Received 29 January 2024, accepted 5 February 2024, date of publication 9 February 2024, date of current version 15 February 2024.
Digital Object Identifier 10.1109/ACCESS.2024.3364396

THEORY

Static Iterative Learning Control Over Finite Frequency Domain for Actuator Failure Uncertain Systems

LIANGXIN DONG[1], **LEI WANG**[2], **FENG GAO**[2], **AND YIYANG CHEN**[3], **(Member, IEEE)**
[1]School of Automation, Nanjing University of Information Science and Technology, Wuxi 214105, China
[2]School of Automation, Wuxi University, Wuxi 214105, China
[3]School of Mechanical and Electrical Engineering, Soochow University, Suzhou 215137, China

Corresponding author: Yiyang Chen (yychen90@suda.edu.cn)

This work was supported in part by the Natural Science Foundation of the Higher Education Institutions of Jiangsu Province under Grant 22KJB120009; in part by the Natural Natural Science Foundation of China under Grant 62103293; in part by the Natural Science Foundation of Jiangsu Province, China, under Grant BK20210709; and in part by the Start-Up Fund for Introducing Talent of Wuxi University under Grant 2021r045.

ABSTRACT The main objective of this article is to investigate the stability and synthesis of controllers for discrete linear repetitive processes that exhibit polyhedral uncertainty. A condition is proposed, which utilizes a parameter-dependent Lyapunov function to alleviate conservatism resulting from uncertainty. The solution involves the use of a Lyapunov function that depends on uncertain parameters represented by a polyhedron, obtained through Linear Matrix Inequality (LMI) conditions. In practical scenarios, it is often the case that the frequency range of reference signals, noise, and interference is predefined, rendering full frequency range controller synthesis inadequate. To tackle this issue, we introduce a finite frequency controller using the generalized Kalman-Yakubovich-Popov (KYP) lemma. This approach enables designers to specify the desired control performance within a designated frequency range, which can be determined by analyzing the available signal spectrum.

INDEX TERMS Finite frequency range, uncertain systems, linear matrix inequality.

I. INTRODUCTION

Iterative learning control (ILC) is well-suited for systems exhibiting cyclical motion and inherent periodicity, enabling the achievement of precise tracking within a finite time interval [1]. By manipulating the input of the controlled system based on the error signal derived from the disparity between the system output and the desired trajectory, ILC effectively rectifies imperfect control signals, thereby enhancing the overall tracking performance of the system [2], [3].

2D models find extensive applications across diverse industries, including areas such as image data manipulation and communication [4], heat treatment [5], and robot industry [6]. Linear repetitive process is a 2D Linear system with a strong engineering practical background. The characteristic of this process is that it is a special 2D system composed of a series of repetitive actions (that is, information spreads in two independent directions) [7]. These repetitive actions are executed within a fixed limited duration, that is, a certain type of repetitive actions are executed repeatedly within a fixed time. These characteristics are consistent with the time dimension and iteration number dimension in ILC [8]. In recent years, study of ILC schemes for linear repetitive processes has become an important research direction, which can improve the control performance in terms of time and number of iterations [9], [10], [11]. Article [12] presented a novel approach for ILC through the development of an auxiliary 2D model structured as a discretely repeated process. This method encompassed the derivation of stability condition for auxiliary model, employing Lyapunov vector function's divergence method. The study improved the

The associate editor coordinating the review of this manuscript and approving it for publication was Laura Celentano.

learning process' performance by establishing stability of the auxiliary model to ensure convergence.

In general, industrial control processes' models often contain uncertainty factors, especially in the parameters that sensors and actuators require for operation [13], [14]. Due to imperfections in the measurement system, these models will inevitably be subject to errors, which, if large, can lead to unacceptable performance degradation. Therefore, it is critical to develop fault-tolerant control schemes that are effective in maintaining system stability and ensuring satisfactory overall system performance [15], [16], [17].

Reference [18] presents a novel algorithm for detecting anomalies in discrete time-varying systems. The proposed algorithm employs a discrete iterative learning method to effectively identify or estimate faults. An important feature of the algorithm is the use of threshold limiting technology. Within optimal time regions, the algorithm utilizes residual signals and iterative learning rules to correct virtual faults, aligning them with actual faults occurring in the system. This iterative process is repeated within the remaining optimal time zone, ultimately achieving the primary goal of fault diagnosis. For iterative learning control of actuator faults, [19] utilized a combination of adaptive dynamic planning techniques and fault compensation to iteratively compute an approximate optimal fault-tolerant control using real-time input/output data. References [20] and [21] designed a fault detection scheme which calls on the memory of learned knowledge to react quickly to faults and improve the performance of the fault diagnostic.

Incorporating time delays and actuator faults, [22] proposes an innovative fault-tolerant control technique for batch processes, utilizing a 2D fuzzy composite iterative learning approach. The proposed approach employs a 2D T-S fuzzy model and a local sector nonlinear method for constructing the model. Drawing on the principles of two-dimensional Lyapunov stability theory, this study establishes system stability conditions. Additionally, a control law is developed to guarantee long-term stability and performance, even in the presence of system failures across multiple time periods and batches. The existence of the controller is examined through an analysis of sufficient conditions using LMI. The designed time-domain iterative learning fault-tolerant control approach specifically caters to actuator fault intermittent processes that experience simultaneous uncertainties in system parameters. It presents a methodology for obtaining the control gain matrix of the controller through convex optimization, subject to the LMI constraint. This approach guarantees the achievement of fault tolerance performance in the system, considering both the temporal and batch processing directions [23]. Uncertain discrete T-S fuzzy for actuator faults was addressed by [24] and [25]. A new H∞ performance analysis criterion is proposed by constructing a segmented Markov Lyapunov-Krasovskii generalized function.

Most of the existing studies in the field primarily focus on analyzing the performance of ILC systems in the time domain, often incorporating various intelligent control algorithms. It must be recognized, however, that in many cases the frequency ranges of reference signals, noise and interference are predetermined. Consequently, it becomes viable to investigate the conditions necessary for the controller's existence within a specified frequency range. This exploration can significantly expand the potential application scenarios of the algorithm. For example, [26], [27], and [28] provide insight into how to solve ILC problem by considering stability and robustness performance metrics restricted to a specific frequency range. They proposed an algorithm utilizing the KYP Lemma, which formulates the control strategy as an associated LMI. Additionally, they explore extending the robust control law to uncertain systems to accommodate norm-bounded uncertainties in the system parameters. To overcome the limitations of time delay in repetitive control of fractional-order linear systems, paper [29] introduces a novel approach that designs distinct ILC rates for input and state time delays. By determining the convergence frequency range of the feedback controller within a specified frequency range, the controller's design is optimized to effectively mitigate the impact of time delay and ensure desired output performance. However, it is important to note that existing research has not yet thoroughly investigated the fault-tolerant control problem of ILC technology specifically in the finite frequency domain.

In light of these considerations, this paper aims primarily to address issue of static iterative fault-tolerant control in discrete linear systems, specifically focusing on finite frequency domains.

