IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received 13 January 2024, accepted 29 January 2024, date of publication 1 February 2024, date of current version 8 February 2024.

Digital Object Identifier 10.1109/ACCESS.2024.3361034

RESEARCH ARTICLE

Output Tracking Control of a Nonlinear System Based on Takagi-Sugeno Fuzzy Model: Generalized Partial Eigenstructure Assignment Approach

FARZAD SOLTANIAN[1,2], MOKHTAR SHASADEGHI[2], SALEH MOBAYEN[3], (Senior Member, IEEE), AND PAWEŁ SKRUCH[4], (Senior Member, IEEE)

[1]Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB T6G 2M9, Canada
[2]Electrical Engineering Faculty, Shiraz University of Technology, Shiraz 71557-13876, Iran
[3]Graduate School of Intelligent Data Science, National Yunlin University of Science and Technology, Douliu, Yunlin 64002, Taiwan
[4]Department of Automatic Control and Robotics, AGH University of Science and Technology, 30-059 Kraków, Poland

Corresponding author: Saleh Mobayen (mobayens@yuntech.edu.tw)

ABSTRACT This paper introduces an innovative optimal control approach to achieve output tracking while incorporating H_2-performance specifications in a specific class of nonlinear dynamics modeled by the Takagi-Sugeno fuzzy model (TSFM). The primary innovation lies in extending partial eigenstructure assignment to TSFM-based nonlinear systems within the framework of sliding mode control (SMC). We propose a two-step methodology for designing optimal sliding surface gains. Initially, optimal state feedback gains are computed for each rule consequence containing a linear subsystem, adhering to predetermined eigenvalues and satisfying H_2-performance criteria. Subsequently, using a convex combination, the overall state feedback gain is calculated and utilized to design sliding matrix gains. The sliding matrix gains are then determined by strategically combining previously calculated state-feedback gains in a convex optimization problem. We reframe the output tracking strategy as a stabilization problem using a virtual control input and reformulate the optimization task concerning tracking state errors. This process yields state feedback gains, sliding gains, and the formulation of the virtual control input. The effectiveness of our approach is verified through comprehensive simulations, emphasizing its capability in addressing output tracking challenges within nonlinear systems.

INDEX TERMS Eigen-structure assignment, generalized partial Eigen-structure assignment, H_2-characterization, LMI approach, output tracking control, sliding mode control, Takagi-Sugeno fuzzy model.

I. INTRODUCTION

The intricacies associated with solving the output tracking control problem in nonlinear systems have posed significant challenges in the realms of both control theory and practical applications [1], [2], [3], [4]. One effective avenue involves the design of output feedback control laws, guiding the nonlinear system towards a desired reference signal. This approach proves particularly beneficial due to its capability to address the complexities arising from unmeasurable state signals [2], [5]. In a preceding study [6], the development of H_∞ observers for fuzzy systems with unmeasurable state variables marked a significant advancement. This achievement was accomplished by introducing a novel Lyapunov function, adeptly managing unmatched premise variables and external disturbances. Similarly, another research endeavor

The associate editor coordinating the review of this manuscript and approving it for publication was Xiaojie Su.

introduced an H_∞ event-triggered filter tailored specifically for a class of Takagi-Sugeno (T-S) fuzzy systems [7]. This innovative contribution encompassed the implementation of an observer-based piecewise fuzzy filter, synchronizing harmoniously with the premise variables within the networked environment of the plant trajectory.

The TSFM stands as a versatile tool commonly harnessed for the representation of intricate nonlinear systems [8], [9]. Through the adoption of the sector nonlinearity approach [10], a precise characterization of the nonlinear system emerges, eloquently expressed as a fuzzy amalgamation of local models situated within a convex framework. This unique capability permits the application of various potent techniques from the realm of linear control theory, each directed at the individual local linear models that constitute the foundation of the fuzzy rule consequences. The amalgamation of these local control laws culminates in the design of an encompassing nonlinear control law that effectively governs the overarching nonlinear system.

Concurrently, research endeavors have frequently centered around linear systems or the TSFM framework, which serves as an interpolation of several local linear subsystems. Combining H_∞ performance metrics with pole placement objectives addresses robust performance and the desirable transient behavior of linear systems confronted by uncertainty or disturbances [11]. In the context of [12], the utilization of linear matrix inequalities (LMIs) facilitates the description of desired performance, ensuring the placement of closed-loop poles from local subsystems within a predefined region of the s-plane. Additionally, [13] presents the proposal of a constrained H_2-control system for TSFMs encompassing stringent constraints and disturbances characterized as impulses.

The efficacy of SMC in tackling uncertainties and disturbances within practical systems has been well-established [14], [15], [16], [17], [18], [19]. The pivotal role of defining the sliding surface in influencing control system performance has been recognized [20], [21]. In a pioneering work [22], a fresh perspective on sub-optimal sliding surface design and gain determination is introduced. This involves a two-tiered control design strategy, initially ascertaining state feedback gain to satisfy performance specifications. Subsequently, the determination of the sliding surface matrix employs both direct and indirect methodologies. Although the enhancement of transient time responses has garnered substantial attention within the literature [23], the intricate relationship between transient response profiles and system parameters remains largely unexplored in the context of SMC for nonlinear system design.

This paper charts a new course by extending the concept of eigenstructure assignment to nonlinear systems, with the specific aim of optimizing transient response characteristics. The proposed approach encompasses the definition of sliding surfaces and control inputs that align with desired eigenvalues corresponding to local subsystems within fuzzy rule consequences. A synthesis of H_2-performance metrics and partial eigenstructure assignment yields suboptimal gains for each fuzzy rule consequence. By combining these consequence-specific control laws via a convex combination, a generalized partial eigenstructure assignment control law is derived, tailored to the native nonlinear system.

In the pursuit of practicality, H_2-optimization techniques are harnessed throughout the control design process. This entails deploying optimization methodologies to evaluate a multi-objective function that encapsulates H_2-performance metrics and generalized partial eigenstructure assignments. The outcome of this endeavor is an optimal enhancement of transient response characteristics. The contribution of this paper is the development of an innovative approach to address the challenging problem of output tracking control in nonlinear systems modeled using the TSFM. It introduces the concept of extending eigenstructure assignment to nonlinear systems, enabling the optimization of transient response characteristics by defining sliding surfaces and control inputs aligned with desired eigenvalues corresponding to local subsystems within fuzzy rule consequences. This paper also introduces the novel idea of generalized partial eigenstructure assignment, where suboptimal gains are computed for each fuzzy rule consequence and then combined to derive a control law customized to the specific nonlinear system. Throughout the control design process, H_2-optimization techniques are applied to enhance transient response characteristics. Empirical validation using a mass-spring system beside a half vehicle suspension model demonstrate the effectiveness of the proposed approach, with potential implications for future research in this field.

The study further delves into the design of output tracking controllers, a realm where the objective is to harness the principles of linear control theory to craft optimal controllers for nonlinear systems while considering specific constraints.

The structure of this paper unfolds as follows: Section II formulates a nonlinear system beset by uncertainty through the lens of TSFM. In Section III, the design process for a sliding mode controller is meticulously outlined. A novel LMI representation for H_2-problems is introduced, extending the concept of partial eigenstructure assignment to nonlinear systems and uniting it with H_2-LMI representation to introduce the notion of generalized eigenstructure assignment. The convex optimization of sliding mode surface gain is elucidated. Furthermore, the conversion of output tracking into a stabilization problem is detailed, with the assessment of tracking error dynamic stability anchored by a theorem. Section IV lends empirical validation to the proposed approach by considering a mass-spring system and half vehicle suspension model. Finally, Section V concludes the paper, offering insights into future directions for research and exploration.

Hint 1: $herm\,(\Delta) = \Delta + \Delta^*$, where Δ is a square matrix and Δ^* stands for the conjugate transpose of Δ.

II. PROBLEM STATEMENT AND PRELIMINARIES

Consider the following nonlinear system [2], [24]:

$$\dot{x}(t) = f(x(t)) + g_1(x(t))\,u(t) + g_2(x(t), u(t), t)$$
$$\bar{y}(t) = h(x(t))$$
$$z(t) = \Phi(x(t), u(t)) \tag{1}$$

where $x\epsilon\mathbb{R}^n$ and $u, \bar{y}\epsilon\mathbb{R}^m$, and $z\epsilon\mathbb{R}^q$ are the state vector, the control input vector, the measured output vector, and the H_2-performance output vector of the system, respectively. $f(\cdot), g_1(\cdot), g_2(\cdot), h(\cdot)$ and $\Phi(\cdot)$ are nonlinear functions with appropriate dimensions where $g_2(\cdot)$ denotes the possible model uncertainties and/or disturbances with a Euclidean norm which is bounded by a known function $\rho(x, u, t)$.

In light of the T-S fuzzy model's benefits of the nonlinear system, the nonlinear system given in (1) can be represented by the fuzzy system [10] as:

Plant Rule i:

If $\mu_1(t)$ is $F_{1,i}$ and …and $\mu_g(t)$ is $F_{g,i}$, Then

$$\dot{x}(t) = A_i x(t) + B_{1,i}u(t) + f_i;\ i = 1, \ldots, r$$
$$z(t) = C_i x(t) + D_i u(t) \tag{2}$$

where $\mu_j(t)$ and $F_{j,i}$ $(j = 1, \ldots, g)$ are the premise variables and fuzzy sets, respectively; r denotes the number of fuzzy rules; A_i, $B_{1,i}$, C_i and D_i are the system matrices with appropriate dimensions, and f_i includes system uncertainty and external disturbance. Without loss of generality, we suppose that the matched uncertainty f_i can be represented by $f_i = B_{1,i}\bar{f}_i$, where $\bar{f}_i$ are constant matrices with Euclidean norm bounded by a known function $\rho(x, u, t)$. Using a singleton fuzzifier, product inference, and center-average defuzzifier, the overall T-S fuzzy model is inferred as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(\mu)\left\{A_i x(t) + B_{1,i}(u(t) + \bar{f}_i)\right\} \tag{3}$$
$$z(t) = \sum_{i=1}^{r} h_i(\mu)\left\{C_i x(t) + D_i u(t)\right\}. \tag{4}$$

where $h_i(\mu)$ are the normalized fuzzy membership functions defined as

$$h_i(\mu) = \frac{\prod_{j=1}^{g} F_{j,i}(\mu_j(t))}{\sum_{i=1}^{r}\prod_{j=1}^{g} F_{j,i}(\mu_j(t))}$$

satisfying $h_i(\mu) \geq 0$ and $\sum_{i=1}^{r} h_i(\mu) = 1$ and $F_{j,i}(\mu_j(t))$ are the grades of membership of $\mu_j(t)$ in $F_{j,i}$. The same method is used to generate the T-S fuzzy controller. The fuzzy controller $u(t)$ will be designed based on Parallel Distributed Compensator (PDC) [10] which can be written as

Controller Rule i:

If $\mu_1(t)$ is $F_{1,i}$ and …and $\mu_g(t)$ is $F_{g,i}$, Then

$$u(t) = K_i x(t) \quad i = 1, \ldots, r. \tag{5}$$

where K_i indicates the feedback gains. The overall PDC controller can be represented as

$$u(t) = \sum_{i=1}^{r} h_i(\mu(t))K_i x(t). \tag{6}$$

where $h_i(\mu)$ denotes the normalized fuzzy membership functions, in which $h_i(\mu) \geq 0$ and $\sum_{i=1}^{r} h_i(\mu) = 1$.

We will design a SMC for dynamics (3)-(4) based on the multi-channel H_2 performance characterization and the proposed generalized eigenvalue assignment. Moreover, we will provide an extension to the output tracking problem.

III. MAIN RESULTS

This paper aims to design a multi-channel H_2-based SMC for the system dynamics (3)-(4). The following subsections illustrate the details of the design procedure.

A. SLIDING MODE CONTROLLER DESIG

Towards this end, primarily, here, for the system dynamics (3), a sliding mode controller is proposed in two steps. The first step deals with a sliding mode controller for each fuzzy rule. In the second step, by inspiring the fuzzy blending, a sliding mode controller is proposed for the overall T-S fuzzy system.

