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RESEARCH ARTICLE

New Method for Measuring Diagnosis Indexes Based on Concept Lattices in Dam Health Diagnosis

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ABSTRACT With multi-layers and multi-indexes, dam health diagnosis is an important way to diagnose the structural safety and health operation of dams. This study focuses on the measurement of diagnosis indexes in dam health diagnosis. The existing methods of constructing diagnosis indexes are mainly based on the subjective judgment of expert knowledge, and lack of consideration of the internal mapping relationship between diagnosis indexes and dam health levels. Therefore, it is necessary to introduce new theories and new methods to study the objective measurement of diagnostic indexes in combination with the characteristics of dam health diagnosis. Based on the Dempster–Shafer (D–S) evidence theory, a new concept lattices-based model for building basic probability assignments (BPAs) is proposed in this study. First, concept lattices under hesitant fuzzy linguistic term sets (HFLTSs) were established to formalize the qualitative and quantitative expression of expert knowledge. Then we defined a new distance of HFLTSs, named *NWD*. *NWD* is strict in mathematical definition and considers the non-overlapping HFLTSs. Based on *NWD*, the weight of the monitoring point for each health level was obtained through similarity analysis and finally transformed into the corresponding BPA. An engineering project demonstrated that the BPAs developed in this study could adequately describe the attributes of diagnosis indexes, forming reliable bases for the comprehensive diagnosis fusion. Simultaneously, the proposed method of building the BPA can significantly improve assignment efficiency, which can shed light on the development of dam operation behavior modelling.

INDEX TERMS Basic probability assignment, concept lattice, dam health diagnosis, diagnosis index, D–S evidence theory.

I. INTRODUCTION

As an important way to monitor the operational behavior of dams and evaluate their health status, dam health diagnosis is a multi-index comprehensive diagnosis method integrating multiple monitoring points, multiple effect quantities and other multi-source monitoring information.

Dam health diagnosis mainly includes three aspects: the establishment of diagnosis index system, the measurement of diagnosis indexes and the fusion analysis of diagnosis indexes. The diagnosis system mainly refers to the indexes that describe the characteristics of a dam based on the structural features and monitoring arrangement of the dam. A diagnosis system is usually a recursive structure with multiple layers, including state layer (diagnosis target), section layer, project layer, effect quantity layer, and monitoring point layer [1], [2], [3]. Based on the monitoring data of effect quantities, the measurement of diagnosis indexes refers to the membership of diagnosis indexes to the health levels. Different fusion methods correspond to

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different measurements of indexes, e.g., conditional probabilities in the Bayesian network, degrees of membership in fuzzy mathematics, and Basic Probability Assignments (BPAs) in the Dempster–Shafer (D–S) evidence theory [4], [5], membership clouds in the cloud model, and so on. The fusion analysis of diagnosis indexes refers to comprehensive diagnosis and evaluation of multi-source monitoring indexes with some intelligent and information-based theories and methods such as modern mathematics, system engineering and information science.

At present, the measurement methods of diagnosis indexes are mainly divided into three categories: (1) subjective methods based on expert knowledge or engineering experience; the most commonly used is experts grading method [6], [7], [8]. This method show strong subjective randomness and may lead to uncoordinated or inconsistent grades with limitation of expert knowledge. (2) semi-empirical method combined with monitoring data analysis; based on the qualitative and quantitative analysis of single-point monitoring data, diagnosis indexes are jointly given by expert experience and theoretical calculation, e.g., threshold methods based on structural calculation [9] and statistical analysis [10], [11], comparative analysis based on curve fitting of monitoring models [12], [13], methods combined with weight coefficients [2], [3], [14], etc. (3) The third method is not to directly determine the values of diagnosis indexes, but to establish a mathematical model to automatically generate diagnosis indexes, such as fuzzifying [15], [16], intervalizing [16], [17], [18], and other ways. Although this method is flexible, relatively scientific and objective to some extent, the research results are still few.

As a representative method in the field of information fusion, D–S evidence theory can directly express uncertainties and unknowns without prior information [19], [20]. This paper studies the method of building BPAs under the framework of the D–S evidence theory.

Building BPAs is closely related to the practical application, so it is difficult to develop a uniform standard. Scholars have developed practical solutions based on their respective fields of study, which are mainly divided into two categories: (1) Qualitative analysis and expert evaluation [21], [22]; these methods are subjective; (2) Quantitative methods with objective data, which need a large sample [23], [24]. However, these methods neglect the characteristics of dam health diagnosis. First, dam health diagnosis provides an early warning before accidents. Dam failure cannot be risked because of the severity of the disaster [25], [26]. Therefore, for dam health diagnosis, repeated failure experiments cannot be conducted to obtain sufficient data to establish BPAs, as in other fields. Second, the application of other historical dam failures to actual situations is difficult owing to the complexity and uncertainty of the dam itself. The data in the dam health diagnosis represent only the external operations of the monitoring points. In this case, methods for building BPAs in dam health diagnosis cannot rely on historical failure data and must also consider the intrinsic relationship between operation behaviors and health levels.

Concept lattices [27] are powerful tools for data analysis and information acquisition and can describe the formal relationships between objects and attributes. Based on this, concept lattices were first introduced into dam health diagnosis in this study. First, we built concept lattices under Hesitant Fuzzy Linguistic Term Sets (HFLTSs) [28] to formalize the mapping between the operation state of the monitoring points and health evaluation level. The purpose of using HFLTSs is to be more semantically similar to the expression habits of experts. Combined with Wasserstein distance, a new distance for HFLTSs (*NWD*) was define. *NWD* is strict in mathematical definition and it considers the non-overlapping HFLTSs, which is often neglected in other distances measures. Subsequently, an algorithm for similarity analysis and assignment of BPAs was proposed under the framework of concept lattices. Finally, the obtained BPAs participated in the fusion calculation to diagnose the health of the dam.

