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RESEARCH ARTICLE

Fuzzy Decision Support Systems for Selection of NEA Detection Technologies Under Non-Linear Diophantine Fuzzy Hamacher Aggregation Information

MARIA SHAMS[1], SALEEM ABDULLAH[1], FAISAL KHAN[2], RIFAQAT ALI[3], AND SHAKOOR MUHAMMAD[1]

[1]Department of Mathematics, Faculty of Physical and Numerical Sciences, Abdul Wali Khan University Mardan, Mardan, Khyber Pakhtunkhwa 23200, Pakistan
[2]Department of Electrical and Electronic Engineering, College of Science and Engineering, National University of Ireland Galway, Galway, H91 TK33 Ireland
[3]Department of Mathematics, College of Science and Arts in Muhayil, King Khalid University, Abha 61413, Saudi Arabia

Corresponding authors: Saleem Abdullah (saleemabdullah@awkum.edu.pk) and Faisal Khan (f.khan4@nuigalway.ie)

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ABSTRACT The Objectives of this study is to extend the concept of q-rung linear Diophantine fuzzy sets (q-RLDFSs), followed by the Near-Earth Asteroids (NEAs) deflection detector. The q-RLDFS is more superior than linear Diophantine fuzzy sets (LDFSs) because of the qth power of reference parameters (RPs). In the present work, first we have recall the q-RLDFS and named it Non-linear Diophantine Fuzzy set (N-LDFS) further, introduce some operational rules on N-LDFS under Hamacher sum and Hamacher product. Hamacher Norms were commonly referred to as Hamacher operations, but their applications might be better expressed if they are presented with a new level of flexibility within the general parameter. Hamacher operations have not yet been applied for N-LDFS in a suitable form. Therefore we have apply the Hamacher operators for N-LDFS and develop a new area of research in decision making problems. We have apply the Hamacher operators to develop Non-linear Diophantine fuzzy aggregation operators from geometric point of view such as non-linear Diophantine fuzzy Hamacher weighted geometric (N-LDFHWG), non-linear Diophantine fuzzy Hamacher ordered weighted geometric (N-LDFHOWG) and non-linear Diophantine fuzzy Hamacher hybrid weighted geometric (N-LDFHHWG) aggregation operators. We will also establish commutativity, idempotency and monotonicity properties which are the most desirable properties for proposed operators. Ultimately, we have implement a case study regarding the decision support method to pick the best NEA deflection detector. We construct an algorithm to solve the problem of multi-attribute decision-making (MADM) problem, followed by a fun of application using the N-LDFHWG operator. A comparison between the proposed and existing methods is perform from the geometric point of view. Finally, the comparisons demonstrate the effectiveness and superiority of the proposed method.

INDEX TERMS Hamacher norms, q-rung linear diophantine fuzzy set (q-RLDFS), non-linear diophantine fuzzy Hamacher weighted geometric (N-LDFHWG) operators, near-earth asteroid (NEA), MADM.

I. INTRODUCTION

A steroids are rocky worlds that are too small, revolving around the sun, to be considered planets. Sometimes, they are also referred to as planetoids or minor planets. Asteroids were leftovers from the emergence of our solar system about 4.6 billion years ago [1]. Early on, in the distance between Mars and Jupiter, the birth of Jupiter stopped any planetary bodies from developing, causing the little objects that were there to intersect with each other and fragment into the

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asteroids found nowadays [2]. Meteoroids are small rocks and fragile aggregates that form as asteroids and comets decay and fall to Earth. As a result, the microscopic dust that hits the planet every day amounts to roughly 100 tons [3] on average. Larger objects are indeed unlikely to enter the Earth's orbit, and a possible impactor could have a dramatic impact on the planet's life and atmosphere. The break-up of the object at an altitude of 30 km caused more than 1500 injuries [4], [5], [6]. Asteroids whose perihelion length is less than 1.3 AU are Near Earth Objects (NEOs) (about 195 million km). In this way, the ultimate aim of the researchers is on detecting near-Earth asteroids (NEAs) [4], [7].

The study of NEAs is sponsored by three main factors, namely planetary defense, scientific awareness (e.g. deepening our origins in the Solar System), and mining. The current article is dedicated to planetary defense in this regard. It is known that several thousands of asteroids large enough to survive the Earth's atmosphere and hit the surface of the planet are within 0.05AU (about 7.5 million kilometers) of Earth's orbit as they orbit around the sun [8]. These Near-Earth Asteroids (NEAs) are known as "Potentially Hazardous Objects" because of the perceived threat of colliding with our planet one day [9]. George E. Brown, Jr. was conducted by a research team organized by NASA's Office of Program and Evaluation in reaction to a Congressional mandate [10]. The NEO Survey Program aims to provide guidelines for a survey of Near-Earth Objects (NEOs, i.e. asteroids and comets within 1.3 AU of Earth) and to examine potential ways of diverting an object to Earth on a probable collision course, in 2007 they issued a report [9].

A study sent to Congress at the beginning of March 2007 was the result of this directive. This was a review of the Analysis of Alternatives (AoA) guided by the Program Analysis and Evaluation (PA&E) office of NASA with the help of external consultants, the Aerospace Corporation, the Langley Research Center (LaRC) of NASA, and the SAIC Research Center (LaRC) (amongst others) [9]. The Double Asteroid Redirection Test (DART) is a planned space probe that visits the Double Asteroid Didymos and shows the kinetic effects for planetary defense purposes of crashing an impactor spacecraft into an asteroid moon [11]. The mission is conducted to examine whether the impact of a spacecraft on a collision course with Earth will successfully deflect an asteroid. An asteroid deflection demonstration is a hugely important test that NASA and other agencies want to conduct until there is an actual need for planetary protection. DART is a NASA-Johns Hopkins Applied Physics Laboratory (APL) joint project and is being established under the auspices of the Planetary Defense Coordination Office of NASA. NASA approved the project in August 2018 to start the final phase of design and assembly [12].

The principal objective of this study is to test five deflection technologies namely, kinetic impactor KI), enhanced gravity tractor (EGT), ion beam deflection (IBD), laser ablation (LA), and conventional rocket engine (CRE) concerning the following list of criteria: NEA deflection technology maturity level, the structure of asteroid, composition of the asteroid, shape of the asteroid, and mission risk [13]. With this aim, we shall apply a novel combination of MADM with the N-LDF approach to the context of Hamacher operators. Many researchers have recently made interesting case study related to their work which improved the research field and give us rich knowledge some of them is cited as [14], [15], [16], [17], and [18].

Literature Review:

In 1965, Zadeh [19] introduced the fuzzy set (FS) notion with membership grade (MG), which is a helpful tool for handling ambiguous and uncertain information in daily life. Apart from FSs, he also presented a significant term called linguistic variables (LVs) [20]. Using LVs, we can convert verbal information into mathematical expressions and solve MADM problems with various mathematical approaches. Hamacher-norms [21], which extend algebraic and Einstein-norms [22], [23], are more general and versatile. Hamacher operations [24], namely Hamacher sum and product, are strong alternatives respectively to the algebraic product and sum. A lot of researchers have been researching the Hamacher aggregation operators and their implementations to multiple attribute group decision-making problems (MAGDMPs) in recent years [21], [25]. In 2021, Wang et al. [26] developed an interactive Hamacher operation for Pythagorean fuzzy set (PyFS). Akram et al. [27] described the complex IF Hamacher. Garg et al. presented [28] for Hamacher Norms. Various notions of Fermatean FSs with Hamacher operators were introduced by Hadi et al. [29].

A meaningful work is how to generalize the Hamacher operations to develop the Non-linear Diophantine fuzzy (N-LDF) information, which is the primary subject of this paper.

An extension of FS called intuitionistic FS (IFS) [30] specifies the MG and non-membership grade (NMG) with the restriction that the sum of MG and NMG bound between [0, 1]. IFS defines the human perspective as yes or no. Atanassov also describes the geometric representation for IF objects [31]. Several scholars have used the idea of IFS, including [32], [33], [34] such as interval-valued IFS (IVIFS) and Einstein Norms. Aside from this, it is probable that IFS worked the past thirty years and collecting the particular attention of the researchers [35], [36], [37], and [38]. Yang et al. [39] proposed Belief and Plausibility Measures for IFSs, Ali et al. [40] proposed Hausdroff distance for single-valued neutrosophic sets (SVNSs) and Ali et al. [41] presented the idea of Correlation Coefficient for T-Spherical FS. Yager [42], [43] introduced the Pythagorean fuzzy set (PyFS), which is the generalized version of IFS that satisfies the restrictions that the square's sum of MG and NMG bound between 0 and 1. Yager and Abbasov studied [44] for PyFNs. Furthermore, Yager [43] presented some aggregation operators (AOs) based on PyFS. Some advanced forms of PyFNs are described in [45] and [46]. The concept of correlation and correlation coefficients of PyFSs were defined by Garg [47]. Zeng provided [48] and Garg presented [49], [50] on the based of PyF information. Different PyF Dombi operators were proposed by Akram et al. [51], who also investigated how they may be used in MCDM. In decision-making, Shahzadi et al. [52] presented PyF Yager operators. In a complex PyF environment, Akram and Naz [53], [54] suggested Dombi aggregating operators. The Hamacher aggregation operator (AO) were established by Mahmood et al. [55] using bipolar complex fuzzy (BCF) data. The Dombi AO for BCF information were also provided by Mahmood and Ur Rehman [56]. Additionally, Mahmood et al. [57] derived the Aczel-Alsina AO under the BCFS model. Rehman et al. [58] looked up an AHP technique for BCFS depending on Frank AO.

Yager [59] and other scholars [60], [61] proposed another concept named q-rung orthopair fuzzy set (q-ROFS), which is a more powerful method for defining the data vagueness and extending the space for IFS and PyFS. q-ROFS is also defined by MG and NMG, but with qth power restriction, i.e. $0 \leq MG^q + NMG^q \leq 1, q \geq 1$. It is clear that q-ROFS is more generalized than IFS and PyFS, and that by fixing $q = 1$ and $q = 2$ reduces the corresponding set to IFSs and PyFSs. Many types of research have been done on q-ROFSs recently, few of them are cited as [62] and [63] which is based on the basic operational laws of q-ROFNs. On the base of aggregation operators (AOs) q-ROFNs has also a rich contribution described in [64] and [65]. Many researchers have applied the traditional methods for the ranking of alternatives, such as [66], distance measures [67], similarity measures [68], and in MAGDM problems [69], [70]. Similarly the weighted Heronian mean (HM) [71], the weighted partitioned HM [72], the weighted Maclaurin symmetric mean (MSM) [73], the weighted power partitioned MSM [74] and the weighted point operators [65].

Riaz and Hashmi [75] developed the Linear Diophantine Fuzzy Set (LDFS), which solves the limitation of existing methods by adding the reference parameters (RPs) to MG and NMG. The model of LDFS is more accurate and effective than other fuzzy models due to the presence of RPs. The sum of RPs with product to MG and NMG, respectively, is bounded between 0 and 1 in LDFSs. However, in some real-life problems, the sum of the RPs that an alternative satisfies the criteria achieved by DM is often greater than one, so LDFS has restricted itself to achieving its RPs target.

The concept of q-rung linear Diophantine fuzzy set (q-RLDFS) were proposed by Almagrabi et al. [76]. They performed it by adding the qth power to reference parameters (RPs) that covered the space of the existing structure of MG and NMG related to RPs. The concept of q-RLDFS is also described by the MG, NMG, and the RPs, whose sum of the qth power to RPs related to MG and NMG is bound between zero and one. In the case of LDFS, the sum of RPs given by DM may be greater than one, i.e. $\alpha + \beta > 1$, which violates the LDFS restriction and limits the MADM problem. The concept of q-RLDFS is capable of dealing with this condition to eradicate the LDFS contradiction. Note that as the qth increases, the space of acceptable Diophantine increases, and more Diophantine satisfy the boundary constraint. The key benefit of the q-RLDF approach is that it takes into account the qth power of RPs, indicating that it is suitable for dealing with real-world DM problems. The concept of Complex LDFSs (CLDFSs) were presented by [77] in 2022. The idea of LDFSs has been employed by many scholars in a range of domains, including [78], [79], [80], [81], [82]. In LDFS, the RPs have their own restrictions. Almagrabi et al. [76] suggested a novel LDF extension known as the q-rung linear Diophantine fuzzy set (q-RLDFS) and examined its key characteristics. An innovative method for describing uncertainty in decision-making is the idea of q-RLDFSs. The q-RLDFS is more flexible and reliable than existing q-ROFSs, and LDFSs because it combines the qth power of RFs with MG and NMG also presented in [83]. Later in [84] give it the name as Non-Linear Diophantine fuzzy set (N-LDFS). Shams et al. [84] offered the theory of CN-LDFS with complex valued-qth power of RPs (CV-RPs) related with there exponential, to overcome the restrictions of complex LDFS. The concept of CN-LDFS eliminates the limits for MG/NMG and RPs, and the expert makers are freely select the desired grades with CV-RPs. Further in 2023, Shams and Abdullah [85] extended the concept of CN-LDFS [84] to Dombi Norms and developed the CN-LDF Dombi operators for decision making problems. So we motivated from the said literature review and applied the Hamacher operators on N-LDFNs and extend it to N-LDF Hamacher weighted operators.

Review of Non-linear Diophantine fuzzy set:

Recall that the framework presented by [76] is identical to the well-known LD equation $ax + by = c$ in number theory, and the addition of the qth power of RPs gives it the name q-RLDFS, which is the most suitable name for the developed framework, further this name is extended to N-LDFS.

A question arises why we needed the N-LDFS or what are the boundaries of LDFSs that leads us to N-LDFSs.? The limitation of LDFSs is its restriction, i.e. $0 \leq \alpha u_{D(\hbar)} + \beta \Re_{D(\hbar)} \leq 1$ because this condition does not support decision-makers to give their consent to MG and NMG values. In a specific domain, the decision-makers are somehow bound so the need of N-LDFS is developed. We will use the Hamacher operators for N-LDF data as a result of the above discussion's motivation and inspiration, so a new series of aggregation N-LDF data is developed in this study. Based on Hamacher Norms, novel Non-linear Diophantine fuzzy Hamacher aggregation operators are established. We note that there are no Hamacher aggregation operators in the literature for dealing with ambiguity in practical problems using novel N-LDFSs, so we are going to present this. Three objectives associated with our presented approach are to clarify the concept of N-LDFSs:

(1) With the latest N-LDFS method, our first objective is to address this information gap. This approach, which is influenced by RPs, can cover the limitation of existing methods. Suppose an example that explains the concept of the N-LDFS, $(0.85)(0.6)+(0.95)(0.7) > 1$ which limited the LDF concept, by putting $q = 2$ to the RPs (e.g $(0.85)(0.6)^2 + (0.95)(0.7)^2 < 1$), where $\langle 0.6, 0.7\rangle$ is RPs, for MG and NMG respectively, which fulfill and handle the lack of qth power. Let us consider another example that explains N-LDFS, $(1)(0.75) + (0.87)(0.9) > 1$ which contradicts the condition of LDFS, by setting $q = 4$ on the RPs (e.g $(1)(0.75)^4 + (0.87)(0.9)^4 < 1$), where $\langle 0.75, 0.9\rangle$ is RPs, for MG and NMG respectively which fulfill the lack of qth power. That's why we need N-LDFS, which can handle the limitation of LDFS with the help of the qth power of RPs. (2) The implementation of the qth power of RPs in N-LDFS is the second objective. If we take $q = 1$, then N-LDFS will reduced to LDFS. Furthermore, as the rung q increases, the diophantine space expands, provide the boundary limits a larger search space to express a wider range of fuzzy data.

(3) The third goal is to establish a strong link between the current research and MADM problems. We have developed algorithm to deal with multi-attribute complexities in a parametric way. Surprisingly, all N-LDFHWG aggregation operators by applying the suggested algorithm have the same outcome.

Objectives of study:

The following are the summarized objectives of this work:

(i) To construct a new notion of Non-Linear Diophantine fuzzy sets (N-LDFSs) based on Hamacher Norms and to construct their operational laws.

(ii) To create a list of aggregation operators from geometric point of view based on Hamacher norms, as well as explain the associated properties.

(iii) To develop a decision-making (DM) methodology using proposed aggregation operators to aggregate the uncertain information in DM real-world problems.

(iv) To show the effectiveness and flexibility of the proposed method, a numerical case study of a real-life problem regarding to the selection of the best NEA deflection detector technologies is addressed.

Contribution of the study: It can be concluded from the aforementioned literature study that there are no specific implementations of N-LDFS based on the Hamacher t-norm and t-conorm to rocky world decision support models for choosing the best NEA deflection detector. To address ambiguity and uncertainty in N-LDFS contexts, this research aims to improve N-LDFS aggregation operators using Hamacher norms. There are three primary phases in MADM problem, where the decision-making process determines the best alternative. The decision model's structure, which is used to collect data information for each alternative based on defined criteria by each decision expert, is the first step in the MADM process. The second process then starts with the data information of all alternatives based on predetermined criteria provided by each expert's decision matrix, and this will be normalized if necessary. Making a final decision regarding the best alternative is the last phase in the decision-making process. The following are the paper's key contributions:

(1) The fundamental operations of Non-Linear Diophantine fuzzy numbers (N-LDFNs) are taken into account, and they are improved from the earlier Hamacher T-norm and T-conorm to the modified Hamacher for N-LDFNs.

(2) In addition, the suggested Hamacher operational laws-based aggregation operators for N-LDFNs are provided and establish Non-linear Diophantine fuzzy Hamacher weighted geometric aggregation operators.

(3) To determine the result for decision-makers as well as for alternatives, the score, quadratic, and expectation functions are developed.

(4) The N-LDFNs is used to evaluate the multi-attribute decision-making (MADM) problem assuming weight information is known.

(5) We create MADM problems which are automated decision-making processes based on input data.

(6) Based on the defined alternatives, the N-LDF-Hamacher aggregation operator is taken into account and choose the best NEA deflection detector technology.

Novelties of the study: We have extended the concept of N-LDFS by using Hamacher norms, which become a more generalized concept because Hamacher operations have not yet been applied for non-linear Diophantine fuzzy sets (N-LDFSs) in a suitable form. Therefore we applied the Hamacher operators for N-LDFS and developed a new area of research in decision-making problems. The above study and discussion make it abundantly evident that Hamacher operations have a built-in capacity for modification and resilience, enabling them to more successfully illustrate both the data and ambiguous real-life challenges. In Hamacher operations, the behavior of the standard operational parameter gamma's is more significant in expressing the decision maker's mind. When using the proposed technique, different values are employed for the operational parameter to evaluate the professional experts' ranking results. To the best of our collective understanding, no implementation of Hamacher operators with the hybrid study of N-LDFS by using N-LDF geometric aggregation operators (AOs) has been established in an N-LDF environment. Through MADM, the effectiveness of the created N-LDF Hamacher geometric operators is demonstrated. The combination of Hamacher Norms and N-LDFS distinguishes the proposed technique from others. In light of this, the current study was motivated to analyze geometric operators, such as the N-LDFHWG, N-LDFHOWG, and N-LDFHHWG AOs, and to thoroughly examine their desired qualities. The suggested aggregation operators (AOs) successfully capture the link between multiple attributes by adding extra qth power to reference parameters alongside with parameter gamma in Hamacher t-norm and t-conorm procedures. The suggested AOs' adaptability comes from their capacity to set the parameters q and gamma to certain values, offering DMs a variety of alternatives. The N-LDF Hamacher operator when combined with AOs is a concept that is introduced in the article. To express data complexity, N-LDFH is used as a combination of N-LDF and Hamacher, the qth power increasing the dominance of the N-LDFS and this concept extended the research area in fuzzy modeling because in FS there is a lack of non-membership, similarly IFS, PyFS and q-ROFS have no reference parameter. So the development of LDFS cover all the limitation of existing methods which introduced the reference parameters, but LDFS is also limited to the RPs, therefore q-RLDFS were developed in which the introduction of qth power increased the Diophantine space. Thus we have extend the concept of q-RLDFS to Hamacher Norms and named it N-LDF Hamacher operators. To evaluate the usefulness of the proposed N-LDFH model, a numerical analysis is presented as a case study for selecting the best detection technology for Near-Earth Asteroids (NEA). Furthermore, a comparative analysis of the existing methods with proposed operators and a sensitivity analysis is conducted to investigate their superiority and flexibility.

