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RESEARCH ARTICLE

Adaptive Output Feedback Control for Uncertain Nonlinear Systems Subject to Deferred State Constraints

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ABSTRACT This paper addresses the tracking control problem for a class of uncertain nonlinear systems with full state deferred constraints. The constraints considered occur after a period of system operation rather than from the beginning. An attempt has been made to use a shifting function to realize the purpose of deferred constraint. Moreover, the difficulty of asymmetric constraint on states comes from the design on barrier function. Due to this, an unified barrier function is proposed to prevent system states from violating the constraint range, indirectly. Meanwhile, it effectively removes the restriction condition used in existing results that the upper and lower bound signs of constraint functions are the same. Consequently, the goals of tracking a reference signal and state constraints are achieved through the control scheme designed. Theoretical analysis and simulations are presented to demonstrate the efficacy of the proposed control strategy.

INDEX TERMS Barrier Lyapunov function, deferred constraints, shifting function.

I. INTRODUCTION

As a fundamental and important problem in the filed of control, tracking control problem has emerged as an exciting hot topic because of its widespread applications, such as robotics [1], [2], [3], autonomous vehicles [4], [5], unmanned aerial vehicles [7]. Early efforts mainly focused on the tracking problem of linear system, while most practical plants can be modeled as nonlinear dynamical systems. For this reason, many scholars have proposed various methods to solve these problems, such as backstepping method [8], [9], sliding mode control [10], adaptive control [12] and so on. Such results, however, focus on designing state feedback controller for nonlinear systems, which are dependent on the system state. Since the information of system state is embedded in the controller designed, such a structure will lead to the controller fail to work properly or even lead to the system instability once the system state is unmeasurable. In this case, the unmeasurable state is usually estimated by designing linear or nonlinear observers. Based on the framework of designing observer, so far, some meaningful results on output feedback control [9], [14], [15] have been reported. For example, in [14], a new output feedback controller was presented to suppress the influence of unmeasurable states for switched systems. Furthermore, in [15], Li et al. extended the result of [14] to address the optimal tracking problem for large-scale systems.

However, the approaches mentioned above are not provably effective in controlling constrained systems. Note that safety requirements, which can be regarded as constraint

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problem, are critical for practical applications. For example, robot vacuum cleaners may need to no touch furniture when moving in apartments. Driverless cars need to ensure that they do not hit the sidewalk when turning at intersections. If these restrictions are violated, it will lead to inaccurate control, system instability, and even accidents. Therefore, from the perspective of maneuverability or safety, it is meaningful to develop control strategy that can guarantee the system stability and avoid violating the constraint on system state or output. At present, a lot of effective control algorithms on constraint problem have been developed, such as model predictive control [17], prescribed performance control [18], barrier Lyapunov method [19], integral barrier Lyapunov method [20] and so on. The property of barrier function can be viewed as a significant advantage in terms of handling constrained problem. For instances, in [19], Tee et al. firstly proposed a barrier function method to solve the constraint problem for nonlinear systems with lower triangular structure. Based on the result of [19], the authors in [21] creatively applied the output constraint control method to crane systems. With its advantages, a great number of results on constraints control based on BLF approach for nonlinear system have been extensively studied and applied in practical systems, such as hypersonic hight vehicle [22], robot joint [24], unmanned vehicle [25]. To relax such a limitation that the constraint functions are constant, the authors in [26] developed a new barrier function, which can handle the case of time-varying [27] constraint as well as constant ones. Furthermore, the authors extend the result in [26] to uncertain robotic manipulators. Recently, the authors in [28] developed a cooperative learning control for nonlinear systems with time-varying output constraints by introducing time-varying BLFs. Most of the above mentioned results put the emphasis on dealing with the case the upper and lower bound constraint function is different sign, which may not match the actual application environment. Even though, the work in [27] designed a general barrier function, which has no requirements on the upper and lower bounds of the constrained function, the result can not be applied to nonlinear system with unmeasurable state.