It is worth mentioning that our study aims at proposing a new measurement of diagnosis indexes and it is not necessarily superior to other methods of building BPAs, but it is more suitable for dam health diagnosis to some extent. We just provide a new way to model dam operation behavior.

The paper is organized as follows. In Section II, we discuss some preliminaries of BPA, HFLTS and concept lattices. In Section III, a concept lattices-based model for building BPAs in dam health diagnosis is presented, in which we define a new distance of HFLTSs under the framework of concept lattices in Section III-A. In Section IV, an engineering example of dam health diagnosis is provided to verify the validity and effectiveness of the proposed model. Section V concludes the study.

II. PRELIMINARIES

In this section, some preliminaries, such as BPA, concept lattices and HFLTS, are briefly introduced.

A. BPA

Let a Frame of Discernment (FOD) $\Theta = \{A_1, A_2, \ldots, A_{\mathrm{N}}\}$, be a finite non-empty set of mutually exhaustive and exclusive hypotheses. A BPA, also called a mass function, is a mapping m from 2^{Θ} to [0, 1] that satisfies the following conditions:

$$m(\emptyset) = 0 \quad \text{and} \sum_{A \in 2^{\Theta}} m(A) = 1, \tag{1}$$

where $m(A)$ represents the degree of belief that one is willing to commit exactly to A for each $A \subseteq \Theta$. $m(A)$ ranges from 0 to 1.

Under the fusion framework of the D–S evidence theory, the measurement of underlying diagnosis indexes is to determine the BPAs. Dempster's combination rule can be used to fuse multiple BPAs, which is defined as follows:

$$m(A) = \begin{cases} \sum_{A_i \cap A_j = A} \dfrac{m_1(A_i)m_2(A_j)}{1-K} & A \neq \emptyset \\ 0 & A = \emptyset, \end{cases} \tag{2}$$

$$K = \sum_{A_i \cap A_j = \emptyset} m_1(A_i)m_2(A_j), \tag{3}$$

where m_1 and m_2 are the BPAs derived from two pieces of evidence over the FOD Θ. K represents the degree of conflict between the two pieces of evidence. K ranges from 0 to 1, i.e., $K \in [0, 1]$. Higher values of K indicate a greater degree of conflict.

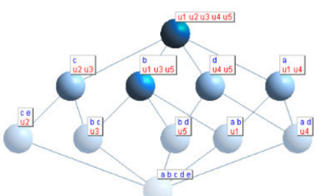
B. CONCEPT LATTICES

First proposed by Wille [27], concept lattice is a core tool for data analysis and processing in Formal Concept Analysis (FCA). Through the formal hierarchical structure between lattices, the concept lattice can mine data with association rules by clearly displaying the generalization and specialization relationships among concepts [29]. Concept lattices have been widely used in software engineering [30], information retrieval [31], case-based reasoning [32], medical diagnosis [33], and other fields. The basic definitions of concept lattices are presented below.

A formal context is a triple (U, A, I), where U and A refer to a set of objects and a set of attributes, respectively. The binary relationship between U and A is denoted by I. uIa is interpreted as object u with attribute a. A formal context can be described as a cross table in which the rows and columns represent different objects and attributes, respectively. If an object in a row has the attributes in a column, an "X" is added at the intersection of the row and column. One of the most widely used formal concept analysis tools is *Lattice Miner* [34], which can draw Hasse graphs and derive rules that satisfy conditions with a progressive construction algorithm. The formal context and concept lattices are shown in Fig. 1, where Fig. 1(a) shows a simple formal context and Fig. 1(b) shows the Hasse graph of the concept lattices.

U/A	a	b	c	d	e
u1	x	x			
u2			x		x
u3		x	x		
u4	x			x	
u5		x		x	

(a)



(b)

FIGURE 1. (a) Formal context and (b) concept lattices.

In dam health diagnosis, the set of objects U refers to all monitoring points. The set of attributes A refers to the qualitative and quantitative diagnosis indexes which combine the monitoring data and expert knowledge, such as the annual amplitude change of a single monitoring point, trend change of a single monitoring point, and coordination of multiple monitoring points.

Given a set of objects $X \subseteq U$ and a set of attributes $Y \subseteq A$, we define the operators:

$$X^* = \{a \in A | u\, Ia,\ \forall u \in X\}, \tag{4}$$

$$Y^* = \{u \in U | u\, Ia,\ \forall a \in Y\}. \tag{5}$$

If $X^* = Y$ and $Y^* = X$, then (X, Y) is a formal concept. X is the *Intent* of (X, Y) and Y is the *Extent* of (X, Y). If an object belongs to the concept (X, Y), it must have every attribute in the *Intent* X. Similarly, if an attribute belongs to the concept (X, Y), it must be owned by every object in the *Extent* Y. Each node of the concept lattice represents a formal concept.

C. HFLTS

HFLTS is a fuzzy linguistic approach to express qualitative decision information with multiple consecutive linguistic terms. It can improve the flexibility and expressive force of linguistic information and allow experts to evaluate linguistic variables when hesitating among several values. This is more semantically similar to the simple but elaborate linguistic expressions of human beings.

