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RESEARCH ARTICLE

H_∞ Synchronization for Chaotic Lur'e System With Uncertainty Based on Memory-Based Sampled-Data Control

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ABSTRACT In this paper, the synchronization issue of chaotic Lur'e system (CLs) is investigated under the coupling memory sampled-data controller (SDC) with parameter uncertainties. First, using a Bernoulli distributed sequence, a more universal coupling controller that involves the factor of data transmission delay is proposed. Based on these vectors, an augmented Lyapunov-Krasovskii functional (LKF) is constructed for the CLs, where the LKF is derived as delay-dependent. The LKF is also based on the information of entire sampling interval and non-linear functional vector. Meanwhile, a relaxed second-stage affine Bessel–Legendre inequality (BLIY) is introduced to estimate integral terms generated during the derivation. On account of the improved functional and the relaxed integral inequality, an admissibility criteria via a coupling controller is acquired for the synchronous error system with less conservative and the framework of linear matrix inequalities (LMIs). Then, the memory SDC is designed to synchronize the driving and responding CLs. Finally, a simulation example is provided to verify the validity and superiority of the proposed design scheme.

INDEX TERMS Lyapunov-Krasovskii functional, memory sampled-data controller, Bessel–Legendre inequality, Lur'e system.

I. INTRODUCTION

Over past decades, chaotic systems have gradually become the emphasis of our research, and the exploration of chaotic synchronization (CCs) has become more and more profound. Up to now, many studies was reported for CCs [1]. Moreover, it is also universal to employ Lur'e systems model to express nonlinear dynamic systems, such as Lorenz's system and hyperchaotic attractors. Hence, some effective methods to research the synchronization of CLs have been established, for instance, time-delay feedback control [5], adaptive control [6], impulsive control [7], SDC [8] and so on.

As network technology evolves at a rapid pace, the SDC has grown and gained popularity among researchers as it is a beneficial tool to reduce the complexity and cost of hardware implementation while maintaining the required level of control performance. In many systems, the continuous-time control method may become computationally expensive and even infeasible due to the need for high-performance computing hardware, sensor and actuator hardware, and communication infrastructure. On the other hand, SDC provides a way to achieve the necessary control performance using low-cost hardware by discretizing the continuous-time systems and applying the control action at specific time intervals. By SDC, the continuous-time signals of a system are periodically sampled and converted into discrete-time signals. The input delay SDC is a control method in which the input to a system is updated periodically at discrete time intervals. This new approach is introduced by Fridman et al. [9]. In literature, researchers have conducted numerous studies to evaluate the performance of the SDC scheme in stabilizing nonlinear systems, and the results have been promising. For example, the synchronization of CLs under traditional SDC was established in [10]. Larger sample interval was acquired by using an augmented LKF method. The advantages of the method was proved by the simulation

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of Chua's electric circuit. Under the action of the sampler, the updating signal was successfully sent to the controller and the zero-order holder. However, in the actual situation, the memory-based SDC is widely used considering the transmission delay. Therefore, under the control of memory-based SDC, the H_∞ synchronization issue of CLs is studied in [11] and [12]. We design a memory-based coupled SDC that is a composite of conventional SDC, memory-based SDC and Bernoulli sequence to further improve the flexibility of the controller in the present study.

Recently, for the sake of deducing the stability criteria with lower conservatism, some improved inequalities and some general functions have been put forward, such as time-dependent LKF and looped functional and so on. In the SDC system, the conservative of stability criteria is reduced by applying the looped functional [13], which takes full account of the information in the sampling interval. In the presence of disturbance, we acquire a lower conservative condition in Lur'e system [14]. Nevertheless, time delay is often encountered in many real systems, and its existence may cause system performance degradation or even make the system unstable [15]. Therefore, the problems related to time-delay system have received extensive attention over the past few decades [16], [17], [18], [19]. Note that the LKF approach is the most effective tool for studying the stability of systems with time delay. In addition, the estimation of the derivative terms of LKF is closely related to the conservatism of the results. So various inequality methods have been proposed as free-matrix-based integral inequality [20], generalized reciprocally convex inequality [21], Wirtinger inequality [22], auxiliary-function-based inequalities [23], BLIY [24], etc. The stability requirements of time-varying delay models are investigated in [20]. However, it is found that the sampling information from t_k to t is ignored, so we still need to further decrease the conservative of this condition. On the previous basis, the concise form of BLIY is derived [25]. But the vector in BLIY may not be considered, and the derived conditions are still conservative [26].

Inspired by these treatises, in this work, investigates the synchronization of CLs with uncertain parameters is investigated via coupling memory SDC scheme and improved Lyapunov functional. The innovations are summarized:

i) As the first attempt, by utilizing a more general coupling memory SDC scheme, the issue of synchronization for CLs is studied, in which the Bernoulli distributed sequence is proposed to design a coupling memory SDC.

ii) The improved LKF considers not only system state information, but also the information about the nonlinear section constraint, which further reduces the conservatism of criteria.

iii) To estimate integral terms, a relaxed integral inequality second-order canonical BLIY is utilized to calculate the bound of quadratic integral term. This inequality not only contains the system state information, but also introduces some free weight matrices, which provides advantages for obtaining improved results.

Notations: $\mathbb{R}^n$ represents the Euclidean space with n dimension, $\mathbb{R}^{m\times n}$ denotes the set of $m \times n$ real matrices. For a matrix Y, Y^{-1} and Y^T are the inverse and transpose of matrix Y, respectively. The symbol $*$ is the symmetric terms in a symmetric matrix. $Y > (\geq 0)$ means that Y is a positive definite (positive semi-definite) matrix. I is an identity matrix with a proper dimension; $\mathbf{E}\{\cdot\}$ indicates the mathematical expectation operator. $diag\{\cdot\}$ refers to a block-diagonal matrix.