The layout is structured for this paper as; Extension of FS (PyFS, q-ROFS, LDFS and q-RLDFS) are offers in Section II. "Section III" is about the concept of N-LDFS, and we also developed N-LDFS operations based on Hamacher Norms. "Section IV provides" N-LDF Hamacher aggregation operators such as N-LDFHWG, N-LDFHOWG, and N-LDFHHWG operators and some desirable properties of the proposed operators. "Section V presents" the novel algorithm for N-LDF data based on Hamacher operators. "Section VI addressed" the decision frames and a case study related to the assessment of the NEA deflection detectors technology problem. And there is also a numerical example which demonstrate the application of the proposed method by using the proposed algorithm based on Hamacher operators under the N-LDF environment. "Section VII describes" an overview comparison of the proposed approach with some existing method. "Section VIII" discusses the conclusion and future directions. Following Table 1(a) represented the detailed description of acronym used in this work. Similarly, Table 1(b) summarized the representation of all the variables and parameters used in this paper.

Following Table 1(b) represented the variables/parameters and their representation and symbols in this work.

TABLE 1. (a) Description of acronyms used in this work. (b) Representation of variable/parameters used in this work.

(a)

Acronym	Description
NEAs	Near-Earth Asteroids
MADM	Multi attribute decision making
DM	Decision-maker
FS	Fuzzy Set
MG	Membership grade
NMG	Non-membership grade
AOs	Aggregation operators
IFS	Intuitionistic fuzzy set
IVIFS	Interval-valued IFS
PyFS	Pythagorean fuzzy set
BCFS	Biopolar complex fuzzy set
q-ROFS	q-rung orthopair fuzzy set
q-ROFN	q-rung orthopair fuzzy number
HM	Heronian mean
MSM	Maclaurin symmetric mean
LDFS	Linear Diophantine fuzzy set
q-RLDFS	q-rung linear Diophantine fuzzy set
N-LDFS	N-linear Diophantine fuzzy set
N-LDFHAOs	N-LDF Hamacher AOs
N-LDFHWG	N-LDF Hamacher weighted geometric
N-LDFHOWG	N-LDF Hamacher ordered weighted geometric
N-LDFHHWG	N-LDF Hamacher hybrid weighted geometric
LDFWG	LDF weighted geometric
LDFOWG	LDF ordered weighted geometric
LDFHWG	LDF hybrid weighted geometric
S.F/A.F	Score function/Accuracy function
Q.S.F/Q.A.F	Quadratic score F/Quadratic accuracy F
E.S.F	Expectation score function
RPs	Reference parameters

(b)

Variables/parameters	Representation
Z	Fixed non-empty Set
l	$l \in Z$
S_P/B	PyFS/q-ROFS
$A_{(\ell)}$	Membership grade
$\Re_{(\ell)}$	Non-membership grade
$\mathbb{N}$	Natural numbers
G_d	LDFS
α, β	Reference Parameters (RPs)
Y_{dq}	q-rung linear Diophantine FS
Υ	Non-linear Diophantine FN
$q, K, \psi, \Delta \in \mathbb{N}$	Positive scalar
α	RP related to membership grade
β	RP related to non-membership grade
$\kappa_\Upsilon/\delta_\Upsilon$	S.F/A.F
$\varpi_\Upsilon/\chi_\Upsilon$	Q.S.F/Q.A.F
F_Υ	E.S.F
R_γ	Hamacher t-norm
R_γ^*	Hamacher t-conorm
$\gamma, \lambda > 0$	Positive scalar number
Ω	weight vector
Υ^c	Complement of N-LDF numbers
$\cup, \cap$	Union, Intersection
$\oplus, \otimes$	Addition, Multiplication
D_k	Decision matrix
$T_i, \breve{G}_j$	Alternatives, Criteria

II. PRELIMINARIES

The extensions of fuzzy sets i.e. (PyFS, q-ROFS, LDFS and q-RLDFS or N-LDFS) are offer in the current section.

Definition 1 [42], [43]: A PyFS S_P over a fixed set Z is defined as:

$$S_P = \{(\ell, A_{SP(\ell)}, \Re_{SP(\ell)}) | \ell \in Z\}, \tag{1}$$

where $A_{SP(\ell)}$ and $\Re_{SP(\ell)}$ are MG and NMG respectively. $Z :\to [0, 1]$ i.e.

$$(A_{SP(\ell)})^2 + (\Re_{SP(\ell)})^2 \leq 1.$$

Definition 2 [59]: A q-ROFS B over a fixed set Z is defined as:

$$B = \{(\ell, A_{q(\ell)}, \Re_{q(\ell)}) : \ell \in Z\} \quad (2)$$

where $A_{q(\ell)}$ and $\Re_{q(\ell)}$ are MG and NMG respectively. $Z :\rightarrow [0, 1]$ i.e.

$0 \leq (A_{q(\ell)})^q + (\Re_{q(\ell)})^q \leq 1; q \geq 1$.

Definition 3 [75]: A LDFS G_d over a fixed non-empty reference set Z is defined as:

$$G_d = \{(\ell, \langle A_{d(\ell)}, \Re_{d(\ell)} \rangle, \langle \alpha, \beta \rangle) : \ell \in Z\} \quad (3)$$

where the MG, NMG and RPs are represented by $A_{d(\ell)}, \Re_{d(\ell)}, \alpha, \beta$ respectively and $\in [0, 1]$, which satisfy the condition that $0 \leq \alpha A_{d(\ell)} + \beta \Re_{d(\ell)} \leq 1 \forall \ell \in Z$ with $0 \leq \alpha + \beta \leq 1$.

III. NON-LINEAR DIOPHANTINE FUZZY SET

Under certain actual problems, the sum of RPs for which an alternative fulfills DM's criteria may be greater than one, so LDFS did not meet the RPs target. To overcome this inconsistency, [76] proposed the idea of N-LDFS, which has the ability to deal with such conditions.

Definition 4 [76]: A q-rung linear Diophantine fuzzy set (q-RLDFS) Y_{dq} over a fixed non-empty reference set Z is defined as:

$$Y_{dq} = \{(\ell, \langle A_{dq(\ell)}, \Re_{dq(\ell)} \rangle, \langle \alpha, \beta \rangle) : \ell \in Z\} \quad (4)$$

where $A_{dq(\ell)}, \Re_{dq(\ell)}, \alpha, \beta \in [0, 1]$ are MG, NMG and RPs respectively, which fulfill the restriction;

$$0 \leq (\alpha)^q A_{dq(\ell)} + (\beta)^q \Re_{dq(\ell)} \leq 1 \forall \ell \in Z, q \geqslant 1, \quad (5)$$

with $0 \leq \alpha^q + \beta^q \leq 1, q \geq 1$. These RPs can be useful in describing or classifying a particular model. The degree of hesitation is defined as follows:

$$\Gamma \pi_d = \sqrt[q]{1 - ((\alpha)^q A_{dq(\ell)} + (\beta)^q \Re_{dq(\ell)})}. \quad (6)$$

where Γ represents the RPs related with the degree of hesitation. The RPs identify and characterize a specific system, and they also affect the physical nature of the system. They enlarge the q-RLDFS grade space and remove restrictions. They generalized the LDFS to q-RLDFS by extending the RPs and describing them as: $\alpha^q + \beta^q \in [0, 1]$. By assigning different types of values to RPs (α, β), this structure explains the decision problem. Because of the qth power of RPs, the proposed q-RLDF (N-LDF) method is more efficient and versatile than other methods.

Definition 5 [76]: A collection of q-rung linear Diophantine fuzzy number (q-RLDFN) is defined as:

$$\Upsilon = \{\langle A_{dq}, \Re_{dq} \rangle, \langle \alpha, \beta \rangle\} \quad (7)$$

where Υ denote the q-RLDFN which satisfy the following restriction;

$$(i)\ 0 \leq (\alpha)^q + (\beta)^q \leq 1, q \geqslant 1,$$
$$(ii)\ 0 \leq (\alpha)^q A_{dq(\ell)} + (\beta)^q \Re_{dq(\ell)} \leq 1$$
$$(iii)\ 0 \leq (\alpha), A_{dq(\ell)}, (\beta), \Re_{dq(\ell)} \leq 1 \quad (8)$$

TABLE 2. (a) Difference between LDFS and N-LDFS. (b) Tabular form of N-LDF parameters.

(a)

$LDFS$	$N-LDFS$
$\alpha + \beta \leq 1$	$\alpha + \beta \leq 1$ or $\alpha + \beta \geq 1$
$0 \leq \alpha + \beta \leq 1$	$0 \leq (\alpha)^q + (\beta)^q \leq 1$
$(\alpha)A + (\beta)\Re \leq 1$	$(\alpha)A + (\beta)\Re \leq 1$ or $(\alpha)A + (\beta)\Re \geq 1$
$0 \leq (\alpha)A + (\beta)\Re \leq 1$	$0 \leq (\alpha)^q A + (\beta)^q \Re \leq 1$
$0 \leq \alpha, A, \beta, \Re \leq 1$	$0 \leq \alpha, A, \beta, \Re \leq 1$
$\Gamma = 1 - (\alpha)A - (\beta)\Re$	$\Gamma = \sqrt[q]{1 - ((\alpha)^q A + (\beta)^q \Re)}$
$\Gamma + (\alpha)A + (\beta)\Re = 1$	$(\Gamma)^q + (\alpha)^q A + (\beta)^q \Re = 1$

(b)

Ist group N-LDFS,q=4	2nd group N-LDFS, q=4
$Y_{dq} = (\langle A, \Re \rangle, \langle \alpha, \beta \rangle)$	$Y_{dq} = (\langle A, \Re \rangle, \langle \alpha, \beta \rangle)$
$(\langle 1, 1 \rangle, \langle .9, .7 \rangle)$	$(\langle 1, .87 \rangle, \langle .75, .9 \rangle)$
$(\langle .87, .9 \rangle, \langle .75, .9 \rangle)$	$(\langle 1, 1 \rangle, \langle .7, .9 \rangle)$
$(\langle 1, 1 \rangle, \langle .8, .8 \rangle)$	$(\langle .87, 1 \rangle, \langle .9, .75 \rangle)$
$(\langle .95, 1 \rangle, \langle .75, .85 \rangle)$	$(\langle .9, .9 \rangle, \langle .85, .8 \rangle)$
$(\langle .8, .9 \rangle, \langle .85, .8 \rangle)$	$(\langle 1, 1 \rangle, \langle .8, .8 \rangle)$
$(\langle .9, 1 \rangle, \langle .9, .75 \rangle)$	$(\langle .8, .9 \rangle, \langle .85, .8 \rangle)$

Example 1: Let $A \in N - LDFS(Z)$. Suppose $A_A(x) = 0.85, \Re_A(x) = 0.95, \alpha = 0.6$ and $\beta = 0.7$ for $Z = \{x\}$. Clearly, $(0.85)(0.6) + (0.95)(0.7) \nleq 1$, and also $(0.6)^2 + (0.7)^2 \nleq 1$, but $(0.85)(0.6)^2 + (0.95)(0.7)^2 \leq 1$. and $(0.6)^2 + (0.7)^2 \leq 1$. Thus for $q = 2$, $N - LDFS$ become quadratic Diophantine FS.

Table. 2(a) explain the difference between $LDFS$ and $N - LDFS$ with conditions.

A. DIAGNOSIS OF DEPRESSION

Depression is a disease that affects millions of people across the globe. More than 300 million human beings of all ages suffer from depression worldwide. Depression is an epidemic of the 21st century. Depression's causes are ambiguous and not well known. It is believed to occur as a result of a mixture of brain chemical imbalances, biology and personal issues. There are so many communities in this contemporary world that believe depression is not a mental illness. Depression, that is the worst form of this disorder, can proceed to suicide Popular questions arise, what exactly is depression? "Depression is a common and extreme psychiatric disorder that negatively affects how you feel how you think, and how you behave" [86]. In diagnosis, we use the concept of N-LDFSs.

Suppose that Z is the of group of depressed patients and is the universe of discourse. More extreme symptoms, reduced quality of life, and a high risk of revictimization are linked with the victimization of depression. Let $Z = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ be a patient gender category where

x_1: female, x_2: male, x_3: young, x_4 : mature, x_5: old and x_6: unemployed people. And due to depression, i.e., all patients reported physical change or behavioral change Hopeless, no longer going out, loss of interest, no pleasant hobbies, sleep issues, stressful, feeling down, lonely, loss or change of unexpected weight problem, guilty, asking for support in all matters, lack of trust, dissatisfied. Let three psychiatrists (decision makers) evaluate the patient groups based on associated criteria Since psychiatrists are qualified medical doctors, they can recommend medicine and as a course of treatment, they spend most of their time with patients on medication administration. Patients suffering from depression are diagnosed by a team of three psychiatrists.

We represent this evaluation using N-LDFS whose tabular representation is given in Table 2(b). For N-LDFS structure first consider RPs (α, β). Let (α, β) represent the treatments for reported depressed physical or behavioral change. we can consider α = methods of treatment, analytical testing, problem-solving techniques, and β = psychological theory, and behavioral therapy. For "$q = 4$" Table 2(b) Ist group lists the numerical form of such N-LDFS parameter. For the 2nd group, let α = psychological theory, behavioral therapy, and β = methods of treatment, analytical testing, and problem-solving techniques. The RPs play an important role in the diagnosis of depression. They represent some special and particular treatments and suitable medicine for the patient about his depression. Also Table 2(b) lists the numerical form of N-LDFS for the 2nd group RPs. The functions $A_{dq(\ell)}$ and $\Re_{dq(\ell)}$ denoted the patient's depressive symptoms and gender, which demonstrates how many symptoms are present in them, whereas RPs demonstrate how a patient should be treated in a best and suitable way and $q \in \mathbb{N}$. The decision-maker will choose the parameters, while attribute grades are determined from the data collected. On the same reference set Z, we can easily describe different N-LDFS for different sets of parameters. Our mathematical model becomes more spatial as a result of these parameters.

B. SCORE AND ACCURACY FUNCTION

Next is about certain score function (S.F) and accuracy function (A.F) which is presented by [88] and [89], hence this concept of different S.F and A.F is updated by [76] with the qth power of RPs.

Definition 6 [76]: Consider $\Upsilon = \{\langle A, \Re\rangle, \langle \alpha, \beta\rangle\} \in N-LDFN$, then score function (S.F) $\kappa : N-LDFN(Z) \longrightarrow [-1, 1]$ with $q \geq 1$ is defined as;

$$\kappa_{\Upsilon} = \left[\frac{(A - \Re) + (\alpha^q - \beta^q)}{2}\right]. \tag{9}$$

Definition 7 [76]: The accuracy function (A.F) $\delta : N-LDFN(Z) \longrightarrow [0, 1]$ is defined as;

$$\delta_{\Upsilon} = \left[\left(\frac{A + \Re}{4}\right) + \left(\frac{\alpha^q + \beta^q}{2}\right)\right] \tag{10}$$

Definition 8 [76]: Let Υ_1 and Υ_2 be two $N-LDFNs$, then the two $N-LDFNs$ can be easily compared by using the S.F and A.F:

(i) if $\kappa_{\Upsilon_1} < \kappa_{\Upsilon_2}$ then $\Upsilon_1 < \Upsilon_2$,
(ii) if $\kappa_{\Upsilon_1} > \kappa_{\Upsilon_2}$ then $\Upsilon_1 > \Upsilon_2$,
(iii) if $\kappa_{\Upsilon_1} = \kappa_{\Upsilon_2}$ then,
(a) if $\delta_{\Upsilon_1} < \delta_{\Upsilon_2}$ then $\Upsilon_1 < \Upsilon_2$,
(b) if $\delta_{\Upsilon_1} > \delta_{\Upsilon_2}$ then $\Upsilon_1 > \Upsilon_2$,
(c) if $\delta_{\Upsilon_1} = \delta_{\Upsilon_2}$ then $\Upsilon_1 \approx \Upsilon_2$.

Next definition is about quadratic score function (Q.S.F).

Definition 9 [76]: The quadratic score function (Q.S.F) $\varpi : N-LDFN(Z) \longrightarrow [-1, 1]$ for $N-LDFN$ is defined as;

$$\varpi_{\Upsilon} = \left(\frac{(A^2 - \Re^2) + ((\alpha^q)^2 - (\beta^q)^2)}{2}\right). \tag{11}$$

Definition 10 [76]: The quadratic accuracy function (Q.A.F) $\chi : N-LDFN(Z) \longrightarrow [0, 1]$ for $N-LDFN$ is defined as;

$$\chi_{\Upsilon} = \left[(\frac{A^2 + \Re^2}{4}) + (\frac{(\alpha^q)^2 + (\beta^q)^2}{2})\right]. \tag{12}$$

Definition 11 [76]: Let Υ_1 and Υ_2 be two $N-LDFNs$, then the two $N-LDFNs$ can be easily compared by using the Q.S.F and Q.A.F:

(i) if $\varpi_{\Upsilon_1} < \varpi_{\Upsilon_2}$ then $\Upsilon_1 < \Upsilon_2$,
(ii) if $\varpi_{\Upsilon_1} > \varpi_{\Upsilon_2}$ then $\Upsilon_1 > \Upsilon_2$,
(iii) if $\varpi_{\Upsilon_1} = \varpi_{\Upsilon_2}$ then,
(a) if $\chi_{\Upsilon_1} < \chi_{\Upsilon_2}$ then $\Upsilon_1 < \Upsilon_2$,
(b) if $\chi_{\Upsilon_1} > \chi_{\Upsilon_2}$ then $\Upsilon_1 > \Upsilon_2$,
(c) if $\chi_{\Upsilon_1} = \chi_{\Upsilon_2}$ then $\Upsilon_1 \approx \Upsilon_2$.

Definition 12 [76]: Suppose $\Upsilon = \{\langle A, \Re\rangle, \langle \alpha, \beta\rangle\} \in N-LDFN$, then expectation score function (E.S.F) $F : N-LDFN(Z) \longrightarrow [0, 1]$ can be defined as;

$$F_{\Upsilon} = \left[\frac{(A - \Re + 1)}{4} + \frac{(\alpha^q - \beta^q + 1)}{4}\right]. \tag{13}$$

The value of E.S.F belong to $[0, 1]$ rather than $[-1, 1]$. We have no need of expectation accuracy function (E.A.F).

Definition 13 [76]: Let Υ_1 and Υ_2 be two $N-LDFNs$ then the two $N-LDFNs$ can be easily compared by using the E.S.F as:

(i) if $F_{\Upsilon_1} < F_{\Upsilon_2}$ then $\Upsilon_1 < \Upsilon_2$,
(ii) if $F_{\Upsilon_1} > F_{\Upsilon_2}$ then $\Upsilon_1 > \Upsilon_2$,
(iii) if $F_{\Upsilon_1} = F_{\Upsilon_2}$ then $\Upsilon_1 = \Upsilon_2$.

C. HAMACHER T-NORM AND T-CONORM

In FS theory, Triangular-norms are an important notion used to defined universal union and intersection of FSs [89]. In 1998, Roychowdhury and Wang presented [90] and Deschrijver and Kerre presented [91]. Further, generalized t-norms is presented by Hamacher [24]. The product Hamacher is t-norm and the sum Hamacher is t-conorm with condition that $\gamma > 0$;

$$R_{\gamma}(g, r) = g \otimes r = \frac{gr}{\gamma + (1 - \gamma)(g + r - gr)}. \tag{14}$$

$$R^*_\gamma(g,r) = g \oplus r = \frac{(g+r-gr)-(1-\gamma)gr}{1-(1-\gamma)gr}. \qquad (15)$$

Especially, when $\gamma = 1$, then Hamacher-norms will reduce to Algebraic-norms as follow;

$$R(g,r) = g \otimes r = gr$$
$$R^*(g,r) = g \oplus r = g + r - gr$$

when $\gamma = 2$, then Hamacher-norms will reduce to Einstein-norms [92].

$$R(g,r) = g \otimes r = \frac{gr}{1+(1-g)(1-r)}$$
$$R^*_\gamma(g,r) = g \oplus r = \frac{g+r}{1+gr}$$

D. HAMACHER OPERATIONS OF N-LDFNS

The operational rules based on Hamacher-norms:

We define the Hamacher product and sum for N-LDFNs.

