IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received 17 November 2023, accepted 23 January 2024, date of publication 2 February 2024, date of current version 8 February 2024.

Digital Object Identifier 10.1109/ACCESS.2024.3361830

RESEARCH ARTICLE

Optimal Flow Factor Determination in Vanadium Redox Flow Battery Control

ALEXANDER MOROZOV[1], MIKHAIL PUGACH[1], ANDREY POLYAKOV[2], PAVEL OSINENKO[1], ANTON BOLYCHEV[1], VLADIMIR TERZIJA[3], (Fellow, IEEE), AND SERGEI PARSEGOV[4,1]

[1]Skolkovo Institute of Science and Technology, 121205 Moscow, Russia
[2]Inria Lille-Nord Europe, 59650 Lille, France
[3]School of Engineering, Newcastle University, NE1 7RU Newcastle upon Tyne, U.K.
[4]Moscow Institute of Physics and Technology, Dolgoprudny, 141700 Moscow, Russia

Corresponding author: Sergei Parsegov (s.e.parsegov@gmail.com)

This work was supported by the Russian Science Foundation under Project 23-29-00807 (https://rscf.ru/en/project/23-29-00807/).

ABSTRACT The optimization of vanadium redox flow batteries (VRFBs) is closely related to the flow rate control: a proper regulation of the electrolyte flow rate reduces losses and prolongs battery lifetime. To this end, a *flow factor* control strategy in VRFBs was proposed in the literature provided with numerical/experimental validations. Yet, a theoretical justification of this approach was lacking. The respective control law is a generalization of Faraday's law of electrolysis since it employs a special scaling parameter referred to as the *flow factor*. In this paper, we show that this coefficient is directly related to the conversion rate of electrolyte in the cell. Furthermore, we pose an optimal control problem with maximization of total battery power integrated over time to determine an optimal flow factor. To this end, we use a simple stochastic policy gradient algorithm. The case studies illustrate the application of the computed optimal controller under various load currents and demonstrate that there is no single flow factor for all modes, the optimal performance of the battery can only be guaranteed by different values of this parameter. As a result, the proposed control strategy can be used for advanced control and monitoring tools for industrial VRFB systems.

INDEX TERMS Vanadium redox flow battery, flow rate control, optimal flow factor.

I. INTRODUCTION

High-capacity and high-power storage systems are becoming ever more relevant in modern power sector. This trend is driven by a number of transformations in the structure and operation of the electrical grid, namely the deployment of renewable energy sources, as well as the desire to optimize costs, depending on the changing energy prices, e. g., see [1]. Vanadium redox flow batteries (VRFBs), although being a relatively young technology, nevertheless have a number of advantages in this regard, i. e., good scalability and long cycling life and low capacity degradation, see [2] and [3].

Electrochemical reactions in redox flow batteries take place in liquid electrolytes rather than on solid electrodes as in traditional lithium-ion batteries. These electrolytes are stored in separate tanks, resulting in minimal self-discharge. However, the presence of a hydraulic system with pumps makes it challenging to maintain optimal operation. Power engineering journals have proposed various methods for controlling electrolyte flow that go beyond the traditional constant flow rate approach. The first "steps away" from constant flow rates, involved simple controllers switching pumps on/off, see [4]. Other approaches known from the literature include the use of sliding modes, model predictive and proportional-integral controllers, online optimization, fuzzy logic, etc. (see, *e.g.*, the following papers [4], [5], [6], [7], [8], [9], [10], [11], [12], [13]). However, it is most natural to use a feedback controller based on the scaling of the well-known Faraday's law of electrolysis using a gain referred to as the flow factor (or stoichiometric factor), e. g., see [14] and

The associate editor coordinating the review of this manuscript and approving it for publication was Akshay Kumar Saha.

references therein. The following recent review articles on vanadium batteries, their features and control, namely [15] and [16], should also be mentioned here.

It is worth noting that previous research on VRFBs has focused heavily on the flow factor parameter, often resulting in heuristic adjustments and modeling of the system's operation, along with consideration of loss functions. Several publications emphasize the significance of accurately selecting the flow factor value based on experimental findings, while others discuss the importance of the conversion rate parameter and offer suggestions for its selection. In our study, we establish a connection between these two parameters and justify one through an optimization problem, thereby justifying the other as well. At the same time, neither formulated nor solved are: the problem of studying the relation of the flow factor with VRFB system parameters (a), nor the problem of optimal gain selection (b). Our work aims to solve the outlined interrelated problems.

The paper is organized as follows. Section II introduces an overview of vanadium batteries, their operating principles and models. Besides, a flow factor based controller is described and the problem statement is formulated. The theoretical results that justify such a controller structure and reveal the relationship with battery parameters and conversion rate are given in Section III. Moreover, in the same section, an integral criterion for battery performance is formulated on the basis of the total battery power function derived by the authors. Based on this, the optimal control problem is solved to find the best flow factor in the sense of the introduced criterion. A simple stochastic policy gradient is employed to this end.

The theoretical results are accompanied with simulations and discussion presented in Section IV. Finally, conclusions are given.