Consider a finite linguistic term set $S = \{s_0, s_1, \ldots, s_{n_{-1}}\}$. Here, n is odd and represents the number of linguistic terms, which is also called the granularity of the term set. S satisfies:

1. A negation operator $Neg(s_i) = s_j$ where $j = n$ - 1 - i,
2. In order: $s_i \ \leq s_j \leftrightarrow i \leq j$.

An HFLTS H_S is an ordered finite subset of consecutive linguistic terms of S. For example, a set of five terms can be expressed as $S = \{s_0$: very low, s_1: low, s_2: medium, s_3: high, s_4: very high$\}$, and $H_S = \{s_1, s_2\}$ is an HFLTS for S.

In order to more effectively apply HFLTS to solve multi-source evidence decision-making problems, the distance and similarity measures of HFLTS are often paid more attention. Based on the Hamming distance and Euclidean distance and their generalized forms, Liao et al. [35] developed a family of distance measures between two HFLTSs for discrete and continuous cases, respectively. However, the triangle inequality is not always satisfied in some cases [36]. Farhadinia [37] and Gou et al. [38] introduced a series entropy-based measures for HFLTSs. Tang and Liao [39] proposed the inclusion measure between HFLTSs and two clustering algorithms based on correlation measure and distance measure. Wu et al. [36] developed a uniform HFLTSs distance measure considering hesitance degree and linguistic terms values, which were applied to the field of judicial execution. However, all these distance measures neglect the non-overlapping HELTSs.

D. CONCEPT LATTICES UNDER HFLTSS

A formal context under HFLTSs can be expressed as (U, A, V, f), where U and A refer to a set of objects and set of attributes, respectively. V is a cluster of linguistic terms describing the relationship between U and A, while f is the mapping from

$U \times A$ to V. For $u \in U$ and $a \in A$, $f(u, a) \in V$ means that the membership degree of object u with attribute a is $f(u, a)$.

In dam health diagnosis, the HFLTS for the annual amplitude can be represented as A = {a_1 = Small, a_2 = General, a_3 = Large}. The HFLTS for the regularity of the changes can be represented as B = {b_1 = Relatively consistent with the objective law, b_2 = General, b_3 = Less consistent with the objective law}. When diagnosing health operations, the health evaluation level can be expressed as H = {h_1 = Normal, h_2 = Nearly normal, h_3 = Mildly abnormal, h_4 = Severely abnormal, h_5 = Malignant abnormal} [40]. More details regarding the contents of the attributes and corresponding HFLTS are introduced in Section III-C.

III. NEW CONCEPT LATTICES-BASED MODEL FOR BUILDING BPAS

In this section, we first define a new distance for HFLTSs and propose a concept lattices-based model for building BPAs for dam health diagnosis.

A. NEW DISTANCE FOR HFLTSS

A new distance measure for HFLTSs, named *NWD*, is proposed in this section. It is obtained by combining Wasserstein distance with HFLTSs.

First proposed by Peleg et al. [41] in 1989, Wasserstein distance is derived from the Monge–Kantorovich problem in the optimal transportation theory. The Wasserstein distance of distributions μ and υ is defined in the following mathematical expectation form:

$$WD(\mu, \nu) = \inf_{\pi} E_{(\mathrm{x},\mathrm{y})\sim\pi}[||x-y||], \tag{6}$$

where π is the set of probabilistic couplings on (μ, ν). The lower bound of the expectations of the distances between samples x and y in all possible probabilistic couplings is the Wasserstein distance of μ and υ. Wasserstein distance is also known as Earth Mover's distance or Monge-Kantorovich distance.

Definition: Two experts are invited to evaluate the attribute separately using a set of n linguistic terms, $S = \{s_0, s_1, \ldots, s_{n-1}\}$. According to their evaluations, the two HFLTSs on S are expressed as $S^K = \{s_p, s_{p+1}, \ldots, s_q\}$ $(0 \le p \le q \le n\text{-}1)$ and $S^L = \{s_r, s_{r+1}, \ldots, s_t\}$ $(0 \le r \le s \le n\text{-}1)$. The new distance of S^K and S^L is

$$NWD(S^K, S^L) = WD[DU(s_p, s_q), DU(s_r, s_t)]. \tag{7}$$

Here, $WD[\mu, \nu]$ is the Wasserstein distance of the distributions μ and ν, $DU[a, b]$ represents the discrete uniform distribution in the interval $[a, b]$.

Equation (7) can be intuitively expressed as follows: the membership degrees of S^K and S^L are regarded as two discrete uniform probability distributions, namely,

$$F^K = \{(s_p, \frac{1}{q-p+1}), (s_{p+1}, \frac{1}{q-p+1}), \ldots, (s_q, \frac{1}{q-p+1})\},$$

$$F^L = \{(s_r, \frac{1}{t-r+1}), (s_{r+1}, \frac{1}{t-r+1}), \ldots, (s_t, \frac{1}{t-r+1})\}.$$

The Wasserstein distance between these two distributions is the distance between the two HFLTSs S^K and S^L.

Compared with other distance measure, NWD exhibits certain advantages.

(1) *NWD* is strict in mathematical definition.

NWD is a strict diatance and meets the basic properties emphasized by Li et al. [42].

A strict distance [43] must satisfy four requirements: symmetry, non-negativity, non-degeneracy, and triangle inequality. Rubner et al. [44] proved that Wasserstein distance has the above four properties and is a strict distance satisfying the axiomatic definition. In the above calculation process, HFLTS is discretized into a uniform distribution on the corresponding interval. Triangular or parabolic distributions can also be used according to the needs of practical engineering applications. *NWD* uses Wasserstein distance to calculate the difference between the two distributions, so it does not change the excellent mathematical properties of the Wasserstein distance. Therefore, *NWD* is also a strict distance in a mathematical sense.

