IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received 6 November 2023, accepted 19 December 2023, date of publication 29 January 2024, date of current version 5 February 2024.

Digital Object Identifier 10.1109/ACCESS.2024.3359413

RESEARCH ARTICLE

Autoencoder-Enhanced Clustering: A Dimensionality Reduction Approach to Financial Time Series

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This was supported in part by the NEOMA Business School under Grant 416004. The work of Daniel González Cortés, Laura Trinchera, and Jian Wu was supported by the Data Science for Insight and Value Creation, Research Group of the AE AI, Data Science and Business, NEOMA Business School.

ABSTRACT While Machine Learning significantly boosts the performance of predictive models, its efficacy varies across different data dimensions. It is essential to cluster time series data of similar characteristics, particularly in the financial sector. However, clustering financial time series data poses considerable challenges due to the market's inherent complexity and multidimensionality. To address these issues, our study introduces a novel clustering framework that leverages autoencoders for a compressed yet informative representation of financial time series. We rigorously evaluate our approach through multiple dimensionality reduction and clustering algorithms, applying it to key financial indices, including IBEX-35, CAC-40, DAX-30, S&P 500, and FTSE 100. Our findings consistently demonstrate that incorporating autoencoders significantly enhances the granularity and quality of clustering, effectively isolating distinct categories of financial time series. Our findings carry significant ramifications for the financial industry. By refining clustering methodologies, we set the stage for increasingly accurate financial predictive models, offering valuable insights for optimizing investment strategies and enhancing risk management.

INDEX TERMS Clustering methods, data compression, financial data processing, neural network applications, time series.

I. INTRODUCTION

Financial time series are challenging because they are inherently non-stationary, and it is common to find seasonal variations and long-term trends. In addition, price information derived from financial markets at any given time is often presented as a series of opening, high, low, and Closing Prices (CP) and transaction volume. This information provides a comprehensive view of market activity and sentiment over a certain period. The opening price represents the initial transaction of the period, establishing the starting sentiment. The high and low prices represent the highest and lowest price variations, reflecting the level of market volatility and the span of trading activities. The CP is often deemed the most significant price, represents the final consensus value for the given period, and is frequently employed in trend research. However, this multidimensional nature hinders efficient examination of the data and impedes the proper use of clustering techniques [1].

The importance of clustering methods applied to time series is gaining traction due to the increase in technological applications that require, generate, and store information in real time. They play an essential role in different domains such as the internet of things [2], [3], autonomous vehicles [4], medicine [5], genetics [6] and finance [7], [8], where data usually is complex, non-stationary and high dimensional with temporal dependencies.

The associate editor coordinating the review of this manuscript and approving it for publication was Shuihua Wang.

Consequently, clustering large time series with high dimensional data sets is complex, and correct dimensionality reduction and efficient extraction of the important features are essential for clustering. For example, a review by Aghabozorgi et al. [9] found that many authors focused on representing time series data in a lower dimension to be consistent with conventional clustering algorithms. On the other hand, Fawaz et al. [10] in an exhaustive review of deep learning for time series classification initially stated that the common attribute shared by those algorithms that outperform previous implementations is a transformation phase to convert the data series into a new feature space. This attribute raises the need for efficient transformation techniques since the most widely disseminated dimensionality-reduction method is the Principal Component Analysis (PCA); however, this technique is inherently linear and can only model linear interdependencies between the data set features [11].

This research intends to display a method to cluster multivariate financial data with a previous transformation stage based on a method for dimensionality reduction. We have proposed a reduction technique to compress multivariate time series in order to classify daily stock market activity and then use this compacted data representation in an efficient clustering process. This novel hybrid technique to cluster financial data can significantly improve the time and accuracy of classifying financial activity in an unsupervised manner. The first contribution made by the authors of this paper is to establish a concrete way of dealing with non-linear relationships to decrease the multi-dimensionality of a time series with opening, high, low and closing prices. The second contribution is implementing and comparing four different reduction techniques with six different technique algorithms to classify intraday activity from the IBEX-35 (IBEX), CAC-40 (CAC), DAX-30 (DAX), S&P 500 (SPX), and the FTSE 100 index (UKX). This review can help the decision-maker to better understand the daily stock market conditions or to use this method to classify time series, avoiding manual labelling.

The rest of the paper is organized as follows: In Section II, we provide a concise overview of research related to the reduction of multidimensional data, including the introduction of the Long Short-Term Memory Autoencoder-based model for reducing the dimensions of financial time series. Following this, Section III offers a brief review of the literature about clustering across various domains. Subsequently, Section IV details the research proposal, outlining its specific settings and performance metrics. In Section V, we will present and discuss the results obtained from each algorithm. Finally, the conclusions of our study are summarized in Section VI.

