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RESEARCH ARTICLE

Decision Algorithm for q-Rung Orthopair Fuzzy Information Based on Schweizer-Sklar Aggregation Operators With Applications in Agricultural Systems

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ABSTRACT The multi-attribute decision-making (MADM) technique is a dominant process for resolving genuine real-life applications and investigating an ideal solution by considering appropriate criteria or attributes. The operational laws of Schweizer-Sklar t-norms and t-conorms are more feasible aggregation operators to serve this purpose. The prioritized aggregation operators also capture single-term aggregated information from given evidence or collected data. In this article, we explore the theory of the q-rung orthopair fuzzy (q-ROF) information to handle awkward and uncertain information of human opinion. Motivated by the significance of the Schweizer-Sklar t-norms and prioritized aggregation operators, we derive a family of mathematical approaches for q-rung orthopair fuzzy information, including q-rung orthopair fuzzy Schweizer-Sklar prioritized average (q-ROFSSPA), q-rung orthopair fuzzy Schweizer-Sklar prioritized weighted average (q-ROFSSPWA), q-rung orthopair fuzzy Schweizer-Sklar prioritized geometric (q-ROFSSPG) and q-rung orthopair fuzzy Schweizer-Sklar prioritized weighted geometric (q-ROFSSPWG) operators. Some notable properties and characteristics are also explored to show the applicability of developed approaches. An application for improving the economic growth of the agriculture sector and a decision algorithm is also discussed under the q-rung orthopair fuzzy environment. With the help of invented mathematical approaches, we resolved a numerical example to choose a suitable crop under reliable characteristics or attributes. To show the reliability and applicability of initiated methodologies, we demonstrate a comparison technique to contrast the results of pioneered aggregation operators with prevailing strategies in the literature.

INDEX TERMS q-rung orthopair fuzzy value, Schweizer-Sklar t-norms, improvement in agriculture and multi-attribute decision-making process.

I. INTRODUCTION

In a variety of areas of life, comparison is required to address several challenges, such as machine learning, decision-making and multi-attribute decision-making (MADM). For decision science, which aims to extract the optimal option from a set of comparable options, MADM is crucial. To achieve the evaluation's goal, MADM first had to assess the other possibilities using a variety of different criteria, such as single, span, and similar ones. However, it is often

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the endeavor to lead for MADM in a novel way in a variety of additional circumstances. There are several approaches to address the issues mentioned above; however, excellent operators are identified when the data are fuzzy. Since the subject under discussion is a generalized production of intuitionistic fuzzy sets (IFSs) and fuzzy sets (FSs), it primarily relates to picture fuzzy (PF) operators. Consequently, it is fitting to highlight the pioneers and current research. Consequently, in terms of the creation and uses of FS and IFS, it is important to discuss the pioneers and contemporary research. No shortage of imprecise, confusing, and unreliable facts exists in real life. In order to deal with this kind of scenario, Zadeh [1] developed FS, in which a membership grade (MG), represented by the letter ($\overline{\overline{u}}$, is given to each element of uncertainty. In FSs, only the MG is recorded, and a one-minus grade is recorded as a non-membership grade (NMG). Therefore, by considering the MG, it is certain to find the NMG. However, one is uncertain about the NMG in real life because of the understanding of MG. It is advised that there be a free NMG function in these situations. In order to address the circumstances, Atanassov Atanassov [2] created the concept of IFS, whereby every element is designated with MG and NMG, indicated as as ($\overline{\overline{v}}$, under the constraint $0 \leq \overline{\overline{u}}+\overline{\overline{v}} \leq 1$. Yager [3] presented a robust theory of the pythagorean fuzzy set (PyFS) by exploring the MG and NMG with the characteristics of the sum of MG and NMG. The mathematical shape of PyFS is described as follows: $0 \leq \overline{\overline{u}}^2+\overline{\overline{v}}^2 \leq 1$. Sometime, the sum of MG and NMG bigger than one and decision maker unable to process given information with PyF environments. To handle such circumstances, Yager [4] developed a new theory of q-rung orthopair fuzzy set (q-ROFS) by generalizing the characteristics of pythagorean fuzzy theory. The mathematical structure of q-ROFS is characterized as $0 \leq \overline{\overline{u}}^{\eta}+\overline{\overline{v}}^{\eta} \leq 1,\ \eta \in \mathbb{Z}^{+}$. Motivated by aforementioned theories, several mathematicians utilized above theories to resolve different crucial applications related to the real life [5], [6], [7].

An important component of MADM, AOs aggregate the results of many criteria into a single score. Alternatives performance is summed up using AOs across many parameters. They create a single score that reflects the alternative's overall performance by adding the individual scores for each criterion. Because they allow decision-makers to assess options using a single performance metric, AOs are crucial to MADM. It would be hard to evaluate alternatives that do well on certain criteria but badly on others without aggregation operators. Aggregation operators allow decision-makers to balance the significance of many criteria, which is one of their advantages. Decision-makers give each criterion in MADM a weight according to its relative value. AOs use these weights to aggregate the results of every criterion into a single score. Decision makers may ensure that their priorities and preferences are reflected in the final choice by giving each criterion weight. First of all, Xu [8] developed important operations and methods for intuitionistic fuzzy information. By using the characteristics of algebraic t-norm and theoretical concepts of geometric operators, Xu and Chen [9] initiated AOs to resolve the decision-making process under the system of interval-valued IFSs. Some attractive operational laws of Aczel Alsina aggregation tools with intuitionistic fuzzy rough theory presented by Ahmmad et al. [10]. Garg [11] utilized operations of the exponential algorithm by incorporating q-ROF information to overcome the drawbacks of existing approaches. Hussain et al. [12] discussed an innovative algorithm for the decision-making process and also resolved an application of vendor management enterprises. Hussain et al. [13] demonstrated Aczel Alsina operations for choosing the best electric cars with the system of complex spherical fuzzy situations. The robust mathematical strategies of Aczel Alsina t-norms with Hamy mean models were developed by Hussain et al. [14]. Characteristics of Dombi t-norms with PyF information applied to graph theory by Akram and Shahzadi [15]. Ali et al. [16] resolved an application of supply chain enterprises based on Einstein aggregation tools. In order to reduce the influence of existing mathematical approaches, Dey et al. [17] developed new AOs for solving an application of medical diagnosis. Ali and Mahmood [18] introduced an innovative mathematical strategy based on Maclaurin symmetric mean operators with complex q-ROF information. Akram et al. [19] established aggregation approaches for complex pythagorean fuzzy information with prioritized operators. Garg [20] explored an advanced decision-making process for interval-valued q-ROF information. Dong et al. [21] adopted realistic operations of Hamacher t-norms and t-conorms to derive mathematical approaches based on complex intuitionistic fuzzy information. Akram and Bilal [22] deduced a series of new AOs to obtain an analytical solution with bipolar fuzzy theory and decision-making technique. Motivated by the significance of Dombi aggregation tools, Mahmood and Rehman [23] resolved complicated real-life applications using the decision-making process. Garg and Chen [24] initiated new mathematical approaches by using basic operations of neutrality aggregation tools for q-ROF information. In order to explore closeness among different arguments, Jiang [25] developed AOs for the decision-making process under the q-ROF framework. Alcantud [26] also generalized algebraic t-norm for geometric aggregation tools with intuitionistic fuzzy situations. Hussain et al. [27] applied some reliable operations of Dombi aggregation operators to resolve complicated real-life challenges. We also studied several mathematical approaches under different fuzzy domains developed by different research scholars [28], [29], [30].

Schweizer and Sklar [31] generalized concepts of triangular norms to develop some flexible operations known as Schweizer and Sklar t-norms by involving a parameter $Ћ$, whose range from $[-\infty, 0]$. By changing the parametric values of Schweizer and Sklar t-norms, we can easily deduce the Hamacher t-norm and nilpotent t-norm. Schweizer and Sklar [32] also explored the characteristics of developed aggregation tools for resolving crucial real-life challenges. With time, the theory of Schweizer-Sklar t-norm and t-conorm became more flexible and attracted a lot of attention from numerous research scholars. For instance, Chen et al. [33] developed some dominant strategies with Schweizer-Sklar t-norm and properties of Hamy mean models. Garg et al. [34] utilized the theory of prioritized operators for intuitionistic fuzzy domains and decision-making processes. Based on the developed operation of Schweizer-Sklar t-norms, Khan et al. [35] introduced new results for resolving real-life applications under the system of the decision-making process. To show the robustness of Schweizer-Sklar aggregation tools, Wang and Liu [36] presented AOs of maclaurin symmetric mean operators, and Khan et al. [37] also generalized some flexible mathematical strategies under considering intuitionistic fuzzy information.

It is clear that a variety of advantages of discussed research work and mathematical aggregation operators. However, many experts may face several challenges during the aggregation process of uncertain information and decision analysis. To handle such situations, Schweizer-Sklar t-norms and t-conorms have remarkable capabilities to achieve smooth and accurate aggregated information. To overcome the impact of limitations and drawbacks of existing strategies, we invented an innovative research work of q-rung orthopair fuzzy information with Schweizer-Sklar t-norms and t-conorms. The key features of this research work are initiated as follows: we explore the notion of q-rung orthopair fuzzy theory and its reliable features. Some operational laws of Schweizer-Sklar t-norms and t-conorms are also adopted to aggregate ambiguous information about human opinion. We derived a family of mathematical approaches of q-rung orthopair fuzzy theory, namely q-ROFSSPA, q-ROFSSPWA, q-ROFSSPG, and q-ROFSSPWG operators. It can identify the best option, extract it from seemingly ambiguous events, and generate a ranking without the need for weight information. Additionally, a decision algorithm for q-rung orthopair fuzzy information is also expressed. A numerical example is also resolved to evaluate a suitable optimal option based on derived mathematical approaches. Finally, a comprehensive comparative study is proposed to contrast the aggregated outcomes of pioneered approaches with existing aggregation operators.

The structure of the proposed research work is maintained as follows: section II explores some basic notions and fundamental rules necessary for improving this research work. We derived new AOs in the light of Schweizer-Sklar t-norms, namely q-ROFSSPA and q-ROFSSPWA operators, with notable characteristics in section III. Section IV also carried out some robust mathematical approaches like q-ROFSSPG and q-ROFSSPWG operators. Section V established an innovative approach to the MADM technique in the light of q-ROF information. With the help of numerical examples, we show the flexibility and effectiveness of derived strategies. The advantages and validity of proposed mathematical methods are verified by the contrasting results of previous procedures with currently developed AOs in section VI. Additionally, final remarks about our proposed research work are presented in section VII.

II. PRELIMINARIES

The main of this section is to present fundamental notions of IFSs and q-ROFSs with their dominant operations. These preliminaries are useful and essential for the improvement of this research work.

Definition 1: [2] Let $\mathcal{T}$ be a universal set and an IFS $\overline{\overline{\mathfrak{D}}}$ is expressed as follows:

$$\overline{\overline{\mathfrak{D}}} = \{\gimel,\ \left(\overline{\overline{u}}\left(\gimel\right), \overline{\overline{v}}\left(\gimel\right)\right) | \gimel \in \mathcal{T}\}$$

Here, $\overline{\overline{u}}\left(\gimel\right) \in [0, 1]$ and $\overline{\overline{v}}\left(\gimel\right) \in [0, 1]$ represent the PV and NV respectively with subject to the $0 \leq \overline{\overline{u}}\left(\gimel\right) + \overline{\overline{v}}\left(\gimel\right) \leq 1$. Moreover, the hesitancy value of $\overline{\overline{\mathfrak{D}}}$ is defined as $r = (1 - (\overline{\overline{u}}\left(\gimel\right) + \overline{\overline{v}}\left(\gimel\right)))$.

Definition 2 ([4]): Let $\mathcal{T}$ be a universal set and a q-ROFS $\overline{\overline{\mathfrak{D}}}$ is given by:

$$\overline{\overline{\mathfrak{D}}} = \{\gimel,\ \left(\overline{\overline{u}}\left(\gimel\right), \overline{\overline{v}}\left(\gimel\right)\right) | \gimel \in \mathcal{T}\}$$

Here, $\overline{\overline{u}}\left(\gimel\right) \in [0, 1]$ and $\overline{\overline{v}}\left(\gimel\right) \in [0, 1]$ represent the PV and NV respectively with subject to the $0 \leq \overline{\overline{u}}^{\eta}\left(\gimel\right) + \overline{\overline{v}}^{\eta}\left(\gimel\right) \leq 1,\ \eta \in \mathbb{Z}^{+}$. The hesitancy value of $\overline{\overline{\mathfrak{D}}}$ is defined as $r = \left(\left(1 - \left(\overline{\overline{u}^{\eta}}\left(\gimel\right) + \overline{\overline{v}^{\eta}}\left(\gimel\right)\right)\right)\right)^{\frac{1}{\eta}}$. A pair $\overline{\overline{\Re}} = \left(\overline{\overline{u}}\left(\gimel\right), \overline{\overline{v}}\left(\gimel\right)\right)$ Indicates the q-rung orthopair fuzzy value (q-ROFV).