(2) *NWD* is continuous when HFLTSs are non-overlapping.

In a complex environment of uncertainty, it is possible for decision-makers to disagree completely. If this happens, HFLTSs are non-overlapping and separable. Considering that the number of HFLTSs is large in some cases, simply relying on manual selection will show strong subjectivity and consume too much manpower and material resources. This extreme case is often neglected in previous distance measures. An extremely useful property of Wasserstein distance is that it is well-defined for distributions with non-overlapping supports [45]. When two HFLTSs have no intersection, Wasserstein distance can still provide a meaningful gradient and accurately measure the distance between them, which is unrealizable for Kullback–Leibler and Jensen–Shannon divergences.

B. OPERATING PROCESS AND MAJOR STEPS

As shown in Fig. 2, the proposed model comprises several major steps.

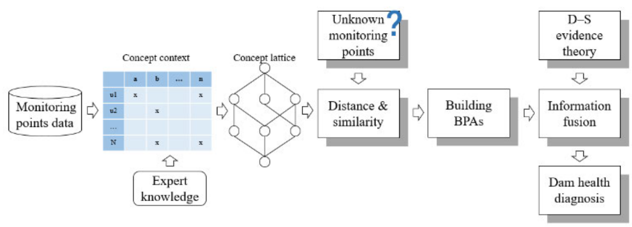


FIGURE 2. Architecture of the model proposed in this work.

(1) Establishment of formal context: In this step, based on the numerical performance and trend changes of the monitor-

ing data, formal contexts (U, A, V, f) and (U, D, W, g) are established.

(2) FCA: In this step, the Hesitant Fuzzy Linguistic Lattices (HFLLs) $HFLL(U, A, V, f)$ and $HFLL(U, D, W, g)$ are constructed using the generation tool of FCA, *Lattice Miner* [34], and the corresponding Hasse graphs are drawn.

(3) A new distance and similarity analyses: In this step, based on *NWD*, several similarity measures are considered for calculating the similarity between the attributes of the unknown monitoring points and concepts in the decision concept lattices. They are used as the weighting membership coefficients of the decision attribute.

(4) Transformation to BPAs: In this step, the weighted decision attribute is calculated to obtain the membership of the health levels, which is the BPA corresponding to an unknown monitoring point.

C. ESTABLISHMENT OF FORMAL CONTEXTS

When establishing formal contexts, the first step is to clarify the objects, attributes, and HFLTSs describing their relations. In dam health diagnosis, the set of objects U refers to all monitoring points. The set of attributes A refers to the qualitative and quantitative diagnosis indexes combined with monitoring data and expert knowledge.

As a scientific criterion to judge whether the engineering operation is normal, the diagnosis indexes can be divided into quantitative numerical and qualitative analysis indexes. The quantitative numerical indexes, which focus on the value size and change trend of the monitoring quantities at a single monitoring point, monitor the local health state. The qualitative analysis indexes integrate multi-source information of multi-points, multi-quantities, and inspection results, thereby monitoring the global health state. Cooperation between the two can provide a technical guarantee for safe engineering operations. Therefore, considering the above reasons and combining them with the method described in the literature [46], the conditional attributes are determined. These include the *Annual amplitude*, *Regularity of changes*, *Trend change*, *Coordination of multiple monitoring points*, *Correlation of multiple monitoring quantities*, and *Inspection conditions*. The decision attribute is the *Health evaluation level*. Among them, *Annual amplitude*, *Regularity of changes*, and *Trend change* are the diagnosis indexes of single monitoring points; *Coordination of multiple monitoring points* and *Correlation of multiple monitoring quantities* are the diagnosis indexes of multiple monitoring points. The HFLTSs for each attribute are presented in Tab. 1.

The selection and division of the attributes are simple descriptions of the monitoring points. The attributes contained in the formal context established in practical engineering applications should be selected in combination with the specific situation of monitoring quantities. Simultaneously, the listed attributes can be divided in more detail to improve accuracy. The determination of the type and number

TABLE 1. HFLTSs of attributes.

Type of attributes	Name of attributes	HFLTS
Conditional attributes	A: Annual amplitude	$\{a_1$ = Small, a_2 = General, a_3 = Large$\}$
	B: Regularity of changes	$\{b_1$ = Relatively consistent with the objective law, b_2 = General, b_3 = Less consistent with the objective law$\}$
	C: Trend change	$\{c_1$ = No significant trend, c_2 = Slight trend, c_3 = Significant trend$\}$
	D: Coordination of multiple monitoring points	$\{d_1$ = Relatively coordinated, d_2 = General, d_3 = Less coordinated$\}$
	E: Correlation of multiple monitoring quantities	$\{e_1$ = Weak, e_2 = General, e_3 = Strong$\}$
	F: Inspection conditions	$\{f_1$ = No significant abnormality, f_2 = Slight abnormality, f_3 = Significant abnormality$\}$
Decision attribute	H: Health evaluation level	$\{h_1$ = Normal, h_2 = Nearly normal, h_3 = Mildly abnormal, h_4 = Severely abnormal, h_5 = Malignant abnormal$\}$

of term sets depends on the actual situation. Tab. 1 is only an example.

The process of establishing a formal context includes the following steps: First, the objects and attributes that match an actual engineering project must be clarified. Then, experts are invited to discuss and evaluate the values and trend performances (attributes) of the selected monitoring points (objects) using the HFLTSs. By analyzing the evaluation results, formal contexts (U, A, V, f) and (U, D, W, g) are obtained, where U, A, and D represent the sets of objects, condition attributes, and decision attributes respectively, while V and W are the linguistic sets describing the binary relations between $U \times A$ and $U \times D$, respectively. f and g denote mappings from $U \times A$ to V, and $U \times D$ to W, respectively.