This paper presents the primary contributions as follows. (1) By designing over a limited frequency range, it is possible to more accurately control the response of the system at a specific frequency. (2) The performance of the system at different frequencies can be more easily tuned to meet specific requirements and specifications. (3) Finite frequency domain design usually improves the robustness of the system to parameter variations and uncertainties.

II. PROBLEM DESCRIPTION

A. THE STABILITY THEORY FOR LINEAR REPETITIVE SYSTEM

Given is state space model [32] that represents a standard discrete linear repetitive process

$$\begin{aligned} X_{k+1}(p+1) &= AX_{k+1}(p) + BU_{k+1}(p) + B_0 y_k(p) \\ y_{k+1}(p) &= CX_{k+1}(p) + DU_{k+1}(p) + D_0 y_k(p) \end{aligned} \tag{1}$$

among the recurrence period $0 \leq p \leq \alpha$, iteration times are defined as $k \geq 0$, in the equation $X_k(p)$, $y_k(p)$, and $U_k(p)$ represent system state, input, and output during their k^{th} operation.

To achieve this procedure, system's boundary conditions are established

$$X_{k+1}(0) = d_{k+1}, \quad k \geq 0$$
$$y_0(p) = f(p), \quad 0 \leq p \leq \alpha - 1 \tag{2}$$

d_{k+1} and $f(p)$ vector function is known.

Lemma 1 ([30]): Given matrices of appropriate dimensions X, Z, Σ, matrix Y is existing to satisfy

$$sym(X^T YZ) + \Sigma < 0 \tag{3}$$

the above projection inequalities are necessary and sufficient conditions to validate the following assertion:

$$X^{\perp T} \Sigma X < 0, Z^{\perp T} \Sigma Z < 0. \tag{4}$$

Lemma 2 ([31]): Given matrices R, F, M. R and M are positive definite, the inequality $F^T RF - M < 0$ can be stated as equivalent to the following condition.

$$\begin{bmatrix} -R & F^T \\ F & -M^{-1} \end{bmatrix} < 0 \tag{5}$$

or

$$\begin{bmatrix} -M^{-1} & F \\ F^T & -M \end{bmatrix} < 0. \tag{6}$$

Lemma 3 ([32]): Given matrices of appropriate dimensions $\Gamma = \Gamma^T, X, Y$, for any matrix Δ which satisfy $\Delta^T \Delta \leq I$, then the sufficient condition for

$$\Gamma + X\Delta Y + Y^{\mathrm{T}}\Delta^{\mathrm{T}}X^{\mathrm{T}} < 0 \tag{7}$$

is the presence of $\varepsilon > 0$ such that:

$$\Gamma + \varepsilon^2 XX^{\mathrm{T}} + \varepsilon^{-2} Y^{\mathrm{T}} Y < 0. \tag{8}$$

Lemma 4 ([33]): Assuming that coefficients A, B_0 are controllable, coefficients C, A are observable. The stability of the linear repetitive process, as expressed by equations (1) and (2), is contingent upon the satisfaction of the following inequality.

(i) $\rho(D_0) < 1$;

(ii) $\rho(A) < 1$;

(iii) The transfer function $G(z) = C(zI - A)^{-1}B_0 + D_0$ has all eigenvalue magnitudes smaller than 1 on the unit circle $|z| = 1$.

Lemma 5 ([21]): For the $G(z)$ and frequency response matrix$G\left(e^{j\theta}\right) = C\left(e^{j\theta}I - A\right)^{-1}B + D$:

(i) Frequency domain inequality

$$\begin{bmatrix} G\left(e^{j\theta}\right) \\ I \end{bmatrix}^T \Pi \begin{bmatrix} G\left(e^{j\theta}\right) \\ I \end{bmatrix} < 0, \forall\theta \in \Theta \tag{9}$$

matrix Π is a given real symmetric matrix, Θ represents a frequency range with following properties.

(ii) The below LMI holds:

$$\begin{bmatrix} A & B \\ I & 0 \end{bmatrix}^T \Xi \begin{bmatrix} A & B \\ I & 0 \end{bmatrix} + \begin{bmatrix} C & D \\ 0 & I \end{bmatrix}^T \Pi \begin{bmatrix} C & D \\ 0 & I \end{bmatrix} < 0 \tag{10}$$

TABLE 1. System frequency range.

Symbol	Low Freq.	Mid Freq.	High Freq.
Π	$\lvert\theta\rvert < \theta_l$	$\theta_1 < \theta < \theta_2$	$\lvert\theta\rvert \geq \theta_h$

where $Q > 0, P = P^T$, and Ξ can be divided to

$$\Xi = \left[\begin{array}{c|c} \Xi_1 & \Xi_2 \\ \hline \Xi_2^T & \Xi_3 \end{array}\right] \tag{11}$$

and the matrix Ξ is:

$$\Xi = \begin{cases} \left[\begin{array}{c|c} -P & Q \\ \hline Q & P - 2\cos(\theta_l)Q \end{array}\right], \\ if\,|\theta| < \theta_1; \\ \left[\begin{array}{c|c} -P & e^{j(\theta_1+\theta_2)/2}Q \\ \hline e^{-j(\theta_1+\theta_2)/2}Q & P - 2\cos\left((\theta_2-\theta_1)/2\right)Q \end{array}\right], \\ if\,\theta_1 \leq \theta \leq \theta_2; \\ \left[\begin{array}{c|c} -P & -Q \\ \hline -Q & P + 2\cos(\theta_h)Q \end{array}\right], \\ if\,|\theta| > \theta_h. \end{cases} \tag{12}$$

let $\bar{\sigma}\left(G\left(e^{j\theta}\right)\right)$ denote maximum singular value of matrix $G\left(e^{j\theta}\right)$, and select $\Pi = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}$, the for (9), the below equation holds:

$$G\left(e^{j\theta}\right)^T G\left(e^{j\theta}\right) < I, \forall\theta \in \Theta \tag{13}$$

or

$$\bar{\sigma}\left(G\left(e^{j\theta}\right)\right) < 1, \forall\theta \in \Theta \tag{14}$$

which means for $\theta \in [0, 2\pi]$, it satisfies

$$\rho\left(G\left(e^{j\theta}\right)\right) \leq \bar{\sigma}\left(G\left(e^{j\theta}\right)\right). \tag{15}$$

By utilizing the aforementioned inequalities, establish condition (iii) of Lemma 4 within a limited frequency range, thereby obtaining practical relevant mathematical methods.

B. SYSTEM DESCRIPTION

Consider the linear repetitive system presented below:

$$X_k(p+1) = AX_k(p) + BU_k(p)$$
$$y_k(p) = CX_k(p) + DU_k(p) \tag{16}$$

where the recurrence period $0 \leq p \leq T - 1$, iteration times are defined as $k \geq 0$; at the pth moment of the kth iteration, $X_k(p)$, $y_k(p)$, and $U_k(p)$ represent system state, input, and output, respectively. The boundary condition $x_k(0) = x_0$ represents the system initial condition in the iteration times k, and A, B, C and D are corresponding system matrix.