Definition 3 ([38]): Let $\overline{\overline{\Re}}_{\iota} = \left(\overline{\overline{u}}_{\iota}\left(\gimel\right), \overline{\overline{v}}_{\iota}\left(\gimel\right)\right), \iota = 1,\ 2$ be any two q-ROFVs. Then:

a) $\overline{\overline{\Re}}_1 \oplus \overline{\overline{\Re}}_2 = \left(\sqrt[\eta]{\overline{\overline{u}}_1^{\eta} + \overline{\overline{u}}_2^{\eta} - \overline{\overline{u}}_1^{\eta}.\overline{\overline{u}}_2^{\eta}}, \overline{\overline{v}}_1.\overline{\overline{v}}_2\right)$

b) $\overline{\overline{\Re}}_1 \otimes \overline{\overline{\Re}}_2 = \left(\overline{\overline{u}}_1.\overline{\overline{u}}_2, \sqrt[\eta]{\overline{\overline{v}}_1^{\eta} + \overline{\overline{v}}_2^{\eta} - \overline{\overline{v}}_1^{\eta}.\overline{\overline{v}}_2^{\eta}}\right)$

c) $\Psi\, \overline{\overline{\Re}} = \left(\sqrt[\eta]{1 - \left(1 - \overline{\overline{u}}^{\eta}\right)^{\Psi}}\, \overline{\overline{v}}^{\Psi}\right),\ \Psi > 0$

d) $\overline{\overline{\Re}}^{\Psi} = \left(\overline{\overline{u}}^{\Psi}, \sqrt[\eta]{1 - \left(1 - \overline{\overline{v}}^{\eta}\right)^{\Psi}}\right),\ \Psi > 0$

Now, we study the notion of Schweizer-Sklar t-norm and t-conorm given by Schweizer and Sklar [31] in 1960.

Definition 4 ([31]): The theory of Schweizer-Sklar t-norms is expressed as follows:

$$U\left(\alpha,\ \beta\right) = \left(\alpha^{\hbar} + \beta^{\hbar} - 1\right)^{\frac{1}{\hbar}}$$

$$T\left(\alpha,\ \beta\right) = 1 - \left(\left(1 - \alpha\right)^{\hbar} + \left(1 - \beta\right)^{\hbar} - 1\right)^{\frac{1}{\hbar}}$$

where $\alpha,\ \beta \in [0,\ 1]$ and $\hbar < 0$.

Definition 5 ([33]): Let $\overline{\overline{\Re}}_{\iota} = \left(\overline{\overline{u}}_{\iota}\left(\gimel\right), \overline{\overline{v}}_{\iota}\left(\gimel\right)\right), \iota = 1, 2$ be any two q-ROFVs with $\Psi > 0$ and $\hbar < 0$. Then, some basic operations of Schweizer-Sklar t-norms are characterized as follows:

a)

$$\overline{\overline{\mathfrak{R}}}_1 \oplus \overline{\overline{\mathfrak{R}}}_2 = \left(\sqrt[\eta]{1 - \left(\left(1 - \overline{\overline{u}}_1^{\eta}\right)^{\text{Ђ}} + \left(1 - \overline{\overline{u}}_2^{\eta}\right)^{\text{Ђ}} - 1 \right)^{\frac{1}{\text{Ђ}}}}, \sqrt[\eta]{\left(\left(\overline{\overline{\upsilon}}_1^{\eta}\right)^{\text{Ђ}} + \left(\overline{\overline{\upsilon}}_2^{\eta}\right)^{\text{Ђ}} - 1 \right)^{\frac{1}{\text{Ђ}}}} \right)$$

b)

$$\overline{\overline{\mathfrak{R}}}_1 \otimes \overline{\overline{\mathfrak{R}}}_2 = \left(\sqrt[\eta]{\left(\left(\overline{\overline{u}}_1^{\eta}\right)^{\text{Ђ}} + \left(\overline{\overline{u}}_2^{\eta}\right)^{\text{Ђ}} - 1 \right)^{\frac{1}{\text{Ђ}}}}, \sqrt[\eta]{1 - \left(\left(1 - \overline{\overline{\upsilon}}_1^{\eta}\right)^{\text{Ђ}} + \left(1 - \overline{\overline{\upsilon}}_2^{\eta}\right)^{\text{Ђ}} - 1 \right)^{\frac{1}{\text{Ђ}}}} \right)$$

c)

$$\Psi \overline{\overline{\mathfrak{R}}} = \left(\sqrt[\eta]{1 - \left(\Psi \left(1 - \overline{\overline{u}}^{\eta}\right)^{\text{Ђ}} - (\Psi - 1) \right)^{\frac{1}{\text{Ђ}}}}, \sqrt[\eta]{\left(\Psi \left(\overline{\overline{\upsilon}}^{\eta}\right)^{\text{Ђ}} - (\Psi - 1) \right)^{\frac{1}{\text{Ђ}}}} \right), \Psi > 0$$

d)

$$\overline{\overline{\mathfrak{R}}}^{\Psi} = \left(\sqrt[\eta]{\left(\Psi \left(\overline{\overline{u}}^{\eta}\right)^{\text{Ђ}} - (\Psi - 1) \right)^{\frac{1}{\text{Ђ}}}}, \sqrt[\eta]{1 - \left(\Psi \left(1 - \overline{\overline{\upsilon}}^{\eta}\right)^{\text{Ђ}} - (\Psi - 1) \right)^{\frac{1}{\text{Ђ}}}} \right), \Psi > 0$$

Definition 6: For any q-ROFV $\overline{\overline{\mathfrak{R}}} = \left(\overline{\overline{u}}(\gimel), \overline{\overline{\upsilon}}(\gimel)\right)$. The score function $\aleph(\mathfrak{R})$ and accuracy function $H\left(\overline{\overline{\mathfrak{R}}}\right)$ are given as follows:

$$\aleph\left(\overline{\overline{\mathfrak{R}}}\right) = \frac{1 + \overline{\overline{u}}^{\eta}(\gimel) - \overline{\overline{\upsilon}}^{\eta}(\gimel)}{2}, \aleph\left(\overline{\overline{\mathfrak{R}}}\right) \in [0, 1]$$

$$H\left(\overline{\overline{\mathfrak{R}}}\right) = \frac{\overline{\overline{u}}^{\eta}(\gimel) + \overline{\overline{\upsilon}}^{\eta}(\gimel)}{2}, H\left(\overline{\overline{\mathfrak{R}}}\right) \in [0, 1]$$

Consider $\overline{\overline{\mathfrak{R}}}_1 = \left(\overline{\overline{u_1}}(\gimel), \overline{\overline{\upsilon_1}}(\gimel)\right)$ and $\overline{\overline{\mathfrak{R}}}_2 = \left(\overline{\overline{u_2}}(\gimel), \overline{\overline{\upsilon_2}}(\gimel)\right)$ are two q-ROFVs. Then $\overline{\overline{\mathfrak{R}}}_1$ is preferable over $\overline{\overline{\mathfrak{R}}}_2$ if $\aleph\left(\overline{\overline{\mathfrak{R}}}_1\right) > \aleph\left(\overline{\overline{\mathfrak{R}}}_2\right)$ and $\overline{\overline{\mathfrak{R}}}_2$ is preferable over $\overline{\overline{\mathfrak{R}}}_1$ if $\aleph\left(\overline{\overline{\mathfrak{R}}}_2\right) > \aleph\left(\overline{\overline{\mathfrak{R}}}_1\right)$. When $\aleph\left(\overline{\overline{\mathfrak{R}}}_1\right) = \aleph\left(\overline{\overline{\mathfrak{R}}}_2\right)$ Then, we move to the accuracy function for the comparison technique. If $\overline{\overline{\mathfrak{R}}}_1 > \overline{\overline{\mathfrak{R}}}_2$ *then* $H\left(\overline{\overline{\mathfrak{R}}}_1\right) > H\left(\overline{\overline{\mathfrak{R}}}_2\right)$.

Definition 7 ([39]): If $\overline{\overline{\mathfrak{R}}}_\iota$, $\iota = 1, 2, \ldots, n$ be a set of positive integers. Then, the Prioritized average (PA) operator is expressed as follows:

$$PA\left(\overline{\overline{\mathfrak{R}}}_1, \overline{\overline{\mathfrak{R}}}_2, \ldots, \overline{\overline{\mathfrak{R}}}_n\right) = \mathop{\oplus}_{\iota=1}^{n} \omega_\iota \overline{\overline{\mathfrak{R}}}_\iota = \omega_1 \overline{\overline{\mathfrak{R}}}_1 \oplus \omega_2 \overline{\overline{\mathfrak{R}}}_2 \oplus, \ldots, \oplus \omega_2 \mathfrak{R}_n$$

Note that $\omega_\iota = \frac{\mathfrak{E}_\iota}{\sum_{\iota=1}^{n} \mathfrak{E}_\iota}$, $\mathfrak{E}_1 = 1$ and $\mathfrak{E}_\iota = \otimes_{\iota=1}^{k-1} \aleph(\mathfrak{R}_\iota)$, $k = 2, 3, \ldots, n$.

III. q-RUNG ORTHOPAIR FUZZY SCHWEIZER-SKLAR AGGREGATION OPERATORS BASED ON q-ROF INFORMATION

Motivated by the robustness of Schweizer-Sklar t-norm and their operations, we developed some decent mathematical approaches, such as q-ROFSSPA and q-ROFSSPWA operators with notable characteristics

Definition 8: For a set of q-ROFVs $\mathfrak{R}_\iota = \left(\overline{\overline{u}}_\iota(\gimel), \overline{\overline{\upsilon}}_\iota(\gimel)\right)$, $\iota = 1, 2, \ldots, n$. Then, the q-ROFSSPA operator is expressed as follows:

$$q - ROFSSPA\left(\overline{\overline{\mathfrak{R}}}_1, \overline{\overline{\mathfrak{R}}}_2, \ldots, \overline{\overline{\mathfrak{R}}}_n\right) = \omega_1 \overline{\overline{\mathfrak{R}}}_1 \oplus \omega_2 \overline{\overline{\mathfrak{R}}}_2 \oplus, \ldots, \oplus \omega_2 \overline{\overline{\mathfrak{R}}}_n = \mathop{\oplus}_{\iota=1}^{n} \omega_\iota \overline{\overline{\mathfrak{R}}}_\iota$$

Note that $\omega_\iota = \frac{\mathfrak{E}_\iota}{\sum_{\iota=1}^{n} \mathfrak{E}_\iota}$, $\mathfrak{E}_1 = 1$ and $\mathfrak{E}_\iota = \otimes_{\iota=1}^{k-1} \aleph\left(\overline{\overline{\mathfrak{R}}}_\iota\right)$, $k = 2, 3, \ldots, n$.

Theorem 1: For any set of q-ROFVs $\overline{\overline{\mathfrak{R}}}_\iota = \left(\overline{\overline{u}}_\iota(\gimel), \overline{\overline{\upsilon}}_\iota(\gimel)\right)$, $\iota = 1, 2, \ldots, n$. the integrated values of the q-ROFSSPA operator is still a q-ROFV are given by:

$$q - ROFSSPA\left(\overline{\overline{\mathfrak{R}}}_1, \overline{\overline{\mathfrak{R}}}_2, \ldots, \overline{\overline{\mathfrak{R}}}_n\right) = \left(\sqrt[\eta]{1 - \left(\sum_{\iota=1}^{n} \omega_\iota \left(1 - \overline{\overline{u}}_\iota^{\eta}\right)^{\text{Ђ}} - \sum_{\iota=1}^{n} \omega_\iota + 1 \right)^{\frac{1}{\text{Ђ}}}}, \sqrt[\eta]{\left(\sum_{\iota=1}^{n} \omega_\iota \left(\overline{\overline{\upsilon}}_\iota^{\eta}\right)^{\text{Ђ}} - \sum_{\iota=1}^{n} \omega_\iota + 1 \right)^{\frac{1}{\text{Ђ}}}} \right)$$

Note that $\omega_\iota = \frac{\mathfrak{E}_\iota}{\sum_{\iota=1}^{n} \mathfrak{E}_\iota}$, $\mathfrak{E}_1 = 1$ and $\mathfrak{E}_\iota = \otimes_{\iota=1}^{k-1} \aleph\left(\overline{\overline{\mathfrak{R}}}_\iota\right)$, $k = 2, 3, \ldots, n$.