II. PRELIMINARIES AND PROBLEM STATEMENT

A. VRFB STRUCTURE AND OPERATION

The diagram of a typical VRFB is presented in Fig. 1. The electrical energy is stored in electrolytes in two separate tanks, each tank has a volume V_{tk}. The electrolyte contains salts of vanadium dissolved in solutions of sulphuric acid. During the operation, the electrolyte is pumped into a stack consisting of cells with volume V_c. Thus, the total volume of the stack is $n_c V_c$, where n_c is a number of cells. The electrochemical reactions in a cell can be described as follows [17]:

$$\text{Negative electrode: } V^{3+} + e^- \leftrightarrow V^{2+}, \tag{1}$$

$$\text{Positive electrode: } VO^{2+} + H_2O \leftrightarrow VO_2^+ + 2H^+ + e^-. \tag{2}$$

Considering the reactions described above and taking into account mass-conservation law in all compartments of the battery, one can obtain a dynamical model for vanadium ions concentrations in the form of ordinary differential equations, presented in the next subsection.

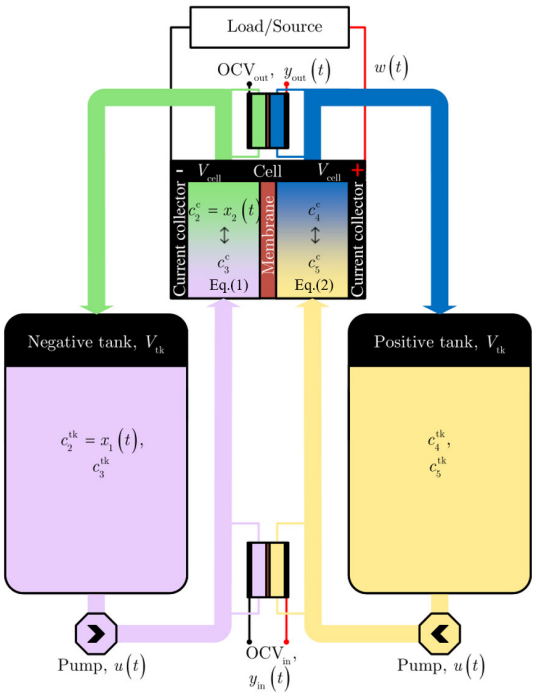


FIGURE 1. VRFB-based storage configuration extended with two OCV cells. Here, c_2, c_3, c_4, and c_5 are concentrations of the corresponding ions in the tanks and half-cells.

B. SECOND-ORDER MODEL OF VRFB

In this subsection we will consider the second-order model that describes the dynamics of vanadium redox flow battery considering evolution of one of vanadium ions in the cell and in the tank. This model can be considered as one of the most simple while it can provide accurate results within several cycles [18], taking into account the influence of external conditions (load current $w(t) \in \mathbb{R}$), as well as the mixing rate determined by the battery size and electrolyte flow rate $u(t) \geq 0$:

$$\begin{cases} \dot{x}_1 = (-\alpha x_1 + \alpha x_2)u, \\ \dot{x}_2 = (\beta x_1 - \beta x_2)u + dw, \end{cases} \tag{3}$$

where x_1, x_2 are the concentrations of V^{2+} in the tanks and stack, respectively, $\alpha = 1/V_{tk}$, $\beta = 1/(n_c V_c)$, and $d = 1/(FV_c)$. The volume of the tank V_{tk} is typically greater than the volume of the stack $V_{st} = n_c V_c$, therefore $\alpha < \beta$.

In addition, we will consider the system also that includes two special cells to measure open-circuit voltages at the input and output of the tanks (OCV_{in} and OCV_{out} cells in Fig. 1, respectively), which are defined by the Nernst equation and have the following form:

$$y_{in} = y_0 + 2y_1 \ln \frac{x_1}{c_b - x_1}, \tag{4}$$

$$y_{out} = y_0 + 2y_1 \ln \frac{x_2}{c_b - x_2}, \tag{5}$$

where the constant coefficients y_0, y_1 are known: y_0 is the formal potential ($y_0 = 1.4\ V$) and $y_1 = RT/F$; $R =$

8.314 $\frac{\text{J}}{\text{K mol}}$ is the gas constant, $F = 96485.332\ \frac{\text{C}}{\text{mol}}$ is the Faraday constant, and $T = 298$ K is the temperature.

Thus, the states of the system can be found as follows:

$$x_1 = \frac{c_b e^{\frac{y_{in}-y_0}{2y_1}}}{1+e^{\frac{y_{in}-y_0}{2y_1}}}, \quad x_2 = \frac{c_b e^{\frac{y_{out}-y_0}{2y_1}}}{1+e^{\frac{y_{out}-y_0}{2y_1}}}. \tag{6}$$

The parameters of the lab-scale VRFB setup used to solve the problem of determining the optimal controller are listed in Table 1, see also [19].

TABLE 1. Parameters of the VRFB setup.