II. PROBLEM STATEMENT

Consider a synchronization plan for CLs with uncertainty via SDC:

$$\mathcal{M}:\begin{cases}\dot{x}(t)=\bar{\mathcal{A}}_\sigma x(t)+\bar{\mathcal{B}}_\sigma f(\mathcal{C}x(t)),\\ \mathfrak{p}(t)=\mathcal{H}x(t),\end{cases}$$
$$\mathcal{F}:\begin{cases}\dot{y}(t)=\bar{\mathcal{A}}_\sigma y(t)+\bar{\mathcal{B}}_\sigma f(\mathcal{C}y(t))+u(t)+\mathcal{D}\omega(t),\\ \mathfrak{q}(t)=\mathcal{H}y(t).\end{cases}\tag{1}$$

The system is composed of the primary system $\mathcal{M}$ and the secondary system $\mathcal{F}$ with their corresponding vectors x, y. $\mathfrak{q}(t), \mathfrak{p}(t) \in \mathbb{R}^l$ indicate the output of the primary-secondary system, $\omega(t) \in \mathbb{R}^k$ is the external disturbance that is covered $\mathcal{L}_2[0,\infty)$, $u(t) \in \mathbb{R}^n$ denotes the control input. $\bar{\mathcal{A}}_\sigma = \mathcal{A} + \Delta\mathcal{A}(t)$, $\bar{\mathcal{B}}_\sigma = \mathcal{B} + \Delta\mathcal{B}(t)$, $\mathcal{A} \in \mathbb{R}^{n\times n}, \mathcal{H} \in \mathbb{R}^{l\times n}, \mathcal{C} \in \mathbb{R}^{n_q\times n}$, $\mathcal{D} \in \mathbb{R}^{n\times k}$ and $\mathcal{B} \in \mathbb{R}^{n\times n_q}$ are some constant matrices, $\Delta\mathcal{A}(t)$ and $\Delta\mathcal{B}(t)$ are unknown matrices characterizing the uncertainty of time-varying parameters, and their structure is characterized by the below equation:

$$[\Delta\mathcal{A}(t), \Delta\mathcal{B}(t)] = \mathcal{J}\hat{\Delta}(t)[\mathcal{E}_c, \mathcal{E}_d],\tag{2}$$

where $\mathcal{J}, \mathcal{E}_c, \mathcal{E}_d$ are known matrices, and parameter uncertainty $\hat{\Delta}(t)$ is unknowable and satisfies $\hat{\Delta}^T(t)\hat{\Delta}(t) < I$.

It is supposed that $f(\cdot) : \mathbb{R}^{n_q} \to \mathbb{R}^{n_q}$ is a nonlinear function belonging to $[\omega_i^-, \omega_i^+]$ with $i = 1, 2, \cdots, n_q$ and the following condition is true:

$$[f_i(s) - \omega_i^+ s][f_i(s) - \omega_i^- s] \leq 0.\tag{3}$$

Denote the error signal as $e(t) = x(t) - y(t)$, one can acquire:

$$\dot{e}(t) = \bar{\mathcal{A}}_\sigma e(t) + \bar{\mathcal{B}}_\sigma g(\mathcal{C}e(t)) - u(t) - \mathcal{D}\omega(t),\tag{4}$$

where $g(\mathcal{C}_\sigma e(t), y(t)) = f(\mathcal{C}_\sigma e(t) + \mathcal{C}_\sigma y(t)) - f(\mathcal{C}_\sigma y(t))$.

Consider the memory-based coupling SDC as:

$$\begin{aligned}u(t) &= \check{\varepsilon}(t)\mathcal{K}(\mathfrak{p}(t_k - \underline{\tau}) - \mathfrak{q}(t_k - \underline{\tau}))\\ &\quad + (1 - \check{\varepsilon}(t))\mathcal{T}(\mathfrak{p}(t_k) - \mathfrak{q}(t_k))\\ &= \check{\varepsilon}(t)\mathcal{K}\mathcal{H}e(t_k - \underline{\tau}) + (1 - \check{\varepsilon}(t))\mathcal{T}\mathcal{H}e(t_k),\end{aligned}\tag{5}$$

where $\mathcal{K}$ and $\mathcal{T}$ are appropriate dimension gain matrices and $\underline{\tau}$ is constant delay. Here $\check{\varepsilon}(t)$ is the Bernoulli probability variable coupling the traditional SDC and memory-based SDC with $\Pr\{\check{\varepsilon}(t) = 1\} = \mathbf{E}\{\check{\varepsilon}(t)\} = \check{\varepsilon}$ and $\Pr\{\check{\varepsilon}(t) = 0\} = 1 - \mathbf{E}\{\check{\varepsilon}(t)\} = 1 - \check{\varepsilon}$, where $\check{\varepsilon} \in [0, 1]$. Moreover,

$$0 \leq t_{k+1} - t_k = \hat{h}_k \leq \hat{h}_m,\tag{6}$$

where $\hat{h}_m$ is the largest upper bound (LUB) of the sampling period.

By substituting (5) into (4), then

$$\begin{aligned}\dot{e}(t) = \bar{\mathcal{A}}_\sigma e(t) + \bar{\mathcal{B}}_\sigma g(Ce(t)) - \mathcal{K}\mathcal{H}\breve{\varepsilon}(t)e(t_k - \underline{\tau}) \\ - \mathcal{T}\mathcal{H}(1-\breve{\varepsilon}(t))e(t_k) - \mathcal{D}\omega(t). \qquad (7)\end{aligned}$$

Remark 1: Note that in the present article, we employ Bernoulli sequences to integrate conventional SDC. Then, the coupling memory SDC can be used to study the H_∞ synchronization issue for Lur'e systems. Moreover, $\breve{\varepsilon}(t)$ is a stochastic variable coupling both sampled-data proportional control and memory sampled-data control. It is convenient to observe that when the probability variable $\breve{\varepsilon}(t)$ is 1 or 0, the control input is equivalent to the controller in [11], [27], and [28].

From (3), one may acquire that

$$\begin{aligned}&[g_i(\mathcal{C}_i e(t), y(t)) - \omega_i^+ \mathcal{C}_i e(t)][g_i(\mathcal{C}_i e(t), y(t)) \\ &-\omega_i^- \mathcal{C}_i e(t)] \le 0, \quad i = 1, 2, \cdots, n_q. \qquad (8)\end{aligned}$$

Definition 1 [29]: For any matrice $S > 0$, error system (7) satisfies the H_∞ performance constraint if

$$\mathbf{E}\Big\{\int_0^\infty e^T(s)Se(s)ds\Big\} < \gamma^2 \mathbf{E}\Big\{\int_0^\infty \omega^T(s)\omega(s)ds\Big\} \qquad (9)$$

is valid for the specified positive scalar γ.