For iterative learning process of system (16), introduce reference trajectory $y_d(p)$ while defining tracking error of experiment k

$$e_k(p) = y_d(p) - y_k(p). \tag{17}$$

Given state space model (16), this paper proposes control inputs $\{u_k(p), 0 \leq p \leq T-1\}$ to ensure that $e_k(p)$ of the controlled system monotonically converges with the batch direction within the specified finite frequency domain range. Simultaneously ensuring the stability of control system along time direction.

III. DESIGN OF STATIC ITERATIVE LEARNING CONTROL SYSTEM

System (16), following ILC law is designed

$$\begin{cases} u_k(p) = u_{k-1}(p) + \Delta u_k(p) \\ u_0(p) = 0, p \in [0, T-1] \end{cases} \tag{18}$$

among them $u_0(p)$ is initial value, $\Delta u_k\ (n)$ is called a correction calculated using previous experimental information.

Simultaneously defining

$$\delta_k\ (f_k(p)) = f_k(p) - f_{k-1}(p) \tag{19}$$

in the equation, $\delta_k\ (f_k(p))$ represents the error of $f_k(p)$ along the direction of the batch k.

A. STABILITY ANALYSIS OF NORMAL SYSTEM

Based on system (16), the state equation of iterative learning controller is

$$\hat{x}_k(p+1) = A_c\hat{x}_k(p) + B_c\delta_k(x_k(p)) + D_c e_{k-1}(p) \tag{20}$$

following update law is adopted:

$$\Delta u_k(p) = K_1\delta_k(x_k(p)) + K_2 e_{k-1}(p). \tag{21}$$

In order to convert a static ILC system, the definition of

$$X\ (p,k) = \begin{bmatrix} \delta_k\ (x\ (p,k)) \\ x_c\ (p,k) \end{bmatrix} \tag{22}$$

thus obtaining following 2D dynamic model

$$\begin{cases} X_k(p+1) = A_1 X_k(p) + B_1 e_{k-1}(p) \\ e_k(p) = C_1 X_k(p) + D_1 e_{k-1}(p) \end{cases} \tag{23}$$

where $A_1 = \begin{bmatrix} A+BK_1 & 0 \\ B_c & A_c \end{bmatrix}$, $B_1 = \begin{bmatrix} BK_2 \\ D_c \end{bmatrix}$, $C_1 = \begin{bmatrix} -C-DK_1 & 0 \end{bmatrix}$, $D_1 = I - DK_2$.

Theorem 1: Consider Iterative learning control systems (16), (18) and (20), if there is matrices $\hat{S} = diag\left\{\hat{S}_1, \hat{S}_2\right\} > 0$, $\hat{P} = diag\left\{\hat{P}_1, \hat{P}_2\right\} > 0$, $\hat{Q} = diag\left\{\hat{Q}_1, \hat{Q}_2\right\} > 0$, well dimensional invertible matrix $\hat{W} = diag\left\{\hat{W}_1, \hat{W}_2\right\} > 0$, and matrices Y_1, Y_2, Y_3 to make the below LMIs hold

$$H_{11} = \begin{bmatrix} \hat{S}_1 - sym\left(\hat{W}_1\right) & 0 & A\hat{W}_1 + BY_1 & 0 \\ * & \hat{S}_2 - sym\left(\hat{W}_2\right) & Y_3 & Y_2 \\ * & * & -\hat{S}_1 & 0 \\ * & * & * & -\hat{S}_2 \end{bmatrix} < 0 \tag{24}$$

$$H_{12} = \begin{bmatrix} \hat{\Xi}_{11} & 0 & \hat{\Xi}_{21} - \hat{W}_1^T & 0 & 0 & 0 \\ * & \hat{\Xi}_{12} & 0 & \hat{\Xi}_{22} - \hat{W}_2^T & 0 & 0 \\ * & * & \hat{\Xi}_{31} + sym\left(A\hat{W}_1 + BY_1\right) & Y_3^T & BK_2 & \left(-C\hat{W}_1 - DY_1\right)^T \\ * & * & * & \hat{\Xi}_{32} + sym\left(Y_{Ac}\right) & D_c & 0 \\ * & * & * & * & -I & (I - DK_2)^T \\ * & * & * & * & * & -I \end{bmatrix} < 0 \tag{25}$$

In order to simplify the analysis process, suppose that $\hat{\Xi}_1 = diag\left\{\hat{\Xi}_{11}, \hat{\Xi}_{12}\right\}$, $\hat{\Xi}_2 = diag\left\{\hat{\Xi}_{21}, \hat{\Xi}_{22}\right\}$, $\hat{\Xi}_3 = diag\left\{\hat{\Xi}_{31}, \hat{\Xi}_{32}\right\}$, then, the system under ILC is stable on both the time axis and the batch axis, and the tracking error of system converges monotonically. Furthermore, controller matrix of system can be obtained as

$$K_1 = Y_1\hat{W}_1^{-1}, A_c = Y_2\hat{W}_2^{-1}, B_c = Y_3\hat{W}_1^{-1} \tag{26}$$

Proof 1: If inequality (25) holds, then according to the definitiion of negative definite matrices, the following inequality also holds

$$\begin{bmatrix} -I & D_1^T \\ * & -I \end{bmatrix} < 0 \tag{27}$$

then obviously $\rho(D_1) < 1$ which meets the condition (i) of Lemma 4.

According to Lemma 5, inequality (13) is established, which means that there are symmetric matrices $P = diag\{P_1, P_2\} > 0$ $Q = diag\{Q_1, Q_2\} > 0$ to make the following inequality also hold true:

$$\begin{bmatrix} A_1 & B_1 \\ I & 0 \end{bmatrix}^T \Xi \begin{bmatrix} A_1 & B_1 \\ I & 0 \end{bmatrix} + \begin{bmatrix} C_1 & D_1 \\ 0 & I \end{bmatrix}^T \Pi \begin{bmatrix} C_1 & D_1 \\ 0 & I \end{bmatrix} < 0 \tag{28}$$

The expression (28) can be reformulated as follows:

$$\left(\Lambda_1^{\perp}\right)^T \Psi_1 \Lambda_1^{\perp} < 0 \tag{29}$$

where

$$\Lambda_1^{\perp}=\begin{bmatrix}A_1 & B_1\\ I & 0\\ 0 & I\end{bmatrix}$$

$$\Psi_1=\begin{bmatrix}\Xi_1 & \Xi_2 & 0\\ * & \Xi_3+C_1^TC_1 & C_1^TD_1\\ * & * & -I+{D_1}^TD_1\end{bmatrix}.$$

In order to use Lemma 1, select $\Lambda_1=\begin{bmatrix}-I & A_1 & B_1\end{bmatrix}$, $\Sigma_1=\begin{bmatrix}0 & I & 0\end{bmatrix}$, then $\Sigma_1^{\perp}=\begin{bmatrix}I & 0\\ 0 & 0\\ 0 & I\end{bmatrix}$, we can obtain

$$\left(\Sigma_1^{\perp}\right)^T\Psi_1\Sigma_1^{\perp}=\begin{bmatrix}\Xi_1 & 0\\ * & {D_1}^TD_1-I\end{bmatrix}. \tag{30}$$

Because $\Xi_1{=}{-}P<0$, it is known from equation (27) that ${D_1}^TD_1{-}I<0$, so $\left(\Sigma_1^{\perp}\right)^T\Psi_1\Sigma_1^{\perp}<0$ holds.