Parameter	Unit	Value
Formal potential, y_0	V	1.4
Tank volume, V_{tk}	m^3	$4 \cdot 10^{-4}$
Total concentration, c_b	$\text{mol} \cdot \text{m}^{-3}$	2000
Initial concentration, $x_1(0) = x_2(0)$	$\text{mol} \cdot \text{m}^{-3}$	1600
Final concentration, x_2^*	$\text{mol} \cdot \text{m}^{-3}$	400
Number of cells, n_c	-	10
Half-cell volume, V_c	m^3	$7.5 \cdot 10^{-6}$
Membrane surface area, A_m	m^2	0.15
Cross-section area of electrode, A_{ed}	m^2	$9 \cdot 10^{-4}$
Mass transfer coefficient, ν	-	$1.6 \cdot 10^{-4}$
Cell length, L	m	0.5
Electrolyte viscosity, μ	$\text{Pa} \cdot \text{s}$	$4.928 \cdot 10^{-3}$
Pump efficiency, ε_p	-	0.9
Average square resistance, $\bar{R}$	$\Omega \cdot \text{m}^2$	$2 \cdot 10^{-4}$

C. ELECTROLYTE FLOW RATE CONTROL

As noted in the introduction, flow rate control is an important problem because over- and under-supply of electrolyte can lead to energy losses as well as malfunction and failure due to more significant effects of reversible and irreversible parasitic processes, such as crossover of ions across the membrane, water transport, hydrogen release, etc. As in any control task, we must first decide on the type of controller.

A special class of feedback control laws based on a scaling of Faraday's law of electrolysis should be distinguished here. This law allows determining the sufficient flow rate for chemical conversion of all reactants. As a result, both the state of the system and the external perturbation are taken into account (it should be noted that the load current is always measured, as it is necessary for proper operation of the battery and for monitoring of the overall battery state-of-charge [20]). In practical applications the Faraday's law is usually extended with multiplication factor (known as the flow factor), that is normally attributed to additional hydraulic losses in the VRFB system [19], [21].

The flow factor based control law is as follows

$$u = \begin{cases} \kappa_1(x_1) = -f\dfrac{n_c w}{F x_1} & \text{during discharge, } w < 0, \\ \kappa_2(x_1) = f\dfrac{n_c w}{F(c_b - x_1)} & \text{during charge, } w > 0. \end{cases} \tag{7}$$

When $f = 1$, we get the classical Faraday's law, which states that *the amount of chemical change being produced by a current at an electrode-electrolyte interface is proportional to the quantity of electricity used*, e. g., see [22] and [23] for the details.

The works devoted to the flow factor are reduced to the selection of some value corresponding to some good or acceptable behavior of the lab setup. The relationship of this variable to the parameters of the battery, to the best of authors' knowledge, has not yet been discussed in any way.

In this paper, we pursue two goals, namely

- the analytical derivation of the relation between the flow factor and the conversion rate;
- the optimal choice of this parameter to maximize the overall battery power, taking into account internal losses.

III. MAIN RESULTS

A. THE MEANING OF THE FLOW FACTOR

To interpret the flow factor and study its relation to the asymptotic dynamics of the system and its parameters, it is important to introduce the following definition.

Definition 1: The conversion rate (or conversion per pass) equals the amount of active ions participating in the electrochemical reactions during a single-pass of electrolyte through the cell fraction. Within our model, this parameter is defined as follows:

$$\gamma = \begin{cases} \dfrac{x_1 - x_2}{x_1} & \text{for discharge, } w < 0, \\ \dfrac{x_2 - x_1}{c_b - x_1} & \text{for charge, } w > 0. \end{cases} \tag{8}$$

It should be noted that for safe battery operation it is important to keep the conversion rate γ significantly less than unity, because this ensures the availability of the necessary amount of reactants in the cell in the case of a sudden change in load current w.

Next, we demonstrate that the flow factor control strategy is a special nonlinear feedback control law aimed to keep a certain value of the conversion rate γ in system (3).

Let us denote a desired conversion rate by $\gamma^* \in (0,\ 1]$ and suppose the flow rate $u > 0$ to have no upper bound. We are now poised to show the following relation of the flow factor to the conversion rate (cf. [24]):

Theorem 1: For any initial condition $\gamma(0) \in (-\infty,\ 1]$, the conversion rate $\gamma(t)$ of system (3) converges asymptotically to γ^*, if the control u obeys (7) with f selected as follows

$$f = \frac{1}{\gamma^*((1-\gamma^*)\frac{n_c V_c}{V_{tk}} + 1)}. \tag{9}$$

Proof: For the sake of simplicity we consider the discharge mode only, $w < 0$. The results for the charge can be proven in the same way.

The proof is based on the analysis of the behavior of γ and its stability properties with respect to the adjustable parameter $k = \frac{fn_c}{F}$ introduced for convenience.

Since $u > 0$, let us introduce new time scale in (3), such that

$$d\tau = u^{-1}dt \Rightarrow u d\tau = dt \tag{10}$$

and then change the variables like so:

$$\dot{x_1} = -\alpha z, \quad \dot{z} = -(\alpha + \beta)z - d\frac{w}{u}, \tag{11}$$

where $z = x_1 - x_2$.

Substituting the control signal u by $u = -k\frac{w}{x_1}$, $k > 0$, into (11) we find

$$\dot{x_1} = -\alpha z, \quad \dot{z} = cx_1 - (\alpha + \beta)z, \quad c = d/k. \tag{12}$$

The dynamics of (12) does not depend neither on u, nor on w. According to (8), $\gamma = z/x_1$, therefore

$$\dot{\gamma} = \frac{\dot{z}}{x_1} - \frac{z\dot{x_1}}{x_1^2}, \tag{13}$$

or

$$\dot{\gamma} = c - (\alpha + \beta)\gamma + \alpha\gamma^2. \tag{14}$$

Let us find stable equilibria of the last system. The following equation

$$\alpha\gamma^2 - (\alpha + \beta)\gamma + c = 0 \tag{15}$$

has the roots:

$$\gamma_{1,2} = \frac{1}{2\alpha}(\alpha + \beta \pm \sqrt{(\alpha + \beta)^2 - 4\alpha c}). \tag{16}$$

