IEEE Access
Multidisciplinary ⁝ Rapid Review ⁝ Open Access Journal

Received 22 January 2024, accepted 2 February 2024, date of publication 6 February 2024, date of current version 13 February 2024.

Digital Object Identifier 10.1109/ACCESS.2024.3362826

RESEARCH ARTICLE

Evolutionary Game Analysis on Safety Supervision for Coal Mine Considering Speculative Behavior

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This work was supported in part by the National Natural Science Foundation of China under Grant 51704140, in part by the Natural Science Foundation of Liaoning Province under Grant 2021-MS-340, and in part by the Foundation of the Education Department of Liaoning Province under Grant LJKZ0347.

ABSTRACT This paper considers the speculative behavior of coal mining enterprises and regulatory authorities, and uses evolutionary game theory to study safety supervision in production process of coal mine. It describes the game strategies of coal mining enterprises and regulatory authorities as safety production investment and safety supervision investment respectively. The four game scenarios are defined based on the different speculations of the two parties. The evolutionary game model is established and solved. The evolutionary game analysis is performed, and the conditions that the system evolves to four stable points are obtained. The numerical simulations are used to further analyze the influence of the parameters. The results show that when the parameters meet different conditions, the system evolves to four different stable points. The smaller the number of individuals in the regulatory authorities who choose speculative strategy, the more favorable it is for coal mining enterprises to make normal investment. The more underestimated the supervision power of the regulatory authorities, the more inclined the coal mining enterprises are to adopt speculative strategies. The higher the regulatory authorities' assessment of the safety investment of coal mining enterprises, the more inclined regulatory authorities are to adopt speculative strategies.

INDEX TERMS Evolutionary game, coal mining production, investment level, safety supervision, speculative behavior.

I. INTRODUCTION

Safety work plays a pivotal role in the production process of coal mines, and the safety of coal mining production is related to the national economy and people's livelihood. Due to the huge demand for coal and the harsh environment of coal mining, the safety production and supervision of coal mines is a prominent problem and challenge [1]. The unsafe production behavior of coal mining enterprises and the ineffective supervision behavior of regulatory authorities will lead to frequent occurrence of coal mining production accidents [2]. Coal mine safety issues are prevalent in major coal producing countries, including China [3], [4], the United States [5], Spain [6], etc. Therefore, it is of great significance to study the safety production incentives of coal mining enterprises and the effective supervision mechanism of regulatory authorities.

The safety production of coal mines have received extensive attention from academia and industry [7], [8], [9], [10], [11]. Zhang et al. [12] identified the characteristics of safety culture deficiencies driving typical coal accidents by using the accident analysis pathway of the 24Model. They emphasized the role of departments, safety communication, safety participation and supervision climate

The associate editor coordinating the review of this manuscript and approving it for publication was Gongbo Zhou.

in influencing and improving the safety culture to further reduce industrial accidents. Yu et al. [13] considered the unsafe behavior of coal miners, and established an index system of influencing factors of unsafe behavior of miners and a system dynamics model of unsafe behavior of coal miners. Liu et al. [14] used evolutionary game theory to analyze the safety partner management system of coal mining enterprises, and the research results showed that increasing rewards and punishments for miners would be beneficial in guiding them to take up safe behavior. Gao et al. [15] applied the analysis process of the Functional Resonance Analysis Method to the Chinese safety regulatory system to classify and evaluate governmental safety regulatory functions. They argued the potential performance variability in the outputs of regulatory functions influenced other functions through up-down coupling which can lead to major accidents.

Ever since the mathematical framework of game theory was applied to evolution, the research on cooperation attracted the attention of fields as varied as biology, psychology, economics, physics, and others [16], [17]. Coal mine safety supervision and game related issues have received more and more attention [18], [19]. Chen et al. [20] constructed the recognition game model, analyzed the influence of comprehensive output attribute, conditional output attribute and safety supervision authority on coal mine safety output. Liu et al. [21] found the safety regulatory authorities and mining enterprises have rent-seeking behavior under the condition of fluke mentality or interest balance, which may lead to the occurrence of accidents. Chen et al. [22] analyzed the tracking mechanism using the dynamic game model with incomplete information, and indicated that the probability of national supervision is influenced by penalties and bribery. Chen et al. [23] simulated the impact of rent-seeking from each level of the regulatory authorities on coal mining productivity in different scenarios. They found rent-seeking had no significant influence on the average level of material productivity but it had an adverse effect on the average level of mental productivity. Li et al. [24] thought that under the mode of on-site supervision, increasing safety investment, that is, increasing the supervision and punishment of coal mining enterprises, can urge enterprises to produce safety.

In the process of coal mine safety production and supervision, coal mining enterprises and regulatory authorities often have bounded rational behavior [25]. Capraro and Perc [26] demonstrated the importance of statistical physics methods in studying cooperative evolution in social dilemma games. Cheng et al. [27] used a multi-agent modeling and simulation method to construct a dynamic evolutionary model of a coal mine safety system based on complex adaptive system theory. The authors found safety leadership, safety management, behavioral safety all directly or indirectly affected system safety. You et al. [28] found that increasing the static reward and punishment intensity can quickly reduce unsafe behavior ratio but increase the fluctuation in the game. Liu et al. [29] explored the use of evolutionary game theory to describe the interactions between the stakeholders in China's coal mining safety inspection system, and explored dynamic simulations of the evolutionary game model to analyze the stability of stakeholder interactions and to identify equilibrium solutions. Liu et al. [30] explored the use of evolutionary game theory to describe the long term dynamic process of multi-player game playing in coal-mine safety regulation under the condition of bounded rationality. Enterprises and regulatory authorities may not conduct safety production supervision due to overconfidence in the safety production level of enterprises or the risk of luck in the loss of safety accidents in coal mines [31]. Therefore, this paper uses normality and speculation to describe the evolutionary game strategy of coal mining enterprises and regulatory authorities.

Safety investment/effort is an important way to improve the safety of coal mining production. Winkler et al. [32] presented a game theory model that captures the decision processes of a manager and an employee with regard to the reporting of a near-miss event for reducing the likelihood of a future accident. Ma and Zhao [33] analyzed the interaction between the government's safety regulation efforts and a company's safety efforts, based on a case in China. He and Qin [34] considered factors such as gambling psychology and supervisory intensity. They found that the safety effort behavior (increasing the supervision frequency) can stabilize the system in the optimal state. Therefore, it is of great significance to continuously describe the safety production behavior of coal mining enterprises and the supervision behavior of regulatory authorities. In fact, continuous characterization of human behavior in games has achieved research results in many other fields [35], [36], [37]. This paper uses two continuous variables to describe the safety production behavior of coal mining enterprises and the supervision behavior of regulatory authorities, that is, the safety investment of coal mining enterprises and the supervision investment of regulatory authorities. The safety investment of coal mining enterprises consists of those costs incurred by safety personnel, safety equipment and facilities, compulsory safety training courses, and so on [38], [39].

