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RESEARCH ARTICLE

A Novel Hybrid Approach for Hydrothermal Scheduling Using Mathematical and Metaheuristic Search Methods

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ABSTRACT Hydrothermal scheduling is a significant concern in the field of power system economics that seeks to reduce the overall cost of generation by optimizing the hourly output of generators. However, this scheduling process suffers from a non-linear and complex problem due to set of uncertainty constraints from hydro and thermal units. Hence ϑ-constraint method has been proposed in which ramp constraints are considered that supply a feasible power region of the thermal units and minimize the uncertainty constraints. In addition, the existing solution techniques of hydro and thermal unit, commitment problems have not considered thermal constraints and their losses. This results in the frequent swapping of hydro units which decreases the energy conversion efficiency by a percentage at each repetition and increases the cost of the system. Therefore, a novel Chaotic geometric programming and chaotic approximation approach has been proposed that balance water discharge level without frequency swapping based on Armijo's rule and Hazen Williams's rule. Further, dynamic indexing is performed to increase the energy conversion efficiency. These balanced values are given to multiple-wave neural networks, to solve the hydro and thermal unit commitment problems. Furthermore, the cost of the system is minimized using Enhanced Ebola Optimization Search (EEOS) Algorithm in which the S_{rate} parameter has been modified, thereby normalizing the hydrothermal scheduling problem for matured convergence. The proposed mathematical model has been implemented in the MATLAB R2022b on a 2.10 GHz, AMD Ryzen 5 3500U with Raden Vega Mobile Gfx processor, 8 GB RAM, 64 bit personal computer, and the results obtained show better performance than the previous approaches in terms of mean, median and standard deviation.

INDEX TERMS Hydrothermal scheduling, hydro constraints, thermal constraints, Ebola optimization, mathematical approach, chaotic programming.

I. INTRODUCTION

Hydrothermal scheduling (HTS) which aims to optimize performance of hydrothermal power plants affects operation of power systems. It has a nonlinear objective function and several constraints [1], [2], [3], [4], [5], [6]. The HTS issue releases reservoir water hourly and obtains thermal unit power in a specified time interval to reduce generating costs. The volume of reservoir in HTS requires constant reservoir water levels for hydropower generation and it needs to be

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monitored as supply system cannot store energy [3], [4], [5], [6]. The volume of reservoir is effected by upstream reservoir's natural inflow, spillage, and discharge. Hydro scheduling is effected by floods, draughts, agricultural water navigational requirements and other natural disasters. The problem of HTS is complicated due to inclusion of constraints related to prohibited discharge zones [7], water availability, power balance, hydro discharge, valve point loading effect, and generation limits [2]. These constraints make HTS problems nonlinear, making classical optimization difficult to solve them [7], [9].

HTS has been solved using dynamic programming (DP), nonlinear programming (NLP), progressive optimality algorithm (POA), mathematical decomposition (MD), gradient search (GS), Newton's technique, Network flow method (NFM), Lagrange relaxation (LR), and mixed-integer programming (MIP). To address the shortcomings of traditional optimization methods, the authors proposed evolutionary algorithms like simulated annealing (SA), genetic algorithm (GA), evolutionary programming (EP), Hopfield neural network (HNN) [10], [11], [12], and differential evolution (DE). Using the approximation method, each reservoir needs an initial workable timetable [13]. While maintaining the schedules of the other reservoirs, one reservoir is scheduled at a time, switching between them until the necessary number of iterations or the cost difference between the last two iterations is within the tolerances [14]. Since it only dispatches one reservoir at a time and optimization techniques are uncertain and expensive, these approaches cannot handle coupling limitations. Integrating neural networks into hydrothermal planning provides valuable benefits by making the optimization process more data-driven and adaptable to changing conditions. Neural networks can capture complex interactions between parameters and adapt to dynamic factors such as weather, water flow and energy demand, improving planning accuracy and efficiency.

Salkuti et al. [15] proposed multi-function global particle swarm optimization (MFGPSO) for optimal short-term hydro-thermal scheduling (ST-HTS). This ideal ST-HTS optimizes hydro and thermal generator generating schedules to lower thermal power plant fuel costs. The MF-GPSO algorithm's performance is assessed using a sample test system and three case studies. Crisscross Differential Evolution (CCDE) algorithm for Constrained Hydrothermal Scheduling is proposed by Kaur et al [16]. The CCDE algorithm's global search is tested on two hydrothermal systems of different dimensions.

Alquthami et al. [17] proposed ABC algorithm for short-term hydro-thermal scheduling. This study considers transmission losses and hydro-thermal power plant economic scheduling. The proposed technique significantly lowers thermal power plant operating costs compared to alternatives and achieves the fastest convergence, offering cost-effective optimization methods for hydroelectric power plants and potential benefits to the power industry. It's essential to balance economic benefits with environmental and social considerations while meeting energy demands efficiently. Hence the thermal power plant optimization reduces fuel costs, while hydropower plant optimization is cost-effective. Das et al. [18] proposed optimal hydrothermal system scheduling considering water transportation delay variability using Grasshopper algorithm. Hydropower production, net head, and water discharge rate are nonlinearly related. The difficult head-sensitive water-to-power conversion and piecewise output restriction are considered. Zeng et al. [19] recommended a grasshopper optimization algorithm for short-term hydrothermal scheduling.

Castaño et al. [20] proposed Short-Term Hydrothermal Scheduling with Solar and Wind Farms Using Second-Order Cone Optimization with Chance-Box Constraints. The proposed approach rigorously addresses the nonlinear relationship between water discharge, reservoir volume, and hydropower output. Chance-box restrictions robustly model the effects of wind and solar energy on the electrical grid. Numerical results show that the chance-box constraint approach yields a robust solution and that the Second-Order Cone approximation is more accurate and faster than recent methods. Fakhar et al. [21] proposed Conventional and Metaheuristic Optimization Algorithms for Short-Term Hydrothermal Scheduling. Optimization strategies reduced STHTS costs. The MOSTHTS algorithms tried to reduce CO_2 and gasoline prices. Naik et al. [22] proposed a Modified Social Group Optimization meta-heuristic algorithm for Short-term Hydrothermal Scheduling. The acquisition phase of Modified Social Group Optimization (MSGO) is enhanced. Fakhar et al. [23] proposed Accelerated Particle Swarm Optimization (APSO) and improved APSO for Non-Cascaded and Cascaded Short-Term Hydrothermal Scheduling. The adaptive and variable local and global search coefficients greatly improve the suggested APSO's performance in finding the best solution. Helseth et al. [24] proposed Convex Relaxations of the Short-Term Hydrothermal Scheduling Problem. Starting as a mixed-integer programming problem, it is approximated by employing Lagrangian and linear relaxation techniques.

From aforementioned literature, it is found that computation capacity and execution time is increased in [15] and [16] breaks equality requirements while changing the violated dependent variables, [17] slows multimodal optimization convergence, in [18] swarm does not coalesce to a single spot, [20] cannot determine optimal scheduling, [21] does not address the objective function properly, in [22] algorithm is highly complex, [23] ignores social and cognitive factors for the PSO technique while [24] takes more computational time. The Hamilton and Egarch algorithm is a computational approach for optimizing the control of water flow in hydraulic systems based on certain criteria and constraints [25], [26]. The Hazen-Williams Rule, a widely used formula in hydraulic engineering, estimates water flow in pipelines based on pipe characteristics and fluid properties and is often used to estimate pressure loss and flow rate [27].