II. DIMENSIONALITY REDUCTION TECHNIQUES

In this section we will discuss the different reduction techniques that have been used in this research. We will first briefly introduce the autoencoder and then go on to describe other commonly used reduction techniques.

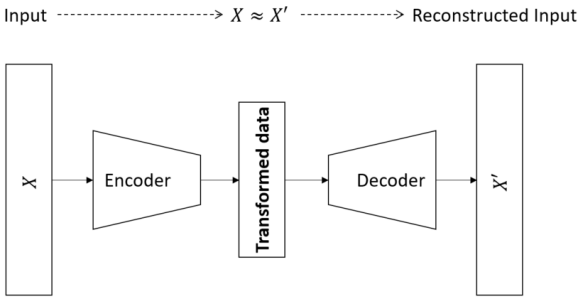

FIGURE 1. The structure of an AE.

A. AUTOENCODERS

An autoencoder (AE) is a form of an artificial neural network composed of two elements; an encoder and a decoder. It is possible to define it as a learning circuit with the primary goal of converting inputs into outputs with the lowest error or the least possible amount of distortion [12]. The function of the encoder is to reduce the raw input multidimensional data into a lower dimension. The decoder takes this transformed data and reconstructs it as accurately as possible by minimizing the reconstruction error using the chain rule to backpropagate error derivatives. The general structure of an AE is shown in Figure 1.

There are multiple forms of AE [12], such as those that are constructed using restricted Boltzmann machine [13] and deep learning structures [14] which change according to the complexity and arrangement of the data. Examples included fully connected deep AEs, convolutional neural networks AEs and recurrent neural network (RNN) AEs, which can be applied to different tasks and fields such as feature extraction for econometrics [15], fault diagnosis [16], fraud detection [17], genetics [18], image processing [19], language translation [20], remote sensing [21], robotics [22], and security [23] among others.

The RNN is a type of neural network that can deal efficiently with temporal data given that the output of the network is fed back into the network together with the new input through a sequential process. As such, an AE based on RNN can efficiently learn a vector representation of sequential data or time-series data because the architecture of this network is specially designed to deal with concatenations of data inputs. As shown in Figure 2, an RNN can be shown as a sequence of inputs, where A represents a layer of the network that contains vectors x with hidden state vectors h on time t.

There are different types of RNNs; one of them is the long short-term memory (LSTM) network created by Hochreiter and Schmidhuber [24] to improve some of the difficulties that classical RNN architecture encounters when trying to achieve an efficient learning process of sequential data. This situation arises because sequential data often struggle with long-term information retention and are susceptible to the vanishing gradient problem, a common challenge in the backpropagation process of artificial neural network training.

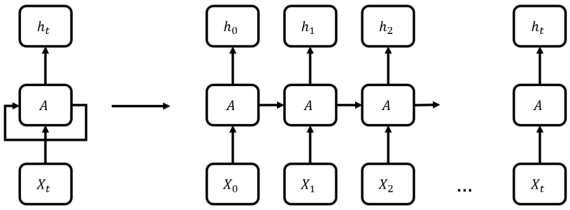

FIGURE 2. The structure of a RNN.

To overcome the difficulties of dealing with long-term data from classical RNN, LSTM networks augment explicit memory by using hidden units to recall short and long-term values. The different units of the LSTM are arranged to form a network with an input node and input, output, and forget gates to regulate the flow of information. The AE that uses LSTM has been successfully applied to various tasks. For example, Jung and Choi [25] use this method to forecast foreign exchange volatility. In addition, Maleki et al. [26] developed an enhanced LSTM AE for anomaly detection in sequential data.

B. PRINCIPAL COMPONENT ANALYSIS

The PCA is one of the most famous and widely used reduction techniques in multivariate statistical analysis. It can be applied to considerably diminish a large dataset's dimensions into a smaller one in an interpretable way, while maintaining as much statistical information as possible and efficiently dealing with data that might have multicollinearity or missing values.

This reduction technique can be defined in different ways, however there are two main descriptions that are frequently used [27]. The first one labels the PCA as an orthogonal projection of the information onto a principal subspace with a linear space of different dimensions to maximize the projected data. This method can also be described as the minimized squared distance between the project and the data points.

The maximum variance formulation considers a set of data $X = \{x_1, \ldots, x_N\}$, with m dimensions and $x_i \in \mathbb{R}^d$. The objective is to obtain a projection with a lower dimension d in $\mathbb{R}^m$, where $m < d$.