Proof: Since for any set of q-ROFVs $\overline{\overline{\mathfrak{R}}}_\iota = \left(\overline{\overline{u}}_\iota(\gimel), \overline{\overline{\upsilon}}_\iota(\gimel)\right)$, $\iota = 1, 2, \ldots, n$. We can prove the above expression by using basic operations of Schweizer-Sklar t-norms as follows:

$$\omega_1\overline{\overline{\Re}}_1 = \left(\sqrt[\eta]{1-\left(\omega_1\left(1-\overline{\overline{u}}_1^{\eta}\right)^{Ћ}-\left(\omega_1-1\right)\right)^{\frac{1}{Ћ}}},\ \sqrt[\eta]{\left(\omega_1\left(\overline{\overline{v}}_1^{\eta}\right)^{Ћ}-\left(\omega_1-1\right)\right)^{\frac{1}{Ћ}}}\right)$$

$$\omega_2\overline{\overline{\Re}}_2 = \left(\sqrt[\eta]{1-\left(\omega_2\left(1-\overline{\overline{u}}_2^{\eta}\right)^{Ћ}-\left(\omega_2-1\right)\right)^{\frac{1}{Ћ}}},\ \sqrt[\eta]{\left(\omega_2\left(\overline{\overline{v}}_2^{\eta}\right)^{Ћ}-\left(\omega_2-1\right)\right)^{\frac{1}{Ћ}}}\right)$$

$$\begin{aligned}
&\omega_1\overline{\overline{\Re}}_1\oplus\omega_2\overline{\overline{\Re}}_1 \\
&= \left(\sqrt[\eta]{1-\left(\omega_1\left(1-\overline{\overline{u}}_1^{\eta}\right)^{Ћ}-\left(\omega_1-1\right)\right)^{\frac{1}{Ћ}}},\ \sqrt[\eta]{\left(\omega_1\left(\overline{\overline{v}}_1^{\eta}\right)^{Ћ}-\left(\omega_1-1\right)\right)^{\frac{1}{Ћ}}}\right) \oplus \left(\sqrt[\eta]{1-\left(\omega_2\left(1-\overline{\overline{u}}_2^{\eta}\right)^{Ћ}-\left(\omega_2-1\right)\right)^{\frac{1}{Ћ}}},\ \sqrt[\eta]{\left(\omega_2\left(\overline{\overline{v}}_2^{\eta}\right)^{Ћ}-\left(\omega_2-1\right)\right)^{\frac{1}{Ћ}}}\right) \\
&= \left(\sqrt[\eta]{1-\left(\left(1-1+\left(\omega_1\left(1-\overline{\overline{u}}_1^{\eta}\right)^{Ћ}-\left(\omega_1-1\right)\right)^{\frac{1}{Ћ}}\right)^{Ћ}+\left(1-1+\left(\omega_2\left(1-\overline{\overline{u}}_2^{\eta}\right)^{Ћ}-\left(\omega_2-1\right)\right)^{\frac{1}{Ћ}}\right)^{Ћ}-1\right)^{\frac{1}{Ћ}}},\ \sqrt[\eta]{\left(\left(\left(\omega_1\left(\overline{\overline{v}}_1^{\eta}\right)^{Ћ}-\left(\omega_1-1\right)\right)^{\frac{1}{Ћ}}\right)^{Ћ}\mathcal{c}\left(\left(\omega_2\left(\overline{\overline{v}}_2^{\eta}\right)^{Ћ}-\left(\omega_2-1\right)\right)^{\frac{1}{Ћ}}\right)^{Ћ}\mathbf{-1}\right)^{\frac{1}{Ћ}}}\right) \\
&= \left(\sqrt[\eta]{1-\left(\left(\left(\omega_1\left(1-\overline{\overline{u}}_1^{\eta}\right)^{Ћ}-\left(\omega_1-1\right)\right)^{\frac{1}{Ћ}}\right)^{Ћ}+\left(\left(\omega_2\left(1-\overline{\overline{u}}_2^{\eta}\right)^{Ћ}-\left(\omega_2-1\right)\right)^{\frac{1}{Ћ}}\right)^{Ћ}-1\right)^{\frac{1}{Ћ}}},\ \sqrt[\eta]{\left(\left(\omega_1\left(\overline{\overline{v}}_1^{\eta}\right)^{Ћ}-\left(\omega_1-1\right)\right)+\left(\omega_2\left(\overline{\overline{v}}_2^{\eta}\right)^{Ћ}-\left(\omega_2-1\right)\right)-1\right)^{\frac{1}{Ћ}}}\right) \\
&= \left(\sqrt[\eta]{1-\left(\left(\omega_1\left(1-\overline{\overline{u}}_1^{\eta}\right)^{Ћ}-\left(\omega_1-1\right)\right)+\left(\omega_2\left(1-\overline{\overline{u}}_2^{\eta}\right)^{Ћ}-\left(\omega_2-1\right)\right)-1\right)^{\frac{1}{Ћ}}},\ \sqrt[\eta]{\left(\left(\omega_1\left(\overline{\overline{v}}_1^{\eta}\right)^{Ћ}-\left(\omega_1-1\right)\right)+\left(\omega_2\left(\overline{\overline{v}}_2^{\eta}\right)^{Ћ}-\left(\omega_2-1\right)\right)-1\right)^{\frac{1}{Ћ}}}\right)
\end{aligned}$$

$$= \left(\sqrt[\eta]{1 - \left(\omega_1 \left(1 - \bar{\bar{u}}_1^{\eta}\right)^{\mathit{Ib}} - \omega_1 + 1 + \omega_2 \left(1 - \bar{\bar{u}}_2^{\eta}\right)^{\mathit{Ib}} - \omega_2 + 1 - 1 \right)^{\frac{1}{\mathit{Ib}}}}, \sqrt[\eta]{\left(\left(\omega_1 \left(\bar{\bar{v}}_1^{\eta}\right)^{\mathit{Ib}} - \omega_1 + 1 \right) + \left(\omega_2 \left(\bar{\bar{v}}_2^{\eta}\right)^{\mathit{Ib}} - \omega_2 + 1 \right) - 1 \right)^{\frac{1}{\mathit{Ib}}}} \right)$$

$$= \left(\sqrt[\eta]{1 - \left(\omega_1 \left(1 - \bar{\bar{u}}_1^{\eta}\right)^{\mathit{Ib}} - \omega_1 + 1 + \omega_2 \left(1 - \bar{\bar{u}}_2^{\eta}\right)^{\mathit{Ib}} - \omega_2 \right)^{\frac{1}{\mathit{Ib}}}}, \sqrt[\eta]{\left(\omega_1 \left(\bar{\bar{v}}_1^{\eta}\right)^{\mathit{Ib}} - \omega_1 + 1 \, \mathcal{C} \, \omega_2 \left(\bar{\bar{v}}_2^{\eta}\right)^{\mathit{Ib}} - \omega_2 \right)^{\frac{1}{\mathit{Ib}}}} \right)$$

$$= \left(\sqrt[\eta]{1 - \left(\omega_1 \left(1 - \bar{\bar{u}}_1^{\eta}\right)^{\mathit{Ib}} + \omega_2 \left(1 - \bar{\bar{u}}_2^{\eta}\right)^{\mathit{Ib}} - \omega_1 - \omega_2 + 1 \right)^{\frac{1}{\mathit{Ib}}}}, \sqrt[\eta]{\left(\omega_1 \left(\bar{\bar{v}}_1^{\eta}\right)^{\mathit{Ib}} \, \mathcal{C} \, \omega_2 \left(\bar{\bar{v}}_2^{\eta}\right)^{\mathit{Ib}} - \omega_1 - \omega_2 + 1 \right)^{\frac{1}{\mathit{Ib}}}} \right)$$

$$= \left(\sqrt[\eta]{1 - \left(\sum\nolimits_{\iota=1}^{2} \omega_\iota \left(1 - \bar{\bar{u}}_\iota^{\eta}\right)^{\mathit{Ib}} - \sum\nolimits_{\iota=1}^{2} \omega_\iota + 1 \right)^{\frac{1}{\mathit{Ib}}}}, \sqrt[\eta]{\left(\sum\nolimits_{\iota=1}^{2} \omega_\iota \left(\bar{\bar{v}}_\iota^{\eta}\right)^{\mathit{Ib}} - \sum\nolimits_{\iota=1}^{2} \omega_\iota + 1 \right)^{\frac{1}{\mathit{Ib}}}} \right)$$

Firstly, we verify for $n = 2$ and, based on this analysis, and then considered it for $n = k$ appropriately, such as:

$$\bar{\bar{\mathfrak{R}}}_\iota = \left(\bar{\bar{u}}_\iota^{\eta} (\gimel) \; \bar{\bar{v}}_\iota^{\eta} (\gimel) \right), \ \iota = 1, 2, \ldots, n$$

$$q - ROFSSPA \left(\bar{\bar{\mathfrak{R}}}_1, \bar{\bar{\mathfrak{R}}}_2, \ldots, \bar{\bar{\mathfrak{R}}}_n \right)$$

$$= \left(\sqrt[\eta]{1 - \left(\sum_{\iota=1}^{k} \omega_\iota \left(1 - \bar{\bar{u}}_\iota^{\eta}\right)^{\mathit{Ib}} - \sum_{\iota=1}^{k} \omega_\iota + 1 \right)^{\frac{1}{\mathit{Ib}}}}, \sqrt[\eta]{\left(\sum_{\iota=1}^{k} \omega_\iota \left(\bar{\bar{v}}_\iota^{\eta}\right)^{\mathit{Ib}} - \sum_{\iota=1}^{k} \omega_\iota + 1 \right)^{\frac{1}{\mathit{Ib}}}} \right)$$

Now, we prove it for $n = k + 1$ such as:

$$q - ROFSSPA \left(\bar{\bar{\mathfrak{R}}}_1, \bar{\bar{\mathfrak{R}}}_2, \ldots, \bar{\bar{\mathfrak{R}}}_{n+1} \right)$$

$$= \omega_1 \bar{\bar{\mathfrak{R}}}_1 \oplus \omega_2 \bar{\bar{\mathfrak{R}}}_2 \oplus \ldots \oplus \omega_k \bar{\bar{\mathfrak{R}}}_k \oplus \omega_{k+1} \bar{\bar{\mathfrak{R}}}_{k+1}$$

$$= \oplus_{\iota=1}^{k} \omega_\iota \bar{\bar{\mathfrak{R}}}_\iota \oplus \omega_{k+1} \bar{\bar{\mathfrak{R}}}_{k+1}$$

$$= \left(\sqrt[\eta]{1 - \left(\sum_{\iota=1}^{k} \omega_\iota \left(1 - \bar{\bar{u}}_\iota^{\eta}\right)^{\mathit{Ib}} - \sum_{\iota=1}^{k} \omega_\iota + 1 \right)^{\frac{1}{\mathit{Ib}}}}, \sqrt[\eta]{\left(\sum_{\iota=1}^{k} \omega_\iota \left(\bar{\bar{v}}_\iota^{\eta}\right)^{\mathit{Ib}} - \sum_{\iota=1}^{k} \omega_\iota + 1 \right)^{\frac{1}{\mathit{Ib}}}} \right)$$

$$\oplus \left(\sqrt[\eta]{1 - \left(\omega_{k+1} \left(1 - \bar{\bar{u}}_{k+1}^{\eta}\right)^{\mathit{Ib}} - \left(\omega_{k+1} - 1\right) \right)^{\frac{1}{\mathit{Ib}}}}, \sqrt[\eta]{\left(\omega_{k+1} \left(\bar{\bar{v}}_{k+1}^{\eta}\right)^{\mathit{Ib}} - \left(\omega_{k+1} - 1\right) \right)^{\frac{1}{\mathit{Ib}}}} \right)$$

$$= \left(\sqrt[\eta]{1 - \left(\sum_{\iota=1}^{k+1} \omega_\iota \left(1 - \bar{\bar{u}}_\iota^{\eta}\right)^{\mathit{Ib}} - \sum_{\iota=1}^{k+1} \omega_\iota + 1 \right)^{\frac{1}{\mathit{Ib}}}}, \sqrt[\eta]{\left(\sum_{\iota=1}^{k+1} \omega_\iota \left(\bar{\bar{v}}_\iota^{\eta}\right)^{\mathit{Ib}} - \sum_{\iota=1}^{k+1} \omega_\iota + 1 \right)^{\frac{1}{\mathit{Ib}}}} \right)$$

Property 1: Consider a finite set of q-ROFVs $\bar{\bar{\mathfrak{R}}}_\iota = \left(\bar{\bar{u}}_\iota, \bar{\bar{v}}_\iota \right), \ \iota = 1, 2, \ldots, n$ implies that $\bar{\bar{\mathfrak{R}}}_\iota = \bar{\bar{\mathfrak{R}}}$. Then:

$$q - ROFSSPA \left(\bar{\bar{\mathfrak{R}}}_1, \bar{\bar{\mathfrak{R}}}_2, \ldots, \bar{\bar{\mathfrak{R}}}_n \right) = \bar{\bar{\mathfrak{R}}}$$