From the linearized system it follows that we have a pair of stable/unstable fixed points, where a locally stable equilibrium is

$$\gamma = \gamma_1 = \frac{1}{2\alpha}(\alpha + \beta - \sqrt{(\alpha + \beta)^2 - 4\alpha c}) \tag{17}$$

provided that

$$(\alpha + \beta)^2 - 4\alpha c > 0, \quad \text{or, equivalently,} \quad c < \frac{(\alpha + \beta)^2}{4\alpha}.$$

Let $V = \frac{1}{2}(\gamma - \gamma_1)^2$ be a Lyapunov function candidate. The set Γ where $V > 0, \dot{V} < 0\ \forall\gamma \in \Gamma$, except for γ_1, where $\dot{V}(\gamma_1) = 0$, is the domain of attraction we are seeking for. Evidently, the function V is positive $\forall\gamma \neq \gamma_1$. It can be shown, that $\dot{V} \leq 0\ \forall\gamma \leq \gamma_2$, since $\dot{V} = \alpha(\gamma - \gamma_1)^2(\gamma - \gamma_2)$. On the other hand, all the initial conditions $\gamma(0) \geq \gamma_2$ have no physical meaning, due to the fact that $\gamma_2 > 1$. Since $\gamma_1 \leq 1$, we arrive at the condition on k to be satisfied:

$$\sqrt{(\alpha + \beta)^2 - 4\alpha c} \geq \beta - \alpha \tag{18}$$

$$(\alpha + \beta)^2 - 4\alpha c \geq (\beta - \alpha)^2 \tag{19}$$

$$\alpha^2 + 2\alpha\beta + \beta^2 - 4\alpha c \geq \beta^2 - 2\alpha\beta + \alpha^2 \tag{20}$$

$$\beta \geq c \tag{21}$$

$$k \geq \frac{d}{\beta}, \tag{22}$$

or equivalently

$$f \geq 1. \tag{23}$$

The last inequality illustrates the fact, that the minimal required flow rate is defined by Faraday's law of electrolysis, $f = 1$. Thus, we can conclude, that for any initial condition $\gamma(0) \in (-\infty,\ 1]$ the solution of (14) converges to stable equilibrium γ_1 if condition (23) is satisfied. Let us now look at system (14) and find the flow factor as a function of the target conversion rate γ^*. Evidently, the stable equilibrium point $\gamma_1 = \gamma^*$ can be found from

$$\gamma^* k((1 - \gamma^*)\alpha + \beta) - d = 0, \tag{24}$$

that gives

$$k = \frac{d}{\gamma^*((1 - \gamma^*)\alpha + \beta)}. \tag{25}$$

Finally, we find the flow factor f as follows:

$$f = \frac{1}{\gamma^*((1 - \gamma^*)\frac{n_c V_c}{V_{tk}} + 1)}. \tag{26}$$

Evidently, for any $\gamma^* \in (0,\ 1]$ the following inequality holds:

$$f \geq 1. \tag{27}$$

This completes the proof. ■

A detailed analysis of function (9), its sensitivity to the battery size and the desired conversion ratio, is given in [24].

Remark 1: As follows from relation (9), the control in accordance with Faraday's law is a special case for the target conversion rate equal to one, $\gamma^* = 1$.

Remark 2: The obtained result shows that the flow factor does not need to be selected empirically, it can be calculated, knowing the parameters of the system and the desired conversion rate. The resulting control law neither takes into account the practical constraints on the control signal, nor the constraints on the states/disturbances. On the one hand, this one is limiting and confusing: in this case, at the end of the discharge, when $x_1 \to 0$, the flow rate grows infinitely. On the other hand, in practice, the system operates over a range of states-of-charge $\approx 20\% - 80\%$, thus never reaching extremely low x_1 values.

The derived relationship between the gain f and the conversion rate γ^* sheds light on the sense of the flow factor, but does not provide guidelines for its selection. In what follows, we present an approach to determining the optimal values of the flow factor.

B. TOTAL BATTERY POWER FUNCTION

To derive the total power function of the VRFB system, it is first necessary to consider the total battery voltage function taking losses into account. The battery voltage y_{bat} can be represented as the sum of the equilibrium potential $y_{eq} = y_{out}$ and the internal losses (ohmic and concentration losses), see [25]:

$$y_{bat} = n_c(y_{eq} + sign(w)(y_{ohm} + y_{conc})), \tag{28}$$

where $w > 0$ when charging the battery and $w < 0$ when discharging. Given the notations used in this paper, we can write the voltage function for the discharge process (see [26])

as follows:

$$y_{bat} = n_c(y_0 + 2y_1 \ln \frac{x_2}{c_b - x_2} - 0.5\bar{R}A_m|w| - 2y_1 \left| \ln \left| 1 - \frac{|w|A_{ed}^{0.4}}{\nu F x_2 u^{0.4}} \right| \right|). \quad (29)$$

Thus, the total power of the VRFB storage can be calculated as the difference of the stack electrical power equal to the product of the battery voltage by the load current, and the power consumed by the pumps, which can be obtained from the pump loss model proposed in [11]:

$$P = n_c \Big((y_0 + 2y_1 \ln \frac{x_2}{c_b - x_2})|w| - 0.5\bar{R}A_m w^2 - 2y_1 \left| \ln \left| 1 - \frac{|w|A_{ed}^{0.4}}{\nu F x_2 u^{0.4}} \right| \right| |w| \Big) - 3\frac{\mu L u^2}{n_c \kappa A_{ed} \varepsilon_p}. \quad (30)$$