By considering $m = 1$, which needs a projection vector $u_i \in \mathbb{R}^d$, each point x_i projects to $u^T x_i$ and the variance of the projected data is defined as,

$$\frac{1}{N}\sum_{N}^{n=1}(u_1^T x_n - u_1^T \bar{x})^2 = u_1^T S u_1 \tag{1}$$

where,

$$S = \frac{1}{N}\sum_{n=1}^{N}(x_n - \bar{x})(x_n - \bar{x})^T \tag{2}$$

$$\bar{x} = \frac{1}{N}\sum_{N}^{n=1} X_n \tag{3}$$

Then $u_1^T S u_1$, which is the projected variance that needs to be maximized with respect to u_1. The $\|u_1\| \to \infty$ maximization of the constraint u_1 also needs to be prevented with a normalization constrain $u_1^T u_1 = 1$.

This technique has been applied to reduce large datasets in multiple disciplines such as medicine [28], robotics [29], and statistical process control [30]. In financial markets, this technique is applied to predict the stock market in lower dimensions using multiple assets [31], [32] and to create models to select stocks [33]. In addition, Pasini [34] applied PCA to different subgroups of stocks efficiently deal with portfolio management; likewise, [35], using macroeconomic and institutional data from emerging markets using PCA, presented an equity asset pricing model to generate dynamic trading strategies.

The iPCA is a modification and substitution of the PCA to deal with large datasets that struggle with memory management. The iPCA uses an amount of memory that is independent of the number of input samples and constructs a low-rank approximation for the input data. This adaptation has the advantage of allowing sparse inputs and being more memory efficient than the traditional PCA model and has been used successfully in different fields such as medicine [36], chemistry [37], and biometric systems [38].

C. FAST FOURIER TRANSFORM

The Fast Fourier Transform (FFT), takes a signal from its original data domain to a different representation, decomposing a series of values into components of different frequencies and expressing it as a function that sums all the periodic components.

The FFT H_k of N points h_k is given by the formula

$$H_k = \sum_{n=0}^{N-1} \mathrm{e}^{-2\pi j \frac{kn}{N}} h_k \tag{4}$$

While the inverse transform is,

$$h_k = \frac{1}{N}\sum_{k=0}^{N-1} \mathrm{e}^{-2\pi j \frac{kn}{N}} H_n \tag{5}$$

as provided by [39].

The FFT presents a significant advancement over previous approaches. While the Discrete Fourier Transform is effective, it is computationally expensive. In contrast, the FFT algorithm can compute these transformations rapidly, reducing the complexity. The FFT's efficiency gain comes from an algorithmic design that reduces the calculations needed for transformation. This efficiency has led to its wide application in various fields, particularly in image processing, where its success is well-documented [40].

III. CLUSTERING TECHNIQUES

Clustering is one of the so-called unsupervised learning methods in data mining. The objective of clustering methods is descriptive rather than predictive to group data instances

into subsets to gather similar instances while different instances belong to different groups. To achieve efficient separation of groups, clustering methods rely on measuring distance between data samples and mechanisms to measure differences among data within similar clusters and differences with others [41].

In this section, we will describe the different cluster techniques used in this research. First, a brief introduction to agglomerative clustering (AGGLO) is given, followed by an overview of Balanced Iterative Reduction and Clustering by Hierarchy Clustering (BIRCH), and then k-means clustering algorithms with Euclidean distances (KNN-EUC) and Dynamic Temporal Warping (KNN-DTW). Finally, we examine the MiniBatch (MNBT) and Spectral (SPCT) techniques.

A. AGGLOMERATIVE CLUSTERING

Agglomerative clustering is one of the most common types of hierarchical clustering. This method initially assumes that each element represents an individual cluster. Then, in an iterative procedure, the two most similar groups are combined according to a criterion to measure distances between groups. The primary methods for gauging distance between groups include single, complete, average, and Ward's linkage, as outlined in [42]. For this experimentation, we will specifically employ Ward's linkage method. The agglomerative clustering results in a tree-based representation of all the unions called a dendrogram. After that, a cut of the dendrogram is performed to obtain the desired number of groups.

The formula for Ward's linkage method, representing the distance $D(w, v)$ between two clusters u and v is as follows, according to [43]:

$$D(u, v) = \sqrt{\frac{|v| + [s]}{T} D(v, s)^2 + \frac{|v| + [t]}{T} D(v, s)^2 - \frac{|v|}{T} D(v, s)^2} \tag{6}$$

Given a dataset that holds N elements, while s and t are the new pair of joined clusters, where $T = |v| + |s| + |t|$

Agglomerative clustering has shown excellent performance in different tasks and has performed comparatively superior to other algorithms [44].