On the other hand, in [21] and [28], the constraint requirements for nonlinear systems are needed at the beginning. However, in practical applications, the constraint requirement does not start from the beginning in some situations, and the system output or state is limited only when the system works for a certain finite time. A example is that a driverless vehicle starts from an unconstrained area and travels a certain distance to enter a constrained area to avoid collisions. Another example is that a common operation in a robotic arm, the arm reaches out to grab an object from a container along a different path. Such all examples can be regard as the deferred constrained problem. Compared with most existing constrained control studies in [29], the key difference and challenge lies in the deferred constraints and unknown initial conditions. To solve such problems, the authors in [30] developed a new shifting function for nonlinear systems with deferred full state constraint. The authors in [31] proposed a shifting function for nonlinear system with output deferred constraint. Furthermore, the authors in [32] extend the result of [31] to stochastic nonlinear multiagent systems. Note that it is a challenging aspect regarding deferred constraints function is the design of shifting function. In order to remove the strict feasibility condition, the authors in [33], proposed an adaptive full-state-constraint tracking algorithm based on non-BLF. To relaxes the control gain functions to be unknown, a new type of time-varying asymmetric integral barrier Lyapunov function was proposed in [34]. However, in the above results, the convergence rate of the shifting function is fixed and cannot be adjusted appropriately due to the demand. In order to meet the actual needs, how to design a flexible shifting function is very meaningful. As far as we know, the problem of deferred constraint for nonlinear systems with unmeasurable state is still open and unsolved.

Inspired by the observation, in this paper, we present a method to design output feedback controllers while considering the deferred constraint on system state, indirectly. The objective of the present paper is to use backstepping technique and unified barrier Lyapunov function for solving the problem of asymmetric deferred state constraints on full state for uncertain nonlinear systems.

1) The difficulty of asymmetric constraint on states comes from the design on barrier Function. Due to this, an unified barrier Function is proposed to prevent system states from violating the constraint range, indirectly. Meanwhile, it effectively eliminates the restriction condition for the upper and lower bounds constraint functions being the same comparing with [27].

2) Unlike early results on deferred constraints in [27] and [30], a new shifting function is designed in this paper, which can adjust the convergence rate of the shifting function by adjusting its parameter.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider a class of nonlinear systems in strict feedback form as follows:

$$\begin{cases} \dot{\chi} = A\chi + \psi(y) + \Phi(y)a + B\sigma(y)u \\ y = C\chi \end{cases} \tag{1}$$

where $\chi = [\chi_1, \cdots, \chi_n]^T \in R^n$ is the state vector, u is the control input, and y is the output of system, respectively. $A = \begin{bmatrix} 0_{n-1\times 1} & I_{n-1} \\ 0_{1\times 1} & 0_{n-1\times 1} \end{bmatrix} \in R^{n\times n}$ is a real matrix; $\psi(y) = [\psi_1(y), \psi_2(y), \cdots, \psi_n(y)]^T \in R^n$ is a known smooth function; $\Phi(y) = [\Phi_1(y), \Phi_2(y), \cdots, \Phi_n(y)]^T \in R^n$, $\Phi_i(y) = [\Phi_{i1}(y), \Phi_{i2}(y), \cdots, \Phi_{iq}(y)] \in R^{1\times q}$ is a known smooth function, $a = [a_1, a_2, \cdots, a_q]^T \in R^q$ is an unknown constant vector; $B = [0, \cdots, 0, \underbrace{b_{q_\lambda} \cdots, b_0}_{n-\lambda}]^T \in R^n$ is a known constant vector, $C = [1, 0, \cdots, 0] \in R^n$. Here, assume that the system states x_j are unmeasurable. At the same time, we expect all states to be strictly within the expected range after a period of time, that is, $\underline{R}_{a_j} < \chi_j < \overline{R}_{a_j}$ holds after $t \geq T$, where $\underline{R}_{a_j}, \overline{R}_{a_j}$ are known functions.

Objectives: Firstly, by designing appropriate shifting functions and barrier function, the nonlinear systems with more general deferred constraints is transformed into an equivalent unconstrained system. Then, an effective output feedback control strategy is designed to ensure that all states of the system are strictly limited within the specified range, indirectly. In addition, all other variables in the system are ensured to be bounded.

In this paper, the authors followed the statement of the following assumption.