We now regard (30) as the instantaneous *reward*.[1] With this at hand, let us formulate the following optimal control problem:

$$\max_f J_t(f) \quad (31)$$

which depends on the flow factor f as follows:

$$J_t(f) := \int_0^{\bar{T}} P(x_1, x_2, w, f, t)dt \quad s.t. \ (3), (7), \quad (32)$$

where in turn $\bar{T}$ is the time at which the state x_2 (concentration in the cell) reaches a value of x_2^* (corresponding to the state-of-charge of 20%). Basically, we consider the problem of total reward optimization.

Note that unlike the most related papers [10], [19], [21], our approach assumes automatic synthesis of the controller (without manual adjustment of f), and aims at the maximization of the available stored energy $J_t(f)$ as a criterion, see Table 2. Along with that, we also derive the relationship between the flow factor and the desired conversion rate.

TABLE 2. Overview of the control strategies.

Reference	Design	f as $f(\gamma^*)$	Performance indicator
[19]	manual	-	loss function
[21]	manual	-	round-trip efficiency
[10]	manual	-	conversion rate tracking error
Our approach	automatic	+	available stored energy

We now proceed to the description of the routine to compute the optimal flow factor using a simple policy gradient algorithm.

[1] We do not consider the losses separately as by reducing the losses through adjusting the gain, one can also reduce the overall output power. That is why it is necessary to consider the difference between the product of the battery OCV by the current, and the power of losses.

TABLE 3. Results for different loads.

w	−70 A	−95 A	−120 A	−145 A	−170 A
f	11.02	10.37	9.62	9.1	9
γ^*	0.083	0.091	0.096	0.101	0.102
$\mathbf{J}_\mathbf{t}^*$, MW·s	33.7	33.5	33.4	33.3	33.2
N	10^4	$9 \cdot 10^3$	$6 \cdot 10^3$	$5 \cdot 10^3$	$4 \cdot 10^3$

C. COMPUTATION OF OPTIMAL FLOW FACTOR

To solve the problem of selecting the optimal controller gain f, we will use typical load profiles, similarly to those in [19]. We maximize, for each current profile, (32) following a basic policy gradient routine (see [27], [28]). We take the control law (7) as the basis and add a small exploration noise to it. Specifically, we consider the policy as the following probability distribution:

$$\rho^f(u|x) = \mathcal{N}(\kappa_1(x_1), \sigma^2), \quad (33)$$

where σ is the exploration noise standard deviation. We then sample actions from ρ^f at each control time step. One could take a plain deterministic policy, but it would require computation of the gradients of the total rewards which is, on one hand, cumbersome and faces nonsmoothness issues, on the other. This is seen from the expression (29). Using the stochastic policy ρ^f, we can avoid computation of the gradient of the rewards and only use gradients of the policy itself, which is straightforward from the (33) formula. Summarizing, the flow factor learning amounts to the following:

$$f_{i+1} \leftarrow f_i + \alpha_{\text{learn}} \sum_{j=0}^{J} P_j \nabla_f \rho_j^{f_i}, \quad (34)$$

where i is the learning iteration index, α is the (in general iteration-varying) learning rate, j is the time step index running up to J which corresponds to $\bar{T}$. Notice in formula (34), we do not use the gradients of the rewards, only the values at time steps j themselves, i. e., P_j. Only the gradients of the policy, i. e., $\rho_j^{f_i}$ are used.

IV. SIMULATIONS AND DISCUSSION

We programmed the vanadium redox flow battery setup along with the optimal flow factor learning algorithm using the Python programming language (3.9.16) and a workstation with a 3,3 GHz CPU, 16 Gb RAM and 16 Gb video RAM GPU. The maximum number of learning iterations was set equal 50, whereas σ was set equal $1.67 \cdot 10^{-7}$. We have examined the system on a single discharge cycle from 80% to

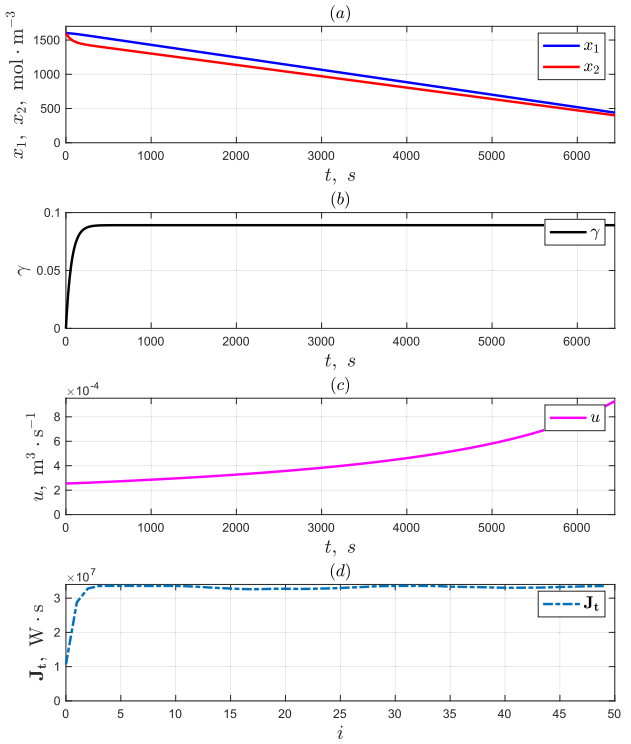


FIGURE 2. Evolution of concentrations x_1, x_2 (a); evolution of conversion rate γ (b); flow rate u (c); total reward learning behavior J_t (d). Load current $w = -95$ A.