B. BIRCH CLUSTERING

The BIRCH technique is another type of hierarchical clustering that can deal quickly and efficiently with large repositories. This algorithm creates a more diminutive representation of the large dataset that summarizes the large group, holding as much information as possible.

The BIRCH cluster is formed by creating a clustering feature tree, composed of different nodes that facilitate calculating inter-cluster and intra-cluster distances between two clusters i and j. The centroid Euclidean distance between these clusters can be calculated as:

$$DO_{if} = \sqrt{\left(\left(\frac{LS_i}{N_i} - \frac{LS_j}{N_j}\right)^2\right)} \tag{7}$$

$$LS_i = \sum_{N}^{i=1} x_i \tag{8}$$

With N_i number of points, where x_i is a point in the cluster feature i. To calculate LS_j, the same procedure given for Equation 8 needs to performed.

This clustering technique has been used successfully in big data analysis [45], [46]. In addition, it has been used in finance to cluster investment recommendations [47], [48], risk management [49], text data mining [50], stock data prediction model [51] and customer segmentation [52].

C. K-MEANS CLUSTERING

K-means clustering is one of the most popular methods in machine learning to create clusters. It is a vector quantization method that seeks to split a dataset into k clusters. The goal is to set each observation in a specific group with the nearest mean to a cluster centroid. One crucial factor to consider when building a cluster is using a proper metric to measure the distance between all the data points to secure regularity and similitude among all the observations. There are two essential distance measures to consider when dealing with k-means; Euclidean (EUC) and Dynamic Time Warping (DTW).

EUC is a distance function that is popularly used when using the k-means technique, which is based on measuring the distance of a line segment between two points in Euclidean space. This function is represented in Equation 9, as the square root of the sum of the distance between vectors x and y where the algorithm aims to split different sample sets into dissociated clusters.

$$EUC(x, y) = \sqrt{\sum_{j=1}^{k} \left(x_j - y_j\right)^2} \tag{9}$$

However, one of the main problems with EUC is that it is not a normalized metric, and in high dimensional spaces, it tends to be augmented; therefore, a previous reduction technique is needed to avoid the curse of dimensionality. K-means clustering has been applied in finance to classify stock performance [53], hedging strategies [54], and financial time series forecasting [55], [56].

Time series should receive special treatment for clustering because of the need to adapt the clustering techniques to time shifts. The DTW is a suitable measure to address this because it breaks the limitation of other one-to-one alignment metrics. This method attempts to discover all the possible paths using the dynamic programming technique, picking the one that renders the minimum distance between two series and building a matrix with the cumulative distances.

The DTW formula measures the distance between two time series x and y as,

$$DTW(x, y) = \min_{\pi} \sqrt{\sum_{(i,j)\in\pi} d(x_i, y_j)^2} \tag{10}$$

where the path π satisfies additional boundary, continuity, and monotonicity constraints, since this algorithm has been developed primarily to deal with time series, applications are focused on classifying these datasets. This algorithm has been used for application in finance for financial time series clustering [57], [58], [59] and financial credit analysis [60].

D. MINIBATCH CLUSTERING

The MNBT clustering technique is an innovative alternative to the traditional k-means algorithm designed to mitigate its spatial and temporal limitations. Despite k-means' widespread adoption, its memory efficiency remains a notable shortfall. In response, MNBT's core objective is to minimize memory usage. This is achieved by generating small, fixed-size random batches from the entire dataset through an iterative method. Each iteration uses a fresh random sample to update the cluster. The computational efficiency of MNBT has facilitated its widespread application in diverse big data projects [61], [62] as well as in industrial settings [63], demonstrating its versatility and effectiveness.

E. SPECTRAL CLUSTERING

The SPCT clustering technique is based on algebraic graph theory using information from the eigenvalues of matrices constructed from the graph. The resulting Laplacian matrix is defined as $L = D - A$, where A_{ij} measures the affinity between a data points x_i and x_j and D is

$$D = \sum_{j} A_{ij} \tag{11}$$

This method performs better than others in non-convex sample spaces by not quickly falling into the local optimum. This clustering algorithm has attracted attention due to its remarkable performance and reliable theoretical foundation, and has been widely used in finance [64], [65].

IV. PROPOSAL

This paper uses AEs as a dimensionality reduction as a pre-treatment before six clustering techniques: AGGLO, BIRCH, KNN-EUC, KNN-DTW, MNBT, and SPCT.[1] The data utilized in this research is intraday data from IBEX, CAC, DAX, SPX, and UKX. This research aims to reduce the dimension of the financial time series into one measure instead of four measures (the open, high, low, and closing price) to avoid the curse of dimensionality of the clustered object. Another objective is to find an ideal method that allows multidimensional financial data to deal with traditional clustering techniques. The purpose of the technique proposed in this paper is to convert the original 10-second price series into a more extensive time series, in this case into 1, 5, 15 and 30 minute time series, and to compare the performance of the AE to other state-of-the-art reduction techniques such as PCA, iPCA, and FF. We also want to investigate if the reduction techniques are effective without losing information by comparing them to the closing price, representing an input without any reduction. The general scheme of the proposal is shown in Figure 3.