The two main contributions of this manuscript are as follows. On the one hand, this paper continuously describes the safety production behavior of coal mining enterprises and the supervision behavior of the regulatory authority, and defines the safety probability of coal mining production and the probability of problems found in supervision. On the other hand, the speculative behavior of coal mining enterprises and regulatory authorities is introduced, and an evolutionary game model with speculative behavior is established and solved. The effect of speculative behavior on evolutionary stable strategies is analysed. The position of this study is presented in Table 1.

The remainder of this paper is organized as follows. Section II describes the problem and assumes models and symbols. Section III gives the evolutionary game model and analyzes the optimal results of investment level and profit. Section IV establishes and solves replication dynamic equations, and then the stability analysis is carried out.

TABLE 1. Position of this study.

References	Supervision game	Bounded rational behavior	Safety investment/effort
Chen et al. [20]	√		
Chen et al. [22]	√		
Chen et al. [23]	√		
Cheng et al. [27]		√	
Liu et al. [29]	√	√	
Liu et al. [30]	√	√	
Liu et al. [31]	√	√	
Winkler et al. [32]			√
Ma and Zhao [33]	√		√
He and Qin [34]	√	√	√
This study	√	√	√

Section V gives the evolution process of the stable points under different initial values. Section VI presents the conclusion of this paper and future research directions.

II. PROBLEM DESCRIPTION AND ASSUMPTIONS

Coal mining enterprises and regulatory authorities are two groups of players in the evolutionary game of safety supervision of coal mine. Both sides of the game have normal and speculative strategies. The coal mining enterprises who hold normal strategy put the the normal safety investment into the production process of coal mine. When coal mining enterprises are speculative, they will not make safety investments. When regulatory authorities are speculative, they will not make regulatory investments. Speculation in this paper is narrowly defined. In coal mining, the normal strategy is to use investment to achieve a safety project, while the speculative strategy is to abandon the project. The normal strategy of the regulatory authorities refers to the normal supervision on the coal mining enterprises. The regulatory authorities who hold speculative strategy do not conduct safety supervision because they are overconfident in the safety status of the coal mine. Overconfidence is a limited rational behavior of regulators, which may stem from two beliefs. On the one hand, regulators trust coal mining enterprises to make safe investments. On the other hand, regulators are confident that safety accidents will not occur in coal mines, regardless of whether coal mining enterprises make safety investments.

Without loss of generality, the following assumptions are made for facilitating model construction and result interpretation:

1. The safety production effort of coal mining enterprises can be measured by safety investment, and the supervision effort of regulatory authorities can be measured by supervision investment;
2. The safety probability of coal mine depends on the safety investment level of coal mining enterprises. The higher the level of safety investment, the greater the safety probability;
3. The probability that the supervisory authorities find that coal mining enterprises have safety problems depends on supervisory investment and the safety probability of coal mine;
4. The speculative behavior makes coal mining enterprises underestimate the safety supervision investment level of the regulatory authorities, and the regulatory authorities overestimate the safety investment level of coal mining enterprises. At the same time, both parties will underestimate the losses caused by the accident of coal mine.

The coal mine is safe when there are no coal mine accidents. Therefore, the safety probability of coal mine is equivalent to the probability of no safety accidents occurring in coal mine in a certain sense. The occurrence of coal mine accidents usually follows a Poisson distribution. Correspondingly, the interval time between coal mine accidents follows an exponential distribution. In this paper, safety investment is used to increase the probability of safety. The greater the investment in safety, the higher the probability of safety. Then, the number of safety accidents that occur in coal mine within a certain time range is reduced, and the time interval between accidents is extended. Variants of exponential distribution is used as a continuous describer in this paper. The definition of safety probability of coal mine reflects the practice of coal mine production and also facilitates the mathematical processing of the model. The safety investment of coal mining enterprises and the regulatory investment of regulatory authorities are denoted by z_1 and z_2, respectively. Therefore, the safety probability of coal mine is defined as $p_1 = 1 - e^{-k_1 z_1}$, where k_1 is safety characteristic coefficient of coal mine. The probability of unsafe production in coal mine is $1 - p_1$. When the safety investment is 0, the safety probability is 0. Then, the safety accidents of coal mine are bound to occur.

The probability that regulatory authorities find a safety problem in coal mine is defined as $p_2 = (1 - p_1)(1 - e^{-k_2 z_2})$, where k_2 is capability characteristic coefficient of regulatory authorities. It can be seen that the greater the regulatory investment, the greater the probability of finding problems in the supervision. When the safety probability of coal mine is 1, no matter how the regulatory authorities supervise, the supervision will not find any problem. When the safety probability is 0, the probability of finding problems in the supervision depends entirely on the level of regulatory investment.

Due to the speculative behavior of coal mining enterprises and regulatory authorities, they may underestimate or overestimate the investment level of the other party and the losses caused by coal mining accidents, respectively. In this paper, we use λ_i to describe the coefficient of speculative behavior, $i = 1, 2, 3, 4$. Specifically, the degree of underestimation of the regulatory investment by coal mining enterprises is recorded as λ_1, and $\lambda_1 \in (0, 1)$. The degree of overestimation of the safety investment of coal mining enterprises by the regulatory authorities is recorded as λ_2, and $\lambda_2 > 1$. The degrees of underestimation of losses caused by safety accidents by coal mining enterprises and regulatory authorities are recorded as λ_3 and λ_4, $\lambda_3, \lambda_4 \in (0, 1)$.

We consider that the coal mining production will bring benefits to coal mining enterprises and regulatory authorities, which are denoted as A and B, respectively. When regulatory authorities find that coal mining enterprises have safety problems, the fine imposed on coal mining enterprises is recorded as H. When a safety accident occurs in the coal mine, the losses suffered by coal mining enterprises and regulatory authorities are recorded as M and N, respectively.

III. EVOLUTIONARY GAME MODEL

A. PROFIT ANALYSIS

Considering that both coal mining enterprises and regulatory authorities have normal and speculative strategies to choose from, the game problem of coal mine safety supervision is divided into four decision-making scenarios. The four sets of strategy pairs of enterprises and regulators are (*Normal*, *Normal*), (*Speculation*, *Normal*), (*Normal*, *Speculation*) and (*Speculation*, *Speculation*). Let i denotes the decision-making scenario, $i = 1, 2, 3, 4$. Let j denotes decision maker, $j = 1$ denotes coal mining enterprises, $j = 2$ denotes regulatory authorities. The z_{ij} represents the investment level of the *jth* decision maker under the *ith* decision-making scenario. When coal mining enterprises and regulatory authorities are normal, they will make continuous safety investment to maximize their expected profits. Then, in four decision-making scenarios, the investment of coal mining enterprises and regulatory authorities is either 0 or an optimized determined value. Therefore, the z_{ij} variables can be considered discrete.

Under the scenario of normal safety investment by coal mining enterprises and normal supervision by regulatory authorities (Scenario 1), the profit of coal mining enterprises is as follows:

$$\Pi_1 = A - (1 - p_1)M - z_{11} - p_2 H, \tag{1}$$

and the profit of regulatory authorities under scenario 1 is as follows:

$$\pi_1 = B - (1 - p_1)N - z_{12} + p_2 H. \tag{2}$$

Considering that Π_1 and π_1 are concave with respect to z_{11} and z_{12}, the optimal investment level for both parties can be obtained using first-order conditions. Thus, we can obtain Proposition 1. Proofs of all formal results are presented in a sequence of appendices that conclude the paper.