Step 1. Sliding mode control design for fuzzy rule consequence

Suppose that the linear model of the i^{th} fuzzy rule consequence subsystems introduced in (3) can be rewritten as

$$\dot{x}(t) = A_i x(t) + B_{1,i}(u(t) + \bar{f}_i) \tag{7}$$

Then, consider a linear switching surface for the system dynamics of (7) such that

$$\mathfrak{H}_i = \left\{x(t) : \sigma_i(t) \triangleq S_i x(t) = 0\right\} \tag{8}$$

where $S_i\epsilon\mathbb{R}^{m\times n}$ are full-rank matrices to be designed such that the appropriate dynamics for the related reduced-order sliding motions [25] are obtained. To guarantee the convergence of the system states of (7) to the sliding surface in (8), consider a control law as

$$u_i(t) = u_{eq,i}(t) + u_{n,i}(t) \tag{9}$$

where $u_{eq,i}(t)$ is the equivalent control input and is defined as [22]

$$u_{eq,i}(t) = -(S_i B_{1,i})^{-1}(S_i A_i - \varphi S_i)x(t) \tag{10}$$

where $\varphi\epsilon\mathbb{R}^{m\times m}$ stands for a Hurwitz matrix, and $u_{n,i}(t)$ is the switching control input described as [22]

$$u_n(t) = -(S_i B_{1,i})^{-1}\rho(x, u, t)\frac{\sigma_i}{\|\sigma_i\|}, \quad if\ \sigma_i(t) \neq 0 \tag{11}$$

where the scalar function $\rho(\cdot)$ satisfies $\rho(x, u, t) \geq \left\|S_i B_{1,i}\bar{f}_i\right\|$. Also, we suppose that $\varphi = \lambda I_m$, where λ is a predetermined negative scalar value. Therefore, by considering $\varphi = \lambda I_m$, the control law (10) can be rewritten as

$$u_{eq,i}(t) = (S_i B_{1,i})^{-1}(S_i A_{\lambda,i})x(t) \tag{12}$$

where $A_{\lambda,i} = \lambda I_n - A_i$.

Lemma 1: consider the linear system (7) with the given controller (9) as

$$u_i(t) = (S_i B_{1,i})^{-1}\left\{(S_i A_{\lambda,i})x(t) - \rho(x, \tau, t)\frac{\sigma_i}{\|\sigma_i\|}\right\};$$
$$if\ \sigma_i(t) \neq 0 \tag{13}$$

where the scalar function $\rho(\cdot)$ *satisfies* $\|\rho(x,u,t)\| \geq \|S_iB_{1,i}\bar{f}_i\|$ *. Then, the associated reduced-order sliding mode dynamic is asymptotically stable.*

Proof: Consider a candidate Lyapunov function as:

$$V_{1,i} = \frac{1}{2}\sigma_i^T\sigma_i \tag{14}$$

Now, calculating the time-derivative of the Lyapunov function (14) leads to

$$\dot{V}_{1,i} = \sigma_i^T\dot{\sigma}_i = (S_ix(t))^T(S_i\dot{x}(t)) \tag{15}$$

substituting $\dot{x}(t)$ from (7) in (15) yields

$$\dot{V}_{1,i} = x(t)^T S_i^T \left(S_iA_ix(t) + S_iB_{1,i}u_i(t) + S_iB_{1,i}\bar{f}_i\right) \tag{16}$$

substituting $u_i(t)$ from (13) in (16) results in

$$\begin{aligned}\dot{V}_{1,i} &= x(t)^T S_i^T (S_iA_ix(t) + S_iB_{1,i}(S_iB_{1,\mathrm{i}})^{-1}(S_i\varphi x(t) \\ &\quad - S_iA_ix(t) - \rho(x,\mathrm{u},t)\frac{\sigma_i}{\|\sigma_i\|}) + S_iB_{1,i}\bar{f}_i) \\ &= x(t)^T S_i^T \left(S_i\varphi x(t) - \rho(x,\mathrm{u},t)\,sgn(\sigma_i) + S_iB_{1,i}\bar{f}_i\right)\end{aligned} \tag{17}$$

Then, suppose $\varphi = \lambda\mathrm{I}_m$, where λ is a pre-determined negative scalar value, (17) is rewritten as

$$\dot{V}_{1,i} = x(t)^T S_i^T \left\{S_i\lambda x(t) - \rho(\cdot)\,sgn(\sigma_i) + S_iB_{1,i}\bar{f}_i\right\} \tag{18}$$

Using the uncertainty boundary assumption, i.e., $\|\rho(x,u,t)\| \geq \|S_iB_{1,i}\bar{f}_i\|$, the derivative of the Lyapunov function (18) along the state trajectory $x(t)$ can be expressed as

$$\begin{aligned}\dot{V}_{1,i} &\leq x(t)^T S_i^T\{\varphi S_ix(t)\} + \left|x(t)^T S_i^T S_iB_{1,i}\bar{f}_i\right| \\ &\quad - \rho(x,\mathrm{u},t)\,|S_ix(t)| \\ &\leq x(t)^T S_i^T\{\varphi S_ix(t)\} + \|S_ix(t)\|\,\|S_iB_{1,i}\bar{f}_i\| \\ &\quad - \rho(x,\mathrm{u},t)\,\|S_ix(t)\|\end{aligned} \tag{19}$$

Since we supposed that $\varphi = \lambda\mathrm{I}_m$ and $\|S_iB_{1,i}\bar{f}_i\| \leq \rho(x,u,t)$, we can rewrite (19) as

$$\dot{V}_{1,i} \leq \lambda\|S_ix(t)\|^2 < 0 \tag{20}$$

in which the reachability condition is held and the proof is completed. ∎

Step 2. Fuzzy slidin mode contro design

Inspiring the fuzzy combination in TSFM, the following switching surface and control law are proposed for the nonlinear system (3):

$$\mathfrak{H} = \left\{x(t) : \sigma(t) \triangleq Sx(t) = 0\right\} \tag{21}$$

$$u(t) = u_{eq}(t) + u_n(t) \tag{22}$$

$$u_{eq}(t) = -\sum\nolimits_{i=1}^{r} h_i(\mu)(S_iB_{1,i})^{-1}(S_iA_i - \varphi S_i)x(t) \tag{23}$$

$$u_n(t) = -\sum\nolimits_{i=1}^{r} h_i(\mu)(S_iB_1)^{-1}\rho(x,\mathrm{u},t)\frac{\sigma_i}{\|\sigma_i\|} \quad if\,\sigma_i(t) \neq 0 \tag{24}$$

where $S\epsilon\mathbb{R}^{m\times n}$ denotes the full rank sliding matrix gain to be designed. Also, $S = \sum_{i=1}^{r} h_i(\mu)S_i$, $\sigma(t) = \sum_{i=1}^{r} h_i(\mu)\sigma_i(t)$, $u_{eq}(t) = \sum_{i=1}^{r} h_i(\mu)u_{eq,i}(t)$, $u_n(t) = \sum_{i=1}^{r} h_i(\mu)u_{n,i}(t)$, and $u(t) = \sum_{i=1}^{r} h_i(\mu)u_i(t)$ are the sliding matrix, the switching surface, the equivalent control input, the switching control input, and the overall control input, respectively.

By employing $\varphi = \lambda\mathrm{I}_m$, the control law (22) can be expressed as

$$u(t) = \sum\nolimits_{i=1}^{r} h_i(\mu)\left\{(S_iB_{1,\mathrm{i}})^{-1}(S_iA_{\lambda,i})x(t)\right\} \tag{25}$$

To formulate the disturbance attenuation property of the proposed approach for the nonlinear TS fuzzy model (3)-(4), we let $\bar{f}(\cdot) = 0$, $u_n(t) = 0$, and we have added an artificial disturbance to the TSFM (3), and then, we proceed with the equivalent control part in (23). Therefore, the system dynamics (3)-(4) and the signal controller (6) are rewritten as:

$$\dot{x}(t) = \sum\nolimits_{i=1}^{r} h_i(\mu)\left\{A_ix(t) + B_{1,i}u(t) + B_{2,i}w(t)\right\} \tag{26}$$

$$z(t) = \sum\nolimits_{i=1}^{r} h_i(\mu)\left\{C_ix(t) + D_iu(t)\right\} \tag{27}$$

$$u(t) = \sum\nolimits_{i=1}^{r} h_i(\mu(t))K_ix(t). \tag{28}$$

where $B_{2,i}$ is the matrix with appropriate dimension, $w \in \mathbb{R}^q$ is the artificial/external disturbance input with bounded energy, and K_i is the state feedback gain to be designed. Without loss of generality, it is also assumed that $rank\left(B_{1,i}\right) = m, i = 1,\ldots,r$, the matrix pair $(A_i;B_{1,i})$ is controllable and$m \leq q \leq n$ [22].

B. OPTIMAL SLIDING GAIN DESIGN USING GENERALIZED PARTIAL EIGENSTRUCTURE ASSIGNMENT

Our aim is to determine the most suitable sliding matrix denoted as 'S' in equation (21). This S serves a dual purpose: firstly, it ensures that the state trajectories, when reduced, stay on a stable surface; secondly, it satisfies the H_2-performance criterion, helping to handle disturbances effectively. Achieving this involves solving a complex challenge known as the multi-channel H_2-state-feedback problem. The solution to this challenge provides us with an optimal state feedback gain, a critical element in the search for the optimal sliding matrix gain.

To guarantee the viability of a solution, we must carefully position m eigenvalues within each local linear subsystem. These eigenvalues need to align with a predefined negative value referred to as λ. This condition forms the bedrock for the subsequent steps in our methodology.

The next phases of our approach encompass designing the most effective sliding gain. This step is pivotal as it guides us toward crafting the optimal sliding mode control. This endeavor involves a well-structured sequence of actions:

1) H_2-LMI CHARACTERIZATION

In this subsection, for the TSFM (26)-(27), using LMI characterization, an H_2 performance is presented from the disturbance w to the output z.

Lemma 2: For the nonlinear system described by TSFM dynamics (26)-(27), the following statements are equivalent:

a) $\exists K_j, j = 1,\ldots,r$, *such that* $A_i + B_{1,i}K_j$ *is stable and the* H_2 *performance from the disturbance* w *to the output* z *is less than* γ.

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\mu)h_j(\mu)\{||(C_i + D_iK_j) \times \left(SI - \left(A_i + B_{1,i}K_j\right)\right)^{-1} B_{2,i}||_2^2\} < \gamma \quad (29)$$

b) $\exists X > 0$ *and* $Z > 0$ *such that:*

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_ih_j \begin{bmatrix} A_iX + B_{1,i}Y_j + XA_i^T + Y_j^TB_{1,i}^T & * \\ C_iX + D_iY_j & -\gamma I \end{bmatrix} < 0 \quad (30)$$

$$\sum_{i=1}^{r} h_i\left(\begin{bmatrix} -Z & * \\ B_{2,i} & -X \end{bmatrix}\right) < 0 \quad (31)$$

$$trace\,(Z) < 1 \quad (32)$$

where $Y_j = K_jX$.

c) $\exists X > 0, Z > 0$ *and nonsingular matrix* G_j *such that*

$$\begin{bmatrix} -\left(G_j + G_j^T\right) & * & * \\ A_iG_j + B_{1,i}Y_j + X + G_j & -2X & * \\ C_iG_j + D_iY_j & 0 & -\gamma I \end{bmatrix} < 0;\ for\ i,j = 1,\ldots,r \quad (33)$$

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_ih_j\left(\begin{bmatrix} X & * \\ B_{2,i}^T & Z \end{bmatrix}\right) < 0 \quad (34)$$

$$trace\,(Z) < 1 \quad (35)$$

where $Y_j = K_jG_j$ *and* $X > 0, Z > 0$ *are positive semi-definite matrices and* G_j *are general nonsingular matrix variables.*

Proof: Note that parts $(a\&b)$ are equivalent forms of the standard H_2 state-feedback synthesis which is developed by Lemma (1) in ref [13] for the T-S fuzzy system. By making use Schur complement lemma, (33) is rewritten as

$$\begin{bmatrix} \left\{\begin{matrix} -\left(G_j + G_j^T\right) + \\ \gamma^{-1}\left(C_iG_j + D_iY_j\right)^T\left(C_iG_j + D_iY_j\right) \end{matrix}\right\} & * \\ A_iG_j + B_{1,i}Y_j + X + G_j & -2X \end{bmatrix} < 0 \quad (36)$$

which $G_j+G_j^T > 0$. Applying the congruence transformation $\begin{bmatrix} G_j^{-T} & 0 \\ 0 & X^{-1} \end{bmatrix}$ on (36) results in

$$\begin{bmatrix} \left\{\begin{matrix} -\left(\tilde{G}_j + \tilde{G}_j^T\right) + \\ \gamma^{-1}(C_i^TC_i + K_j^TD_i^TD_iK_j + \\ C_i^TD_iK_j + K_j^TD_i^TC_i) \end{matrix}\right\} & * \\ \tilde{X}(A_i + B_{1,i}K_j) + \tilde{X} + \tilde{G}_j & -2\tilde{X} \end{bmatrix} < 0 \quad (37)$$

where $\tilde{G}_j = G_j^{-1}$, $\tilde{X} = X^{-1}$, and $K_j = Y_jG_j^{-1}$. Then, (37) can be rewritten as

$$\begin{bmatrix} \left\{\gamma^{-1}\begin{pmatrix} C_i^TC_i + K_j^TD_i^TD_iK_j \\ +C_i^TD_iK_j + K_j^TD_i^TC_i \end{pmatrix}\right\} & * \\ \tilde{X}(A_i + B_{1,i}K_j) + \tilde{X} & -2\tilde{X} \end{bmatrix} + herm(\begin{bmatrix} -I \\ I \end{bmatrix}\tilde{G}_{ij}^T\begin{bmatrix} I & 0 \end{bmatrix}) < 0 \quad (38)$$