D. DISTANCE AND SIMILARITY ANALYSES

In this section, several similarities are introduced based on the *NWD*.

1) ATTRIBUTE SIMILARITY OF OBJECTS

For two objects u_x and u_y in a formal context (U, A, V, f), the HFLTSs of attribute A_i are S_i^x and S_j^y. The attribute similarity, $Sim_A(u_x, u_y)$, of u_x and u_y is defined as

$$[Sim_A(u_x, u_y)]^{-1} = Normal \sum_{i=1}^{m} \left[\alpha_i \cdot NWD(S_i^x, S_i^y)\right]. \quad (8)$$

Here, $NWD(S_i^x, S_i^y)$ is the *NWD* of the HFLTSs S_i^x and S_j^y (according to Eq. 7); m is the number of attributes, $1 \leq i \leq m$; α_i is the weight of attribute A_i among all the conditional attributes; and *Normal* indicates normalization of the result. Eq. 8 can be intuitively expressed as follows: first, the distance between two HFLTSs under the same attribute is calculated and then is weighted according to the attribute weight. The reciprocal of the normalization result is the attribute similarity of the two objects.

2) SIMILARITY OF ATTRIBUTES AND A CONCEPT

We consider the HFLTSs of all the conditional attributes of an unknown monitoring point u and their importance weights, α_i. A formal concept of $HFLL(U, D, W, g)$ is written as (X_j, Y_j) $(1 \leq j \leq N)$. Here, N is the number of concept nodes in $HFLL(U, D, W, g)$, where neither the set of attributes nor the set of objects is empty. NI is the number of objects in the Intent of the concept (X_j, Y_j). The similarity between the attributes of u and concept (X_j, Y_j) is defined as

$$Sim_{u,j} = \frac{1}{NI}\sum_{u_k \subseteq X_j}^{m} Sim_A(u, u_k). \tag{9}$$

By calculating the similarity of the attributes of u and every concept according to Eq. 9, the ordered similarity set $[Sim_{u,D}] = \{Sim_{u,1}, Sim_{u,2}, \ldots, Sim_{u,N}\}$ is obtained after sorting from largest to smallest. A threshold, e, is defined such that if the first N' similarities in the set $[Sim_{u,D}]$ satisfy $N'/N \leq e$, the N' similarities and their corresponding formal concepts are selected to the process of building a BPA.

3) BUILDING A BPA OF AN UNKNOWN MONITORING POINT

First, the HFLTS of each object u_k $(u_k \in U)$ in the formal decision context (U, D, W, g) is uniformly mapped to the membership degree $M(h_{k,L})$ $(L = 1\text{–}5)$ on [0, 1] corresponding to the health level h_L. For example, $\{h_1, h_2\} \rightarrow \{0.5, 0.5, 0, 0, 0\}$.

Within the concept (X_j, Y_j), $Sim_A(u,u_k)$ (Eq. 8) is used as the weighting coefficient to obtain the membership degree of the unknown monitoring point u to the health level h_L under the concept (X_j, Y_j).

$$m'(h_L)|(X_j, Y_j) = \sum_{u_k \subseteq X_j}^{NI} \left[M(h_{k,L}) \cdot Sim_A(u, u_k)\right]. \tag{10}$$

Within the N' concepts selected, $Sim_{u,j}$ (Eq. 9) is used as the weighting coefficient to obtain the membership degree of unknown monitoring point u to the health level h_L under the N' concepts.

$$m'(h_L) = \sum_{j, H_L \cap Y_j = h_L}^{N'} \left[Sim_{u,j} \cdot m'(h_L)|(X_j, Y_j)\right]. \tag{11}$$

The results of Eq. 11 are normalized to obtain the BPA of the unknown monitoring point u:

$$m(h_L) = Normal[m'(h_L)]. \tag{12}$$

The scheme for building a BPA is shown in Fig. 3, where the calculation process is based on establishing a formal context. Several key similarities are presented.

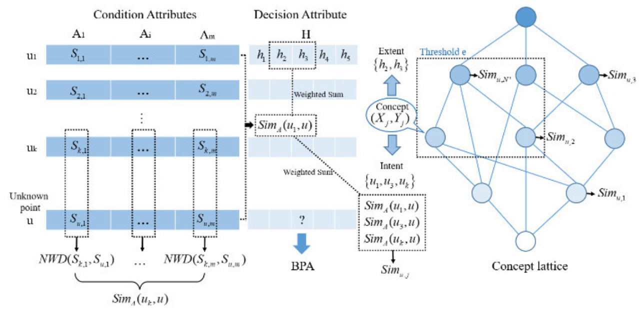


FIGURE 3. Scheme for building a BPA.

IV. ENGINEERING EXAMPLE

The project analyzed is a medium-sized hydropower station involved in flood control, irrigation, and aquaculture, in addition to hydropower generation. The hydropower station is primarily composed of a concrete gravity dam and a water diversion and power generation system, with a total power of 210 MW. The dam is divided into eight sections:1# and 2# are non-overflow dam sections near the left bank; 3#–7# are overflow sections; 7# is a dam section with a sediment discharge bottom outlet; 8# is a non-overflow dam section near the right bank.

To monitor dam safety in real-time, the dam is equipped with relatively perfect monitoring facilities, such as deformation, seepage, and stress–strain facilities, with a total of 352 monitoring points. The deformation monitoring items include horizontal displacement, vertical displacement, bedrock deformation and joint deformation, etc. Seepage monitoring items include foundation uplift pressure and seepage around the dam, etc. Stress–strain monitoring items include the concrete temperature and concrete strain, etc.