Then according to Lemma 1, there is an invertible matrix $W=diag\left\{W_1,W_2\right\}>0$ meet the below inequality:

$$\Psi_1+sym\left\{\Lambda_1^TW\Sigma_1\right\}<0. \tag{31}$$

Applying Lemma 2 to Inequality (31) yields

$$\begin{bmatrix}\Xi_1 & \Xi_2-W & 0 & 0\\ * & \Xi_3+sym\left(A_1^TW\right) & W^TB_1 & C_1^T\\ * & * & -I & D_1^T\\ * & * & * & -I\end{bmatrix}<0. \tag{32}$$

Assuming $\hat{W}=W^{-1}$, inequality (33) is left multiplied and right multiplied respectively with $diag\left\{\hat{W}^T,\hat{W}^T,I,I\right\}$ and $diag\left\{\hat{W},\hat{W},I,I\right\}$, it can be obtained that

$$\begin{bmatrix}\hat{\Xi}_1 & \hat{\Xi}_2-\hat{W}^T & 0 & 0\\ * & \hat{\Xi}_3+sym\left(A_1\hat{W}\right) & B_1 & \left(C_1\hat{W}\right)^T\\ * & * & -I & D_1^T\\ * & * & * & -I\end{bmatrix}<0. \tag{33}$$

Finally, introduce the variable $\hat{S}=\hat{W}^TS\hat{W}$, $\hat{P}=\hat{W}^TP\hat{W}$, $\hat{Q}=\hat{W}^TQ\hat{W}$, $\hat{\Xi}_1=\hat{W}^T\Xi_1\hat{W}$, $\hat{\Xi}_2=\hat{W}^T\Xi_2\hat{W}$, $\hat{\Xi}_3=\hat{W}^T\Xi_3\hat{W}$, we can obtain $H_{12}<0$ of inequality (25), satisfy the condition (iii) of Lemma 4.

By multiplying inequality (24) by $diag\left\{W^T,W^T\right\}$ and $diag\left\{W,W\right\}$ respectively on the left and right, it can be obtained that

$$\begin{bmatrix}S & 0\\ * & -S\end{bmatrix}+sym\left\{\begin{bmatrix}-I\\ A_1^T\end{bmatrix}W\begin{bmatrix}I & 0\end{bmatrix}\right\}<0. \tag{34}$$

According to Lemma 1, taking $\Psi_2=\begin{bmatrix}S & 0\\ & -S\end{bmatrix}$, $\Sigma_2=\begin{bmatrix}I & 0\end{bmatrix}$, $\Lambda_2=\begin{bmatrix}-I & A_1\end{bmatrix}$, then $\left(\Sigma_2^{\perp}\right)^T=\begin{bmatrix}0 & I\end{bmatrix}$, $\left(\Lambda_2^{\perp}\right)^T=\begin{bmatrix}A_1^T & I\end{bmatrix}$ and $\Psi_2>0$, $S>0$, we can obtain

$$\left(\Sigma_2^{\perp}\right)^T\Psi_2\Sigma_2^{\perp}=-S<0. \tag{35}$$

$$\left(\Lambda_2^{\perp}\right)^T\Psi_2\Lambda_2^{\perp}=A_1^TSA_1{-}S<0. \tag{36}$$

The establishment of inequality (36) guarantees the establishment of $\rho(A_1)<1$, it satisfies the condition (i) of Lemma 4.

B. STABILITY ANALYSIS OF UNCERTAIN SYSTEM

Furthermore, based on Theorem 1, consider that linear repetitive processes (16) contain norm uncertainty

$$\left[\begin{array}{c|c}\hat{A} & \hat{B}\\ \hat{C} & \hat{D}\end{array}\right]=\left[\begin{array}{c|c}A{+}\Delta A & B{+}\Delta B\\ C{+}\Delta C & D{+}\Delta D\end{array}\right] \tag{37}$$

among ΔA, ΔB, ΔC, ΔD stand for allowable norm bounded uncertainty, which satisfies

$$\begin{bmatrix}\Delta A & \Delta B\\ \Delta C & \Delta D\end{bmatrix}=\begin{bmatrix}E_1\\ E_2\end{bmatrix}\Omega\begin{bmatrix}F_1 & F_2\end{bmatrix} \tag{38}$$

among them, E_1, E_2, F_1, F_2 are well-defined constant matrices with known dimensions, matrix $\Omega=\begin{bmatrix}\Delta_1 & 0\\ 0 & \Delta_2\end{bmatrix}$ represents the uncertainty perturbation and satisfies

$$\Omega^T\Omega\leq I. \tag{39}$$

For the input $u_k(p)$, $u_k^F(p)$ represents output in presence of actuator faults, define actuator model is [22]:

$$u_k^F(p)=\Gamma_k(p)u_k(p) \tag{40}$$

where $\Gamma_k(p)$ is an unknown value which is time variant, its varies that $0\leq\Gamma_k(p)\leq 1$. If $\Gamma_k(p)=1$, the actuator is normal $u_k^F(p)=u_k(p)$. If $\Gamma_k(p)=0$, it represents actuator failure. If $\Gamma_k(p)$>0, it indicates a partial malfunction of the actuator.

Then system (16) can be rewritten as

$$\begin{aligned}X_k(p+1)&=(A+\Delta A)X_k(p)+\Gamma_k(p)(B+\Delta A)U_k(p)\\ y_k(p)&=(C+\Delta C)X_k(p)+\Gamma_k(p)(D+\Delta D)U_k(p).\end{aligned} \tag{41}$$

Theorem 2: Considering the static Iterative learning control system (16), (18) and (20), it is assumed that the model matrix of the system has uncertainties (38) and (39). If there is matrices $\hat{S}=diag\left\{\hat{S}_1,\hat{S}_2\right\}>0$, $\hat{P}=diag\left\{\hat{P}_1,\hat{P}_2\right\}>0$, $\hat{Q}=diag\left\{\hat{Q}_1,\hat{Q}_2\right\}>0$, well dimensional invertible matrix $\hat{W}=diag\left\{\hat{W}_1,\hat{W}_2\right\}>0$, and matrices Y_1, Y_2, Y_3, scalar $\lambda_1>0$, $\lambda_2>0$ to make the below LMIs hold

$$\mathrm{H'}_{11}=$$

$$\begin{bmatrix}\hat{S}_1-sym\left(\hat{W}_1\right) & 0 & A\hat{W}_1+\Gamma BY_1\\ * & \hat{S}_2-sym\left(\hat{W}_2\right) & Y_3\\ * & * & -\hat{S}_1\\ * & * & *\\ * & * & *\\ * & * & *\end{bmatrix}\rightarrow$$

$$\leftarrow \begin{matrix} 0 & \lambda_1 E_1 & 0 \\ Y_2 & 0 & 0 \\ 0 & 0 & \left(F_1\hat{W}_1 + F_2 Y_1\right)^T \\ -\hat{S}_2 & 0 & 0 \\ * & -\lambda_1 I & 0 \\ * & * & -\lambda_1 I \end{matrix}\Bigg] < 0 \quad (42)$$