Definition 14: Let $\Upsilon_1 = \left(\left\langle {}^1A, {}^1\Re\right\rangle, \left\langle {}^1\alpha, {}^1\beta\right\rangle\right)$ and $\Upsilon_2 = \left(\left\langle {}^2A, {}^2\Re\right\rangle, \left\langle {}^2\alpha, {}^2\beta\right\rangle\right)$ be any two N-LDFNs, with $\gamma > 0$, $\lambda > 0$ and $q \geq 1$, then we have define basic Hamacher operations for N-LDFNs as follow;

(i) $\Upsilon_1^c = \left(\left\langle {}^1\Re, {}^1A\right\rangle, \left\langle {}^1\beta, {}^1\alpha\right\rangle\right)$

(ii) $\Upsilon_1 \oplus \Upsilon_2$

$$= \left[\begin{array}{c} \left(\dfrac{({}^1A)+({}^2A)-({}^1A)({}^2A)-(1-\gamma)({}^1A)({}^2A)}{1-(1-\gamma)({}^1A)({}^2A)}, \dfrac{({}^1\Re)({}^2\Re)}{\gamma+(1-\gamma)(({}^1\Re)+({}^2\Re)-({}^1\Re)({}^2\Re))}\right), \\ \left(\sqrt[q]{\dfrac{({}^1\alpha)^q+({}^2\alpha)^q-({}^1\alpha)^q({}^2\alpha)^q-(1-\gamma)({}^1\alpha)^q({}^2\alpha)^q}{1-(1-\gamma)(({}^1\alpha)^q({}^2\alpha)^q)}}, \dfrac{({}^1\beta)({}^2\beta)}{\sqrt[q]{\gamma+(1-\gamma)(({}^1\beta)^q+({}^2\beta)^q-({}^1\beta)^q({}^2\beta)^q)}}\right) \end{array}\right]$$

(iii) $\Upsilon_1 \otimes \Upsilon_2$

$$= \left[\begin{array}{c} \left(\dfrac{({}^1A)({}^2A)}{\gamma+(1-\gamma)(({}^1A)+({}^2A)-({}^1A)({}^2A))}, \dfrac{({}^1\Re)+({}^2\Re)-({}^1\Re)({}^2\Re)-(1-\gamma)({}^1\Re)({}^2\Re)}{1-(1-\gamma)({}^1\Re)({}^2\Re)}\right), \\ \left(\dfrac{({}^1\alpha)({}^2\alpha)}{\sqrt[q]{\gamma+(1-\gamma)(({}^1\alpha)^q+({}^2\alpha)^q-({}^1\alpha)^q({}^2\alpha)^q)}}, \sqrt[q]{\dfrac{({}^1\beta)^q+({}^2\beta)^q-({}^1\beta)^q({}^2\beta)^q-(1-\gamma)({}^1\beta)^q({}^2\beta)^q}{1-(1-\gamma)({}^1\beta)^q({}^2\beta)^q}}\right) \end{array}\right]$$

(iv) $\lambda\Upsilon_1$

$$= \left[\begin{array}{c} \left(\dfrac{(1+(\gamma-1){}^1A)^\lambda-(1-{}^1A)^\lambda}{(1+(\gamma-1){}^1A)^\lambda+(\gamma-1)(1-{}^1A)^\lambda}, \dfrac{\gamma({}^1\Re)^\lambda}{(1+(\gamma-1)(1-{}^1\Re))^\lambda+(\gamma-1)({}^1\Re)^\lambda}\right), \\ \left(\sqrt[q]{\dfrac{(1+(\gamma-1)({}^1\alpha)^q)^\lambda-(1-({}^1\alpha)^q)^\lambda}{(1+(\gamma-1)({}^1\alpha)^q)^\lambda+(\gamma-1)(1-({}^1\alpha)^q)^\lambda}}, \dfrac{\sqrt[q]{\gamma}({}^1\beta)^\lambda}{\sqrt[q]{(1+(\gamma-1)(1-({}^1\beta)^q))^\lambda+(\gamma-1)(({}^1\beta)^q)^\lambda}}\right) \end{array}\right]$$

(v) Υ_1^λ

$$= \left[\begin{array}{c} \left(\dfrac{\gamma(A)^\lambda}{(1+(\gamma-1)(1-{}^1A))^\lambda+(\gamma-1)({}^1A)^\lambda}, \dfrac{(1+(\gamma-1)({}^1\Re))^\lambda-(1-{}^1\Re)^\lambda}{(1+(\gamma-1){}^1\Re)^\lambda+(\gamma-1)(1-{}^1\Re)^\lambda}\right), \\ \left(\dfrac{\sqrt[q]{\gamma}({}^1\alpha)^\lambda}{\sqrt[q]{(1+(\gamma-1)(1-({}^1\alpha)^q))^\lambda+(\gamma-1)(({}^1\alpha)^q)^\lambda}}, \sqrt[q]{\dfrac{(1+(\gamma-1)({}^1\beta)^q)^\lambda-(1-({}^1\beta)^q)^\lambda}{(1+(\gamma-1)({}^1\beta)^q)^\lambda+(\gamma-1)(1-({}^1\beta)^q)^\lambda}}\right) \end{array}\right]$$

Definition 15: Let $\Upsilon_\psi = \left(\left\langle {}^\psi A_{dq}, {}^\psi\Re_{dq}\right\rangle, \left\langle {}^\psi\alpha, {}^\psi\beta\right\rangle\right)$ for $\psi \in \Delta$ be an assembling of N-LDFNs, then the following properties can be easily satisfied based on Hamacher-norms;

(1) $\bigcup_{\psi\in\Delta} \Upsilon_\psi = \left((\sup_{\psi\in\Delta} {}^\psi A_{dq}, \inf_{\psi\in\Delta} {}^\psi\Re_{dq}), (\sup_{\psi\in\Delta} {}^\psi\alpha, \inf_{\psi\in\Delta} {}^\psi\beta)\right)$;

(2) $\bigcap_{\psi\in\Delta} \Upsilon_\psi = \left((\inf_{\psi\in\Delta} {}^\psi A_{dq}, \sup_{\psi\in\Delta} {}^\psi\Re_{dq}), (\inf_{\psi\in\Delta} {}^\psi\alpha, \sup_{\psi\in\Delta} {}^\psi\beta)\right)$.

Definition 16: Let $\Upsilon_1 = \left(\left\langle {}^1A_{dq}, {}^1\Re_{dq}\right\rangle, \left\langle {}^1\alpha, {}^1\beta\right\rangle\right)$ and $\Upsilon_2 = \left(\left\langle {}^2A_{dq}, {}^2\Re_{dq}\right\rangle, \left\langle {}^2\alpha, {}^2\beta\right\rangle\right)$ be any two N-LDFNs, with $\gamma > 0$ and $\lambda > 0$, then

(1) $\Upsilon_1 = \Upsilon_2 \iff {}^1A_{dq} = {}^2A_{dq}, {}^1\Re_{dq} = {}^2\Re_{dq}, {}^1\alpha = {}^2\alpha, {}^1\beta = {}^2\beta$;

(2) $\Upsilon_1 \subseteq \Upsilon_2 \iff {}^1A_{dq} \leq {}^2A_{dq}, {}^1\Re_{dq} \geq {}^2\Re_{dq}, {}^1\alpha \leq {}^2\alpha, {}^1\beta \geq {}^2\beta$.

Proposition 1: Let Υ_1 and Υ_2 belong to N-LDFNs with real numbers $\lambda > 0$ and $\gamma > 0$; then Υ_1 and Υ_2 are still N-LDFNs after applying the operation that is Υ_1^c, $\Upsilon_1 \cup \Upsilon_2$, $\Upsilon_1 \cap \Upsilon_2$, $\Upsilon_1 \oplus \Upsilon_2$, $\Upsilon_1 \otimes \Upsilon_2$, $\lambda\Upsilon_1$ and Υ_1^λ are also N-LDFNs.

Proof The above Definition 14, 15, and 16 can easily be used to prove this result.

Proposition 2: Consider three N-LDFNs; $\Upsilon_1 = (\left\langle {}^1A_{dq}, {}^1\Re_{dq}\right\rangle, \left\langle {}^1\alpha, {}^1\beta\right\rangle)$, $\Upsilon_2 = (\left\langle {}^2A_{dq}, {}^2\Re_{dq}\right\rangle, \left\langle {}^2\alpha, {}^2\beta\right\rangle)$ and $\Upsilon_3 = (\left\langle {}^3A_{dq}, {}^3\Re_{dq}\right\rangle, \left\langle {}^3\alpha, {}^3\beta\right\rangle)$ with $\gamma > 0$ and $\lambda > 0$, then the following cases are satisfied;

(1) if $\Upsilon_1 \subseteq \Upsilon_2$ and $\Upsilon_2 \subseteq \Upsilon_3$ then $\Upsilon_1 \subseteq \Upsilon_3$
(2) $\Upsilon_1 \cup \Upsilon_2 = \Upsilon_2 \cup \Upsilon_1$
(3) $\Upsilon_1 \cap \Upsilon_2 = \Upsilon_1 \cap \Upsilon_2$
(4) $\Upsilon_1 \cup (\Upsilon_2 \cup \Upsilon_3) = (\Upsilon_1 \cup \Upsilon_2) \cup \Upsilon_3$
(5) $\Upsilon_1 \cap (\Upsilon_2 \cap \Upsilon_3) = (\Upsilon_1 \cap \Upsilon_2) \cap \Upsilon_3$
(6) $\Upsilon_1 \cup (\Upsilon_2 \cap \Upsilon_3) = (\Upsilon_1 \cup \Upsilon_2) \cap (\Upsilon_1 \cup \Upsilon_3)$
(7) $\Upsilon_1 \cap (\Upsilon_2 \cup \Upsilon_3) = (\Upsilon_1 \cap \Upsilon_2) \cup (\Upsilon_1 \cap \Upsilon_3)$
(8) $(\Upsilon_1 \cup \Upsilon_2)^c = \Upsilon_1^c \cap \Upsilon_2^c$
(9) $(\Upsilon_1 \cap \Upsilon_2)^c = \Upsilon_1^c \cup \Upsilon_2^c$

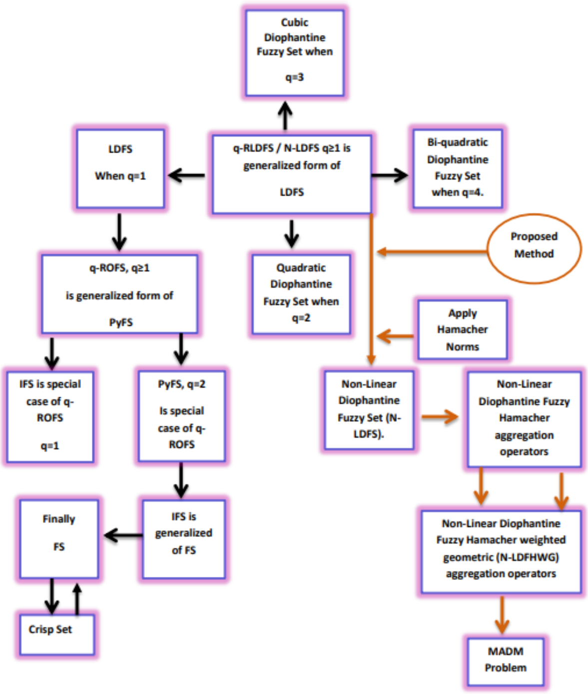

FIGURE 1. **Framework of proposed method.**

Proof: Proof of the above statements are obvious.

Following Fig 1. represented the framework of proposed N-LDFHWG aggregation operators, which also explained the difference between the existing method and proposed method.

IV. N-LDF HAMACHER AGGREGATION OPERATORS

In the current section, with the help of the Hamacher operations, we develop the N-LDF aggregation operators.

A. N-LDF HAMACHER WEIGHTED GEOMETRIC AGGREGATION (N-LDFHWGA) OPERATOR

In this subsection we have define N-LDF Hamacher weighted geometric aggregation operators i.e. N-LDFHWG, N-LDFHOWG and N-LDFHHWG aggregation operators.

Definition 17: Let $\Upsilon_{dq\psi} = \{(\langle {}^\psi A_{dq}, {}^\psi \Re_{dq}\rangle, \langle {}^\psi\alpha, {}^\psi\beta\rangle) : \psi \in \mathbb{N}\}$ be a family of N-LDFNs over the fixed set Z and $\Omega = (\Omega_1, \Omega_2, \ldots, \Omega_n)^T$ are the weights with $\sum_{\psi=1}^{n} \Omega_\psi = 1, q \geq 1$; then we define the N-LDF Hamacher weighted geometric (N-LDFHWG) operator as follows and let the transformation $\theta : N-LDFN(Z) \longrightarrow N-LDFN(Z)$

$$N-LDFHWG(\Upsilon_{dq1}, \Upsilon_{dq2}, \ldots, \Upsilon_{dqn}) = \bigotimes_{\psi=1}^{n}(\Upsilon_{dq\psi}^{\Omega_\psi}) = \Upsilon_{dq1}^{\Omega_1} \bigotimes \Upsilon_{dq2}^{\Omega_2} \bigotimes \ldots \bigotimes \Upsilon_{dqn}^{\Omega_n}. \tag{16}$$

In N-LDFHWG operator, A denote MG and $\Re$ denote NMG, α, β denote the RPs and $q \geq 1$. Weights are denoted by Ω, $\Upsilon_{dq\psi}$ are the N-LDFNs, where $\psi \in \mathbb{N}$ and N-LDFN(Z) combines all N-LDFNs.

Based on Hamacher product operations rules for N-LDFNs, we have capture the result displayed in theorem 1.

Theorem 1: Let $\Upsilon_{dq\psi} = \{(\langle {}^\psi A_{dq}, {}^\psi \Re_{dq}\rangle, \langle {}^\psi\alpha, {}^\psi\beta\rangle) : \psi \in \mathbb{N}\}$ be an assembling of N-LDFNs over the fixed set Z and $\Omega = (\Omega_1, \Omega_2, \ldots, \Omega_n)^T$ are the weights with $\sum_{\psi=1}^{n} \Omega_\psi = 1$, and $q \geq 1, \gamma > 0$; then by applying the N-LDFHWG operator their aggregated value is also an N-LDFN, and the transformation $\theta : N-LDFN(Z) \longrightarrow N-LDFN(Z)$ is called N-LDF Hamacher weighted geometric operator (N-LDFHWG) and define as follow;

$$N-LDFHWG(\Upsilon_{dq1}, \Upsilon_{dq2}, \ldots, \Upsilon_{dqn}) = \bigotimes_{\psi=1}^{n}(\Upsilon_{dq\psi})^{\Omega_\psi}$$

$$= \left[\begin{array}{c} \left(\dfrac{\gamma \prod_{\psi=1}^{n} ({}^\psi A_{dq})^{\Omega_\psi}}{\prod_{\psi=1}^{n}(1+(\gamma-1)(1-{}^\psi A_{dq}))^{\Omega_\psi} + (\gamma-1)\prod_{\psi=1}^{n}({}^\psi A_{dq})^{\Omega_\psi}}, \dfrac{\prod_{\psi=1}^{n}(1+(\gamma-1){}^\psi\Re_{dq})^{\Omega_\psi} - \prod_{\psi=1}^{n}(1-{}^\psi\Re_{dq})^{\Omega_\psi}}{\prod_{\psi=1}^{n}(1+(\gamma-1){}^\psi\Re_{dq})^{\Omega_\psi} + (\gamma-1)\prod_{\psi=1}^{n}(1-{}^\psi\Re_{dq})^{\Omega_\psi}}\right), \\ \left(\dfrac{\sqrt[q]{\gamma}\prod_{\psi=1}^{n}({}^\psi\alpha)^{\Omega_\psi}}{\sqrt[q]{\prod_{\psi=1}^{n}(1+(\gamma-1)(1-({}^\psi\alpha)^q))^{\Omega_\psi} + (\gamma-1)\prod_{\psi=1}^{n}(({}^\psi\alpha)^q)^{\Omega_\psi}}}, \sqrt[q]{\dfrac{\prod_{\psi=1}^{n}(1+(\gamma-1)({}^\psi\beta)^q)^{\Omega_\psi} - \prod_{\psi=1}^{n}(1-({}^\psi\beta)^q)^{\Omega_\psi}}{\prod_{\psi=1}^{n}(1+(\gamma-1)({}^\psi\beta)^q)^{\Omega_\psi} + (\gamma-1)\prod_{\psi=1}^{n}(1-({}^\psi\beta)^q)^{\Omega_\psi}}}, \right) \end{array}\right] \tag{17}$$

Proof By induction method we want to prove this theorem. We put $n = 2$ in Eq. (17). So for N-LDFNs based on Hamacher product, we obtained the associated result.

(i): Let $\Upsilon_{dq1} = \{(\langle {}^1A_{dq}, {}^1\Re_{dq}\rangle, \langle {}^1\alpha, {}^1\beta\rangle)$ and $\Upsilon_{dq2} = \{(\langle {}^2A_{dq}, {}^2\Re_{dq}\rangle, \langle {}^2\alpha, {}^2\beta\rangle)$ be two N-LDFNs then built on Hamacher product. The left side of Eq. (17) become

$$N-LDFHWG(\Upsilon_{dq1}, \Upsilon_{dq2}) = \Upsilon_{dq1}^{\Omega_1} \bigotimes \Upsilon_{dq2}^{\Omega_2}$$

The right side of Eq. (17) become,

$$= \left[\begin{array}{c} \left(\dfrac{\gamma({}^1A_{dq})^{\Omega_1}}{(1+(\gamma-1)(1-{}^1A_{dq}))^{\Omega_1}+(\gamma-1)({}^1A_{dq})^{\Omega_1}}, \dfrac{(1+(\gamma-1){}^1\Re_{dq})^{\Omega_1}-(1-{}^1\Re_{dq})^{\Omega_1}}{(1+(\gamma-1){}^1\Re_{dq})^{\Omega_1}+(\gamma-1)(1-{}^1\Re_{dq})^{\Omega_1}}\right) \\ \left(\dfrac{\sqrt[q]{\gamma}({}^1\alpha)^{\Omega_1}}{\sqrt[q]{(1+(\gamma-1)(1-({}^1\alpha)^q))^{\Omega_1}+(\gamma-1)(({}^1\alpha)^q)^{\Omega_1}}}, \sqrt[q]{\dfrac{(1+(\gamma-1)({}^1\beta)^q)^{\Omega_1}-(1-({}^1\beta)^q)^{\Omega_1}}{(1+(\gamma-1)({}^1\beta)^q)^{\Omega_1}+(\gamma-1)(1-({}^1\beta)^q)^{\Omega_1}}}\right) \end{array}\right] \bigotimes$$

$$\left[\begin{array}{c}\left(\frac{\gamma({}^{2}A_{dq})^{\Omega_2}}{(1+(\gamma-1)(1-{}^{2}A_{dq}))^{\Omega_2}+(\gamma-1)({}^{2}A_{dq})^{\Omega_2}},\ \frac{(1+(\gamma-1)^{2}\Re_{dq})^{\Omega_2}-(1-{}^{2}\Re_{dq})^{\Omega_2}}{(1+(\gamma-1)^{2}\Re_{dq})^{\Omega_2}+(\gamma-1)(1-{}^{2}\Re_{dq})^{\Omega_2}}\right),\\ \left(\frac{\sqrt[q]{\gamma}({}^{2}\alpha)^{\Omega_2}}{\sqrt[q]{(1+(\gamma-1)(1-({}^{2}\alpha)^q))^{\Omega_2}+(\gamma-1)(({}^{2}\alpha)^q)^{\Omega_2}}},\ \sqrt[q]{\frac{(1+(\gamma-1)({}^{2}\beta)^q)^{\Omega_2}-(1-({}^{2}\beta)^q)^{\Omega_2}}{(1+(\gamma-1)({}^{2}\beta)^q)^{\Omega_2}+(\gamma-1)(1-({}^{2}\beta)^q)^{\Omega_2}}}\right)\end{array}\right].$$

$$=\left[\begin{array}{c}\left(\frac{\gamma\prod_{\psi=1}^{2}({}^{\psi}A_{dq})^{\Omega_\psi}}{\prod_{\psi=1}^{2}(1+(\gamma-1)(1-{}^{\psi}A_{dq}))^{\Omega_\psi}+(\gamma-1)\prod_{\psi=1}^{2}({}^{\psi}A_{dq})^{\Omega_\psi}},\ \frac{\prod_{\psi=1}^{2}(1+(\gamma-1)^{\psi}\Re_{dq})^{\Omega_\psi}-\prod_{\psi=1}^{2}(1-{}^{\psi}\Re_{dq})^{\Omega_\psi}}{\prod_{\psi=1}^{2}(1+(\gamma-1)^{\psi}\Re_{dq})^{\Omega_\psi}+(\gamma-1)\prod_{\psi=1}^{2}(1-{}^{\psi}\Re_{dq})^{\Omega_\psi}}\right),\\ \left(\frac{\sqrt[q]{\gamma}\prod_{\psi=1}^{2}({}^{\psi}\alpha)^{\Omega_\psi}}{\sqrt[q]{\prod_{\psi=1}^{2}(1+(\gamma-1)(1-({}^{\psi}\alpha)^q))^{\Omega_\psi}+(\gamma-1)\prod_{\psi=1}^{2}(({}^{\psi}\alpha)^q)^{\Omega_\psi}}},\ \sqrt[q]{\frac{\prod_{\psi=1}^{2}(1+(\gamma-1)({}^{\psi}\beta)^q)^{\Omega_\psi}-\prod_{\psi=1}^{2}(1-({}^{\psi}\beta)^q)^{\Omega_\psi}}{\prod_{\psi=1}^{2}(1+(\gamma-1)({}^{\psi}\beta)^q)^{\Omega_\psi}+(\gamma-1)\prod_{\psi=1}^{2}(1-({}^{\psi}\beta)^q)^{\Omega_\psi}}}\right)\end{array}\right].$$