In this study, the monitoring data of the horizontal displacement is used as an example to illustrate the proposed method. The dam is equipped with two tension wire alignments and three groups of vertical lines to monitor horizontal displacement. The two tension wire alignments are located at the dam crest, and the embedded gallery of 365 m. The three groups of vertical lines are located at dam section #3 on the left bank, dam section #5 in the middle of the riverbed, and dam section #7 on the right bank, numbered 1#, 2#, and 3# from left to right.

Twelve monitoring points were selected as objects to establish the formal context, and the overall health status of eight monitoring points (EX1–EX8) of the tension wire alignments located on the dam crest was studied. Considering only one type of monitoring quantity in the monitoring data, four attributes, namely, A: *Annual amplitude*, B: *Regularity of changes*, C: *Trend change*, and D: *Coordination of multiple monitoring points*, were selected as conditional attributes in the formal context of horizontal deformation, without conditional attributes E: *Correlation of multiple monitoring quantities* and F: *Inspection conditions*. Experts were invited to evaluate the above four conditional attributes and

TABLE 2. **Formal context of dam health diagnosis.**

U/A	A			B			C		
	a1	a2	a3	b1	b2	b3	c1	c2	c3
$u1$	X			X			X		
$u2$			X	X			X		
$u3$	X				X				X
$u4$		X				X	X		
$u5$		X			X			X	
$u6$		X			X		X		
$u7$			X	X					X
$u8$			X			X		X	
$u9$		X		X					X
$u10$	X				X		X		
$u11$		X				X		X	
$u12$			X			X			X

Note: The rows in the table represent different objects ($u1$-$u12$); and the columns represent different attributes (A-D, H). For simplicity, the names of the attributes are listed in Tab. 1. An "X" at the intersection of a row and a column indicates that an object in the row has an attribute in the column.

TABLE 3. **Formal context of dam health diagnosis (Continued Table).**

U/A	D			H				
	d1	d2	d3	h1	h2	h3	h4	h5
$u1$	X			X				
$u2$		X			X			
$u3$	X				X	X		
$u4$	X			X	X			
$u5$		X				X		
$u6$	X			X	X			
$u7$			X				X	X
$u8$		X					X	
$u9$		X				X	X	
$u10$			X		X	X		
$u11$		X				X	X	
u12			X					X

the decision attribute H: Health evaluation level. The formal context is presented in Tab. 2 and Tab. 3.

Based on the conditional formal context (U, A, V, f) and decision formal context (U, D, W, g), we constructed $HFLL(U, A, V, f)$ and $HFLL(U, D, W, g)$ using *Lattice Miner* [34]. The Hasse graphs are shown in Fig. 4 and Fig. 5.

Among the four conditional attributes, A, B, and C are indexes of a single monitoring point, whereas D is a comprehensive indicator of multiple monitoring points. Considering that multiple points can better reflect the overall safety state of a certain part of the dam than a single monitoring point, attribute D has the greatest importance weight, α_D. Regularity

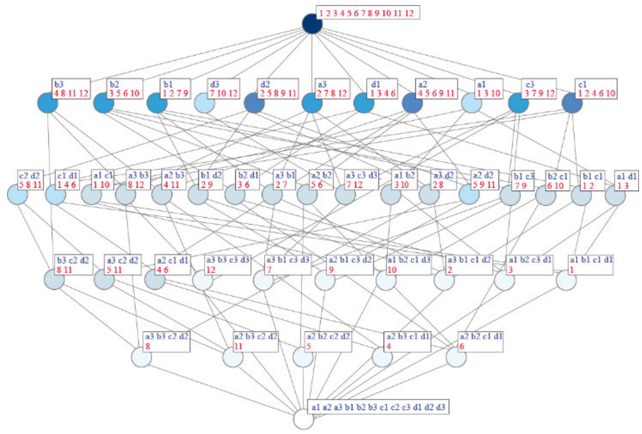


FIGURE 4. **Concept lattices of condition attributes. For simplicity, the names of objects (in red) are shortened to their sequence numbers, e.g. "u1" is shortened to "1." The code names of the attributes (in blue) are listed in Tab. 1. The concept of color intensity depends on the object count, with more objects corresponding to darker nodes.**

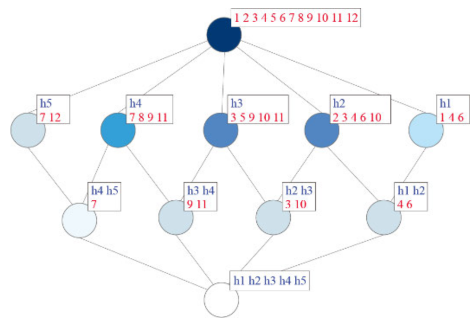


FIGURE 5. **Concept lattices of the decision attribute. For simplicity, the names of objects (in red) are shortened to their sequence numbers, e.g. "u1" is shortened to "1." The code names of the attributes (in blue) are listed in Tab. 1. The concept of color intensity depends on the object count, with more objects corresponding to darker nodes.**

and trend change are important indexes that determine the safety of a project and reflect spatiotemporal variations better than the absolute size of monitoring quantities. Changes in monitoring quantities that do not conform to the objective law or the occurrence of many sharp points and mutations indicate an anomaly in the change process of the monitoring quantities. If a monitoring quantity displays an abnormal condition of long-term non-convergent trend change, it contains information on potential insecurity in the monitoring sequence, which requires more attention. Therefore, the importance weights of the attributes A, B, and C are represented as $\alpha_C > \alpha_B > \alpha_A$. Hence, the importance weights of all the conditional attributes are identified as $\alpha_i = \{0.15, 0.20, 0.30, 0.35\}$. The HFLTSs for the conditional attributes of monitoring points EX1–EX8 are presented in Tab. 4.