Proof: Since a finite set of q-ROFVs $\bar{\bar{\mathfrak{R}}}_\iota = \left(\bar{\bar{u}}_\iota, \bar{\bar{v}}_\iota \right), \ \iota = 1, 2, \ldots, n$ implies that $\bar{\bar{\mathfrak{R}}}_\iota = \bar{\bar{\mathfrak{R}}}$. We can write:

$$q - ROFSSPA \left(\bar{\bar{\mathfrak{R}}}_1, \bar{\bar{\mathfrak{R}}}_2, \ldots, \bar{\bar{\mathfrak{R}}}_n \right)$$

$$= \left(\sqrt[\eta]{1-\left(\sum_{\iota=1}^{n}\omega_\iota\left(1-\overline{\overline{u}}_\iota^{\eta}\right)^{\text{Ђ}}-\sum_{\iota=1}^{n}\omega_\iota+1\right)^{\frac{1}{\text{Ђ}}}},\ \sqrt[\eta]{\left(\sum_{\iota=1}^{n}\omega_\iota\left(\overline{\overline{\varpi}}_\iota^{\eta}\right)^{\text{Ђ}}-\sum_{\iota=1}^{n}\omega_\iota+1\right)^{\frac{1}{\text{Ђ}}}} \right)$$

$$q-ROFSSPA\left(\overline{\overline{\Re}}_1,\overline{\overline{\Re}}_2,\ \ldots,\overline{\overline{\Re}}_n\right)$$

$$= \left(\sqrt[\eta]{1-\left(\left(1-\overline{\overline{u^{\eta}}}\right)^{\text{Ђ}}-1+1\right)^{\frac{1}{\text{Ђ}}}},\ \sqrt[\eta]{\left(\left(\overline{\overline{\varpi^{\eta}}}\right)^{\text{Ђ}}-1+1\right)^{\frac{1}{\text{Ђ}}}} \right),$$

$$\sum_{\iota=1}^{n}\omega_\iota=1$$

$$= \left(\sqrt[\eta]{1-\left(\left(1-\overline{\overline{u^{\eta}}}\right)^{\text{Ђ}}-1+1\right)^{\frac{1}{\text{Ђ}}}},\ \sqrt[\eta]{\left(\left(\overline{\overline{\varpi^{\eta}}}\right)^{\text{Ђ}}-1+1\right)^{\frac{1}{\text{Ђ}}}} \right)$$

$$= \left(\sqrt[\eta]{1-\left(\left(1-\overline{\overline{u^{\eta}}}\right)^{\text{Ђ}}\right)^{\frac{1}{\text{Ђ}}}},\ \sqrt[\eta]{\left(\left(\overline{\overline{\varpi^{\eta}}}\right)^{\text{Ђ}}\right)^{\frac{1}{\text{Ђ}}}} \right)$$

$$= \left(\sqrt[\eta]{1-\left(1-\overline{\overline{u^{\eta}}}\right)},\ \sqrt[\eta]{\left(\overline{\overline{\varpi^{\eta}}}\right)} \right)$$

$$\left(\overline{\overline{u}}\,\overline{\overline{\varpi}}\right) = \overline{\overline{\Re}}$$

Property 2: Consider any two sets of q-ROFVs $\overline{\overline{\Re}}_\iota = (\overline{\overline{u}}_\iota, \overline{\overline{\varpi}}_\iota)$, and $\Xi_\iota = (\mu_\iota, \nu_\iota)\ (\iota = 1, 2, 3, \ldots, n)$. if $\overline{\overline{\Re}}_\iota \leq \Xi_\iota$ such that $\overline{\overline{u}}_\iota \leq \mu_\iota$ and $\overline{\overline{\varpi}}_\iota \geq \nu_\iota$. Then:

$$q-ROFSSPA\left(\overline{\overline{\Re}}_1,\overline{\overline{\Re}}_2,\ \ldots,\overline{\overline{\Re}}_n\right) \leq q-ROFSSPA\left(\Xi_1,\ \Xi_2,\ \ldots,\Xi_m\right)$$

Proof: Suppose that any two sets of q-ROFVs $\overline{\overline{\Re}}_\iota = (\overline{\overline{u}}_\iota, \overline{\overline{\varpi}}_\iota)$, and $\Xi_\iota = (\mu_\iota, \nu_\iota)\ (\iota = 1, 2, 3, \ldots, n)$. if $\overline{\overline{\Re}}_\iota \leq \Xi_\iota$ such that $\overline{\overline{u}}_\iota \leq \mu_\iota$ and $\overline{\overline{\varpi}}_\iota \geq \nu_\iota$. We can write:

$$\overline{\overline{u}}_\iota \leq \mu_\iota \Rightarrow \overline{\overline{u}}_\iota^{\eta} \geq \mu_\iota^{\eta}$$

$$\Rightarrow 1-\overline{\overline{u}}_\iota^{\eta} \geq 1-\mu_\iota^{\eta}$$

$$\Rightarrow \left(1-\overline{\overline{u}}_\iota^{\eta}\right)^{\text{Ђ}} \geq \left(1-\mu_\iota^{2}\right)^{\text{Ђ}}$$

$$\Rightarrow \sum_{\iota=1}^{n}\omega_\iota\left(1-\overline{\overline{u}}_\iota^{\eta}\right)^{\text{Ђ}} \geq \sum_{\iota=1}^{m}\omega_\iota\left(1-\mu_\iota^{\eta}\right)^{\text{Ђ}}$$

$$\Rightarrow \sum_{\iota=1}^{n}\omega_\iota\left(1-\overline{\overline{u}}_\iota^{\eta}\right)^{\text{Ђ}}-\sum_{\iota=1}^{n}\omega_\iota+1 \geq \sum_{\iota=1}^{m}\omega_\iota\left(1-\mu_\iota^{\eta}\right)^{\text{Ђ}} -\sum_{\iota=1}^{m}\omega_\iota+1$$

$$\Rightarrow \left(\sum_{\iota=1}^{n}\omega_\iota\left(1-\overline{\overline{u}}_\iota^{\eta}\right)^{\text{Ђ}}-\sum_{\iota=1}^{n}\omega_\iota+1\right)^{\frac{1}{\text{Ђ}}} \geq \left(\sum_{\iota=1}^{m}\omega_\iota\left(1-\mu_\iota^{\eta}\right)^{\text{Ђ}}-\sum_{\iota=1}^{m}\omega_\iota+1\right)^{\frac{1}{\text{Ђ}}}$$

$$\Rightarrow 1-\left(\sum_{\iota=1}^{n}\omega_\iota\left(1-\overline{\overline{u}}_\iota^{\eta}\right)^{\text{Ђ}}-\sum_{\iota=1}^{n}\omega_\iota+1\right)^{\frac{1}{\text{Ђ}}} \leq 1-\left(\sum_{\iota=1}^{m}\omega_\iota\left(1-\mu_\iota^{\eta}\right)^{\text{Ђ}}-\sum_{\iota=1}^{m}\omega_\iota+1\right)^{\frac{1}{\text{Ђ}}}$$

$$\times \sqrt[\eta]{1-\left(\sum_{\iota=1}^{n}\omega_\iota\left(1-\overline{\overline{u}}_\iota^{\eta}\right)^{\text{Ђ}}-\sum_{\iota=1}^{n}\omega_\iota+1\right)^{\frac{1}{\text{Ђ}}}} \leq \sqrt[\eta]{1-\left(\sum_{\iota=1}^{m}\omega_\iota\left(1-\mu_\iota^{\eta}\right)^{\text{Ђ}}-\sum_{\iota=1}^{m}\omega_\iota+1\right)^{\frac{1}{\text{Ђ}}}}$$

Further, we assume that $\overline{\overline{\varpi}}_\iota \geq \nu_\iota$. Then we have:

$$\overline{\overline{\Re}}_\iota = \left(\overline{\overline{u}}_\iota^{\eta}(\gimel)\ \overline{\overline{\varpi}}_\iota^{\eta}(\gimel)\right) \leq \Xi_\iota = (\mu_\iota, \nu_\iota)$$

$$\overline{\overline{\varpi}}_\iota \geq \nu_\iota \Rightarrow \overline{\overline{\varpi}}_\iota^{\eta} \geq \nu_\iota^{\eta} \Rightarrow \left(\overline{\overline{\varpi}}_\iota^{\eta}\right)^{\text{Ђ}} \geq \left(\nu_\iota^{\eta}\right)^{\text{Ђ}}$$

$$\Rightarrow \sum_{\iota=1}^{n}\omega_\iota\left(\overline{\overline{\varpi}}_\iota^{\eta}\right)^{\text{Ђ}} \geq \sum_{\iota=1}^{m}\omega_\iota\left(\nu_\iota^{\eta}\right)^{\text{Ђ}}$$

$$\sum_{\iota=1}^{n}\omega_\iota\left(\overline{\overline{\varpi}}_\iota^{\eta}\right)^{\text{Ђ}}-\sum_{\iota=1}^{n}\omega_\iota+1 \geq \sum_{\iota=1}^{m}\omega_\iota\left(\nu_\iota^{\eta}\right)^{\text{Ђ}}-\sum_{\iota=1}^{m}\omega_\iota+1$$

$$\Rightarrow \left(\sum_{\iota=1}^{n}\omega_\iota\left(\overline{\overline{\varpi}}_\iota^{\eta}\right)^{\text{Ђ}}-\sum_{\iota=1}^{n}\omega_\iota+1\right)^{\text{Ђ}} \geq \left(\sum_{\iota=1}^{m}\omega_\iota\left(\nu_\iota^{\eta}\right)^{\text{Ђ}}-\sum_{\iota=1}^{m}\omega_\iota+1\right)^{\text{Ђ}}$$

$$\sqrt[\eta]{\left(\sum_{\iota=1}^{n}\omega_\iota\left(\overline{\overline{\varpi}}_\iota^{\eta}\right)^{\text{Ђ}}-\sum_{\iota=1}^{n}\omega_\iota+1\right)^{\text{Ђ}}} \geq \sqrt[\eta]{\left(\sum_{\iota=1}^{m}\omega_\iota\left(\nu_\iota^{\eta}\right)^{\text{Ђ}}-\sum_{\iota=1}^{m}\omega_\iota+1\right)^{\text{Ђ}}}$$

$$q-ROFSSPA\left(\overline{\overline{\Re}}_1,\overline{\overline{\Re}}_2,\ \ldots,\ \overline{\overline{\Re}}_n\right) \leq q-ROFSSPA\left(\Xi_1,\ \Xi_2,\ \ldots,\Xi_m\right)$$

Property 3: For any set of q-ROFVs $\overline{\overline{\Re}}_\iota = (\overline{\overline{u}}_\iota, \overline{\overline{\varpi}}_\iota)$, $(\iota = 1, 2, 3, \ldots, n)$. If $\overline{\overline{\Re}}_\iota^{+} = (max\{\overline{\overline{u}}_\iota\}, min\{\overline{\overline{\varpi}}_\iota\})$ and $\overline{\overline{\Re}}_\iota^{-} = (min\{\overline{\overline{u}}_\iota\}, max\{\overline{\overline{\varpi}}_\iota\})$. Then we can get:

$$\overline{\overline{\Re}}_\iota^{-} \leq q - ROFSSPA\left(\overline{\overline{\Re}}_1, \overline{\overline{\Re}}_2, \ldots, \overline{\overline{\Re}}_n\right) \leq \overline{\overline{\Re}}_\iota^{+}$$

Proof: Here we derived the above properties, under considering of property 1 and 2, we have:

$$\begin{aligned} q - ROFSSPA&\left(\overline{\overline{\Re}}_1^{-}, \overline{\overline{\Re}}_2^{-}, \ldots, \overline{\overline{\Re}}_n^{-}\right) \\ &\leq q - ROFSSPA\left(\overline{\overline{\Re}}_1^{+}, \overline{\overline{\Re}}_2^{+}, \ldots, \overline{\overline{\Re}}_m^{+}\right) \\ &= \overline{\overline{\Re}}_\iota^{+} q - ROFSSPA\left(\overline{\overline{\Re}}_1, \overline{\overline{\Re}}_2, \ldots, \overline{\overline{\Re}}_n\right) \\ &\geq q - ROFSSPA\left(\overline{\overline{\Re}}_1^{-}, \overline{\overline{\Re}}_2^{-}, \ldots, \overline{\overline{\Re}}_n^{-}\right) = \overline{\overline{\Re}}_\iota^{-} \end{aligned}$$

Then:

$$\overline{\overline{\Re}}_\iota^{-} \leq q - ROFSSPA\left(\overline{\overline{\Re}}_1, \overline{\overline{\Re}}_2, \ldots, \overline{\overline{\Re}}_n\right) \leq \overline{\overline{\Re}}_\iota^{+}$$

Definition 9: For any finite set of q-ROFVs $\overline{\overline{\Re}}_\iota = (\overline{\overline{u}}_\iota, \overline{\overline{\varpi}}_\iota)$, $(\iota = 1, 2, 3, \ldots, n)$. Then, the q-ROFSSPWA operator is characterized as follows:

$$\begin{aligned} q - ROFSSPWA&\left(\overline{\overline{\Re}}_1, \overline{\overline{\Re}}_2, \ldots, \overline{\overline{\Re}}_n\right) \\ &= \Sigma_1 \overline{\overline{\Re}}_1 \oplus \Sigma_2 \overline{\overline{\Re}}_2 \oplus, \ldots, \oplus \Sigma_2 \overline{\overline{\Re}}_n = \bigoplus_{\iota=1}^{n} \Sigma_\iota \overline{\overline{\Re}}_\iota \end{aligned}$$

Now $\Sigma_\iota = \frac{\ddot{w}_\iota \mathfrak{E}_\iota}{\sum_{\iota=1}^{n} \mathfrak{E}_\iota}$, where $\mathfrak{E}_1 = 1$ and $\mathfrak{E}_t = \otimes_{k=1}^{\iota-1} \aleph\left(\overline{\overline{\Re}}_\iota\right)$, $(k = 1, 2, 3, \ldots, n)$. Moreover, the representation of the weight vector is stated by $\ddot{w}_\iota \in [0, 1]$ with $\sum_{\iota=1}^{n} \ddot{w}_\iota = 1$.