Therefore, this study proposes a novel mathematical model for hydraulic system scheduling to solve the HTS problem of time minimization for two constraints. In hydrothermal power systems, uncertainty in constraints is minimized using the ϑ – constraint method considering the ramp constraint, cost and speed factors. An Enhanced Ebola Optimization Search Algorithm is proposed to reduce hydrothermal system fuel costs. This algorithm works by modifying S_{rate}. A chaotic geometric programming and chaotic approximation approach have been proposed to regulate water discharge and energy efficiency without net load variations via dynamic indexing. It utilizes multiple waves neural network to eliminate hydro and thermal unit commitment problems. The contributions of this work are summarized as follows:

The ϑ-constraint method is utilized to minimize constraint uncertainty in hydrothermal power systems.

Dynamic indexing with chaotic geometric programming and chaotic approximation is suggested to regulate water discharge and energy efficiency. This technique utilizes multiple-wave neural network to solve hydro and thermal unit commitment issues.

An Enhanced Ebola Optimization Search Algorithm is proposed to minimize the fuel cost of hydrothermal scheduling problem.

Metaheuristic Optimization: The primary benefit of the metaheuristic optimization algorithm EEOS is that it offers a flexible and effective method for resolving challenging optimization issues. It can be used to solve a variety of issues, including task scheduling, transportation routing, heat scheduling, and more. It makes a substantial contribution by being able to locate nearly optimal solutions in an effective computational approach.

Hydrothermal Scheduling: EEOS offers a potent tool for improving the scheduling of power generation from both hydroelectric and thermal power plants in the context of hydrothermal scheduling. Its use in this field may result in lower costs, better use of renewable energy sources, and more effective power production. The standard optimization techniques on which EEOS is based may undergo specific improvements or adjustments.

The content of the paper has been developed as follows section II problem formulation, 3 depicts the proposed methodology for hydrothermal scheduling, section IV chaotic geometric programming and chaotic approximation approach, 5 deliberates the result and comparison and finally, section 6 discusses the conclusion.

II. PROBLEM FORMULATION

Hydrothermal scheduling is a non-linear optimization problem that is intrinsically non-convex and stochastic. Moreover, modern power systems include wind and solar generation, which introduces new challenges such as uncertainty. However, uncertainty due to the integration of wind power with hydraulic power increases the cost of reserve and electricity dissipation in the power system. It is on account of wind power overestimation and underestimation. Previously most of the algorithms attempted to minimize the cost but they have not even moderately considered the ramp constraints thereby resulting in the unfeasible working region for thermal unit and uncertainty constraints. Hence ϑ-constraint method has been proposed that considers ramp constraints for supplying a feasible region of the thermal unit and for the minimization of uncertainty constraint. Also, it utilizes a function for minimization of uncertainty by considering factors such as overestimation cost, underestimation cost, shape factor, scale factor and velocity of current wind speed.

During the HTS process, hydro and thermal unit commitment problem occurs which is previously solved by considering the hydro unit dynamic constraints, but they have not considered thermal constraints and their losses that cause variability in the net load that was never considered before resulting in the frequent swapping of hydro units. This decreases energy conversion efficiency by a percentage at each repetition. Hence a novel Chaotic geometric programming and chaotic approximation approach has been proposed in which net load variability from heretofore unconsidered two-dimensional data are explored with its left endpoint of the allowable range of water discharge as 0 based on Armijo's rule and these constraints are given to convergence for minimization. While considering the variations, the energy control dynamic indexing is performed at approximation case thereby the energy dispersion efficiency does not decrease for each iteration. Then, water discharge of the hydro unit during period t is limited using the Hamiltonian and Egarch algorithm based on the Hazen Williams rule and penstock water balance. These values are given to multiple wave neural networks for bringing up a constraint that was never considered before to solve the HTS problem. Since by applying a ''Multiple Waves Neural Network'', it helps the hydropower facilities to run more efficiently and control how much water is released from reservoirs. With the help of these networks, hydroelectric systems produce energy more effectively and manage their reservoirs more effectively by analyzing and simulating the periodic patterns and changes in water flow.

Moreover, the HTS problem, increased fuel cost and as well as shortcomings in balancing global exploration and local exploitation still exist in the existing stochastic searching algorithms. Hence to solve these issues a novel Enhanced Ebola Optimization Search Algorithm is subjected to perform for the objective function and the constraints stated. The EEOS Algorithm use more effective search and exploration techniques, allowing it to locate ideal or almost ideal solutions more quickly. The Ebola Optimization Search (EOS) Algorithm has been enhanced, and the new algorithm is called the Enhanced Ebola Optimization Search (EEOS). Improvements in search tactics, convergence speed, adaptation to changing surroundings, and handling of restrictions are all included. With these enhancements, the algorithm should be able to solve optimization issues more quickly and effectively. Research papers or other relevant documentation may contain specific details. In Ebola optimization, the parameter S_{rate} has been modified thereby standardizing

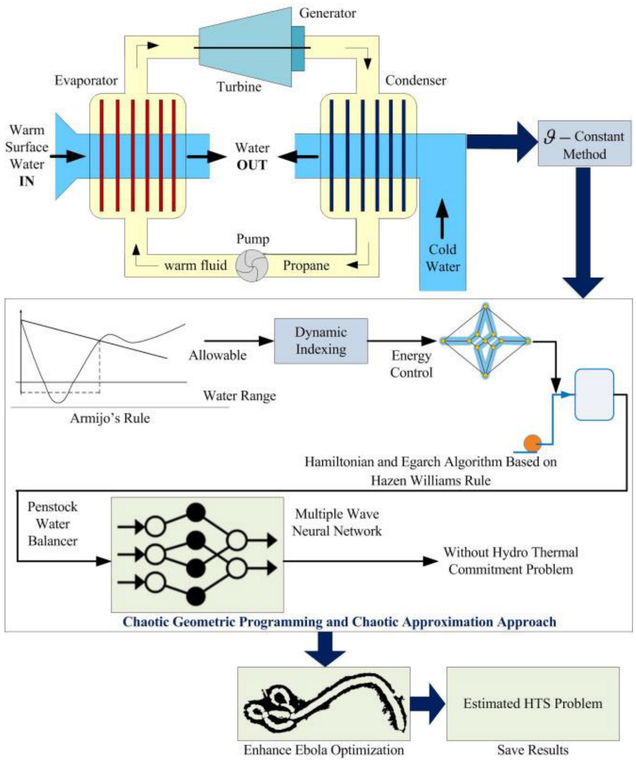

FIGURE 1. Block diagram of the proposed mathematical model.

the hydrothermal scheduling problem for matured convergence. This optimization algorithm has enhanced exploratory and exploitative performance, a high convergence rate than SPPSO with an increase in the diversity of the population. The choice between Extended Ebola Evolutionary Algorithm, PSO, or GA for multi-objective optimization depends on the specific problem, computational resources, desired optimization criteria (e.g., diversity, convergence rate), and the ability of the algorithm to adapt to changing problem conditions. To make an informed decision, it is important to evaluate the effectiveness of each algorithm for the specific problem.