Hence Eq. (17) is true when we put $n = 2$.
(ii): Assume that Eq. (17) holds for $n = K$,

$$N-LDFHWG(\Upsilon_{dq1},\Upsilon_{dq2},\ldots,\Upsilon_{dqk})=$$

$$\left[\begin{array}{c}\left(\frac{\gamma\prod_{\psi=1}^{k}({}^{\psi}A_{dq})^{\Omega_\psi}}{\prod_{\psi=1}^{k}(1+(\gamma-1)(1-{}^{\psi}A_{dq}))^{\Omega_\psi}+(\gamma-1)\prod_{\psi=1}^{k}({}^{\psi}A_{dq})^{\Omega_\psi}},\ \frac{\prod_{\psi=1}^{k}(1+(\gamma-1)^{\psi}\Re_{dq})^{\Omega_\psi}-\prod_{\psi=1}^{k}(1-{}^{\psi}\Re_{dq})^{\Omega_\psi}}{\prod_{\psi=1}^{k}(1+(\gamma-1)^{\psi}\Re_{dq})^{\Omega_\psi}+(\gamma-1)\prod_{\psi=1}^{k}(1-{}^{\psi}\Re_{dq})^{\Omega_\psi}}\right),\\ \left(\frac{\sqrt[q]{\gamma}\prod_{\psi=1}^{k}({}^{\psi}\alpha)^{\Omega_\psi}}{\sqrt[q]{\prod_{\psi=1}^{k}(1+(\gamma-1)(1-({}^{\psi}\alpha)^q))^{\Omega_\psi}+(\gamma-1)\prod_{\psi=1}^{k}(({}^{\psi}\alpha)^q)^{\Omega_\psi}}},\ \sqrt[q]{\frac{\prod_{\psi=1}^{k}(1+(\gamma-1)({}^{\psi}\beta)^q)^{\Omega_\psi}-\prod_{\psi=1}^{k}(1-({}^{\psi}\beta)^q)^{\Omega_\psi}}{\prod_{\psi=1}^{k}(1+(\gamma-1)({}^{\psi}\beta)^q)^{\Omega_\psi}+(\gamma-1)\prod_{\psi=1}^{k}(1-({}^{\psi}\beta)^q)^{\Omega_\psi}}}\right)\end{array}\right].$$

(iii): Now we prove that Eq. (17) holds for $n = K + 1$, Let

$$N-LDFHWG(\Upsilon_{dq1},\Upsilon_{dq2},\ldots,\Upsilon_{dqk+1})$$

$$=\prod_{\psi=1}^{k}\Upsilon_{dq\psi}^{\Omega_\psi}\bigotimes\Upsilon_{dqk+1}^{\Omega_{k+1}}$$

$$=\left[\begin{array}{c}\left(\frac{\gamma\prod_{\psi=1}^{k}({}^{\psi}A_{dq})^{\Omega_\psi}}{\prod_{\psi=1}^{k}(1+(\gamma-1)(1-{}^{\psi}A_{dq}))^{\Omega_\psi}+(\gamma-1)\prod_{\psi=1}^{k}({}^{\psi}A_{dq})^{\Omega_\psi}},\ \frac{\prod_{\psi=1}^{k}(1+(\gamma-1)^{\psi}\Re_{dq})^{\Omega_\psi}-\prod_{\psi=1}^{k}(1-{}^{\psi}\Re_{dq})^{\Omega_\psi}}{\prod_{\psi=1}^{k}(1+(\gamma-1)^{\psi}\Re_{dq})^{\Omega_\psi}+(\gamma-1)\prod_{\psi=1}^{k}(1-{}^{\psi}\Re_{dq})^{\Omega_\psi}}\right),\\ \left(\frac{\sqrt[q]{\gamma}\prod_{\psi=1}^{k}({}^{\psi}\alpha)^{\Omega_\psi}}{\sqrt[q]{\prod_{\psi=1}^{k}(1+(\gamma-1)(1-({}^{\psi}\alpha)^q))^{\Omega_\psi}+(\gamma-1)\prod_{\psi=1}^{k}(({}^{\psi}\alpha)^q)^{\Omega_\psi}}},\ \sqrt[q]{\frac{\prod_{\psi=1}^{k}(1+(\gamma-1)({}^{\psi}\beta)^q)^{\Omega_\psi}-\prod_{\psi=1}^{k}(1-({}^{\psi}\beta)^q)^{\Omega_\psi}}{\prod_{\psi=1}^{k}(1+(\gamma-1)({}^{\psi}\beta)^q)^{\Omega_\psi}+(\gamma-1)\prod_{\psi=1}^{k}(1-({}^{\psi}\beta)^q)^{\Omega_\psi}}}\right)\end{array}\right]\bigotimes$$

$$\left[\begin{array}{c}\left(\frac{\gamma({}^{k+1}A_{dq})^{\Omega_{k+1}}}{(1+(\gamma-1)(1-{}^{k+1}A_{dq}))^{\Omega_{k+1}}+(\gamma-1)({}^{k+1}A_{dq})^{\Omega_{k+1}}},\ \frac{(1+(\gamma-1)^{k+1}\Re_{dq})^{\Omega_{k+1}}-(1-{}^{k+1}\Re_{dq})^{\Omega_{k+1}}}{(1+(\gamma-1)^{k+1}\Re_{dq})^{\Omega_{k+1}}+(\gamma-1)(1-{}^{k+1}\Re_{dq})^{\Omega_{k+1}}}\right),\\ \left(\frac{\sqrt[q]{\gamma}({}^{k+1}\alpha)^{\Omega_{k+1}}}{\sqrt[q]{(1+(\gamma-1)(1-({}^{k+1}\alpha)^q))^{\Omega_{k+1}}+(\gamma-1)(({}^{k+1}\alpha)^q)^{\Omega_{k+1}}}},\ \sqrt[q]{\frac{(1+(\gamma-1)({}^{k+1}\beta)^q)^{\Omega_{k+1}}-(1-({}^{k+1}\beta)^q)^{\Omega_{k+1}}}{(1+(\gamma-1)({}^{k+1}\beta)^q)^{\Omega_{k+1}}+(\gamma-1)(1-({}^{k+1}\beta)^q)^{\Omega_{k+1}}}}\right)\end{array}\right].$$

$$= \left[\begin{array}{c} \left(\begin{array}{c} \frac{\gamma \prod_{\psi=1}^{k+1} ({}^{\psi}A_{dq})^{\Omega_\psi}}{\prod_{\psi=1}^{k+1} (1+(\gamma-1)(1-{}^{\psi}A_{dq}))^{\Omega_\psi} + (\gamma-1) \prod_{\psi=1}^{k+1} ({}^{\psi}A_{dq})^{\Omega_\psi}}, \\ \frac{\prod_{\psi=1}^{k+1} (1+(\gamma-1)^{\psi}\Re_{dq})^{\Omega_\psi} - \prod_{\psi=1}^{k+1} (1-{}^{\psi}\Re_{dq})^{\Omega_\psi}}{\prod_{\psi=1}^{k+1} (1+(\gamma-1)^{\psi}\Re_{dq})^{\Omega_\psi} + (\gamma-1) \prod_{\psi=1}^{k+1} (1-{}^{\psi}\Re_{dq})^{\Omega_\psi}} \end{array} \right), \\ \left(\begin{array}{c} \frac{\sqrt[q]{\gamma} \prod_{\psi=1}^{k+1} ({}^{\psi}\alpha)^{\Omega_\psi}}{\sqrt[q]{\prod_{\psi=1}^{k+1} (1+(\gamma-1)(1-({}^{\psi}\alpha)^q))^{\Omega_\psi} + (\gamma-1) \prod_{\psi=1}^{k+1} (({}^{\psi}\alpha)^q)^{\Omega_\psi}}}, \\ \sqrt[q]{\frac{\prod_{\psi=1}^{k+1} (1+(\gamma-1)({}^{\psi}\beta)^q)^{\Omega_\psi} - \prod_{\psi=1}^{k+1} (1-({}^{\psi}\beta)^q)^{\Omega_\psi}}{\prod_{\psi=1}^{k+1} (1+(\gamma-1)({}^{\psi}\beta)^q)^{\Omega_\psi} + (\gamma-1) \prod_{\psi=1}^{k+1} (1-({}^{\psi}\beta)^q)^{\Omega_\psi}}} \end{array} \right) \end{array} \right]$$

Hence Eq. (17) is true for $n = K + 1$. Which proved the theorem.

Theorem 2 (Idempotency): If $\Upsilon_{dq\psi} = \{(\langle {}^{\psi}A_{dq}, {}^{\psi}\Re_{dq}\rangle, \langle {}^{\psi}\alpha, {}^{\psi}\beta\rangle) : \psi \in \mathbb{N}\}$ be a family of N-LDFNs which are all same, i.e., $\Upsilon_{dq\psi} = \Upsilon_{dq}\ \forall\psi$, then

$$N - LDFHWG_{\Omega}(\Upsilon_{dq1}, \Upsilon_{dq2}, \ldots, \Upsilon_{dqn}) = \Upsilon_{dq}$$

Theorem 3 (Boundedness):. Let $\Upsilon_{dq\psi} = \{(\langle {}^{\psi}A_{dq}, {}^{\psi}\Re_{dq}\rangle, \langle {}^{\psi}\alpha, {}^{\psi}\beta\rangle) : \psi \in \mathbb{N}\}$be a family of N-LDFNs, and consider
$\Upsilon^{-}_{dq} = \min_{\psi} \Upsilon_{dq\psi},\ \Upsilon^{+}_{dq} = \max_{\psi} \Upsilon_{dq\psi}$.
Then

$$\Upsilon^{-}_{dq} \leq N - LDFHWG_{\Omega}(\Upsilon_{dq1}, \Upsilon_{dq2}, \ldots, \Upsilon_{dqn}) \leq \Upsilon^{+}_{dq}$$

Theorem 4 (Monotonicity): Let $\Upsilon_{dq\psi}(\psi \in \mathbb{N})$ and $\Upsilon^{*}_{dq\psi}(\psi \in \mathbb{N})$ belong to N-LDFNs, if $\Upsilon_{dq\psi} \leq \Upsilon^{*}_{dq\psi}, \forall\ \psi$. then

$$N - LDFHWG_{\Omega}(\Upsilon_{dq1}, \Upsilon_{dq2}, \ldots, \Upsilon_{dqn}) \leq N - LDFHWG_{\Omega}(\Upsilon^{*}_{dq1}, \Upsilon^{*}_{dq2}, \ldots, \Upsilon^{*}_{dqn})$$

Further, we develop the N-LDF Hamacher ordered weighted geometric (N-LDFHOWG) operator.

Definition 18: Consider a family of N-LDFNs $\Upsilon_{dq\psi} = \{(\langle {}^{\psi}A_{dq}, {}^{\psi}\Re_{dq}\rangle, \langle {}^{\psi}\alpha, {}^{\psi}\beta\rangle) : \psi \in \mathbb{N}\}$ over the reference set Z and $\Omega = (\Omega_1, \Omega_2, \ldots, \Omega_n)^R$ are the weights with $\sum_{\psi=1}^{n} \Omega_\psi = 1$, $q \geq 1$; then N-LDF Hamacher ordered weighted geometric (N-LDFHOWG) operator is define as follows and let the transformation $\theta : N - LDFN(Z) \longrightarrow N - LDFN(Z)$

$$\begin{aligned} &N - LDFHOWG(\Upsilon_{dq1}, \Upsilon_{dq2}, \ldots, \Upsilon_{dqn}) \\ &= \bigotimes_{\psi=1}^{n} (\Upsilon_{dq(\delta)\psi})^{\Omega_\psi} \\ &= \Upsilon^{\Omega_1}_{dq(\delta)1} \bigotimes \Upsilon^{\Omega_2}_{dq(\delta)2} \bigotimes \ldots \bigotimes \Upsilon^{\Omega_n}_{dq(\delta)n}. \end{aligned}$$

where the arrangement of $(\psi \in \mathbb{N})$ is $(\delta(1), \delta(2), \ldots, \delta(n))$, for which $\Upsilon_{dq\delta(\psi=1)} \geq \Upsilon_{dq\delta(\psi)} \forall(\psi \in \mathbb{N})$.

Theorem 5: Consider a family of N-LDFNs $\Upsilon_{dq\psi} = \{(\langle {}^{\psi}A_{dq}, {}^{\psi}\Re_{dq}\rangle, \langle {}^{\psi}\alpha, {}^{\psi}\beta\rangle) : \psi \in \mathbb{N}\}$ over the fixed set Z and $\Omega = (\Omega_1, \Omega_2, \ldots, \Omega_n)^T$ are the weights with $\sum_{\psi=1}^{n} \Omega_\psi = 1$, and $\gamma > 0$, $q \geq 1$; by applying the N-LDFHOWG operator their aggregated value is also N-LDFN, and the transformation $\theta : N - LDFN(Z) \longrightarrow N - LDFN(Z)$ is known as N-LDF Hamacher ordered weighted geometric operator (N-LDFHOWG) and define as follow;

$$\begin{aligned} &N - LDFHOWG(\Upsilon_{dq1}, \Upsilon_{dq2}, \ldots, \Upsilon_{dqn}) \\ &= \bigotimes_{\psi=1}^{n} (\Upsilon_{dq(\delta)\psi})^{\Omega_\psi} \end{aligned}$$

$$= \left[\begin{array}{c} \left(\begin{array}{c} \frac{\gamma \prod_{\psi=1}^{n} ({}^{\psi}A_{dq(\delta)})^{\Omega_\psi}}{\prod_{\psi=1}^{n} (1+(\gamma-1)(1-{}^{\psi}A_{dq(\delta)}))^{\Omega_\psi} + (\gamma-1) \prod_{\psi=1}^{n} ({}^{\psi}A_{dq(\delta)})^{\Omega_\psi}}, \\ \frac{\prod_{\psi=1}^{n} (1+(\gamma-1)^{\psi}\Re_{dq(\delta)})^{\Omega_\psi} - \prod_{\psi=1}^{n} (1-{}^{\psi}\Re_{dq(\delta)})^{\Omega_\psi}}{\prod_{\psi=1}^{n} (1+(\gamma-1)^{\psi}\Re_{dq(\delta)})^{\Omega_\psi} + (\gamma-1) \prod_{\psi=1}^{n} (1-{}^{\psi}\Re_{dq(\delta)})^{\Omega_\psi}} \end{array} \right), \\ \left(\begin{array}{c} \frac{\sqrt[q]{\gamma} \prod_{\psi=1}^{n} ({}^{\psi}\alpha(\delta))^{\Omega_\psi}}{\sqrt[q]{\prod_{\psi=1}^{n} (1+(\gamma-1)(1-({}^{\psi}\alpha(\delta))^q))^{\Omega_\psi} + (\gamma-1) \prod_{\psi=1}^{n} (({}^{\psi}\alpha(\delta))^q)^{\Omega_\psi}}}, \\ \sqrt[q]{\frac{\prod_{\psi=1}^{n} (1+(\gamma-1)({}^{\psi}\beta(\delta))^q)^{\Omega_\psi} - \prod_{\psi=1}^{n} (1-({}^{\psi}\beta(\delta))^q)^{\Omega_\psi}}{\prod_{\psi=1}^{n} (1+(\gamma-1)({}^{\psi}\beta(\delta))^q)^{\Omega_\psi} + (\gamma-1) \prod_{\psi=1}^{n} (1-({}^{\psi}\beta(\delta))^q)^{\Omega_\psi}}} \end{array} \right) \end{array} \right]. \quad (18)$$

where the arrangement of $(\psi \in \mathbb{N})$ is $(\delta(1), \delta(2), \ldots, \delta(n))$, for which $\Upsilon_{dq\delta(\psi=1)} \geq \Upsilon_{dq\delta(\psi)} \forall(\psi \in \mathbb{N})$.

Theorem 6 (Idempotency): If $\Upsilon_{dq\psi} = \{(\langle {}^{\psi}A_{dq}, {}^{\psi}\Re_{dq}\rangle, \langle {}^{\psi}\alpha, {}^{\psi}\beta\rangle) : \psi = 1, 2, \ldots n\}$ be a family of N-LDFNs which are all same, i.e., $\Upsilon_{dq\psi} = \Upsilon_{dq}$ $\forall\psi$, then

$$N - LDFHOWG_{\Omega}(\Upsilon_{Dq1}, \Upsilon_{dq2}, \ldots, \Upsilon_{dqn}) = \Upsilon_{dq}$$

Theorem 7 (Boundedness): Let $\Upsilon_{dq\psi} = \{(\langle {}^{\psi}A_{dq}, {}^{\psi}\Re_{dq}\rangle, \langle {}^{\psi}\alpha, {}^{\psi}\beta\rangle) : \psi = 1, 2, \ldots n\}$ be a family of N-LDFNs, and consider

$\Upsilon_{dq}^{-} = \min_{\psi} \Upsilon_{dq\psi}$, $\Upsilon_{dq}^{+} = \max_{\psi} \Upsilon_{dq\psi}$, then

$$\Upsilon_{dq}^{-} \leq N-LDFHOWG_{\Omega}(\Upsilon_{dq1}, \Upsilon_{dq2}, \ldots, \Upsilon_{dqn}) \leq \Upsilon_{dq}^{+}$$

Theorem 8 (Monotonicity): Let $\Upsilon_{dq\psi}(\psi = 1, 2, \ldots n)$ and $\Upsilon_{dq\psi}^{*}(\psi = 1, 2, \ldots n)$ belong to N-LDFNs, if $\Upsilon_{dq\psi} \leq \Upsilon_{dq\psi}^{*}$, $\forall\, \psi$. then

$$\begin{aligned} &N-LDFHOWG_{\Omega}(\Upsilon_{dq1}, \Upsilon_{dq2}, \ldots, \Upsilon_{dqn}) \\ &\quad \leq N-LDFHOWG_{\Omega}(\Upsilon_{dq1}^{*}, \Upsilon_{dq2}^{*}, \ldots, \Upsilon_{dqn}^{*}) \end{aligned}$$

We defined the N-LDFHWG and N-LDFHOWG operators in the previous paragraph. Now the N-LDF Hamacher hybrid weighted geometric (N-LDFHHWG) operator is being presented.