A. PROCEDURE

The time series is obtained from the Bloomberg terminal with a frequency of 10 seconds, the lowest available among the previously mentioned indices. The chosen dataset ranges from 01 September 2020 to 16 March 2021. Adding the five indices together, this research amounted to 1,874,423 observations. The empty values are filled with the closest previous value, assuming that if no price is available, the last known value is the current value; subsequently, each data set is normalized. An LSTM structure with a sigmoid activation function for both the encoding and decoding neural networks is applied to create the AE. The input size of the encoder network is equivalent to a three-dimensional matrix, which reflects the total number of observations, including the total intraday prices per day and the number of prices per instrument. Similarly, the encoder aims to generate a compressed data representation characterized by a three-dimensional matrix. The dimensions of this matrix represent the number of observations, the frequency of the data, and the value of a single price. The decoder, on the other hand, uses as input the compressed representation of the information generated by the encoder; it also uses as output a matrix of identical dimensions to the one used as input by the encoder. Once the decoder finishes decompressing the information it was fed, it then compares this information with the initial input of the encoder to measure the error by means of the mean square error (MSE) function. The formula of the MSE is shown in Equation 12, where n represents the number of observations, while y_i and $\hat{y}_i$, represent the observed and the predicted value, respectively.

$$MSE = \frac{1}{n} \sum_{i=1}^{n} \left(y_i - \hat{y}_i\right) \tag{12}$$

B. METRICS OF PERFORMANCE

Once the information has been compressed, it is pass to the AGGLO, BIRCH, KNN-EUC, KNN-DTW, MNBT, and SPCT cluster algorithms. Then, two performance metrics are applied to measure the effectiveness of these fitting techniques.

The first metric is the SilHouette coefficient (SH), which measures how well defined a cluster is, and it is

[1] In this work, ward AGGLO, BIRCH, MNBT, and SPCT techniques are used within the proposed framework, implemented by scikit-learn [66], while the library Tslearn [67] is utilised for KNN-DTW and KNN-EUC algorithm.

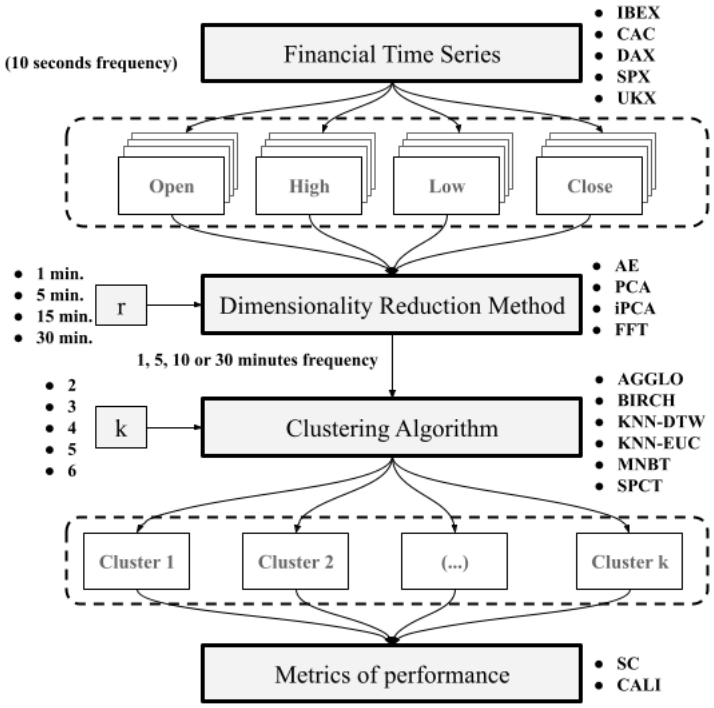

FIGURE 3. The general scheme of the proposal.

defined as:

$$SH_i = \frac{r_i - s_i}{max(s_i, r_i)} \tag{13}$$

where s are the mean distances of a sample and r are all other points of the same class and the next cluster. It should be noted that the SH is framed between the values of -1 and 1, where a negative value means an inaccurate clustering process. Furthermore, a zero implies overlapping clusters, and a value equal to 1 indicates a highly dense and well-separated cluster. Therefore the higher the SC, the better the model's performance.

