\exp^{i2\pi(zero)},0.1\exp^{i2\pi(zero)},0.7\exp^{i2\pi(zero)})}{b},\frac{(0.4\exp^{i2\pi(zero)},0.2\exp^{i2\pi(zero)},0.5\exp^{i2\pi(zero)})}{c}\right\},$$

$$\Box(\delta_2,\delta_3,\delta_5)=\left\{\frac{(0.5\exp^{i2\pi(zero)},0.2\exp^{i2\pi(zero)},0.1\exp^{i2\pi(zero)})}{a},\frac{(0.9\exp^{i2\pi(zero)},0.7\exp^{i2\pi(zero)},0.2\exp^{i2\pi(zero)})}{b},\frac{(0.2\exp^{i2\pi(zero)},0.1\exp^{i2\pi(zero)},0.2\exp^{i2\pi(zero)})}{c}\right\},\Bigg\},$$

and ideal CMFHSS are $(\mathfrak{Z},\mathfrak{K})$

$$=\Bigg\{\mathfrak{Z}(\delta_1,\delta_3,\delta_4)=\left\{\frac{(0.6\exp^{i2\pi(zero)},0.1\exp^{i2\pi(zero)},0.6\exp^{i2\pi(zero)})}{a},\frac{(0.6\exp^{i2\pi(zero)},0.1\exp^{i2\pi(zero)},0.3\exp^{i2\pi(zero)})}{b},\frac{(0.2\exp^{i2\pi(zero)},0.8\exp^{i2\pi(zero)},0.7\exp^{i2\pi(zero)})}{c}\right\},$$

$$\mathfrak{Z}(\delta_1,\delta_3,\delta_5)=\left\{\frac{(0.2\exp^{i2\pi(zero)},0.2\exp^{i2\pi(zero)},0.9\exp^{i2\pi(zero)})}{a},\frac{(0.1\exp^{i2\pi(zero)},0.5\exp^{i2\pi(zero)},0.3\exp^{i2\pi(zero)})}{b},\frac{(0.6\exp^{i2\pi(zero)},0.1\exp^{i2\pi(zero)},0.2\exp^{i2\pi(zero)})}{c}\right\},$$

$$\mathfrak{Z}(\delta_2,\delta_3,\delta_4)=\left\{\frac{(0.5\exp^{i2\pi(zero)},0.1\exp^{i2\pi(zero)},0.2\exp^{i2\pi(zero)})}{a},\frac{(0.1\exp^{i2\pi(zero)},0.4\exp^{i2\pi(zero)})}{b},\frac{(0.7\exp^{i2\pi(zero)},0.1\exp^{i2\pi(zero)},0.2\exp^{i2\pi(zero)})}{c}\right\},$$

$$\mathfrak{Z}(\delta_2,\delta_3,\delta_5)=\left\{\frac{(0.7\exp^{i2\pi(zero)},0.1\exp^{i2\pi(zero)},0.7\exp^{i2\pi(zero)})}{a},\frac{(0.5\exp^{i2\pi(zero)},0.1\exp^{i2\pi(zero)},0.2\exp^{i2\pi(zero)})}{b},\frac{(0.4\exp^{i2\pi(zero)},0.9\exp^{i2\pi(zero)},0.3\exp^{i2\pi(zero)})}{c}\right\},\Bigg\},$$

$$S_1=S((\mathfrak{Z},\mathfrak{K}),(\varkappa,\mathfrak{K}))=0.31,$$
$$S_2=S((\mathfrak{Z},\mathfrak{K}),(\Box,\mathfrak{K}))=0.21,$$
$$S_3=S((\mathfrak{Z},\mathfrak{K}),(\Box,\mathfrak{K}))=0.27.$$