Figure 1 depicts the block diagram of the proposed mathematical model for performing the HTS process in which the hydro and thermal constraints are considered along with uncertainty factors and ramp constraints. Then, water discharge is balanced by considering variation in net load using multiple waves neural network and finally, the fuel cost is minimized by standardizing the HTS problem by modifying the S_{rate} the parameter in enhanced Ebola optimization. Hydrothermal scheduling involves a combination of two sources: hydroelectric (which is usually cheaper and more environmentally friendly) and thermal (which is often more expensive due to fuel costs). EEOS seeks to strike a balance between these sources by optimizing the hourly discharge of reservoirs and the operation of thermal units. This balance helps in minimizing the reliance on costly thermal power generation.

A. OBJECTIVE FUNCTION

The primary goal of HTS is to reduce the total fuel cost of thermal plants by taking into account several equality and inequality constraints, including the power balance constraint, the availability of water constraints, and generator operating limits. The Objective function [1] for HTS is to minimize the total cost of the system which is given in equation (1);

$$\boldsymbol{minF} = \sum\nolimits_{t=1}^{T} \sum\nolimits_{i=1}^{N_s} f_i\left(P_s\left(i,t\right)\right) \tag{1}$$

where T stands for the overall quantity of thermal plants. The power generated by the t the thermal plant at interval i is represented by P_s, N_s stands for the total number of intervals. Equation (2) represents the thermal generator's quadratic fuel cost function as [2];

$$f_i\left(P_s\left(i,t\right)\right) = a_i + b_i P_{i,t} + c_i P_{i,t}^2 \tag{2}$$

where, a_i, b_i and c_i are the coefficients of fuel cost for the t thermal plant at interval i. However, this objective function is subjected to nonconvex problems due to the consideration of variable net load parameters and hence, certain essential constraint has to be considered to minimize the problem in uncertainty constraints.

B. ϑ-CONSTRAINT METHOD

ϑ-constraint method has been used to provide both equality and inequality constraints with the ultimate objective to distribute the power generation of hydro and thermal power plants to the distribution centers to minimize the dispatch cost, which is essentially the fuel cost of the generating units. In the HTS problem, the power produced by the thermal unit and the hourly water discharged from the reservoirs is received in a predetermined period to lower the cost of overall generating. By optimizing the hourly discharge of reservoirs, power system operators can strike a balance between the cost-effective use of both thermal and hydropower sources, ultimately reducing the overall cost of electricity generation. This is crucial for ensuring the efficient and economical operation of power systems while meeting energy demand. But while attempting to minimize cost existing techniques failed to consider ramp constraints in thermal constraints thereby resulting in the unfeasible working regions for thermal units. Hence this ϑ-constraint method considers ramp constraints in thermal constraints to obtain a feasible working region in the thermal unit without uncertainty. The essential thermal constraints in HTS with the consideration of ramp constraints are given in equations (3-10).

AC electricity is primarily used in generation, distribution, and consumption and loss calculations in power systems are done in AC. Transmission line resistance and transformer inefficiency are two causes of these losses. In contrast to DC calculations, which would not adequately account for the complexity of AC power systems, AC calculations are practical and in line with industry standards, ensuring accuracy and compliance with established rules. The power balance constraint is given in equation (3) which is the sum of the output of thermal power units and wind power and that is equal to the sum of the load and the loss of the network. The power loss is evaluated by Loss $= \sum (I_i^2.R_i)$. Where, Loss represents the total power loss in the network, $\sum$ denotes the summation overall network components, I_i^2 is the current flowing through component I and R_i is the resistance of component I [1], [2].

$$\sum_{i=1}^{Ns} P_{i,t} + \sum_{h=1}^{H} P_{h,t} = D_t + Ploss_t \tag{3}$$

where, $P_{(i,t)}$ is the output of thermal power units, $P_{(h,t)}$ is the output of hydropower units, D_t is the load and $P(loss)_t$ is the network loss at the time interval t. N_s stands for the total number of intervals at i and H stands for number of hydro power units at interval h.

The spinning reserve constraint R_t is the amount of unused capacity in online energy assets $r_{(i,t)}$ which compensate for power shortages or frequency drops within a given period which is given in equation (4) [1], [2], [8], [11]:

$$\sum\nolimits_{i=1}^{N_s} \left[r_{i,t} + P_{i,t} \right] + \sum\nolimits_{h=1}^{H} \overline{P_{h,t}} \geq R_t + D_t + Ploss_t \tag{4}$$

The constraint for a capacity limit of generating units in the thermal power plant is the amount of electricity a generator can produce when it's running at full blast. This maximum amount of power P is typically measured in megawatts which are given in equation (5,6) as [1], [2]:

$$P \leq P_{i,t} \leq \bar{P} \tag{5}$$

$$0 \leq P_{i,t} \leq \overline{P_h} \tag{6}$$

Then to maintain a feasible region in the thermal unit, ramp constraint is considered which is a dynamic constraint that restricts drastic change in power generation by a unit in successive time instants. The inclusion of the ramp rate constraint requires the modification of the range of generated power for each unit at every time instant which is given in equation (7) [11]:

$$(P_{i,t+1} - \Delta_i) \leq P_{i,t} \leq (P_{i,t+1} + \Delta_i) \tag{7}$$

After the inclusion of the ramp rate constraint, the constraint for minimum generation for the first and last hour in the thermal unit has been given in equation (8) [11].

$$P_{i,t} \leq \bar{P} \tag{8}$$

The constraints for minimum up time and minimum down time in the thermal unit have been given in equations (9) and (10) [11]:

$$x_{i,t} \geq T_u min_i \tag{9}$$

$$x_{i,t} \geq T_d min_i \tag{10}$$

where T_u and T_d are the uptime and downtime in the thermal unit and are represented in minimum value as min_i. These constraints are considered in the thermal unit with determining factors such as overestimation cost, underestimation cost, shape factor, scale factor and velocity of current wind speed thereby minimizing the uncertainty constraint. Then to solve the hydro and thermal unit commitment problem, hydro constraints have to be considered with balancing the water discharge in the hydro unit which is explained.

Effective improvements in power generation optimization should take into account market dynamics, better data quality, improved modeling approaches, and environmental and regulatory considerations in order to address these limits. Additionally, successful functioning of such systems frequently requires real-time monitoring and response to changing conditions.

The problem is multi-objective because it involves balancing the cost-effective utilization of both thermal and hydroelectric sources. There are multiple conflicting objectives: minimizing dispatch cost (fuel cost) and optimizing the hourly discharge of reservoirs. These objectives are often in conflict with each other, which is a characteristic of multi-objective optimization.

The fitness function (or functions) in multi-objective optimization generally changes as the optimization algorithm investigates the trade-offs between the various objectives. The method looks for a set of solutions that depict a Pareto front, where no option is superior to any other simultaneously across all objectives. In order to balance and optimize the competing objectives, the fitness function may alter with each optimization iteration.