Lemma 1 [30]: For scalars r_1 and r_2 ($r_2 > r_1$), any matrix $\tilde{\mathcal{R}} > 0$, differentiable function $x(s)$ satisfying $\dot{x} : [r_1, r_2] \to \mathbb{R}^n$, then the below inequality is acquired:

$$\begin{aligned}-\int_{r_1}^{r_2} \dot{x}^T(s)\tilde{\mathcal{R}}\dot{x}(s)dr &\le -\frac{1}{r_2 - r_1}\psi_1^T \Theta^T \mathbb{R}\Theta\psi_1 \\ &= -\frac{1}{r_2 - r_1}\psi_2^T \Theta^T \mathbb{R}\Theta\psi_2 \qquad (10)\end{aligned}$$

where $\mathbb{R}$=diag$\Big\{\tilde{\mathcal{R}}, 3\tilde{\mathcal{R}}, 5\tilde{\mathcal{R}}\Big\}$, and

$$\Theta = \begin{bmatrix} I & -I & 0 & 0 \\ I & I & -2I & 0 \\ I & -I & -6I & 12I \end{bmatrix},$$

$$\psi_1 = \begin{bmatrix} x(r_2) \\ x(r_1) \\ \int_{r_1}^{r_2} \frac{x(s)ds}{(r_2-r_1)} \\ \int_{r_1}^{r_2} \frac{(r_2-s)x(s)ds}{(r_2-r_1)^2} \end{bmatrix},$$

$$\psi_2 = \begin{bmatrix} x(r_2) \\ x(r_1) \\ \int_{r_1}^{r_2} \frac{x(r)dr}{(r_2-r_1)} \\ \int_{r_1}^{r_2} \frac{(s-r_1)x(s)ds}{(r_2-r_1)^2} \end{bmatrix}.$$

For any matrix M with adaptive dimensions, the above inequality can be written in an affine form by using the following fundamental inequality:

$$-\frac{1}{r_2 - r_1}\Theta^T\tilde{\mathcal{R}}\Theta \le Sym\{\Theta^T M\} + (r_2 - r_1)M^T\tilde{\mathcal{R}}^{-1}M. \quad (11)$$

Then the above inequalities are changed to the more general form below:

Lemma 2 [13]: For scalars r_1 and r_2 ($r_2 > r_1$), any matrix M, $\tilde{\mathcal{R}} > 0$, differentiable function $x(s)$ satisfying $\dot{x} : [r_1, r_2] \to \mathbb{R}^n$, then the below inequality is acquired:

$$\begin{aligned}&-\int_{r_1}^{r_2} \dot{x}^T(s)\tilde{\mathcal{R}}\dot{x}(s)ds \\ &\quad \le \psi_1^T[Sym\{\Theta^T M\} + (r_2 - r_1)M^T\mathbb{R}^{-1}M]\psi_1 \\ &\quad = \psi_2^T[Sym\{\Theta^T M\} + (r_2 - r_1)M^T\mathbb{R}^{-1}M]\psi_2, \qquad (12)\end{aligned}$$

where M is a free-weighting matrix and Θ, $\mathbb{R}$, ψ_1 and ψ_2 are defined in **Lemma 1**.

Remark 2: In [23] and [32], it was proposed that the general integral inequality and its corresponding affine version integral inequality provide the identical lower bound for the associated integral term. More importantly, **Lemma 2** is more suitable for nonlinear systems than **Lemma 1**, because affine inequalities in **Lemma 2** do not engender reciprocal convexity.

Lemma 3 [31]: Let $\omega(s)$ be a vector function in $[r_1, r_2] \to \mathbb{R}^{n\times n}$, for scalars r_1 and r_2 ($r_2 > r_1$), any matrix $Q > 0$, there exists the following inequality:

$$\begin{aligned}&(r_2 - r_1)\int_{r_1}^{r_2} \omega^T(s)Q\omega(s) \\ &\qquad \ge \Big(\int_{r_1}^{r_2}\omega(s)ds\Big)^T Q\Big(\int_{r_1}^{r_2}\omega(s)ds\Big). \qquad (13)\end{aligned}$$

Lemma 4 [33]: With a positive scalar σ, the matrices $\mathcal{H}$, $\mathcal{L}$ and $\nabla_m(t)$ of adaptive dimensions, if $\nabla_m^T(t)\nabla_m(t) \le I$, the below inequality is acquired:

$$\mathcal{H}\nabla_m(t)\mathcal{L} + \mathcal{L}^T\nabla_m^T(t)\mathcal{H}^T \le \sigma\mathcal{H}\mathcal{H}^T + \sigma^{-1}\mathcal{L}^T\mathcal{L}. \qquad (14)$$

III. MAIN RESULTS

In this short section, stability criteria and sufficient conditions of the error model are provided. For brevity, the signs below are defined as:

$$\begin{aligned}
&e_i = [0_{n\times(i-1)n}, I, 0_{n\times(15-i)n}](i = 1, 2, \cdots, 15),\\
&\mathcal{W}_1 = diag\{\omega_1^+, \omega_2^+, \cdots, \omega_{n_q}^+\},\\
&\mathcal{W}_2 = diag\{\omega_1^-, \omega_2^-, \cdots, \omega_{n_q}^-\},\\
&\xi_N(t) = col\{\xi_1(t), \xi_2(t)\},\\
&\xi_1(t) = col\{e(t), \dot{e}(t), e(t_k), e(t_{k+1}), e(t-\underline{\tau}), e(t_k - \underline{\tau})\},\\
&\xi_2(t) = col\{(t-t_k)\hat{\eth}(t), (t_{k+1}-t)\breve{\Im}(t), \hat{\eth}(t), \breve{\Im}(t), g(Ce(t))\},\\
&\hat{\eth}(t) = col\{\hat{\eth}_1(t), \hat{\eth}_2(t)\}, \breve{\Im}(t) = col\{\breve{\Im}_1(t), \breve{\Im}_2(t)\},\\
&\hat{\eth}_i(t) = \int_{t_k}^{t}\frac{(t-s)^{i-1}}{(t-t_k)^i}e(s)ds, (i = 1, 2),\\
&\breve{\Im}_i = \int_{t}^{t_k+1}\frac{(s-t)^{i-1}}{(t_{k+1}-t)^i}e(s)ds, (i = 1, 2),\\
&\eta_1(t) = col\{e(t), e(t_k), e(t_{k+1}), (t-t_k)\hat{\eth}(t), (t_{k+1}-t)\breve{\Im}(t)\},\\
&\dot{\eta}_1(t) = col\{\dot{e}(t), 0, 0, e(t), \hat{\eth}_1(t) - \hat{\eth}_2(t), -e(t), -\breve{\Im}_1(t)\\
&\quad + \breve{\Im}_2(t)\},
\end{aligned}$$

$\eta_2(t_k) = col\{e(t_k), e(t_{k+1}), e(t_k - \underline{\tau})\}$,
$E_1 = [I \quad 0], E_2 = [0 \quad I]$.