TABLE 2. **Similarity measures.**

$S^{r}_{l=1}((\mathfrak{Z},\mathfrak{K}),(\varkappa,\mathfrak{K}))$	0.42
$S^{r}_{l=2}((\mathfrak{Z},\mathfrak{K}),(\varkappa,\mathfrak{K}))$	0.33
$S^{r}_{l=3}((\mathfrak{Z},\mathfrak{K}),(\varkappa,\mathfrak{K}))$	0.61
$S^{r}_{l=4}((\mathfrak{Z},\mathfrak{K}),(\varkappa,\mathfrak{K}))$	0.54
$S^{\omega}_{l=1}((\mathfrak{Z},\mathfrak{K}),(\varkappa,\mathfrak{K}))$	4.2
$S^{\omega}_{l=2}((\mathfrak{Z},\mathfrak{K}),(\varkappa,\mathfrak{K}))$	3.6
$S^{\omega}_{l=3}((\mathfrak{Z},\mathfrak{K}),(\varkappa,\mathfrak{K}))$	1.2
$S^{\omega}_{l=4}((\mathfrak{Z},\mathfrak{K}),(\varkappa,\mathfrak{K}))$	1.5
$S^{r}_{l=1}((\mathfrak{Z},\mathfrak{K}),(\Box,\mathfrak{K}))$	0.72
$S^{r}_{l=2}((\mathfrak{Z},\mathfrak{K}),(\Box,\mathfrak{K}))$	0.51
$S^{r}_{l=3}((\mathfrak{Z},\mathfrak{K}),(\Box,\mathfrak{K}))$	0.71
$S^{r}_{l=4}((\mathfrak{Z},\mathfrak{K}),(\Box,\mathfrak{K}))$	0.91
$S^{\omega}_{l=1}((\mathfrak{Z},\mathfrak{K}),(\Box,\mathfrak{K}))$	3.2
$S^{\omega}_{l=2}((\mathfrak{Z},\mathfrak{K}),(\Box,\mathfrak{K}))$	3.43
$S^{\omega}_{l=3}((\mathfrak{Z},\mathfrak{K}),(\Box,\mathfrak{K}))$	4.71
$S^{\omega}_{l=4}((\mathfrak{Z},\mathfrak{K}),(\Box,\mathfrak{K}))$	5.2
$S^{r}_{l=1}((\mathfrak{Z},\mathfrak{K}),(\Box,\mathfrak{K}))$	0.51
$S^{r}_{l=2}((\mathfrak{Z},\mathfrak{K}),(\Box,\mathfrak{K}))$	0.64
$S^{r}_{l=3}((\mathfrak{Z},\mathfrak{K}),(\Box,\mathfrak{K}))$	0.65
$S^{r}_{l=4}((\mathfrak{Z},\mathfrak{K}),(\Box,\mathfrak{K}))$	0.71
$S^{\omega}_{l=1}((\mathfrak{Z},\mathfrak{K}),(\Box,\mathfrak{K}))$	3.12
$S^{\omega}_{l=2}((\mathfrak{Z},\mathfrak{K}),(\Box,\mathfrak{K}))$	2.61
$S^{\omega}_{l=3}((\mathfrak{Z},\mathfrak{K}),(\Box,\mathfrak{K}))$	4.61
$S^{\omega}_{l=4}((\mathfrak{Z},\mathfrak{K}),(\Box,\mathfrak{K}))$	2.81

D. USING A TOPSIS-BASED OPTIMISED CMFHSS CLASSIFIER FOR EVALUATIONS OF CARBON EMISSIONS IN THE PETROCHEMICAL INDUSTRY

1) STRATEGIC INITIATIVES FOR REDUCING CARBON EMISSIONS IN THE PETROCHEMICAL INDUSTRY AND ENHANCING SUSTAINABILITY PRACTICES

One of the paramount concerns in environmental research revolves around the reduction of carbon emissions. Mitigating carbon emissions is a critical aspect in combating climate change and its detrimental effects. Rather than focusing solely on Renewable Energy (RE), it's imperative to explore a spectrum of techniques aimed at curbing carbon emissions across various sectors. These techniques encompass a wide array of strategies, ranging from energy-efficient practices in households, industries, and organizations to innovative technological advancements fostering cleaner and more sustainable energy sources. To address this multifaceted challenge, a comprehensive approach utilizing a merged fuzzy MCDM framework based on the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) is proposed. This framework aims to evaluate and prioritize carbon emission reduction techniques suitable for implementation. The assessment involves a meticulous analysis that considers diverse factors including social, institutional, technological, financial, and environmental aspects. Please see 2 for more detail on the eight various kinds of carbon emission reduction techniques resources that are explored.