Definition 19: Let $\Upsilon_{dq\psi} = \{(\langle {}^{\psi}A_{dq}, {}^{\psi}\Re_{dq}\rangle, \langle {}^{\psi}\alpha, {}^{\psi}\beta\rangle) : \psi = 1, 2, \ldots n\}$ be an assembling of N-LDFNs over the fixed set Z and $\Omega = (\Omega_1, \Omega_2, \ldots, \Omega_n)^T$ are the weights with $\sum_{\psi=1}^{n} \Omega_{\psi} = 1$, and $\gamma > 0$, $q \geq 1$; then we define the N-LDF Hamacher hybrid weighted geometric (N-LDFHHWG) operator as follows and let the mapping $\theta : N-LDFN(Z) \longrightarrow N-LDFN(Z)$

$$\begin{aligned} &N-LDFHHWG(\Upsilon_{dq1}, \Upsilon_{dq2}, \ldots, \Upsilon_{dqn}) \\ &\quad = \bigotimes_{\psi=1}^{n} (\Upsilon_{(\delta)dq\psi}^{\star})^{\Omega_{\psi}} \\ &\quad = (\Upsilon_{(\delta)dq1}^{\star})^{\Omega_1} \bigotimes (\Upsilon_{(\delta)dq2}^{\star})^{\Omega_2} \bigotimes \ldots \bigotimes (\Upsilon_{(\delta)dqn}^{\star})^{\Omega_n}. \end{aligned} \tag{19}$$

where $\Upsilon_{\delta dq(\psi)}^{\star}$ is the ψth largest weighted N-LDF values $\Upsilon_{dq(i)}^{\star}(\Upsilon_{dq(\psi)}^{\star} = (\Upsilon_{dq(\psi)})^{n\Omega_{\psi}}, \psi \in \mathbb{N})$ and the weights of $\Upsilon_{dq(\psi)}^{\star}$ are $\Omega = (\Omega_1, \Omega_2, \ldots, \Omega_n)^T$ by mean of $\Omega > 0$ with $\sum_{\psi=1}^{n} \Omega_{\psi} = 1$.

Theorem 9: Consider a family of N-LDFNs $\Upsilon_{dq\psi} = \{(\langle {}^{\psi}A_{dq}, {}^{\psi}\Re_{dq}\rangle, \langle {}^{\psi}\alpha, {}^{\psi}\beta\rangle) : \psi \in \mathbb{N}\}$ over the reference set Z and $\Omega = (\Omega_1, \Omega_2, \ldots, \Omega_n)^T$ are the weight with $\sum_{\psi=1}^{n} \Omega_{\psi} = 1$, also $q \geq 1$; by applying the N-LDFHHWG operator their aggregated value is also an N-LDFN, and the transformation $\theta : N-LDFN(Z) \longrightarrow N-LDFN(Z)$ are known as N-LDF Hamacher hybrid weighted geometric operator (N-LDFHHWG) and define as follow;

$$N-LDFHHWG(\Upsilon_{dq1}, \Upsilon_{dq2}, \ldots, \Upsilon_{dqn}) = \bigotimes_{\psi=1}^{n} (\Upsilon_{(\delta)dq\psi}^{\star})^{\Omega_{\psi}}$$

$$= \left[\begin{array}{c} \left(\dfrac{\gamma \prod_{\psi=1}^{n} ({}^{\psi}A_{(\delta)dq}^{\star})^{\Omega_{\psi}}}{\prod_{\psi=1}^{n} (1 + (\gamma - 1)(1 - {}^{\psi}A_{(\delta)dq}^{\star}))^{\Omega_{\psi}} + (\gamma - 1) \prod_{\psi=1}^{n} ({}^{\psi}A_{(\delta)dq}^{\star})^{\Omega_{\psi}}}, \dfrac{\prod_{\psi=1}^{n} (1 + (\gamma - 1){}^{\psi}\Re_{(\delta)dq}^{\star})^{\Omega_{\psi}} - \prod_{\psi=1}^{n} (1 - {}^{\psi}\Re_{(\delta)dq}^{\star})^{\Omega_{\psi}}}{\prod_{\psi=1}^{n} (1 + (\gamma - 1){}^{\psi}\Re_{(\delta)dq}^{\star})^{\Omega_{\psi}} + (\gamma - 1) \prod_{\psi=1}^{n} (1 - {}^{\psi}\Re_{(\delta)dq}^{\star})^{\Omega_{\psi}}} \right), \\ \left(\dfrac{\sqrt[q]{\gamma} \prod_{\psi=1}^{n} ({}^{\psi}\alpha^{\star}(\delta))^{\Omega_{\psi}}}{\sqrt[q]{\prod_{\psi=1}^{n} (1 + (\gamma - 1)(1 - ({}^{\psi}\alpha^{\star}(\delta))^{q}))^{\Omega_{\psi}} + (\gamma - 1) \prod_{\psi=1}^{n} (({}^{\psi}\alpha^{\star}(\delta))^{q})^{\Omega_{\psi}}}}, \sqrt[q]{\dfrac{\prod_{\psi=1}^{n} (1 + (\gamma - 1)({}^{\psi}\beta^{\star}(\delta))^{q})^{\Omega_{\psi}} - \prod_{\psi=1}^{n} (1 - ({}^{\psi}\beta^{\star}(\delta))^{q})^{\Omega_{\psi}}}{\prod_{\psi=1}^{n} (1 + (\gamma - 1)({}^{\psi}\beta^{\star}(\delta))^{q})^{\Omega_{\psi}} + (\gamma - 1) \prod_{\psi=1}^{n} (1 - ({}^{\psi}\beta^{\star}(\delta))^{q})^{\Omega_{\psi}}}} \right) \end{array} \right]. \tag{20}$$

where $\Upsilon_{\delta dq(\psi)}^{\star}$ is the ψth biggest weighted N-LDF values $\Upsilon_{dq(i)}^{\star}(\Upsilon_{dq(\psi)}^{\star} = (\Upsilon_{dq(\psi)})^{n\Omega_{\psi}}, \psi \in \mathbb{N})$ and be the weights of $\Upsilon_{dq(\psi)}^{\star}$ is $\Omega = (\Omega_1, \Omega_2, \ldots, \Omega_n)^T$ with condition $\Omega > 0$, $\sum_{\psi=1}^{n} \Omega_{\psi} = 1$.

If $\Omega = (\frac{1}{\Omega}, \frac{1}{\Omega}, \ldots, \frac{1}{\Omega})$, then N-LDFHWG and N-LDFHOWG operators are supposed to be a particular case N-LDFHHWG. So from this it obtained that N-LDFHHWG operator is the generalized version of N-LDFHWG and N-LDFHOWG operators.

V. MADM MODEL USING N-LDF INFORMATION

Throughout this section, a novel approach to MADM is introduced, which is based on Hamacher operators and is called N-LDFHWG, N-LDFHOWG, and N-LDFHHWG. As a result, we proposed a methodology for numerical modeling and applying different S.F and A.F and also construct and describes a case study related to the assessment of the NEA deflection technologies.

A. ALGORITHM BASED ON N-LDFHWG AGGREGATION OPERATOR

We have develop an algorithm based on N-LDFHWG aggregation operators throughout this subsection. Multi-attribute decision making (MADM) problem is applied to identify and select the best NEA detector technologies for N-LDF data based on Hamacher operators. Consider a set of alternatives $T = \{T_1, T_2, T_3, \ldots, T_m\}$ and $\breve{G} = \{\breve{G}_1, \breve{G}_2, \breve{G}_3, \ldots \breve{G}_n\}$ be a set of criteria. Suppose the weight of criteria $\Omega_\psi(\psi = 1, 2, \ldots, n)$ are $\Omega = (\Omega_1, \Omega_2, \ldots, \Omega_n)^T$ with $\Omega_\psi > 0$, $\sum_{\psi=1}^{n} \Omega_\psi = 1$. Suppose the N-LDF $DM = (\langle {}^{g\psi}A_{dqK}, {}^{g\psi}\Re_{dqK}\rangle, \langle {}^{g\psi}\alpha, {}^{g\psi}\beta\rangle)_{m\times n}$, where ${}^{g\psi}A_{dqK}$ is the MG, ${}^{g\psi}\Re_{dqK}$ is the NMG and ${}^{g\psi}\alpha, {}^{g\psi}\beta$ are the RPs for which the alternative (T_ψ) fulfill the criteria $(\breve{G}_\psi)$, where ${}^{g\psi}A_{dq}, {}^{g\psi}\Re_{dq}, {}^{g\psi}\alpha, {}^{g\psi}\beta \subset [0, 1]$ such that $0 \leq^{g\psi} ((\alpha)^q A_{dq}) +^{g\psi} ((\beta)^q \Re_{dq}) \leq 1, (g = 1, 2, \ldots, m)$. Based on above information we solve MADM problem with N-LDFNs based on Hamacher operators by mean of N-LDFHWG, N-LDFHOWG and N-LDFHHWG aggregation operators.

Input:

Step 1: For an acceptable number of alternatives and criteria, construct a DMs group of N-LDF information. Here the decision maker group are represented by $DM = \{DM_1, DM_2, \ldots, DM_u, DM_l\}$ with weight vector Ω. Each DMs are evaluated by N-LDFNs based on Hamacher operators.

Step 2: Normalization of N-LDF input information

To obtain the most accurate results, it is necessary to normalize the input data before starting the calculations. As a result, the N-LDF analysis can be standardized by using the following formula:

$$R_{Dq\psi} = \begin{cases} (\langle {}^{\psi}A_{dqK}, {}^{\psi}\Re_{dqK}\rangle, \langle {}^{\psi}\alpha, {}^{\psi}\beta\rangle); & \text{Same data} \\ (\langle {}^{\psi}\Re_{dqK}, {}^{\psi}A_{dqK}, \rangle, \langle {}^{\psi}\beta, {}^{\psi}\alpha\rangle); & \text{Different data} \end{cases}$$

Here we have the same input data for all criteria so there is no need to apply this step.

Step 3: Select the weights for each decision-maker's opinion.

Calculations:

Step 4: By applying **Eq 21** of N-LDF Hamacher aggregation operators with weights $\Omega_\psi(\psi = 1, 2, 3)$ of criteria $\breve{G}_j$ to combine the decision information presented in matrix $DM_k(k = 1, 2, 3)$ into the collective N-LDF DM.

$$N - LDFHWG(\Upsilon^1_{dq(g\psi)}, \Upsilon^2_{dq(g\psi)}, \ldots, \Upsilon^n_{dq(g\psi)}) =$$

$$\left[\begin{array}{c} \left(\begin{array}{c} \dfrac{\gamma \prod_{\psi=1}^{n} ({}^{\psi}A^{g\psi}_{dq})^{\Omega_\psi}}{\prod_{\psi=1}^{n}(1+(\gamma-1)(1-{}^{\psi}A^{g\psi}_{dq}))^{\Omega_\psi} + (\gamma-1)\prod_{\psi=1}^{n}({}^{\psi}A^{g\psi}_{dq})^{\Omega_\psi}}, \\ \dfrac{\prod_{\psi=1}^{n}(1+(\gamma-1)({}^{\psi}\Re^{g\psi}_{dq}))^{\Omega_\psi} - \prod_{\psi=1}^{n}(1-{}^{\psi}\Re^{g\psi}_{dq})^{\Omega_\psi}}{\prod_{\psi=1}^{n}(1+(\gamma-1)({}^{\psi}\Re^{g\psi}_{dq}))^{\Omega_\psi} + (\gamma-1)\prod_{\psi=1}^{n}(1-{}^{\psi}\Re^{g\psi}_{dq})^{\Omega_\psi}} \end{array}\right), \\ \left(\begin{array}{c} \dfrac{\sqrt[q]{\gamma}\prod_{\psi=1}^{n}({}^{\psi}\alpha^{g\psi})^{\Omega_\psi}}{\sqrt[q]{\prod_{\psi=1}^{n}(1+(\gamma-1)(1-({}^{\psi}\alpha^{g\psi})^q))^{\Omega_\psi} + (\gamma-1)\prod_{\psi=1}^{n}(({}^{\psi}\alpha^{g\psi})^q)^{\Omega_\psi}}}, \\ \sqrt[q]{\dfrac{\prod_{\psi=1}^{n}(1+(\gamma-1)({}^{\psi}\beta^{g\psi})^q)^{\Omega_\psi} - \prod_{\psi=1}^{n}(1-({}^{\psi}\beta^{g\psi})^q)^{\Omega_\psi}}{\prod_{\psi=1}^{n}(1+(\gamma-1)({}^{\psi}\beta^{g\psi})^q)^{\Omega_\psi} + (\gamma-1)\prod_{\psi=1}^{n}(1-({}^{\psi}\beta^{g\psi})^q)^{\Omega_\psi}}} \end{array}\right) \end{array}\right]. \quad (21)$$

Similarly, apply the N-LDFHOWG and N-LDFHHWG operator. Thus for $\mho = 1, 2, 3, \ldots, g$; $\phi = 1, 2, 3, \ldots, m$; $\psi = 1, 2, 3, \ldots, n$, we get the aggregated decision matrix. Also for order and hybrid AOs.

Step 5: By applying the above equation of N-LDF Hamacher operators, calculate the collective aggregated value for each criteria with weights $\Omega_\psi(\psi = 1, 2, 3, 4, 5)$.

Step 6: Determine the scores of each alternatives by applying above definition of S.F, Q.S.F and E.S.F.

Output:

Step 7: Based on the values of S.F, Q.S.F, and E.S.F, rank the alternatives.

Step 8: The alternative with the greatest score has the highest rank and must be chosen for the final decision. End.

Figure 2. show the proposed algorithm steps for N-LDF data based on Hamacher operators.

VI. NEA DEFLECTION DETECTORS TECHNOLOGY PROBLEM

The aim of this research is to conduct a fuzzy MADM analysis in order to evaluate the NEA deflection detectors technology: KI, EGT, IBD, CRE and LA. With respect to

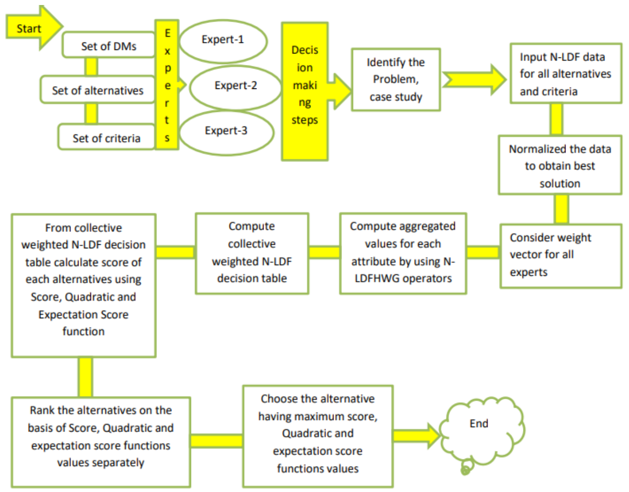

FIGURE 2. **Flow chart for proposed N-LDFS algorithm based on Hamacher operators.**

the 5 criteria the alternatives will be determined. In addition, using the data offered by various experts, we will be able to measure the aggregate relative value of a given alternative for each criteria in terms of Hamacher operators associated with N-LDFNs in order to deal with that task. With some previous methods, we relate our findings.

A. PROBLEM DESCRIPTION: A CASE STUDY RELATED TO THE ASSESSMENT OF THE NEA DEFLECTION TECHNOLOGIES

An asteroid's deflection consists of accelerating the object just enough to reach the Earth's orbit by a minimal distance from the point it would have been crossed by the NEA, providing it was not deflected. Our study's assumptions, which were revealed to the team of experts, were as described. In the 2013 Congress of the United States, NASA will need at least five years of planning before an asteroid intercept mission could be launched [93], [94]. Before deciding which strategy is sufficient, it is also useful to find out the material composition of the object. Valuable guidance about what to expect has been provided by missions like the 2005 Deep Impact probe. We therefore briefly provide a summary of each alternative.

B. DESCRIPTION OF THE ALTERNATIVES FOR NEA DEFLECTION

1) CONVENTIONAL ROCKET ENGINE (CRE) (T_1)

It will have a similar effect of giving a push to attach some spacecraft propulsion device, possibly pushing the asteroid into a trajectory that leads it away from Earth. An in-space rocket engine able to produce a pulse of 106 N· s (e.g. adding 1 km/s to a spacecraft weighing 1000 kg) would have a relatively small impact on a relatively small asteroid weighing approximately a million times as much. In [95], deflections are determined using the latest chemical rockets supplied to the asteroid. The use of highly-efficient electrically driven spacecraft propulsion, such as ion thrusters or VASIMR, is usually suggested for such direct force rocket engines.

2) ION BEAM DEFLECTION (IBD) (T_2)

The IBD technology consists primarily of an ion thruster on board a spacecraft (called the ''shepherd'') that at the NEA points to a strongly collimated high-speed ion beam. Simultaneously, to preserve a uniform distance from the asteroid, a secondary thruster points in the opposite direction [1], [96]. In this way, a hovering distance of twice the diameter of the target asteroid makes it possible to neglect the NEA gravitational force [97]. Interestingly, the IBD rendezvous spacecraft can be sent to the NEA in advance, thereby reducing the uncertainty about the asteroid's orbit. In comparison to the KI technique, this might be seen as an advantage of the IBD. Furthermore, the IBD allows for accurate targeting of the asteroid's impact position, which is especially important for massive asteroids that may only be deflected by a few Earth radii [98].

3) ENHANCED GRAVITY TRACTOR (EGT) (T_3)

The Gravity Tractor (GT) is a spacecraft that hovers over a target NEA and uses the gravitational force between the asteroid and the spacecraft to change its trajectory. It's worth noting that the GT is an observer strategy in and of itself. The EGT builds its mass by removing rocks or regolith from the NEA it is targeting. That mass is estimated in such a way that when the spaceship's engines are turned on full power and pointed in the general direction of the NEA, the asteroid and the spaceship distance do not increase. The thrusters must slowly impulse the entire system to reduce the NEA's velocity or to improve its velocity, to maintain a uniform distance between the spacecraft and the targeted asteroid [1], [99].

4) LASER ABLATION (LA) (T_4)

The energy from a set of phase-locked laser amplifiers is continually impinged on the NEA, ejecting some material away from its surface and altering the velocity of the targeted asteroid [1], [97], [100].

5) KINETIC IMPACTOR (KI) (T_5)

The Kinetic Impactor (KI) is a spacecraft that is sent on a collision course with an NEA. This would change the asteroid's momentum and velocity [1], [97], [100]. It's worth noting that, as NASA's Deep Impact mission reported in 2005, it's already possible to crash into an asteroid at a high velocity. One of the advantages of the KI deflection technology, according to the space science community, is its quick effect, as well as the high degree of momentum that can be supplied to the targeted asteroid [98].

C. SELECTED CRITERIA

All of the criteria listed below will be examined in this research using N-LDFN-based significance scales. It is

TABLE 3. N-LDF decision matrix 1.

$q=4$	$\breve{G}_1$	$\breve{G}_2$	$\breve{G}_3$	$\breve{G}_4$	$\breve{G}_5$
T_1	$\left(\langle 1,1\rangle,\langle .9,.7\rangle\right)$	$\left(\langle 1,.87\rangle,\langle .75,.9\rangle\right)$	$\left(\langle .95,1\rangle,\langle .75,.85\rangle\right)$	$\left(\langle .8,.9\rangle,\langle .85,.8\rangle\right)$	$\left(\langle 1,1\rangle,\langle .7,.9\rangle\right)$
T_2	$\left(\langle .87,.9\rangle,\langle .75,.9\rangle\right)$	$\left(\langle 1,1\rangle,\langle .7,.9\rangle\right)$	$\left(\langle .87,1\rangle,\langle .9,.75\rangle\right)$	$\left(\langle 1,1\rangle,\langle .9,.7\rangle\right)$	$\left(\langle .87,1\rangle,\langle .9,.75\rangle\right)$
T_3	$\left(\langle 1,1\rangle,\langle .8,.8\rangle\right)$	$\left(\langle .87,1\rangle,\langle .9,.75\rangle\right)$	$\left(\langle .9,.9\rangle,\langle .85,.8\rangle\right)$	$\left(\langle 1,1\rangle,\langle .85,.75\rangle\right)$	$\left(\langle 1,1\rangle,\langle .7,.9\rangle\right)$
T_4	$\left(\langle .95,1\rangle,\langle .75,.85\rangle\right)$	$\left(\langle .9,.9\rangle,\langle .85,.8\rangle\right)$	$\left(\langle .8,.9\rangle,\langle .85,.8\rangle\right)$	$\left(\langle .87,.1\rangle,\langle .9,.75\rangle\right)$	$\left(\langle .87,.9\rangle,\langle .75,.9\rangle\right)$
T_5	$\left(\langle 1,.87\rangle,\langle .75,.9\rangle\right)$	$\left(\langle 1,1\rangle,\langle .9,.7\rangle\right)$	$\left(\langle .95,1\rangle,\langle .75,.85\rangle\right)$	$\left(\langle 1,1\rangle,\langle .8,.8\rangle\right)$	$\left(\langle .87,.9\rangle,\langle .75,.9\rangle\right)$

TABLE 4. N-LDF decision matrix 2.