Theorem 1: For given scalars $\hat{h}_m > 0, \underline{\tau} > 0, \check{\varepsilon}, \epsilon_1, \epsilon_2$ and ϵ_3, the system (1) is robustly $\mathcal{H}_\infty$ synchronous for any $\hat{h}_k \in (0, \hat{h}_m]$, if there exists $P > 0, S > 0, S_1 > 0, S_2 > 0, \tilde{\mathcal{R}}_1 > 0, \tilde{\mathcal{R}}_2 > 0, Q_1, Q_2 = Q_2^T, \Lambda_1 = diag\{\lambda_1, \cdots, \lambda_{n_q}\} > 0, \Lambda_2 = diag\{\delta_1, \cdots, \delta_{n_q}\} > 0, U = diag\{u_1, \cdots, u_{n_q}\}$, any appropriate dimension matrices $\mathcal{J}, G, L_1, L_2, \hat{\mathbb{L}}, \hat{\mathbb{W}}, \mathbb{N}, \mathbb{M}, \mathbb{Y}$, and scalars $\sigma > 0, \gamma$, the following LMIs hold,

$$\begin{bmatrix} \Phi_0 & \sigma\Gamma_1^T G\mathcal{J} & \mathcal{E}_n^T & \Gamma_1^T G\mathcal{D} \\ * & -\sigma I & 0 & 0 \\ * & * & -\sigma I & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0 \tag{15}$$

$$\begin{bmatrix} \Phi_0 + h\Phi_1 & \sigma\Gamma_1^T G\mathcal{J} & \mathcal{E}_n^T & \Gamma_1^T G\mathcal{D} & \hat{h}_m\mathbb{Y}^T \\ * & -\sigma I & 0 & 0 & 0 \\ * & * & -\sigma I & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & -\hat{h}_m\mathbb{R}_2 \end{bmatrix} < 0 \tag{16}$$

$$\begin{bmatrix} \Phi_0 + h\Phi_2 & \sigma\Gamma_1^T G\mathcal{J} & \mathcal{E}_n^T & \Gamma_1^T G\mathcal{D} & \hat{h}_m\mathbb{M}^T \\ * & -\sigma I & 0 & 0 & 0 \\ * & * & -\sigma I & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & -\hat{h}_m\mathbb{R}_1 \end{bmatrix} < 0 \tag{17}$$

where

$$\begin{aligned}
\Phi_0 &= 2e_1^T P e_2 + 2e_1^T \mathcal{C}^T(\mathcal{W}_1\Lambda_1 - \mathcal{W}_2\Lambda_2)\mathcal{C}e_2 - e_5^T S_1 e_5 \\
&\quad + 2e_{11}^T(\Lambda_2 - \Lambda_1)\mathcal{C}e_2 + e_1(S + S_1)e_1 + \tau^2 e_2^T S_2 e_2 \\
&\quad - (e_1 - e_5)^T S_2(e_1 - e_5) + 2\ell_1^T\mathbb{N}\ell_2 + 2e_2^T Q_1 E_2\ell_2 \\
&\quad - 2e_2^T Q_1^T E_1\ell_2 + 2(\Pi_1^T\Theta^T\mathbb{M} + \Pi_2^T\Theta^T\mathbb{Y}) - 2e_{11}^T U e_{11} \\
&\quad + 2e_1^T\mathcal{C}^T(\mathcal{W}_1 + \mathcal{W}_2)Ue_{11} - e_1^T\mathcal{C}^T\mathcal{W}_1 U\mathcal{W}_2\mathcal{C}e_1 \\
&\quad + 2\hat{\mathbb{L}}e_7 + 2\hat{\mathbb{W}}e_8 + 2\Gamma_1^T\Gamma_2, \\
\Phi_1 &= 2\ell_1^T\mathbb{N}E_1^T e_1 - 2\ell_3\mathbb{N}E_1^T E_1\ell_2 + \ell_4^T Q_2\ell_4 \\
&\quad + e_2^T\mathcal{R}_1 e_2 - 2\hat{\mathbb{W}}e_{10}, \\
\Phi_2 &= 2\ell_1^T\mathbb{N}E_2^T e_1 + 2\ell_3\mathbb{N}E_2^T E_2\ell_2 - \ell_4^T Q_2\ell_4 \\
&\quad + e_2^T\mathcal{R}_1 e_2 - 2\hat{\mathbb{L}}e_{10},
\end{aligned}$$

$\ell_1 = col\{e_1, e_3, e_4, e_7, e_8\}, \ell_2 = col\{e_3 - e_1, e_1 - e_4\}$,
$\ell_3 = col\{e_2, 0, 0, e_1, (E_1 - E_2)e_9, (-E_1 + E_2)e_{10}\}$,
$\ell_4 = col\{e_3, e_4, e_6\}, \mathbb{R}_i = diag\{\tilde{\mathcal{R}}_i, 3\tilde{\mathcal{R}}_i, 5\tilde{\mathcal{R}}_i\}(i = 1, 2)$,
$\Pi_1 = col\{e_1, e_3, E_1e_9, E_2e_9\}, \mathcal{E}_n = [\mathcal{E}_c, \mathcal{E}_d]$,
$\Pi_2 = col\{e_4, e_1, E_1e_{10}, E_2e_{10}\}$,
$\Gamma_1 = [\epsilon_1 I, \epsilon_3 I, 0, \epsilon_2 I, \underbrace{0, \cdots, 0}_{11}]$,
$\Gamma_2 = [\mathcal{A}, -I, -(1-\check{\varepsilon})L_2\mathcal{H}, 0, 0, -\check{\varepsilon}L_1\mathcal{H}, \underbrace{0, \cdots, 0}_{8}, \mathcal{B}]$,
$\Gamma_3 = [\Delta\mathcal{A}, \underbrace{0, \cdots, 0}_{13}, \Delta\mathcal{B}]$.

Moreover, the SDC matrices are given as $\mathcal{K} = G^{-1}L_1$ and $\mathcal{T} = G^{-1}L_2$.