$$\mathrm{H}'_{12} = \begin{bmatrix} \hat{\Xi}_{11} & 0 & \hat{\Xi}_{21} - \hat{W}_1^T & 0 \\ * & \hat{\Xi}_{12} & 0 & \hat{\Xi}_{22} - \hat{W}_2^T \\ * & * & \hat{\Xi}_{31} + sym\left(A\hat{W}_1 + \Gamma B Y_1\right) & Y_3^T \\ * & * & * & \hat{\Xi}_{32} + sym\left(Y_2\right) \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \rightarrow$$

$$\leftarrow \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ BK_2 & \left(-C\hat{W}_1 - \Gamma D Y_1\right)^T & \lambda_2 E_1 & \left(F_1\hat{W}_1 + F_2 Y_1\right)^T \\ D_c & 0 & 0 & 0 \\ -I & (I - \Gamma D K_2)^T & 0 & (F_2 K_2)^T \\ * & -I & -\lambda_2 E_2 & 0 \\ * & * & -\lambda_2 I & 0 \\ * & * & * & -\lambda_2 I \end{bmatrix} < 0 \quad (43)$$

among $\hat{\Xi}_1 = diag\left\{\hat{\Xi}_{11}, \hat{\Xi}_{12}\right\}$, $\hat{\Xi}_2 = diag\left\{\hat{\Xi}_{21}, \hat{\Xi}_{22}\right\}$, $\hat{\Xi}_3 = diag\left\{\hat{\Xi}_{31}, \hat{\Xi}_{32}\right\}$, then the system under static iterative learning control is stable on both the time axis and the batch axis, and the tracking error of system is monotonic convergence. Furthermore, controller matrix of system can be obtained as

$$K_1 = Y_1\hat{W}_1^{-1}, A_c = Y_2\hat{W}_2^{-1}, B_c = Y_3\hat{W}_1^{-1}. \quad (44)$$

Proof 2: The following proves the establishment of $\mathrm{H}'_{11} < 0$ in inequality (42), then $\mathrm{H}'_{12} < 0$ can be proven using the same method. According to Theorem 1, after adding system uncertainty and actuator fault, H_{11} becomes to be

$$\mathrm{H}'_{11} = \mathrm{H}_{11} + sym\left\{X_1\Omega N_1\right\} < 0. \quad (45)$$

in the equation, $X_1 = \left[E_1^T\ 0\ 0\ 0\right]^T$, $N_1 = \left[0 0 F_1\hat{W}_1 + F_2 Y_1 0\right]^T$. Using Lemma 3, it can be seen that the sufficient condition for equation (45) to hold is that it exists $\lambda_1 > 0$ to make

$$\mathrm{H}_{11} + \lambda_1 X_1 X_1^T + \lambda_1^{-1} N_1^T N_1 < 0. \quad (46)$$

Next, applying Lemma 2 to inequality (46), we can obtain

$$\begin{bmatrix} \mathrm{H}_{11} & * \\ \begin{bmatrix} \lambda_1^{1/2} X_1^T \\ \lambda_1^{-1/2} N_1 \end{bmatrix} & -I \end{bmatrix} < 0. \quad (47)$$

Multiply the left matrix of inequality (47) by the diagonal matrix $diag\left\{I, I, I, I, \lambda_1^{1/2} I, \lambda_1^{1/2} I\right\}$ on left and right to get in inequality (42), then it's proved. Here is a step-by-step description of the algorithm main steps in Algorithm 1.

Algorithm 1 Linear Repetitive Process Static ILC Algorithm Over Finite Frequency

Input: System array A, B, C, D, $E1$, $E2$, $F1$, $F2$, actuator fault factor Ω, initial input $u_0(p)$, target output $y*(p)$, p is the operational discrete steps for each iteration.

Output: Static ILC law factor K_1,K_2, controller input $u_k(p)$, actual output $y_k(p)$.

1: **if** $(\Gamma = 0)$: Substitute matrices A, B, C, D into Theorem 1 to Obtain Static ILC Law K_1,K_2.
2: **else**: Substitute matrices A, B, C, D, $E1$, $E2$, $F1$, $F2$ into Theorem 2 to Obtain Static ILC Law K_1,K_2.
3: **initialization:** Iteration number $k = 0$.
4: Obtain the output $y_k(p)$ and error $e_k(p)$ with input $u_k(p)$.
5: Update the input $u_{k+1}(p)$ with $K1$ and $K2$ according to Equation (21).
6: Send the input signal $u_{k+1}(p)$ to the controller and measure the output $y_{k+1}(p)$.
7: **return** $u_k(p)$ and $y_k(p)$.

IV. SIMULATION RESULTS

To assess the effectiveness of ILC algorithm proposed in this study, simulations are conducted to compare its performance with the traditional static ILC algorithm as described in article [23]. In order to quantify the tracking error of system, the Root-mean-square (RMS) deviation measure is employed.

$$RMS(k) = \sqrt{\frac{1}{T}\sum_{p=1}^{T-1} e^2{}_k(p)}. \quad (48)$$

A. NORMAL SYSTEM

A second-order mechanical displacement system composed of unit mass spring damper is considered, the Equation of state of system (16) is as illustrated below

$$\begin{cases} \dot{x}(p) = A_p x(p) + B_p u(p) \\ y(p) = C_p x(p) + D_p u(p) \end{cases} \quad (49)$$

the parameter matrix of the system is $A_p = \begin{bmatrix} -2 & 3 \\ 4 & -5 \end{bmatrix}$, $B_p = \begin{bmatrix} 0 \\ 1.5 \end{bmatrix}$, $C_p = \begin{bmatrix} 5 & 0 \end{bmatrix}$, $D_p = 1$. By Discretization the system with sampling time of $T_s = 0.01$, the following discrete state space model as shown below: $A = \begin{bmatrix} 0.9808 & 0.0290 \\ 0.0386 & 0.9518 \end{bmatrix}$, $B = \begin{bmatrix} 0.0002 \\ 0.0146 \end{bmatrix}$, $C = \begin{bmatrix} 5 & 0 \end{bmatrix}$, $D = 1$.

The reference locus of the defined system and the responding spectrum are shown in Figure 1, which represents the spectrum obtained through Fast Fourier Transform (FFT). From the figure, it is evident that the effective harmonics of

