IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received 30 January 2024, accepted 15 February 2024, date of publication 21 February 2024, date of current version 1 March 2024.

Digital Object Identifier 10.1109/ACCESS.2024.3368806

RESEARCH ARTICLE

A New Event-Triggered Distributed Fixed-Time Consensus Strategy for Multi-Agent Systems With Nonlinear Dynamics and Uncertain Disturbances

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This work was supported in part by the National Natural Science Foundation of China (NSFC) under Grant 51975542, in part by the University-Industry Collaborative Education Program of Ministry of Education under Grant 230803117035132, and in part by the Scientific Research Project of Hubei University of Science and Technology under Grant 2024-25X06.

ABSTRACT This paper considers the event-triggered distributed fixed-time consensus control problem of leader-following multi-agent systems with nonlinear dynamics and inherent disturbances. First, an event-triggered distributed strategy, that is used to decide when to update states by agents, is proposed. Under the proposed strategy, each agent receives its neighbors' state to make decisions only if the measurement error exceeds a certain threshold. Second, we design a nonsingular terminal sliding mode consensus protocol so that all followers reach the leader's state in a fixed-time. Particularly, the gain in the consensus protocol is determined by the settling time, which makes devising and tuning the gain conveniently. Simulation examples are worked out to demonstrate the effectiveness of our theoretical results.

INDEX TERMS Multi-agent systems, event-triggered control, consensus, dynamics and disturbances, fixed-time.

I. INTRODUCTION

In recent years, the cooperative control problem of multi-agent systems has received significant attention within control community [1], [2], [3]. The core task of cooperative control in multi-agent systems is to design suitable control scheme to achieve the consensus agreement on the states or outputs for each agent. Distributed control algorithms have been used in the literature [4] to study the asymptotic consensus of multi-agent systems. The literature [5] proposed a consensus fuzzy control method with preset performance for high-order strict feedback multi-agent systems.

However, for practical systems, the design of the control protocol should consider the speed of convergence. Finite-time convergence is always expected, rather than asymptotic convergence [4], [5], which has been discussed for a long time in system control. To achieve finite-time convergence in system control, many efficient methods have been proposed [6], [7], [8], [9], [10]. For example, [6] studied the distributed finite-time consensus protocol in a time-varying topology of first-order linear multi-agent systems. Reference [7] designed a non-smooth sampled-data control method for second-order multi-agent systems. References [8] and [9] respectively designed different finite-time consensus control methods for nonlinear systems based on robust fuzzy control and neural networks. Reference [10] studied the finite-time consensus control method for second-order nonlinear multi-agent switching systems. Although many invariants have been proposed to improve convergence speed, the convergence time usually depends on the initial conditions and can be arbitrarily large, which is not advisable. Therefore, researchers and scholars are paying more attention to the

The associate editor coordinating the review of this manuscript and approving it for publication was Qiang Li.

convergence of fixed-time, where there is an upper bound on the convergence time with arbitrary initial conditions [11]. In addition, note that most of existing results are related to linear systems, it is difficult to directly apply the existing results of linear systems to nonlinear systems. Therefore, the fixed-time consensus of uncertain nonlinear systems has become an attractive and hot issue within control research area. Recently, several fixed-time consensus algorithms for nonlinear multi-agent systems were proposed in [12], [13], [14], [15], [16], and [17]. Among them, [12] and [13] studied the fixed-time consensus tracking problem of first-order nonlinear multi-agent systems. Reference [14] designed an observer-based consensus control method for second-order multi-agent systems with disturbance. Reference [15] studied a fixed-time consensus tracking control method for heterogeneous multi-agent systems. In [16] and [17], fixed-time consensus control for high-order nonlinear multi-agent systems were developed.

On the other hand, due to the highly complexity and inter-connectivity of multi-agent systems, in practical applications where resources and communication bandwidth are limited, researchers and scholars have developed two effective control strategies to decrease the communication burden: time-triggered control strategy [18] and event-triggered control strategy [19], [20]. Between them, the time-triggered scheme depends on the sampling time period for achieving the control objectives. However, it often leads to unnecessary sampling, which in turn results in additional communication burden and cost. Subsequently, an event-triggered strategy is developed, in which triggering control tasks are only executed when certain predefined conditions are triggered. Some significative results for reducing the communication burden have been proposed, which focus on the event-triggered control strategy of multi-agent systems with static event-triggered strategies [21], [22] and dynamic event-triggered strategies [23], [24], [25], [26], [27]. Among them, [22], [23], and [24] studied uncertain nonlinear multi-agent systems based on event-triggered control mechanisms. References [21], [25], [26], and [27] proposed event-triggering strategies for linear multi-agent systems. These researches include leader-following problems, switching topologies, system disturbances, and so on. There are still many issues that require further research. This paper will further focus on the event-triggered fixed- time consensus control for multi-agent systems with nonlinear uncertainties.

Motivated by above discussions, this paper investigates the event-triggered distributed fixed-time consensus control problem of leader-following multi-agent systems with nonlinear dynamics and inherent disturbances. Under the framework of Lyapunov stability theorem, a novel distributed consensus control scheme and event-triggered control strategy is developed. The main contribution of this article is:

1) Different from [20], we design a stable event-triggered mechanism, i.e., frequent switching of the controller will not cause undesirable phenomena such as chattering.

2) A new event-triggered distributed fixed-time consensus method is developed, where controller updates are event-driven under a predetermined event-triggered strategy. The proposed control scheme effectively reduces the communication burden between the agent and the controller, while ensuring the stability of the system and achieving the goal of consensus tracking control.

3) This article provides a new controller design method to reduce the conservatism of convergence time bound estimation, which can reduce controller gain and avoid actuator saturation as much as possible, and the minimum setting value is obtained and less conservative.

4) The control gain can be directly determined from the specified time, which is beneficial for the design and tuning of the control gain.

Finally, simulation examples verified the effectiveness of the theoretical results.

Notations: Throughout this paper, the symbol $\mathbb{R}$ is the set of real numbers. The symbol $\mathbb{R}^+$ is the set of nonnegative real numbers. $D^*\omega(t)$ denotes the upper right-hand derivative of a function $\omega(t)$, i.e. $D^*\omega(t) := suplim_{h\to 0^+}\frac{\omega(t+h)-\omega(t)}{h}$.$\mathbf{1}_M$ denotes M-dimensional vectors whose all elements are 1. Denote $x_i = [x_{i1}, \cdots, x_{iM}]^T \in\mathbb{R}^M$, $and\ sign(x_i) = [sign(x_{i1}), \cdots, sign(x_{iM})]^T$, where $sign(x_i)$ is the signum function. If $s> 0$, $sign(s) = 1$; else if $s< 0$, $sign(s) = -1$; else $sign(s) = 0$. For $z\in\mathbb{R}^+$, we define $z^{[k]} = sign(\cdot)|z|^k$. Given a matrix P, P^T, $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ respresent its transposition, its maximum and minimum eigenvalues respectively.

II. PREPARATION AND PROBLEM FORMULATIONS

A. GRAPH THEORY

The interaction among followers can be modeled by a graph $G = (V, E, A)$, where $V = \{v_1, v_2, \cdots v_N\}$ is the set of nodes, and $E \subseteq V \times V$ is the set of edges. A directed edge $(v_j, v_i) \in E$ indicates that node v_i can receive information from node v_j, but not necessarily vice versa. Among them, node v_j is called parent node, node v_i is called child node. Denote $A = [a_{ij}] \in \mathbb{R}^{N\times N}$ as the adjacency matrix, whose elements can be defined as that $a_{ij} > 0$ if $(v_j, v_i) \in E$ and $a_{ij} = 0$ otherwise. The degree matrix is defined as $D = diag\{d_1, d_2, \cdots, d_N\}$, where $d_i = \sum_{j=1,j\neq i}^N a_{ij}$. The Laplacian matrix $L = [l_{ij}]_{N\times N}$ of graph G is defined as $L = D - A$.