The optimization problem involves two concurrent objectives, initially it minimizes dispatch cost by reducing fuel expenses of generating units, particularly thermal power plants, to enhance the cost-efficiency of electricity production. And secondly it optimizes hourly reservoir discharge to maximize hydroelectric power generation, aiming to find the most efficient strategy for releasing water from reservoirs at specific intervals. Thus it achieves a balance between minimizing costs and maximizing hydroelectric power generation, ensuring a cost-effective and sustainable operation of the power generation system.

III. CHAOTIC GEOMETRIC PROGRAMMING AND CHAOTIC APPROXIMATION APPROACH

In the Chaotic geometric programming and chaotic approximation approach, the hydro constraints are considered to solve the hydro and thermal unit commitment problem. The hydro constraints considered in the hydro unit have been given in the equations. In a hydro unit, the hydroelectric reservoir volume limit is a large collection of water behind a hydroelectric dam that makes use of potential energy v_r of water for generating electricity. This water is held back by the dam and a small amount is allowed to fall down the base of the dam to generate electricity when it is needed. Hence the condition for this reservoir volume limit $v_{(r,t)}$ is given in equation (11):

$$v_r^{\min} \leq v_{r,t} \leq v_r^{\max} \tag{11}$$

At hydropower plants, water flows through a pipe pushes against and turns blades in a turbine to spin a generator to produce electricity. Hence there is a need to balance water flow in the reservoir by using the condition for water flow balance which also includes the discharge of the hydro unit ($Q_{(r,t)}$) and it is given in equation (12);

$$v_{r,t} - v_{r,t-1} + c_1 \left[Q_{r,t} + S_{r,t} - \sum_{m \in R_r^{up}} (Q_{m,t-\tau_{mr}} + S_{m,t-\tau_{mr}}) \right] = c_1 y_{rt} \tag{12}$$

The condition for penstock water balance on each reservoir is given in equation (13) in which water discharge of hydro unit during period t is given as q_{irt}.

$$\sum_{i=1}^{n_{rt}} q_{irt} - Q_{irt} = 0 \tag{13}$$

The hydro units discharged out flow limits constraints with considering is given in equation (14);

$$u_{irt}.q_{ir}^{\min}(v_{rt}, Q_{rt}, S_{rt}, q_{irt}) \le q_{irt} \le u_{irt}.q_{ir}^{\max}(v_{rt}, Q_{rt}, S_{rt}, q_{irt}) \tag{14}$$

where, Q_{rt} is the reservoir r and stage t's discharged outflow (in m3/s), S_{rt} are the reservoir r and stage t's spillage (in m3/s), v_{rt} is the volume of the reservoir r and stage t (hm3) and u_{irt} is the binary variable that indicates if unit i is operating ($u_{irt} = 1$) or not ($u_{irt} = 0$) during stage t. The losses of hydropower plants have been calculated by using equation (15):

$$\boldsymbol{p_{hirt} - rp\,(v_{rt}, Q_{rt}, S_{rt}, q_{irt}) + tml_{irt}(p_{hirt}) + ggl_{irt}(p_{hirt}) = 0} \tag{15}$$

where, the power used by the mechanical friction in the guide bearings, thrust bearing, and shaft seals is referred to as the turbine mechanical loss (tml), which is measured in MW. A set of points (tml, gp) can be obtained through unit field tests and then a polynomial function is modified which is given in equation (16).

$$tml = b_0 + b_1.gp + b_2gp^2 \tag{16}$$

where b0, b1, and b2 are constants and gp is the power output at generator terminals. The generator's mechanical and electrical losses are referred to as the global losses which are denoted as ggl. Through field tests, a set of points (ggl, gp) is yielded that represent the mechanical losses of the turbine, from which the exponential function is adjusted which is given in equation (17).

$$ggl = a_0.e^{a1.gp} \tag{17}$$

where a0 and a1 are constants. Then, the mechanical power transferred through the coupling of the runner and the turbine shaft r_pirt is given in equation (18).

$$rp_{irt} = G.\eta_{rt}.nh_{irt}.q_{irt} \tag{18}$$

where, $G = 9.76102^{*}10^{-3}$ and η is the turbine hydraulic efficiency, and is given in equation (19).

$$\eta = C_0 + C_1.q_{irt} + C_2.nh_{irt} + C_3.q_{irt}.nh_{irt} + C_4.q_{irt}^2 + C_5.nh_{irt}^2 \tag{19}$$

where, $\boldsymbol{nh_{irt}}$ is the net head i.e., the part of the $\boldsymbol{gh}$ that is available for the turbine which is given in equation (20).

$$\begin{aligned} nh_{irt} &= gh_{irt} - (k_0q^2 + k_1q^2) \\ nh_{irt} &= fbl_{irt}(v) - trl_{irt}(Q, s) - (k_0q^2 + k_1q^2) \end{aligned} \tag{20}$$

where the functions $\boldsymbol{k_0q^2}$ and $\boldsymbol{k_1q^2}$ represent the head loss resulting from penstock water friction and the hydraulic energy lost between the tailrace level and the low-pressure reference section of the turbine, respectively. Additionally, net load variability in the form of two-dimensional data is explored with its left endpoint of the allowable range of water discharge in Armijo's rule that determines step size in some descent methods to solve unconstrained local optimization. Hence based on the Armijo rule, the allowable range of water discharge is considered as fact 0. Along with the Armijo rule, energy control dynamic indexing has been performed that recreates the index provided from the water discharge allowable range regularly and if there aren't many changes over time, there are enough resources to create a new index while the old one is still searchable. Thereby, this energy control dynamic indexing increases energy efficiency in each iteration. From equations (12) and (13), water discharge of hydro unit during period $\boldsymbol{t}$ is given as $\boldsymbol{q_{irt}}$ and set up a limit using the Hamiltonian and Egarch algorithms based on the Hazen-Williams rule and then calculate the penstock water balance. This Hazen William rule is given in equation (21).

$$\boldsymbol{H = \frac{4.73^{*}L^{*}\left(\frac{q_{irt}}{C}\right)^{1.852}}{d^{4.87}}} \tag{21}$$

where $\boldsymbol{H}$ is friction head loss, $\boldsymbol{L}$ and $\boldsymbol{d}$ are the length and diameter of the pipe, and C = Hazen-Williams C coefficient, dimensionless. Hence based on Hazen William's rule, the penstocks have been limited with an allowable C factor of 130-140. This model for the hydropower function incorporates the turbine-mechanical generator's electrical losses and takes into account hydraulic losses in the turbine suction tube and conduits, tailrace functions, hydraulic efficiency and mechanical losses in the turbine, as well as mechanical and electrical losses in the generator. These calculated water discharge and balance values are given as input to multiple waves neural networks to regulate the scheduling allowing acceptable water discharge. The architecture of multiple-wave neural networks is shown in Figure 2 [12].