Proof: For the purpose of theoretical analysis, we select the following LKF:

$$\mathcal{V}(t) = \sum_{r=1}^{6}\mathcal{V}_r(t), \tag{18}$$

where

$$\begin{aligned}
\mathcal{V}_1(t) &= e^T(t)Pe(t) \\
&\quad + 2\sum_{i=1}^{n_q}\int_0^{\mathcal{C}_\sigma e(t)}[\lambda_i(\omega_i^+ s - g_i(s)) + \delta_i(g_i(s) - \omega_i^- s)]ds, \\
\mathcal{V}_2(t) &= \int_{t-\underline{\tau}}^{t} e^T(s)S_1e(s)ds + \underline{\tau}\int_{t-\underline{\tau}}^{t}\int_{t+\theta}^{t}\dot{e}^T(s)S_2\dot{e}(s)dsd\theta, \\
\mathcal{V}_3(t) &= 2\eta_1^T(t)\mathbb{N}\begin{bmatrix}(t_{k+1} - t)(e(t) - e(t_k)) \\ (t - t_k)(e(t) - e(t_{k+1}))\end{bmatrix}, \\
\mathcal{V}_4(t) &= 2[e(t) - e(t_k)]^T Q_1[e(t) - e(t_{k+1})], \\
\mathcal{V}_5(t) &= (t_{k+1} - t)(t - t_k)\eta_2^T(t_k)Q_2\eta_2(t_k), \\
\mathcal{V}_6(t) &= (t_{k+1} - t)\int_{t_k}^{t}\dot{e}^T(s)\tilde{\mathcal{R}}_1\dot{e}(s)ds \\
&\quad - (t - t_k)\int_{t}^{t_{k+1}}\dot{e}^T(s)\tilde{\mathcal{R}}_2\dot{e}(s)ds.
\end{aligned}$$

In functional (18), $\mathcal{V}_n(t)$(n=3,4,5,6) make full use of information about the intervals t_k to t and t to t_{k+1} and satisfy the looped-functional condition $\mathcal{V}_n(t_k) = \mathcal{V}_n(t_{k+1}) = 0$.

$\check{\mathcal{L}}$ is defined as a weak infinitesimal operator, then

$$\begin{aligned}
\check{\mathcal{L}}\mathcal{V}_1(t) &= 2e^T(t)P\dot{e}(t) + 2e^T(t)\mathcal{C}^T(\mathcal{W}_1\Lambda_1 - \mathcal{W}_2\Lambda_2)\mathcal{C}\dot{e}(t) \\
&\quad + 2g^T(\mathcal{C}e(t))(\Lambda_2 - \Lambda_1)\mathcal{C}\dot{e}(t), \\
\check{\mathcal{L}}\mathcal{V}_2(t) &= e^T S_1 e(t) - e^T(t - \underline{\tau})S_1 e(t - \underline{\tau}) + \underline{\tau}^2\dot{e}^T(t)S_2\dot{e}(t) \\
&\quad - \underline{\tau}\int_{t-\underline{\tau}}^{t}\dot{e}^T(s)S_2\dot{e}(s)ds, \\
\check{\mathcal{L}}\mathcal{V}_3(t) &= 2\eta_1^T(t)\mathbb{N}\begin{bmatrix}(t_{k+1} - t)\dot{e}(t) - (e(t) - e(t_k)) \\ (t - t_k)\dot{e}(t) + (e(t) - e(t_{k+1}))\end{bmatrix} \\
&\quad + 2\dot{\eta}_1^T(t)\mathbb{N}\begin{bmatrix}(t_{k+1} - t)(e(t) - e(t_k)) \\ (t - t_k)(e(t) - e(t_{k+1}))\end{bmatrix}, \\
\check{\mathcal{L}}\mathcal{V}_4(t) &= 2\dot{e}^T(t)Q_1[e(t) - e(t_{k+1})] + 2[e(t) - e(t_k)]^T Q_1\dot{e}(t), \\
\check{\mathcal{L}}\mathcal{V}_5(t) &= [(t_{k+1} - t) - (t - t_k)]\eta_2^T(t_k)Q_2\eta_2(t_k), \\
\check{\mathcal{L}}\mathcal{V}_6(t) &= \dot{e}(t)[(t_{k+1} - t)\tilde{\mathcal{R}}_1 + (t - t_k)\tilde{\mathcal{R}}_2]\dot{e}(t) \\
&\quad - \int_{t_k}^{t}\dot{e}^T(s)\tilde{\mathcal{R}}_1\dot{e}(s)ds - \int_{t}^{t_{k+1}}\dot{e}^T(s)\tilde{\mathcal{R}}_2\dot{e}(s)ds.
\end{aligned}$$

Applying the Lemma 3, it is easy to get:

$$\begin{aligned}
\underline{\tau}\int_{t-\underline{\tau}}^{t}\dot{e}^T(s)S_2\dot{e}(s)ds &\geq \int_{t-\underline{\tau}}^{t}\dot{e}^T(s)dsS_2\int_{t-\underline{\tau}}^{t}\dot{e}(s)ds \\
&= (e(t) - e(t - \underline{\tau}))^T S_2(e(t) - e(t - \underline{\tau})).
\end{aligned} \tag{19}$$

By using (10) and (12), one can get

$$
\begin{aligned}
&-\int_{t_k}^{t} \dot{e}^T(s)\tilde{\mathcal{R}}_1\dot{e}(s)ds \\
&\leq \xi_N^T(t)[Sym\{\Pi_1^T\Theta^T\mathbb{M}\} + (t-t_k)\mathbb{M}^T\mathbb{R}_1^{-1}\mathbb{M}]\xi_N(t), \quad (20)\\
&-\int_{t}^{t_{k+1}} \dot{e}^T(s)\tilde{\mathcal{R}}_2\dot{e}(s)ds \\
&\leq \xi_N^T(t)[Sym\{\Pi_2^T\Theta^T\mathbb{Y}\} + (t_{k+1}-t)\mathbb{Y}^T\mathbb{R}_2^{-1}\mathbb{Y}]\xi_N(t). \quad (21)
\end{aligned}
$$

Now, for any adaptive dimension matrix G and scalars $\epsilon_1, \epsilon_2, \epsilon_3$, we may acquire with ease:

$$
\begin{aligned}
0 &= 2(\epsilon_1 e^T(t) + \epsilon_2 e^T(t_k) + \epsilon_3\dot{e}^T(t))G(-\dot{e}(t) + \bar{\mathcal{A}}_\sigma e(t) \\
&\quad + \bar{\mathcal{B}}_\sigma g(\mathcal{C}e(t)) - \breve{\varepsilon}(t)\mathcal{K}\mathcal{H}e(t_k - \underline{\tau}) \\
&\quad - (1-\breve{\varepsilon}(t))\mathcal{T}\mathcal{H}e(t_k) - \mathcal{D}\omega(t)) \\
&= 2\xi_N^T(t)(\Gamma_1^T\Gamma_2 + \Gamma_1^T G\mathcal{J}\hat{\Delta}(t)\mathcal{E}_n)\xi_N(t) \\
&\quad - 2\xi_N^T(t)\Gamma_1^T G\mathcal{D}\omega(t). \qquad (22)
\end{aligned}
$$