By using the Projection lemma [26], inequality (38) holds if the following inequalities are satisfied

$$\begin{bmatrix} I \\ I \end{bmatrix}^T \begin{bmatrix} \left\{\gamma^{-1}\begin{pmatrix} C_i^TC_i+ \\ K_j^TD_i^TD_iK_j \\ +C_i^TD_iK_j \\ +K_j^TD_i^TC_i \end{pmatrix}\right\} & * \\ \tilde{X}(A_i + B_{1,i}K_j) + \tilde{X} & -2\tilde{X} \end{bmatrix}\begin{bmatrix} I \\ I \end{bmatrix} < 0 \quad (39)$$

$$\begin{bmatrix} 0 \\ I \end{bmatrix}^T \begin{bmatrix} \left\{\gamma^{-1}\begin{pmatrix} C_i^TC_i+ \\ K_j^TD_i^TD_iK_j \\ +C_i^TD_iK_j \\ +K_j^TD_i^TC_i \end{pmatrix}\right\} & * \\ \tilde{X}(A_i + B_{1,i}K_j) + \tilde{X} & -2\tilde{X} \end{bmatrix}\begin{bmatrix} 0 \\ I \end{bmatrix} < 0 \quad (40)$$

Inequality (39) indicates $-\tilde{X} < 0$ and inequality (40) is equivalent to

$$\tilde{X}(A_i + B_{1,i}K_j) + (A_i + B_{1,i}K_j)^T\tilde{X}^T + \gamma^{-1}(C_i^TC_i + K_j^TD_i^TD_iK_j + C_i^TD_iK_j + K_j^TD_i^TC_i) < 0 \quad (41)$$

By multiplying both sides of (41) by $X = \tilde{X}^{-1}$, one obtains:

$$A_iX + B_{1,i}Y_j + (A_iX + B_{1,i}Y_j)^T + \gamma^{-1}(XC_i^TC_iX + Y_j^TD_i^TD_iY_j + XC_i^TD_iY_j + Y_j^TD_i^TC_iX) < 0 \quad (42)$$

where $Y_j = K_jX$. From the Schur complement, (42) is equivalent to (30). The proof is completed. ■

C. MULTI-CHANNEL H_2 STATE-FEEDBACK

Now, we propose T_{wz} as an H_2- performance from the input w to the output z with control input $u(t) = \sum_{j=1}^{r} h_jK_jx(t)$. We aim to design K_j such that the following performance characterization are satisfied

$$\begin{aligned} &minimize\ \left\|T_{w_\psi z_\psi}\right\|_2 \\ &subject\ to\ \left\|T_{w_1z_1}\right\|_2^2 < \gamma_1,\ldots,\left\|T_{w_{\psi-1}z_{\psi-1}}\right\|_2^2 < \gamma_{\psi-1}, \\ &\qquad \left\|T_{w_{\psi+1}z_{\psi+1}}\right\|_2^2 < \gamma_{\psi+1},\ldots,\left\|T_{w_\aleph z_\aleph}\right\|_2^2 < \gamma_\aleph \end{aligned} \quad (43)$$

where $\left\|T_{w_\psi z_\psi}\right\|_2 := \left\|L_\psi T_{wz}R_\psi\right\|_2$, such that L_ψ and R_ψ are used to select the appropriate input/output channels or constrained channels [27]. And, $\aleph$ stands for the different Lyapunov constraints or the numeral of channels. Moreover, $T_{w_\psi z_\psi}$ is obtained from substituting $B_{2,i}$, C_i and

D_i in (26)-(27) with $B_{2\psi,i}, C_{\psi,i}$ and $D_{\psi,i}, \psi = 1,\ldots\aleph$, respectively. The H_2 -performance can be guaranteed by minimizing the H_2-norm associated with signals $w_\psi = R_\psi w$ and $z_\psi = L_\psi z$. Now, consider a set of LMI characterizations of (33)-(35) for each channel. Then, the state-feedback LMI characterization for the multi-channel system can be synthesized by assigning global variables G_j, Y_j for entire channels, and a distinct Lyapunov variable $X_\psi > 0$ to ψ^{th} channel. As a result, the LMI characterization for ψ^{th} channel trough extension part (c) of lemma 1 can be formulated as

$$\begin{bmatrix} -\left(G_j + G_j^T\right) & * & * \\ A_iG_j + B_{1,i}Y_j + X_\psi + G_j & -2X_\psi & * \\ C_{\psi,i}G_j + D_{\psi,i}Y_j & 0 & -\gamma I \end{bmatrix} < 0; \quad (44)$$

$$\sum_{i=1}^{r} h_i\left(\begin{bmatrix} X_\psi & * \\ B_{2\psi,i}^T & Z \end{bmatrix}\right) < 0 \quad (45)$$

$$trace(Z) < 1 \quad (46)$$

where $X_\psi > 0, Z > 0, G_j$ and Y_j are global LMI decision variables and $Y_j = K_jG_j$. Then, one may modify the optimization problem (43) as

$$\begin{aligned} &minimize\ \gamma_\psi \\ &subject\ to\ (44), (45), and\ (46)\ for\ \psi - th\ channel, \\ &\qquad (44), (45), and\ (46)\ for\ \theta - th\ channel \\ &\qquad with\ given\ \gamma_\theta; \theta \neq \psi; \theta = 1,\ldots,\aleph \end{aligned} \quad (47)$$

D. GENERALIZED PARTIAL EIGENSTRUCTURE ASSIGNMEN

In this part, assignments of partial eigenstructures which are combined with H_2 performance is proposed. Assignment of m poles of each rule consequence linear subsystem (26)-(27) to a pre-determined negative value through the PDC controller can be performed by using the LMI conditions (44)-(46). The assignment of the following partial eigenstructures

$$\overbrace{\{\lambda,\ldots,\lambda\}}^{m\ times} \quad (48)$$

is done by the state feedback. This problem can be divided into two steps as follows

1-Compute the base vector $\left[M_{\lambda,j}\ N_{\lambda,j}\right]^T$ of the null space $\left[A_j - \lambda I\ B_{1,j}\right]$, where $j = 1,\ldots,r$.

2- With arbitrary $\eta_1,\ldots,\eta_m \epsilon \mathbb{R}^m$, the state feedback can be obtained as $K_j = Y_jG_j^{-1}$ with

$$Y_j = N_j\Sigma_N, G_j = M_j\Sigma_M \quad (49)$$

where

$$N_j := \left[\overbrace{N_{\lambda,j,\ldots,N_{\lambda,j\ldots,}}}^{m\ times} \quad \overbrace{\{I,\ldots,I\}}^{(n-m)\ times}\right]$$

$$M_j := \left[\overbrace{M_{\lambda,j,\ldots,M_{\lambda,j\ldots,}}}^{m\ times} \quad \overbrace{\{I,\ldots,I\}}^{(n-m)\ times}\right]$$

$$\begin{aligned} \Sigma_N &:= diag\left[\eta_1,\ldots,\eta_m,k_1,\ldots,k_{n-m}\right] \\ \Sigma_M &:= diag\left[\eta_1,\ldots,\eta_m,l_1,\ldots,l_{n-m}\right] \end{aligned} \quad (50)$$

such that $k_1,\ldots,k_{n-m}\epsilon\mathbb{R}^n$, and $l_1,\ldots,l_{n-m}\epsilon\mathbb{R}^n$. Note that, some arrays of Σ_M and Σ_M are dependent on the assignment of m eigenstructure to a predetermined value λ. In other words, other arrays which are not used for eigenvalue assignment can be employed to reach further constraints. Hence, the first step in designing an H_2 based SMC is recasting (47) by the LMI characterizations (44)-(46) with $X > 0, Z > 0$, $\Sigma_{N,j}$, $\Sigma_{M,j}$ and $\gamma_i > 0$ as below

$$\begin{aligned} &minimize\ \gamma_\psi \\ &subject\ to\ (44), (45), (46), and\ (49)\ for\ \psi - th\ channel, \\ &\qquad (44), (45), (46), and\ (49)\ for\ \theta - th\ channel \\ &\qquad with\ given\ \gamma_\theta; \theta \neq \psi; \theta = 1,\ldots,\aleph \end{aligned} \quad (51)$$

The following lemma demonstrates the conditions that the consequences subsystems contain eigenvalues (48) for each local subsystem with PDC controller (28) defined by (49).

Lemma 3: For each fuzzy rule with the state feedback $K_j = Y_jG_j^{-1}$, where Y_j and G_j presented in (49), the set of $A_j + B_{1,j}K_j$ eigenvalues contain the subset (48).

Proof: The base vector $\left[M_{\lambda,j}\ N_{\lambda,j}\right]^T$ of the null space $\left[A_j - \lambda I\ B_{1,j}\right]$ is obtained as follows:

$$\left[A_j - \lambda I \quad B_{1,j}\right]\left[M_{\lambda,j} \quad N_{\lambda,j}\right]^T = 0 \quad (52)$$

where $M_{\lambda,j}$ and $N_{\lambda,j}$ are the j^{th} eigenstructure vectors associated with the system state and the control input respectively, where $j = 1,\ldots,r$. One can rewrite (52) as

$$A_jM_{\lambda,j} + B_{1,j}N_{\lambda,j} = \lambda M_{\lambda,j} \quad (53)$$

Furthermore, a set of η_i always exists such that $A_j + B_{1,j}K_j$ has m independent eigenvectors associated with λ, i.e. $M_{\lambda,j}\eta_i, i = 1,\ldots,m$. By utilizing (49), one can form (53) as

$$\begin{aligned} &\left(A_j + B_{1,j}K_j\right)M_{\lambda,j}\eta_i \\ &\quad = \left[A_j + B_{1,j}\left(N_j\Sigma_{N,j}\right)\left(M_j\Sigma_{M,j}\right)^{-1}\right]M_{\lambda,j}\eta_i \\ &\quad = \left[A_j + B_{1,j}\left(N_j\Sigma_{N,j}\right)\left(M_j\Sigma_{M,j}\right)^{-1}\right]\left(M_j\Sigma_{M,j}\right)e_i \end{aligned} \quad (54)$$

where $i = 1,\ldots,m$, and e_i denotes the canonical or standard basis of $\mathbb{R}^n$ [28]. By using (50) and (53), equation (54) can be rewritten as

$$\begin{aligned} \left(A_j + B_{1,j}K_j\right)M_{\lambda,j}\eta_i &= \left[A_j\left(M_j\Sigma_{M,j}\right) + B_{1,j}\left(N_j\Sigma_{N,j}\right)\right]e_i \\ &= A_jM_{\lambda,j}\eta_i + B_{1,j}N_{\lambda,j}\eta_i \\ &= \lambda M_{\lambda,j}\eta_i \end{aligned} \quad (55)$$

which shows that the set of $A_j + B_{1,j}K_j$ eigenvalues contain the subset (54).