20%.[2] This corresponds to the initial concentrations $x_1(0) = x_2(0) = 1600$ mol · m $^{-3}$ and final concentration $x_2^* = 400$ mol · m^{-3}. A range of loads w allowed for this battery was considered, in increments of 25 A. The constant current load profiles were taken as the most representative ones for analyzing the effective operation of the storage system. Table 3 shows the main parameters of the optimizations performed to find the optimal flow factor f, target conversion rate γ^*, best learned total reward J_t^*, number of iterations N.

Example battery discharge dynamics under a load current $w = -95$ A and optimal learned flow factor can be seen in Figure 2. It can be seen that the concentrations decrease as the battery discharges, with the electrolyte flow rate (control u) increasing significantly as the battery approaches a 20% discharge (by more than four times compared to the initial value). At the same time, the conversion ratio tends to the equilibrium state $\gamma^* = 0.091$; convergence to the neighborhood of the desired value γ^* is achieved in $t \approx 200$ seconds.

Note that in the known literature some universal values of flow factor f, equally suitable for all load currents, are proposed. For example, in [19] an interval of values $7 - 8$ is given. A comparative analysis with regard to our proposed criterion shows that e. g., at $w = -70$ A and $f = 7.5$ the system operates for three minutes less than with the flow factor we found equal to 11.02; the stored available energy $J_t(7.5) = 33.2 \cdot 10^6$ is also lower in this case.

[2]Note that the developed method is equivalently applicable for controller design in discharge and charge modes (in general case for any current profile). In calculations and simulations, the discharge mode is considered for convenience.

V. CONCLUSION

In this work, a design of an electrolyte flow rate controller for a vanadium redox flow battery based on the flow factor concept was suggested. It is a special feedback controller that takes into account the measured disturbance (load current). It found application in the control of electrolyte flow rate in redox flow batteries, but its use was limited to the experimental selection of the gain (flow factor) providing correct operation of the battery. The task of studying the relation of this gain to the system parameters and its influence on the asymptotic behavior was neither posed nor solved. The problem of optimal adjustment of the flow factor was not solved either. This paper gives answers to these questions. Firstly, it was shown that this factor was not just an arbitrary adjustable parameter but was closely related to the conversion rate of the battery. Next, it was demonstrated that the problem of finding the best flow factor could be posed and solved as an optimal feedback control problem. The available stored energy (total battery power integrated over time) was derived and used as the objective function, whereas typical load profiles were considered in the design and simulations. A simple stochastic policy gradient algorithm was employed to compute an optimal flow factor. To conclude, the proposed control law can be used for advanced control and monitoring tools for VRFB storages. Previous research on the given topic was regarded to selecting a flow factor based on either the empirical suggestions, or based on the conversion coefficient; in our work we clearly stated the interconnection between these two parameters and suggested a method, which would allow optimal computation for both of them. It is important to note that the suggested method and the results were obtained only for a determined set of load profiles, while it is planned to consider time-varying load profiles $w(t)$ and solve optimization problems for these cases, the practical implementation and validation of the proposed control is also expected.

REFERENCES

[1] A. Lucas and S. Chondrogiannis, "Smart grid energy storage controller for frequency regulation and peak shaving, using a vanadium redox flow battery," *Int. J. Electr. Power Energy Syst.*, vol. 80, pp. 26–36, Sep. 2016.

[2] M. Aneke and M. Wang, "Energy storage technologies and real life applications—A state of the art review," *Appl. Energy*, vol. 179, pp. 350–377, Oct. 2016.

[3] V. Viswanathan, A. Crawford, D. Stephenson, S. Kim, W. Wang, B. Li, G. Coffey, E. Thomsen, G. Graff, P. Balducci, M. Kintner-Meyer, and V. Sprenkle, "Cost and performance model for redox flow batteries," *J. Power Sources*, vol. 247, pp. 1040–1051, Feb. 2014.

[4] R. L. Largent, M. Skyllas-Kazacos, and J. Chieng, "Improved PV system performance using vanadium batteries," in *Proc. Conf. Rec. 23rd IEEE Photovoltaic Spec. Conf.*, May 1993, pp. 1119–1124.

[5] C. Y. Ling, H. Cao, M. L. Chng, M. Han, and E. Birgersson, "Pulsating electrolyte flow in a full vanadium redox battery," *J. Power Sources*, vol. 294, pp. 305–311, Oct. 2015.