The input Y1,Y2…Yn represents calculated water discharge and balance values and these values are processed in the hidden layer by considering multiple wavelet functions and updating the weight parameter to choose the best wavelet function. Then, in the output layer based on these optimized functional values, the output allowable water flow is attained as Z1,Z2,….,Zm without a hydro commitment

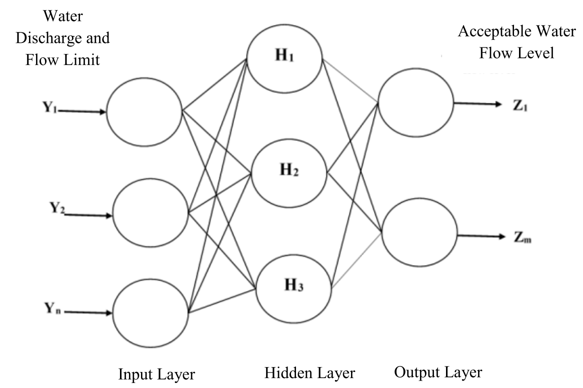

FIGURE 2. Architecture of multiple-wave neural networks.

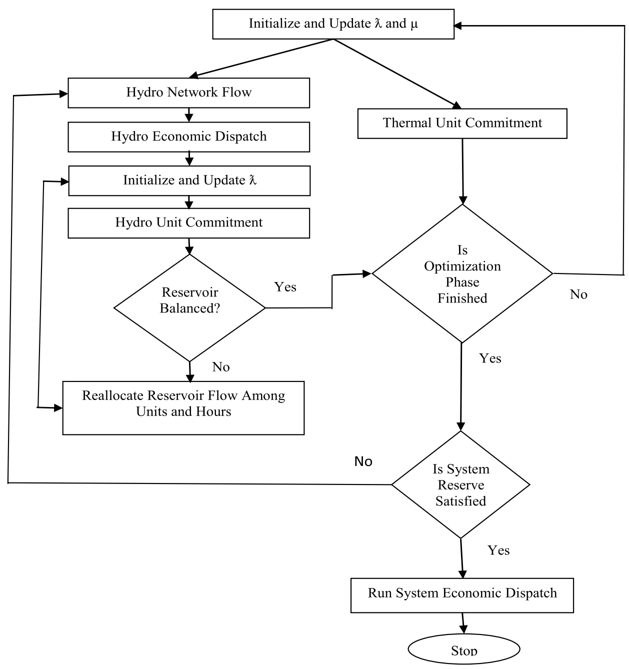

FIGURE 3. Flow chart of the chaotic geometric programming with multiple waves neural network.

problem. Thereby minimizing the hydro unit commitment problems by providing regulated scheduling of water flow in all penstock units.

The problem of both hydro and thermal unit commitment is solved by independently optimizing hydro and thermal commitment functions λ and μ via implementing Chaotic geometric programming with multiple waves neural network. The flow chart of the Chaotic geometric programming for solving the combined hydro and thermal unit commitment is shown in Figure 3.

Feature extraction, multi-resolution analysis, noise reduction, signal compression, and time-frequency analysis are some of the unique ways that wavelet functions are applied in the proposed method's Multiple Waves Neural Network. Wavelets improve data preprocessing and representation, allowing the network to more effectively comprehend and model complex wave-like patterns and time-dependent aspects in the data, especially in the context of applications like hydroelectric power production and signal processing. For the purpose of resolving hydro and thermal unit commitment, the chaotic geometric programming with multiple waves neural network's computational process is divided into different parts.

i λ and μ of the system should be initialized at the master coordinator.
ii Run Thermal Unit Commitment to obtain the commitment per unit and the generating schedules for each thermal unit.
iii Run the hydro network flow programming for watersheds to provide all reservoirs with the water release schedules.
iv Run the hydro Chaotic geometric programming with the reservoir water release schedules derived from multiple waves neural network flow to initialize the marginal water values.
v Run hydro unit commitment to obtain the generating schedules and unit commitments for each hydro unit in the reservoir.
vi Verify the balance of the reservoir's input and outflow. Additionally, determine if the absolute difference between the marginal water values at two distinct hours is less than a predetermined threshold. If so, go to step 7. If not, adjust water releases and update λ.
vii Verify the hydrothermal unit commitment is optimal. If the number of this phase's iterations reaches a predetermined minimum number and the difference between the system λ and μ in successive iterations is small enough, the chaotic approximation phase ends. If not, update λ and μ, then repeat steps 2 to 6.
viii Reallocate flow among units and hours if the system reserve requirements are met.
ix If not, repeat steps 2 through 8 again.
x Run the system economic dispatch via Chaotic geometric programming with multiple waves neural network to plan the committed units' power generation and halt computation.

Hence, the water flow discharge is balanced even with net load variation by the Armijo rule and the losses in penstocks are limited by the Hazen-William rule. Then, hydro and thermal commitment problems are solved using Chaotic geometric programming with multiple waves neural networks. Furthermore, there is a need to reduce the system cost without the HTS problem with an optimization scheme which is explained in the next subsection.

A. ENHANCED EBOLA OPTIMIZATION SEARCH ALGORITHM

In Enhanced Ebola Optimization Search Algorithm, the short-distance movement has been modified to standardize the hydrothermal scheduling problem for matured with a high convergence rate. The fitness function of this EEOS algorithm is given in equation (22).

$$F = \min_{i,t\epsilon T,N_s} f_i\left(P_s\left(i,t\right)\right) \tag{22}$$

Then to update the position of each parameter considered in the hydro and thermal units, the EEOS algorithm applies equation (23)

$$ML_i^{t\mathcal{C}1} = ML_i^t \mathcal{C} \rho M\left(L\right) \tag{23}$$

where $ML_i^{(t+1)}$ and $ML_i^{(t)}$ are updated and original position of calculated unit parameters, ρ is the scalar factor of displacement of each parameter and M(L) is the movement rate of water in the hydro unit, and M(T) is the movement rate of power spinning in the thermal unit and it is denoted in equation (24,25) as:

$$M\left(L\right) = S_{rate}{}^{*}rand\left(0,1\right)\mathcal{C}M\left(S_{best}\right) \tag{24}$$

$$M\left(T\right) = I_{rate}{}^{*}rand\left(0,1\right)\mathcal{C}M\left(I_{best}\right) \tag{25}$$

Hence, the exploitation stage is built on the presumption that the allowable water flow as well as power generation either stays within a distance of zero or is displaced within a range that does not exceed S_{rate}, where S_{rate} stands for short-distance movement. The fact that the allowable water flow as well as power generation outside of the typical neighborhood range I_{rate} serves as the basis for the exploration phase. The neighborhood parameter controls the S_{rate} and I_{rate} in such a way that when the neighborhood ≥ 0.5, the unit parameters have left the neighborhood, causing the excess water flow and power generation; otherwise, it stays within the neighborhood, which prevents excess water flow and power generation. In this EEOS algorithm, this S_{rate} has been modified by adjusting its limit using equation (26).

$$V_r^{min} \leq S_{rate} \leq V_r^{max} \tag{26}$$

In equation (26), V_r^{min} and V_r^{max} are the minimum and maximum limits for reservoir volume. By modifying the S_{rate} parameter based on reservoir limits, the best solutions for hydro and thermal units $\left(M\left(S_{best}\right), M\left(I_{best}\right)\right)$ has been obtained that standardizes the hydrothermal scheduling problem with matured convergence. Also, the enhanced exploratory and exploitative performance with suitable neighborhood parameter control provides a high convergence rate. Since the EEOS algorithm controls and maintains both water flow and power generation in the allowable range, the fuel cost of the system is reduced.