Theorem 2: For any finite set of q-ROFVs $\Re_\iota = (\overline{\overline{u}}_\iota, \overline{\overline{\varpi}}_\iota)$, $(\iota = 1, 2, 3, \ldots, n)$ with $\hbar < 0$. Then the integrated values of the q-ROFSSPWA operator still q-ROFVs are given by:

$$\mathrm{q - ROFSSPWA}\left(\overline{\overline{\Re}}_1, \overline{\overline{\Re}}_2, \ldots, \overline{\overline{\Re}}_n\right) = \left(\begin{array}{c} \sqrt[\eta]{1 - \left(\sum_{\iota=1}^{n} \Sigma_\iota \left(1 - \overline{\overline{u}}_\iota^{\eta}\right)^{\hbar} - \sum_{\iota=1}^{n} \Sigma_\iota + 1\right)^{\frac{1}{\hbar}}}, \\ \sqrt[\eta]{\left(\sum_{\iota=1}^{n} \Sigma_\iota \left(\overline{\overline{\varpi}}_\iota^{\eta}\right)^{\hbar} - \sum_{\iota=1}^{n} \Sigma_\iota + 1\right)^{\frac{1}{\hbar}}} \end{array}\right)$$

Proof: Similar to the theorem 1.

Property 4: Consider a finite set of q-ROFVs $\overline{\overline{\Re}}_\iota = (\overline{\overline{u}}_\iota, \overline{\overline{\varpi}}_\iota)$, $\iota = 1, 2, \ldots, n$ implies that $\overline{\overline{\Re}}_\iota = \overline{\overline{\Re}}$. Then:

$$q - ROFSSPWA\left(\overline{\overline{\Re}}_1, \overline{\overline{\Re}}_2, \ldots, \overline{\overline{\Re}}_n\right) = \overline{\overline{\Re}}$$

Property 5: Consider any two sets of q-ROFVs $\overline{\overline{\Re}}_\iota = (\overline{\overline{u}}_\iota, \overline{\overline{\varpi}}_\iota)$, and $\Xi_\iota = (\mu_\iota, \nu_\iota)$ $(\iota = 1, 2, 3, \ldots, n)$. if $\overline{\overline{\Re}}_\iota \leq \Xi_\iota$ such that $\overline{\overline{u}}_\iota \leq \mu_\iota$ and $\overline{\overline{\varpi}}_\iota \geq \nu_\iota$. Then:

$$\begin{aligned} q - ROFSSPWA&\left(\overline{\overline{\Re}}_1, \overline{\overline{\Re}}_2, \ldots, \overline{\overline{\Re}}_n\right) \\ &\leq q - ROFSSPWA\left(\Xi_1, \Xi_2, \ldots, \Xi_m\right) \end{aligned}$$

Property 6: For any set of q-ROFVs $\overline{\overline{\Re}}_\iota = (\overline{\overline{u}}_\iota, \overline{\overline{\varpi}}_\iota)$, $(\iota = 1, 2, 3, \ldots, n)$. If $\overline{\overline{\Re}}_\iota^{+} = (max\{\overline{\overline{u}}_\iota\}, min\{\overline{\overline{\varpi}}_\iota\})$ and $\Re_\iota^{-} = (min\{\overline{\overline{u}}_\iota\}, max\{\overline{\overline{\varpi}}_\iota\})$. Then we can get:

$$\overline{\overline{\Re}}_\iota^{-} \leq \boldsymbol{q - ROFSSPWA}\left(\overline{\overline{\Re}}_1, \overline{\overline{\Re}}_2, \ldots, \overline{\overline{\Re}}_n\right) \leq \overline{\overline{\Re}}_\iota^{\mathcal{C}}$$

IV. q-RUNG ORTHOPAIR FUZZY SCHWEIZER-SKLAR GEOMETRIC AGGREGATION OPERATORS BASED ON q-ROF INFORMATION

This section aims to develop reliable strategies using the properties of Schweizer-Sklar t-norms, namely q-ROFSSPG and q-ROFSSPWG operators in light of q-ROF information.

Definition 10: For any set of q-ROFVs $\overline{\overline{\Re}}_\iota = (\overline{\overline{u}}_\iota(\lambda), \overline{\overline{\varpi}}_\iota(\lambda))$, $(\iota = 1, 2, 3, \ldots, n)$. Then, the q-ROFSSPG operator is characterized as follows:

$$\begin{aligned} q - ROFSSPG&\left(\overline{\overline{\Re}}_1, \overline{\overline{\Re}}_2, \ldots, \overline{\overline{\Re}}_n\right) \\ &= \overline{\overline{\Re}}_1^{\omega_1} \otimes \overline{\overline{\Re}}_2^{\omega_2} \otimes \ldots \otimes \overline{\overline{\Re}}_n^{\omega_n} \end{aligned}$$

Now $\omega_\iota = \frac{\ddot{w}_\iota \mathfrak{E}_\iota}{\sum_{\iota=1}^{n} \mathfrak{E}_\iota}$, where $\mathfrak{E}_1 = 1$ and $\mathfrak{E}_t = \otimes_{\iota=1}^{k-1} \aleph\left(\overline{\overline{\Re}}\right)$, $(k = 2, 3, \ldots, n)$.

Theorem 3: For any set of q-ROFVs $\overline{\overline{\Re}}_\iota = (\overline{\overline{u}}_\iota, \overline{\overline{\varpi}}_\iota)$, $(\iota = 1, 2, 3, \ldots, n)$ with $\hbar < 0$. Then, the integrated values of the q-ROFSSPWG operator is still a q-ROFV is given by:

$$q - ROFSSPG\left(\overline{\overline{\Re}}_1, \overline{\overline{\Re}}_2, \ldots, \overline{\overline{\Re}}_n\right) = \left(\begin{array}{c} \sqrt[\eta]{\left(\sum_{\iota=1}^{n} \omega_i \left(\overline{\overline{u}}_\iota^{\eta}\right)^{\hbar} - \sum_{\iota=1}^{n} \omega_\iota + 1\right)^{\frac{1}{\hbar}}}, \\ \sqrt[\eta]{1 - \left(\sum_{\iota=1}^{n} \omega_i \left(1 - \overline{\overline{\varpi}}_\iota^{\eta}\right)^{\hbar} - \sum_{\iota=1}^{n} \omega_\iota + 1\right)^{\frac{1}{\hbar}}} \end{array}\right)$$

Similar to the theorem 1.

Property 7: Consider a finite set of q-ROFVs $\overline{\overline{\Re}}_\iota = (\overline{\overline{u}}_\iota, \overline{\overline{\varpi}}_\iota)$, $\iota = 1, 2, \ldots, n$ implies that $\overline{\overline{\Re}}_\iota = \overline{\overline{\Re}}$. Then:

$$q - ROFSSPG\left(\overline{\overline{\Re}}_1, \overline{\overline{\Re}}_2, \ldots, \overline{\overline{\Re}}_n\right) = \overline{\overline{\Re}}$$

Property 8: Consider any two sets of q-ROFVs $\overline{\overline{\Re}}_\iota = (\overline{\overline{u}}_\iota, \overline{\overline{\varpi}}_\iota)$, and $\Xi_\iota = (\mu_\iota, \nu_\iota)$ $(\iota = 1, 2, 3, \ldots, n)$. if $\overline{\overline{\Re}}_\iota \leq \Xi_\iota$ such that $\overline{\overline{u}}_\iota \leq \mu_\iota$ and $\overline{\overline{\varpi}}_\iota \geq \nu_\iota$. Then:

$$q - ROFSSPG\left(\overline{\overline{\Re}}_1, \overline{\overline{\Re}}_2, \ldots, \overline{\overline{\Re}}_n\right)$$

$$\leq q-ROFSSPG\left(\Xi_1,\ \Xi_2,\ \ldots,\Xi_m\right)$$

Property 9: For any set of q-ROFVs $\overline{\overline{\Re}}_\iota = \left(\overline{\overline{u}}_\iota,\ \overline{\overline{v}}_\iota\right),\ (\iota = 1,\ 2,\ 3,\ \ldots, n)$. If $\overline{\overline{\Re}}_\iota^{+} = \left(max\left\{\overline{\overline{u}}_\iota\right\},\ min\left\{\overline{\overline{v}}_\iota\right\}\right)$ and $\Re_\iota^{-} = \left(min\left\{\overline{\overline{u}}_\iota\right\},\ max\left\{\overline{\overline{v}}_\iota\right\}\right)$. Then we can get:

$$\overline{\overline{\Re}}_\iota^{-} \leq q-ROFSSPG\left(\overline{\overline{\Re}}_1,\ \overline{\overline{\Re}}_2,\ \ldots,\ \overline{\overline{\Re}}_n\right) \leq \overline{\overline{\Re}}_\iota^{+}$$

Definition 11: For any q-ROFVs $\overline{\overline{\Re}}_\iota = \left(\overline{\overline{u}}_\iota\left(\gimel\right),\ \overline{\overline{v}}_\iota\left(\gimel\right)\right)$, $\iota = 1, 2, \ldots,\ n$. Then, the q-ROFSSPWG operator is characterized as follows:

$$q-ROFSSPWG\left(\overline{\overline{\Re}}_1,\ \overline{\overline{\Re}}_2,\ \ldots,\ \overline{\overline{\Re}}_n\right) = \overset{n}{\underset{\iota=1}{\otimes}} \overline{\overline{\Re}}_\iota^{\Sigma_\iota} = \overline{\overline{\Re}}_1^{\Sigma_1} \otimes \overline{\overline{\Re}}_2^{\Sigma_2} \otimes \ldots \otimes \overline{\overline{\Re}}_n^{\Sigma_n}$$

Now $\Sigma_\iota = \frac{\ddot{w}_\iota \mathfrak{E}_\iota}{\sum_{\iota=1}^{n} \mathfrak{E}_\iota}$, where $\mathfrak{E}_1 = 1$ and $\mathfrak{E}_t = \otimes_{k=1}^{\iota-1} \aleph\left(\overline{\overline{\Re}}_\iota\right), (k = 1,\ 2,\ 3,\ \ldots, n)$. Moreover, the representation of the weight vector is stated by $\ddot{w}_\iota \in [0,\ 1]$ with $\sum_{\iota=1}^{n} \ddot{w}_\iota = 1$.

Theorem 4: For any q-ROFVs $\overline{\overline{\Re}}_\iota = \left(\overline{\overline{u}}_\iota\left(\gimel\right),\ \overline{\overline{v}}_\iota\left(\gimel\right)\right)$, $\iota = 1, 2, \ldots,\ n$. with $\hbar < 0$. Then, the integrated values of the q-ROFSSPWG operator is still a q-ROFV is given by:

$$\mathrm{q-ROFSSPWG}\left(\overline{\overline{\Re}}_1,\ \overline{\overline{\Re}}_2,\ \ldots,\ \overline{\overline{\Re}}_n\right) = \left(\begin{array}{c} \sqrt[\eta]{\left(\sum_{\iota=1}^{n} \Sigma_\iota\left(\overline{\overline{u}}_\iota^{\eta}\right)^{\hbar} - \sum_{\iota=1}^{n} \Sigma_\iota + 1\right)^{\frac{1}{\hbar}}}, \\ \sqrt[\eta]{1-\left(\sum_{\iota=1}^{n} \Sigma_\iota\left(1-\overline{\overline{v}}_\iota^{\eta}\right)^{\hbar} - \sum_{\iota=1}^{n} \Sigma_\iota + 1\right)^{\frac{1}{\hbar}}} \end{array}\right)$$

Proof: Similar to the theorem 1.