The second metric, Calinski and Harabasz (CALI), represents the average similarity between clusters and measures the ratio between the dispersion within and between the clusters, where compact and properly separated groups should maximize this ratio. It is specified as:

$$CALI = \frac{n_d - n_k}{n_k - 1} \cdot \frac{tr(B_g)}{tr(W_g)} \tag{14}$$

With n_d representing the number of elements of a dataset d and n_k the number of clusters. While $tr(W_g)$ and $tr(B_g)$ represents the trace of the with-in cluster and between-group dispersion matrices, denoted as:

$$W_g = \sum_{j=1}^{g} \sum_{x \epsilon M_j} (x - m_j)(x - m_j)^T \tag{15}$$

$$B_g = \sum_{j=1}^{g} n_j (m_j - m_D)(m_j - m_D)^T \tag{16}$$

TABLE 1. Table with the principal configurations of the different reduction techniques.

Cluster Technique	Parameters
AGGLO	Linkage = Ward Affinity = Euclidean
BIRCH	Threshold = 0.01 Branching factor = 50
KNN-DTW	Distance = DTW
KNN-EUC	Distance = Euclidean
MNBT	Initialization = k-means++
SPCT	Eigen solver = arpack

The set of points in cluster j is defined as M_j and with cluster j and its center m_j. Where the center of D is defined as m_d and n_j is the number of observations in cluster j. A high CALI score implies a better performance of the cluster technique with well-separated and dense clusters.

V. EXPERIMENTATION

While the previous sections have described the techniques used to efficiently develop a reduction model for clustering data, this section will show our results on different data sets and measure them against the metrics described above. In Section IV, experimentation was carried out to validate the performance of the presented framework. We then analyzed the impact of autoencoders on six clustering techniques described below in Table 1, where each configuration has $k = \{2, 3, 4, 5, 6\}$ number of clusters. In addition, we compared this execution with CP, PCA, iPCA and FF, which are

TABLE 2. Comparative outcomes of various clustering methods on compressed multi-index data: Optimal CALI and SH scores for each condition.

Exchange	Time Frame	Red_Cali	Cluster_Cali	k	CALI Score	Red_SH	Cluster_SH	k	SH Score
IBEX	1 minute	AE	MNBT	6	1673.31	PCA	AGGLO	2	0.8355
IBEX	5 minutes	AE	KNN-EUC	6	1871.78	PCA	AGGLO	2	0.8365
IBEX	15 minutes	AE	SPCT	6	1726.64	PCA	AGGLO	2	0.8388
IBEX	30 minutes	AE	KNN-EUC	6	1728.58	PCA	AGGLO	2	0.8418
CAC	1 minute	AE	SPCT	6	1163.60	PCA	KNN-EUC	2	0.8564
CAC	5 minutes	AE	MNBT	6	1182.68	PCA	KNN-EUC	2	0.8572
CAC	15 minutes	AE	SPCT	6	1185.16	PCA	KNN-EUC	2	0.8593
CAC	30 minutes	AE	KNN-EUC	6	1203.66	PCA	KNN-EUC	2	0.8620
DAX	1 minute	AE	MNBT	6	682.53	PCA	MNBT	2	0.7611
DAX	5 minutes	AE	MNBT	6	678.69	PCA	MNBT	2	0.7622
DAX	15 minutes	AE	SPCT	6	676.54	PCA	SPCT	2	0.7651
DAX	30 minutes	AE	MNBT	6	677.89	PCA	KNN-EUC	2	0.7736
SPX	1 minute	AE	SPCT	6	992.24	AE	MNBT	2	0.6591
SPX	5 minutes	AE	SPCT	6	1002.10	AE	KNN-EUC	2	0.6626
SPX	15 minutes	AE	SPCT	6	1008.60	AE	KNN-EUC	2	0.6628
SPX	30 minutes	AE	MNBT	6	1002.67	AE	KNN-EUC	2	0.6640
UKX	1 minute	FF	KNN-EUC	6	1167.09	AE	KNN-EUC	2	0.7549
UKX	5 minutes	AE	MNBT	6	1185.57	PCA	SPCT	2	0.7682
UKX	15 minutes	AE	SPCT	6	1213.87	AE	KNN-EUC	2	0.7605
UKX	30 minutes	AE	KNN-EUC	6	1232.10	AE	KNN-EUC	2	0.7680

state-of-the-art reduction techniques previously explained in Section III.

Each reduction technique was tested on the following datasets: IBEX, CAC, DAX, SPX, and UKX, by converting the original 10-second price series into a reduced 1, 5, 15 and 30 minutes time series and was then tested by six different clustering algorithms. Each configuration was executed four times with different random seed numbers to validate the results statistically, with five different numbers of clusters and four epochs, obtaining a total number of 9600 experiments to analyze.