Overall, the proposed EEOS-based mathematical approach for hydrothermal scheduling has been presented to eliminate the problems in HTS by reducing fuel costs. Initially, the uncertainty constraints in hydrothermal power systems have been minimized by considering overestimated and underestimated cost functions as well as ramp constraints via the ϑ-constraint method. Then, thermal and hydro constraints are considered in Chaotic geometric programming and chaotic approximation approach to eliminate hydro and thermal commitment problems. Also, this approximation approach regulates the net load variability and water discharge based on Armijo's rule and Hazen Williams's rule. Finally, the fuel cost in HTS has been minimized by enhanced Ebola optimization that also eliminates the HTS problem. The next section explains the result obtained from EEOS based mathematical approach for hydrothermal scheduling and discusses it in detail.

IV. RESULTS AND DISCUSSION

A. TEST SYSTEM

The proposed EEOS-based mathematical approach for hydrothermal scheduling has been tested in Gem 5 simulator, and obtained optimal results for both hydro and thermal units without HTS problems. The test system consists of 5 Thermal units and 5 Hydro units. The scheduled period is divided into 24 periods with time resolution of one hour in day-ahead. A thermal unit's fuel cost function is a quadratic, hence ϑ-constraint method, chaotic geometric programming and chaotic approximation approach and Enhanced Ebola Optimization Search Algorithm have been used to solve the problem. The Thermal unit parameters are set to point, temperature, pressure, mass flow rate and max power attains at 12.5, 400K, 0.05 to 300 bar, 1000 kg/s, and 300 MW respectively. Forecasted water inflows are assumed to be constant across the whole time range. The V_{min} for three reservoirs are $131hm^3$, $84hm^3$ and $403hm^3$. The V_{max} for three reservoirs are $246hm^3$, $133hm^3$ and $280hm^3$. For thermal power plants, the essential inputs include the generation capacity, fuel costs, operating constraints (min/max output, startup/shutdown times, ramping rates), emission constraints, and the current state of the power plant, all of which play a critical role in optimizing power generation.

B. SIMULATED OUTPUT

The optimal discharge and power output obtained from the proposed mathematical model have been presented in Table 1. The hydro discharge from five hydro plants has been provided in Table 1. Gem5 simulator is used and the results were found to lie within optimal limit due to the incorporated Enhanced Ebola Optimization Search Algorithm in the proposed model and the Hydropower generation from five hydro plants and thermal power generation from one thermal plant are determined in an hourly basis and their values are maintained in optimum levels using ϑ-constraint method, and Chaotic geometric programming and chaotic approximation approach.

In order to achieve operational criteria such as minimum and maximum power output levels, environmental restrictions, and other requirements, the whole cost of generating, which may include fuel expenses for thermal generation, must be kept as low as possible.

Based on these inputs shown in Table 1, the mathematical model uses mathematical optimization techniques to identify the ideal schedule for both thermal and hydropower generating, which eventually results in the ideal discharge and power output for each source. This aids in striking a balance between

TABLE 1. Optimal discharge and power output obtained from a proposed mathematical model.

Hour	Hydro discharges (m3)					Hydropower generation (MW)						Thermal generation (MW)
	D1	D2	D3	D4	D5	Hyp1	Hyp2	Hyp3	Hyp4	Hyp4	Hyp5	Thp1
1	95010	58020	110000	100215	120654	85	50	59	69	55	65	996
2	93230	60601	115041	102325	121546	83	52	62	73	58	66	998
3	91050	62000	120000	104435	122438	81	54	65	77	61	67	1000
4	89008	64000	125013	106545	123330	79	56	68	81	64	68	1025
5	87000	66000	130000	108655	124222	77	58	71	85	67	69	1050
6	85010	76060	135000	110765	125114	75	60	74	89	70	70	1075
7	86000	75000	230085	112875	126006	72	62	77	93	65	55	1124
8	87000	74000	210000	130254	126898	69	80	80	97	57	58	1173
9	88000	73000	190000	129124	127790	66	81	75	85	49	61	1222
10	58000	72008	170506	127994	156842	63	82	72	84	41	64	1271
11	59000	71000	150000	126864	152156	65	83	69	83	33	67	1320
12	60000	105000	150000	125734	147470	64	84	66	82	25	70	1369
13	61000	101000	160000	124604	142784	63	85	63	81	17	73	1418
14	62005	98012	170000	123474	138098	62	90	60	80	9	89	1467
15	63000	77000	180000	122344	133412	61	86	57	79	75	91	1516
16	64000	75000	190000	121214	128726	60	82	54	78	77	93	1684
17	98000	73000	200000	120105	124040	50	78	51	58	79	95	1852
18	95000	71000	210568	119654	135218	52	74	85	62	81	97	2020
19	92000	102070	220000	119203	136025	54	70	86	66	86	99	1954
20	89000	100000	210000	118752	136832	56	85	87	70	89	101	1888
21	86000	98000	190014	118301	137639	58	88	88	74	92	99	1822
22	83080	96001	170000	117850	138446	60	81	89	78	95	95	1756
23	80015	94000	150000	117399	139253	55	56	90	82	98	97	1690
24	77047	92000	130000	116948	140060	50	31	91	86	101	99	1624

the efficient use of both sources and the maintenance of a steady supply of power.

The simulated output for maintaining water level without the commitment problem has been shown in Table 2, in which the water discharge is maintained in the allowable range by step response in line search mechanism of Armijo rule. Thus the optimal values of water level have been obtained using multiple waves neural network in Chaotic geometric programming and chaotic approximation approach. The step response, feasible solution, and optimal solution for maintaining the water discharge level within the permissible level have been obtained by Chaotic geometric programming and chaotic approximation approach which also generate a penalty rate when the water discharge level exceeds the permissible level.

Table 3 depicts the simulated output of the Enhanced Ebola Optimization Search Algorithm with the minimum fuel cost by performing optimization for 19 iterations. The optimized objective function and step response of the Enhanced Ebola Optimization Search Algorithm with the best short distance S_{rate} is obtained. Using the classical benchmark function, the best and worst values are determined for all 16 iterations in which the 6, 11,16 and 19 produce the best results, and are found to have minimum fuel cost as per the objective. From

TABLE 2. Simulated output of Chaotic geometric programming and chaotic approximation approach.

Major	Step	Feasible	Optimal	Penalty
0	0.0E+00	1.3E+02	2.4E+00	0.0E+00
1	0.22E-01	5.3E+01	1.3E+00	1.9E-03
2	2.4E-0.1	2.8E+01	3.0E-01	1.9E-03
3	1.0E+00	3.3E+00	1.2E-06	1.9E-03
4	6.9E-08	3.3E+00	8.1E+00	1.9E-03
5	8.5E-01	3.3E+00	4.0E-01	1.6E+04
6	1.0E+00	3.3E+00	6.4E-03	2.7E+04
7	1.0E+00	3.3E+00	2.5E-10	2.7E+04

TABLE 3. Simulated output of enhanced ebola optimization search algorithm.