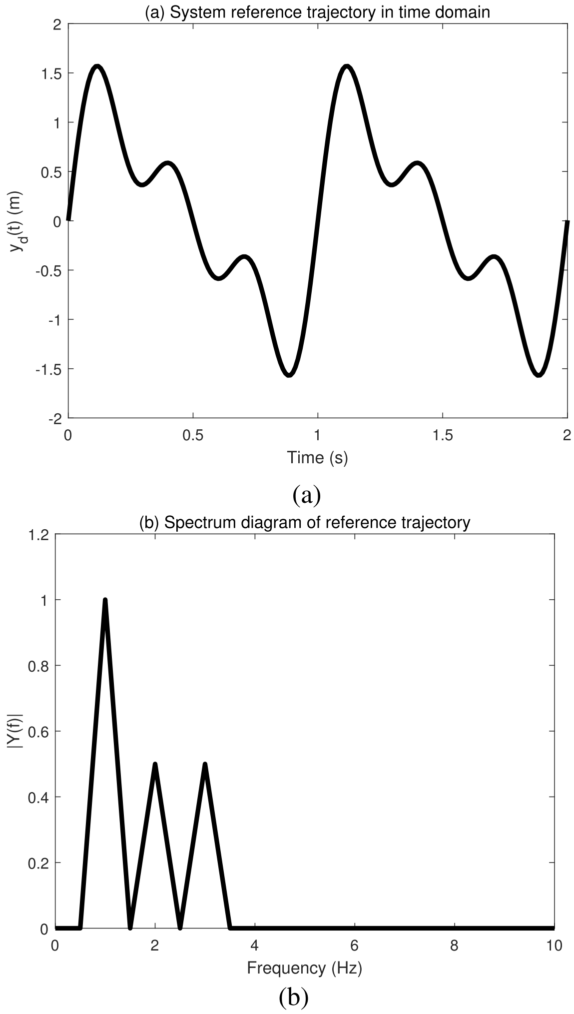

FIGURE 1. (a) Reference locus. (b) Corresponding frequency spectrum.

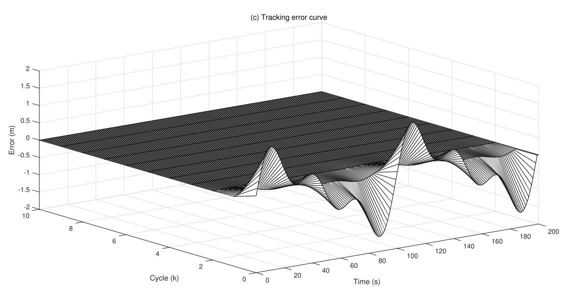

FIGURE 2. Reference track diagram of control system.

the reference trajectory range from 0 to 5 Hz in the low-frequency range. Hence, we have $\theta_l = 0.3142$.

For static ILC systems (16), (18) and (21), we can get following results by using formulas (24), (25), (26) in Theorem 1: $K_1 = \begin{bmatrix} -5.00 & -0.0245 \end{bmatrix}$, $K_2 = 0.9069$. Figure 2 shows the variation in trace error, the plot illustrates control ability of static ILC system on two dimensions.

The given parameters for traditional static ILC law are $L_1 = \begin{bmatrix} -5.00 & 0 \end{bmatrix}$, $L_2 = 0.5395$. The variation of the root mean square tracking error of system with the number of iterations is illustrated in Figure 3 considering the influence of both control laws, the figure reveals that the ILC significantly improves the convergence capability of the normal system.

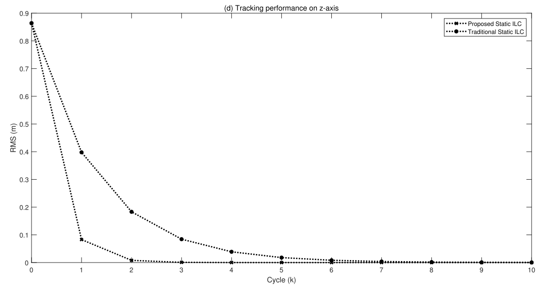

FIGURE 3. Reference track diagram of control system.

B. UNCERTAIN SYSTEM

In our study [35], an injection molding machine was chosen as the experimental subject, with the nozzle pressure being selected as the control parameter.

An ARX model is established to depict correlation between valve aperture and nozzle pressure:

$$\left\{1 - 1.607(\pm 5\%)z^{-1} + 0.6086(\pm 5\%)z^{-2}\right\} y_k(p) = \left\{1.239(\pm 5\%)z^{-1} - 0.9282(\pm 5\%)z^{-2}\right\} u_k(p) \quad (50)$$

the parameter uncertainty is represented by $\pm 5\%$, indicating a relative uncertainty in the model parameter.

The output of the system also incorporates same uncertainty parameter, By transforming Equation (50), we can express it in the following:

$$\begin{cases} x_k(p+1) = \left(\begin{bmatrix} 1.607 & 1 \\ -0.6086 & 0 \end{bmatrix} + \Delta A(p)\right) x_k(p) \\ \qquad + \left(\begin{bmatrix} 1.239 \\ -0.9282 \end{bmatrix} + \Delta B(p)\right) u_k(p) \\ y_k(p) = \left(\begin{bmatrix} 1 & 0 \end{bmatrix} + \Delta C(p)\right) x_k(p) \end{cases} \quad (51)$$

The parameters of each uncertainty as per (40) are

$$\begin{aligned} \Delta A(p) &= \begin{bmatrix} 0.0804\delta_1(p) & 0 \\ -0.0304\delta_2(p) & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Delta_1(p) \begin{bmatrix} 0.0804 & 0 \\ -0.0304 & 0 \end{bmatrix} \\ \Delta B(p) &= \begin{bmatrix} 0.062\delta_1(p) \\ -0.0464\delta_2(p) \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Delta_1(p) \begin{bmatrix} 0.062 \\ -0.0464 \end{bmatrix} \\ \Delta C(p) &= \begin{bmatrix} 0.05\delta_3(p) & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0.6219 & 0 \end{bmatrix} \Delta_2(p) \begin{bmatrix} 0.0804 & 0 \\ -0.0304 & 0 \end{bmatrix} \end{aligned}$$

where $|\delta_i(p)| \leq 1 (i = 1, 2, 3)$.