In Table 2, we can see the best results in terms of CALI and SH, given a particular index and a specific time frame. Moreover, it is possible to observe that the results vary according to the metric by which they are measured. On the one hand, we can see that, when using CALI, the reduction technique that concentrates the highest values is consistently given by the AE, except for the clustering of the UKX in 1 minute time series, where the reduction technique with better results turned out to be FF. In the same way, if we continue analyzing the results in terms of CALI, we see that the cluster techniques with the highest values do not focus specifically on specific algorithms but vary according to the index and the time frame, without a clear pattern. However, by observing the ideal number of clusters, denoted as k, we found that six clusters uniformly give the highest CALI values.

When analyzing the results in terms of SH, we do not necessarily see association with the best combinations measured by CALI. Instead, we find that the best-performing reduction and cluster techniques in a specific index are never the same if we compare the performance of the reduction and cluster combinations in terms of CALI and SH, with the exception of the cluster performed in a 30 minutes compressed representation of the UKX, where the highest values were found using AE and KKN-EUC.

Furthermore, by measuring the performance according to SH, it is possible to observe that in thirteen of the twenty combinations presented in Table 2, the highest value is given by PCA, followed by AE. The type of index seems to explain this change; for example, by clustering IBEX, CAC and DAX data, the highest values are found using PCA, while using SPX and UKX, the highest values are found by using AE. In the same way, the best cluster techniques are mainly differentiated by the type of data that they compress; for instance, the groups created in IBEX, with the highest values of SH, are formed using AGGLO techniques in the same way KNN-EUC is the predominant technique in those formed with CAC data. Additionally, similarly to those results measured with CALI, there is only one ideal value of k, and it takes the value of two.

To make an aggregate comparison of the different clusters, we have created Table 3, which counts the times that a given reduction technique performs better, the same and worse than the others by the results in percentage terms. In the lower part of Table 3, under the diagonal that represents the comparison of the same techniques, the results are presented in terms of CALI, while in the upper part, those are measured by SH. Table 4, which compares the different clustering techniques, is structured in the same way. In both tables, the first number represents the percentage by which the technique performed better or the same or worse than the others, respectively.

By revising the dissimilarities between the different reduction techniques shown in Table 3, we see that AE outperformed the others by a considerable percentage, especially in terms of CALI. Subsequently, the second-best reduction technique is iPCA, which performs better than CP, FFT and PCA, and finally, CP performs better than FFT and PCA in general percentage terms. Finally, when analyzing the results with respect to SH, we can see that they closely follow those presented above, where AE produces the best

TABLE 3. Comparative analysis of reduction techniques - Frequency of superior, equivalent, and inferior performance in percentage terms, with CALI scores below the diagonal and SH scores above.

	AE	CP	FFT	PCA	iPCA
AE	-	0.89, 0.0, 0.11	0.95, 0.0, 0.05	0.96, 0.0, 0.04	0.87, 0.0, 0.13
CP	0.24, 0.0, 0.76	-	0.95, 0.0, 0.05	0.95, 0.0, 0.05	0.17, 0.0, 0.83
FFT	0.0, 0.0, 1.0	0.0, 0.0, 1.0	-	0.59, 0.0, 0.41	0.05, 0.0, 0.95
PCA	0.0, 0.0, 1.0	0.0, 0.0, 1.0	0.31, 0.0, 0.69	-	0.05, 0.0, 0.95
iPCA	0.25, 0.0, 0.75	0.7, 0.0, 0.3	1.0, 0.0, 0.0	1.0, 0.0, 0.0	-

TABLE 4. Comparative analysis of clustering techniques - Frequency of outperforming, matching, and underperforming in percentage terms.

	AGGLO	BIRCH	KNN-DTW	KNN-EUC	MNBT	SPCT
AGGLO	-	0.09, 0.83, 0.07	0.48, 0.1, 0.42	0.4, 0.09, 0.51	0.39, 0.09, 0.52	0.39, 0.1, 0.51
BIRCH	0.04, 0.83, 0.13	-	0.47, 0.1, 0.43	0.39, 0.1, 0.51	0.38, 0.1, 0.52	0.38, 0.1, 0.51
KNN-DTW	0.44, 0.1, 0.46	0.45, 0.1, 0.45	-	0.18, 0.3, 0.51	0.33, 0.14, 0.53	0.34, 0.14, 0.52
KNN-EUC	0.7, 0.09, 0.2	0.71, 0.1, 0.19	0.61, 0.3, 0.08	-	0.42, 0.18, 0.4	0.44, 0.18, 0.39
MNBT	0.67, 0.09, 0.23	0.68, 0.1, 0.22	0.6, 0.14, 0.26	0.31, 0.18, 0.51	-	0.43, 0.16, 0.41
SPCT	0.66, 0.1, 0.24	0.67, 0.1, 0.23	0.59, 0.14, 0.27	0.32, 0.18, 0.5	0.42, 0.16, 0.43	-

results, although to a lesser extent. Similarly, the second best technique is iPCA, followed by CP.