$q=4$	$\breve{G}_1$	$\breve{G}_2$	$\breve{G}_3$	$\breve{G}_4$	$\breve{G}_5$
T_1	$\left(\langle 1,.87\rangle,\langle .75,.9\rangle\right)$	$\left(\langle 1,1\rangle,\langle .9,.7\rangle\right)$	$\left(\langle .95,1\rangle,\langle .75,.85\rangle\right)$	$\left(\langle 1,1\rangle,\langle .8,.8\rangle\right)$	$\left(\langle .87,.9\rangle,\langle .75,.9\rangle\right)$
T_2	$\left(\langle .9,.9\rangle,\langle .85,.8\rangle\right)$	$\left(\langle .95,1\rangle,\langle .75,.85\rangle\right)$	$\left(\langle .87,1\rangle,\langle .9,.75\rangle\right)$	$\left(\langle .87,.9\rangle,\langle .75,.9\rangle\right)$	$\left(\langle .8,.9\rangle,\langle .85,.8\rangle\right)$
T_3	$\left(\langle 1,1\rangle,\langle .9,.7\rangle\right)$	$\left(\langle 1,.87\rangle,\langle .75,.9\rangle\right)$	$\left(\langle .8,.9\rangle,\langle .85,.8\rangle\right)$	$\left(\langle 1,1\rangle,\langle .7,.9\rangle\right)$	$\left(\langle .95,1\rangle,\langle .75,.85\rangle\right)$
T_4	$\left(\langle .87,1\rangle,\langle .9,.75\rangle\right)$	$\left(\langle .87,.9\rangle,\langle .75,.9\rangle\right)$	$\left(\langle 1,1\rangle,\langle .9,.7\rangle\right)$	$\left(\langle .87,.1\rangle,\langle .9,.75\rangle\right)$	$\left(\langle 1,.87\rangle,\langle .75,.9\rangle\right)$
T_5	$\left(\langle .9,.9\rangle,\langle .9,.75\rangle\right)$	$\left(\langle 1,1\rangle,\langle .65,.9\rangle\right)$	$\left(\langle 1,1\rangle,\langle .7,.9\rangle\right)$	$\left(\langle 1,1\rangle,\langle .65,.9\rangle\right)$	$\left(\langle .9,.9\rangle,\langle .85,.8\rangle\right)$

utilized the information offered by our group of experts to achieve the goal.

1) COMPOSITION OF ASTEROID ($\breve{G}_1$)

Note that this criterion can strongly focus on the efficiency of many NEA deflection approaches. For example, when applied to metallic surfaces, LA can not function properly because the heat produced can be carried away.

2) STRUCTURE OF ASTEROID ($\breve{G}_2$)

Rather than the asteroid's surface material structure or friability, this is associated with the object's porosity and interior structure. It's worth noting that KI is affected by the object's internal structure and porosity, both of which might affect momentum transmission. EGT's ability to collect material from the NEA's surface may be affected as well.

3) SHAPE OF ASTEROID ($\breve{G}_3$)

In targeted NEAs a large variety of irregular contours may appear.

4) TECHNOLOGICAL READINESS LEVEL (TRL) ($\breve{G}_4$)

NASA recommended this standardized scale about the target maturity level for that method to assess the current level of progress of the technology. Targeted maturity in this article means a redirection technology for asteroids that is ready to be proven at the next stage in space, which is similar to TRL [97].

5) MISSION RISK ($\breve{G}_5$)

It takes into account the probability of a technical failure concerning the asteroid deflection mission or an unsuccessful outcome. To define certain unique risks that can occur when applying each NEA deflection technique, this is quantified separately from the TRL. It is worth noting that to resolve the risk evaluation, a scale based on the Goddard risk matrix has been suggested [97].

D. GROUP OF EXPERTS

The questionnaires sent by the authors were completed by a group of three (3) researchers whose areas of expertise include NEA deflection technologies, thus providing some useful information for the alternatives and parameters are involved in our analysis. As follows, their affiliations were: Department of Physics (one expert), Department of Mathematics (one researcher), and Laboratory of Applied Physics (one scientist).

Assume that there are five deflection technologies $T = \{T_1, T_2, T_3, T_4, T_5\}$ for the evaluation of NEA. The decision criteria set are $\breve{G} = \{\breve{G}_1, \breve{G}_2, \breve{G}_3, \breve{G}_4, \breve{G}_5\}$. Consider that the expert's weight is: $\Omega = (\Omega_1, \Omega_2, \Omega_3) = (0.4, 0.35, 0.25)$. The NEA deflection technology evaluations are performed by the expert and Table 3 to Table 5 represented the Non-linear Diophantine fuzzy matrix in tabular form. We apply the above algorithm to select the best technology for active NEA deflection under N-LDF data based on Hamacher operators.

Solution By Using Above Algorithm

TABLE 5. N-LDF decision matrix 3.

$q=4$	$\breve{G}_1$	$\breve{G}_2$	$\breve{G}_3$	$\breve{G}_4$	$\breve{G}_5$
T_1	$\left(\langle .9,1\rangle,\langle .9,.75\rangle\right)$	$\left(\langle .87,1\rangle,\langle .9,.75\rangle\right)$	$\left(\langle 1,1\rangle,\langle .65,.9\rangle\right)$	$\left(\langle .87,.9\rangle,\langle .75,.9\rangle\right)$	$\left(\langle .8,.9\rangle,\langle .85,.8\rangle\right)$
T_2	$\left(\langle 1,.87\rangle,\langle .75,.9\rangle\right)$	$\left(\langle 1,1\rangle,\langle .9,.7\rangle\right)$	$\left(\langle .95,1\rangle,\langle .75,.85\rangle\right)$	$\left(\langle 1,1\rangle,\langle .8,.8\rangle\right)$	$\left(\langle 1,1\rangle,\langle .7,.9\rangle\right)$
T_3	$\left(\langle 1,1\rangle,\langle .9,.7\rangle\right)$	$\left(\langle 1,.87\rangle,\langle .75,.9\rangle\right)$	$\left(\langle .8,.9\rangle,\langle .85,.8\rangle\right)$	$\left(\langle 1,1\rangle,\langle .7,.9\rangle\right)$	$\left(\langle 1,1\rangle,\langle .65,.9\rangle\right)$
T_4	$\left(\langle .9,.9\rangle,\langle .9,.75\rangle\right)$	$\left(\langle .9,1\rangle,\langle .9,.75\rangle\right)$	$\left(\langle 1,1\rangle,\langle .9,.7\rangle\right)$	$\left(\langle .87,.9\rangle,\langle .75,.9\rangle\right)$	$\left(\langle 1,.87\rangle,\langle .75,.9\rangle\right)$
T_5	$\left(\langle 1,1\rangle,\langle .8,.8\rangle\right)$	$\left(\langle .87,1\rangle,\langle .9,.75\rangle\right)$	$\left(\langle .87,.9\rangle,\langle .75,.9\rangle\right)$	$\left(\langle .95,1\rangle,\langle .75,.85\rangle\right)$	$\left(\langle 1,1\rangle,\langle .9,.7\rangle\right)$

TABLE 6. The collective N-LDFHWG Decision matrix.

$q=4,\gamma=1$	$\breve{G}_1$	$\breve{G}_2$	$\breve{G}_3$
T_1	$(\langle 0.974,1\rangle,\langle 0.0528,0.1098\rangle)$	$(\langle 0.966,1\rangle,\langle 0.0523,0.115\rangle)$	$(\langle 0.962,1\rangle,\langle 0.045,0.1399\rangle)$
T_2	$(\langle 0.9116,0.893\rangle,\langle 0.049,0.1461\rangle)$	$(\langle 0.982,1\rangle,\langle 0.0477,0.132\rangle)$	$(\langle 0.889,1\rangle,\langle 0.054,0.0937\rangle)$
T_3	$(\langle 1,1\rangle,\langle 0.0537,0.0783\rangle)$	$(\langle 0.946,1\rangle,\langle 0.0504,0.137\rangle)$	$(\langle 0.839,0.9\rangle,\langle 0.053,0.1024\rangle)$
T_4	$(\langle 0.9088,1\rangle,\langle 0.0523,0.1019\rangle)$	$(\langle 0.889,1\rangle,\langle 0.0516,0.123\rangle)$	$(\langle 0.915,1\rangle,\langle 0.055,0.0783\rangle)$
T_5	$(\langle 0.9638,1\rangle,\langle 0.0508,0.1248\rangle)$	$(\langle 0.966,1\rangle,\langle 0.0502,0.11\rangle)$	$(\langle 0.946,1\rangle,\langle 0.046,0.1519\rangle)$
$q=4$	$\breve{G}_4$		$\breve{G}_5$
T_1	$(\langle 0.883,1\rangle,\langle 0.0504,0.121\rangle)$		$(\langle 0.901,1\rangle,\langle 0.047,0.152\rangle)$
T_2	$(\langle 0.952,1\rangle,\langle 0.0512,0.115\rangle)$		$(\langle 0.875,1\rangle,\langle 0.0518,0.113\rangle)$
T_3	$(\langle 1,1\rangle,\langle 0.0473,0.137\rangle)$		$(\langle 0.982,1\rangle,\langle 0.044,0.154\rangle)$
T_4	$(\langle 0.87,1\rangle,\langle 0.0537,0.106\rangle)$		$(\langle 0.946,0.883\rangle,\langle 0.0469,0.164\rangle)$
T_5	$(\langle 0.987,1\rangle,\langle 0.0458,0.134\rangle)$		$(\langle 0.912,1\rangle,\langle 0.0513,0.123\rangle)$

Step-1: Here we are going to apply the above algorithm on the given input data. Three NEA experts have been assigned to rate the five NEA detector $T_i(i=1,2,3,4)$ alternatives in terms of five criteria $\breve{G}_j(j=1,2,\ldots,5)$, and the decision matrices $D_k(k=1,2,3)$ have been listed in **Table 3 to Table 5**.

Step-2: First, we have finalize the input data by assigning weights to each expert. At this point, every NEA expert's viewpoint is important in making a final decision. $\Omega=(\Omega_1,\Omega_2,\Omega_3)=(0.4,0.35,0.25)^T$ are the weights for N-LDF input data.

Step-3: In this step we applied N-LDFHWG, N-LDFHOWG, and N-LDFHHWG operators by using **Eq. 21** on input N-LDF- data as shown in **Table 3**, **Table 4** and **Table 5**. We aggregated the individual decision matrix $DM_k(k=1,2,3)$ via **Eq 21** into the collective N-LDFHWG matrix as given in **Table 6**.

Step-4: Now for **Table 6** we choose different five weights, consider $\Omega=(0.35,0.25,0.2,0.1,0.1)^T$ are the five weights for **Table 6** of N-LDFHWG data.

Step-5: To aggregate **Table 6** alternative wise we repeat above **step-3** by applying **Eq 21** with weights $\ell=(0.35,0.25,0.2,0.1,0.1)^T$. As a result we obtained the aggregated N-LDFHWG, N-LDFHOWG and N-LDFHHWG listed in **Table 7.**

TABLE 7. Aggregated N-LDFHWG.

$\gamma=1$	Aggregated Alternatives
T_1	$(\langle 0.9527,1\rangle,\langle 0.0515,0.00024\rangle)$
T_2	$(\langle 0.9244,1\rangle,\langle 0.0490,0.00028\rangle)$
T_3	$(\langle 0.9504,1\rangle,\langle 0.0502,0.00021\rangle)$
T_4	$(\langle 0.9047,1\rangle,\langle 0.0542,0.00019\rangle)$
T_5	$(\langle 0.9577,1\rangle,\langle 0.0511,0.00028\rangle)$

Step-6: By applying S.F, Q.S.F and E.S.F via above definition we calculated the score values, so we get **Table 8;**

For $\gamma=1$, we have obtained Table 8 as follows; **Table 9** represent the ranking of N-LDFHWG operator. From Table 9, we concluded that T_3 which is Enhanced Gravity Tractor (EGT) selected the best overall NEA deflection detector obtained from N-LDFHWG operator.

TABLE 8. Score values of N-LDFHWG.

$\gamma=1$	S.F(κ)	Q.S.F(ϖ)	E.S.F(F)
T_1	$-.0236$	$-.0462$	$.4882$
T_2	$-.0377$	$-.0727$	$.4811$
T_3	$-.0211$	$-.0413$	$.4885$
T_4	$-.0476$	$-.0907$	$.4762$
T_5	$-.0264$	$-.0414$	$.4884$

TABLE 9. Ranking of N-LDFHWG.

$q=4,\gamma=1$	N-LDFHWG
S.F(κ)	$T_3>T_1>T_5>T_2>T_4$
Q.S.F(ϖ)	$T_3>T_5>T_1>T_2>T_4$
E.S.F(F)	$T_3>T_5>T_1>T_2>T_4$

TABLE 10. Aggregated N-LDFHOWG.

$\gamma=1$	Aggregated Alternatives
T_1	$(\langle 0.9414, 1\rangle, \langle 0.00319, 0.0002\rangle)$
T_2	$(\langle 0.9295, 1\rangle, \langle 0.00317, 0.00025\rangle)$
T_3	$(\langle 0.9501, 1\rangle, \langle 0.0032, 0.0002\rangle)$
T_4	$(\langle 0.9109, 1\rangle, \langle 0.00332, 0.00016\rangle)$
T_5	$(\langle 0.9512, 1\rangle, \langle 0.0031, 0.00024\rangle)$

TABLE 11. Score values of N-LDFHOWG.

$\gamma=1$	S.F(κ)	Q.S.F(ϖ)	E.S.F(F)
T_1	$-.0293$	$-.0569$	$.4854$
T_2	$-.0353$	$-.0680$	$.4824$
T_3	$-.0242$	$-.0466$	$.4878$
T_4	$-.0446$	$-.0851$	$.4777$
T_5	$-.0243$	$-.0476$	$.4878$

Step-7: For N-LDFHOWG, first of all we calculate the order of the three experts individually by applying E.S.F which is given in the form of **Table 3** to **Table 5**. The reordering phase is the most important part of the Order weighted operator; it reorders all the input data in descending number. We continue the steps of the aforementioned Algorithm to obtain the following N-LDFHOWG results, as shown in **Table 10**. **Table 11** represent the different score values of N-LDFHOWG operator. **Table 12** shows the ranking Table of N-LDFHOWG operator for $\gamma = 1$. From Table 12, we conclude that T_3 which is Enhanced Gravity Tractor (EGT) selected the best overall NEA deflection detector technology obtained from N-LDFHOWG operator.

TABLE 12. Ranking of N-LDFHOWG.

$q=4,\gamma=1$	N-LDFHOWG
S.F(κ)	$T_3>T_5>T_1>T_2>T_4$
Q.S.F(ϖ)	$T_3>T_5>T_1>T_2>T_4$
E.S.F(F)	$T_3=T_5>T_1>T_2>T_4$

TABLE 13. Aggregated N-LDFHHWG.

$\gamma=1$	Aggregated Alternatives
T_1	$(\langle 0.95449, 1\rangle, \langle 0.0023, 0.00026\rangle)$
T_2	$(\langle 0.94555, 1\rangle, \langle 0.00334, 0.00023\rangle)$
T_3	$(\langle 0.97398, 1\rangle, \langle 0.00352, 0.00013\rangle)$
T_4	$(\langle 0.92132, 1\rangle, \langle 0.00504, 0.23579\rangle)$
T_5	$(\langle 0.96588, 1\rangle, \langle 0.00427, 0.00016\rangle)$

Step-8: We also applied the same steps of above Hamacher-N-LDFNs algorithm for N-LDFHHWG operator but in hybrid case one more step is added in the above algorithm that is, we first aggregate three decision makers tables by means of N-LDFHWG operator, as same as we aggregate in **step-3** of N-LDFHWG after this we apply N-LDFHWG operator individually in each number of N-LDF aggregated table as we obtained from **step-3** for this aggregation table we choose another five weights for five criteria which is (0.3, 0.25, 0.2, 0.15, 0.1) and every weight is multiply by 5, so for N-LDFHhybridWG operator the weight vector is (1.5, 1.25, 1, 0.75, 0.5). We get N-LDFH hybrid weighted geometric table after this we apply S.F on this table and reorder the criteria to get hybrid aggregated table, this whole process is considered in **step-3**. Then moving to the next steps and use the same steps of above algorithm after **step-3,** so we get N-LDFHHWG table as given below in Table 13 for $\gamma = 1$. Table 14 represent the score values of N-LDFHHWG operator for $\gamma = 1$. Table 15 is the ranking Table of N-LDFHHWG operator for $\gamma = 1$; Overall ranking of N-LDFHWG, N-LDFHOWG and N-LDFHHWG operators for $q = 4$ and $\gamma = 1$ are follow as in Table 16.

From Table 16, we concluded that T_3 which is Enhanced Gravity Tractor (EGT) selected the best overall NEA deflection detector obtained from N-LDFHWG, N-LDFHOWG and N-LDFHHWG operators. It is important to keep in mind that the final result obtained from proposed algorithm is identical for all score functions. Following Figure 3. shows the graphical ranking of N-LDFHWG operator for NEA deflection technologies in which the Enhanced Gravity Tractor become at the top of all alternatives.

TABLE 14. Score values of N-LDFHHWG.

$\gamma = 1$	S.F(κ)	Q.S.F(ϖ)	E.S.F(F)
T_1	$-.023$	$-.045$	$.489$
T_2	$-.027$	$-.053$	$.486$
T_3	$-.013$	$-.026$	$.494$
T_4	$-.041$	$-.076$	$.479$
T_5	$-.017$	$-.034$	$.492$

TABLE 15. Ranking of N-LDFHHWG.

$q = 4, \gamma = 1$	N-LDFHHWG
S.F(κ)	$T_3 > T_5 > T_1 > T_2 > T_4$
Q.S.F(ϖ)	$T_3 > T_5 > T_1 > T_2 > T_4$
E.S.F(F)	$T_3 > T_5 > T_1 > T_2 > T_4$

TABLE 16. Overall ranking of N-LDFHWG operators.

$q = 4, \gamma = 1$	S.F(κ)	Q.S.F(ϖ)
N-LDFHWG	$T_3 > T_1 > T_5 > T_2 > T_4$	$T_3 > T_5 > T_1 > T_2 > T_4$
N-LDFHOWG	$T_3 > T_5 > T_1 > T_2 > T_4$	$T_3 > T_5 > T_1 > T_2 > T_4$
N-LDFHHWG	$T_3 > T_5 > T_1 > T_2 > T_4$	$T_3 > T_5 > T_1 > T_2 > T_4$
$q = 4, \gamma = 1$	E.S.F(F)	
N-LDFHWG	$T_3 > T_5 > T_1 > T_2 > T_4$	
N-LDFHOWG	$T_3 = T_5 > T_1 > T_2 > T_4$	
N-LDFHHWG	$T_3 > T_5 > T_1 > T_2 > T_4$	

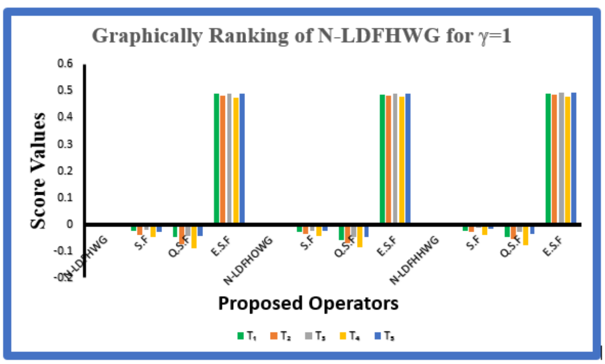

FIGURE 3. Graphical Ranking of N-LDFHWG operator.