For a leader-following multi-agent systems, assume that the leader is labeled as v_0 and the follower is represented by $v_1, v_2, \cdots v_N$. The directed graph consisting of $N + 1$ agents can be represented as $\hat{G}$, and $N + 1$ agents can be viewed as $N + 1$ nodes. $\hat{V} = \{v_0, \cdots, v_N\}$ represents the set of nodes, $\hat{E}\in\hat{V}\times\hat{V}$ represents the set of edges. Let $\hat{L}$ denote the Laplacian matrix of $\hat{G}$. Graph $\hat{G}$ can be represented by a weight matrix $\hat{L} = L + B$, where $B = diag\{a_{10}, a_{20} \cdots, a_{N0}\}$. A path from node v_i to v_j refers to a sequence of edges $\{(v_j, v_{i1}), (v_{j1}, v_{i2}), \cdots, (v_{jl}, v_j)\}$ composed of different nodes $v_{ik}, k = 1, 2, \cdots, l$. If a root node exists so that it has at least a directed path to each other node, then graph $\hat{G}$ is said to have a spanning tree.

B. FIXED-TIME STABILITY

For a general differential equation system:

$$\dot{x}(t) = g(t, x(t)), \quad x(0) = x_0 \tag{1}$$

where $x(t) \in \mathbb{R}^M$ is the state vector and $g(t, x(t)) : \mathbb{R}^+ \times \mathbb{R}^M \to \mathbb{R}^M$ is a nonlinear function. Suppose that the orgin zero is an equilibrium point of (1).

Definition 1 ([6]): The equilibrium point of system (1) is said to be globally finite-time stable if it is globally Lyapunov stable and any solution of the equation reaches equilibrium at a finite time, i.e., $x(t, x_0) = 0, \forall t \geq T(x_0)$, where $x(t, x_0) \in \mathbb{R}^M$ is the solution of system (1) and $T(x_0) : \mathbb{R}^M \to \mathbb{R}^+ \cup \{0\}$ is the setting time function.

Definition 2 ([22]): The equilibrium point of system (1) is said to be globally fixed-time stable if it is globally Lyapunov stable with bounded time $T(x_0)$, i.e., $\exists T_{\max} > 0 : T(x_0) \leq T_{\max}, \forall x_0 \in \mathbb{R}^M$.

Remark 1: Consider systems $\dot{z} = -z^{1/3}$, $z(0) = z_0$ and $\dot{y} = -y^{1/3} - y^3$, $y(0) = y_0$, respectively, an arbitrary solution $z(t, z_0) = 0$ for $\forall t \geq T(z_0) = (3/4) z_0^{4/3}$ is finite-time stable and $y(t, y_0)$ for $\forall t \geq T(y_0)$, $T(y_0) \leq T_{\max} = \left(3\sqrt{2}\pi\right)/8$ is fixed-time stable, respectively. As can be seen from the above definitions and examples, the main difference between finite-time stability and fixed-time stability is the convergence time, i.e., the convergence time of finite-time stability depends on the initial value, while the upper bound of the convergence time for fixed-time stability is a constant.

Lemma 1 ([22]): If there exists a continuous radially unbounded function $V : \mathbb{R}^M \to \mathbb{R}^+ \cup \{0\}$ such that $V(x(t)) = 0 \Leftrightarrow x(t) = 0$ and any solution $x(t)$ satisfies the inequality $D^* V(x(t)) \leq -\alpha V(x(t))^p - \beta V(x(t))^q$ for some constants α, $\beta > 0$, $p = 1 - \frac{1}{\mu}$, $q = 1 + \frac{1}{\mu}$, $\mu > 1$, then globally fixed-time stable can be achieved and the settling time T holds:

$$T(x_0) \leq \frac{\pi \mu}{2\sqrt{\alpha\beta}}.$$

Lemma 2 ([18]): For $w_1, w_2, \cdots, w_M \geq 0$ and $0 < b \leq 1$, we have

$$\sum_{i=1}^{M} w_i^b \geq \left(\sum_{i=1}^{M} w_i\right)^b.$$

Lemma 3 ([18]): For $w_1, w_2, \cdots, w_M \geq 0$ and $c > 1$, one has

$$\left(\sum_{i=1}^{M} w_i\right)^c \geq \sum_{i=1}^{M} w_i^c \geq M^{1-c}\left(\sum_{i=1}^{M} w_i\right)^c.$$

C. PROBLEM FORMULATION

Consider a directed connected graph $\hat{G}$ with N followers and one leader. The dynamic model of each follower can be described as

$$\dot{x}_i(t) = u_i(t) + g(t, x_i(t)) + d_i(t, x_i(t)), i \in \{1, \cdots, N\}, \tag{2}$$

where $x_i(t)$ denotes the state of the agent i, $u_i(t)$ is the ith agent' control input. $g : \mathbb{R}^+ \times \mathbb{R}^M \to \mathbb{R}^M$ is the nonlinear function, and $d_i : \mathbb{R}^+ \times \mathbb{R} \to \mathbb{R}^M$ is the uncertain disturbance.

The dynamic model of a leader is represented as

$$\dot{x}_0(t) = u_0(t) + g(t, x_0(t)), \tag{3}$$

where $x_0(t)$ is the leader's state. $g(t, x_0(t))$ is the nonlinear function, and $u_0(t)$ is the control input for the leader.

Assumption 1: The leader's control input is bounded, i.e., one can find a constant $\theta \in \mathbb{R}^+$ such that

$$|u_0| \leq \theta. \tag{4}$$

Assumption 2: There exist $\rho > 0$ and $d_{\max} > 0$ satisfying

$$|g(t, \omega) - g(t, \varsigma)| \leq \rho |\omega - \varsigma|, \forall \omega, \varsigma \in \mathbb{R}, \forall t \geq 0. \tag{5}$$

and

$$d_i(t, \zeta) \leq d_{\max}, i = 0, 1, 2, \cdots, N. \tag{6}$$

where the uncertain disturbance $d_i(t, \zeta)$ is a continuous function with respect to time t and the state ζ.

Definition 3: For given input $u_i(t), i = 1, 2, \cdots, N$, if there exists a setting time function $T(x_0) \in [0, \infty]$ bounded by some positive number $T_{\max}$, i.e., $T(x_0) < T_{\max}$, $\forall x(0) \in \mathbb{R}$ regardless of initial states, such that

$$\lim_{t \in T(x_0)} \left|x_i(t) - x_j(t)\right| = 0, x_i(t) = x_0(t), \forall t \geq T(x_0). \tag{7}$$

The fixed-time consensus tracking of the multi-agent system (2), (3) is achieved.

D. EVENT-TRIGGERED DISTRIBUTED FIXED-TIME CONSENSUS ALGORITHMS

The event-triggered function is proposed as

$$\varphi_i(t) = |e_i(t)| - \ell |\chi_i(t)|, \tag{8}$$

where $|e_i(t)| = \left|x_i\left(t_{k'}^i\right) - x_i(t)\right|$, it represents the error value when the event action is triggered. $|\chi_i(t)| = \left|\sum_{j=0}^{N} a_{ij}\left(x_i(t) - x_j(t)\right)\right|$ represents the total errors of all agents. ℓ is a positive constant and will be designed later. $k' \triangleq \operatorname{argmin}_{l \in \mathbb{R}^+ : t > t_l^j}\left(t - t_l^j\right)$ refers to the event time of agent j.