Table 4 shows a comparison similar to that of Table 3, however this table focuses on comparing the different cluster techniques instead of reduction techniques. It can be seen that AGGLO performs irregularly compared to the others; for example, 83 % of the time it is equal to BIRCH in terms of CALI and SH; however, it does not show any significant improvement compared to the other techniques. Also, BIRCH performs similarly to KNN-DTW but worse than KNN-EUC, MNBT and SPCT using CALI and to a lesser extent when using SH. On the other hand, the KNN-EUC technique outperforms all other techniques, both using CALI and to a lesser extent using SH; moreover, the MNBT technique ranks second as a clustering technique and, subsequently, the SPCT technique performs better than AGGLO, BIRCH and KNN-DTW in both metrics.

VI. CONCLUSION

Finding ways to reduce and classify the information in the financial markets is crucial. This research intends to contribute to these efforts to find a suitable technique for efficiently classifying multidimensional financial data. In addition to proposing a novel hybrid technique for clustering financial time series using autoencoder-based compression and evaluating multiple clustering algorithms, the current paper contributes to the literature on unsupervised approaches that deal with the difficulties of processing high-dimensional data [68], [69] and time series classification [70], [71]. Furthermore, it is pertinent and key to discover how algorithms can help stakeholders automate financial research that involves high volatility and large volumes of data, especially during short and fast periods, such as the intraday sessions of the stock market.

In this investigation, different reduction techniques were used; we tested the suitability of machine and deep learning algorithms, showing the efficiency of AE in intraday financial time series. In addition, this study demonstrated that AE outperformed other reduction techniques by allowing the creation of more compact and well-separated clusters by testing our techniques on five different indexes and four different time frames, contributing to the clustering intraday financial time series literature [72]. Thus, those interested in developing strategies based on information obtained from intraday time series should apply AE with another clustering technique rather than just using traditional techniques such as PCA or relying only on the closing price.

Despite the proven efficiency and superiority of AE and KNN-EUC, it is possible to observe diffuse results when comparing the results of the clustering techniques with different exchanges and time frames. Furthermore, there may be specific cases where the maximum values found do not necessarily coincide with the averages; for example, the highest values in terms of SH for certain exchanges and time frames were found using PCA, even when, on average overall, EA proved to be the most successful technique by a wide margin.

The partial inconsistency of the AE method across different financial exchanges and time frames, which can be attributed to each dataset's distinct characteristics and market dynamics, can be considered one limitation of this methodology. Financial indices exhibit unique behaviors influenced by different factors, such as economic sectors and geographical regions. These factors could contribute to significant variations in how data clusters are formed using AE. Additionally, the time frame of data aggregation affects the granularity of data and the visibility of trends. Shorter time frames might show more volatility and noise, impacting the effectiveness of some clustering algorithms, whereas longer time frames might smooth these fluctuations, possibly favoring different algorithms. Nevertheless, this study establishes the suitability of AE to deal with multidimensional intraday data; therefore, future research can only focus on the search for the most suitable clustering technique for time series using compressed data from AE.

By measuring the performance of cluster techniques with reduced data, we can clearly understand the practical use of reduction techniques such as AE and its superiority over other methods. Furthermore, acknowledging these processes allows us to investigate larger volumes of information, expanding research toward models capable of integrating multiple and diverse variables and creating the opportunity to expand the analysis capacity of investors and scholars. As such, AE can not only be used to decrease the dimensionality of intraday prices but also to reduce large datasets covering different prices from diverse instruments and markets. Based on our investigation, we can use reduction techniques to develop further investment strategies that determine different classifications to simulate more realistic scenarios, which is part of our future research.

ACKNOWLEDGMENT

The views, opinions, and conclusions expressed in this article solely reflect those of the authors.

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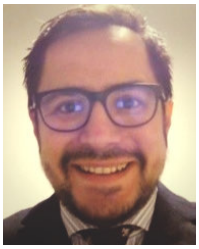

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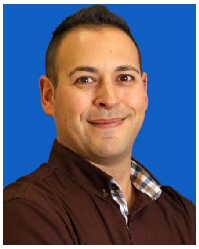

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