Proposition 1: Under the scenario of normal safety investment of coal mining enterprises and normal supervision of regulatory authorities, the optimal investment level of coal mining enterprises is $z_{11}^* = \ln(k_1(M + H))/k_1$, and the optimal investment level by regulatory authorities is $z_{12}^* = \ln(k_2 H/(k_1(M+H)))/k_2$. The maximum profit of coal mining enterprises is $\Pi_1^* = A + 1/k_2 - (1 + \ln(k_1(M + H)))/k_1$, and the maximum profit of regulatory authorities is $\pi_1^* = B - (N - H)/(k_1(M + H)) - (1 + \ln((k_2 H)/(k_1 M + k_1 H)))/k_2$.

As can be seen from Proposition 1, to ensure that the optimal investment is not less than 0, we need $1 < k_1(M + H) < k_2 H$. As H and M increase, z_{11}^* becomes larger and larger. For coal mining enterprises, the greater the regulatory penalty or the greater the accident losses, the higher the investment level for coal mine safety. For $\partial z_{11}^*/\partial k_1 > 0$ when $k_1 < e/(M + H)$. This indicates that when the safety characteristic coefficient of coal mining enterprises is small, coal mining enterprises will increase investment with the increase of safety characteristic coefficient. For $\partial z_{12}^*/\partial k_2 > 0$ when $k_2 < k_1 e(M + H)/H$. At this time, z_{12}^* increases with k_2. We also have $\partial z_{12}^*/\partial H > 0$, which shows regulators raise investment level of regulatory as fine rises.

Under the scenario of speculative safety investment of coal mining enterprises and normal supervision of regulatory authorities (Scenario 2), $z_{21} = 0$. Then, the profit of coal mining enterprises is as follows:

$$\Pi_2 = A - (1 - p_1)M - p_2 H, \tag{3}$$

and the profit of regulatory authorities under scenario 2 is as follows:

$$\pi_2 = B - (1 - p_1)N - z_{22} + p_2 H. \tag{4}$$

By solving the above model, we can obtain Proposition 2.

Proposition 2: Under the scenario of speculative safety investment of coal mining enterprises and normal supervision of regulatory authorities, the optimal investment level of coal mining enterprises is $z_{21}^* = 0$, and the optimal investment level by regulatory authorities is $z_{22}^* = \ln(k_2 H)/k_2$. The maximum profit of coal mining enterprises is $\Pi_2^* = A - M - H + 1/k_2$, and the maximum profit of regulatory authorities is $\pi_2^* = B - N - (1 + \ln(k_2 H))/k_2 + H$.

It can be seen from Proposition 2 that z_{22}^* increases with H. This shows that the investment level of regulators increases with the fine. For $\partial z_{22}^*/\partial k_2 < 0$ when $k_2 H > e$. Then, z_{22}^* decreases with k_2. Otherwise, z_{22}^* increases with k_2 when $k_2 H > e$.

Under the scenario of normal safety investment of coal mining enterprises and speculative supervision of regulatory authorities (Scenario 3), $z_{32} = 0$. Then, the profit of coal mining enterprises is as follows:

$$\Pi_3 = A - (1 - p_1)M - z_{31}, \tag{5}$$

and the profit of regulatory authorities under scenario 3 is as follows:

$$\pi_3 = B - (1 - p_1)N. \tag{6}$$

By solving the above model, we can obtain Proposition 3.

Proposition 3: Under the scenario of normal safety investment of coal mining enterprises and speculative supervision of regulatory authorities, the optimal investment level of coal mining enterprises is $z_{31}^* = \ln(k_1 M)/k_1$, and the optimal investment level by regulatory authorities is $z_{32}^* = 0$. The maximum profit of coal mining enterprises is $\Pi_3^* = A - (1 + \ln(k_1 M))/k_1$, and the maximum profit of regulatory authorities is $\pi_3^* = B - N/(k_1 M)$.

It can be seen from Proposition 3 that z^*_{31} increases with M. This indicates that the greater the loss of coal mine safety accidents to coal mining enterprises, the higher the investment level of coal mining enterprises. For $\partial z^*_{31}/\partial k_1 < 0$ when $k_1 M > e$. Then, z^*_{31} decreases with k_1. Otherwise, z^*_{31} increases with k_1 when $k_1 M < e$.

Under the scenario of speculative safety investment of coal mining enterprises and speculative supervision of regulatory authorities (Scenario 4), the investment level of both parties is 0. Then, the profits of coal mining enterprises and regulatory authorities are $\Pi_4 = A - M$ and $\pi_4 = B - N$. By observation, we can directly obtain Proposition 4.

Proposition 4: Under the scenario of speculative safety investment of coal mining enterprises and speculative supervision of regulatory authorities, the optimal investment level of coal mining enterprises is $z^*_{41} = 0$, and the optimal investment level by regulatory authorities is $z^*_{42} = 0$. The maximum profit of coal mining enterprises is $\Pi^*_4 = A - M$, and the maximum profit of regulatory authorities is $\pi^*_4 = B - N$.

B. GAME MODEL

Under the scenario of speculative safety investment of coal mining enterprises and normal supervision of regulatory authorities, coal mining enterprises will underestimate the level of regulatory investment due to speculation. Therefore, coal mining enterprises have a greater utility than actual profit, i.e.,

$$U^*_2 = A - M - (1 - e^{-k_2 \lambda_1 z^*_{22}})H, \tag{7}$$

where λ_1 indicates the degree to which coal enterprises underestimate the level of regulatory investment, and z^*_{22} denotes the actual regulatory investment by the regulator.

Under the scenario of normal safety investment of coal mining enterprises and speculative supervision of regulatory authorities, regulatory authorities will overestimate the safety investment of coal mining enterprises due to speculation. Therefore, regulatory authorities have a greater utility than actual profit, i.e.,

$$u^*_3 = B - e^{-k_1 \lambda_2 z^*_{31}} N, \tag{8}$$

where λ_2 indicates the degree to which regulatory authorities overestimate the level of safety investment, and z^*_{31} denotes the actual safety investment by the coal mining enterprises.

In the scenario where both coal mining enterprises and regulatory authorities adopt speculative strategies, both parties will underestimate the losses caused by coal mine safety accidents. Therefore, both coal mining enterprises and regulatory authorities have a greater utility than the actual income, that is, $U^*_4 = A - \lambda_3 M$ and $u^*_4 = B - \lambda_4 N$.

We assume that the proportion of the population that chooses the normal investment strategy in the coal mining enterprises group is x, and the proportion of the population that chooses the speculative strategy is $1 - x$. We assume that the proportion of the population that chooses the normal supervision strategy in the regulatory authorities group is y, and the proportion of the population that chooses the speculative supervision strategy is $1-y$. To sum up, the payoff matrix of evolutionary game can be obtained, which is shown in Table 2.

TABLE 2. The payoff matrix of evolutionary game.