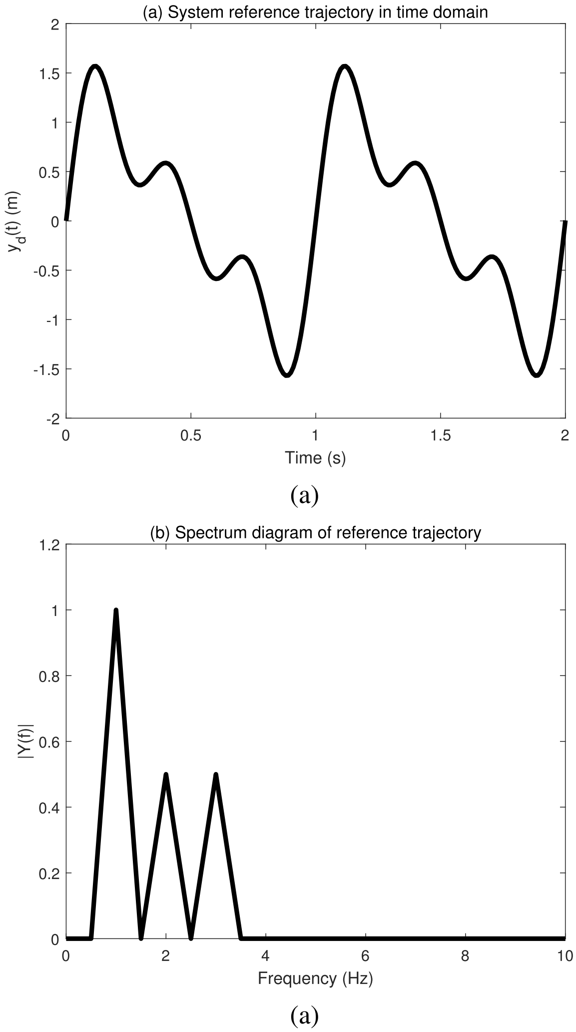

FIGURE 4. **(a) Reference locus. (b) Corresponding frequency spectrum.**

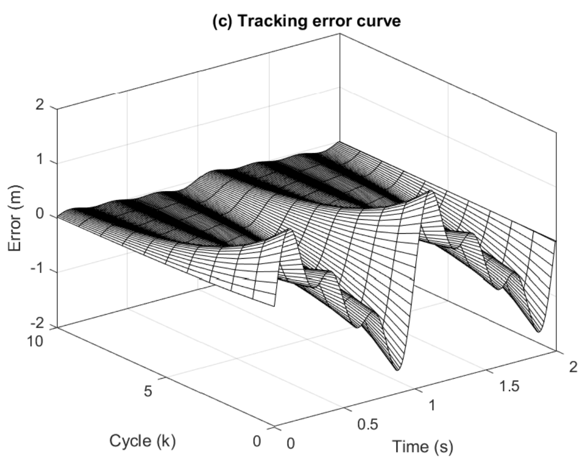

FIGURE 5. **Reference track diagram of control system.**

Assuming an initial condition of $x(0, k) \equiv \begin{bmatrix} 0 & 0 \end{bmatrix}^T$, system encounters a known actuator fault denoted by $\Gamma = 0.85$. The

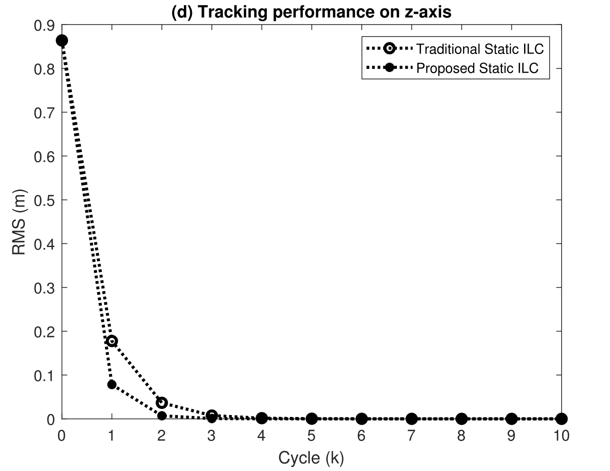

FIGURE 6. **Reference track diagram of control system.**

output target value is chosen to be the same as the previous simulation:

$$y_d(p) = \sin(2\pi p) + 0.5\sin(4\pi p) + 0.5\sin(6\pi p), \quad 0 \leq p \leq 2\,s. \tag{52}$$

The reference locus of the defined system and the responding spectrum are shown in Figure 4 which is same with normal system simulation, then we have $\theta_l = 0.3142$. For static ILC systems (16), (18) and (21), the following results can be derived using the formulas (24), (25), (26) in Theorem 1: $K_1 = \begin{bmatrix} -1.3080 & -0.8090 \end{bmatrix}$, $K_2 = 0.8974$. Figure 5 shows the variation in trace error, demonstrating the control ability of the static ILC system in two dimensions.

The given parameters for traditional static ILC law are $L_1 = \begin{bmatrix} -1.3107 & -0.8117 \end{bmatrix}$ and $L_2 = 0.9951$. Figure 6 illustrates the variation of the root mean square tracking error of the system with the number of iterations considering the influence of both control laws, the figure demonstrates that ILC significantly enhances the convergence capability of the uncertain system.

V. CONCLUSION AND FUTURE WORK

This study focuses on addressing the problem of ILC in discrete linear repetitive system within limited frequency range. By leveraging 2D system theory, a static iterative learning controller is developed for both nominal and uncertain systems, with a finite frequency range constraint. The controller's existence and the associated gain matrix are derived through the utilization of generalized KYP lemma, in the form of LMI. These conditions ensure the system's dynamic performance. Additionally, by conducting simulations on the truss robot model, the efficacy of suggested method in this article is compared to that of the static ILC algorithm, revealing its enhanced capabilities. However, the proposed method is effective for specific types of systems and require significant computational resources, while it is short of detecting the time of fault occurrence and the severity of the failure. Our next step is to study the fault diagnostic and fault severity estimation, which will extend the use cases of the proposed static ILC algorithm.

REFERENCES

[1] Y. Liu, X. Ruan, and X. Li, "Optimized iterative learning control for linear discrete-time-invariant systems," *IEEE Access*, vol. 7, pp. 75378–75388, 2019.

[2] I. Hussain, X. Ruan, C. Liu, and Y. Liu, "Linearly monotonic convergence and robustness of P-type gain-optimized iterative learning control for discrete-time singular systems," *IEEE Access*, vol. 9, pp. 58337–58350, 2021.

[3] X. Li, X. Ruan, and Y. Liu, "Learning-gain-adaptive iterative learning control to linear discrete-time-invariant systems," *IEEE Access*, vol. 7, pp. 98934–98945, 2019.

[4] C. E. de Souza, L. Xie, and D. F. Coutinho, "Robust filtering for 2-D discrete-time linear systems with convex-bounded parameter uncertainty," *Automatica*, vol. 46, no. 4, pp. 673–681, Apr. 2010.

[5] L. Blanken and T. Oomen, "Multivariable iterative learning control design procedures: From decentralized to centralized, illustrated on an industrial printer," *IEEE Trans. Control Syst. Technol.*, vol. 28, no. 4, pp. 1534–1541, Jul. 2020.

[6] Y. Chen, B. Chu, and C. T. Freeman, "Iterative learning control for path-following tasks with performance optimization," *IEEE Trans. Control Syst. Technol.*, vol. 30, no. 1, pp. 234–246, Jan. 2022.

[7] Y. T. Shen, L. Wang, J. Yu, R. Zhang, and F. Gao, "A hybrid 2D fault-tolerant controller design for multi-phase batch processes with time delay," *J. Process Control*, vol. 69, pp. 138–157, Sep. 2018.

[8] L. Wu and W. X. Zheng, "On design of reduced-order H_2 filters for discrete repetitive processes," in *Proc. IEEE Int. Symp. Circuits Syst. (ISCAS)*, May 2011, pp. 2137–2140.

[9] O. Khan, G. Mustafa, A. Q. Khan, M. Abid, and M. Ali, "Fault-tolerant robust model-predictive control of uncertain time-delay systems subject to disturbances," *IEEE Trans. Ind. Electron.*, vol. 68, no. 11, pp. 11400–11408, Nov. 2021.

[10] Y. Chen, B. Chu, and C. T. Freeman, "Iterative learning control for robotic path following with trial-varying motion profiles," *IEEE/ASME Trans. Mechatronics*, vol. 27, no. 6, pp. 4697–4706, Dec. 2022.

[11] Y. Chen, B. Chu, C. T. Freeman, and Y. Liu, "Generalized iterative learning control with mixed system constraints: A gantry robot based verification," *Control Eng. Pract.*, vol. 95, Feb. 2020, Art. no. 104260.

[12] P. V. Pakshin and J. P. Emelianova, "Iterative learning control design for switched systems," *Autom. Remote Control*, vol. 81, no. 8, pp. 1461–1474, Aug. 2020.