Assumption 1: [30] Let $-\underline{y}_d(t) \leq y_d(t) \leq \bar{y}_d(t)$, where $\underline{y}_d(t)$ and $\bar{y}_d(t)$ are continuous positive functions and bounded as $\underline{H}_{c_1}(t) > \underline{y}_d(t)$ and $\bar{H}_{c_1}(t) > \bar{y}_d(t)$. Here, the signals $\underline{y}_d(t), y_d(t), \bar{y}_d(t)$ are C^n and bounded. Besides, for $j = 1, \ldots, n$, $\underline{H}_{c_j}(t)$ and $\bar{H}_{c_j}(t)$ are C^{n-j+1}.

A. OBSERVER DESIGN

Since the system states are unmeasurable, the following observer is introduced to accurately estimate them,

$$\dot{\hat{\chi}}_j = \hat{\chi}_{j+1} + l_j(y - \hat{\chi}_1) + \psi_j(y), \quad j = 1, 2, \ldots, \lambda \tag{2}$$

$$\dot{\hat{\chi}}_j = \hat{\chi}_{j+1} + l_j(y - \hat{\chi}_1) + \psi_j(y) + b_{q_j}\sigma(y)u, \quad j = \lambda, \ldots, n-1 \tag{3}$$

$$\dot{\hat{\chi}}_n = b_0\sigma(y)u + l_n(y - \hat{\chi}_1) + \psi_n(y) \tag{4}$$

where $\hat{\chi}_j$ denotes the estimation of χ_j, $q_j = n - j$, $j = \lambda, \cdots, n-1$.

The matrix form of the observer model designed above can rewritten as

$$\dot{\hat{\chi}} = A\hat{\chi} + L(y - C\hat{\chi}) + \psi(y) + B\sigma(y)u \tag{5}$$

with $L = [l_1, l_2, \cdots, l_n]^T \in R^n$ being the designed parameter vector.

Define $\tilde{\chi} = \chi - \hat{\chi}$ as the observer error. Taking the difference between (1) and (5), this results the following equation,

$$\dot{\tilde{\chi}} = A_l\tilde{\chi} + \Phi(y)a \tag{6}$$

where $A_l = A - LC$.

For a given $Q = Q^T > 0$, there exists a positive definite matrix $P = P^T$ satisfying

$$A_l^T P + PA_l = -Q$$

Aim for ensuring the stability of the designed observer, select the Lyapunov candidate function as follows:

$$V_0 = \frac{\tilde{\chi}^T P \tilde{\chi}}{2} \tag{7}$$

Its derivative with respect to time is

$$\dot{V}_0 = \frac{\tilde{\chi}^T \left(A_l^T P + PA_l\right)\tilde{\chi}}{2} + \tilde{\chi}^T P\Phi(y)a \tag{8}$$

Applying Young's inequality to the last term in (8), one has

$$\tilde{\chi}^T P\Phi(y)a \leq \frac{\tilde{\chi}^T\tilde{\chi}}{2} + \frac{\|P\|^2\|\Phi(y)\|^2\check{a}^2}{2} \tag{9}$$

where $\check{a}^2 \geq \|a\|^2$ is a bounded constant.

Putting (9) into (8), one gets

$$\dot{V}_0 \leq -\bar{Q}V_0 + \bar{P} \tag{10}$$

where $\bar{Q} = Q - I$, $\bar{P} = \frac{\|P\|^2\|\Phi(y)\|^2\check{a}^2}{2}$.

III. CONTROLLER DESIGN AND STABILITY ANALYSIS

In this section, in order to indirectly ensure that all state deferred constraints are within the specified range, some shifting functions are introduced. In addition, to ensure the system stability, it is required to design adaptive controllers and the adaptive laws for system (1) based on the backstepping technology.

Consider a problem of stability for system (1) by backstepping technology, the coordinate transformation is firstly defined as follows:

$$z_1 = \chi_1 - y_d \tag{11}$$

$$z_j = \hat{\chi}_j - \alpha_{j-1}, j = 2, \ldots, \lambda \tag{12}$$

where α_{j-1} is the virtual controller to be designed later.

A. SHIFTING FUNCTION

To achieve the goal of deferred constraint on full state, the following shifting function is designed,

$$\varpi(t) = \begin{cases} e^{-\frac{\left(\tan\left(\frac{\pi(T-t)}{2T}\right)\right)^{2n}}{2\Upsilon_1^{2n}}}, & 0 \leq t < T \\ 1, & t \geq T \end{cases} \tag{13}$$

where T is a prespecified finite setting time, Υ_1 is a designed positive constant, which has a great influence on the convergence rate of the shifting function (13), n represents the system order.