For consensus tracking algorithms triggered by distributed events, each agent has its own sampling time series, which is determined by its own distributed event triggering function. Suppose that graph $\hat{G}$ contains a directed spanning tree, where the root node is the leader and the subgraph formed by N followers is a strongly connected graph. For any follower $i\,(i = 1, 2, \cdots, N)$, we construct the following controller

$$u_i(t) = -k_1\left(\sum_{j=0}^{N} a_{ij}\left(x_i\left(t_k^i\right) - x_j\left(t_{k'}^j\right)\right)\right)^{\left[\frac{\mu}{2q-\mu}\right]}$$

$$
\begin{aligned}
&- k_2 \left(\sum_{j=0}^{N} a_{ij}\left(x_i\left(t_k^i\right) - x_j\left(t_{k'}^j\right)\right)\right)^{\left[\frac{2q-\mu}{\mu}\right]} \\
&- \gamma\, sign \left(\sum_{j=0}^{N} a_{ij}\left(x_i\left(t_k^i\right) - x_j\left(t_{k'}^j\right)\right)\right) \\
&-2\ell \left(\sum_{j=0}^{N} a_{ij}\left(x_i\left(t_k^i\right) - x_j\left(t_{k'}^j\right)\right)\right), \qquad (9)
\end{aligned}
$$

where k_1, k_2, and ℓ are positive constants, $\gamma = d_{max} + \theta$. μ and q are the odd numbers, where $q > \mu$, and $s^{[k]} = sign(\cdot)\,|s|^k$. For each $t \in \left[t_k^i, t_{k+1}^i\right)$, $t_{k'}^j$ is the last event time of agent j. Agent i only updates the control input at time t_0^i, $t_1^i, \cdots$, similarly, agent j only updates the control input at time t_0^j, $t_1^j, \cdots$, where agent j is a neighbor of agent i. Note that the leader is independent and the trajectory is not influenced by their followers. Therefore, it has no triggering moment when $j = 0$, $x_0\left(t_{k'}^0\right) = x_0(t)$.

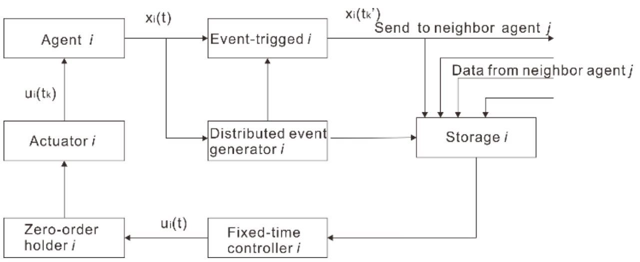

FIGURE 1. Diagram of event-triggered control.

The distributed event triggering algorithm is to pre-establish an event-triggering function and then design appropriate control inputs to ensure consensus tracking characteristics. The diagram of event-triggered consensus control strategy is given in Figure 1. For each agent i, an event is triggered when $\varphi_i(t)$ exceeds zero. At the nth event moment of intelligent agent i, it will sample its state and update its controller by using the newly sampled state. Then, it will send the state to its neighbors, who will use the received state to update their controller.

III. MAIN RESULTS

Our main result will be stated and the proofs will be given in detail.

Lemma 4 ([12]): If a directed graph composed of N followers is strongly connected and one can find a path between each follower and the leader in $\hat{G}$, then $\hat{L}$ is nonsingular matrix. Further, define

$$
\begin{aligned}
&[\varepsilon_1, \varepsilon_2 \cdots \varepsilon_N]^T H^T = 1_N \\
&W = diag\{\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_N\} \\
&Q = W\hat{L} + \hat{L}^T W, \qquad (10)
\end{aligned}
$$

then the positive definite W and Q can be satisfied.

Theorem 1: Consider a directed connected graph $\hat{G}$, and suppose Assumptions 1- 2 hold under communication rule (2) - (3) and control input (9), if the following condition is designed as:

$$
\frac{N\rho}{\lambda_2\left(\hat{L}\right)} \le \ell. \qquad (11)
$$

and the parameters satisfy

$$
\begin{aligned}
c_1 &= \frac{h}{2\left(1+2^{q-\mu}\right)} \left(\frac{2q}{(2q-\mu)\,\varepsilon_{\max}}\right)^{\frac{\mu}{q}} \\
c_2 &= \frac{h}{2\left(1+2^{q-\mu}\right)} \left(\frac{2q}{(2q-\mu)\,\varepsilon_{\max}}\right)^{\frac{2q-\mu}{q}} \\
&\begin{cases} \min\left\{k_2 N^{3-\frac{4q}{\mu}}, k_1\right\}, & if\, 0 < \frac{2\mu}{2q} - \mu < 1 \\ \min\left\{k_2 N^{3-\frac{4q}{\mu}}, k_1 N^{1-\frac{2\mu}{2q-\mu}}\right\}, & if\, \frac{2\mu}{2q} - \mu \ge 1 \end{cases} \\
Q &= W\hat{L} + \hat{L}^T W
\end{aligned}
$$

with positive-definite matrices Q and $W = diag\{\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_N\}$ and $\varepsilon_{\max} = \max\{\varepsilon_i\}$, $\lambda_2\left(\hat{L}\right)$ is the second smallest eigenvalue of matrix $\hat{L}$ and the settling time T is bounded by:

$$
T \le T_{\max} := \frac{q\pi}{\left[(q-\mu)\,\lambda_{\min}(Q)\sqrt{c_1 c_2}\right]}. \qquad (12)
$$

Proof. Construct a candidate of Lyapunov function as

$$
V(t) = \sum_{i=1}^{N} \varepsilon_i \frac{2q-\mu}{2q} |\chi_i|^{\frac{2q}{2q-\mu}} + \sum_{i=1}^{N} \varepsilon_i \frac{\mu}{2q} |\chi_i|^{\frac{2q}{\mu}}. \qquad (13)
$$

Differentiating (13), there holds

$$
\begin{aligned}
\dot{V}(t) &= \sum_{i=1}^{N} \varepsilon_i sig(\chi_i)^{\frac{\mu}{2q-\mu}} \dot{\chi}_i + \sum_{i=1}^{N} \varepsilon_i sig(\chi_i)^{\frac{2q-\mu}{\mu}} \dot{\chi}_i \\
&= \left(\chi^{\left[\frac{\mu}{2q-\mu}\right]}\right)^T W\dot{\chi}_i + (\chi^{\left[\frac{2q-\mu}{\mu}\right]})^T W\dot{\chi}_i \\
&= \left(\left(\chi^{\left[\frac{\mu}{2q-\mu}\right]}\right)^T + (\chi^{\left[\frac{2q-\mu}{\mu}\right]})^T\right) W\dot{\chi}_i. \qquad (14)
\end{aligned}
$$

where $\chi(t) = [\chi_1(t), \cdots, \chi_N(t)]^T$.

The tracking error of agent i is defined as

$$
\hat{x}_i(t) = x_i(t) - x_0(t), \qquad (15)
$$

and define $\chi_i(t) = \sum_{j=0}^{N} a_{ij}\left(x_i(\mathrm{t}) - x_j(\mathrm{t})\right)$. Let $\hat{x}(t) = \left[\hat{x}_1(t), \cdots, \hat{x}_N(t)\right]^T$, then we have $\chi(t) = \hat{L}\hat{x}(t)$.