E. THE OPTIMAL SWITCHING GAIN DESIG

This part proposes an LMI optimization approach to obtain the sliding gain S associated with the state feedback designed in (51). To obtain the i^{th} sliding gain S_i, consider the state feedback (28), which is equivalent to the sliding mode controller (13) as follows

$$(S_jB_{1,j})^{-1}S_jA_{\lambda,j} = K_j, \quad for\ j = 1,\ldots,r \tag{56}$$

Since the matrix $S_jB_{1,j}$ should be an invertible matrix, without loss of generality, let us suppose that $S_j = B_{1,j}^T P_j$, where P_j are semi-positive definite matrices to be obtained. To analyze (56), one can use a simple relaxation method as

$$\begin{aligned} &minimize\ \delta_j \\ &subject\ to\ \left\| \left(B_1^T P_j(A_{\lambda,j} - B_{1,j}K_j)\right) \right\| < \delta_j,\ for\ j = 1,\ldots,r \end{aligned} \tag{57}$$

where $\delta_j > 0$ are scalar variables, and K_j are the j^{th} state-feedback obtained from (51), which makes ensuring that m poles of each rule consequence subsystem are exactly located at λ. Using LMI optimization, the minimization problem (57) is rewritten as

$$\begin{aligned} &minimise\ \delta_j\ for\ j = 1,\ldots,r \\ &subject\ to \begin{bmatrix} -\delta_j I & * \\ B_{1,j}^T P_j(A_{\lambda,j} - B_{1,j}K_j) & -\delta_j I \end{bmatrix} < 0 \end{aligned} \tag{58}$$

Finally, inspired by the convex structure in TSFM, the switching matrix gain is proposed to obtain as

$$S = \sum\nolimits_{i=1}^{r} h_i(\mu) B_{1,j}^T P_j. \tag{59}$$

F. THE OUTPUT TRACKING CONTROL

In this subsection, the output tracking problem of the nonlinear systems, described by TSFM (3)-(4), is studied while some performance specifications are under control. For output tracking control, we suppose that the H_2-performance output vector is only dependent on the state vector; i.e., $z(t) = \Phi(x(t))$. Furthermore, it is necessary that the tracking error between the measured output and the desired trajectory finally meets zero. To convert the output tracking problem into a stabilization problem, a set of the desired variable $x_d(t)$ which must be tracked by the state variable $x(t)$ is introduced. Let $\tilde{x}(t) := x(t) - x_d(t)$ and $\tilde{z}(t) := z(t) - z_d(t)$ which $\tilde{x}(t)$ and $\tilde{z}(t)$ denote the tracking state error of the state variables and the tracking error of the H_2-performance output vector, respectively. The time derivative of $\tilde{x}(t)$ results in

$$\dot{\tilde{x}}(t) = \dot{x}(t) - \dot{x}_d(t) \tag{60}$$

Substitute $\dot{x}(t)$, given in (3), in (60) yields

$$\dot{\tilde{x}}(t) = \sum\nolimits_{i=1}^{r} h_i(\mu)\{A_i x(t) + B_{1,i}(u(t) + \bar{f}_i)\} - \dot{x}_d(t) \tag{61}$$

Now, we introduce a new control signal $\tau(t)$ satisfying the following equation [29]

$$\sum\nolimits_{i=1}^{r} h_i(\mu)B_{1,i}\tau(t) \triangleq \sum\nolimits_{i=1}^{r} h_i(\mu)B_{1,i}u(t) + \sum\nolimits_{i=1}^{r} h_i(\mu)A_i x_d(t) - \dot{x}_d(t) \tag{62}$$

where $\tau(t)$ is a new control input that is to be yet acquired. Therefore, the state-tracking error dynamic is

$$\dot{\tilde{x}}(t) = \sum\nolimits_{i=1}^{r} h_i(\mu)\left\{A_i\tilde{x}(t) + B_{1,i}(\tau(t) + \bar{f}_i)\right\} \tag{63}$$

$$\tilde{z}(t) = \sum\nolimits_{i=1}^{r} h_i(\mu)\, C_i\tilde{x}(t) \tag{64}$$

The control purpose is to enforce $\tilde{x}(t)$ to converge zero, i.e., tracking the desired state $x_d(t)$ by the state trajectory $x(t)$. Next, the novel fuzzy controller $\tau(t)$ is proposed relying upon PDC and can be written as

Controller Rule i:

If $\mu_1(t)$ is $F_{1,i}$ and ... and $\mu_g(t)$ is $F_{g,i}$, Then

$$\tau(t) = \mathfrak{K}_i\tilde{x}(t); \qquad i = 1,\ldots,r \tag{65}$$

where $\mathfrak{K}_i$ denotes the feedback gain. Then, using the singleton fuzzifier, product inference, and center-average defuzzifier, the overall new control signal in PDC form can be expressed as

$$\tau(t) = \sum\nolimits_{i=1}^{r} h_i(\mu(t))\mathfrak{K}_i\tilde{x}(t) \tag{66}$$

where $h_i(\mu)$ denotes the normalized fuzzy membership functions, in which $h_i(\mu) \geq 0$ and $\sum_{i=1}^{r} h_i(\mu) = 1$.

The output tracking problem while satisfying the H_2-performance specification combined with the generalized partial eigenstructure assignment can be implemented in two steps. In the first step, the optimization problem introduced in (51) is recast for the tracking state error (63) and the H_2-performance output tracking error (64) with the overall new control signal (66). In the next step, the control input of the output tracking problem will be obtained based on the new control signal (66) introduced for the tracking state error (63).

The remaining part to solve the output tracking problem is to determine the desired state $x_d(t)$. Toward this end, we use the fact that $g_1(x(t)) = \sum_{i=1}^{r} h_i(\mu)B_{1,i}$, one may rewrite (62) as

$$g_1(x(t))(\tau(t) - u(t)) = \sum\nolimits_{i=1}^{r} h_i(\mu)A_i x_d(t) - \dot{x}_d(t) \tag{67}$$

The existence of the new control signal $\tau(t)$ relates to the pattern of $g_1(x(t))$. Hereon, we suppose the matrix of the input coefficient $g_1(x(t))$ as a full-column rank matrix with the following form

$$g_1(x(t)) = \left[\, 0_{(n-m)\times m}\ B(x) \,\right]^T \tag{68}$$

where $0_{(n-m)\times m} \epsilon \mathbb{R}^{(n-m)\times m}$ and $B(x) \epsilon \mathbb{R}^{m\times m}$ are a zero matrix and nonsingular matrix, respectively. Similarly, $A(x) = \sum_{i=1}^{r} h_i(\mu)A_i$ and $x_d(t)$ are partitioned as follows

$$A(x) = \begin{bmatrix} A(x)_{(n-m)\times n} \\ A(x)_{m\times n} \end{bmatrix}, \quad x_d(t) = \begin{bmatrix} x_d(t)_{n-m} \\ x_d(t)_m \end{bmatrix} \tag{69}$$

Substituting (67)-(68) in (66), one has

$$\begin{bmatrix} 0_{n-m} \\ B(x)\left(\tau\left(t\right)-u\left(t\right)\right) \end{bmatrix} = \begin{bmatrix} A\left(x\right)_{(n-m)\times n}x_d\left(t\right)-\dot{x}_d(t)_{n-m} \\ A\left(x\right)_{m\times n}x_d\left(t\right)-\dot{x}_d(t)_m \end{bmatrix} \tag{70}$$

As a consequence, using the output equation (67) and the condition in (70), the desired state variables are determined by the following dynamic [29]:

$$\dot{x}_d(t)_{n-m} = A\left(x\right)_{(n-m)\times n}x_d\left(t\right) \tag{71}$$

$$\bar{y}\left(t\right) = h(x_d\left(t\right)) \tag{72}$$

Also, from (66) and (70), the practical control input is obtained as

$$u\left(t\right) = \tau\left(t\right) - B^{-1}(A\left(x\right)_{m\times n}x_d\left(t\right)-\dot{x}_d(t)_m) \tag{73}$$

The desired variables $x_d\left(t\right)$ for many physical systems can be determined by the equations (71)-(72). The stability and performance of the suggested control system were described in Theorem 1.

Theorem 1: Suppose that for some $\gamma_\psi > 0, \psi = 1, \ldots \aleph$, *the state feedback* $\mathfrak{K}$ *is a solution to the optimization problem introduced in (51). Then, the multi-channel* H_2 *-performance constraints* $\left\|T_{w_\psi \emptyset_\psi}\right\|_2^2 < \gamma_\psi, \psi = 1, \ldots \aleph$ *are guaranteed, and the new control input*

$$\tau\left(t\right) = \sum\nolimits_{i=1}^{r} h_i\left(z\left(t\right)\right)\mathfrak{K}_i\tilde{x}\left(t\right) + \tau_n\left(t\right) \tag{74}$$

where

$$\tau_n\left(t\right) = -\sum\nolimits_{i=1}^{r} h_i\left(\mu\right)(S_iB_1)^{-1}\rho\left(\tilde{x},\tau,t\right)\frac{\sigma_i}{\|\sigma_i\|} \quad if\ \sigma_i(t) \neq 0 \tag{75}$$

In this case, the switching control law $\sigma_i\left(t\right) \triangleq S_i\tilde{\boldsymbol{x}}\left(t\right)$, *stabilizes the obtained sliding mode dynamics in an asymptotic manner.*

Proof: Select a candidate Lyapunov function as

$$\begin{aligned} V_2 &= \frac{1}{2}\sigma^T\sigma \\ &= \frac{1}{2}\sum\nolimits_{i=1}^{r}\sum\nolimits_{j=1}^{r} h_i\left(\mu\right)h_j\left(\mu\right)\tilde{x}\left(t\right)^T S_i^T S_j\tilde{x}\left(t\right) \\ &\leq \frac{1}{2}\sum\nolimits_{i=1}^{r} h_i\left(\mu\right)\tilde{x}\left(t\right)^T S_i^T S_i\tilde{x}\left(t\right) \end{aligned} \tag{76}$$

Now, applying the time derivative of the Lyapunov function (76) leads to

$$\dot{V}_2 = \sigma^T\dot{\sigma} \leq \sum\nolimits_{i=1}^{r} h_i\left(\mu\right)\tilde{x}\left(t\right)^T S_i^T S_i\dot{\tilde{x}}\left(t\right) \tag{77}$$

substituting $\dot{\tilde{x}}\left(t\right)$ from (63) in (77) yields

$$\dot{V}_2 \leq \sum\nolimits_{i=1}^{r} h_i\left(\mu\right)\tilde{x}\left(t\right)^T S_i^T S_i \sum\nolimits_{i=1}^{r} h_i\left(\mu\right) \times \left\{A_i\tilde{x}\left(t\right) + B_{1,i}\tau\left(t\right) + B_{1,i}\bar{f}_i\right\} \tag{78}$$

Then, (78) is reformulated as follows

$$\dot{V}_2 \leq \sum\nolimits_{i=1}^{r}\sum\nolimits_{j=1}^{r} h_i\left(\mu\right)h_j\left(\mu\right)\tilde{x}\left(t\right)^T S_i^T S_i \times \{A_j\tilde{x}\left(t\right) + B_{1,j}\tau\left(t\right) + B_{1,j}\bar{f}_j\} \tag{79}$$

substituting $\tau\left(t\right)$ from (74) in (79) yields

$$\begin{aligned} \dot{V}_2 &\leq \sum\nolimits_{i=1}^{r}\sum\nolimits_{j=1}^{r}\sum\nolimits_{l=1}^{r} h_i\left(\mu\right)h_j\left(\mu\right)h_l\left(\mu\right)\tilde{x}\left(t\right)^T S_i^T S_i \\ &\quad \times \{(A_j + B_{1,j}\mathfrak{K}_l)\tilde{x}\left(t\right) - \rho\left(\tilde{x},\tau,t\right)\frac{\sigma_l}{\|\sigma_l\|} + B_{1,j}\bar{f}_j\} \\ &\leq \sum\nolimits_{i=1}^{r}\sum\nolimits_{j=1}^{r}\sum\nolimits_{l=1}^{r} h_i\left(\mu\right)h_j\left(\mu\right)h_l\left(\mu\right)\tilde{x}\left(t\right)^T S_i^T S_i \\ &\quad \times \{(A_j + B_{1,j}\mathfrak{K}_l)\tilde{x}\left(t\right)\} \end{aligned} \tag{80}$$

In this paper, the statement $A_i + B_{1,i}\mathrm{K}_j$ are Hurwitz matrices, therefore (80) leads to

$$\dot{V}_2 < \sum\nolimits_{i=1}^{r}\sum\nolimits_{j=1}^{r}\sum\nolimits_{l=1}^{r} h_ih_jh_l\lambda_{max,jl}\left\|S_i\tilde{x}\left(t\right)\right\|^2 < 0; \tag{81}$$

where $\lambda_{max,jl}$ are the maximal eigenvalues of the Hurwitz matrices $A_j + B_{1,j}\mathrm{K}_l$, and $\lambda_{max,jl}$ locate on the left side of the s-plane, which guarantees the reachability condition. ■

Finally, the proposed approach for the output tracking problem of a nonlinear system based on TSFM can be implemented by Algorithm 1.

Hint 2: In the process of designing the controller, its parameters are optimized by solving the associated LMI optimization problem. Furthermore, the computational load associated with this approach closely aligns with that of a comparable study [30]. This similarity arises from the fact that both approaches involve solving the controller parameters and designing it through the resolution of offline Linear Matrix Inequalities (LMIs) only once.

Hint 3: The article [31] opens avenues for future research by overcoming assumptions on state variables and constraints on slack matrices. The exploration of these principles, DVDP and VSP, presents an opportunity to further reduce conservativeness in filter design. Additionally, the removal of assumptions on state variables, achieved through a recursive method, provides a more general and practical approach. Future research directions may involve extending these methodologies to broader applications and exploring ways to eliminate assumptions in finite-time extended dissipative analysis.