- Carbon Capture and Storage
- Energy Efficiency Improvements
- Optimization of Processes
- Emission Control Technologies
- Carbon Offsetting and Renewable
- Bio-based Alternatives
- Improved Manufacturing Processes
- Product Innovation and Recycling

2) ALGORITHM

1) Initially, the conversion of the CMFHS collection into a FHSS aims to derive weighted aggregation values. This process can be approached in two distinct

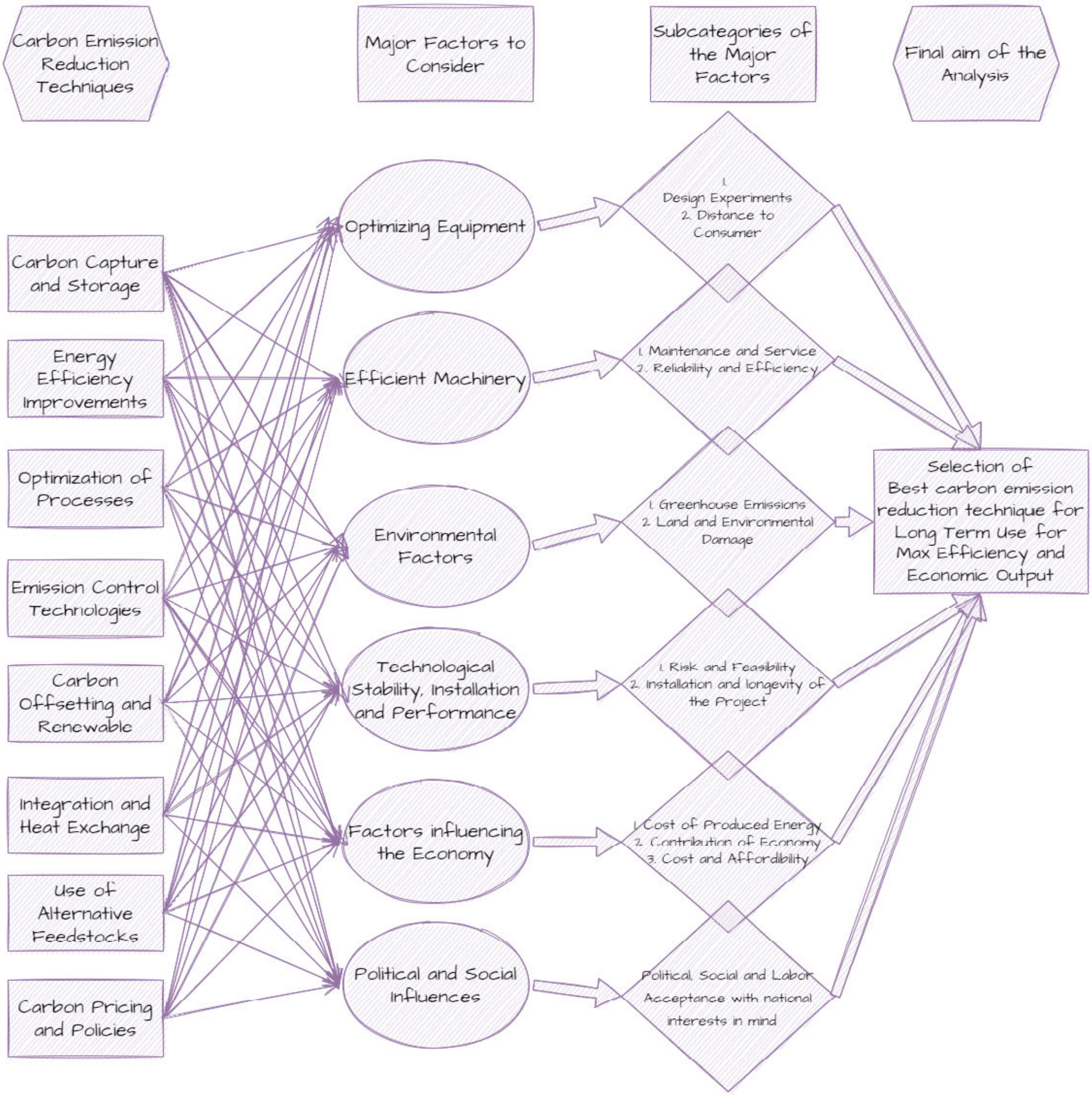

FIGURE 2. Flow chart for different renewable energy resources.

TABLE 3. The combined viewpoints from all specialists.

Types of Energy Sources/Criteria	1	2	3	4	5	6	7	8
Carbon Capture and Storage	0.05	0.7	0.34	0.1	0.4	0.4	0.1	0.6
Energy Efficiency Improvements	0.7	0.4	0.6	0.2	0.6	0.9	0.4	0.5
Optimization of Processes	0.2	0.3	0.2	0.61	0.85	0.7	0.6	0.5
Emission Control Technologies	0.3	0.1	0.4	0.6	0.4	0.2	0.6	0.1
Carbon Offsetting and Renewable	0.3	0.9	0.52	0.3	0.7	0.6	0.7	0.4
Bio-based Alternatives	0.3	0.4	0.9	0.1	0.6	0.7	0.4	0.2
Improved Manufacturing Processes	0.6	0.7	0.1	0.07	0.4	0.01	0.4	0.8
Product Innovation and Recycling	0.5	0.5	0.9	0.1	0.7	0.4	0.8	0.4