The derivative of $\hat{x}_i(t)$ is

$$
\begin{aligned}
\dot{\hat{x}}(t) &= \dot{x}_i(t) - \dot{x}_0(t) \\
&= -k_1 \left(\sum_{j=0}^{N} a_{ij}\left(x_i\left(t_k^i\right) - x_j\left(t_{k'}^j\right)\right)\right)^{\left[\frac{\mu}{2q-\mu}\right]}
\end{aligned}
$$

$$
\begin{aligned}
&- k_2 \left(\sum_{j=0}^{N} a_{ij} \left(x_i \left(t_k^i \right) - x_j \left(t_{k'}^j \right) \right) \right)^{\left[\frac{2q-\mu}{\mu} \right]} \\
&- \gamma sign \left(\sum_{j=0}^{N} a_{ij} \left(x_i \left(t_k^i \right) - x_j \left(t_{k'}^j \right) \right) \right) \\
&-2\ell \left(\sum_{j=0}^{N} a_{ij} \left(x_i \left(t_k^i \right) - x_j \left(t_{k'}^j \right) \right) \right) \\
&+ \hat{g}(t) + \hat{d}(t) - 1_N u_0. \qquad (16)
\end{aligned}
$$

The measurement error of agent i is defined as

$$
\begin{aligned}
e_i(t) = k_1 &\left(\sum_{j=0}^{N} a_{ij} \left(x_i \left(t_k^i \right) - x_j \left(t_{k'}^j \right) \right) \right)^{\left[\frac{\mu}{2q-\mu} \right]} \\
&- k_1 \chi_i(t)^{\left[\frac{\mu}{2q-\mu} \right]} \\
&+ k_2 \left(\sum_{j=0}^{N} a_{ij} \left(x_i \left(t_k^i \right) - x_j \left(t_{k'}^j \right) \right) \right)^{\left[\frac{2q-\mu}{\mu} \right]} \\
&- k_2 \chi_i(t)^{\left[\frac{2q-\mu}{\mu} \right]} \\
&-2\ell \left(\sum_{j=0}^{N} a_{ij} \left(x_i \left(t_k^i \right) - x_j \left(t_{k'}^j \right) \right) \right) + 2\ell \chi_i(t) \\
&- \gamma sign \left(\sum_{j=0}^{N} a_{ij} \left(x_i \left(t_k^i \right) - x_j \left(t_{k'}^j \right) \right) \right) \\
&+ \gamma sign(\chi_i(t)). \qquad (17)
\end{aligned}
$$

From (3), (4), (16) and (17), there holds

$$
\begin{aligned}
\dot{\chi}(t) = &-e(t) - k_1 \chi(t)^{\left[\frac{\mu}{2q-\mu} \right]} - k_2 \chi(t)^{\left[\frac{2q-\mu}{\mu} \right]} \\
&- \gamma sign(\chi(t)) - 2\ell \chi(t) + \hat{g}(t) + \hat{d}(t) - 1_N u_0. \qquad (18)
\end{aligned}
$$

where $\hat{g}(t) = \left[\hat{g}(x_1, t) - \hat{g}_0(x_0, t), \cdots, \hat{g}(x_N, t) - \hat{g}_0(x_N, t) \right]^T$, $\hat{d}(t) = [d(x_1, t), \cdots, d(x_N, t)]^T$.

Furthermore, (14) is equivalent to:

$$
\begin{aligned}
&\dot{V}(t) \\
&\leq \left(\chi(t)^{\left[\frac{\mu}{2q-\mu} \right]} + \chi(t)^{\left[\frac{2q-\mu}{\mu} \right]} \right)^T \frac{W(-L) + (-L)^T W}{2} \\
&\quad \times \left(k_1 \chi(t)^{\left[\frac{\mu}{2q-\mu} \right]} + k_2 \chi(t)^{\left[\frac{2q-\mu}{\mu} \right]} + e(t) + \gamma sign(\chi(t)) \right. \\
&\quad \left. + 2\ell \chi(t) - \hat{g}(t) - \hat{d}(t) + 1_N u_0 \right) \\
&\leq -\frac{1}{2} \lambda_{\min}(Q) \left(\chi(t)^{\left[\frac{\mu}{2q-\mu} \right]} + \chi(t)^{\left[\frac{2q-\mu}{\mu} \right]} \right)^T \\
&\quad \times \left(k_1 \chi(t)^{\left[\frac{\mu}{2q-\mu} \right]} + k_2 \chi(t)^{\left[\frac{2q-\mu}{\mu} \right]} \right. \\
&\quad \left. + e(t) + \gamma sign(\chi(t)) + 2\ell \chi(t) - \hat{g}(t) - \hat{d}(t) + 1_N u_0 \right)
\end{aligned}
$$

$$
\begin{aligned}
&\leq -\frac{1}{2} \lambda_{\min}(Q) \left(k_1 \sum_{i=1}^{N} |\chi_i(t)|^{\frac{2\mu}{2q-\mu}} + k_2 \sum_{i=1}^{N} |\chi_i(t)|^{\frac{2(2q-\mu)}{\mu}} \right. \\
&\quad \left. + (k_1 + k_2) \sum_{i=1}^{N} |\chi_i(t)|^{\frac{\mu}{2q-\mu} + \frac{2q-\mu}{\mu}} \right) \\
&\leq -\frac{1}{2} \lambda_{\min}(Q) \left(k_1 \sum_{i=1}^{N} |\chi_i(t)|^{\frac{2\mu}{2q-\mu}} + k_2 \sum_{i=1}^{N} |\chi_i(t)|^{\frac{2(2q-\mu)}{\mu}} \right. \\
&\quad \left. + (k_1 + k_2) \sum_{i=1}^{N} |\chi_i(t)|^{\frac{\mu}{2q-\mu} + \frac{2q-\mu}{\mu}} \right) \\
&\quad + \frac{1}{2} \lambda_{\min}(Q) \sum_{i=1}^{N} \left(\hat{d}_i(t) - u_0 - \gamma \right) \\
&\quad \times \left(|\chi_i(t)|^{\frac{\mu}{2q-\mu}} + |\chi_i(t)|^{\frac{2q-\mu}{\mu}} \right) \\
&\quad + \frac{1}{2} \lambda_{\min}(Q) \sum_{i=1}^{N} \left| \hat{g}_i(t) - \hat{g}_0(t) \right| \\
&\quad \times \left(|\chi_i(t)|^{\frac{\mu}{2q-\mu}} + |\chi_i(t)|^{\frac{2q-\mu}{\mu}} \right) \\
&\quad - \frac{1}{2} \lambda_{\min}(Q) \sum_{i=1}^{N} |e_i(t)| \left(|\chi_i(t)|^{\frac{\mu}{2q-\mu}} + |\chi_i(t)|^{\frac{2q-\mu}{\mu}} \right) \\
&\quad - \frac{1}{2} \lambda_{\min}(Q) 2\ell \sum_{i=1}^{N} |\chi_i(t)| \left(|\chi_i(t)|^{\frac{\mu}{2q-\mu}} + |\chi_i(t)|^{\frac{2q-\mu}{\mu}} \right). \qquad (19)
\end{aligned}
$$

Note that

$$
\begin{aligned}
V(t) &= \sum_{i=1}^{N} \varepsilon_i \frac{2q-\mu}{2q} |\chi_i|^{\frac{2q}{2q-\mu}} + \sum_{i=1}^{N} \varepsilon_i \frac{\mu}{2q} |\chi_i|^{\frac{2q}{\mu}} \\
&\leq \frac{2q-\mu}{2q} \varepsilon_{\max} \left(\sum_{i=1}^{N} |\chi_i|^{\frac{2q}{2q-\mu}} + \sum_{i=1}^{N} |\chi_i|^{\frac{2q}{\mu}} \right). \qquad (20)
\end{aligned}
$$

Let

$$
V_1(t) = \sum_{i=1}^{N} |\chi_i|^{\frac{2q}{2q-\mu}} + \sum_{i=1}^{N} |\chi_i|^{\frac{2q}{\mu}}. \qquad (21)
$$

Invoking Lemmas 2 and 3, $\frac{2q}{2q} - \mu > 1$, $\frac{2(2q-\mu)}{\mu} > 1$, let $g_1(\chi) = V_1(t)^{\frac{\mu}{q}} + V_1(t)^{2q - \frac{\mu}{q}}$, we have

$$
\begin{aligned}
g_1(\chi) &= V_1(t)^{\frac{\mu}{q}} + V_1(t)^{\frac{2q-\mu}{q}} \\
&= \left(\sum_{i=1}^{N} |\chi_i|^{\frac{2q}{2q-\mu}} + \sum_{i=1}^{N} |\chi_i|^{\frac{2q}{\mu}} \right)^{\frac{\mu}{q}} \\
&\quad + \frac{1}{N^{\frac{\mu-q}{q}}} \left(\sum_{i=1}^{N} |\chi_i|^{\frac{2q}{2q-\mu}} + \sum_{i=1}^{N} |\chi_i|^{\frac{2q}{\mu}} \right)^{\frac{2q-\mu}{q}} \\
&\leq \left(\sum_{i=1}^{N} |\chi_i| \right)^{\frac{2\mu}{2q-\mu}} + \left(1 + 2^{\frac{q-\mu}{q}} \right) \left(\sum_{i=1}^{N} |\chi_i| \right)^2
\end{aligned}
$$

$$+2^{\frac{q-\mu}{q}}\left(\sum_{i=1}^{N}|\chi_i|\right)^{\frac{2(2q-\mu)}{\mu}}. \tag{22}$$