Regulatory authorities	Coal mining enterprises: Normal (x)	Coal mining enterprises: Speculative ($1 - x$)
Normal (y)	(Π^*_1, π^*_1)	(U^*_2, π^*_2)
Speculative ($1 - y$)	(Π^*_3, u^*_3)	(U^*_4, u^*_4)

IV. THE SOLUTION OF GAME MODEL

A. REPLICATION DYNAMIC EQUATIONS

We denote the expected profits of coal mining enterprises and regulatory authorities under normal and speculative strategies as E_x, E_{1-x}, E_y and E_{1-y}, respectively. We can get $E_x = y\Pi^*_1 + (1 - y)\Pi^*_3$, $E_{1-x} = yU^*_2 + (1 - y)U^*_4$, $E_y = x\pi^*_1 + (1 - x)\pi^*_2$ and $E_{1-y} = xu^*_3 + (1 - x)u^*_4$.

According to evolutionary game theory, the replication dynamic equations of coal mining enterprises and regulatory authorities can be obtained as

$$F(x) = \frac{dx}{dt} = x(1 - x)(y(\Pi^*_1 - U^*_2) + (1 - y)(\Pi^*_3 - U^*_4)), \tag{9}$$

$$G(y) = \frac{dy}{dt} = y(1 - y)(x(\pi^*_1 - u^*_3) + (1 - x)(\pi^*_2 - u^*_4)). \tag{10}$$

B. MODEL SOLVING

Coal mining enterprises and regulatory authorities are two different populations. The coal mining enterprise in the coal mining enterprises population can choose normal or speculative strategies at a certain moment. The regulatory authority in the regulatory authorities population can choose between normal or speculative strategies at a certain moment. The number of coal mining enterprises and regulatory authorities that choose speculative strategies during the evolution process will change. Depending on the initial values and system parameters of x and y, the evolution process may converge to a stable strategy or not converge. Moreover, evolutionary stability strategies are not unique. The system replication dynamic equations reflect the dynamic adjustment of strategy selection for the coal mining enterprises population and the regulatory authorities population in the game process. Let $F(x) = 0$ and $G(y) = 0$, we can obtain the five equilibrium points of the system as (0, 0), (0, 1), (1, 0), (1, 1), and (x^*, y^*), where $x^* = (u^*_4 - \pi^*_2)/(\pi^*_1 + u^*_4 - u^*_3 - \pi^*_2)$ and $y^* = (U^*_4 - \Pi^*_3)/(\Pi^*_1 + U^*_4 - \Pi^*_3 - U^*_2)$.

Next, the Jacobian matrix of replication dynamic equations is

$$J = \begin{pmatrix} \frac{\partial F(x)}{\partial x} & \frac{\partial F(x)}{\partial y} \\ \frac{\partial G(y)}{\partial x} & \frac{\partial G(y)}{\partial y} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \tag{11}$$

TABLE 3. Stability analysis of equilibrium points.

Condition	Equilibrium point	det J	Sign of det J	tr$J$	Sign of tr$J$	Stability
$\Delta_1 < 0$ and $\Delta_3 < 0$	$(0,0)$	$\Delta_2\Delta_4$	*INF*	$-\Delta_2-\Delta_4$	*INF*	*SP*
	$(0,1)$	$-\Delta_1\Delta_4$	*INF*	$\Delta_4-\Delta_1$	*INF*	*SP*
	$(1,0)$	$-\Delta_2\Delta_3$	*INF*	$\Delta_2-\Delta_3$	*INF*	*SP*
	$(1,1)$	$\Delta_1\Delta_3$	$+$	$\Delta_1+\Delta_3$	$-$	*ESS*
	(x^*,y^*)	$-CD$	*INF*	0	/	*UP*
$\Delta_1 > 0$ and $\Delta_4 < 0$	$(0,0)$	$\Delta_2\Delta_4$	*INF*	$-\Delta_2-\Delta_4$	*INF*	*SP*
	$(0,1)$	$-\Delta_1\Delta_4$	$+$	$\Delta_4-\Delta_1$	$-$	*ESS*
	$(1,0)$	$-\Delta_2\Delta_3$	*INF*	$\Delta_2-\Delta_3$	*INF*	*SP*
	$(1,1)$	$\Delta_1\Delta_3$	*INF*	$\Delta_1+\Delta_3$	*INF*	*SP*
	(x^*,y^*)	$-CD$	*INF*	0	/	*UP*
$\Delta_2 < 0$ and $\Delta_3 > 0$	$(0,0)$	$\Delta_2\Delta_4$	*INF*	$-\Delta_2-\Delta_4$	*INF*	*SP*
	$(0,1)$	$-\Delta_1\Delta_4$	*INF*	$\Delta_4-\Delta_1$	*INF*	*SP*
	$(1,0)$	$-\Delta_2\Delta_3$	$+$	$\Delta_2-\Delta_3$	$-$	*ESS*
	$(1,1)$	$\Delta_1\Delta_3$	*INF*	$\Delta_1+\Delta_3$	*INF*	*SP*
	(x^*,y^*)	$-CD$	*INF*	0	/	*UP*
$\Delta_2 > 0$ and $\Delta_4 > 0$	$(0,0)$	$\Delta_2\Delta_4$	$+$	$-\Delta_2-\Delta_4$	$-$	*ESS*
	$(0,1)$	$-\Delta_1\Delta_4$	$-$	$\Delta_4-\Delta_1$	*INF*	*SP*
	$(1,0)$	$-\Delta_2\Delta_3$	$-$	$\Delta_2-\Delta_3$	*INF*	*SP*
	$(1,1)$	$\Delta_1\Delta_3$	$+$	$\Delta_1+\Delta_3$	$+$	*SP*
	(x^*,y^*)	$-CD$	*INF*	0	/	*UP*

where $a_{11} = \frac{\partial F}{\partial x} = (1-2x)(y(\Pi_1^*-U_2^*)+(1-y)(\Pi_3^*-U_4^*))$, $a_{12} = \frac{\partial F}{\partial y} = x(1-x)(\Pi_1^*+U_4^*-U_2^*-\Pi_3^*)$, $a_{21} = \frac{\partial G}{\partial x} = y(1-y)(\pi_1^*+u_4^*-u_3^*-\pi_2^*)$, and $a_{22} = \frac{\partial G}{\partial y} = (1-2y)(x(\pi_1^*-u_3^*)+(1-x)(\pi_2^*-u_4^*))$.

The equilibrium point obtained from replication dynamic equations is not necessarily the evolutionary stable point of the system. According to the method proposed by Friedman, the stability of the equilibrium point can be derived from the local stability analysis of the Jacobian matrix of the system. If the equilibrium point satisfies the formula (12), it is the evolutionary stable point.

$$\begin{cases} a_{11}+a_{22} < 0 \\ \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22}-a_{12}a_{21} > 0 \end{cases} \tag{12}$$

For the sake of brevity, we let $\Delta_1 = U_2^*-\Pi_1^*$, $\Delta_2 = U_4^*-\Pi_3^*$, $\Delta_3 = u_3^*-\pi_1^*$, $\Delta_4 = u_4^*-\pi_2^*$, $C = x^*(1-x^*)(\Delta_2-\Delta_1)$, and $D = y^*(1-y^*)(\Delta_4-\Delta_3)$.