IV. SIMULATION RESULTS

In this section, we assess how well the suggested optimal sliding matrix control system design works and whether it is appropriate. Initially, we examine the benchmark problem that deals with a mass-spring mechanical system. Following that, we explore the second example, which involves considering a half-vehicle suspension model.

Example 1: Consider a mass-spring mechanical system shown in Fig. 1, where $(x_1, x_2), (F_{f_1}, F_{f_2})$, (F_{s_1}, F_{s_2}), (F_{μ_1}, F_{μ_2}), (m_1, m_2), and u denote the displacement from

Algorithm 1 The Proposed Control System Design

1) Define Y_j and G_j according to (49)-(50), and the state feedback gain $K_j = Y_j G_j^{-1}$.
If system (1) is SISO,
 2) Solve (33)-(35) to obtain Y_j and G_j and therefore K_j.
 Go to STEP 4.
Else if the system (1) is MIMO,
 3) Solve GEVP (51) to minimize γ_ψ, and to obtain Y_j and G_j and therefore K_j.
 Go to STEP 4.
End
% Optimal sliding mode control design
4) Solve minimization problem (58).
5) Define $S_i = B_{1,j}^T P_j$
6) Calculate the switching matrix gain from (59).
7) Define the linear switching surface $\sigma_i(t) \triangleq S_i x(t)$.
8) Derive the control laws $u_{eq}(t)$ and $u_n(t)$ from (23)-(24).
% Output tracking control
9) Solve (33)-(35) with $D_j = 0$, to obtain Y_j and G_j and therefore $\mathfrak{K}_j = Y_j G_j^{-1}$.
10) Determine the desired states $x_d(t)$ by (71)-(72) and $\tau_n(t)$ by (75).
11) Obtain $\tau(t)$ by (74).
12) Find the practical control input $u(t)$ with (73)

the reference points, the viscous damping forces, the restoring forces of the springs, the kinetic friction forces, the masses, and the external input, respectively. By definition $X_1 = x_1 - x_2$, $X_2 = \dot{X}_1$, $X_3 = x_2$, and $X_4 = \dot{X}_3$, the nonlinear system is represented in the following state-space form:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -k - ka^2X_1^2 & -c & 0 & 0 \\ 0 & 0 & 0 & 1 \\ k + ka^2X_1^2 & c & -k - ka^2X_3^2 & -c \end{bmatrix} \times \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + [0, 1, 0, 0]^T (u(t) + g(x)) + [0, -1, 0, -1]^T w(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_3 \end{bmatrix}, \; z_1(t) = X_1, \; z_2(t) = X_3 \quad (82)$$

Using the sector nonlinearity approaches [10], the fuzzy rule matrices are obtained. Suppose that, we select the pre-determined eigenvalue as $\lambda = -10$, the actuator as $g(x) = 0.1\, X_1(t)$, and the periodic external disturbance caused by a bump (or a rough surface) with 10 cm widths

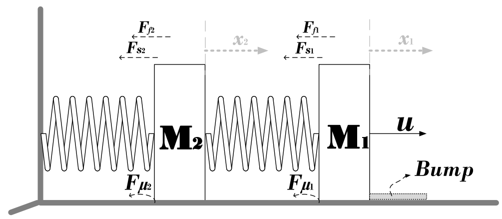

FIGURE 1. Mass-spring mechanical system.

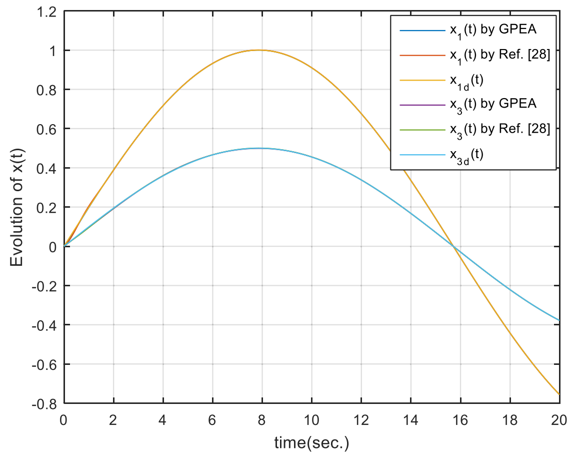

FIGURE 2. Responses of x_1, x_3 and x_{1d}, x_{3d}.

which is modeled as

$$\mathrm{w(t)} = 0.01\sin(\pi X_1(t)), \quad \text{if } 0 \le X_1(t) \le \frac{1}{10}.$$

Then, using the LMI optimization problem (51), we obtain the H_2-performance from the external disturbance vector to the H_2 output vector; i.e., from $w(t)$ to $z(t)$, as $\gamma = 13.4741$.

To compare the proposed approach efficiency with similar works, [30] has utilized an integral-type sliding-surface function for the TSFM, and their approach shows that the H_∞ performance $\gamma_\infty = 2.5891$ is guaranteed. The illustrative comparison simulations are brought in Figs. 2-9. In the other scenario, we devote different Lyapunov matrix variables for each channel in (51). Also, the pre-determined value $\gamma = 2.5891$ is considered for the H_2-performance from the external disturbance vector to the first H_2 output vector; i.e. $z_2(t)$. Then, the results show that the H_2-performance $\gamma = 2.7982$ is obtained for the first H_2 output vecto; i.e. $z_1(t)$.

The comparison study between the proposed approach and [30] are shown in Figs. 2-8. Fig. 5 illustrates that the approach phase is done faster and the trajectory stays on the surface. The control effort has been reduced and the chattering phenom is avoided compared with the work [30] as shown in Fig. 6. The error evolutions of the H_2-outputs tracking control are depicted in Figs. 7-8. The results demonstrate that the first and the second state trajectories of the errors have been decreased compared with [30] and their transient time performances are improved.

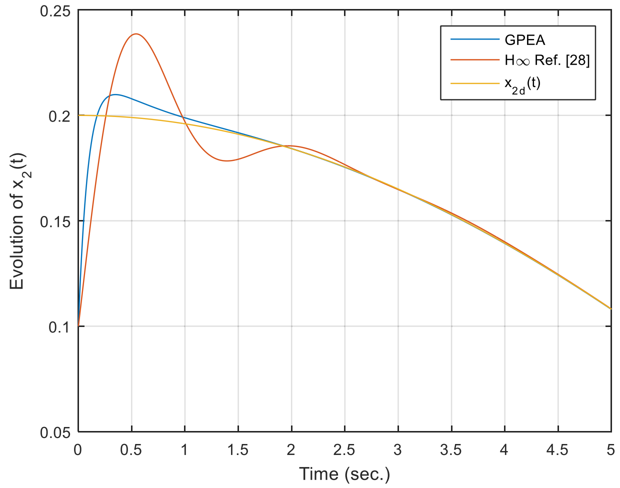

FIGURE 3. Responses of x_2 and x_{2d}.

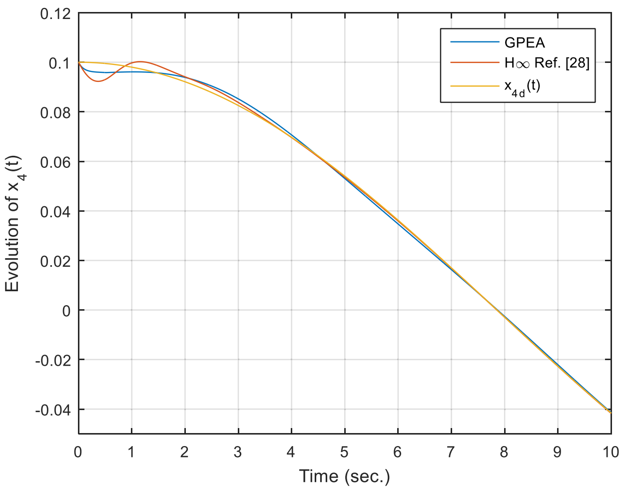

FIGURE 4. Responses of x_4 and x_{4d}.

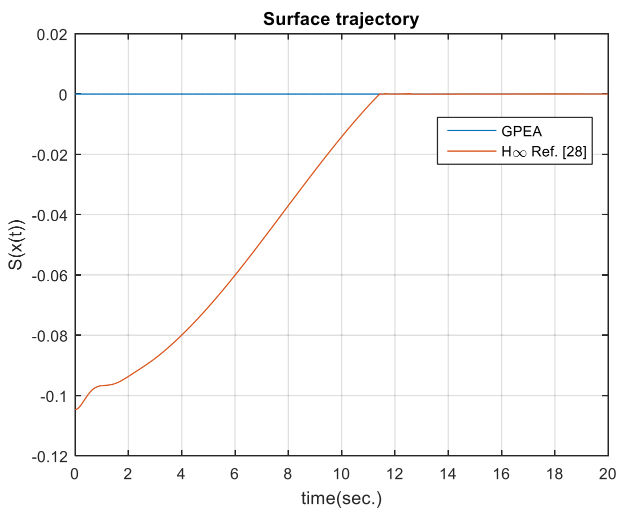

FIGURE 5. The evolution of the sliding surface.

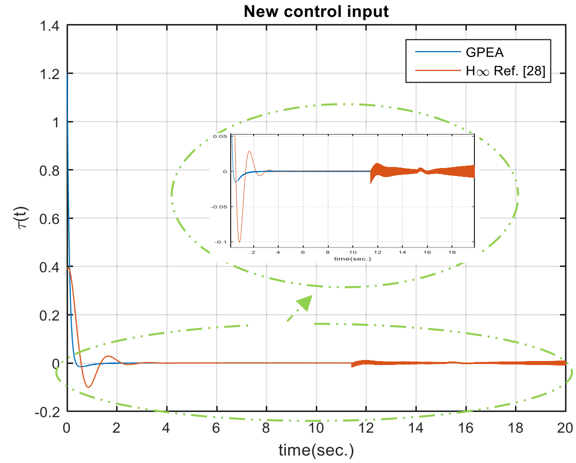

FIGURE 6. The new control input signal $\tau(t)$.

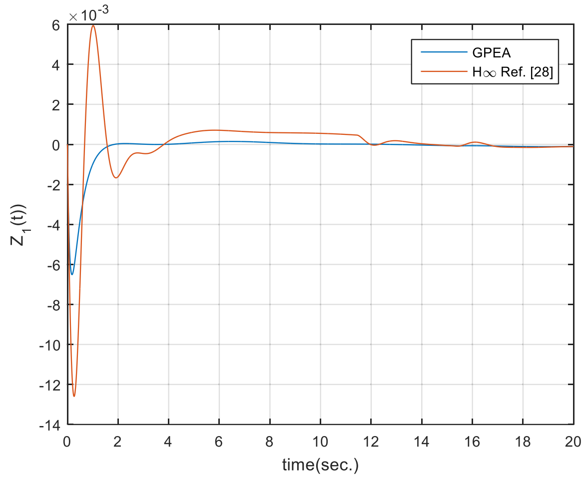

FIGURE 7. The trajectory of $z_1(t)$.

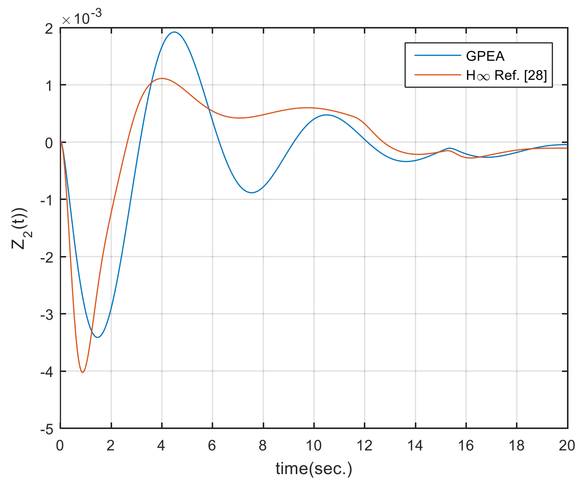

FIGURE 8. The trajectory of $z_2(t)$.

Example 2: Consider the half-vehicle model depicted in Fig. 9 [30]. In this model, $Z_{sf}(t)$ represents the front body displacement, $Z_{sr}(t)$ denotes the rear body displacement, l_1 signifies the distance between the front axle and the center of mass, l_2 indicates the distance between the rear axle and the center of mass, $\varphi(t)$ represents the pitch angle, and

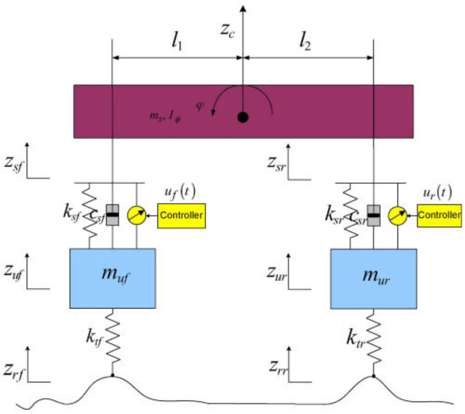

FIGURE 9. Half-vehicle model [30].