Based on $-2\tilde{\mathcal{X}}^T\tilde{\mathcal{Y}} \leq \tilde{\mathcal{X}}^T\mathcal{Z}\tilde{\mathcal{X}} + \tilde{\mathcal{Y}}^T\mathcal{Z}^{-1}\tilde{\mathcal{Y}}$ for any $\mathcal{Z} > 0$, the last term of (22) could be described as:

$$
\begin{aligned}
&-2\xi_N^T(t)\Gamma_1^T G\mathcal{D}\omega(t) \\
&\leq \gamma^2\omega^T(t)\omega(t) + \gamma^{-2}\xi_N^T(t)\Gamma_1^T G\mathcal{D}\mathcal{D}^T G^T\Gamma_1\xi_N(t). \quad (23)
\end{aligned}
$$

For any adaptive dimension matrices $\hat{\mathbb{L}}$ and $\hat{\mathbb{W}}$, we may acquire:

$$
0 = 2\xi_N^T(t)\hat{\mathbb{L}}[e_7 - (t-t_k)e_9]\xi_N(t), \quad (24)
$$
$$
0 = 2\xi_N^T(t)\hat{\mathbb{W}}[e_8 - (t_{k+1}-t)e_{10}]\xi_N(t). \quad (25)
$$

For any $U = diag\{u_1, \cdots, u_{n_h}\} \geq 0$, the below inequality holds,

$$
2[e^T(t)\mathcal{C}^T\mathcal{W}_1 - g^T(\mathcal{C}e(t))]U[g(\mathcal{C}e(t)) - \mathcal{W}_2\mathcal{C}e(t)] \geq 0. \quad (26)
$$

Combining $\breve{\mathcal{L}}\mathcal{V}(t)$ with (19)–(26), one can get:

$$
\begin{aligned}
&\mathbf{E}\{\breve{\mathcal{L}}\mathcal{V}(t)\} + \mathbf{E}\{e^T(t)Se(t)\} - \mathbf{E}\{\gamma^2\omega^T(t)\omega(t)\} \\
&\quad \leq \xi_N^T(t)\Psi(t)\xi_N(t), \qquad (27)
\end{aligned}
$$

where

$$
\begin{aligned}
\Psi(t) &= \Psi_0 + (t_{k+1}-t)\Psi_1 + (t-t_k)\Psi_2, \\
\Psi_0 &= \Phi_0 + 2\Gamma_1^T G\mathcal{J}\Delta(t)\mathcal{E}_n + \gamma^{-2}\Gamma_1 GDD^T G^T\Gamma_1, \\
\Psi_1 &= \Phi_1 + \mathbb{Y}^T\mathbb{R}_2^{-1}\mathbb{Y}, \\
\Psi_2 &= \Phi_2 + \mathbb{M}^T\mathbb{R}_1^{-1}\mathbb{M}.
\end{aligned}
$$

Since $\Psi(t) < 0$, one may draw a conclusion from (27)

$$
\mathbf{E}\{\breve{\mathcal{L}}\mathcal{V}(t)\} + \mathbf{E}\{e^T(t)Se(t)ds\} - \gamma^2\mathbf{E}\{\omega^T(t)\omega(t)ds\} < 0. \quad (28)
$$

Integrating on both sides of (28) from 0 to ∞, it may be derived that

$$
\begin{aligned}
&\mathcal{V}(\infty) - \mathcal{V}(0) \\
&< \gamma^2\mathbf{E}\Big\{\int_0^\infty \omega^T(t)\omega(t)ds\Big\} - \mathbf{E}\Big\{\int_0^\infty e^T(t)Se(t)ds\Big\}. \quad (29)
\end{aligned}
$$

Under the zero-initial condition, (29) guarantees that

$$
\mathbf{E}\Big\{\int_0^\infty e^T(t)Se(t)ds\Big\} < \gamma^2\mathbf{E}\Big\{\int_0^\infty \omega^T(t)\omega(t)ds\Big\}. \quad (30)
$$

So that H_∞ performance is formed. This completes the proof.

Remark 3: In contrast to the present literature, a novel two-sided looped-function is formulated in (18). This functional makes the most of the information from t_k to t and t to t_{k+1}. Consequently, our result is believed to produce less conservatism.

Remark 4: Recently, new stability conditions of networked systems under denial-of-service attack was proposed in [34], in which the affine BLIY was used for handling the quadratic integral term. Inspired by the above literature, Lemma 2 is used for handling the term -$\int_{t_k}^{t}\dot{e}^T(s)\tilde{\mathcal{R}}_1\dot{e}(s)ds$ and -$\int_{t}^{t_{k+1}}\dot{e}^T(s)\tilde{\mathcal{R}}_2\dot{e}(s)ds$.

Remark 5: As it is known from [26], the conservatism of the stability results is not significantly enhanced by applying the Wirtinger inequality and BLIY. Therefore, we structure the $\mathcal{V}_3(t)$. $\mathcal{V}_3(t)$ makes the most of the information about the second order BLIY. It makes the vector in the BLIY emerge in the derivative of LKF, thus avoiding the problem of creating the same conservative condition as using the Wirtinger inequality.

When the uncertainty is not taken into account in systems (1), systems (1) can be rewritten as

$$
\begin{aligned}
&\mathcal{M}: \begin{cases} \dot{x}(t) = \mathcal{A}x(t) + \mathcal{B}f(\mathcal{C}x(t)), \\ \mathfrak{p}(t) = \mathcal{H}x(t), \end{cases} \\
&\mathcal{F}: \begin{cases} \dot{y}(t) = \mathcal{A}y(t) + \mathcal{B}f(\mathcal{C}y(t)) + u(t) + \mathcal{D}\omega(t), \\ \mathfrak{q}(t) = \mathcal{H}y(t). \end{cases} \qquad (31)
\end{aligned}
$$

Therefore, the corresponding error system is represented as

$$
\begin{aligned}
\dot{e}(t) &= \mathcal{A}e(t) + \mathcal{B}g(\mathcal{C}e(t)) - \mathcal{K}\mathcal{H}\breve{\varepsilon}(t)e(t_k - \underline{\tau}) \\
&\quad - \mathcal{T}\mathcal{H}(1-\breve{\varepsilon}(t))e(t_k) - \mathcal{D}\omega(t). \qquad (32)
\end{aligned}
$$

Based on Theorem 1, the relevant criterion is given.