C. STABILITY ANALYSIS

For five equilibrium points of the replication dynamic equations, the determinant ($\det J$) and trace (trJ) of the Jacobian matrix are discussed respectively, and the results of stability analysis are shown in Table 3. The equilibrium point is the evolutionary stable point of the system (*ESS*), the sign of the determinant or trace of a Jacobian matrix is indefinite (*INF*), the equilibrium point is the saddle point (*SP*), the equilibrium point is unstable (*UP*).

Table 3 shows that when $\Delta_1 < 0$ and $\Delta_3 < 0$, equilibrium point $(1,1)$ is the stable point. Therefore, the evolution stability strategy of coal mining enterprises and regulatory authorities is (*normal*, *normal*), that is, coal mining enterprises and regulatory authorities make normal safety investment and normal regulatory investment, respectively. At this time, the normal strategy of both parties makes their expected utility greater than the speculative strategy. When $\Delta_1 > 0$ and $\Delta_4 < 0$, equilibrium point $(0,1)$ is the stable point. Therefore, the evolution stability strategy of coal mining enterprises and regulatory authorities is (*speculative*, *normal*), that is, coal mining enterprises choose a speculative strategy and do not make safety investments, while the regulatory authorities conduct normal regulatory investment. At this time, the speculative strategy allows coal mining enterprises to obtain greater expected utility, while the normal strategy allows the regulator to obtain greater expected utility.

When $\Delta_2 < 0$ and $\Delta_3 > 0$, equilibrium point $(1,0)$ is the stable point. Therefore, the evolution stability strategy of coal mining enterprises and regulatory authorities is (*normal*, *speculative*), that is, coal mining enterprises make normal safety investment, while regulators choose speculative strategies and do not make regulatory investments. At this time, the normal strategy enables coal mining enterprises to obtain greater expected utility, while the speculative strategy enables the regulator to obtain greater expected utility. When $\Delta_2 > 0$ and $\Delta_4 > 0$, equilibrium point $(0,0)$ is the stable point. Therefore, the evolution stability strategy of coal mining enterprises and regulatory authorities is (*speculation*, *speculation*), that is, coal mining enterprises and regulatory authorities both choose a speculative strategy and do not make safety investment or regulatory investment. At this time, the speculative strategies of both parties can make their expected utility greater.

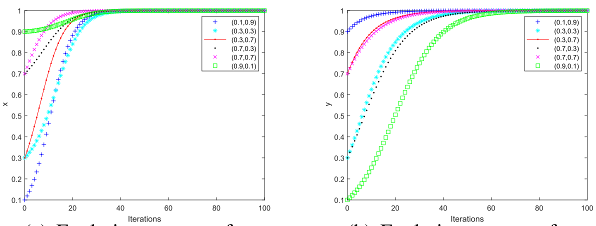
V. NUMERICAL ANALYSIS

In this section, the numerical simulation is used to further study the stability of the equilibrium points, and the evolution process of the stable points under different initial values is given and the influence of system parameters on the evolution process is analyzed. In order to improve the accuracy of the experiment, this paper provides different initial values, that

TABLE 4. Parameter settings.

Setting	Equilibrium point	A	B	H	k_1	k_2	M	N	λ_1	λ_2	λ_3	λ_4
1	(1,1)	5	3	0.4	6	11	0.2	0.2	0.99	1.2	0.9	0.9
2	(0,1)	5	3	0.3	11	75	0.1	0.2	0.001	1.1	0.5	0.5
3	(1,0)	5	3	0.35	4	9	0.3	0.2	0.99	100	0.99	0.5
4	(0,0)	5	3	1	6	13	1	2	0.4	8	0.04	0.6

is, the proportion of the population choosing normal and speculative strategies is different. For the four evolutionary stable points obtained in the theoretical analysis process, four sets of parameters are given respectively, as shown in Table 4. The algorithm program based on Matlab software is compiled to reveal the evolution law of different stable points.

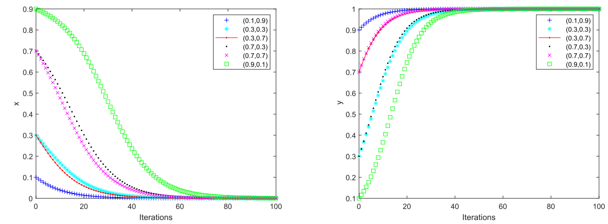


(a) Evolution process of x (b) Evolution process of y

FIGURE 1. The evolutionary process of stable point (1,1) where coal mining enterprises and regulatory authorities choose normal strategies.

When $(1, 1)$ is the stable point, the evolution process is plotted based on parameter setting 1, and the result is shown in Figure 1. At this point, both coal mining enterprises and regulatory authorities choose normal strategies. Figure 1(a) shows the evolution process of the population proportion of coal mining enterprises that choose the normal strategy under different initial values. It can be seen that x with different initial values can converge to 1 after a few iterations. When the initial value of x is fixed, the larger the initial value of y is, the faster the convergence speed of x is. When the initial value of y is fixed, the larger the initial value of x, the faster it converges to 1. A large initial value of $y(y = 0.9)$ can speed up the convergence of $x(x = 0.1)$, while a small initial value of $y(y = 0.1)$ can slow down the convergence of $x(x = 0.9)$. The disparity in the values of x and y has two effects. On the one hand, one side converges in a smaller number of iterations, which promotes the other side to speed up the convergence. On the other hand, one side needs to go through a large number of iterations to converge, resulting in slower convergence of the other side. For example, when there are fewer individuals adopting the speculative strategy in the regulatory authorities population, the regulatory authorities population will converge to the normal strategy at a faster rate, which in turn will prompt the individuals who adopt the speculative strategy in the coal mining enterprises to switch to the normal strategy more quickly, and vice versa.

Figure 1(b) shows the evolution process of the population proportion of regulatory authorities who choose the normal strategy under different initial values. It is easy to see that y with different initial values can converge to 1 after a few iterations. The larger the initial value of y, the faster it converges to 1. When the initial value of y is fixed, the larger the initial value of x is, the slower the convergence speed of y is. When the initial value of x is fixed, the larger the initial value of y, the faster it will converge to 1. When the initial value of (x, y) is $(0.9, 0.1)$, the convergence speed of y is fast in the early stage of the iteration and slow in the later stage.

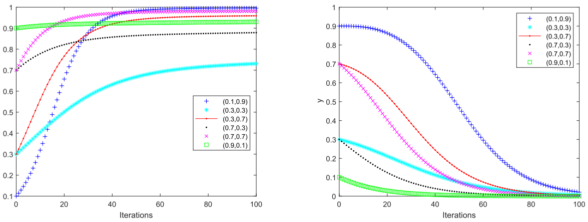


(a) Evolution process of x (b) Evolution process of y

FIGURE 2. The evolutionary process of stable point (0,1) where coal mining enterprises choose speculative strategies and regulatory authorities choose normal strategies.