Iteration	S_{rate}	Objective	Step
6	3	4758139.6091	1.0
11	3	4386497.8528	0.0000017
16	4	4366944.1734	1.0
19	4	4366944.1598	1.0

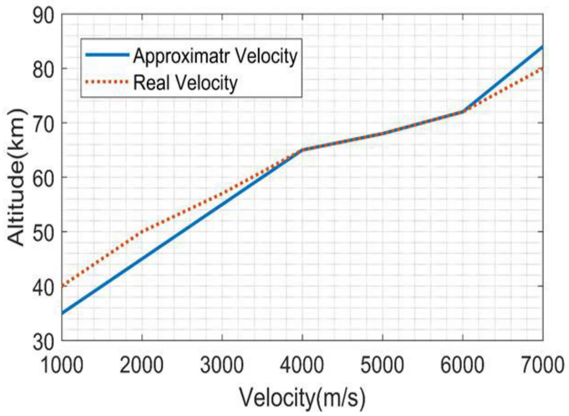

FIGURE 4. Velocity of the proposed mathematical model.

Table 3, it is clear that S_{rate} can be modified by adjusting its limits (minimum and maximum limits for reservoir volume). Thus by modifying S_{rate}, the best solution is obtained for the HTS problem. This optimization produces optimum fuel cost with high convergence by modifying the S_{rate} between 3 and 4 based on the reservoir limit.

C. PERFORMANCE METRICS OF THE PROPOSED SYSTEM

The performance calculation is known as the estimation of the outcome, which provides accurate information on the efficiency of the method proposed. The parameters such as velocity, water discharge, energy, reservoir storage volume, active power, fuel cost, and generation cost are discussed below to determine the performance of the proposed model. Thus, the relation between the input value and the output value of the proposed system is known.

The approximate velocity of the proposed mathematical model with real velocity and also both the velocities ranges from 1000 to 7000 but, the attitude range varies from 30 to 90 km as shown in Figure 4. The approximate velocity of the proposed model rises with the increase in the real velocity. Hence, the approximate velocity and real velocity values are more or less the same. It is to be noted that the proposed mathematical model is improved by Chaotic geometric programming and Chaotic approximation approach that solve net load variability and uncertainty in hydrothermal scheduling.

The water discharge of the proposed model and the scheduling interval of four units in penstock water flow during the period of 5 to 24 hours have been shown in Figure 5. Each unit of water discharge varies slightly with the scheduling interval. The water discharge of the hydro unit during 5-hour intervals has been calculated using Hamiltonian and Egarch algorithms based on Hazen Williams's rule and then Penstock Water balance is calculated to balance the water discharge in all four units thereby solving the HTS problem.

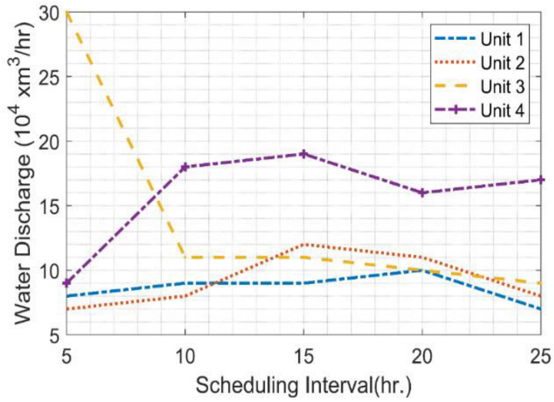

FIGURE 5. Water discharge of the proposed model.

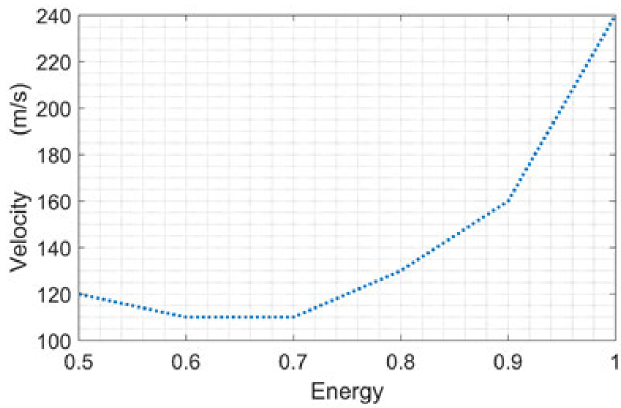

FIGURE 6. Energy of proposed model.

The energy consumption of the proposed model with varying velocities in which the energy varies from 0.5 to 1 and the velocity varies from 120 to 240 m/s has been illustrated in Figure 6. The energy increases with an increase in velocity but the energy is maintained within the acceptable level by using Chaotic geometric programming and chaotic approximation approach in which energy control dynamic indexing regulate energy level with increasing energy efficiency in each iteration.

The reservoir storage volume of the proposed model during the period between 1 to 24 hours is shown in Figure 7. The reservoir storage volume has been maintained between 1.4×10^6 m^2 to 1.2×10^6 m^2 for the time interval from 1 to 24 hours. The reservoir storage volume has been maintained at the optimum level by Chaotic geometric programming and a chaotic approximation approach in which chaotic geometric programming upholds reservoir storage limit constraint for each hour.

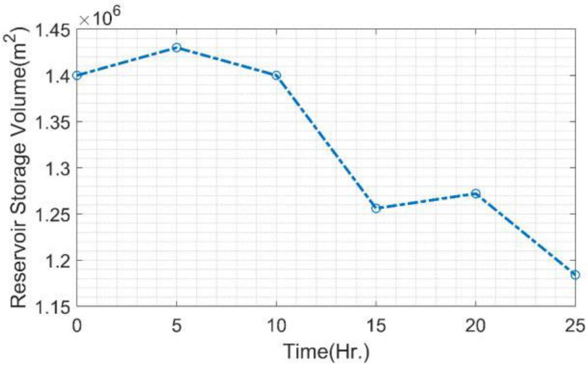

FIGURE 7. Reservoir storage volume of the proposed model.

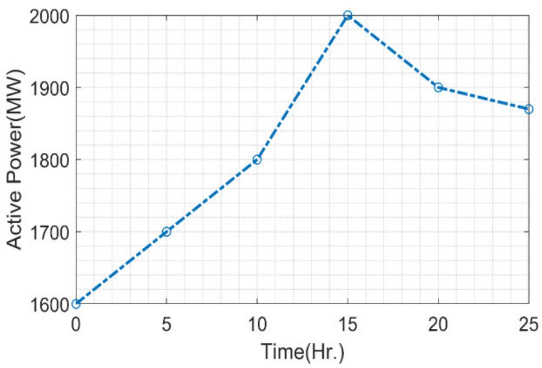

FIGURE 8. Active power of the proposed model.