Remark 1: Motivated by this idea in [30], [31], [33], and [34], the above prescribed-time shifting function is designed. By introducing such a function, it is possible for all states of the system to converge in the desired range in finite time and the violation of deferred constraint is prevented. Unlike the shifting functions designed in [30], [31], [33], and [34], the shifting function designed above can change the convergence by adjusting the parameter Υ_1, and can be adjusted appropriately as needed.

Remark 2: The convergence rate of function $\varpi(t)$ on the interval $0 \leq t < T$ is affected by the value of parameter Υ_1. The smaller the value of parameter Υ_1 is, the faster the convergence rate of function $\varpi(t)$ is. To observe the effect of parameter Υ_1 on the convergence rate of the shifting function $\varpi(t)$, the trajectories of shifting functions with different parameters are provided in Fig.1. Here, the setting time $T = 1$.

From Fig.1, it shows that when Υ_1 is the small enough, the shift function $\varpi(t)$ can quickly converge to 1 in the interval

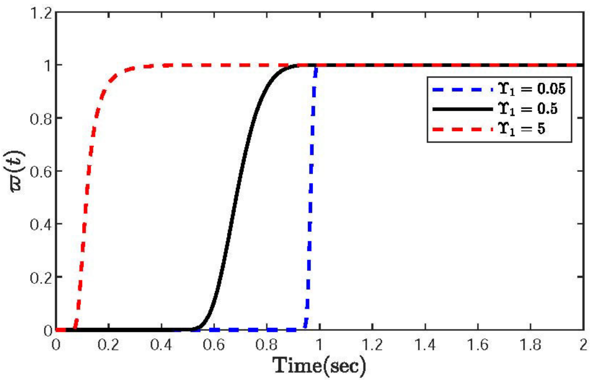

FIGURE 1. Trajectories of shifting function $\varpi(t)$ with different parameters.

of $0 \leq t < T$. Next, some properties of shift function $\varpi(t)$ are given in the following lemma.

Lemma 1: The properties of shifting function $\varpi(t)$ are as follows:

- Shifting function $\varpi(t)$ is a constant when $t \geq T$;
- Shifting function $\varpi(t)$ increases monotonically when $0 \leq t < T$, and $\lim_{t\to T^-} \varpi(t) = 1$;
- The derivative of $\varpi(t)$ is continuous in the interval $t \in [0, +\infty)$.

In order to deal with the deferred constraint problem in two different cases that the upper and lower bounds of the constraint function are the same sign and the different sign, the following shifting functions describe all the cases mentioned above:

$$\zeta_j(t) = \varpi(t)(z_j - \bar{\delta}_j(t)) = \varpi(t)\check{z}_j, \; j = 1, \cdots, \lambda \tag{14}$$

where $\bar{\delta}_j(t)$ will be given later.

To solve the problem of asymmetric state constraints, the barrier function is designed as follows:

$$\begin{aligned}\bar{z}_j = \frac{\bar{F}_{j,1}\bar{F}_{j,2}\zeta_j}{(\bar{F}_{j,1} - \zeta_j)(\bar{F}_{j,2} + \zeta_j)}, \; j = 1, \cdots, \lambda \\ -\bar{F}_{j,2}(0) < \delta_j(0) < \bar{F}_{j,1}(0)\end{aligned} \tag{15}$$

where

$$\bar{F}_{j,1}(t) = \begin{cases} F_{j,1}(t), & F_{j,1}(t)F_{j,2}(t) > 0 \\ F_{j,1}(t) - \delta_j(t), & F_{j,1}(t)F_{j,2}(t) < 0 \end{cases}$$

$$\bar{F}_{j,2}(t) = \begin{cases} F_{j,2}(t), & F_{j,1}(t)F_{j,2}(t) > 0 \\ \delta_j(t) + F_{j,2}(t), & F_{j,1}(t)F_{j,2}(t) < 0 \end{cases}$$

$$\bar{\delta}_j(t) = \begin{cases} 0, & F_{j,1}(t)F_{j,2}(t) > 0 \\ \delta_j(t), & F_{j,1}(t)F_{j,2}(t) < 0 \end{cases}$$

where $F_{j,1}(t)$, $F_{j,2}(t)$ and $\delta_j(t)$ $(j = 1, \cdots, \lambda)$ are known functions, satisfying $-F_{j,2}(t) < \delta_j(t) < F_{j,1}(t)$.