methods. The first method involves employing the formula $\Re z'(s')(p) = w1\mu_{z'(s')}(p) + t_2(\frac{1}{2\pi})\sigma_{z'(s')}(p)$ [18], where the weights are designated as $t_1 = 0.2$ and $t_2 = 0.4$. This method emphasizes the evaluation of individual aspects, harnessing the collaboration among professionals within the FHS assembly to construct an average decision matrix for each available option.

2) Create a mean decision grid for each option based on the combined viewpoint of experts within the FHS structure. Utilize the Standardized Precipitation Fuzzy

TABLE 4. Normalized.

Types of Energy Sources/Criteria	1	2	3	4	5	6	7	8
Carbon Capture and Storage	0.3	0.9	0.8	0.3	0.1	0.30	0.9	0.5
Energy Efficiency Improvements	0.1	0.7	0.1	0.7	0.4	0.4	0.7	0.6
Optimization of Processes	0.6	0.8	0.35	0.8	0.3	0.7	0.1	0.7
Emission Control Technologies	0.1	0.2	0.2	0.5	0.4	0.3	0.4	0.4
Carbon Offsetting and Renewable	0.8	0.4	0.2	0.3	0.9	0.6	0.7	0.4
Bio-based Alternatives	0.01	0.5	0.7	0.2	0.2	0.4	0.2	0.9
Improved Manufacturing Processes	0.4	0.3	0.4	0.2	0.5	0.5	0.6	0.4
Product Innovation and Recycling	0.7	0.5	0.8	0.2	0.4	0.7	0.4	0.2

TABLE 5. Matrix with normalized weights.