By the first term in (19), one has

$$g_2(\chi)=k_1\sum_{i=1}^{N}|\chi_i(t)|^{\frac{2\mu}{2q-\mu}}+(k_1+k_2)\sum_{i=1}^{N}|\chi_i(t)|^{\frac{\mu}{2q-\mu}+\frac{2q-\mu}{\mu}} +k_2\sum_{i=1}^{N}|\chi_i(t)|^{\frac{2(2q-\mu)}{\mu}}. \tag{23}$$

1) If $\frac{2\mu}{2q}-\mu>1$, $\frac{2(2q-\mu)}{q}>1$, then using Lemmas 2 and 3 yields

$$\begin{aligned} g_2(\chi)\geq{}& k_1N^{\frac{2q-3\mu}{2q-\mu}}\left(\sum_{i=1}^{N}|\chi_i(t)|\right)^{\frac{2\mu}{2q-\mu}} \\ &+k_2N^{3-\frac{4q}{\mu}}\left(\sum_{i=1}^{N}|\chi_i(t)|\right)^{\frac{2(2q-\mu)}{\mu}} \\ &+(k_1+k_2)N^{1-\frac{\mu}{2q-\mu}+\frac{2q-\mu}{\mu}} \\ &\times\left(\sum_{i=1}^{N}|\chi_i(t)|\right)^{\frac{\mu}{2q-\mu}+\frac{2q-\mu}{\mu}}. \end{aligned} \tag{24}$$

If $\sum_{i=1}^{N}|\chi_i(t)|\geq 1$, then

$$g_2(\chi)\geq k_2N^{\frac{3-4q}{\mu}}\left(\sum_{i=1}^{N}|\chi_i(t)|\right)^{\frac{2(2q-\mu)}{\mu}},$$

and the equation holds by the facts that $\frac{2\mu}{2q}-\mu<2<\frac{2(2q-\mu)}{\mu}$, then

$$g_1(\chi)\leq 2\left(1+2^{q-\mu}\right)\left(\sum_{i=1}^{N}|\chi_i(t)|\right)^{\frac{2(2q-\mu)}{\mu}}.$$

It implies that

$$\begin{aligned} g_2(\chi)&\geq k_2N^{3-\frac{4q}{\mu}}\left(\sum_{i=1}^{N}|\chi_i(t)|\right)^{\frac{2(2q-\mu)}{\mu}} \\ &\geq\frac{k_2N^{3-\frac{4q}{\mu}}}{2\left(1+2^{q-\mu}\right)}\left(V_1(t)^{\frac{\mu}{q}}+V_1(t)^{\frac{2q-\mu}{q}}\right). \end{aligned} \tag{25}$$

If $\sum_{i=1}^{N}|\chi_i(t)|\leq 1$, then

$$g_2(\chi)\geq k_1N^{\frac{2q-3\mu}{2q-\mu}}\left(\sum_{i=1}^{N}|\chi_i(t)|\right)^{\frac{2\mu}{2q-\mu}},$$

and

$$g_1(\chi)\leq 2\left(1+2^{q-\mu}\right)\left(\sum_{i=1}^{N}|\chi_i(t)|\right)^{\frac{2\mu}{2q-\mu}}.$$

It implies that

$$\begin{aligned} g_2(\chi)&\geq k_1N^{\frac{2q-3\mu}{2q-\mu}}\left(\sum_{i=1}^{N}|\chi_i(t)|\right)^{\frac{2\mu}{2q-\mu}} \\ &\geq\frac{k_1N^{\frac{2q-3\mu}{2q-\mu}}}{2\left(1+2^{q-\mu}\right)}\left(V_1(t)^{\frac{\mu}{q}}+V_1(t)^{\frac{2q-\mu}{q}}\right). \end{aligned} \tag{26}$$

Therefore, combing all these cases results in that

$$\begin{aligned} g_2(\chi)\geq{}&\frac{h}{2\left(1+2^{q-\mu}\right)}\left(V_1(t)^{\frac{\mu}{q}}+V_1(t)^{\frac{2q-\mu}{q}}\right) \\ \geq{}&\frac{h}{2\left(1+2^{q-\mu}\right)}\left(\left(\frac{2q}{(2q-\mu)\varepsilon_{\max}}V(t)\right)^{\frac{\mu}{q}}\right. \\ &\left.+\left(\frac{2q}{(2q-\mu)\varepsilon_{\max}}V(t)\right)^{\frac{2q-\mu}{q}}\right) \\ \geq{}&\frac{h}{2\left(1+2^{q-\mu}\right)}\left(\frac{2q}{(2q-\mu)\varepsilon_{\max}}\right)^{\frac{\mu}{q}}V(t)^{\frac{\mu}{q}} \\ &+\frac{h}{2\left(1+2^{q-\mu}\right)}\left(\frac{2q}{(2q-\mu)\varepsilon_{\max}}\right)^{\frac{2q-\mu}{q}}V(t)^{\frac{2q-\mu}{q}} \\ \geq{}& c_1V(t)^{\frac{\mu}{q}}+c_2V(t)^{\frac{2q-\mu}{q}}, \end{aligned} \tag{27}$$

where

$$\begin{aligned} c_1&=\frac{h}{2\left(1+2^{q-\mu}\right)}\left(\frac{2q}{(2q-\mu)\varepsilon_{\max}}\right)^{\frac{\mu}{q}} \\ c_2&=\frac{h}{2\left(1+2^{q-\mu}\right)}\left(\frac{2q}{(2q-\mu)\varepsilon_{\max}}\right)^{2q-\frac{\mu}{q}} \\ h&=\min\left\{k_2N^{3-\frac{4q}{\mu}},k_1N^{\frac{2q-3\mu}{2q-\mu}}\right\}. \end{aligned}$$

2) If $\frac{2\mu}{2q}-\mu<1$, $\frac{2(2q-\mu)}{q}>1$. Exploiting Lemma 2 and Lemma 3 similarly, consider $\sum_{i=1}^{N}|\chi_i(t)|\leq 1$ and $\sum_{i=1}^{N}|\chi_i(t)|\geq 1$ separately, we have

$$g_2(\chi)\geq c_1V(t)^{\frac{\mu}{q}}+c_2V(t)^{2q-\frac{\mu}{q}},$$

where

$$\begin{aligned} c_1&=\frac{h}{2\left(1+2^{q-\mu}\right)}\left(\frac{2q}{(2q-\mu)\varepsilon_{\max}}\right)^{\frac{\mu}{q}} \\ c_2&=\frac{h}{2\left(1+2^{q-\mu}\right)}\left(\frac{2q}{(2q-\mu)\varepsilon_{\max}}\right)^{2q-\frac{\mu}{q}} \\ h&=\min\left\{k_2N^{3-\frac{4q}{\mu}},k_1\right\}. \end{aligned}$$