$Z_c(t)$ corresponds to the displacement of the center of mass. Additionally, m_s stands for the mass of the car body, while m_{uf} and m_{ur} denote the unsprung masses on the front and rear wheels, respectively. I_φ denotes the pitch moment of inertia about the center of mass. $Z_{uf}(t)$ and $Z_{ur}(t)$ represent the front and rear unsprung mass displacements, respectively. $Z_{rf}(t)$ and $Z_{rr}(t)$ denote the front and rear terrain height displacements, respectively. c_{sf} and c_{sr} are the stiffnesses of the passive elements of the front and rear wheels, and k_{sf} and k_{sr} represent the front- and rear-tire stiffnesses. $u_f(t)$ and $u_r(t)$ are the front and rear actuator force inputs, respectively.

By introducing the following scale factors:

$$a_1 = \frac{1}{m_s} + \frac{l_1^2}{I_\varphi}, a_2 = \frac{1}{m_s} - \frac{l_1 l_2}{I_\varphi}, a_3 = \frac{1}{m_s} + \frac{l_2^2}{I_\varphi}$$

and defining the following variables:

$x_1(t)$- suspension deflection of the front car body,
$x_2(t)$- suspension deflection of the rear car body,
$x_3(t)$- tire deflection of the front car body,
$x_4(t)$- tire deflection of the rear car body,
$x_5(t)$- vertical velocity of the front car body,
$x_6(t)$- vertical velocity of the rear car body,
$x_7(t)$- vertical velocity of the front wheel,
$x_8(t)$- vertical velocity of the rear wheel,

And considering the disturbance input as $w^T(t) = \left[\dot{Z}_{rf}(t)\ \dot{Z}_{rr}(t)\right]$, this paper aims to optimize the control output vector $z^T(t) = \left[\ddot{Z}_c(t)\ \ddot{\varphi}(t)\right]$, encompassing both heave and pitch accelerations, to enhance passenger ride comfort.

The half-vehicle suspension system can be represented as follows:

$$\begin{aligned} \dot{x}(t) &= \mathrm{A(t)}x(t) + B_1(t)\tau(t) + B_2(t)w(t) \\ z(t) &= \mathrm{C(t)}x(t) + \mathcal{D}(\mathrm{t})\tau(t) \end{aligned} \tag{83}$$

Here, t is used to account for parameter uncertainty resulting from variations in vehicle load. Additionally,

$$x^T = \begin{bmatrix} x_1(t) & \ldots & x_8(t) \end{bmatrix}, \quad u(t) = \begin{bmatrix} u_f(t) \\ u_r(t) \end{bmatrix}, \text{ and}$$

$$A(t) = \begin{bmatrix} 0_{4\times4} & a_{12}(\mathrm{t}) \\ a_{21}(\mathrm{t}) & a_{22}(\mathrm{t}) \end{bmatrix}, \ a_{12}(\mathrm{t}) = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$a_{21}(t) = \begin{bmatrix} -a_1 k_{sf} & -a_2 k_{sr} & 0 & 0 \\ -a_2 k_{sf} & -a_3 k_{sr} & 0 & 0 \\ \frac{k_{sf}}{m_{uf}} & 0 & -\frac{k_{tf}}{m_{uf}} & 0 \\ 0 & \frac{k_{sr}}{m_{ur}} & 0 & -\frac{k_{tr}}{m_{ur}} \end{bmatrix}$$

$$a_{22}(t) = \begin{bmatrix} -a_1 c_{sf} & -a_2 c_{sr} & a_1 c_{sf} & a_2 c_{sr} \\ -a_2 c_{sf} & -a_3 c_{sr} & a_2 c_{sf} & a_3 c_{sr} \\ \frac{c_{sf}}{m_{uf}} & 0 & -\frac{c_{sf}}{m_{uf}} & 0 \\ 0 & \frac{c_{sr}}{m_{ur}} & 0 & -\frac{c_{sr}}{m_{ur}} \end{bmatrix}$$

$$B_1(t) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ a_1 & a_2 \\ a_2 & a_3 \\ -\frac{1}{m_{uf}} & 0 \\ 0 & -\frac{1}{m_{ur}} \end{bmatrix}, \ B_2(t) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -1 & 0 \\ 0 & -1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$C^T(t) = \begin{bmatrix} -\frac{k_{sf}}{m_s} & \frac{l_1 k_{sf}}{I_\varphi} \\ -\frac{k_{sr}}{m_s} & -\frac{l_2 k_{sr}}{I_\varphi} \\ 0 & 0 \\ 0 & 0 \\ -\frac{c_{sf}}{m_s} & \frac{l_1 c_{sf}}{I_\varphi} \\ -\frac{c_{sr}}{m_s} & -\frac{l_2 c_{sr}}{I_\varphi} \\ \frac{c_{sf}}{m_s} & -\frac{l_1 c_{sf}}{I_\varphi} \\ \frac{c_{sr}}{m_s} & \frac{l_2 c_{sr}}{I_\varphi} \end{bmatrix}, \ D(t) = \begin{bmatrix} \frac{1}{m_s} & \frac{1}{m_s} \\ -\frac{l_1}{I_\varphi} & \frac{l_2}{I_\varphi} \end{bmatrix}$$

In this example, it is observed that the suspension system constitutes an uncertain model, with the sprung mass m_s and the front and rear wheel unsprung masses m_{uf} and m_{ur} varying within specified ranges. Additionally, when constructing the model for the suspension systems, it is crucial to

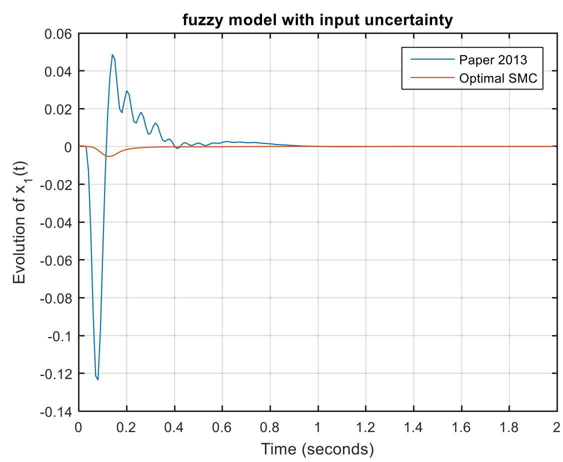

FIGURE 10. Response of the front suspention deflection constraints.

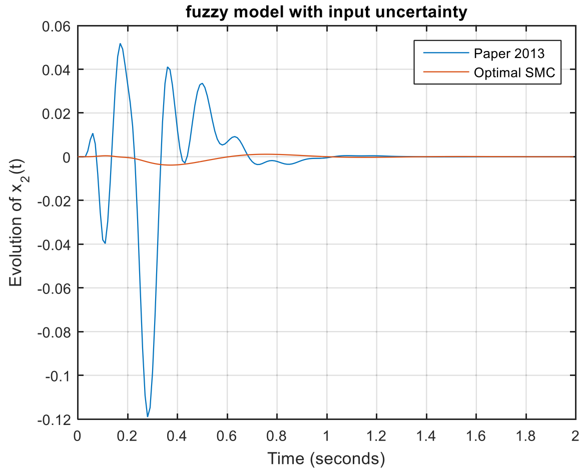

FIGURE 11. **Response of the rear suspension deflection constraints.**

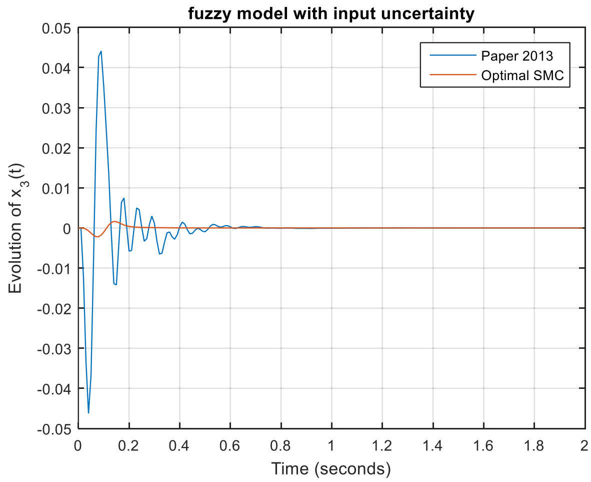

FIGURE 12. **Response of the dynamic front-tire stroke constraints.**

consider actuator uncertainty. In this context, a nonlinear term $g(x) = [0.5x_1(t), 0.5x_1(t)]^T$ is assumed.

The T–S fuzzy model for the half-vehicle suspension system can be formulated as follows

$$\begin{aligned} &\dot{x}(t) \\ &= \sum_{i=1}^{8} h_i(\mu)\left\{A_i x(t) + B_{1,i}(\tau(t) + g(x)) + B_{2,i} w(t)\right\} \\ &z(t) \\ &= \sum_{i=1}^{r} h_i(\mu)\{C_i x(t) + \mathcal{D}_i(\tau(t) + g(x))\} \end{aligned} \tag{84}$$

In this simulation, the parameters of the half-car model are detailed in Table 1 [30]. It is assumed that $m_s \in [621\text{kg}\ 759kg]$, $m_{uf} \in [39.6\text{ kg}\ 40.4kg]$, and $m_s \in [44.55\text{ kg}\ 45.45kg]$. The membership functions are calculated as follows:

$$M_1\left(\frac{1}{m_s}\right) = \frac{\frac{1}{m_s} - \frac{1}{759}}{\frac{1}{621} - \frac{1}{759}},$$

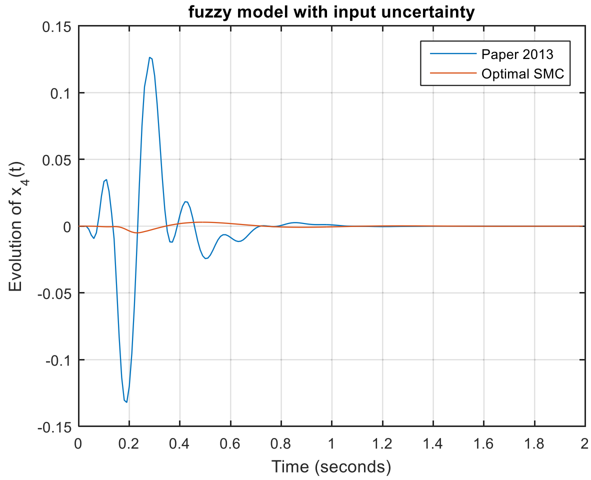

FIGURE 13. **Response of the dynamic rear-tire stroke constraints.**

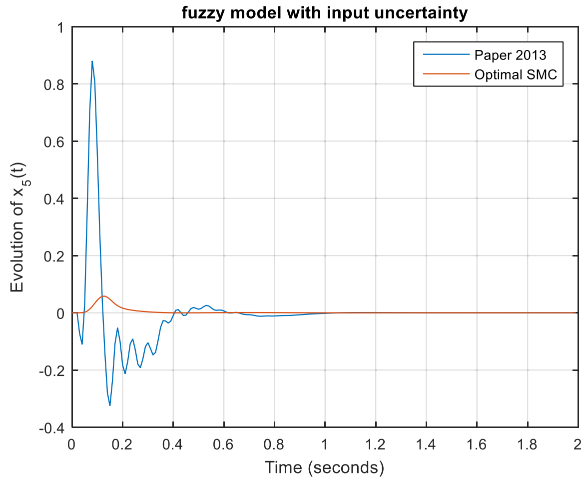

FIGURE 14. **Response of the vertical velocity of the front car body.**

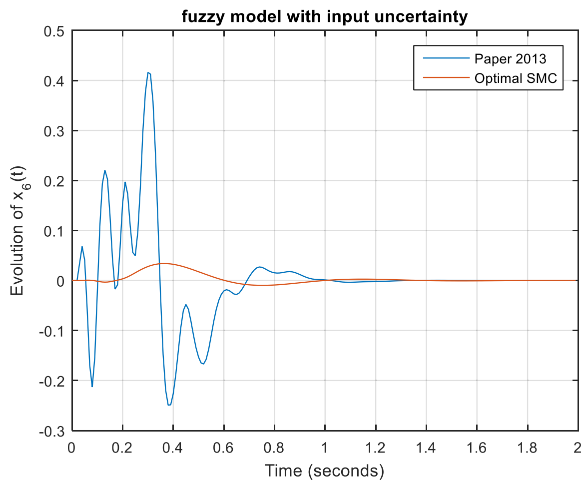

FIGURE 15. **Response of the vertical velocity of the rear car body.**

$$M_2\left(\frac{1}{m_s}\right) = \frac{\frac{1}{621} - \frac{1}{m_s}}{\frac{1}{621} - \frac{1}{759}}$$

$$N_1\left(\frac{1}{m_{uf}}\right) = \frac{\frac{1}{m_{uf}} - \frac{1}{40.4}}{\frac{1}{39.6} - \frac{1}{40.4}},$$

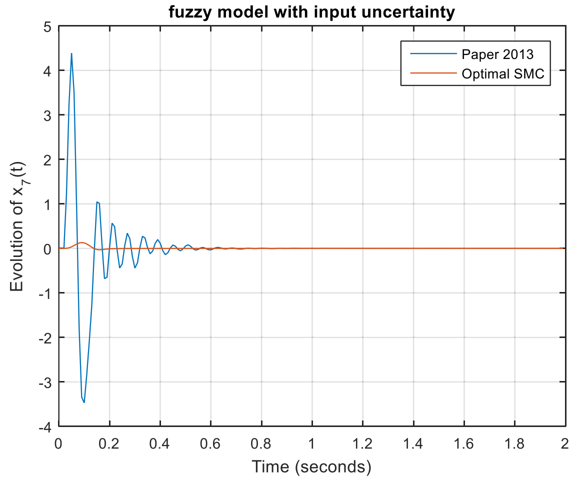

FIGURE 16. Response of the vertical velocity of the front wheel.