Types of Energy Sources/Criteria	1	2	3	4	5	6	7	8
Carbon Capture and Storage	0.1	0.2	0.2	0.2	0.4	0.4	0.3	0.3
Energy Efficiency Improvements	0.1	0.2	0.4	0.5	0.4	0.5	0.7	0.1
Optimization of Processes	0.7	0.8	0.8	0.3	0.05	0.4	0.2	0.6
Emission Control Technologies	0.7	0.9	0.5	0.1	0.02	0.1	0.8	0.4
Carbon Offsetting and Renewable	0.2	0.4	0.7	0.2	0.8	0.7	0.4	0.2
Bio-based Alternatives	0.5	0.2	0.8	0.3	0.1	0.4	0.24	0.4
Improved Manufacturing Processes	0.7	0.2	0.4	0.2	0.67	0.1	0.3	0.5
Product Innovation and Recycling	0.1	0.6	0.3	0.8	0.2	0.1	0.4	0.4

TABLE 6. Matrix of ultimate rankings.

Energy Sources/Criteria	1	2	3	4	5	6	7	8	Rank
Carbon Capture and Storage	0.5	0.1	0.82	0.1	0.3	0.7	0.24	0.7	6
Energy Efficiency Improvements	0.1	0.6	0.3	0.3	0.4	0.7	0.1	0.2	4
Optimization of Processes	0.6	0.8	0.1	0.7	0.4	0.4	0.7	0.2	7
Emission Control Technologies	0.2	0.4	0.2	0.8	0.1	0.6	0.8	0.1	8
Carbon Offsetting and Renewable	0.83	0.99	0.42	0.21	0.39	0.62	0.61	0.64	5
Bio-based Alternatives	0.2	0.4	0.8	0.1	0.5	0.9	0.8	0.7	3
Improved Manufacturing Processes	0.4	0.1	0.2	0.2	0.4	0.4	0.7	0.7	1
Product Innovation and Recycling	0.6	0.1	0.2	0.4	0.11	0.4	0.2	0.5	2

TABLE 7. Positive ideal solution.

C_1	0.12
C_2	0.15
C_3	0.05
C_4	0.04
C_5	0.082
C_6	0.043
C_7	0.041
C_8	0.07

TABLE 8. Negative ideal solution.

C_1	0.0078
C_2	0.026
C_3	0.013
C_4	0.0041
C_5	0.017
C_6	0.0021
C_7	0.0081
C_8	0.002

TABLE 9. Separation from positive ideal.

Carbon Capture and Storage	0.52
Energy Efficiency Improvements	0.71
Optimization of Processes	0.63
Emission Control Technologies	0.27
Carbon Offsetting and Renewable	0.03
Bio-based Alternatives	0.09
Improved Manufacturing Processes	0.07
Product Innovation and Recycling	0.02

Conceptual Framework as a standardized decision grid well-known for its normalization procedure.

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{1}^{m} x_{ij}^{2}}}; \tag{34}$$

TABLE 10. Separation from negative ideal.

Carbon Capture and Storage	0.23
Energy Efficiency Improvements	0.81
Optimization of Processes	0.38
Emission Control Technologies	0.19
Carbon Offsetting and Renewable	0.13
Bio-based Alternatives	0.24
Improved Manufacturing Processes	0.28
Product Innovation and Recycling	0.17

TABLE 11. Preference values.

Carbon Capture and Storage	0.21
Energy Efficiency Improvements	0.28
Optimization of Processes	0.14
Emission Control Technologies	0.13
Carbon Offsetting and Renewable	0.23
Bio-based Alternatives	0.3
Improved Manufacturing Processes	0.71
Product Innovation and Recycling	0.45

3) The computation of the weighted normalized fuzzy control matrix is based on the evaluation of the weighted normalized assessment (y_{ij}):

$$y_{ij} = w_i r_{ij} \tag{35}$$

with $i = 1, 2, \ldots, m$; and $j = 1, 2, \ldots, n$.

4) The procedure to determine the most advantageous positive and unfavorable solutions involves creating the positive ideal solution matrix using equation 36. Additionally, the negative ideal solution matrix is

TABLE 12. Comparative analysis of similarity measures: Proposed CMFHSS versus existing SM.