From the inequalities above, it follows that

$$\begin{aligned} &\dot{V}(t) \\ &\leq-\frac{1}{2}\lambda_{\min}(Q)c_1V(t)^{\frac{\mu}{q}}-\frac{1}{2}\lambda_{\min}(Q)c_1V(t)^{\frac{2q-\mu}{q}} \\ &\quad+\frac{1}{2}\lambda_{\min}(Q)|e_i(t)|\sum_{i=1}^{N}\left|\left(|\chi_i(t)|^{\frac{\mu}{2q-\mu}}+|\chi_i(t)|^{\frac{2q-\mu}{\mu}}\right)\right| \end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{2}\lambda_{\min}(Q)\sum_{i=1}^{N}\left|\hat{g}_i(t)-\hat{g}_0(t)\right| \\
&\times\left(|\chi_i(t)|^{\frac{\mu}{2q-\mu}}+|\chi_i(t)|^{\frac{2q-\mu}{\mu}}\right) \\
&-\frac{1}{2}\lambda_{\min}(Q)*2\ell\sum_{i=1}^{N}\chi_i(t)\left(|\chi_i(t)|^{\frac{\mu}{2q-\mu}}+|\chi_i(t)|^{\frac{2q-\mu}{\mu}}\right) \\
\leq&-\frac{1}{2}\lambda_{\min}(Q)c_1V(t)^{\frac{\mu}{q}}-\frac{1}{2}\lambda_{\min}(Q)c_1V(t)^{\frac{2q-\mu}{q}} \\
&+\frac{1}{2}\lambda_{\min}(Q)\left|\frac{e_i(t)}{\chi_i(t)}\right|\left|\sum_{i=1}^{N}\left(|\chi_i(t)|^{\frac{2q}{2q-\mu}}+|\chi_i(t)|^{\frac{2q}{\mu}}\right)\right| \\
&+\frac{1}{2}\lambda_{\min}(Q)\rho\sum_{i=1}^{N}|\bar{x}_i(t)|\left(|\chi_i(t)|^{\frac{\mu}{2q-\mu}}+|\chi_i(t)|^{\frac{2q-\mu}{\mu}}\right) \\
&-\frac{1}{2}\lambda_{\min}(Q)\times2\ell\sum_{i=1}^{N}\chi_i(t)\left(|\chi_i(t)|^{\frac{\mu}{2q-\mu}}+|\chi_i(t)|^{\frac{2q-\mu}{\mu}}\right) \\
\leq&\frac{1}{2}\lambda_{\min}(Q)\left|\frac{e_i(t)}{\chi_i(t)}-\ell\right|\left|\sum_{i=1}^{N}\left(|\chi_i(t)|^{\frac{2q}{2q-\mu}}+|\chi_i(t)|^{\frac{2q}{\mu}}\right)\right| \\
&-\frac{1}{2}\lambda_{\min}(Q)c_1V(t)^{\frac{\mu}{q}}-\frac{1}{2}\lambda_{\min}(Q)c_1V(t)^{\frac{2q-\mu}{q}} \\
&+\frac{1}{2}\lambda_{\min}(Q)\frac{N\rho}{\lambda_2\left(\hat{L}\right)}\left|\sum_{i=1}^{N}\left(|\chi_i(t)|^{\frac{2q}{2q-\mu}}+|\chi_i(t)|^{\frac{2q}{\mu}}\right)\right| \\
&-\frac{u}{4}\lambda_{\min}(Q)\left(|\chi_i(t)|^{\frac{\mu}{2q-\mu}}+|\chi_i(t)|^{\frac{2q-\mu}{\mu}}\right) \\
\leq&\frac{1}{2}\lambda_{\min}(Q)\left(\frac{N\rho}{\lambda_2\left(\hat{L}\right)}-\ell\right)\left|\sum_{i=1}^{N}\left(|\chi_i(t)|^{\frac{2q}{2q-\mu}}+|\chi_i(t)|^{\frac{2q}{\mu}}\right)\right| \\
&-\frac{1}{2}\lambda_{\min}(Q)c_1V(t)^{\frac{\mu}{q}}-\frac{1}{2}\lambda_{\min}(Q)c_2V(t)^{\frac{2q-\mu}{q}}. \qquad (28)
\end{aligned}
$$

By (11), we have

$$
\dot{V}(t)\leq-\frac{1}{2}\lambda_{\min}(Q)c_1V(t)^{\frac{\mu}{q}}-\frac{1}{2}\lambda_{\min}(Q)c_2V(t)^{\frac{2q-\mu}{q}}. \qquad (29)
$$

By (29), there holds

$$
\begin{aligned}
\frac{dV}{dt}V(t)^{-\frac{\mu}{q}}&=-\frac{1}{2}\lambda_{\min}(Q)c_1-\frac{1}{2}\lambda_{\min}(Q)c_2V(t)^{\frac{2q-2\mu}{q}}, \\
\Rightarrow\frac{dV^{1-\frac{\mu}{q}}}{dt}&=-\left(1-\frac{\mu}{q}\right)(\frac{1}{2}\lambda_{\min}(Q)c_1 \\
&\quad+\frac{1}{2}\lambda_{\min}(Q)c_2V(t)^{2\left(1-\frac{\mu}{q}\right)}). \qquad (30)
\end{aligned}
$$

Let $Z=V^{1-\frac{\mu}{q}}$, (30) can be written as

$$
dZ=-\left(1-\frac{\mu}{q}\right)\left(\frac{1}{2}\lambda_{\min}(Q)c_1+\frac{1}{2}\lambda_{\min}(Q)c_2Z^2\right)dt. \qquad (31)
$$

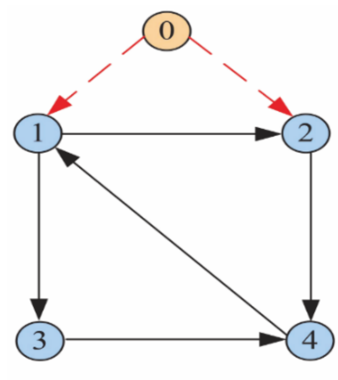

FIGURE 2. The communication graph.

Integrating both sides of (31) yields

$$
\begin{aligned}
&\frac{1}{\sqrt{\frac{1}{2}\lambda_{\min}(Q)c_1\frac{1}{2}\lambda_{\min}(Q)c_2}}\tan^{-1}\left(\sqrt{\frac{\frac{1}{2}\lambda_{\min}(Q)c_2}{\frac{1}{2}\lambda_{\min}(Q)c_1}}z(t)\right) \\
&=\frac{1}{\frac{1}{2}\lambda_{\min}(Q)\sqrt{c_1c_2}}\tan^{-1}\left(\sqrt{\frac{c_2}{c_1}}z(0)\right)-\frac{q-\mu}{q}t. \qquad (32)
\end{aligned}
$$

As a result, the convergence time bound is achieved as follows

$$
\begin{aligned}
t_0&\leq\lim_{z(0)\in\infty}\frac{q}{q-\mu}\frac{1}{\frac{1}{2}\lambda_{\min}(Q)\sqrt{c_1c_2}}\tan^{-1}\left(\sqrt{\frac{c_2}{c_1}}z(0)\right) \\
&=\frac{q}{q-\mu}\frac{\pi}{\lambda_{\min}(Q)\sqrt{c_1c_2}}. \qquad (33)
\end{aligned}
$$

Remark 2: The convergence time is independent of the initial conditions, but depends on the positive definite matrix Q given in Lemma 1 and the parameters in the controller (9). In practical applications, when the convergence time of the system is required, the gain of the controller can be adjusted according to equation (12). The convergence speed can also be adjusted by changing the parameters of the controller, such as $c1$, $c2$, μ, and q.

Theorem 2: On the basis of Assumptions 1-2, considering the multi-agent system (2)-(3) by using the control protocols (9) and the event-triggering function (8), and the parameters satisfy the condition (11), then the agent does not have Zeno behavior under any initial conditions.