[6] H.-F. Shen, X.-J. Zhu, M. Shao, and H.-F. Cao, "Neural network predictive control for vanadium redox flow battery," *J. Appl. Math.*, vol. 2013, pp. 1–7, 2013.
[7] S. König, M. R. Suriyah, and T. Leibfried, "Innovative model-based flow rate optimization for vanadium redox flow batteries," *J. Power Sources*, vol. 333, pp. 134–144, Nov. 2016.
[8] M. Averbukh, A. Pozin, and S. Sukoriansky, "Electrolyte pumping optimization in already manufactured vanadium redox battery based on experimentally determined electrical and hydrodynamic losses," *J. Energy Eng.*, vol. 143, no. 2, pp. 1–12, Apr. 2017.
[9] Y. Li, X. Zhang, J. Bao, and M. Skyllas-Kazacos, "Studies on optimal charging conditions for vanadium redox flow batteries," *J. Energy Storage*, vol. 11, pp. 191–199, Jun. 2017.
[10] Y. Li, X. Zhang, J. Bao, and M. Skyllas-Kazacos, "Control of electrolyte flow rate for the vanadium redox flow battery by gain scheduling," *J. Energy Storage*, vol. 14, pp. 125–133, Dec. 2017.
[11] M. Pugach, S. Parsegov, E. Gryazina, and A. Bischi, "Output feedback control of electrolyte flow rate for vanadium redox flow batteries," *J. Power Sources*, vol. 455, Apr. 2020, Art. no. 227916.
[12] R. Badrinarayanan, K. J. Tseng, B. H. Soong, and Z. Wei, "Modelling and control of vanadium redox flow battery for profile based charging applications," *Energy*, vol. 141, pp. 1479–1488, Dec. 2017.
[13] B. Xiong, J. Tang, Y. Li, C. Xie, Z. Wang, X. Zhang, and H. Beng Gooi, "Design of a two-stage control strategy of vanadium redox flow battery energy storage systems for grid application," *IEEE Trans. Sustain. Energy*, vol. 13, no. 4, pp. 2079–2091, Oct. 2022.
[14] A. Trovò, "Battery management system for industrial-scale vanadium redox flow batteries: Features and operation," *J. Power Sources*, vol. 465, Jul. 2020, Art. no. 228229.
[15] A. Aluko and A. Knight, "A review on vanadium redox flow battery storage systems for large-scale power systems application," *IEEE Access*, vol. 11, pp. 13773–13793, 2023.
[16] H. Wang, S. A. Pourmousavi, W. L. Soong, X. Zhang, and N. Ertugrul, "Battery and energy management system for vanadium redox flow battery: A critical review and recommendations," *J. Energy Storage*, vol. 58, Feb. 2023, Art. no. 106384.
[17] M. Pugach, M. Kondratenko, S. Briola, and A. Bischi, "Numerical and experimental study of the flow-by cell for vanadium redox batteries," *Energy Proc.*, vol. 142, pp. 3667–3674, Dec. 2017.
[18] S. Bogdanov, M. Pugach, S. Parsegov, V. Vlasov, F. M. Ibanez, K. J. Stevenson, and P. Vorobev, "Dynamic modeling of vanadium redox flow batteries: Practical approaches, their applications and limitations," *J. Energy Storage*, vol. 57, Jan. 2023, Art. no. 106191.
[19] A. Tang, J. Bao, and M. Skyllas-Kazacos, "Studies on pressure losses and flow rate optimization in vanadium redox flow battery," *J. Power Sources*, vol. 248, pp. 154–162, Feb. 2014.
[20] S. Parsegov, M. Pugach, V. Erofeeva, F. Ibáñez, and V. Terzija, "Critical assessment of the explicit methods for state of charge calculation in vanadium redox flow batteries," in *Proc. IEEE Belgrade PowerTech*, Jun. 2023, pp. 1–6.
[21] M. Guarnieri, A. Trovò, and F. Picano, "Enhancing the efficiency of kW-class vanadium redox flow batteries by flow factor modulation: An experimental method," *Appl. Energy*, vol. 262, Mar. 2020, Art. no. 114532.
[22] M. Faraday, "Experimental researches in electricity. Seventh series," *Phil. Trans. Roy. Soc. London*, vol. 124, pp. 77–122, Dec. 1834.
[23] B. Sundén, *Hydrogen, Batteries and Fuel Cells*. Cambridge, MA, USA: Academic, 2019.
[24] S. Parsegov, M. Pugach, A. Polyakov, and F. Ibáñez, "Analysis of flow factor control strategy in vanadium redox flow batteries," *IFAC-PapersOnLine*, vol. 55, no. 9, pp. 187–192, 2022.
[25] M. Pugach, M. Kondratenko, S. Briola, and A. Bischi, "Zero dimensional dynamic model of vanadium redox flow battery cell incorporating all modes of vanadium ions crossover," *Appl. Energy*, vol. 226, pp. 560–569, Sep. 2018.
[26] A. A. Kurilovich, A. Trovò, M. Pugach, K. J. Stevenson, and M. Guarnieri, "Prospect of modeling industrial scale flow batteries–from experimental data to accurate overpotential identification," *Renew. Sustain. Energy Rev.*, vol. 167, pp. 1–19, Feb. 2022.
[27] R. S. Sutton and A. G. Barto, *Reinforcement Learning: An Introduction*. Cambridge, MA, USA: A Bradford Book, 2018.
[28] D. Silver, G. Lever, N. Heess, T. Degris, D. Wierstra, and M. Riedmiller, "Deterministic policy gradient algorithms," in *Proc. Int. Conf. Mach. Learn.*, 2014, pp. 387–395.