When $(0, 1)$ is the stable point, the evolution process is plotted based on parameter setting 2, and the result is shown in Figure 2. At this point, coal mining enterprises choose speculative strategies and regulatory authorities choose normal strategies. Figure 2(a) shows the evolution process of the population proportion of coal mining enterprises that choose the normal strategy under different initial values. It can be seen that x with different initial values can converge to 0 after a few iterations. The smaller the initial value of x, the faster it converges to 0. When the initial value of x is fixed, the larger the initial value of y is, the faster the convergence speed of x is. When the initial value of (x, y) is $(0.9, 0.1)$, the convergence speed of x is the fastest in the intermediate stage of the iteration.

Figure 2(b) shows the evolution process of the population proportion of regulatory authorities that choose the normal strategy under different initial values. It is easy to see that y with different initial values can converge to 1 after a few iterations. The larger the initial value of y, the faster it converges to 1. When the initial value of y is fixed, the larger the initial value of x is, the faster the convergence speed of y is. It should be pointed out that the effect of the initial value of x on the convergence speed of y is not as significant as the effect of initial value of y on the convergence speed of x. It shows that the tendency of the population of the regulatory authorities to adopt the normal strategy is less affected by the changes of the strategies of the coal mining enterprises, while the tendency of the population of the coal mining enterprises to adopt the speculative strategy is greatly affected by the changes of the strategies of the regulatory authorities.



(a) Evolution process of x (b) Evolution process of y

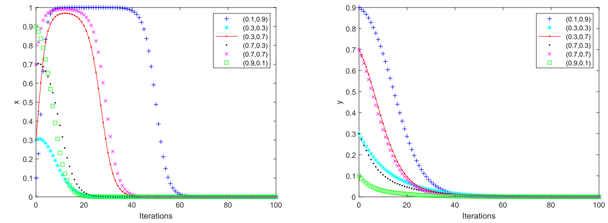
FIGURE 3. The evolution process of stable point (1,0) where coal mining enterprises choose normal strategies and regulatory authorities choose speculative strategies.

When $(1, 0)$ is the stable point, the evolution process is plotted based on parameter setting 3, and the result is shown in Figure 3. At this point, coal mining enterprises choose normal strategies and regulatory authorities choose speculative strategies. Figure 3(a) shows the evolution process of the population proportion of coal mining enterprises that choose the normal strategy under different initial values. After evolution (4000 iterations), different initial values of x eventually converge to 1. Due to space limitations, only the evolution of 100 generations is intercepted here.

Figure 3(a) shows that the evolution iterations required for different initial values of x to converge to 1 are different. When the initial value of x is fixed, the larger the initial value of y is, the faster the convergence speed of x is. When y takes a large initial value ($y = 0.9$), it takes a small iterations (80) for x to converge to 1. When y takes a small initial value ($y = 0.1$), it takes a large iterations (4000) to converge to 1. When the initial value of y is fixed, the larger the initial value of x, the faster it converges to 1. Compared with the initial value $(0.7, 0.3)$, a larger initial value of $y(y = 0.7)$ can make the smaller initial value of $x(x = 0.3)$ converge to 1 faster. The lower the ratio of speculative strategies adopted in the initial stage of the population of regulatory authorities, the faster the evolution stability of the population of coal mining enterprises to the normal strategy.

Figure 3(b) shows the evolution process of the population proportion of regulatory authorities that choose the normal strategy under different initial values. It can be seen that y with different initial values can converge to 0 after a few iterations. The smaller the initial value of y, the faster the convergence speed. When the initial value of y is fixed, the smaller the initial value of x is, the slower the speed of y converging to 0 is. When the initial value of x is fixed, the smaller the initial value of y is, the faster the convergence speed of y is. When the initial value of (x, y) is $(0.1, 0.9)$, y converges fastest in the intermediate stage of the iteration.

When $(0, 0)$ is the stable point, the evolution process is plotted based on parameter setting 4, and the result is shown in Figure 4. At this point, both coal mining enterprises and regulatory authorities choose speculative strategies. Figure 4(a) shows the evolution process of the population proportion of coal mining enterprises that choose the normal strategy under different initial values. It can be seen that x with different initial values can converge to 0 after a few iterations. Interestingly, different initial values have different convergence trends. When the initial value of y ($y \geq 0.7$) is large, x goes through the process of first rising and then falling, and finally converges to 0. When the initial value of y ($y \leq 0.3$) is small, x will converge to 0 directly after a few iterations. When the initial speculation population ratio of coal mining enterprises is relatively large and the initial speculation population ratio of regulatory authorities is relatively small, coal mining enterprises will first tend to the normal strategy, and then stabilize to the speculative strategy. When the initial value of x is fixed, the larger the initial value of y, the slower the convergence speed of x. When the initial value of y is fixed, the larger the initial value of x, the slower the convergence speed.



(a) Evolution process of x (b) Evolution process of y

FIGURE 4. The evolutionary process of stable point (0,0) where coal mining enterprises and regulatory authorities choose speculative strategies.

Figure 4(b) shows the evolution process of the population proportion of regulatory authorities that choose the normal strategy under different initial values. Different initial values of y can all converge to 0 after a few iterations. The larger the initial value of y, the slower the convergence speed. When the initial value of y is fixed, the larger the initial value of x is, the faster the convergence speed of y is. When the initial value of x is fixed, the larger the initial value of y, the slower the convergence speed.

Next, this paper investigates the impact of the safety characteristic coefficient k_1 of coal mining enterprises and the capability characteristic coefficient k_2 of the regulatory authorities on the evolution process and evolutionary stabile points. After verification, k_1 and k_2 only have a significant effect on the evolution process of $(0, 1)$ and $(1, 0)$, respectively.

When $(0, 1)$ is the stable point, the influence of k_1 on the evolution process is studied. Under parameter setting 2, we let k_1 take 11, 20 and 50 respectively, and the other parameters remain unchanged. The result is shown in Figure 5. Figure 5 shows that the convergence of x to 0 gradually slows down as k_1 increases. In particular, x converges to 1 when $k_1 = 50$. Then, the evolutionary stable point is transformed from $(0, 1)$ to $(1, 1)$. It can be seen that with the increase of the safety characteristic coefficient of coal mining enterprises, the population evolution stability strategy of coal mining enterprises changes from speculation to normal investment.

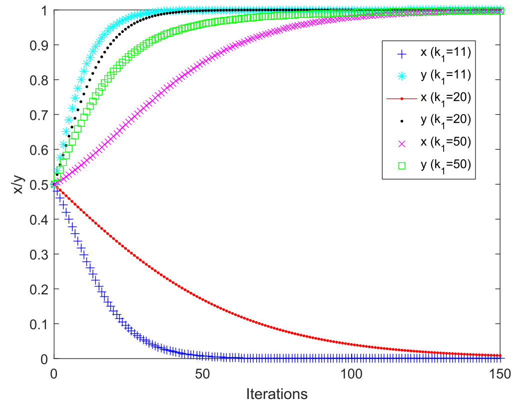


FIGURE 5. The impact of the safety characteristic coefficient k_1 on $(0, 1)$.

As k_1 increases, the rate at which y converges to 1 gradually slows down.