Proof. Define $\psi_i(t)=\left|\frac{e_i(t)}{\chi_i(t)}\right|$, the derivative of $\psi_i(t)$ is

$$
\begin{aligned}
\dot{\psi}_i(t)=&\frac{|e_i(t)|'|\chi_i(t)|-|e_i(t)||\chi_i(t)|'}{|\chi_i(t)|^2} \\
\leq&(\psi_i(t)+k_1\frac{\mu}{2q-\mu}|\chi_i(t)|^{\frac{-2q}{2q-\mu}} \\
&+\frac{2q-\mu}{\mu}k_2|\chi_i(t)|^{\frac{2q-2\mu}{\mu}}+2l+\gamma)\|\hat{L}\|(\psi_i(t) \\
&+k_1\frac{\mu}{2q-\mu}|\chi_i(t)|^{\frac{-2q}{2q-\mu}}+\frac{2q-\mu}{\mu}k_2|\chi_i(t)|^{\frac{2q-2\mu}{\mu}} \\
&+2\ell+\gamma+\rho\hat{x}_i(t)+\hat{d}(t)|\chi_i(t)|^{-1}). \qquad (34)
\end{aligned}
$$

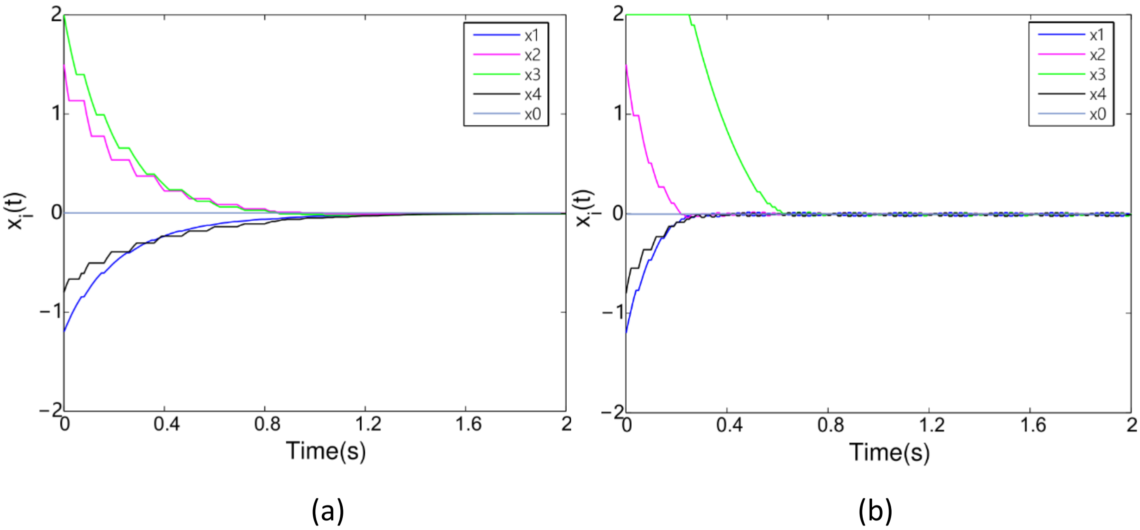

FIGURE 3. State trajectories.(a) Proposed event-triggered strategy (8). (b) Event-triggered strategy in [20].

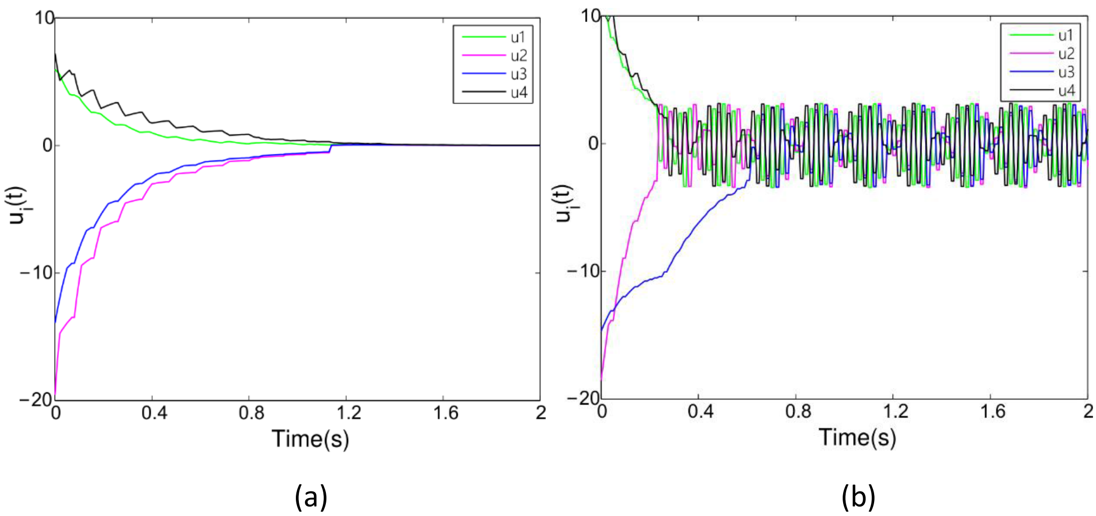

FIGURE 4. Inputs of the followers.(a) Proposed event-triggered strategy (8). (b) Event-triggered strategy in [20].

As $\sum_{i=1}^{N} \chi_i^2(t) = \left(\left(\hat{L}^{\frac{1}{2}}\hat{x}(t)\right)\hat{L}\left(\hat{L}^{\frac{1}{2}}\hat{x}(t)\right)\right) \leq 2\lambda_N\left(\hat{L}\right) V(t) \leq 2\lambda_N\left(\hat{L}\right) V(0)$, then one has

$$|\chi_i(t)| < \|\chi_i(t)\| \leq \sqrt{2\lambda_N\left(\hat{L}\right) V(0)},$$

where $\chi_i(t) \neq 0$, $\lambda_N\left(\hat{L}\right)$ is the largest eigenvalue of matrix $\hat{L}$. ξ_i is defined as

$$\xi_i = k_1\frac{\mu}{2q-\mu}|\chi_i(t)|^{\frac{-2q}{2q-\mu}} + k_2\frac{2q-\mu}{\mu}|\chi_i(t)|^{\frac{2q-2\mu}{\mu}} + \hat{d}(t)|\chi_i(t)|^{-1}.$$

As $\chi_i(t)$ is bounded, ξ_i exists a maximum value. Denote $\bar{\xi} = \max\{\xi_i\}$, then we have

$$\dot{\psi}_i(t) \leq \left(\psi_i(t) + \bar{\xi} + 2\ell + \gamma\right)\|\hat{L}\| \times \left(\psi_i(t) + \bar{\xi} + 2\ell + \gamma + N\rho/\|\hat{L}\|\right) \leq \|\hat{L}\|\left(\psi_i(t) + \bar{\xi} + 2\ell + \gamma + N\rho/\|\hat{L}\|\right)^2. \quad (35)$$

Therefore,

$$\psi_i(t) \leq \phi_i\left(t, \phi_0^i\right). \quad (36)$$

where $\phi_i\left(t, \phi_0^i\right)$ is the solution of the differential equation $\dot{\psi}_i = \beta(\sigma + \psi_i)^2$, $\psi_i\left(0, \psi_0^i\right) = \psi_0^i$, $\beta = \|\hat{L}\|$, $\sigma = \bar{\xi} + 2\ell + \gamma + \frac{N\rho}{\|\hat{L}\|}$.

The solution of the above inequality is $\phi_i(\tau_i, 0) = \frac{\tau_i\sigma^2\beta}{1-\tau_i\sigma\beta}$. Based on the (9), we have $\phi_i(\tau_i, 0) = \ell$.