Theorem 2: For given scalars $\hat{h}_m > 0$, $\underline{\tau} > 0$, $\breve{\varepsilon}$, ϵ_1, ϵ_2 and ϵ_3, the system (32) can achieve robustly $\mathcal{H}_\infty$ synchronous for any $h_k \in (0, h]$, if there exists $P > 0$, $S > 0$, $S_1 > 0$, $S_2 > 0$, $\tilde{\mathcal{R}}_1 > 0$, $\tilde{\mathcal{R}}_2 > 0$, Q_1, $Q_2 = Q_2^T$, $\Lambda_1 = diag\{\lambda_1, \cdots, \lambda_{n_q}\} > 0$, $\Lambda_2 = diag\{\delta_1, \cdots, \delta_{n_q}\} > 0$, $U = diag\{u_1, \cdots, u_{n_q}\}$, any suitable dimensional matrices J, G, L_1, L_2, $\hat{\mathbb{L}}$, $\hat{\mathbb{W}}$, $\mathbb{N}$, $\mathbb{M}$, $\mathbb{Y}$, and scalars $\sigma > 0$, γ such that

$$
\begin{bmatrix} \Phi_0 & \Gamma_1^T G\mathcal{D} \\ * & -\gamma^2 I \end{bmatrix} < 0, \quad (33)
$$

$$
\begin{bmatrix} \Phi_0 + h\Phi_1 & \Gamma_1^T G\mathcal{D} & \hat{h}_m\mathbb{Y}^T \\ * & -\gamma^2 I & 0 \\ * & * & -\hat{h}_m\mathbb{R}_2 \end{bmatrix} < 0, \quad (34)
$$

$$
\begin{bmatrix} \Phi_0 + h\Phi_2 & \Gamma_1^T G\mathcal{D} & \hat{h}_m\mathbb{M}^T \\ * & -\gamma^2 I & 0 \\ * & * & -\hat{h}_m\mathbb{R}_1 \end{bmatrix} < 0, \quad (35)
$$

where the notations can be found in Theorem 1. In addition, the SDC matrices are given as $\mathcal{K} = G^{-1}L_1$ and $\mathcal{T} = G^{-1}L_2$.

When system (32) without the stochastic variable $\underline{\beta}(t)$ and disturbance, an error model can be formulated as:

$$\dot{e}(t) = \mathcal{A}e(t) + \mathcal{B}g(\mathcal{C}e(t)) - \mathcal{T}\mathcal{H}e(t_k). \tag{36}$$

Corollary 1: System (36) can successfully synchronize for any $\hat{h}_k \in (0, \hat{h}_m]$, if there exists $P > 0, \tilde{\mathcal{R}}_1 > 0, \tilde{\mathcal{R}}_2 > 0$, $Q_1, Q_2 = Q_2^T, \Lambda_1 = diag\{\lambda_1, \cdots, \lambda_{n_q}\} > 0, \Lambda_2 = diag\{\delta_1, \cdots, \delta_{n_q}\} > 0, U = diag\{u_1, \cdots, u_{n_q}\}$, for given scalars $h > 0, \epsilon_1, \epsilon_2$ and ϵ_3, any suitable dimensional matrices $G, L_2, \mathbb{L}, \hat{\mathbb{W}}, \mathbb{N}, \mathbb{M}, \mathbb{Y}$, such that the below LMIs hold,

$$\tilde{\Phi}_0 < 0, \tag{37}$$

$$\begin{bmatrix} \tilde{\Phi}_0 + h\tilde{\Phi}_1 & \hat{h}_m\mathbb{Y}^T \\ * & -\hat{h}_m\mathbb{R}_2 \end{bmatrix} < 0, \tag{38}$$

$$\begin{bmatrix} \tilde{\Phi}_0 + h\tilde{\Phi}_2 & \hat{h}_m\mathbb{M}^T \\ * & -\hat{h}_m\mathbb{R}_1 \end{bmatrix} < 0, \tag{39}$$

where

$$\begin{aligned}
\tilde{\Phi}_0 &= 2e_1^TPe_2 + 2e_1^T\mathcal{C}^T(\mathcal{W}_1\Lambda_1 - \mathcal{W}_2\Lambda_2)\mathcal{C}e_2 + e_1Se_1 \\
&\quad + 2e_{11}^T(\Lambda_2 - \Lambda_1)\mathcal{C}e_2 + 2e_2^TQ_1E_2\ell_2 - 2e_2^TQ_1^TE_1\ell_2 \\
&\quad + 2\ell_1^T\mathbb{N}\ell_2 + 2(\Pi_1^T\Theta^T\mathbb{M} + \Pi_2^T\Theta^T\mathbb{Y}) - 2e_{11}^TUe_{11} \\
&\quad + 2e_1^T\mathcal{C}^T(W_1 + W_2)Ue_{11} - e_1^T\mathcal{C}^TW_1UW_2\mathcal{C}e_1 \\
&\quad + 2\hat{\mathbb{L}}e_7 + 2\hat{\mathbb{W}}e_8 + 2\hat{\Gamma}_1^T\hat{\Gamma}_2^T, \\
\tilde{\Phi}_1 &= 2\ell_1^T\mathbb{N}E_1^Te_1 - 2\ell_3\mathbb{N}E_1^TE_1\ell_2 + \tilde{\ell}_4^TQ_2\tilde{\ell}_4 + e_2^T\mathcal{R}_1e_2 \\
&\quad - 2\hat{\mathbb{W}}e_{10}, \\
\tilde{\Phi}_2 &= 2\ell_1^T\mathbb{N}E_2^Te_1 + 2\ell_3\mathbb{N}E_2^TE_2\ell_2 - \tilde{\ell}_4^TQ_2\tilde{\ell}_4 + e_2^T\mathcal{R}_1e_2 \\
&\quad - 2\hat{\mathbb{L}}e_{10}, \\
\hat{\Gamma}_1 &= [\epsilon_1I, \epsilon_3I, 0, \epsilon_2I, \underbrace{0, \cdots, 0}_{9}], \\
\hat{\Gamma}_2 &= [\mathcal{A}, -I, L_2H, \underbrace{0, \cdots, 0}_{9}, \mathcal{B}], \\
\tilde{\ell}_4 &= col\{e_3, e_4\}, \eta_3(t_k) = col\{e(t_k), e(t_{k+1})\}.
\end{aligned}$$

The other notations can be found in Theorem 1. Moreover, the SDC matrices are given as $\mathcal{T} = G^{-1}L_2$.

The LKF can be structured from (18) as shown below:

$$\mathcal{V}(t) = \sum_{r=1}^{l}\mathcal{V}_r(t)(l = 1, 3, 4, 6, 7), \tag{40}$$

where $\mathcal{V}_7(t) = (t_{k+1} - t)(t - t_k)\eta_3^T(t_k)S_2\eta_3(t_k)$.

Proof: The proving process is skipped, and the method is similar to Theorem 1.