Property 10: Consider a finite set of q-ROFVs $\overline{\overline{\Re}}_\iota = \left(\overline{\overline{u}}_\iota,\ \overline{\overline{v}}_\iota\right),\ \iota = 1, 2, \ldots,\ n$ implies that $\overline{\overline{\Re}}_\iota = \overline{\overline{\Re}}$. Then:

$$q-ROFSSPWG\left(\overline{\overline{\Re}}_1,\ \overline{\overline{\Re}}_2,\ \ldots,\ \overline{\overline{\Re}}_n\right) = \overline{\overline{\Re}}$$

Property 11: Consider any two sets of q-ROFVs $\overline{\overline{\Re}}_\iota = \left(\overline{\overline{u}}_\iota,\ \overline{\overline{v}}_\iota\right)$, and $\Xi_\iota = \left(\mu_\iota, \nu_\iota\right)(\iota = 1,\ 2,\ 3,\ \ldots, n)$. if $\overline{\overline{\Re}}_\iota \leq \Xi_\iota$ such that $\overline{\overline{u}}_\iota \leq \mu_\iota$ and $\overline{\overline{v}}_\iota \geq \nu_\iota$. Then:

$$q-ROFSSPWG\left(\overline{\overline{\Re}}_1,\ \overline{\overline{\Re}}_2,\ \ldots,\ \overline{\overline{\Re}}_n\right) \leq q-ROFSSPWG\left(\Xi_1,\ \Xi_2,\ \ldots,\Xi_m\right)$$

Property 12: For any set of q-ROFVs $\overline{\overline{\Re}}_\iota = \left(\overline{\overline{u}}_\iota,\ \overline{\overline{v}}_\iota\right),\ (\iota = 1,\ 2,\ 3,\ \ldots, n)$. If $\overline{\overline{\Re}}_\iota^{+} = \left(max\left\{\overline{\overline{u}}_\iota\right\},\ min\left\{\overline{\overline{v}}_\iota\right\}\right)$ and $\Re_\iota^{-} = \left(min\left\{\overline{\overline{u}}_\iota\right\},\ max\left\{\overline{\overline{v}}_\iota\right\}\right)$. Then we can get:

$$\overline{\overline{\Re}}_\iota^{-} \leq \boldsymbol{q-ROFSSPWG}\left(\overline{\overline{\Re}}_1,\ \overline{\overline{\Re}}_2,\ \ldots,\ \overline{\overline{\Re}}_n\right) \leq \overline{\overline{\Re}}_\iota^{+}$$

V. ASSESSMENT OF THE MADM PROCESS BASED ON q-ROF INFORMATION

An innovative approach of the MADM technique is used to evaluate the finest option from the different available options under certain characteristics or attributes. The main theme of the MADM problem is to explore alternatives or individuals under the system of specific characteristics, identify the type of attributes, aggregate vague type information of human opinion, and choose reliable, optimal options based on score or accuracy values. The decision-making process faces many complicated challenges due to redundant and incomplete information about any object. To address this situation, Consider a finite class of alternative $\{\mathbb{A}_1,\ \mathbb{A}_2,\ \ldots, \mathbb{A}_n\}$ and a set of finite attributes $\left\{\hbar_1,\ \hbar_2,\ \ldots, \hbar_m\right\}$. To identify a suitable option, we need to assign some specific degree to each attribute in each alternative such that a set of weight vector $\ddot{w}_\iota \in [0,\ 1]$ and $\sum_{\iota=1}^{m} \ddot{w}_\iota = 1$. Furthermore, the Decision-maker accumulates information about any object in the form of q-ROFV $\overline{\overline{\Re}}_{i\iota} = \left(\overline{\overline{u}}_{i\iota}\left(\gimel\right),\ \overline{\overline{v}}_{i\iota}\left(\gimel\right)\right),\ i = 1, 2, \ldots,\ n\ \&\ \iota = 1, 2, \ldots,\ m$ with mathematical expression $0 \leq \overline{\overline{u}}_{i\iota}^{\eta}\left(\gimel\right) + \overline{\overline{v}}_{i\iota}^{\eta}\left(\gimel\right) \leq 1$. Decision-maker packed acquired q-ROF information in a single decision matrix as follows $M = \left[\overline{\overline{\Re}}_{i\iota}\right]_{n\times m}$. Here, we also represent the hesitancy value of $\overline{\overline{\Re}}_{i\iota}$ as $\pi_\iota = 1-\left(\overline{\overline{u}}_{i\iota}^{\eta}\left(\gimel\right) + \overline{\overline{v}}_{i\iota}^{\eta}\left(\gimel\right)\right)$. The decision maker integrates q-ROF information by following a robust algorithm for the MADM problem.

A. ALGORITHM

Step 1: First of all, the decision maker acquires information about any object in the form of q-ROFVs. This information is based on human opinion with attributes associated with each alternative.

Step 2: To demonstrate the same type of given information of attributes, follow the following expression to modify the standard decision matrix into a normalized decision matrix:

$$M = \left[\overline{\overline{\Re}}_{i\iota}\right]_{n\times m} = \begin{cases} \left(\overline{\overline{u}}_{i\iota}^{\eta}\left(\gimel\right), \overline{\overline{v}}_{i\iota}^{\eta}\left(\gimel\right)\right) & \textit{for benefit type} \\ \left(\overline{\overline{v}}_{i\iota}^{\eta}\left(\gimel\right), \overline{\overline{u}}_{i\iota}^{\eta}\left(\gimel\right)\right) & \textit{for cost type} \end{cases}$$

Note that the above expression only applies if there is more than one type of information, such as beneficial and cost type.

Step 2: Compute the degree of preferences based on score values $\omega_\iota = \frac{\mathfrak{E}_\iota}{\sum_{\iota=1}^{n}\mathfrak{E}_\iota}$, $\mathfrak{E}_1 = 1$ and $\mathfrak{E}_\iota = \otimes_{\iota=1}^{k-1} \aleph\left(\overline{\overline{\Re}}_\iota\right),\ k = 2,\ 3, \ldots, n$. Additionally, the degree of weighted preferences investigated by using $\omega_\iota = \frac{\ddot{w}_\iota \mathfrak{E}_\iota}{\sum_{\iota=1}^{n} \mathfrak{E}_\iota}$, where $\mathfrak{E}_1 = 1$ and $\mathfrak{E}_t = \otimes_{k=1}^{\iota-1} \aleph\left(\overline{\overline{\Re}}\right), (k = 2,\ 3,\ \ldots, n)$.

Step 3: Aggregate given information based on derived mathematical strategies of the q-ROFSSPA, q-ROFSSWPA, q-ROFSSPG, and q-ROFSSWPG operators.

Step 4: Calculate score values of all alternatives with the help of Definition 6. If the score values of individuals are the same, then we move to the accuracy function.

Step 5: We rank all score values and choose the most suitable alternative.

B. POSITIVE IMPACT OF AGRICULTURE ON PAKISTAN ECONOMY

Agriculture is the mainstay of Pakistan's economy and way of life. The agricultural industry employs more than half of the labor force, contributes one-fifth of the country's gross domestic product (GDP), and provides raw materials to several other industries. Despite the fact that Pakistan's economy clearly depends on agriculture, this sector has not received the support or technological improvement it deserves. If major adjustments are not made, it runs the danger of lagging behind other businesses that are expanding more quickly. The importance of Pakistan's agricultural sector for the nation's economic growth and welfare will be covered in this article, along with the actions that must be made to allow it to reach its full potential and how it guarantees food security and employment.

Pakistan's economy has always been mostly reliant on agriculture. It accounts for a large share of the labor force and provides a significant contribution to the country's GDP. Despite a growing emphasis on other sectors, Pakistan's farmers continue to be the backbone of the country's economy. Pakistan's excellent land and pleasant climate allow it to produce some of the best crops in the world. Pakistani agriculture is a diverse and sustainable sector that includes fisheries, animals, and crops. The Pakistani government's recognition of the importance of agriculture and its efforts to preserve and encourage this vital sector should come as no surprise. Future economic success in Pakistan will largely depend on its ability to sustain and increase its agricultural output.

Pakistan's agricultural sector has faced several difficulties recently, making it more difficult for farmers to earn a livelihood. Climate change has been primarily blamed for the problem of unpredictable weather patterns and extreme temperatures that lead to reduced agricultural output and increasing rates of pest infestation. Another major issue is that farmers are unable to get funding, which keeps them from investing in better equipment, technology, and agricultural practices. These challenges, which have put a great deal of stress on the whole agricultural industry, must be resolved if the agricultural sector is to survive and prosper in the long run.

C. PRACTICAL EXAMPLE

In this numerical case study, we evaluated some dominant crops ($\mathcal{A}_1$, $\mathcal{A}_2$, $\mathcal{A}_3$, $\mathcal{A}_4$, $\mathcal{A}_5$), which are highly profitable and play an efficient role in lifting any country's economy. The decision maker completes this task by considering some reliable characteristics or attributes explored as follows:

1) PROVIDING RAW MATERIALS $Ђ_1$

The term "crop raw materials" describes the plant-based resources extracted from agricultural crops and used as the foundation for various goods and businesses. The plant's seeds, leaves, stems, and roots are among the sections from which these basic resources are derived. A vast array of goods and commercial endeavors are built on crop basic ingredients. Numerous sectors, including agriculture, food processing, textiles, medicines, bioenergy, and more depend on these basic agricultural components. They serve as the foundation for goods that meet consumer requirements, drive economic expansion, and develop technology.

2) JOB CREATION AND ENCOURAGING ECONOMIC DEVELOPMENT $Ђ_2$

A flourishing economy and higher living standards for a society depend on fostering economic growth and creating jobs. These ideas are closely related since economic development is largely dependent on the production of jobs, and jobs themselves are created as a result of economic development. A strong and dynamic economy depends on fostering economic growth and creating jobs.

3) CREATING A STRONG SUPPLY CHAIN $Ђ_3$

Building a robust supply chain is the process of putting in place a coordinated and effective system of people, organizations, tools, and procedures that cooperate to transfer goods or services from suppliers to final consumers. In order to satisfy consumer needs, manage expenses, and maintain market competitiveness, firms need a robust supply chain. Establishing a robust supply chain involves meticulous planning, consistent investment, and continuing improvement. It's a dynamic process. In addition to ensuring prompt and economical product delivery, an organized and successful supply chain also boosts a business's profitability and competitiveness.

4) INDUSTRIAL PRODUCTS $Ђ_4$

Materials, chemicals, or other substances obtained from agricultural crops are referred to as industrial goods created from crops. These products are utilized as inputs or raw materials in a variety of industrial processes and production. These products are often converted into commodities or raw materials for use in non-agricultural sectors.

The evaluation of a dominant crop is demonstrated without a weight vector by using derived approaches of q-ROFSSPA and q-ROFSSPG operators. Furthermore, we also investigated a suitable crop with the help of additional weight vectors (0.25, 0.35, 0.15, 0.25) based on q-ROFSSPWA and q-ROFSSPWG operators. Further proceeding is explored under the system of the MADM problem.

Step 1: To evaluate a suitable crop for economic stability, the decision maker organizes information about different crops under certain criteria or characteristics in Table 1.

Step 2: The discussed experimental case study carries only one type of information, such as beneficial. So, there is no need to normalize the standard decision matrix into a normalized matrix.

TABLE 1. Carries q-rung orthopair fuzzy information in the decision matrix.

Alternatives	$\mathfrak{H}_1$	$\mathfrak{H}_2$	$\mathfrak{H}_3$	$\mathfrak{H}_4$
$\mathbb{A}_1$	$(0.81, 0.65)$	$(0.61, 0.86)$	$(0.73, 0.73)$	$(0.44, 0.93)$
$\mathbb{A}_2$	$(0.61, 0.85)$	$(0.56, 0.93)$	$(0.65, 0.81)$	$(0.49, 0.89)$
$\mathbb{A}_3$	$(0.71, 0.71)$	$(0.85, 0.61)$	$(0.91, 0.44)$	$(0.69, 0.77)$
$\mathbb{A}_4$	$(0.91, 0.45)$	$(0.73, 0.69)$	$(0.77, 0.66)$	$(0.59, 0.96)$
$\mathbb{A}_5$	$(0.53, 0.86)$	$(0.43, 0.91)$	$(0.55, 0.88)$	$(0.78, 0.69)$

TABLE 2. Carries score values of all alternatives and degree of preferences.