As shown in Tab. 4, the same situations for the four monitoring points (EX2, EX3, EX5, and EX8) are not observed in the formal context (U, A, V, f). Therefore, it is necessary to build BPAs using the proposed method.

For example, the HFLTS of the conditional attributes of EX2 is $S_{\text{EX2}} = \{\{\text{a3}\},\{\text{b2}\},\{\text{c1}\},\{\text{d1}\}\}$. First, the distances

TABLE 4. HFLTSs for the conditional attributes of EX1–EX8.

Monitoring points	Conditional attributes				Existence in formal context
	A	B	C	D	
EX1	a3	b1	c1	d2	Yes, u2
EX2	a3	b2	c1	d1	No
EX3	a3	b3	c1	d1	No
EX4	a1	b2	c3	d1	Yes, u3
EX5	a1	b2	c2	d2	No
EX6	a1	b2	c1	d3	Yes, u10
EX7	a2	b1	c3	d2	Yes, u9
EX8	a1	b3	c2	d2	No

$NWD(S_{\text{EX2},S_{ui}})$ (i = 1–12) between S_{EX2} and the HFLTSs of the conditional attributes of objects (u_1–u_{12}) in (U, A, V, f) are calculated respectively according to Eq. 7. Then, the similarities $Sim_A(\text{EX2}, u_i)$ between EX2 and all the objects are calculated according to Eq. 8. There are nine concept nodes (neither the set of attributes nor the set of objects is empty) in $HFLL(U, D, W, g)$: ({1,4,6},$\{h_1\}$), ({4,6},$\{h_1, h_2\}$), ({2,3,4,6,10},$\{h_2\}$), ({3,10},$\{h_2, h_3\}$), ({3,5,9,10,11},$\{h_3\}$), ({9,11},$\{h_3, h_4\}$), ({7,8,9,11},$\{h_4\}$), ({7},$\{h_4, h_5\}$), and ({7,12},$\{h_5\}$). For the concept ({4,6},$\{h_1, h_2\}$), the *Intent* and *Extent* are $\{u_4, u_6\}$ and $\{h_1, h_2\}$, respectively $Sim_{\text{EX2},6}$ = 1/2× [$Sim_A(\text{EX2}, u_4)$ + $Sim_A(\text{EX2}, u_6)$. Subsequently, according to Eq. 9. Using this analogy, we obtain the similarities $Sim_{\text{EX2,j}}$ (j = 1–9) of EX2 and all the nine concepts. If e = 50%, the concept nodes involved in building the BPA are those whose $Sim_{EX2,j}$ are at the top four, namely, ($\{u_4, u_6\}$,$\{h_1, h_2\}$), ($\{u_1, u_4, u_6\}$,$\{h_1\}$), ($\{u_2, u_3, u_4, u_6, u_{10}\}$,$\{h_2\}$), and ($\{u_3, u_{10}\}$,$\{h_2, h_3\}$). For each of the four concept nodes, the membership degree $m'(h_L)|(X_j, Y_j)$ of EX2 to the health level h_L (L = 1–5) under the concept node is obtained according to Eq. 10. Next, the membership degrees of EX2 to the health levels under the four formal concepts are obtained using $Sim_{\text{EX2,j}}$ as the weighting coefficient, according to Eq. 11. Finally, the BPA of EX2 is obtained by normalizing the results of the final step.

The BPAs of EX3, EX5, and EX8 are calculated using the method described above. The calculation results are listed in Tab. 5 and Tab. 6.

Field experts are also invited to evaluate the health status of EX1–EX8 when building the formal context. The HFLTSs for the evaluation of the four monitoring points (EX2, EX3, EX5, and EX8) that are not included in the formal context are H_{EX2} = $\{h_1, h_2\}$, $H_{\text{EX3}} = \{h_2, h_3\}$, $H_{\text{EX5}} = \{h_2, h_3\}$, and H_{EX8} = $\{h_3, h_4\}$. The evaluation results above are broadly consistent with the BPAs listed in Tab. 6, indicating that the proposed method is feasible and practical. In addition, compared with the initial HFLTSs, the BPAs obtained using the proposed method are not simply uniformly distributed and provide more accurate and detailed beliefs at each evaluation level, laying a solid foundation for the subsequent fusion process.

TABLE 5. Calculation process and results for EX2, EX3, EX5, and EX8.

Object	Selected concept	
	(X_i, Y_j)	$Sim_{u,j}$
EX2	($\{u_1,u_4,u_6\}$,$\{h_1\}$)	19.11
	($\{u_4,u_6\}$,$\{h_1, h_2\}$)	26.67
	($\{u_2,u_3,u_4,u_6,u_{10}\}$,$\{h_2\}$)	15.67
	($\{u_3, u_{10}\}$,$\{h_2, h_3\}$)	5.833
EX3	($\{u_2,u_3,u_4,u_6,u_{10}\}$,$\{h_2\}$)	17.81
	($\{u_3,u_5,u_9,u_{10},u_{11}\}$,$\{h_3\}$)	9.13
	($\{u_4,u_6\}$,$\{h_1, h_2\}$)	9.524
	($\{u_3, u_{10}\}$,$\{h_2, h_3\}$)	15.00
EX5	($\{u_2, u_3, u_4, u_6, u_{10}\}$,$\{h_2\}$)	20.13
	($\{u_3,u_5,u_9,u_{10},u_{11}\}$,$\{h_3\}$)	21.41
	($\{u_3, u_{10}\}$,$\{h_2, h_3\}$)	40.00
	($\{u_9,u_{11}\}$,$\{h_3, h_4\}$)	6.86
EX8	($\{u_3,u_5,u_9,u_{10},u_{11}\}$,$\{h_3\}$)	22.93
	($\{u_7,u_8,u_9,u_{11}\}$,$\{h_4\}$)	15.91
	($\{u_3, u_{10}\}$,$\{h_2, h_3\}$)	10.67
	($\{u_9,u_{11}\}$,$\{h_3, h_4\}$)	26.67

TABLE 6. BPAs for EX2, EX3, EX5, and EX8.