The minimum time interval for event-triggering can be obtained:

$$\tau_i = \frac{\ell}{\sigma^2\beta + \ell\sigma\beta}. \quad (37)$$

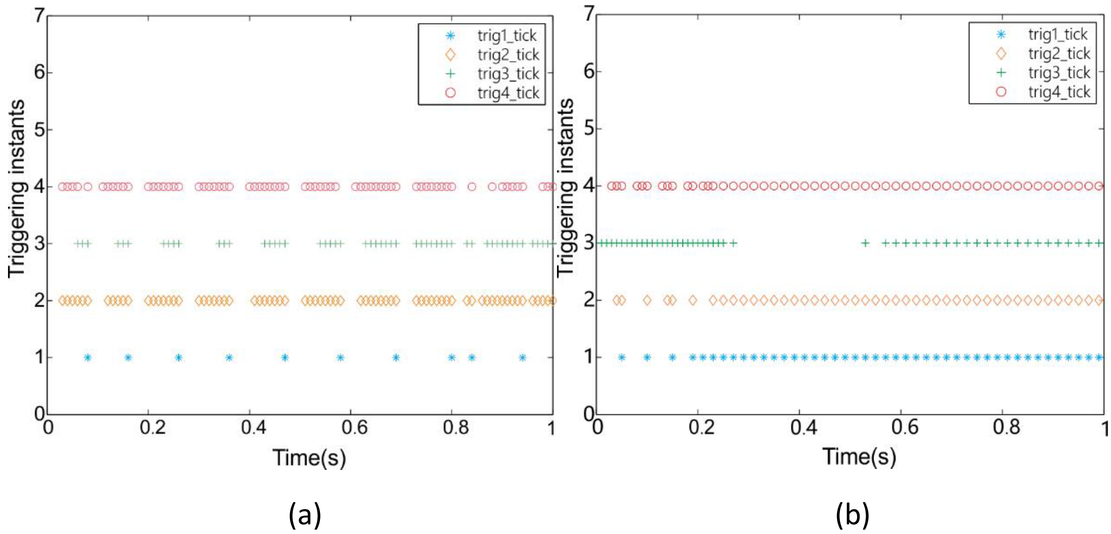

FIGURE 5. Triggering instants of the followers.(a) Proposed event-triggered strategy (8).(b) Event-triggered strategy in [20].

Obviously, τ_i is strictly greater than 0. Then there is no Zeno behavior. Thus, this completes the proof.

Remark 3: According to Theorem 1 and Theorem 2, the problem of distributed event-triggered control design for multi-agent system can be solved in a bound of settling time T.

IV. SIMULATION RESULTS

In this section, we study a simple numerical example to verify the effectiveness of the control algorithm proposed in this paper. Consider a network with four followers and one virtual leader, the communication graph is shown in Figure 2. The relevant parameters are set as follows $g(x_i(t), t) = 0.2x_i(t)$, $g(x_0(t), t) = 0.2x_0(t)$, $d(x_i(t), t) = 0.3\cos(t)$, $k_1 = 0.7$, $k_2 = 1.4$, $\gamma = 0.2$, $\ell = 0.7$, $\mu = 5$, $q = 7$. The simulation is conducted by assuming that the initial states of the followers are $x(0) = [-1.2 1.5 2 - 0.8]^T$ and the initial state of the leader is $x_0(0) = 0$. From Figure 2, $\hat{L}$ is given by

$$\hat{L} = \begin{bmatrix} 2 & 0 & 0 & -1 \\ -1 & 2 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & -1 & 0 & 2 \end{bmatrix}.$$

And there holds $\lambda_2\left(\hat{L}\right) = 0.6753$. Taking the parameters into (12), it can be easily calculated that T_{max} is $= 3.14s$.

The trajectories are shown in Figure 3. (a) and Figure 4. (a). It can be seen that the control input is a piecewise constant value. In addition, when the system error is small, the control input is also small, and when the control input tends to zero, the system reaches an equilibrium state. Obviously, the settling time is about $T = 1.2s$, which is smaller than T_{max}. Figure 5. (a) shows the triggered interval of each agent under control scheme (9). Figure 6 illustrates the error in observation of the four followers under the controllers (9).

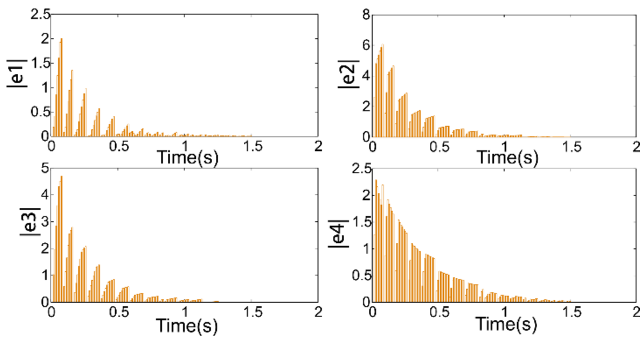

FIGURE 6. The error in observation of the four followers under the controllers (8).

The event-triggered results for agent 1 and agent 3 are shown in Figure 7. It is apparent that the update time of the controller is all less than the sampling times. For comparison, the simulation results with the event-triggered strategy in [20] are also shown in Figure 3, Figure 4 and Figure 5. We can see that the states of the agents versus time and the inputs of the followers versus time are not smooth, but jitter.

Compared with the algorithm in [20], which also studied the fixed-time consensus for nonlinear multi-agent system with event-triggered scheme. It can be observed that the inputs of the followers chattered. Obviously, in this paper, there is no jitter at the inputs of the followers with the proposed control protocol (9). For nonlinear multi-agent systems with disturbances, [2] studied formation control of multi-agent systems with minimum energy constraints. Reference [29] provided a formation control algorithm for the gain matrix of the control protocol by compensating for the disturbance term. However, unlike this, this article focuses on saving communication bandwidth and computing resources, proposing a new event triggering mechanism to determine

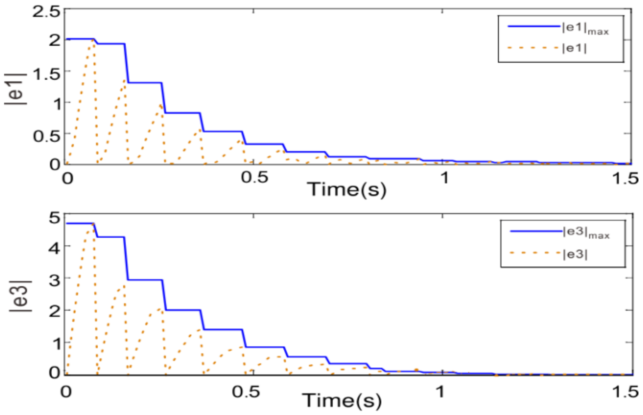

FIGURE 7. Comparison chart of the threshold value and measurement error value of agent 1 and agent 3.

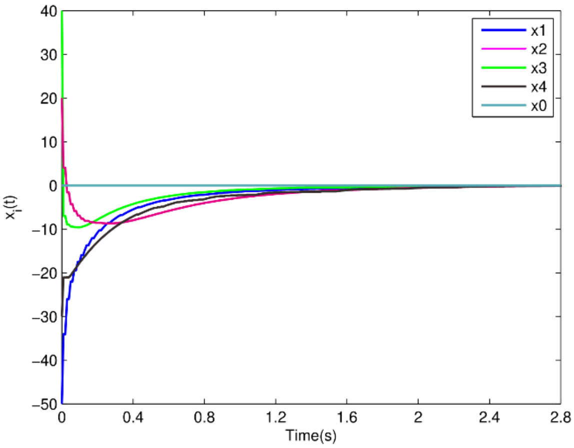

FIGURE 8. The state trajectories of each intelligent agent under another set of initial values.

when control signals should be updated to improve efficiency, enabling the entire system to complete tasks more efficiently.

Finally, to demonstrate that the convergence time no longer depends on the initial state of the system, here selects a larger initial state of the system, $x(0) = [-50\ 40\ 20\ -30]$. Figure 8 shows that the convergence time of the system is not significantly affected by the different initial states.

V. CONCLUSION

A distributed fixed-time event-triggered algorithm is proposed in this article for a class of leader-following multi-agent system with inherent nonlinear dynamics and uncertain disturbances. An event-triggered distributed strategy is proposed, which can substantially reduce energy consumption and the update frequency of the controller. The fixed-time consensus strategy can ensure that the setting time is bounded under any initial condition. Besides, the proposed algorithm can effectively restrain the jittering effect in the control input. Finally, a simulation example is provided to verify the effectiveness of the results.

CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

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