Similarly we give different values to $\gamma's$ for proposed operators and got a little bit different in values but the top one alternative is same in all method. The score values and ranking are shown in the following Table 17 to Table 21.

Following Table 17 listed the score values for $\gamma = 2$ Thus, Table 18 presented the ranking result for

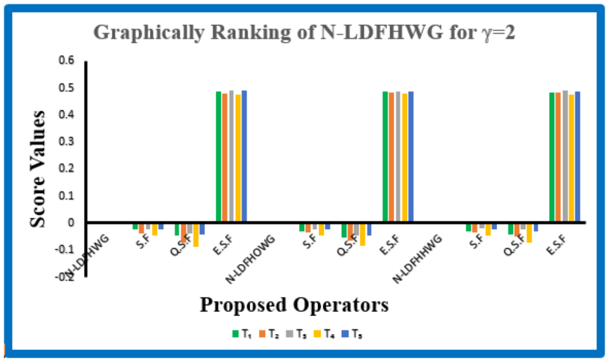

FIGURE 4. Graphical Ranking of N-LDFHWG operator.

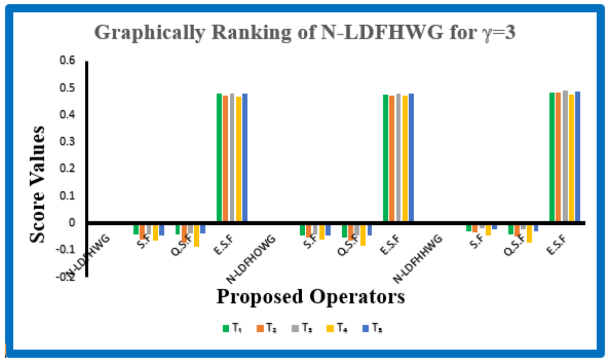

FIGURE 5. Graphical Ranking of N-LDFHWG operator.

$\gamma = 2$ Following Figure 4. shows the graphical ranking of N-LDFHWG operator for NEA deflection technologies in which the Enhanced Gravity Tractor become at the top of all alternatives.

For $\gamma = 3$, we get Table 19 as follows;

Table-20 listed the ranking results for $\gamma = 3$ as follow. Following Figure 5. shows the graphical ranking of N-LDFHWG operator for NEA deflection technologies in which the Enhanced Gravity Tractor become at the top of all alternatives.

Step-9: The alternatives of N-LDFHWG, N-LDFHOWG, and N-LDFHHWG were then rated, so we obtained the final result displayed in **Table 21**.

Step-10: From this we conclude that; T_3 Enhanced Gravity Tractor (EGT) is chose the best NEA deflection alternative other deflection alternatives were ranked near close to each other but far away from EGT. It is important to keep in mind that the final result obtained from proposed algorithm is identical for all score functions.

VII. DISCUSSION AND COMPARISON ANALYSIS

We compare the proposed three N-LDF Hamacher aggregation operators to the existing method [75], [76] in this section, demonstrating their ability to manage daily life DMPs. Because of the qth power of RPs, this method is impressive because it provides the valuation spaces of IFSs,

TABLE 17. Different score values of N-LDFHWG.

$\gamma = 2$	S.F(κ)				
$q = 4$	κ_{σ_1}	κ_{σ_2}	κ_{σ_3}	κ_{σ_4}	κ_{σ_5}
$N-LDFHWG$	-0.0239	-0.0381	-0.0217	-0.0479	-0.0218
$N-LDFHOWG$	-0.0293	-0.0355	-0.0248	-0.0449	-0.0249
$N-LDFHHWG$	-0.0307	-0.0348	-0.0204	-0.0464	-0.0248
$\gamma = 2$	E.S.F(F)				
$q = 4$	$F_{\sigma 1}$	$F_{\sigma 2}$	$F_{\sigma_{\hbar 3}}$	$F_{\sigma_{\hbar 4}}$	$F_{\sigma_{\hbar 5}}$
$N-LDFHWG$	0.4880	0.4809	0.4897	0.4760	0.4892
$N-LDFHOWG$	0.4853	0.4822	0.4876	0.4776	0.4876
$N-LDFHHWG$	0.4846	0.4826	0.4897	0.4768	0.4875
$\gamma = 2$	Q.S.F(ϖ)				
$q = 4$	ϖ_{σ_1}	$\varpi_{\sigma 2}$	ϖ_{σ_3}	ϖ_{σ_4}	ϖ_{σ_5}
$N-LDFHWG$	-0.0444	-0.0711	-0.0401	-0.0894	-0.0405
$N-LDFHOWG$	-0.0549	-0.0664	-0.0461	-0.0838	-0.0462
$N-LDFHHWG$	-0.0434	-0.0514	-0.0243	-0.0736	-0.0325

TABLE 18. Ranking of N-LDFHWG operator.

$q = 4$,$\gamma = 2$	S.F(κ)	Q.S.F(ϖ)
N-LDFHWG	$T_3 > T_5 > T_1 > T_2 > T_4$	$T_3 > T_5 > T_1 > T_2 > T_4$
N-LDFHOWG	$T_3 > T_5 > T_1 > T_2 > T_4$	$T_3 > T_5 > T_1 > T_2 > T_4$
N-LDFHHWG	$T_3 > T_5 > T_1 > T_2 > T_4$	$T_3 > T_5 > T_1 > T_2 > T_4$
$q = 4$,$\gamma = 2$	E.S.F(F)	
N-LDFHWG	$T_3 > T_5 > T_1 > T_2 > T_4$	
N-LDFHOWG	$T_3 = T_5 > T_1 > T_2 > T_4$	
N-LDFHHWG	$T_3 > T_5 > T_1 > T_2 > T_4$	

PyFSs, q-ROFSs, and LDFSs. Table 26 represented the general comparison of suggested and existing concept.

A. COMPARISON WITH LDFS

In this subsection we compare the proposed method with existing LDF method [75] namely LDFWG, LDFOWG and LDFHWG aggregation operators. The different score function values for LDF is given in following Table 22, and Table 23 represented the ranking result which is similar to proposed ranking method.

Table 23 listed the ranking of LDFWG operators.

B. COMPARISON WITH Q-RLDFS

In this subsection we compare the proposed work with existing q-RLDF [76] method namely q-RLDFWG, q-RLDFOWG and q-RLDFHWG aggregation operators. The

TABLE 19. Different score values of N-LDFHWG.

$\gamma = 3$	S.F(κ)				
$q = 4$	κ_{σ_1}	κ_{σ_2}	κ_{σ_3}	κ_{σ_4}	κ_{σ_5}
$N-LDFHWG$	-0.0435	-0.0606	-0.0419	-0.0648	-0.0443
$N-LDFHOWG$	-0.0467	-0.0553	-0.0415	-0.0598	-0.0447
$N-LDFHHWG$	-0.0306	-0.0346	-0.0205	-0.0465	-0.0245
$\gamma = 3$	E.S.F(F)				
$q = 4$	$F_{\sigma 1}$	$F_{\sigma 2}$	F_{σ_3}	F_{σ_4}	F_{σ_5}
$N-LDFHWG$	0.4783	0.4697	0.4790	0.4676	0.4779
$N-LDFHOWG$	0.4767	0.4724	0.4792	0.47009	0.4777
$N-LDFHHWG$	0.4847	0.4827	0.4898	0.4767	0.4877
$\gamma = 3$	Q.S.F(ϖ)				
$q = 4$	ϖ_{σ_1}	$\varpi_{\sigma 2}$	ϖ_{σ_3}	ϖ_{σ_4}	ϖ_{σ_5}
$N-LDFHWG$	-0.0438	-0.0709	-0.0401	-0.0889	-0.0403
$N-LDFHOWG$	-0.0540	-0.0659	-0.0449	-0.0832	-0.0460
$N-LDFHHWG$	-0.0434	-0.0514	-0.0243	-0.0736	-0.0325

TABLE 20. Ranking of N-LDFHWG operator.

$q = 4$,$\gamma = 3$	S.F(κ)	Q.S.F(ϖ)
N-LDFHWG	$T_3 > T_1 > T_5 > T_2 > T_4$	$T_3 > T_5 > T_1 > T_2 > T_4$
N-LDFHOWG	$T_3 > T_5 > T_1 > T_2 > T_4$	$T_3 > T_5 > T_1 > T_2 > T_4$
N-LDFHHWG	$T_3 > T_5 > T_1 > T_2 > T_4$	$T_3 > T_5 > T_1 > T_2 > T_4$
$q = 4$,$\gamma = 3$	E.S.F(F)	
N-LDFHWG	$T_3 > T_1 > T_5 > T_2 > T_4$	
N-LDFHOWG	$T_3 > T_5 > T_1 > T_2 > T_4$	
N-LDFHHWG	$T_3 > T_5 > T_1 > T_2 > T_4$	

different score function values of q-RLDF is given in the following Table 24 and Table 25 represented the ranking result which is similar to proposed ranking method.

Table 25 listed the ranking result of q-RLDFWG operators. "In Tables 27 and Table 28" we can see the ranking results of five alternatives of proposed model and existing approaches. Table 28 listed the decision maker's opinion for the selection of NEA deflection detector based on N-LDFHWG operators, calculated rankings by the proposed and previous method is same, which is approachable and validates the reliability and viability of the recommended work, demonstrating that NEA deflection detector process is very particularly considerable to support companies. The ranking results obtained from the suggested methodology

TABLE 21. Overall ranking of N-LDFHWG operators for different values of operational parameter.

$q=4$	Operators	S.F(κ)	Q.S.F(ϖ)	E.S.F(F)
$\gamma=1$	N-LDFHWG	$T_3>T_1>T_5>T_2>T_4$	$T_3>T_5>T_1>T_2>T_4$	$T_3>T_5>T_1>T_2>T_4$
	N-LDFHOWG	$T_3>T_5>T_1>T_2>T_4$	$T_3>T_5>T_1>T_2>T_4$	$T_3=T_5>T_1>T_2>T_4$
	N-LDFHHWG	$T_3>T_5>T_1>T_2>T_4$	$T_3>T_5>T_1>T_2>T_4$	$T_3>T_5>T_1>T_2>T_4$
$\gamma=2$	N-LDFHWG	$T_3>T_5>T_1>T_2>T_4$	$T_3>T_5>T_1>T_2>T_4$	$T_3>T_5>T_1>T_2>T_4$
	N-LDFHOWG	$T_3>T_5>T_1>T_2>T_4$	$T_3>T_5>T_1>T_2>T_4$	$T_3=T_5>T_1>T_2>T_4$
	N-LDFHHWG	$T_3>T_5>T_1>T_2>T_4$	$T_3>T_5>T_1>T_2>T_4$	$T_3>T_5>T_1>T_2>T_4$
$\gamma=3$	N-LDFHWG	$T_3>T_1>T_5>T_2>T_4$	$T_3>T_5>T_1>T_2>T_4$	$T_3>T_1>T_5>T_2>T_4$
	N-LDFHOWG	$T_3>T_5>T_1>T_2>T_4$	$T_3>T_5>T_1>T_2>T_4$	$T_3>T_5>T_1>T_2>T_4$
	N-LDFHHWG	$T_3>T_5>T_1>T_2>T_4$	$T_3>T_5>T_1>T_2>T_4$	$T_3>T_5>T_1>T_2>T_4$

TABLE 22. Different score values of existing LDFWG.

	S.F(κ)				
Existing method	κ_{σ_1}	κ_{σ_2}	κ_{σ_3}	κ_{σ_4}	κ_{σ_5}
$LDFWG$	-0.03786	-0.05728	-0.02459	-0.03592	-0.04883
$LDFOWG$	-0.03094	-0.04569	-0.01987	-0.01958	-0.03923
$LDFHWG$	-0.01208	0.00279	0.00671	-0.00067	-0.00934
	E.S.F(F)				
	$F_{\sigma 1}$	$F_{\sigma 2}$	F_{σ_3}	F_{σ_4}	F_{σ_5}
$LDFWG$	0.48107	0.47136	0.48770	0.48204	0.47559
$LDFOWG$	0.48453	0.47716	0.49006	0.49021	0.48039
$LDFHWG$	0.49396	0.49967	0.50335	0.50139	0.49533
	Q.S.F(ϖ)				
	ϖ_{σ_1}	$\varpi_{\sigma 2}$	ϖ_{σ_3}	ϖ_{σ_4}	ϖ_{σ_5}
$LDFWG$	-0.06943	-0.10472	-0.04804	-0.07143	-0.08644
$LDFOWG$	-0.05957	-0.08516	-0.04036	-0.04396	-0.07174
$LDFHWG$	-0.02873	-0.00893	-0.00107	-0.00663	-0.02175

and the existing methodology differ slightly, but in terms of overall methodology, the best and first choices are the same. The comparison's ranking results are shown in Table 28 below.

Superiority and comparison between suggested and existing methods: LDFSs [75] have some restrictions on RPs and are unable to handle qth parameterizations. Almagrabi et al. [76] modified the idea of LDFSs and developed q-RLDFSs to cover this research gap, and we applied the Hamacher operators on q-RLDFSs to obtain a more generalized idea which named is N-LDFHWG aggregation operators. There is a close link between the

TABLE 23. Ranking of LDFWG.

Existing method	S.F(κ)	Q.S.F(ϖ)
LDFWG	$T_3 > T_4 > T_1 > T_5 > T_2$	$T_3 > T_1 > T_4 > T_5 > T_2$
LDFOWG	$T_3 > T_4 > T_1 > T_5 > T_2$	$T_3 > T_4 > T_1 > T_5 > T_2$
LDFHWG	$T_3 > T_2 > T_4 > T_5 > T_1$	$T_3 > T_4 > T_2 > T_5 > T_1$
	E.S.F(F)	
LDFWG	$T_3 > T_4 > T_1 > T_5 > T_2$	
LDFOWG	$T_3 > T_4 > T_1 > T_5 > T_2$	
LDFHWG	$T_3 > T_4 > T_2 > T_5 > T_1$	

TABLE 24. Different score values of q-RLDFWG.

	S.F(κ)				
$q = 4$	$\kappa_{\sigma 1}$	$\kappa_{\sigma 2}$	$\kappa_{\sigma 3}$	$\kappa_{\sigma 4}$	$\kappa_{\sigma 5}$
$q - RLDFWG$	-0.0609	-0.09625	-0.0261	-0.0359	-0.0888
$q - RLDFOWG$	-0.0467	-0.0533	-0.0153	-0.02594	-0.0474
$q - RLDFHWG$	-0.0466	-0.3562	-0.0355	-0.0434	-0.0542
	E.S.F(F)				
$q = 4$	$F_{\sigma 1}$	$F_{\sigma 2}$	F_{σ_3}	F_{σ_4}	F_{σ_5}
$q - RLDFWG$	0.4695	0.4519	0.4869	0.4821	0.4556
$q - RLDFOWG$	0.4775	0.4424	0.4927	0.4819	0.4677
$q - RLDFHWG$	0.4475	0.4267	0.4889	0.4836	0.4767
	Q.S.F(ϖ)				
$q = 4$	ϖ_{σ_1}	$\varpi_{\sigma 2}$	ϖ_{σ_3}	ϖ_{σ_4}	ϖ_{σ_5}
$q - RLDFWG$	-0.0758	-0.1241	-0.0496	-0.0803	-0.1039
$q - RLDFOWG$	-0.0594	-0.0683	-0.0532	-0.0854	-0.0760
$q - RLDFHWG$	-0.0334	-0.5462	-0.0143	-0.0752	-0.0775

proposed approach and MADM problems. As a result, when compared to other methods, our N-LDFS-based on Hamacher Norms provides more accurate results because of extended values.

TABLE 25. Ranking of q-RLDFWG.

$q=4$	S.F(κ)	Q.S.F(ϖ)
q-RLDFWG	$T_3 > T_4 > T_1 > T_5 > T_2$	$T_3 > T_1 > T_4 > T_5 > T_2$
q-RLDFOWG	$T_3 > T_4 > T_1 > T_5 > T_2$	$T_3 > T_1 > T_2 > T_5 > T_4$
q-RLDFHWG	$T_3 > T_4 > T_1 > T_2 > T_5$	$T_3 > T_1 > T_2 > T_4 > T_5$
$q=4$	E.S.F(F)	
q-RLDFWG	$T_3 > T_4 > T_1 > T_5 > T_2$	
q-RLDFOWG	$T_3 > T_4 > T_1 > T_5 > T_2$	
q-RLDFHWG	$T_3 > T_4 > T_5 > T_1 > T_2$	

TABLE 26. The comparison study of N-LDFs with previous methods.

Collections	Remarks	Parameterizations
FS [19]	non-membership $(\Re_{(\ell)})$ not satisfies	NO
IFS [30]	cannot deal, $A_{(\ell)} + \Re_{(\ell)} > 1$	NO
PyFS [42], [43]	cannot deal, $A^2_{(\ell)} + \Re^2_{(\ell)} > 1$	NO
q-ROFS [59]	$A^q_{(\ell)} + \Re^q_{(\ell)} > 1$ and $A_{(\ell)} = 1, \Re_{(\ell)} = 1$	NO
LDFS [75]	LDFS covers this situation, $0 \leq (\alpha)A_{d(\ell)} + (\beta)\Re_{d(\ell)} \leq 1,$	YES
q-RLDFS/N-LDFS [76]	q-RLDFS/N-LDFS cover LDFS limitation, $0 \leq (\alpha)^q A_{dq(\ell)} +$ $(\beta)^q \Re_{dq(\ell)} \leq 1, q \geqslant 1.$	YES

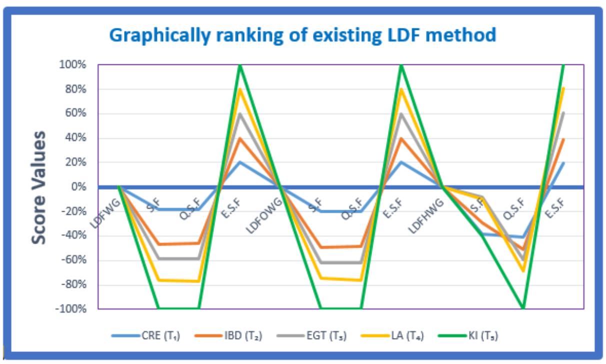

FIGURE 6. Graphical Ranking of existing LDFWG operator.

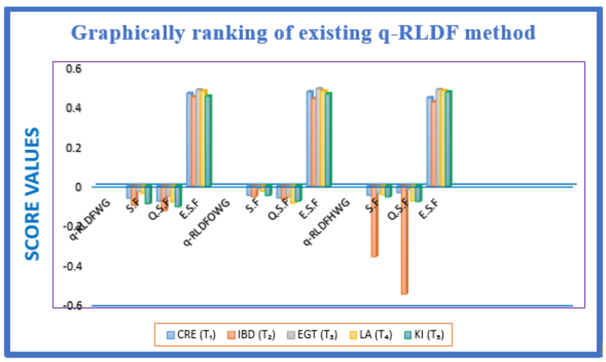

FIGURE 7. Graphical Ranking of existing q-RLDFWG operator.

Following Figure 6. shows the graphical ranking of existing LDFWG, LDFOWG and LDFHWG operators on the base of different score function for NEA deflection technologies in which T_3 is selected again the optimal alternatives and is similar to the ranking of proposed method which showed the superiority of propose method.

Following Figure 7. shows the graphical ranking of existing q-RLDFWG, q-RLDFOWG and q-RLDFHWG operators on the base of different score function for NEA deflection technologies in which again the Enhanced Gravity Tractor become at the top of all alternatives and is similar to the ranking of proposed method which showed the superiority of propose method.

From "Table 28" we concluded that; T_3 which represent the Enhanced Gravity Tractor (EGT) is chose the best NEA deflection alternative other deflection alternatives were

TABLE 27. Different Score values of proposed and existing method.