ALEXANDER MOROZOV received the Bachelor of Engineering (B.E.) degree in mechatronics and robotics from Bauman Moscow State Technical University, in 2015, and the Master of Science (M.Sc.) degree in computer science from the Skolkovo Institute of Science and Technology (Skoltech), where he is currently pursuing the Ph.D. degree with the Artificial Intelligence in Dynamic Action Laboratory. Then, he joined the Center for Computational and Data-Intensive Science and Engineering, Skoltech, to continue his study to the Doctor of Philosophy (Ph.D.) as a member of the iMolecule Laboratory. His current research interests include the area of deep learning and multi-agent reinforcement learning.

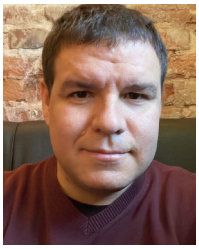
MIKHAIL PUGACH received the master's degree in applied mathematica and physics from the Moscow Institute of Physics and Technology (MIPT), in 2015, and the Ph.D. degree in control of engineering systems from the Skolkovo Institute of Science and Technology (Skoltech), in 2020. In 2016, he joined the Energy Center, Skoltech, where he developed numerical methods for mathematical modeling of physico-chemical processes in various media focusing on modeling of internal processes in energy storage systems based on vanadium redox flow batteries (VRFBs). He developed a dynamic model of VRFBs, with detailed modeling of ion crossover through the membrane. He also investigated the limits of applicability for different modeling approaches of VRFB systems, energy efficiency of such systems under various operating conditions, and challenges related to electrolyte flow control for fail-safe operation of VRFB systems under dynamic load conditions.

ANDREY POLYAKOV received the Ph.D. degree in systems analysis and control from Voronezh State University, Voronezh, Russia, in 2005. From 2004 to 2010, he was a Lecturer and an Associate Professor with Voronezh State University. In 2007 and 2008, he was worked with the CINVESTAV-IPN Center, Mexico City. From 2010 to 2013, he was a Leader Researcher of the Institute of the Control Sciences, Russian Academy of Sciences. In 2013, he was with the Inria, Lille, France. He has coauthored more than 150 publications in control theory and applications, including the books *Attractive Ellipsoids in Robust Control* (Springer, 2014), *Road Map for Sliding Mode Control Design* (Springer, 2020), and *Generalized Homogeneity in Systems and Control* (Springer, 2020). His research interests include the different aspects of nonlinear control and estimation theory, such as finite-time fixed-time stability, generalized homogeneity, sliding modes, and Lyapunov methods for both finite dimensional and infinite dimensional systems.

PAVEL OSINENKO received the diploma (Hons.) in control systems from Bauman Moscow State Technical University, Moscow, Russia, in 2009. In 2014, he defended the Ph.D. thesis in engineering at the Dresden University of Technology. His dissertation work is dedicated to optimal traction control and vehicle dynamic parameter identification. From 2011 to 2020, he worked at German Academic and Industrial Sectors, including such places as Fraunhofer Institute and Chemnitz University of Technology, where he was responsible for project supervision, research coordination, Ph.D. student advising, administration and teaching. Since 2020, he has been an Assistant Professor at Skoltech, Moscow. He reads courses in reinforcement learning and advanced control. His main research interests include reinforcement learning, nonlinear system stabilization, and computational aspects of dynamical systems.

ANTON BOLYCHEV received the master's degree from the Faculty of Mechanics and Mathematics, Moscow State University, in 2020, where he conducted research in advanced probability theory. He is currently pursuing the Ph.D. degree in the artificial intelligence with the Dynamic Action Laboratory, Skolkovo Institute of Science and Technology (Skoltech). He has industrial experience in research and development in quantitative research an machine learning. He has expertise in reinforcement learning with a focus on computationally efficient implementation.

VLADIMIR TERZIJA (Fellow, IEEE) received the Dipl-Ing., M.Sc., and Ph.D. degrees in electrical engineering from the University of Belgrade, Belgrade, Serbia, in 1988, 1993, and 1997, respectively. Since 2023, he has been a Professor of energy systems and networks with Newcastle University. From 2021 to 2023, he was a Full Professor and the Head of Laboratory of Modern Energy Systems, Skoltech, Russia. From 2006 to 2020, he was the EPSRC Chair Professor in power system engineering with the Department of Electrical and Electronic Engineering, The University of Manchester, Manchester, U.K. From 1997 to 1999, he was an Assistant Professor with the University of Belgrade. From 2000 to 2006, he was a Senior Specialist for switchgear and distribution automation with ABB, Ratingen, Germany. His research interests include smart grid applications, wide-area monitoring, protection and control, multi-energy systems, switchgear and transient processes, ICT, data analytics, and digital signal processing applications in power systems. He was a recipient of the prestigious Alexander von Humboldt Fellow. He is the Editor-in-Chief of the *International Journal of Electrical Power and Energy Systems*.



SERGEI PARSEGOV received the master's degree in automation and control from Bauman Moscow State Technical University, Russia, in 2008, and the Ph.D. degree in physics and mathematics from the Institute of Control Sciences of Russian Academy of Sciences (ICS RAS), in 2013. In 2010, he joined the Laboratory of Robust and Adaptive Systems, ICS RAS. In 2018, he joined the Center for Energy Science and Technology, Skolkovo Institute of Science and Technology, Moscow. Since 2023, he has been a leading Researcher with the Laboratory of Mathematical Methods of Optimization, Moscow Institute of Physics and Technology. His research interests include the dynamics and control of network systems, opinion dynamics, decentralized optimization, and power system control.