IV. NUMERICAL EXAMPLES

In this section, we provide a numerical example to demonstrate the effectiveness of the proposed method and the determined sufficient conditions.

Consider the nonlinear CLs with the following parameters:

$$\mathcal{A} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \mathcal{B} = \begin{bmatrix} 1.2 & -1.6 & 0 \\ 1.24 & 1 & 0.9 \\ 0 & 2.2 & 1.5 \end{bmatrix},$$

$$\mathcal{C} = \mathcal{H} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The activation function $f(x_i(t)) = \frac{1}{2}(|x_i(t) + 1| - |x_i(t) - 1|), i = 1, 2, 3$ belongs to sector $[0, 1]$. By setting $\epsilon_1 = 1.4, \epsilon_2 = 1.3, \epsilon_3 = 1.5$ and solving the conditions in Corollary 1, we can obtain $\hat{h}_m = 0.9845$.

Then, the corresponding controller gain matrix is determined by $\mathcal{T} = G^{-1}L_2$, as follows:

$$\mathcal{T} = \begin{bmatrix} 1.4691 & -0.7892 & -0.4439 \\ 0.5301 & 1.6519 & 0.4431 \\ -0.0768 & 1.4064 & 2.1819 \end{bmatrix}.$$

The LUB of the sampling interval acquired by Corollary 1 is listed in Table 1 with the various literature.

TABLE 1. LUB of sampling interval $\hat{h}_m$.

Methods	$\hat{h}_m$
[14]	0.6966
[35]	0.9121
[10]	0.9140
[4]	0.9195
[2]	0.9375
Corollary 1	0.9845

By comparing the data measured in Table 1 above, it can be observed that the sampling interval LUB acquired in Corollary 1 is significantly larger than the methods in [2],

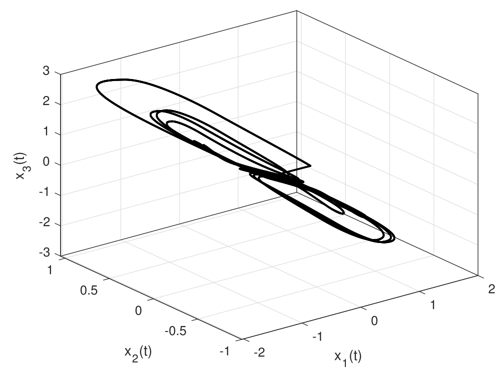

FIGURE 1. Chaotic behavior of primary system $\mathcal{M}$.

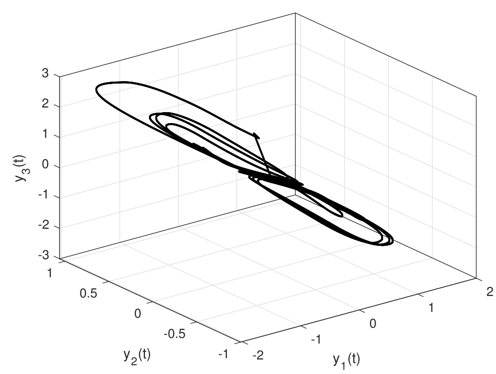

FIGURE 2. Chaotic behavior of secondary system $\mathcal{F}$.

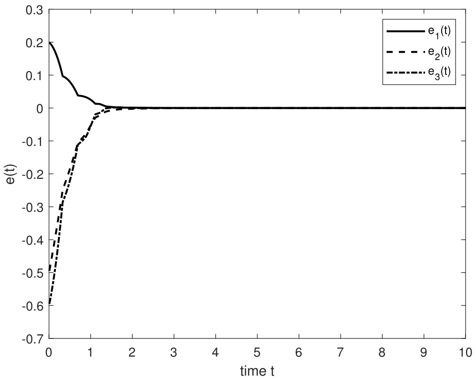

FIGURE 3. Responses of error e(t).

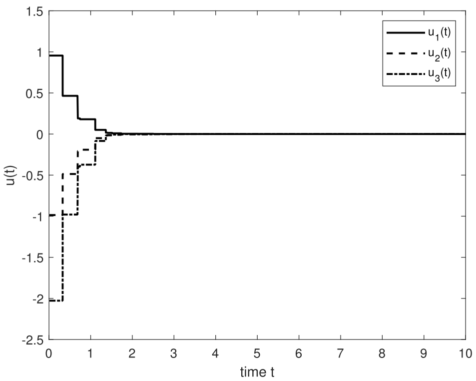

FIGURE 4. Responses of controller u(t).

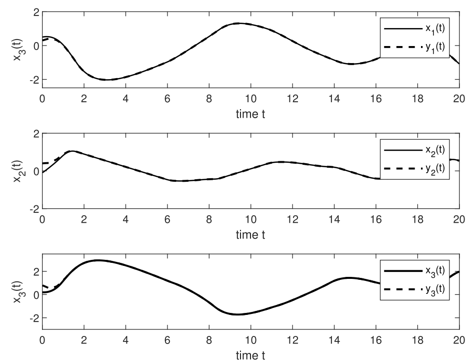

FIGURE 5. State response of x(t) and y(t).

[4], [10], and [35], arguing for the superiority of the algorithm proposed in this article.

By choosing the initial state value x(0)=$[0.4, 0.3, 0.8]^T$ and y(0)=$[0.2, 0.4, 0.9]^T$ for the primary and secondary systems, respectively, the state curves in FIGURE 1 and FIGURE 2 can be plotted based on the system characteristics and initial values. Under the designed controller, FIGURE 3 and FIGURE 4 delineate the trajectory of the error model and the trajectory of the controller, respectively. The responses of the state $x(t)$, $y(t)$ are displayed in FIGURE 5. As we can see in FIGURE 3, the errors are tending to 0, which represents that the proposed algorithm can fulfill the primary-secondary system synchronization.

V. CONCLUSION

In this work, the H_∞ synchronization issue is analyzed for CLs with parameter uncertainties and external distrubance. By considering the signal transmission delay and Bernoulli sequence, a coupling memory SDC strategy that involves time delay effect is derived. Besides, an augmented LKF is constructed by using the information of the sampling interval, nonlinear function and the vector in the affine B-L inequality. In this way, less conservative stabilization criteria of error system under H_∞ performance are acquired. Finally, the simulation example is given to demonstrate the superiority and validity of the theoretical algorithm. Note that the future research topics including memristive neural networks with time-varying delays and practical applications.ddd

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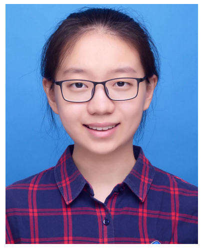

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