SN	References	Parameters	Sub-parameters	Two-Dimensional Data	Multifaceted	Ranking
1	[39]	×	×	×	×	×
2	[40]	×	×	×	×	×
3	[41]	×	×	×	×	×
4	[51]	×	×	×	×	×
5	[56]	✓	×	×	×	×
6	[60]	✓	×	✓	×	×
7	[61]	✓	✓	×	×	×
8	[42]	✓	×	×	×	×
9	[43]	×	×	×	×	×
10	[44]	✓	✓	×	×	×
11	[45]	✓	✓	✓	×	×
12	[46]	✓	✓	×	×	×
13	[54]	✓	×	×	✓	×
14	[34]	✓	✓	✓	×	×
15	[37]	✓	✓	×	×	×
16	[18]	✓	×	✓	✓	×
17	[49]	✓	×	×	×	×
18	[50]	✓	×	×	✓	×
19	Proposed Method in this paper	✓	✓	✓	✓	✓

computed using equation 37 simultaneously.

$$A^{+} = (y_1^{+}, y_2^{+}, \ldots, y_n^{+}); \tag{36}$$

$$A^{+} = (y_1^{+}, y_2^{+}, \ldots, y_n^{+}); \tag{37}$$

5) The following stage entails calculating the disparities among every attribute measurement of each renewable energy source for every criterion, concerning both the positive and negative ideal solutions. Equation 5 represents the gap between alternative A_i and the positive ideal solution.

$$D^{+} = \sqrt{\sum_{j=1}^{n} (y_i^{+} - y_{ij})^2}; \tag{38}$$

$i = 1, 2, 3 \ldots m$ The gap between the alternative A_i and the negative ideal solution can be articulated using equation 37.

$$D^{-} = \sqrt{\sum_{j=1}^{n} (y_i^{-} - y_{ij})^2}; \tag{39}$$

$i = 1, 2, 3 \ldots m$

6) Assigning a value to preferences for individual alternatives is done through the preference value, denoted as (V_i).

$$V_i = \frac{D_i^{-}}{D_i^{-} + D_i^{+}} \tag{40}$$

$i = 1, 2, 3 \ldots m$

7) Organize the choices and pick the optimal one.

E. MATHEMATICAL DEPICTION

1) Define the set of alternatives X as follows: $X = \{a =$ Carbon Capture and Storage, $b =$ Energy Efficiency Improvements, $c =$ Optimization of Processes, $d =$ Emission Control Technologies, $e =$ Carbon Offsetting and Renewable, $f =$ Energy Efficiency Improvements, $g =$ Carbon Capture and Storage, $h =$ Emission Control Technologies$\}$. Let $\delta_1, \delta_2, \delta_3, \delta_4$ represent a group of experts who will evaluate these alternatives. They will assign weights according to the vector $(0.2, 0.3, 0.1, 0.05, 0.15, 0.05, 0.05, 0.1)^T$. Additionally, define distinct features $a_1 =$ Environmental, $a_2 =$ Quality of Energy Source, and $a_3 =$ Economic, each having specific feature values represented by

collections of main components Q_1, Q_2, Q_3. Here, $Q_1 = \{\eta_1 =$ Greenhouse gas emission, $\eta_2 =$ Land requirement, $\eta_3 =$ Urgency for waste clearance, $\eta_4 =$ Ecological devastation$\}$, $Q_2 = \{\eta_5 =$ Durability, $\eta_6 =$ Sustainability$\}$, and $Q_3 = \{\eta_7 =$ Affordability$\}$. The combination of $Q_1 \times Q_2 \times Q_3$ results in the set $C_i, i = 1, 2, 3, \ldots 8$.

2) Generate a decision mean grid for each option based on the combined viewpoints of all specialists within the NHSS group. This requires the application of a set of criteria with their corresponding sub-criteria values (refer to Table 3). Standardize Table 3 using Formula 34 to derive Table 4.
3) Utilize Formula 35 to produce a weighted decision grid for each option, demonstrated in Table 5.
4) Compute the favorable optimal solution utilizing Formula 36 and the adverse ideal solution utilizing Formula 37 to generate Tables 7 and 8, respectively.
5) Determine the proximity of each contender from the favorable and adverse ideal solutions applying Formulas 38 and 39 and exhibit the outcomes in Tables 9 and 10.
6) Assess the proximity of each contender from both the favorable and adverse ideal solutions employing Formulas 38 and 39. Depict the outcomes in Tables 9 and 10 correspondingly.
7) Compute the inclination value for each option using Formula 40, as presented in Table 11.