[13] H. Tao, W. Paszke, E. Rogers, H. Yang, and K. Gałkowski, "Finite frequency range iterative learning fault-tolerant control for discrete time-delay uncertain systems with actuator faults," *ISA Trans.*, vol. 95, pp. 152–163, Dec. 2019.

[14] D. Ye and G.-H. Yang, "Adaptive fault-tolerant tracking control against actuator faults with application to flight control," *IEEE Trans. Control Syst. Technol.*, vol. 14, no. 6, pp. 1088–1096, Nov. 2006.

[15] P. Leissner, S. Gunnarsson, and M. Norrlöf, "Some controllability aspects for iterative learning control," *Asian J. Control*, vol. 21, no. 3, pp. 1057–1063, May 2019.

[16] S. Hao, T. Liu, W. Paszke, and K. Galkowski, "Robust iterative learning control for batch processes with input delay subject to time-varying uncertainties," *IET Control Theory Appl.*, vol. 10, no. 15, pp. 1904–1915, Oct. 2016.

[17] S. Bifaretti, P. Tomei, and C. M. Verrelli, "A global robust iterative learning position control for current-fed permanent magnet step motors," *Automatica*, vol. 47, no. 1, pp. 227–234, Jan. 2011.

[18] W. Cao and M. Sun, "Iterative learning based fault diagnosis for discrete linear uncertain systems," *J. Syst. Eng. Electron.*, vol. 25, no. 3, pp. 496–501, Jun. 2014.

[19] V. D. Phan, C. P. Vo, H. V. Dao, and K. K. Ahn, "Actuator fault-tolerant control for an electro-hydraulic actuator using time delay estimation and feedback linearization," *IEEE Access*, vol. 9, pp. 107111–107123, 2021.

[20] T. Chen, C. Wang, and D. J. Hill, "Fault detection for a class of uncertain sampled-data systems using deterministic learning," *IEEE Trans. Cybern.*, vol. 51, no. 12, pp. 5930–5940, Dec. 2021.

[21] M. Shahriari-Kahkeshi, F. Sheikholeslam, and J. Askari, "Adaptive fault detection and estimation scheme for a class of uncertain nonlinear systems," *Nonlinear Dyn.*, vol. 79, no. 4, pp. 2623–2637, Mar. 2015.

[22] L. Wang, B. Li, J. Yu, R. Zhang, and F. Gao, "Design of fuzzy iterative learning fault-tolerant control for batch processes with time-varying delays," *Optim. Control Appl. Methods*, vol. 39, no. 6, pp. 1887–1903, Nov. 2018.

[23] L. Wang, L. Dong, Y. Chen, K. Wang, and F. Gao, "Iterative learning control for actuator fault uncertain systems," *Symmetry*, vol. 14, no. 10, p. 1969, Sep. 2022.

[24] Y. Wei, J. Qiu, P. Shi, and L. Wu, "A piecewise-Markovian Lyapunov approach to reliable output feedback control for fuzzy-affine systems with time-delays and actuator faults," *IEEE Trans. Cybern.*, vol. 48, no. 9, pp. 2723–2735, Sep. 2018.

[25] J. Qiu, Y. Wei, and L. Wu, "A novel approach to reliable control of piecewise affine systems with actuator faults," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 64, no. 8, pp. 957–961, Aug. 2017.

[26] H. Tao, Q. Wei, W. Paszke, L. Zhou, and H. Yang, "Robust iterative learning control in finite frequency ranges for differential spatially interconnected systems," *Circuits, Syst., Signal Process.*, vol. 39, no. 10, pp. 5104–5126, Oct. 2020.

[27] C. E. Boudjedir and D. Boukhetala, "Adaptive robust iterative learning control with application to a delta robot," *Proc. Inst. Mech. Eng., I, J. Syst. Control Eng.*, vol. 235, no. 2, pp. 207–221, Feb. 2021.

[28] W. Zou, Y. Shen, W. Paszke, and H. Tao, "Robust feedback feed-forward PD-type iterative learning control for uncertain discrete systems over finite frequency ranges," *Trans. Inst. Meas. Control*, vol. 44, no. 14, pp. 2850–2862, Oct. 2022.

[29] Y. Wang, F. Zhou, L. Yin, and F. Wan, "Iterative learning control for fractional order linear systems with time delay based on frequency analysis," *Int. J. Control, Autom. Syst.*, vol. 19, no. 4, pp. 1588–1596, Apr. 2021.

[30] X. Li and H. Gao, "Robust finite frequency H_∞ filtering for uncertain 2-D Roesser systems," *Automatica*, vol. 48, no. 6, pp. 1163–1170, Jun. 2012.

[31] D. A. Bristow, M. Tharayil, and A. G. Alleyne, "A survey of iterative learning control," *IEEE Control Syst. Mag.*, vol. 26, no. 3, pp. 96–114, Jun. 2006.

[32] L. Xie, "Output feedback H_∞ control of systems with parameter uncertainty," *Int. J. Control*, vol. 63, no. 4, pp. 741–750, Mar. 1996.

[33] J. Bolder and T. Oomen, "Inferential iterative learning control: A 2D-system approach," *Automatica*, vol. 71, pp. 247–253, Sep. 2016.

[34] T. Iwasaki and S. Hara, "Feedback control synthesis of multiple frequency domain specifications via generalized KYP lemma," *Int. J. Robust Nonlinear Control*, vol. 17, nos. 5–6, pp. 415–434, Mar. 2007.

[35] L. Yu, "An LMI approach to reliable guaranteed cost control of discrete-time systems with actuator failure," *Appl. Math. Comput.*, vol. 162, no. 3, pp. 1325–1331, Mar. 2005.

LIANGXIN DONG received the bachelor's degree in material science and engineering from the Harbin University of Technology. He is currently pursuing the master's degree in electronic information with the Wuxi Graduate School, Nanjing University of Information Science and Technology. His scientific research achievements include two SCI journal published, two utility model patents have been authorized, and two invention patents have been submitted.

LEI WANG received the Ph.D. degree in control theory and control engineering from Jiangnan University, in 2021. He studied at the Yunlin University of Science and Technology, Taiwan, in 2014. From 2018 to 2019, he studied at the University of Zielona Góra, Poland. He is currently a Lecturer with the School of Automation, Wuxi University, China. His research interest includes iterative learning control.

FENG GAO received the Ph.D. degree in mechanical design and theory from Northeastern University, in 2019. He used to work with BMW Brilliance Automotive Company Ltd., in 2016. His research interests include mechanical system dynamics and vibration control.

YIYANG CHEN (Member, IEEE) received the M.Eng. degree from Imperial College London, London, U.K., in 2013, and the Ph.D. degree from the University of Southampton, Southampton, U.K., in 2017. After that, he was a Research Fellow in control systems, from 2017 to 2018, and in traffic signal control, from 2018 to 2020, with the University of Southampton. He joined the School of Mechanical and Electrical Engineering, Soochow University, in 2020, as an Associate Professor. He has published several papers in top control conferences and journals. His research interests include iterative learning control, optimization, artificial intelligence, image processing, and robotic systems.