$q=4,\gamma=1$	S.F(κ)				
Proposed Method	κ_{σ_1}	κ_{σ_2}	κ_{σ_3}	κ_{σ_4}	κ_{σ_5}
N-LDFHWG	$-.0236$	$-.0377$	$-.0211$	$-.0476$	$-.0264$
N-LDFHOWG	$-.0293$	$-.0353$	$-.0242$	$-.0446$	$-.0243$
N-LDFHHWG	$-.023$	$-.027$	$-.013$	$-.041$	$-.017$
Existing Method	κ_{σ_1}	κ_{σ_2}	κ_{σ_3}	κ_{σ_4}	κ_{σ_5}
LDFWG [75]	-0.0379	-0.0573	-0.0246	-0.0359	-0.0488
q-RLDFWG [76]	-0.0609	-0.09625	-0.0261	-0.0359	-0.0888
LDFOWG [75]	-0.0309	-0.0457	-0.0196	-0.0199	-0.0392
q-RLDFOWG [76]	-0.0467	-0.0533	-0.0153	-0.02594	-0.0474
LDFHWG [75]	-0.01208	0.0028	0.0067	-0.00067	-0.0094
q-RLDFHWG [76]	-0.0466	-0.3562	-0.0355	-0.0434	-0.0542
$q=4,\gamma=1$	Q.S.F(ϖ)				
Proposed Method	ϖ_{σ_1}	$\varpi_{\sigma 2}$	ϖ_{σ_3}	ϖ_{σ_4}	ϖ_{σ_5}
N-LDFHWG	$-.0462$	$-.0727$	$-.0413$	$-.0907$	$-.0414$
N-LDFHOWG	-0.0569	-0.0680	-0.0466	-0.0851	-0.0476
N-LDFHHWG	-0.045	-0.053	-0.026	-0.076	-0.034
Existing Method	ϖ_{σ_1}	$\varpi_{\sigma 2}$	ϖ_{σ_3}	ϖ_{σ_4}	ϖ_{σ_5}
LDFWG [75]	-0.0694	-0.1047	-0.0480	-0.0714	-0.0864
q-RLDFWG [76]	-0.0758	-0.1241	-0.0496	-0.0803	-0.1039
LDFOWG [75]	-0.0596	-0.0852	-0.0404	-0.0439	-0.0717
q-RLDFOWG [76]	-0.0594	-0.0683	-0.0532	-0.0854	-0.0760
LDFHWG [75]	-0.0287	-0.00672	-0.0059	-0.0067	-0.0218
q-RLDFHWG [76]	-0.0334	-0.5462	-0.0143	-0.0752	-0.0775
$q=4,\gamma=1$	E.S.F(F)				
Proposed Method	$F_{\sigma 1}$	$F_{\sigma 2}$	F_{σ_3}	F_{σ_4}	F_{σ_5}
N-LDFHWG	.4882	.4811	.4885	.4762	.4884
N-LDFHOWG	0.4854	0.4824	0.4878	0.4777	0.4878
N-LDFHHWG	0.489	0.486	0.494	0.479	0.492
Existing Method	$F_{\sigma 1}$	$F_{\sigma 2}$	F_{σ_3}	F_{σ_4}	F_{σ_5}
LDFWG [75]	0.4810	0.4714	0.4877	0.4820	0.4756
q-RLDFWG [76]	0.4695	0.4519	0.4869	0.4821	0.4556
LDFOWG [75]	0.4845	0.4772	0.4903	0.4902	0.4804
q-RLDFOWG [76]	0.4775	0.4424	0.4927	0.4819	0.4677
LDFHWG [75]	0.4939	0.5014	0.5504	0.5034	0.4953
q-RLDFHWG [76]	0.4475	0.4267	0.4889	0.4836	0.4767

ranked near close to each other but far away from EGT. It is important to keep in mind that the final result obtained from proposed algorithm and existing method is identical for all types of score functions. Following Figure 8. shows the systematics ranking diagram of five NEA deflection technologies in which the Enhanced Gravity Tractor become at the top of all alternatives, 2nd one is the Kinetic Impactor, 3rd ranked alternatives for NEA deflection technologies is conventional rocket engine, 4th one is Ion Bean deflection

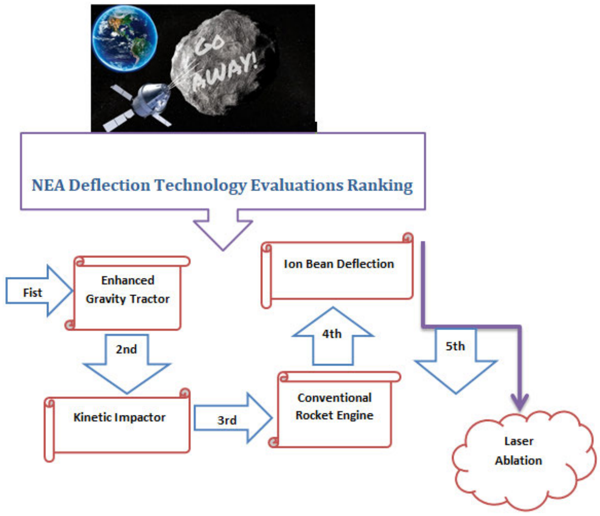

FIGURE 8. Systematics ranking diagram of NEA deflection technologies.

and the last and 5th alternative is laser ablation which ranked at the bottom of list.

C. COMPARISON WITH SPEARMAN'S AND WS COEFFICIENTS OF RANKINGS SIMILARITY

Comparing the accuracy of the two rankings' order is an essential decision. Checking if the ranks are constant or inconsistent is the easiest way to do this. One of the coefficients of monotonous dependency of two variables is used in the largely accepted method, where our variables are the rankings which are obtained for a group of alternatives that are under consideration. The Spearman's coefficient [101] is the most often used symmetrical coefficient of this kind of relationship and may be written as follows in Eq (22):

$$r_s = 1 - \frac{6.\sum d_i^2}{n.(n^2-1)}. \tag{22}$$

where n is the number of objects in the ranking and d_i is defined as the difference between the ranks $d_i = R_{xi} - R_{yi}$. As a percentage of the rank variation of one variable that is explained by the other variable, the Spearman's coefficient is recognized [101].

Given the differences between two ranks in certain locations, we expected the WS indicator to be substantially correlated with the rankings. There's also the supposition that the top of the ranking influences similarity more than the bottom. These presumptions led to the development of a new indicator caused by [102], which is shown by Eq (23):

$$WS = 1 - \sum_{i=1}^{n} \left(2^{-R_{xi}} . \frac{|R_{xi} - R_{yi}|}{\max\left\{|1-R_{xi}|, |N-R_{yi}|\right\}} \right), \tag{23}$$

where N is the ranking length, WS is the similarity coefficient value, and R_{xi} and R_{yi} denote the ranking position of the $i-th$ element in rankings x and y, correspondingly.

Table 29, shows five rankings T_i, including one reference (R_x) and three test rankings such that S.F is represented by

TABLE 28. Ranking of proposed and existing method for different *gamma*, *s*.

$\gamma = 1, q = 4$			
Proposed Method	S.F(κ)	Q.S.F(ϖ)	E.S.F(F)
N-LDFHWG	$T_3 > T_1 > T_5 > T_2 > T_4$	$T_3 > T_5 > T_1 > T_2 > T_4$	$T_3 > T_5 > T_1 > T_2 > T_4$
N-LDFHOWG	$T_3 > T_5 > T_1 > T_2 > T_4$	$T_3 > T_5 > T_1 > T_2 > T_4$	$T_3 = T_5 > T_1 > T_2 > T_4$
N-LDFHHWG	$T_3 > T_5 > T_1 > T_2 > T_4$	$T_3 > T_5 > T_1 > T_2 > T_4$	$T_3 > T_5 > T_1 > T_2 > T_4$
$\gamma = 2$	S.F(κ)	Q.S.F(ϖ)	E.S.F(F)
N-LDFHWG	$T_3 > T_5 > T_1 > T_2 > T_4$	$T_3 > T_5 > T_1 > T_2 > T_4$	$T_3 > T_5 > T_1 > T_2 > T_4$
N-LDFHOWG	$T_3 > T_5 > T_1 > T_2 > T_4$	$T_3 > T_5 > T_1 > T_2 > T_4$	$T_3 = T_5 > T_1 > T_2 > T_4$
N-LDFHHWG	$T_3 > T_5 > T_1 > T_2 > T_4$	$T_3 > T_5 > T_1 > T_2 > T_4$	$T_3 > T_5 > T_1 > T_2 > T_4$
$\gamma = 3$	S.F(κ)	Q.S.F(ϖ)	E.S.F(F)
N-LDFHWG	$T_3 > T_1 > T_5 > T_2 > T_4$	$T_3 > T_5 > T_1 > T_2 > T_4$	$T_3 > T_1 > T_5 > T_2 > T_4$
N-LDFHOWG	$T_3 > T_5 > T_1 > T_2 > T_4$	$T_3 > T_5 > T_1 > T_2 > T_4$	$T_3 > T_5 > T_1 > T_2 > T_4$
N-LDFHHWG	$T_3 > T_5 > T_1 > T_2 > T_4$	$T_3 > T_5 > T_1 > T_2 > T_4$	$T_3 > T_5 > T_1 > T_2 > T_4$
Existing Method	S.F(κ)	Q.S.F(ϖ)	E.S.F(F)
LDFWG [75]	$T_3 > T_4 > T_1 > T_5 > T_2$	$T_3 > T_1 > T_4 > T_5 > T_2$	$T_3 > T_4 > T_1 > T_5 > T_2$
LDFOWG [75]	$T_3 > T_4 > T_1 > T_5 > T_2$	$T_3 > T_4 > T_1 > T_5 > T_2$	$T_3 > T_4 > T_1 > T_5 > T_2$
LDFHWG [75]	$T_3 > T_2 > T_4 > T_5 > T_1$	$T_3 > T_4 > T_2 > T_5 > T_1$	$T_3 > T_4 > T_2 > T_5 > T_1$
q-RLDFWG [76]	$T_3 > T_4 > T_1 > T_5 > T_2$	$T_3 > T_1 > T_4 > T_5 > T_2$	$T_3 > T_4 > T_1 > T_5 > T_2$
q-RLDFOWG [76]	$T_3 > T_4 > T_1 > T_5 > T_2$	$T_3 > T_1 > T_2 > T_5 > T_4$	$T_3 > T_4 > T_1 > T_5 > T_2$
q-RLDFHWG [76]	$T_3 > T_4 > T_1 > T_2 > T_5$	$T_3 > T_1 > T_2 > T_4 > T_5$	$T_3 > T_4 > T_5 > T_1 > T_2$

R_{y1}, Q.S.F is represented by R_{y2} and E.S.F is represented by R_{y3}. The test rankings were created based on Table 21 of three different ranking score function values of five alternatives. We want to remind that we have already determined the ranking of proposed operators by choosing different operational parameters $\gamma = 1, 2, 3$ and have applied different score functions. The ranking results of proposed operator is listed in above **Table 21**, but here we want to find out the similarity of these ranking by applying the Spearman's and WS Coefficients of Rankings Similarity listed in Eq (22) and Eq (23) to choose the best possible solution, and with each place in the ranking, the preferences lose value. An error at the bottom of the ranking should not have the same significance as the difference at the top. Compared to the shifting of the third and fourth positions, the alternative placements from the second and third positions represent a more significant inaccuracy. But according to the coefficient values, there is similarity in the test sets' resemblance between the test ranks and the reference ranking. Following **Table 29** listed the *WS* and r_s coefficients ranking for N-LDFHWG operators with $\gamma = 1$. Similarly we calculated the Spearman's and WS Coefficients raking for N-LDFHOWG and N-LDFHHWG including $\gamma = 1, 2, 3$.

TABLE 29. Spearmans and WS coefficients ranking.

$\gamma = 1$	N-LDFHWG			
T_i	R_{xi}	R_{y1}(S.F)	R_{y2}(Q.S.F)	R_{y3}(E.S.F)
T_1	1	3	3	3
T_2	2	1	5	5
T_3	3	5	1	1
T_4	4	2	2	2
T_5	5	4	4	4
Coefficients	WS	0.49	0.33	0.33
	r_s	0.3	-0.1	-0.1

In short we calculated the *WS* and r_s Coefficients for whole Table 21 listed above, as a result we obtained following Table 30 and Table 31.

Similarly for r_s Coefficient ranking, we obtained following Table 31;

TABLE 30. WS coefficient ranking.

$\gamma = 1$	R_{y1}(S.F)	R_{y2}(Q.S.F)	R_{y3}(E.S.F)
N-LDFHWG	0.49	0.33	0.33
N-LDFHOWG	0.33	0.33	0.33
N-LDFHHWG	0.33	0.33	0.33
$\gamma = 2$	R_{y1}	R_{y2}	R_{y3}
N-LDFHWG	0.33	0.33	0.33
N-LDFHOWG	0.33	0.33	0.33
N-LDFHHWG	0.33	0.33	0.33
$\gamma = 3$	R_{y1}	R_{y2}	R_{y3}
N-LDFHWG	0.49	0.33	0.49
N-LDFHOWG	0.33	0.33	0.33
N-LDFHHWG	0.33	0.33	0.33

TABLE 31. r_s coefficient ranking.

$\gamma = 1$	R_{y1}	R_{y2}	R_{y3}
N-LDFHWG	0.3	-0.1	-0.1
N-LDFHOWG	-0.1	-0.1	-0.1
N-LDFHHWG	-0.1	-0.1	-0.1
$\gamma = 2$	R_{y1}	R_{y2}	R_{y3}
N-LDFHWG	-0.1	-0.1	-0.1
N-LDFHOWG	-0.1	-0.1	-0.1
N-LDFHHWG	-0.1	-0.1	-0.1
$\gamma = 3$	R_{y1}	R_{y2}	R_{y3}
N-LDFHWG	0.3	-0.1	0.3
N-LDFHOWG	-0.1	-0.1	-0.1
N-LDFHHWG	-0.1	-0.1	-0.1

D. THREE CRITERIA BASED ASSESSMENT FOR MADM PROBLEM

The Multiple Attribute Decision Making (MADM) scheme's validity and feasibility have been verified through assessments based on three criteria. The effectiveness and accuracy of the MADM system may be assessed with the use of these evaluation tests. We have supposed the following three criteria while designing evaluation reviews for MADM scheme:

1. **Accuracy**: We have evaluated the accuracy of the MADM scheme by considering the same numerical data and comparing it with some existing methods which show that the MADM scheme gives the best solution using test cases or examples with predefined results, Table 28 represented the ranking accuracy of proposed and existing method.

2. **Consistency**: To demonstrate the consistency of the MADM scheme we conduct multiple tests with the same input data based on different score functions which shows that the ranking results are similar obtained by S.F, Q.S.F and E.S.F by applying the proposed and existing method. This can help assess the reliability and stability of the MADM scheme in different situations, Table 27 and Table 28 listed the consistency of MADM problem.

3. **Sensitivity**: We have applied various operational parameters to test the sensitivity of the MADM scheme and observed how the scheme responds to different decision-making scenarios, in order to improve the robustness and adaptability of the MADM scheme. We have select gamma =1,2 and 3 as different operational parameters for sensitivity analysis, Table 21 represented the proposed sensitive analysis ranking and Table 28 represented the proposed and existing method combined sensitive analysis for MADM problem.

VIII. CONCLUSION

We applied Hamacher operators for q-RLDF concept and we get a more generalized version of FS called N-LDF Hamacher operators. Remind that in N-LDF concept the qth power of RPs play an important role in decision-making problem and also provide a more effective and flexible framework. The geometric aspects of N-LDFS have been proposed, and the concept of N-LDFS has been generalized to N-LDFHWG aggregation operator, which further includes N-LDFHOWG and N-LDFHHWG. We offered an implementation of the suggested MADM problem approaches with the help of a case study for the selection of the best NEA detectors technologies. We conclude that the current decision making method is appropriate and stable, and can be effectively implemented for decision-making problems with multi-attribut/criteria group. For the remainder, Enhanced Gravity Tractor (EGT) has been selected the best overall NEA deflection detector, rankings can be illustrated to understand their shortcomings and follow the logical trend for future intentions. Then we likened the current method to some of previous approaches. The obtained results, demonstrated the advantages and validity of the recommended methodology. We also calculated the Spearman's and WS coefficients of rankings similarity for the validation of proposed method.

This study provides several interesting topics for future research. More Non-linear Diophantine fuzzy decision-making techniques can be used in this study, including N-LDF CODAS, N-LDF VIKOR, N-LDF TOPSIS, N-LDF EDAS, N-LDF GRA and their hybrid methods. Future research will develop on the suggested methodology by using complex numbers which will further extend to complex non-linear Diophantine fuzzy numbers and will develop different decision operators for Dombi, Bonferroni, Aczel-Alsina, Hamacher, and for Einstein aggregation operators. The proposed MADM problem can also be used for other complicated problems like risk evaluation, risk aversion, emerging technology, project installation, and also for medical diagnosis.

CONFLICT OF INTERESTS

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

AUTHORS' CONTRIBUTIONS

All authors have read and agreed to the published version of the manuscript.

AVAILABILITY OF DATA AND MATERIALS

Enquiries about data availability should be directed to the authors.

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MARIA SHAMS was born in Swabi, Pakistan. She received the M.Phil. degree in mathematics from Abdul Wali Khan University Mardan, Mardan, Pakistan, where she is currently pursuing the Ph.D. degree in fuzzy decision support systems with the Department of Mathematics. Her two research articles have been published in high-impact *Journal of Ambient Intelligence and Humanized Computing* and *Complex and Intelligent Systems*. Her research interests include fuzzy logic, fuzzy set theory, non-linear diophantine fuzzy theory, and decision-making problems.

SALEEM ABDULLAH was born in Swabi, Khyber Pakhtunkhwa, Pakistan. He received the M.Phil. and Ph.D. degrees in mathematics from Quaid-i-Azam University, Islamabad, Pakistan. He is currently an Associate Professor of mathematics with the Department of Mathematics, Abdul Wali Khan University Mardan, Mardan, Pakistan. He is recognized as one of the outstanding research fellows of the Department of Mathematics, Abdul Wali Khan University Mardan. He has 230 peer-reviewed articles published in international journals, including ISI-indexed and IF journal publications. Some of our articles have been published in high-impact journals, such as *International Journal of Intelligent Systems*, *Soft Computing*, *International Journal of General Systems*, *Journal of Intelligent and Fuzzy Systems*, *International Journal of Fuzzy Systems*, *Journal of Ambient Intelligence and Humanized Computing*, *Mathematics*, and *Symmetry*, to name just a few of the high-impact journals, where some of our articles have been published. He has mentored six Ph.D. students and nine master's students. His research interests include fuzzy logic, fuzzy set theory, and decision-making.

FAISAL KHAN received the B.S. degree in mathematics from the University of Malakand, Lower Dir, Chakdara, Pakistan, in 2015, and the M.Phil. degree in mathematics from Hazara University Mansehra, Pakistan, in 2017. He is currently pursuing the Ph.D. degree with the National University of Ireland Galway (NUIG). He is also with FotoNatioN/Xperi. His research interests include machine learning using deep neural networks for tasks related to computer vision, including depth estimation and 3D reconstruction.

RIFAQAT ALI received the Ph.D. degree from Aligarh Muslim University, Aligarh, India, in 2012. He is currently an Assistant Professor with the Department of Mathematics, College of Science, King Khalid University, Abha, Saudi Arabia. He is also working as the Principal and the Co-Principal Investigator in several ongoing projects of King Khalid University. He has qualified graduate aptitude test in engineering (GATE) All India rank-270. His research interests include complex analysis, approximation theory, and differential geometry.

SHAKOOR MUHAMMAD was born in Buner, Khyber Pakhtunkhwa, Pakistan. He received the master's degree in mathematics from the University of Peshawar, Khyber Pakhtunkhwa, Pakistan, in 2003, the M.S. degree in mathematics from Quaid-i-Azam University, Islamabad, Pakistan, in 2010, and the Ph.D. degree in applied mathematics and computational systems from the Federal University of Minas Gerais (UFMG), Minas Gerais, Brazil, in 2015. He is currently an Assistant Professor with the Department of Mathematics, Abdul Wali Khan University Mardan, Pakistan. His research interests include a wide variety of subject matters, including computational mathematics, decision-making, optimal energy management systems, dynamic evolutionary optimization, multi-objective optimization, heuristic and metaheuristic approaches, intelligent systems, and mathematical modeling and analysis.