1) COMPARATIVE STUDIES

We examined the effectiveness and superiority of our proposed ENT-driven methodology, integrating SM and TOPSIS within the CMFHSS framework, through various comparisons. These comparisons elucidated both the strengths and weaknesses of our approach when contrasted with established methodologies. Our assessment involved juxtaposing our method with a range of existing techniques across the domain. A notable limitation of current methodologies lies in their incapacity to adequately address the partitioning of attributes into attribute values, particularly when dealing with intricate two-dimensional data, which necessitates considerations of influence degree and total duration. Our proposed ENT-based CMFHSS method adeptly resolves these crucial issues, distinguishing itself from the shortcomings prevalent in established methodologies. For a more detailed insight, please refer to Table 12.

2) SENSITIVITY ANALYSIS

1) Ignoring the theoretical aspects and taking $n = 1$ where the values of $A_1, A_2, A_3, \ldots, A_n$ are equal, the suggested CMFHSS simplifies to a Multi-Fuzzy Soft Set [27].
2) When $k = 1$ and $n = 1$ with the values of $A_1, A_2, A_3, \ldots, A_n$ being equal, the proposed CMFHSS simplifies to CMFSS [18].

IV. DISCUSSION AND CONCLUSION

The petrochemical industry significantly contributes to carbon emissions, but a lack of comprehensive data hinders efforts to understand and implement emission reduction strategies. This study aims to fill this gap by examining the challenges associated with cutting carbon emissions in this sector. It explores various strategies for reducing emissions, offers guidance for investment decisions, and emphasizes the importance of selecting suitable techniques to enhance market competitiveness, reduce costs, and promote emission-free manufacturing. The implications of this research are crucial for procurement analysts, managers, and policymakers in both private and governmental sectors, highlighting the urgency of adopting sustainable practices. Additionally, the research proposes a framework for decision-making that identifies environmentally conscious suppliers in the petrochemical industry while considering potential unintended consequences. Collaborations with external entities can strengthen research efforts, particularly in developing nations seeking to improve efficiency in petrochemical production. This approach facilitates informed decision-making by integrating sustainability reports, evaluating risks, and fostering industry partnerships. Furthermore, the study underscores the role of external entities in advancing companies' capabilities in reducing carbon emissions, enabling progress in the petrochemical industry beyond internal resources. It addresses procurement challenges associated with emission reduction techniques, emphasizing the influence of financial considerations on decision-making. However, it emphasizes the importance of a balanced technological policy, considering factors such as intellectual capabilities and economic demands within the petrochemical industry. Given budget limitations and macroeconomic challenges, the study highlights the potential of emission reduction techniques while stressing the need to analyze the impacts of different scenarios on carbon emissions. It advocates for robust strategies that consider risks associated with emission reduction methods and continuously adapt to technological advancements and regulations. The study recommends using Multi-Criteria Decision Analysis (MCDA) to evaluate alternatives based on economic, environmental, and social criteria, prioritizing effective emission reduction actions. Collaborating with other industries, analyzing external variables like political developments, and conducting comprehensive life cycle assessments (LCAs) are suggested to understand emissions at each stage. Additionally, the study introduces a novel technique, CMFHSS, to comprehensively evaluate factors influencing emission reduction methodologies. It establishes a scientific foundation for managing variability across sectors and broadens the understanding of available options. Mathematical models and comparative analyses with existing models are discussed, offering analytical solutions for practical strategic planning. This research lays the groundwork for addressing uncertainties across various fields and provides a theoretical basis for further applications in physical and natural sciences.

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