Alternatives	Score Values	$\mathfrak{E}_t$	$\sum_{t=1}^{n}\mathfrak{E}_t$	ω_t
$\mathbb{A}_1$	$\aleph(\mathbb{A}_1) = 0.6284$, $\aleph(\mathbb{A}_2) = 0.2955$, $\aleph(\mathbb{A}_3) = 0.5000$, $\aleph(\mathbb{A}_4) = 0.1404$	$\mathfrak{E}_1 = 1$ $\mathfrak{E}_2 = 0.6284$ $\mathfrak{E}_3 = 0.1857$ $\mathfrak{E}_4 = 0.0928$	1.9069	$\omega_1 = 0.5244$ $\omega_2 = 0.3295$ $\omega_3 = 0.0974$ $\omega_4 = 0.0487$
$\mathbb{A}_2$	$\aleph(\mathbb{A}_1) = 0.3064$, $\aleph(\mathbb{A}_2) = 0.1856$, $\aleph(\mathbb{A}_3) = 0.3716$, $\aleph(\mathbb{A}_4) = 0.2063$	$\mathfrak{E}_1 = 1$ $\mathfrak{E}_2 = 0.3064$ $\mathfrak{E}_3 = 0.0569$ $\mathfrak{E}_4 = 0.0211$	1.3844	$\omega_1 = 0.7223$ $\omega_2 = 0.2213$ $\omega_3 = 0.0411$ $\omega_4 = 0.0153$
$\mathbb{A}_3$	$\aleph(\mathbb{A}_1) = 0.5000$, $\aleph(\mathbb{A}_2) = 0.6936$, $\aleph(\mathbb{A}_3) = 0.8342$, $\aleph(\mathbb{A}_4) = 0.4360$	$\mathfrak{E}_1 = 1$ $\mathfrak{E}_2 = 0.5000$ $\mathfrak{E}_3 = 0.3468$ $\mathfrak{E}_4 = 0.2893$	2.1361	$\omega_1 = 0.4681$ $\omega_2 = 0.2341$ $\omega_3 = 0.1623$ $\omega_4 = 0.1354$
$\mathbb{A}_4$	$\aleph(\mathbb{A}_1) = 0.8312$, $\aleph(\mathbb{A}_2) = 0.5303$, $\aleph(\mathbb{A}_3) = 0.5845$, $\aleph(\mathbb{A}_4) = 0.1603$	$\mathfrak{E}_1 = 1$ $\mathfrak{E}_2 = 0.8312$ $\mathfrak{E}_3 = 0.4408$ $\mathfrak{E}_4 = 0.2576$	2.5296	$\omega_1 = 0.3953$ $\omega_2 = 0.3286$ $\omega_3 = 0.1742$ $\omega_4 = 0.1018$
$\mathbb{A}_5$	$\aleph(\mathbb{A}_1) = 0.2564$, $\aleph(\mathbb{A}_2) = 0.1630$, $\aleph(\mathbb{A}_3) = 0.2425$, $\aleph(\mathbb{A}_4) = 0.5730$	$\mathfrak{E}_1 = 1$ $\mathfrak{E}_2 = 0.2564$ $\mathfrak{E}_3 = 0.0418$ $\mathfrak{E}_4 = 0.0101$	1.3083	$\omega_1 = 0.7643$ $\omega_2 = 0.1960$ $\omega_3 = 0.0319$ $\omega_4 = 0.0077$

Step 3: Here, we compute the degree of preferences based on score values by using $\omega_\iota = \frac{\mathfrak{E}_\iota}{\sum_{\iota=1}^{n}\mathfrak{E}_\iota}$, $\mathfrak{E}_1 = 1$ and $\mathfrak{E}_\iota = \otimes_{\iota=1}^{k-1}\aleph(\mathfrak{R}_\iota)$, $k = 2, 3, \ldots, n$ and stated them in Table 2. We also investigate the degree of weighted preferences by using $\sum_\iota = \frac{\ddot{w}_\iota\mathfrak{E}_\iota}{\sum_{\iota=1}^{n}\mathfrak{E}_\iota}$, where $\mathfrak{E}_1 = 1$ and $\mathfrak{E}_t = \otimes_{k=1}^{\iota-1}\aleph\left(\overline{\overline{\mathfrak{R}}}_\iota\right)$, $(k = 1, 2, 3, \ldots, n)$ shown in Table 3.

Step 4: We applied the drive strategies of the q-ROFSSPA, q-ROFSSPWA, q-ROFSSPG, and q-ROFSSPWG operators and aggregated the collective information, we can also see in Table 4.

Step 5: We find the score values with the help of the given information; we can see it in Table 5.

Step 6: Finally, rank all the preferences to find the most appropriate optimal from the collection of available options or individuals. After analysis raking of score values, we captured $\mathbb{A}_3$ and $\mathbb{A}_4$ are best individuals from the derived weighted average and weighted geometric operators respectively. We also explore the ranking of score values in Table 6.

Figure 1 also explored the results of score values in a graphical shape, which are produced by the derived approaches of q-ROFSSPA, q-ROFSSPWA, q-ROFSSPG, and q-ROFSSPWG operators and shown in Table 6.

D. EFFECT OF DIFFERENT PARAMETRIC VALUES ON THE RESULTS OF THE MADM PROBLEM

To see the advantages and reliability of the proposed algorithm of the MADM technique, different mathematicians explored it by using several methods or techniques. This subsection aims to explore the impact of different parametric values of the Schweizer-Sklar t-norms in the MADM technique. By setting different parametric values in the derived

TABLE 3. covers score values and degree of weighted preferences.

Alternative	Score Values	$\mathfrak{E}_t$	$\ddot{w}_t\mathfrak{E}_t$	$\sum_{t=1}^{n}\ddot{w}_t\mathfrak{E}_t$	$\sum_t$
$\mathbb{A}_1$	$\aleph(\mathbb{A}_1) = 0.6284$,	$\mathfrak{E}_1 = 1$	$\ddot{w}_1\mathfrak{E}_1 = 0.2500$	0.5210	$\sum_1 = 0.4798$
	$\aleph(\mathbb{A}_2) = 0.2955$,	$\mathfrak{E}_2 = 0.6284$	$\ddot{w}_2\mathfrak{E}_2 = 0.2199$		$\sum_2 = 0.4222$
	$\aleph(\mathbb{A}_3) = 0.5000$,	$\mathfrak{E}_3 = 0.1857$	$\ddot{w}_3\mathfrak{E}_3 = 0.0279$		$\sum_3 = 0.0535$
	$\aleph(\mathbb{A}_4) = 0.1404$	$\mathfrak{E}_4 = 0.0928$	$\ddot{w}_4\mathfrak{E}_4 = 0.0232$		$\sum_4 = 0.0445$
$\mathbb{A}_2$	$\aleph(\mathbb{A}_1) = 0.3064$,	$\mathfrak{E}_1 = 1$	$\ddot{w}_1\mathfrak{E}_1 = 0.2500$	0.3711	$\sum_1 = 0.6737$
	$\aleph(\mathbb{A}_2) = 0.1856$,	$\mathfrak{E}_2 = 0.3064$	$\ddot{w}_2\mathfrak{E}_2 = 0.1072$		$\sum_2 = 0.2890$
	$\aleph(\mathbb{A}_3) = 0.3716$,	$\mathfrak{E}_3 = 0.0569$	$\ddot{w}_3\mathfrak{E}_3 = 0.0085$		$\sum_3 = 0.0230$
	$\aleph(\mathbb{A}_4) = 0.2063$	$\mathfrak{E}_4 = 0.0211$	$\ddot{w}_4\mathfrak{E}_4 = 0.0053$		$\sum_4 = 0.0142$
$\mathbb{A}_3$	$\aleph(\mathbb{A}_1) = 0.5000$,	$\mathfrak{E}_1 = 1$	$\ddot{w}_1\mathfrak{E}_1 = 0.2500$	0.5493	$\sum_1 = 0.4551$
	$\aleph(\mathbb{A}_2) = 0.6936$,	$\mathfrak{E}_2 = 0.5000$	$\ddot{w}_2\mathfrak{E}_2 = 0.1750$		$\sum_2 = 0.3186$
	$\aleph(\mathbb{A}_3) = 0.8342$,	$\mathfrak{E}_3 = 0.3468$	$\ddot{w}_3\mathfrak{E}_3 = 0.0520$		$\sum_3 = 0.0947$
	$\aleph(\mathbb{A}_4) = 0.4360$	$\mathfrak{E}_4 = 0.2893$	$\ddot{w}_4\mathfrak{E}_4 = 0.0723$		$\sum_4 = 0.1317$
$\mathbb{A}_4$	$\aleph(\mathbb{A}_1) = 0.8312$,	$\mathfrak{E}_1 = 1$	$\ddot{w}_1\mathfrak{E}_1 = 0.2500$	0.6714	$\sum_1 = 0.3723$
	$\aleph(\mathbb{A}_2) = 0.5303$,	$\mathfrak{E}_2 = 0.8312$	$\ddot{w}_2\mathfrak{E}_2 = 0.2909$		$\sum_2 = 0.4333$
	$\aleph(\mathbb{A}_3) = 0.5884$,	$\mathfrak{E}_3 = 0.4408$	$\ddot{w}_3\mathfrak{E}_3 = 0.0661$		$\sum_3 = 0.0985$
	$\aleph(\mathbb{A}_4) = 0.1603$	$\mathfrak{E}_4 = 0.2576$	$\ddot{w}_4\mathfrak{E}_4 = 0.0644$		$\sum_4 = 0.0959$
$\mathbb{A}_5$	$\aleph(\mathbb{A}_1) = 0.2564$,	$\mathfrak{E}_1 = 1$	$\ddot{w}_1\mathfrak{E}_1 = 0.2500$	0.3485	$\sum_1 = 0.7173$
	$\aleph(\mathbb{A}_2) = 0.1630$,	$\mathfrak{E}_2 = 0.2564$	$\ddot{w}_2\mathfrak{E}_2 = 0.0897$		$\sum_2 = 0.2575$
	$\aleph(\mathbb{A}_3) = 0.2425$,	$\mathfrak{E}_3 = 0.0418$	$\ddot{w}_3\mathfrak{E}_3 = 0.0063$		$\sum_3 = 0.0180$
	$\aleph(\mathbb{A}_4) = 0.5730$	$\mathfrak{E}_4 = 0.0101$	$\ddot{w}_4\mathfrak{E}_4 = 0.0025$		$\sum_4 = 0.0073$

TABLE 4. Aggregated results from derived strategies.

Alternatives	q-ROFSSPA	q-ROFSSPG	q-ROFSSPWA	q-ROFSSPWG
$\mathbb{A}_1$	(0.7555, 0.7148)	(0.6744, 0.7984)	(0.7453, 0.7269)	(0.6635, 0.8103)
$\mathbb{A}_2$	(0.6008, 0.8640)	(0.5961, 0.8801)	(0.5968, 0.8698)	(0.5920, 0.8874)
$\mathbb{A}_3$	(0.8167, 0.6038)	(0.7557, 0.6800)	(0.8078, 0.6261)	(0.7558, 0.6827)
$\mathbb{A}_4$	(0.8499, 0.5485)	(0.7635, 0.8004)	(0.8451, 0.5548)	(0.7586, 0.7970)
$\mathbb{A}_5$	(0.5203, 0.8674)	(0.5041, 0.8739)	(0.5147, 0.8701)	(0.4963, 0.8775)

TABLE 5. Carries score values of alternatives obtained by proposed approaches.

Alternatives	q-ROFSSPA	q-ROFSSPG	q-ROFSSPWA	q-ROFSSPWG
$\mathbb{A}_1$	0.5330	0.3989	0.5150	0.3801
$\mathbb{A}_2$	0.2859	0.2650	0.2773	0.2543
$\mathbb{A}_3$	0.6622	0.5586	0.6409	0.5567
$\mathbb{A}_4$	0.7245	0.4661	0.7164	0.4652
$\mathbb{A}_5$	0.2441	0.2303	0.2388	0.2233

approaches, we can reveal the effectiveness of the proposed MADM problem algorithm under the q-ROF information system.

Table 7 ranked score values corresponding to each alternative at different parametric values of the Schweizer-Sklar t-norms in q-ROFSSPWA operators. We clearly noticed the ranking of score values $\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_2 > \mathbb{A}_5$ at $Ћ = -1, -5$. Furthermore, we can examine the ranking of all score values for different parametric values of the Schweizer-Sklar t-norms $Ћ < -15$ in Table 7.

TABLE 6. Ranking of alternatives under considering results of score values.

Aggregation operators	Ranking and ordering
q-ROFSSPA	$\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_2 > \mathbb{A}_5$
q-ROFSSPG	$\mathbb{A}_3 > \mathbb{A}_4 > \mathbb{A}_1 > \mathbb{A}_2 > \mathbb{A}_5$
q-ROFSSPWA	$\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_2 > \mathbb{A}_5$
q-ROFSSPWG	$\mathbb{A}_3 > \mathbb{A}_4 > \mathbb{A}_1 > \mathbb{A}_2 > \mathbb{A}_5$

TABLE 7. Covered ranking results by the q-ROFSSPWA operator at different values of Ђ < 0.