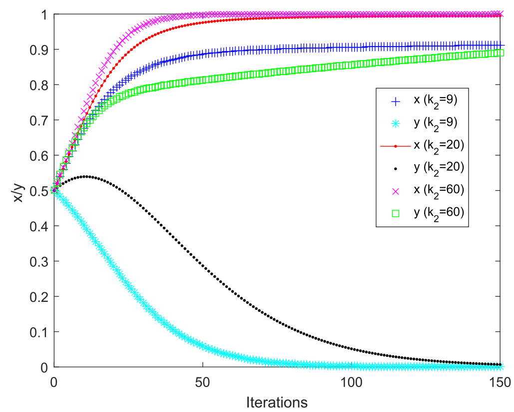


FIGURE 6. The impact of the capability characteristic coefficient k_2 on $(1, 0)$.

When $(1, 0)$ is the stable point, the influence of k_2 on the evolution process is studied. Under parameter setting 3, we let k_2 take 9, 20 and 60 respectively, and the other parameters remain unchanged. The result is shown in Figure 6. Figure 6 shows that as k_2 increases, the speed of the convergence of x to 1 increases gradually. As k_2 increases, the rate at which y converges to 0 gradually slows down. In particular, y converges to 1 when $k_2 = 60$. The evolutionary stable point is transformed from $(1, 0)$ to $(1, 1)$. It can be seen that with the increase of the capacity characteristic coefficient of the regulatory authorities, its evolutionary stabilization strategy gradually changes from speculation to normal supervision.

The effect of the speculative behavior coefficient λ_i on the evolution process and evolutionary stability point is respectively studied in the following. It has been verified that λ_1 has a significant effect on $(0, 1)$, λ_2 has a significant effect on $(1, 0)$, λ_3 and λ_4 have a significant effect on $(0, 0)$.

When $(0, 1)$ is the stable point, the influence of λ_1 on the evolution process is studied. Under parameter setting 2, we let λ_1 take 0.01, 0.1 and 0.8 respectively, and the other parameters remain unchanged. The result is shown

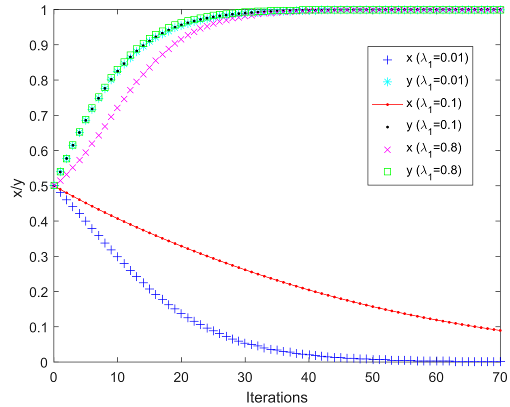


FIGURE 7. The impact of λ_1 on $(0, 1)$.

in Figure 7. Figure 7 shows that the change of λ_1 has a significant effect on the evolution process of x. As λ_1 increases, the speed at which x converges to 0 gradually slows down. In particular, x converges to 1 when $\lambda_1 = 0.8$. The evolutionary stable point is transformed from $(0, 1)$ to $(1, 1)$. The more coal mining enterprises underestimate regulation, the more inclined they are to adopt speculative strategy. The change of λ_1 has no effect on the evolution process of y.

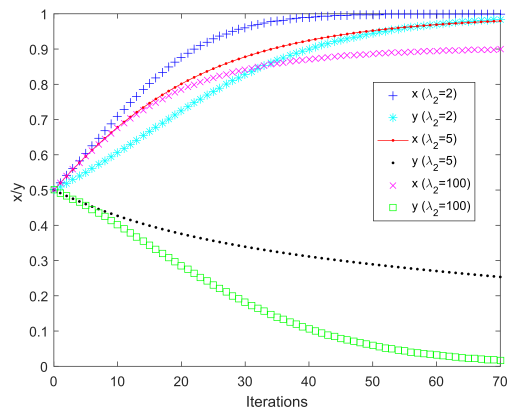


FIGURE 8. The impact of λ_2 on $(1, 0)$.

When $(1, 0)$ is the stable point, the influence of λ_2 on the evolution process is studied. Under parameter setting 3, we let λ_1 take 2, 5 and 100 respectively, and the other parameters remain unchanged. The result is shown in Figure 8. Figure 8 shows that as λ_2 increases, the rate at which x converges to 1 gradually slows down. As λ_2 decreases, the rate at which y converges to 0 gradually slows down. In particular, y converges to 1 when $\lambda_2 = 2$. The evolutionary stable point is transformed from $(1, 0)$ to $(1, 1)$. The more regulatory authorities overestimate the safety investment of coal mining enterprises, the more inclined they are to adopt speculative strategies.

When $(0, 0)$ is the stable point, the influence of λ_3 on the evolution process is studied. Under parameter setting 4, we let λ_3 take 0.04, 0.4 and 0.5 respectively, and the other parameters remain unchanged. The result is shown

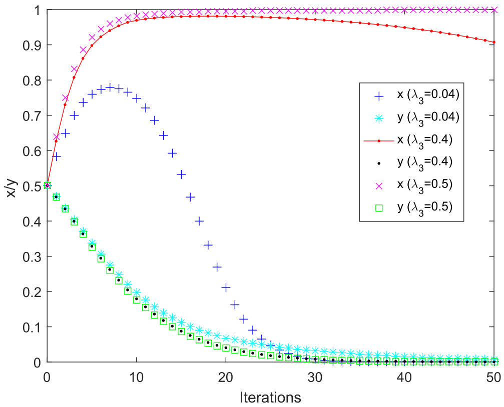


FIGURE 9. The impact of λ_3 on (0, 0).

in Figure 9. Figure 9 shows that the change of λ_3 has a significant effect on the evolution process of x. As λ_3 increases, the speed at which x converges to 0 gradually slows down. In particular, x converges to 1 when $\lambda_3 = 0.5$. The evolutionary stable point is transformed from $(0, 0)$ to $(1, 0)$. The smaller the degree of underestimation of accident losses, the more inclined the coal mining enterprises are to adopt normal strategy. In addition, the change of λ_3 has no significant effect on the evolution process of y.

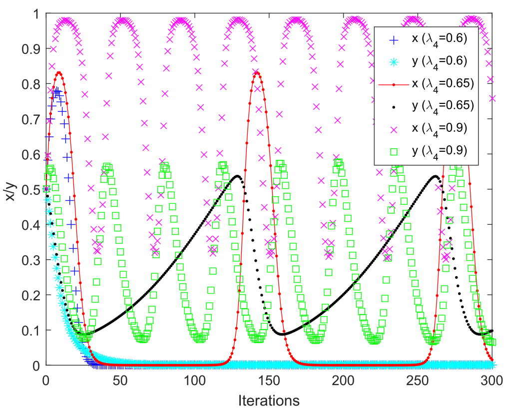
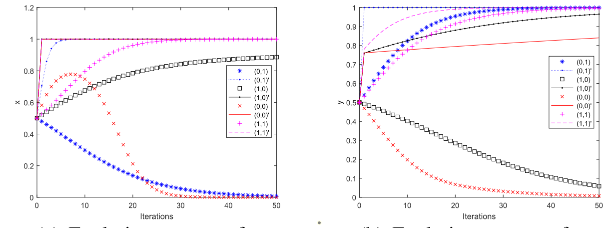


FIGURE 10. The impact of λ_4 on (0, 0).