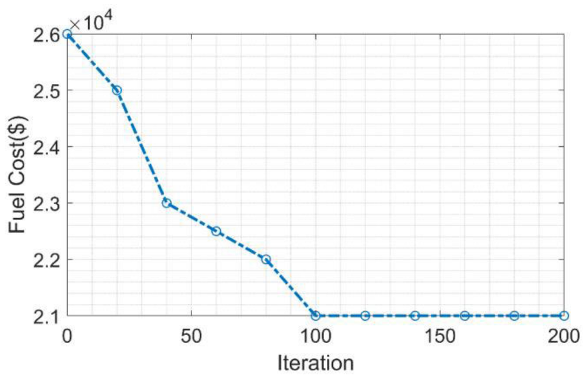

FIGURE 9. Fuel cost of the proposed model.

The active power of the proposed model for time intervals between 1 to 24 hours is shown in Figure 8. The active power of the proposed model lies in the range between 1600 to 2000 MV and it varies for each hour. The active power consumption of the proposed model has been maintained within the permissible level by using Chaotic geometric programming and a chaotic approximation approach in which the energy control dynamic indexing is performed at each time interval thereby active power is controlled effectively.

The fuel cost of the proposed model for the iterations ranges from 1 to 200 and is presented in Figure 9. The

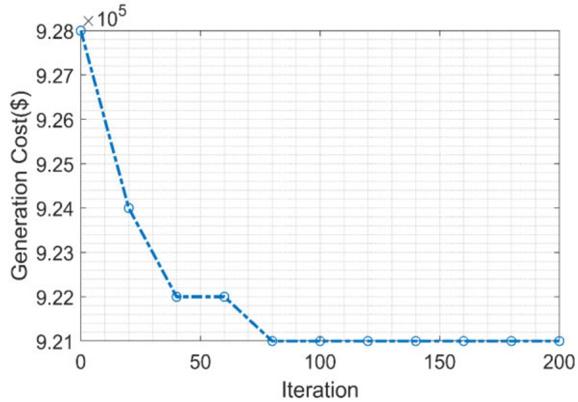

FIGURE 10. Generation cost of the proposed model.

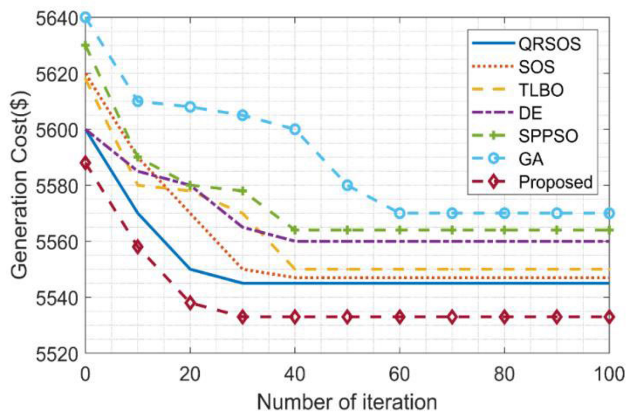

FIGURE 11. Comparison of generation cost.

fuel cost of the proposed model has a minimum value of 2.1×10^4 $ and attain a maximum value of 2.6×10^4 $. The fuel cost of the proposed model is reduced by increasing the number of iterations to 100 and after that, the fuel cost remains constant. The fuel cost of the proposed model is minimized by using the Enhanced Ebola Optimization Search Algorithm that increases the convergence rate by modifying ***Srate*** parameter..

The generation cost of the proposed model for the iterations ranges from 1 to 200 is shown in Figure 10. The generation cost of the proposed model has a minimum value of $9.21\times\mathbf{10^5}$ $ and attain a maximum value of $9.28\times\mathbf{10^5}$ $. The generation cost of the proposed model is reduced by increasing the number of iterations to 80 and after that, the generation cost remains constant. The generation cost of the proposed model is minimized by using Enhanced Ebola Optimization Search Algorithm in which enhanced exploratory and exploitative performance reduces cost with an increase in the diversity of the population.

D. COMPARATIVE ANALYSIS

The generation cost comparison of various optimization methods [1] such as Quantum Random Search Optimization

TABLE 4. Comparison of the existing model with the proposed model.

Technique	Fuel Cost	Operating Cost	Generation Cost	Simulation Time	Average Cost	High no. of hits	Standard Deviation
QRSOS	1.38$	4.6$	5544$	5.5s	9.223$	24	16%
SOS	1.39$	4.5$	5545$	6s	9.223$	24	30%
TLBO	1.36$	4.5$	5550$	10s	9.233$	21	32%
DE	1.37$	4.4$	5560$	7s	9.237$	23	20%
SPPSO	1.35$	4.3$	5535$	16s	9.231$	20	35%
CDE	1.35$	4.3$	5565$	6s	9.231$	23	25%
CCDE	1.34$	4.2$	5543$	9s	9.232$	22	24%
Proposed	1.31$	4$	5530$	5s	9.223$	28	15%

System (QRSOS), Stochastic Optimization System (SOS), Teaching-Learning-Based Optimization (TLBO), Differential Evolution (DE), Standard Particle Swarm Optimization (SPPSO) and Genetic Algorithm (GA) with the proposed model is presented in Figure 11. The generation cost of the proposed system at iteration 100 is 5535 $ whereas the generation cost of the existing techniques [1] such as QRSOS, SOS, TLBO, DE, SPPSO and GA are 5544, 5545, 5550, 5560, 5565 and 5570 $ respectively. Detailed comparison of proposed model with existing techniques is provided in table 4. It is to be noted that the proposed mathematical model performs well as compared with all other existing techniques. These better performances are attained due to the consideration of ramp, hydro and thermal constraints in the ϑ-constraint method and Chaotic geometric programming and chaotic approximation approach as well as performing enhanced Ebola optimization to minimize cost parameters

V. CONCLUSION

Hydrothermal scheduling aims to optimize the use of cheaper and more environmentally friendly hydroelectric power, alongside more expensive thermal power, reducing reliance on the latter's high fuel costs. EEOS-based mathematical approach for hydrothermal scheduling has been presented to address the uncertainty problems, hydrothermal commitment problems, and HTS problems, while optimizing the system cost. Findings by using the EEOS-based approach on the objective function are as follows:

- ϑ-constraint method considers the ramp constraints thereby minimizing uncertainty with a low standard deviation of 15 %. Then, Chaotic geometric programming and chaotic approximation approach maintain the water discharge at an appropriate level by penstock water balance equation in which four units' water flow during the period of 5 to 25 hours are considered with maintaining water discharge in the acceptable limit between 5 to 35 $\times 10^3 m^3/hr$ and also eliminate hydro thermal commitment problem using Multiple waves neural network with optimum reservoir storage limit and active power of $1.4\times10^6 m^2$ and 2000 MW.
- Furthermore, the system cost is reduced by using Enhanced Ebola Optimization in which a low fuel cost of 2.1×10^4 \$, operating cost of 4 \$, generation cost of 9.21×10^5 \$, simulation time of 5 seconds has been obtained by modifying S_{rate} the parameter that also eliminates the HTS problem.
- The proposed mathematical model outperforms the existing techniques with a low average cost of 922,341.16 $ and a high number of hits to the best solution of 28.

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