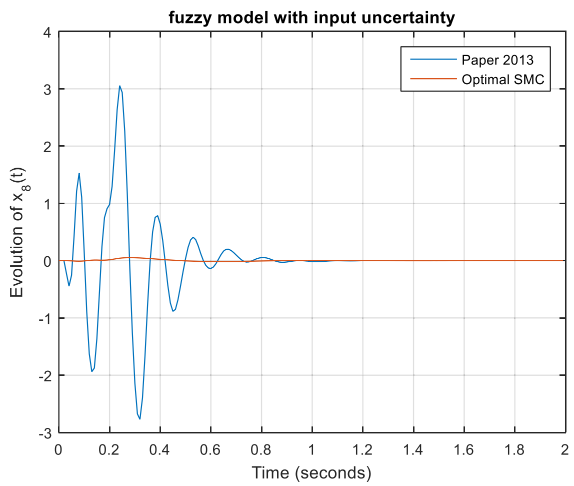

FIGURE 17. Response of the vertical velocity of the rear wheel.

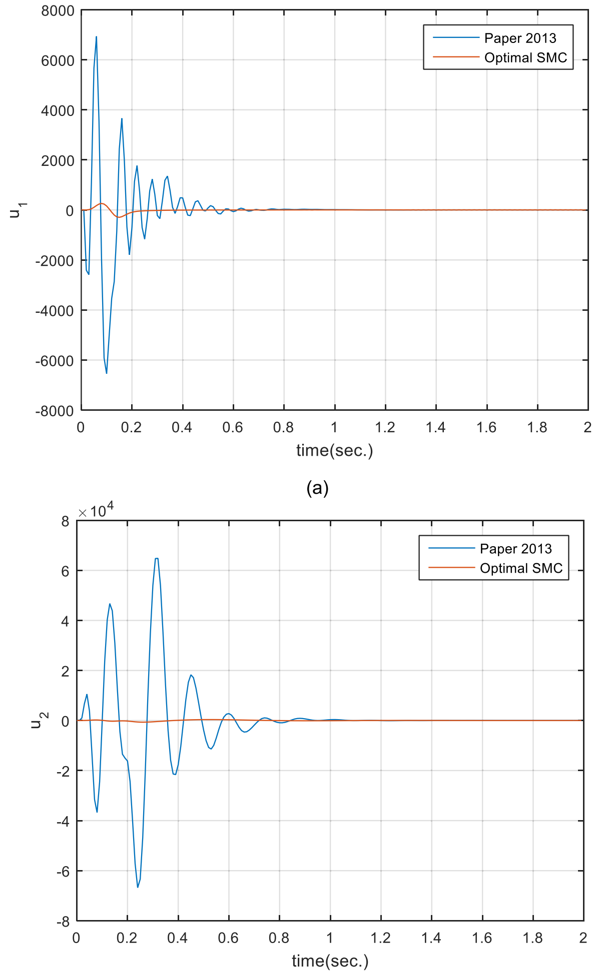

Figure 1. (b)

FIGURE 18. Response of the dynamic front and rear actuator forces. (a):The first vector of the control input and (b): The second vector of the input control input.

$$N_2(\frac{1}{m_{uf}}) = \frac{\frac{1}{39.6} - \frac{1}{m_{uf}}}{\frac{1}{39.6} - \frac{1}{40.4}}$$

$$O_1\left(\frac{1}{m_{ur}}\right) = \frac{\frac{1}{m_{ur}} - \frac{1}{45.45}}{\frac{1}{44.55} - \frac{1}{45.45}},$$

$$O_2(\frac{1}{m_{ur}}) = \frac{\frac{1}{44.55} - \frac{1}{m_{ur}}}{\frac{1}{44.55} - \frac{1}{45.45}}$$

The fuzzy weighting functions $h_i\,(\mu)$ are obtained as:

$$h_1\,(\mu) = M_1(\mu_1) \times N_1(\mu_2) \times O_1(\mu_3)$$

$$h_2\,(\mu) = M_1(\mu_1) \times N_2(\mu_2) \times O_1(\mu_3)$$

$$h_3\,(\mu) = M_2(\mu_1) \times N_1(\mu_2) \times O_1(\mu_3)$$

$$h_4\,(\mu) = M_2(\mu_1) \times N_2(\mu_2) \times O_1(\mu_3)$$

$$h_5\,(\mu) = M_2(\mu_1) \times N_2(\mu_2) \times O_2(\mu_3)$$

$$h_6\,(\mu) = M_2(\mu_1) \times N_1(\mu_2) \times O_2(\mu_3)$$

$$h_7\,(\mu) = M_1(\mu_1) \times N_2(\mu_2) \times O_2(\mu_3)$$

$$h_8\,(\mu) = M_1(\mu_1) \times N_1(\mu_2) \times O_2(\mu_3)$$

where $\mu_1 = \frac{1}{m_s}$, $\mu_2 = \frac{1}{m_{uf}}$, and $\mu_3 = \frac{1}{\text{ur}}$.

The road profile in this example is considered to unveil the transient response characteristic, given by:

$$Z_{rf}\,(t) = \begin{cases} \frac{A}{2}(1-\cos(\frac{2\pi V}{L}t)), & if\ 0 \le t \le \frac{L}{V} \\ 0, & if\ t > \frac{L}{V} \end{cases}$$

Here, A and L are the height and length of the bump, respectively. Assuming $A = 0.1\ m$, $L = 2.5\ m$, and the vehicle forward velocity as $V = 20\ km/h$. In this scenario, the road condition $Z_{rr}\,(t)$ for the rear wheel is assumed to be the same as that for the front wheel but with a time delay of $(l_1 + l_2)/V$. For this example, zero initial conditions are chosen. The desired eigenvalue is selected as $\lambda = -30$. Utilizing the optimization problem introduced in (51), the H_2 performance index from the disturbance w to the output z is

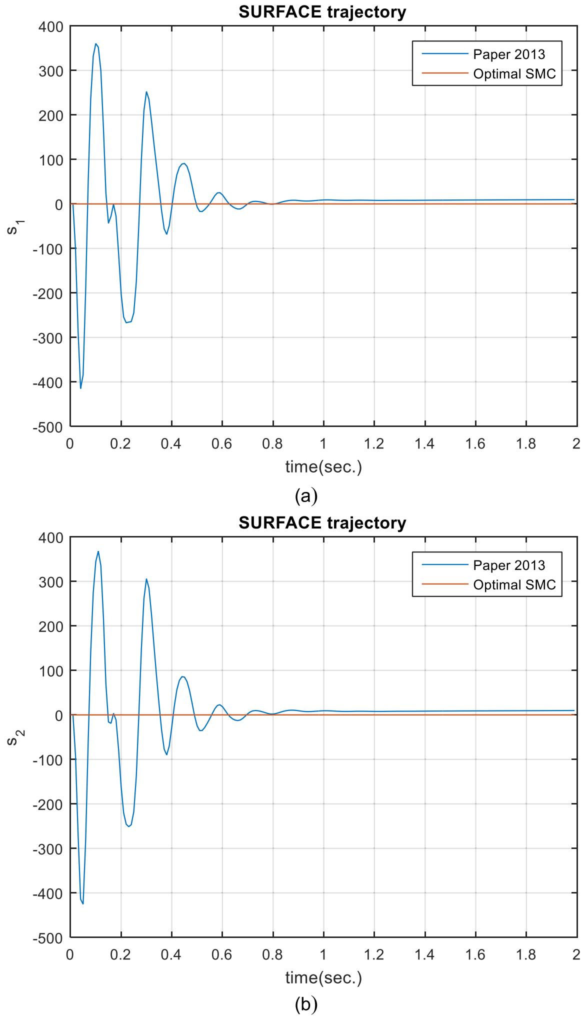

FIGURE 19. Trajectories of sliding variable s(t). (a): S_1 and (b): S_2.

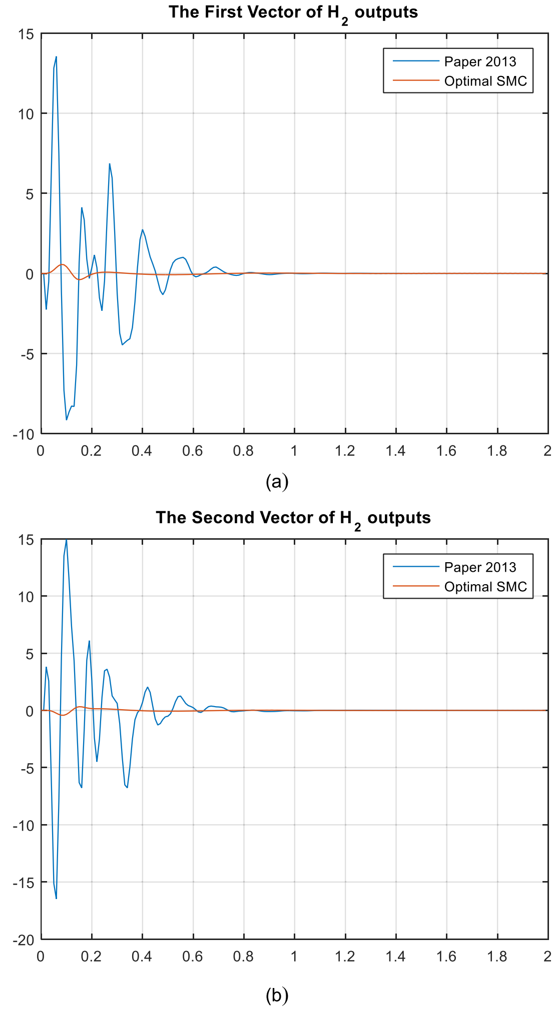

FIGURE 20. Response of the heave and the pitch accelerations. (a):The first vector of H_2 outpout vector and (b): The second vector of H_2 outpout vectot.

obtained to be less than $\gamma = 7.71$. Additionally, the state feedback gains are obtained as:

$$k^T = 10^4 \begin{bmatrix} -6.1721 & 0.4018 \\ -0.0590 & -0.8129 \\ -4.3908 & -2.6817 \\ -0.3251 & -1.2200 \\ -0.8538 & 0.0958 \\ -0.0178 & 1.7968 \\ 0.2999 & -0.0286 \\ -0.0044 & -2.0014 \end{bmatrix}$$

The sliding gain is obtained as:

$$S^T = 10^{-3} \begin{bmatrix} 0.0424 & 0.0061 \\ 0.0091 & 0.1220 \\ -0.0134 & 0.0228 \\ 0.0007 & -0.0093 \\ 0.0036 & -0.0013 \\ 0.0010 & 0.0123 \\ -0.0003 & -0.0006 \\ -0.0003 & -0.0037 \end{bmatrix}.$$

TABLE 1. Half-vehicle model parameters [30].

k_{sf}	k_{tf}	c_{sf}
$18000\ N/m$	$200000\ N/m$	$1000\ Ns/m$
k_{sr}	k_{tr}	c_{sr}
$22000\ N/m$	$200000\ N/m$	$1000\ Ns/m$
I_φ	l_1	l_2
$1222\ kgm^2$	$1.3\ m$	$1.5\ m$

The response of the front and rear suspension deflection constraints is presented in Fig.10 and Fig.11, respectively. Figs.12-17 exhibit the tire deflection of the front car body, the tire deflection of the rear car body, the vertical velocity of the front car body, the vertical velocity of the rear car body, the vertical velocity of the front wheel, and the vertical velocity of the rear wheel, respectively. Fig.18 visualizes the control inputs, computed by integrating the H_2 characterization and the generalized eigenstructure assignment method. Fig.19 illustrates the surface trajectory of the SMC, reaching zero within a finite time horizon. Finally, the response of the heave and pitch accelerations is depicted in Fig.20, demonstrating superior tracking of zero compared to the method proposed in [30]. The obtained results affirm the effectiveness of the proposed method.