Object	BPA				
	$m(h_1)$	$m(h_2)$	$m(h_3)$	$m(h_4)$	$m(h_5)$
EX2	0.46	0.52	0.02	0.00	0.00
EX3	0.04	0.73	0.23	0.00	0.00
EX5	0.00	0.48	0.51	0.01	0.00
EX8	0.00	0.00	0.43	0.56	0.01

TABLE 7. Results of fusion.

Health level	Normal	Nearly normal	Mildly abnormal	Severely abnormal	Malignant abnormal
	$m(h_1)$	$m(h_2)$	$m(h_3)$	$m(h_4)$	$m(h_5)$
BPA	0.0000	0.9146	0.0854	0.0000	0.0000

To diagnose the overall health status of horizontal displacement, the information of all the monitoring points, including the BPAs of the unknown points obtained by the method in this study, and those of points in the formal context provided by experts must be integrated. For more reasonable fusion results in case of the conflict of evidence, the method proposed in the literature [45] is adopted to fuse the BPAs of EX1–EX8 using Eq. 2. The final fusion results are listed in Tab. 7.

As shown in Tab. 7, the maximum degree of membership is at the level of "Nearly normal." Therefore, the overall health state of the horizontal displacement of the tension wire alignment in the dam crest of the hydropower station is diagnosed as "Nearly normal." Experts are invited to comprehensively evaluate the horizontal displacement of the eight monitoring points, and the obtained results are consistent with the calculated results in this study.

For the engineering project described in this section, the formal context contains only four types of attributes, and the number of divisions is small. The number of attributes and their divisions must be increased according to the actual needs to diagnose the health state of the dam more accurately. The number of monitoring points required to establish a formal context must also be adjusted accordingly.

Guo et al. [16] studies a cloud-model-based method to establish BPAs by assuming that the dependency relationship of monitoring values of the diagnosis indexes to each health level meets the distribution of the normal cloud model. Although their method is suitable for the practice of dam health diagnosis, the assumption of data distribution is rather strict. Moreover, the range division of individual diagnosis indexes is completely dependent on expert experience in Guo et al.'s method. The numerical performance and trend of each monitoring point in the monitoring data are not considered. Xu et al. [24] propose a method of constructing BPAs to apply in the classification problems. Their method also assume that the data are normally distributed and neglect the characteristics of dam health diagnosis. Although the construction process is objective, there is a lack of physical explanation for the relationship between BPA and sample features. In contrast to other methods for constructing BPAs, the proposed method does not require a specific distribution, and uses the hierarchy of concept lattices, which can determine the implication relation between the objective attributes of the monitoring points and health levels. This process includes the frequency of historical events as well as, more potential information, and is thus more suitable for engineering applications. In addition, in the case of large amounts of monitoring data in modern dam management, the proposed method can obtain relatively reliable underlying evaluation indexes for unknown monitoring points with less expert knowledge. This knowledge transfer can avoid repetitive consultation processes, improve work efficiency, and reduce the pressure of manual labeling.

V. CONCLUSION

In this study, we studied the measurement method of diagnosis indexes in dam health diagnosis. Based on the D–S evidence theory, a new concept lattices-based model for building BPAs was proposed. For consistency with the linguistic habits of experts in practical applications, we introduced HFLTS to express uncertainty and fuzziness effectively. Concept lattices explored the implication relation between the objective attributes of monitoring points and the health operation state and provided a new method for formalizing potential information. First, combined with the actual needs of dam health diagnosis, a formal context was established under HFLTSs. Then, we defined a new distance of HFLTSs (*NWD*) under the framework of concept lattices. Compared with other distance for HFLTSs, *NWD* is strict in math and considers the non-overlapping HFLTSs. FCA and several similarities based on *NWD* were used to calculate the similarity between the condition attributes and concept nodes of unknown monitoring points. It was then converted to the weights of the corresponding decision attributes, thus obtaining the final BPA for the fusion calculation. The results for the engineering example indicated that: (1) The BPAs obtained by the proposed method were consistent with the health state provided by experts in the field, indicating the feasibility of the proposed method. (2) Compared with initial HFLTSs, the proposed method provided a more detailed distribution of beliefs, which was convenient for fusion calculations using the D–S evidence theory. (3) The proposed method can avoid repetitive consultation processes, improve work efficiency, and adapt to the general situation of dam health diagnosis.

In conclusion, the new proposed method for measuring diagnosis indexes can meet the need of dam health diagnosis and shed light on the development of dam operation behavior modelling. In a future work, more interdisciplinary theories and methods will be involved in our study to improve the calculation speed and render the technique suitable to more applications.

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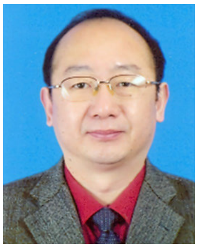

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