When $(0, 0)$ is the stable point, the influence of λ_4 on the evolution process is studied. Under parameter setting 4, we let λ_4 take 0.6, 0.65 and 0.9 respectively, and the other parameters remain unchanged. The result is shown in Figure 10. Figure 10 shows that the evolutionary stable point is $(0, 0)$ when $\lambda_4 = 0.6$. As λ_4 increases, the value of (x, y) oscillates periodically, and the evolution process no longer converges to a stable point. As the regulatory authorities' underestimation of coal mine accident losses decreases, the evolution strategies of coal mining enterprises and regulatory authorities are constantly changing between normal and speculative. In particular, the larger λ_4 is, the smaller the oscillation period is.

Let H take 20 and the corresponding values in parameter settings 1−4, respectively, we can get Figure 11. In Figure 11,



(a) Evolution process of x (b) Evolution process of y

FIGURE 11. The impact of fine H on the strategy choice of coal mining enterprises and regulatory authorities.

$(i, j)'$ represents the evolution process of stable point (i, j) when $H = 20$, where $i = 0, 1$ and $j = 0, 1$.

Figure 11 shows that the change of H has a significant effect on the evolutionary stable point and the evolutionary process. When $H = 20$, the original stable points $(0, 0)$, $(0, 1)$, $(1, 0)$ all converge to the new stable point (1,1). Of course, the original stable point $(1, 1)$ is still the stable point. Furthermore, the convergence speed is accelerated. Figure 11(a) shows that when $H = 20$, x can converge to a stable value of 1 through very few iterations, and the convergence speed is significantly accelerated. In particular, for the original stable points $(0, 0)$, $(1, 0)$, $(1, 1)$, x can converge to 1 after only one iteration. Figure 11(b) shows that when $H = 20$, for the original stable point $(0, 1)$, y can converge to 1 after only one iteration. For the original stable points $(0, 0)$, $(1, 0)$, $(1, 1)$, y undergoes a fast and then slow convergence process. The higher the fine account for coal mine safety supervision, the more it will prompt coal mining enterprises to adopt normal strategy.

VI. CONCLUSION

Based on evolutionary game, this paper studies the game between coal mining enterprises and safety supervision authorities. It continuously describes the strategies of coal mining enterprises and regulatory authorities, and defines the safety probability of coal mining production and the probability of safety problems discovered by supervision. On the basis of introducing the speculative behavior of coal mining enterprises and regulatory authorities, an evolutionary game model is established. By solving the replication dynamic equations, the evolutionary stable points are found and the stability analysis is carried out. The evolution law of the stable points and the influence of system parameters on the evolution process are revealed through sufficient numerical experiments.

The main conclusions are as follows. On the one hand, the speculative behavior coefficients of coal mining enterprises and regulatory authorities can both affect the evolution process and even change the evolution stability point. The more coal mining enterprises underestimate the supervision strength of the regulatory authorities, the more inclined they are to adopt speculative strategies. The more the regulatory authorities overestimate the safety investment of coal mining enterprises, the more inclined they are to adopt speculative strategies. Therefore, in order to improve the safety production and supervision level of coal mines, we should pay attention to the speculative behavior of coal mining enterprises and regulatory authorities, and weaken them. On the other hand, both the safety characteristic coefficient of coal mining enterprises and the capability characteristic coefficient of regulatory authorities can affect the evolution process and even change the evolution stability points. The improvement of the safety characteristic coefficient of coal mining enterprises can prompt them to adopt normal investment strategies. The improvement of the capacity characteristic coefficient of the regulatory authorities can prompt it to adopt the normal regulatory strategy. Therefore, the safety characteristics of coal mining enterprises and the capabilities of the regulatory authorities should be improved.

In order to weaken the speculative behavior of coal mining enterprises and regulatory authorities, regulatory authorities should increase their penalties for coal mining enterprises, the government should increase the revenue generated by regulatory authorities from coal mining production. In order to improve the safety characteristics of coal mining enterprises and the ability of regulatory authorities, coal mining enterprises can increase the number of ventilation points, personnel responsible for coal mine safety and the quality of miners, regulatory authorities can increase the frequency of supervision and investment in regulatory equipment.

This paper assumes that when coal mining enterprises or regulatory authorities choose a speculative strategy, they do not make corresponding safety production investment or regulatory investment at all. Although this assumption is meaningful, it can be considered in further research that when they choose a speculative strategy, the corresponding safety production investment or regulatory investment will be reduced.

Proof of proposition 1

The profit of coal mining enterprises under scenario 1 is described in Equation (1), and the profit of regulatory authorities under scenario 1 is described in Equation (2). For $d^2\pi_1/dz_{12}^2 = -k_2 H k_2 e^{-k_1 z_{11} - k_2 z_{12}} < 0$, we know that π_1 is concave with respect to z_{12}. Thus, using the first order condition $d\pi_1/dz_{12} = -1 + Hk_2 e^{-k_1 z_{11} - k_2 z_{12}} = 0$, we can get $z_{12}^* = (\ln k_2 H - k_1 z_{11})/k_2$. Substituting z_{12}^* into equation (3), we have $\Pi_1 = A - e^{-k_1 z_{11}}(M+H) - z_{11} + 1/k_2$. For $d^2\Pi_1/dz_{11}^2 = -k_1^2 e^{-k_1 z_{11}}(M+H) < 0$, we know Π_1 is concave with respect to z_{11}. Hence, the first order condition $d\Pi_1/dz_{11} = k_1 e^{-k_1 z_{11}}(M+H) - 1 = 0$ is used to obtain $z_{11}^* = \ln[k_1(M+H)]/k_1$. By substituting, other optimal results can be obtained. The proof of Proposition 1 is completed.

Proof of proposition 2

The profit of coal mining enterprises under scenario 2 is described in Equation (3), and the profit of regulatory authorities under scenario 2 is described in Equation (4). For $d^2\pi_2/dz_{22}^2 = -Hk_2^2 e^{-k_2 z_{22}} < 0$, we know that π_2 is concave with respect to z_{22}. Thus, using the first order condition $d\pi_2/dz_{22} = -1 + Hk_2 e^{-k_2 z_{22}} = 0$, we can get $z_{22}^* = \ln(k_2 H)/k_2$. By substituting, other optimal results can be obtained. The proof of Proposition 2 is completed.

Proof of proposition 3

The profit of coal mining enterprises under scenario 3 is described in Equation (5), and the profit of regulatory authorities under scenario 3 is described in Equation (6). For $d^2\Pi_3/dz_{31}^2 = -k_1^2 e^{-k_1 z_{31}} M < 0$, we know that Π_3 is concave with respect to z_{31}. Thus, using the first order condition $d\Pi_3/dz_{31} = k_1 e^{-k_1 z_{31}} M - 1 = 0$, we can get $z_{31}^* = \ln(k_1 M)/k_1$. By substituting, other optimal results can be obtained. The proof of Proposition 3 is completed.

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