V. CONCLUSION AND FUTURE WORKS

In this paper, a novel control strategy has been introduced, tailored for nonlinear systems characterized by TSFM. The objective was to devise an optimal switching surface within the framework of sliding mode control. The strategy involved obtaining state feedback gain through convex optimization, executed individually for each closed-loop local subsystem. This ensured that a predetermined number of eigenvalues aligned with a predefined negative value, while simultaneously adhering to the H_2-norm constraints. Subsequently, the determination of the sliding surface, contingent upon this specialized state feedback, was achieved by addressing another convex optimization challenge. The effectiveness of the proposed method was substantiated through simulation results, confirming its efficiency. Looking ahead, the study opens avenues for future research. The development of an adaptive strategy is envisioned, drawing inspiration from observer-based output tracking control methods for managing unmeasurable state variables, as demonstrated in [32] and [33]. In addition, in [34] future directions involve extending the study to switched neural networks with time-varying delays. Additionally, the authors express interest in exploring more complex application scenarios and considering alternative methods, such as terminal sliding mode and H_∞/H_2 LMI characterization, to achieve optimal performance. The paper also suggests investigating machine learning techniques for defining uncertainty boundaries and developing adaptive strategies inspired by observer-based output tracking control methods. These directions hold the promise of extending the reach and impact of the findings presented in this research.

REFERENCES

[1] Y. Yu, H.-K. Lam, and K. Y. Chan, "T-S fuzzy-model-based output feedback tracking control with control input saturation," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 6, pp. 3514–3523, Dec. 2018.

[2] X. Sun, Y. Gao, and C. Wu, "Output tracking control for a class of continuous-time T-S fuzzy systems," *Neurocomputing*, vol. 152, pp. 199–208, Mar. 2015.

[3] S. Pakkhesal and I. Mohammadzaman, "Less conservative output-feedback tracking control design for polynomial fuzzy systems," *IET Control Theory Appl.*, vol. 12, no. 13, pp. 1843–1852, Sep. 2018.

[4] X. Yu, F. Liao, L. Li, and Y. Lu, "Observer-based decentralized robust H_∞ output tracking control with preview action for uncertain and disturbed nonlinear interconnected systems," *Asian J. Control*, vol. 24, no. 2, pp. 626–641, Mar. 2022.

[5] L. Rajabpour, M. Shasadeghi, and A. Barzegar, "Design of robust H_∞ fuzzy output feedback controller for affine nonlinear systems: Fuzzy Lyapunov function approach," *Int. J. Adv. Intell. Paradigms*, vol. 14, nos. 3–4, pp. 328–344, 2019.

[6] X.-L. Wang and G.-H. Yang, "H_∞ observer design for fuzzy system with immeasurable state variables via a new Lyapunov function," *IEEE Trans. Fuzzy Syst.*, vol. 28, no. 2, pp. 236–245, Feb. 2020.

[7] C. Zhang, J. Hu, J. Qiu, and Q. Chen, "Event-triggered nonsynchronized H_∞ filtering for discrete-time T-S fuzzy systems based on piecewise Lyapunov functions," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 47, no. 8, pp. 2330–2341, Aug. 2017.

[8] F. Soltanian and M. Shasadeghi, "Optimal sliding mode control design for constrained uncertain nonlinear systems using T-S fuzzy approach," *Iranian J. Sci. Technol., Trans. Electr. Eng.*, vol. 46, pp. 549–563, Jan. 2022.

[9] Q. Wang, L. Ma, D. Wang, and X. Ma, "Stabilization bound of singularly perturbed switched nonlinear systems subject to actuator saturation using the Takagi–Sugeno fuzzy model," *Asian J. Control*, vol. 24, no. 1, pp. 415–426, Jan. 2022.

[10] K. Tanaka and H. O. Wang, *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach*. Hoboken, NJ, USA: Wiley, 2004.

[11] M. Chilali and P. Gahinet, "H_∞ design with pole placement constraints: An LMI approach," *IEEE Trans. Autom. Control*, vol. 41, no. 3, pp. 358–367, Mar. 1996.

[12] A. Argha, S. W. Su, A. Savkin, and B. Celler, "H_2/H_∞ based sliding mode control: A partial eigenstructure assignment method," in *Proc. IEEE 55th Conf. Decis. control (CDC)*, Dec. 2016, pp. 5354–5359.

[13] G. Xingquan and G. Shuang, "LMI-based H_2 control for T-S fuzzy system with hard constraints," *Int. J. Control Autom.*, vol. 8, no. 3, pp. 21–30, Mar. 2015.

[14] B. Rahmani, "Robust discrete-time sliding mode control for tracking and model following of uncertain nonlinear systems," *Int. J. Syst. Sci.*, vol. 49, no. 11, pp. 2427–2441, Aug. 2018.

[15] H. Li, J. Wang, H. Du, and H. R. Karimi, "Adaptive sliding mode control for Takagi–Sugeno fuzzy systems and its applications," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 2, pp. 531–542, Apr. 2018.

[16] L. Chaouech, M. Soltani, S. Dhahri, and A. Chaari, "An optimal fuzzy sliding mode controller design based on particle swarm optimization and using scalar sign function," *Iranian J. Fuzzy Syst.*, vol. 14, no. 4, pp. 67–85, 2017.

[17] Z. Kardous and N. B. Braiek, "Stabilizing multimodel sliding mode control for homogeneous TS-bilinear systems," *J. Franklin Inst.*, vol. 352, no. 1, pp. 177–188, Jan. 2015.

[18] F. Soltanian, M. Shasadeghi, S. Mobayen, and A. Fekih, "Adaptive optimal multi-surface back-stepping sliding mode control design for the Takagi–Sugeno fuzzy model of uncertain nonlinear system with external disturbance," *IEEE Access*, vol. 10, pp. 14680–14690, 2022.

[19] C. A. Lúa, S. Di Gennaro, A. N. Guzmán, and J. R. Domínguez, "Digital sliding mode controllers for active control of ground vehicles," *Asian J. Control*, vol. 23, no. 5, pp. 2129–2144, Sep. 2021.

[20] S. H. Rouhani, E. Abbaszadeh, M. A. Sepestanaki, S. Mobayen, C. L. Su, and A. Nemati, "Adaptive finite-time tracking control of fractional microgrids against time-delay attacks," *IEEE Trans. on Ind. Appl.*, 2023, doi: 10.1109/TIA.2023.3312223.

[21] A. Nemati, S. Mobayen, S. H. Rouhani, and C. L. Su, "Design of non-singular second-order terminal sliding mode controller for cyber-physical systems with time-delays and cyber-attack on actuators," *Int. J. Syst. Sci.*, 2024, doi: 10.1080/00207721.2023.2300717.

[22] A. Argha, S. Su, A. Savkin, and B. Celler, "Design of optimal sliding-mode control using partial eigenstructure assignment," *Int. J. Control*, vol. 92, no. 7, pp. 1511–1523, 2019.

[23] H. Behrouz, I. Mohammadzaman, and A. Mohammadi, "Robust static output feedback design with pole placement constraints for linear systems with polytopic uncertainties," *Trans. Inst. Meas. Control*, Jan. 2019, Art. no. 014233121882385.

[24] S. S. Xu and Y. Liu, "Study of Takagi–Sugeno fuzzy-based terminal-sliding mode fault-tolerant control," *IET Control Theory Appl.*, vol. 8, no. 9, pp. 667–674, Jun. 2014.

[25] M. Patil, B. Bandyopadhyay, A. Chalanga, and H. Arya, "Output tracking of nonminimum-phase systems via reduced-order sliding-mode design," *IEEE/ASME Trans. Mechatronics*, vol. 23, no. 4, pp. 1981–1992, Aug. 2018.

[26] C. Scherer and S. Weiland, "Linear matrix inequalities in control," in *Mathematical Methods for Robust and Nonlinear Control* (Lecture Notes in Control and Information Sciences), vol. 3, no. 2. Delft, The Netherlands: Dutch Institute of Systems and Control, 2000.

[27] C. Scherer, P. Gahinet, and M. Chilali, "Multiobjective output-feedback control via LMI optimization," *IEEE Trans. Autom. Control*, vol. 42, no. 7, pp. 896–911, Jul. 1997.

[28] P. Montagnon, "Stability of piecewise deterministic Markovian metapopulation processes on networks," *Stochastic Processes their Appl.*, vol. 130, no. 3, pp. 1515–1544, Mar. 2020.

[29] K.-Y. Lian and J.-J. Liou, "Output tracking control for fuzzy systems via output feedback design," *IEEE Trans. Fuzzy Syst.*, vol. 14, no. 5, pp. 628–639, Oct. 2006.

[30] H. Li, J. Yu, C. Hilton, and H. Liu, "Adaptive sliding-mode control for nonlinear active suspension vehicle systems using T-S fuzzy approach," *IEEE Trans. Ind. Electron.*, vol. 60, no. 8, pp. 3328–3338, Aug. 2013.
[31] Y. Tian and Z. Wang, "Finite-time extended dissipative filtering for singular T-S fuzzy systems with nonhomogeneous Markov jumps," *IEEE Trans. Cybern.*, vol. 52, no. 6, pp. 4574–4584, Jun. 2022.
[32] F. Soltanian, A. A. Alvanagh, and M. J. Khosrowjerdi, "Adaptive locally-linear-models-based fault detection and diagnosis for unmeasured states and unknown faults," in *Proc. 2nd Int. Conf. Control, Instrum. Autom.*, Dec. 2011, pp. 507–512.
[33] F. Soltanian, A. A. Alvanagh, and M. J. Khosrowjerdi, "Adaptive locally-linear-models-based fault tolerant control for humanoid robot with unknown faults," in *Proc. 20th Iranian Conf. Electr. Eng. (ICEE)*, May 2012, pp. 1000–1005.
[34] Y. Tian, X. Su, C. Shen, and X. Ma, "Exponentially extended dissipativity-based filtering of switched neural networks," *Automatica*, vol. 161, Mar. 2024, Art. no. 111465.

FARZAD SOLTANIAN received the B.Sc. degree in electrical engineering from Shahid Rajaee Teacher Training University, Tehran, in 2008, the M.Sc. degree in electrical engineering from the Sahand University of Technology, Tabriz, in 2011, and the Ph.D. degree in electrical engineering from the Shiraz University of Technology, Shiraz, Iran, in 2022. He is currently a Visiting Researcher in biomedical and electrical engineering with the Department of Electrical and Computer Engineering and the Faculty of Medicine, University of Alberta, Edmonton, AB, Canada. His research interests include microwave component for sensor design, Takagi-Sugeno fuzzy, intelligent control, robust control, robotic, and fault-tolerant control.

MOKHTAR SHASADEGHI is currently an Associate Professor with the Shiraz University of Technology, Shiraz, Iran. His research interests include fuzzy control, linear matrix inequalities, and optimization.

SALEH MOBAYEN (Senior Member, IEEE) currently collaborates with the National Yunlin University of Science and Technology as an Associate Professor of the Future Technology Research Center and the Graduate School of Intelligent Data Science. His research interests include control theory, sliding mode control, robust tracking, non-holonomic robots, and chaotic systems.

PAWEŁ SKRUCH (Senior Member, IEEE) received the M.S. degree (Hons.) in automation control from the Faculty of Electrical Engineering, Automatics, Computer Science and Electronics, AGH University of Science and Technology, Kraków, Poland, in 2001, and the Ph.D. degree (summa cum laude) and the D.Sc. (Habilitation) degree in automatics and robotics from the AGH University of Science and Technology, in 2005 and 2016, respectively. He is currently a Professor in control engineering with the AGH University of Science and Technology, and a Principal Engineer AI with the Aptiv Technical Center, Kraków. His current research interests include dynamical systems, autonomous systems, artificial intelligence, machine learning, modeling and simulation, and the applications of control theory to software systems.