Parametric values	Score values	Ranking and ordering
Ђ $= -1$	$\aleph(\mathbb{A}_4) > \aleph(\mathbb{A}_3) > \aleph(\mathbb{A}_1) > \aleph(\mathbb{A}_2) > \aleph(\mathbb{A}_5)$	$\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_2 > \mathbb{A}_5$
Ђ $= -5$	$\aleph(\mathbb{A}_4) > \aleph(\mathbb{A}_3) > \aleph(\mathbb{A}_1) > \aleph(\mathbb{A}_2) > \aleph(\mathbb{A}_5)$	$\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_2 > \mathbb{A}_5$
Ђ $= -15$	$\aleph(\mathbb{A}_4) > \aleph(\mathbb{A}_3) > \aleph(\mathbb{A}_1) > \aleph(\mathbb{A}_5) > \aleph(\mathbb{A}_2)$	$\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_5 > \mathbb{A}_2$
Ђ $= -25$	$\aleph(\mathbb{A}_4) > \aleph(\mathbb{A}_3) > \aleph(\mathbb{A}_1) > \aleph(\mathbb{A}_5) > \aleph(\mathbb{A}_2)$	$\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_5 > \mathbb{A}_2$
Ђ $= -40$	$\aleph(\mathbb{A}_4) > \aleph(\mathbb{A}_3) > \aleph(\mathbb{A}_1) > \aleph(\mathbb{A}_5) > \aleph(\mathbb{A}_2)$	$\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_5 > \mathbb{A}_2$
Ђ $= -55$	$\aleph(\mathbb{A}_4) > \aleph(\mathbb{A}_3) > \aleph(\mathbb{A}_1) > \aleph(\mathbb{A}_5) > \aleph(\mathbb{A}_2)$	$\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_5 > \mathbb{A}_2$
Ђ $= -65$	$\aleph(\mathbb{A}_4) > \aleph(\mathbb{A}_3) > \aleph(\mathbb{A}_1) > \aleph(\mathbb{A}_5) > \aleph(\mathbb{A}_2)$	$\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_5 > \mathbb{A}_2$
Ђ $= -75$	$\aleph(\mathbb{A}_4) > \aleph(\mathbb{A}_3) > \aleph(\mathbb{A}_1) > \aleph(\mathbb{A}_5) > \aleph(\mathbb{A}_2)$	$\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_5 > \mathbb{A}_2$
Ђ $= -90$	$\aleph(\mathbb{A}_3) > \aleph(\mathbb{A}_4) > \aleph(\mathbb{A}_1) > \aleph(\mathbb{A}_5) > \aleph(\mathbb{A}_2)$	$\mathbb{A}_3 > \mathbb{A}_4 > \mathbb{A}_1 > \mathbb{A}_5 > \mathbb{A}_2$
Ђ $= -100$	$\aleph(\mathbb{A}_3) > \aleph(\mathbb{A}_4) > \aleph(\mathbb{A}_1) > \aleph(\mathbb{A}_5) > \aleph(\mathbb{A}_2)$	$\mathbb{A}_3 > \mathbb{A}_4 > \mathbb{A}_1 > \mathbb{A}_5 > \mathbb{A}_2$

FIGURE 1. Shows the findings of derived strategies.

Table 8 also covered the ranking of score values obtained by the q-ROFSSPWG operators for different parametric values of the Schweizer-Sklar t-norms. We noticed that $\mathbb{A}_3$ be an appropriate optimal option acquired by using different parametric values of Ђ in q-ROFSSPWG operators. By using this technique, decision-makers acquired results according to their preferences. Based on the derived approaches of the q-ROFSSPWA and q-ROFSSPWG operators, we noticed that different parametric values play important roles in the decision-making process.

VI. COMPARATIVE STUDY

The aim of this section is to compare the findings of currently proposed mathematical approaches in the light of Schweizer-Sklar t-norms with well-known existing methodologies seen in [38], [40], [41], [42], [43], [44], and [45]. To serve this purpose, we applied aggregation approaches under considering our proposed algorithm of the MADM problem to show the applicability and compatibility of derived strategies. Darko and Liang [38] elaborated the concepts of Hamacher aggregation tools in the light of q-ROF information and developed new AOs such as q-ROF Hamacher weighted average (q-ROFHWA) and q-ROF Hamacher weighted geometric (q-ROFHWG) operators. Some robust AOs based on q-ROF information like q-ROF Frank weighted average (q-ROFFWA) and q-ROF Frank weighted geometric (q-ROFFWG) operators presented by Seikh and Mandal [40]. Jana et al. [41] developed AOs based on the operations of the Dombi aggregation tools, namely q-ROF Dombi weighted average (q-ROFDWA) and q-ROF Dombi weighted geometric (q-ROFDWG) operators. By using dominant operations of Aczel Alsina t-norms q-ROF Aczel Alsina weighted average (q-ROFAAWA) and q-ROF Aczel Alsina weighted geometric (q-ROFAAWG) operators developed by the Khan et al. [43] and Farid and Riaz [42]

TABLE 8. Covered ranking results by the q-ROFSSPWG operator at different values of $\Pi\!\!\!\flat < 0$.

Parametric values	Score values	Ranking and ordering
$\Pi\!\!\!\flat = -1$	$\aleph(A_3) > \aleph(A_4) > \aleph(A_1) > \aleph(A_2) > \aleph(A_5)$	$A_3 > A_4 > A_1 > A_2 > A_5$
$\Pi\!\!\!\flat = -5$	$\aleph(A_3) > \aleph(A_4) > \aleph(A_1) > \aleph(A_2) > \aleph(A_5)$	$A_3 > A_4 > A_1 > A_2 > A_5$
$\Pi\!\!\!\flat = -15$	$\aleph(A_3) > \aleph(A_4) > \aleph(A_2) > \aleph(A_5) > \aleph(A_1)$	$A_3 > A_4 > A_2 > A_5 > A_1$
$\Pi\!\!\!\flat = -25$	$\aleph(A_3) > \aleph(A_4) > \aleph(A_2) > \aleph(A_5) > \aleph(A_1)$	$A_3 > A_4 > A_2 > A_5 > A_1$
$\Pi\!\!\!\flat = -40$	$\aleph(A_3) > \aleph(A_4) > \aleph(A_5) > \aleph(A_2) > \aleph(A_1)$	$A_3 > A_4 > A_5 > A_2 > A_1$
$\Pi\!\!\!\flat = -55$	$\aleph(A_3) > \aleph(A_4) > \aleph(A_5) > \aleph(A_2) > \aleph(A_1)$	$A_3 > A_4 > A_5 > A_2 > A_1$
$\Pi\!\!\!\flat = -65$	$\aleph(A_3) > \aleph(A_5) > \aleph(A_4) > \aleph(A_2) > \aleph(A_1)$	$A_3 > A_5 > A_4 > A_2 > A_1$
$\Pi\!\!\!\flat = -75$	$\aleph(A_3) > \aleph(A_5) > \aleph(A_4) > \aleph(A_2) > \aleph(A_1)$	$A_3 > A_5 > A_4 > A_2 > A_1$
$\Pi\!\!\!\flat = -90$	$\aleph(A_3) > \aleph(A_5) > \aleph(A_4) > \aleph(A_2) > \aleph(A_1)$	$A_3 > A_5 > A_4 > A_2 > A_1$
$\Pi\!\!\!\flat = -100$	$\aleph(A_3) > \aleph(A_5) > \aleph(A_4) > \aleph(A_2) > \aleph(A_1)$	$A_3 > A_5 > A_4 > A_2 > A_1$

TABLE 9. Carries results of score values and their ranking captured by the existing strategies.

Aggregation operators	Score values	Ranking of score values
q-ROFSSPWA	$\aleph(A_4) > \aleph(A_3) > \aleph(A_1) > \aleph(A_2) > \aleph(A_5)$	$A_4 > A_3 > A_1 > A_2 > A_5$
q-ROFSSPWG	$\aleph(A_3) > \aleph(A_4) > \aleph(A_1) > \aleph(A_2) > \aleph(A_5)$	$A_3 > A_4 > A_1 > A_2 > A_5$
q-ROFFWA by Seikh and Mandal **[40]**	$\aleph(A_3) > \aleph(A_4) > \aleph(A_1) > \aleph(A_5) > \aleph(A_2)$	$A_3 > A_4 > A_1 > A_5 > A_2$
q-ROFFWG by Seikh and Mandal **[40]**	$\aleph(A_3) > \aleph(A_1) > \aleph(A_5) > \aleph(A_2) > \aleph(A_4)$	$A_3 > A_1 > A_5 > A_2 > A_4$
q-ROFHWA by Darko and Liang **[38]**	$\aleph(A_3) > \aleph(A_4) > \aleph(A_1) > \aleph(A_5) > \aleph(A_2)$	$A_3 > A_4 > A_1 > A_5 > A_2$
q-ROFHWG by Darko and Liang **[38]**	$\aleph(A_3) > \aleph(A_1) > \aleph(A_5) > \aleph(A_2) > \aleph(A_4)$	$A_3 > A_1 > A_5 > A_2 > A_4$
q-ROFDWA by Jana et al. **[41]**	$\aleph(A_3) > \aleph(A_4) > \aleph(A_1) > \aleph(A_5) > \aleph(A_2)$	$A_3 > A_4 > A_1 > A_5 > A_2$
q-ROFDWG by Jana et al. **[41]**	$\aleph(A_3) > \aleph(A_4) > \aleph(A_1) > \aleph(A_5) > \aleph(A_2)$	$A_3 > A_4 > A_1 > A_5 > A_2$
q-ROFAAWA by Farid and Riaz **[42]**	$\aleph(A_3) > \aleph(A_4) > \aleph(A_1) > \aleph(A_5) > \aleph(A_2)$	$A_3 > A_4 > A_1 > A_5 > A_2$
q-ROFAAWG by Khan et al. **[43]**	$\aleph(A_1) > \aleph(A_3) > \aleph(A_5) > \aleph(A_2) > \aleph(A_4)$	$A_1 > A_3 > A_5 > A_2 > A_4$
Ahmed and Dai **[44]**	Not applicable	
Al-Quran **[45]**	Not applicable	

respectively. However, some previously presented mathematical approaches established by [44] and [45], cannot handle given information discussed in an experimental case study due to incomplete information of the human opinion. Table 9 also carried the results of all existing approaches by different mathematicians.

We noticed that derived approaches of the q-ROFSSPWA and q-ROFSSPWG operators based on Schweizer-Sklar t-norms are more flexible and dominant. It also plays an efficient role in the MADM problem and produces preferable results of alternatives by using parametric values of Schweizer-Sklar t-norms. Figure 2 and Figure 3 show the

FIGURE 2. Shows the findings of existing strategies.

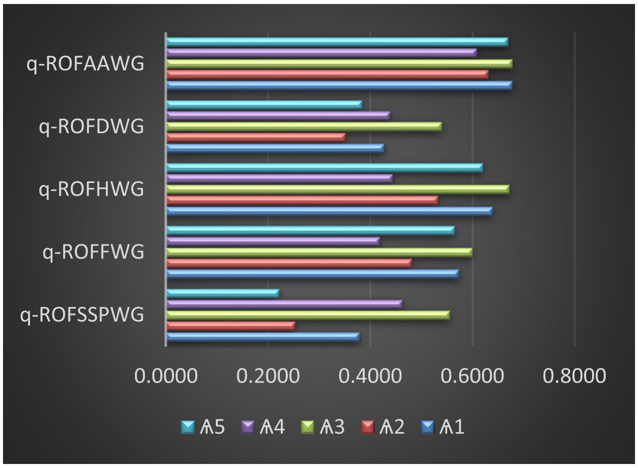

FIGURE 3. Shows the findings of existing strategies.

graphical behavior of score values obtained by the existing weighted average and geometric operators, respectively.

VII. CONCLUSION

The crops are very important for the economy of any nation. They are an important part of the agricultural industry and essential to the economic growth. A nation's economic health largely depends on its agricultural production, which supports millions of people's lives across various industries. Prosperity in general, food security, and economic growth may all be greatly aided by implementing sustainable farming techniques, effective agricultural policy, and investments in crop production. In this proposed research work, we studied innovative approaches to the MADM problem to evaluate a suitable crop under dominant criteria or attributes. Motivated by the theory of prioritized aggregation operators and flexible operations of Schweizer-Sklar t-norms, we developed new AOs in the light of q-ROF information including q-rung orthopair fuzzy Schweizer-Sklar prioritized average (q-ROFSSPA), q-rung orthopair fuzzy Schweizer-Sklar prioritized weighted average (q-ROFSSPWA), q-rung orthopair fuzzy Schweizer-Sklar prioritized geometric (q-ROFSSPG) and q-rung orthopair fuzzy Schweizer-Sklar prioritized weighted geometric (q-ROFSSPWG) operators. Some particular properties and characteristics are also demonstrated to show the flexibility and robustness of derived approaches. In order to explore the applicability of an algorithm of the MADM technique, we established a numerical example to select a suitable crop to improve the financial sector of farmers. The feasibility and robustness of derived approaches are revealed by contrasting the findings of previous mathematical strategies with developed approaches.

We noticed our derived methodologies are preferable. Sometimes, developed strategies cannot handle ambiguous and uncertain information of human opinions when given information in three components. To serve this situation, we will explore proposed research on different fuzzy frameworks like picture fuzzy sets [46], t-spherical fuzzy hypersoft theory [47], spherical fuzzy theory [48] and complex spherical sets [49]. By utilizing developed methodologies, we try to resolve complicated challenges of real life, such as artificial intelligence, game theory, medical diagnosis, green supplier selection, and many other applications.

AUTHOR CONTRIBUTIONS

All authors agreed to the manuscript.

DATA AVAILABILITY STATEMENT

The datasets used and/or analyzed during the current study are available from the corresponding author on reasonable request.

CONFLICTS OF INTEREST

The authors declare no conflict of interest.

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