Remark 3: Then the derivative of $\dot{\bar{z}}_j$ can be written as follows:

$$\begin{aligned}\dot{\bar{z}}_j &= \frac{1}{(\bar{F}_{j,1} + \zeta_j)^2(\bar{F}_{j,2} + \zeta_j)^2}((\bar{F}_{j,1}\bar{F}_{j,2}\zeta_j)'(\bar{F}_{j,1} - \zeta_j) \\ &\quad \times (\bar{F}_{j,2} + \zeta_j) - \bar{F}_{j,1}\bar{F}_{j,2}\zeta_j(\dot{\bar{F}}_{j1} - \dot{\zeta}_j)(\bar{F}_{j,2} + \zeta_j) \\ &\quad - \bar{F}_{j,1}\bar{F}_{j,2}\zeta_j(\dot{\bar{F}}_{j,1} - \dot{\zeta}_j)(\bar{F}_{j,2} + \zeta_j)) \\ &= \Gamma_j\dot{\zeta}_j - \Lambda_j\zeta_j^2 - G_j\zeta_j^3\end{aligned} \tag{16}$$

where $\Gamma_j = \frac{\bar{F}_{j,1}^2\bar{F}_{j,2}^2 + \bar{F}_{j,1}\bar{F}_{j,2}\zeta_j^2}{(\bar{F}_{j,1}+\zeta_j)^2(\bar{F}_{j,2}+\zeta_j)^2}$, $\Lambda_j = \frac{\dot{\bar{F}}_{j,1}\bar{F}_{j,2}^2 - \bar{F}_{j,1}^2\dot{\bar{F}}_{j,2}}{(\bar{F}_{j,1}+\zeta_j)^2(\bar{F}_{j,2}+\zeta_j)^2}$ and $G_j = \frac{\dot{\bar{F}}_{j,1}\bar{F}_{j,2} + \bar{F}_{j,1}\dot{\bar{F}}_{j,2}}{(\bar{F}_{j,1}+\zeta_j)^2(\bar{F}_{j,2}+\zeta_j)^2}$ for $j = 1, \cdots, \lambda$.

Let

$$\beta_j = \frac{(\bar{F}_{j,1} - \zeta_j)(\bar{F}_{j,2} + \zeta_j)}{\bar{F}_{j,1}\bar{F}_{j,2}} \tag{17}$$

Then, the following representation becomes valid,

$$\zeta_j = \bar{z}_j\beta_j, \; j = 1, \cdots, \lambda \tag{18}$$

B. DESIGN OF CONTROLLER

Step 1: In what follows, the following candidate Lyapunov function is chosen as

$$V_1 = \frac{1}{2}\bar{z}_1^2 + \frac{\tilde{a}^T\tilde{a}}{2r_1} \tag{19}$$

where $\tilde{a} = a - \hat{a}$ is an adaptive estimation error.

Furthermore, the derivative of V_1 along (16) can be obtained as

$$\dot{V}_1 = \bar{z}_1(\Gamma_1\dot{\zeta}_1 - \Lambda_1\zeta_1^2 - G_1\zeta_1^3) - \frac{\tilde{a}^T\dot{\hat{a}}}{r_1} \tag{20}$$

Substituting (1), (11), (12), (14) and (16) into (20), one has

$$\begin{aligned}\dot{V}_1 &= \bar{z}_1\Gamma_1(\dot{\varpi}\check{z}_1 + \varpi\check{z}_2 + \varpi(\tilde{\chi}_2 + \alpha_1 + \bar{\delta}_2 + \psi_1(y) - \dot{\bar{\delta}}_1 \\ &\quad + \Phi_1(y)a - \frac{\Lambda_1}{\Gamma_1}\zeta_1\check{z}_1 - \dot{y}_d - \frac{G_1}{\Gamma_1}\zeta_1^2\check{z}_1)) - \frac{\tilde{a}^T\dot{\hat{a}}}{r_1}\end{aligned} \tag{21}$$

Using Young's inequality, one has

$$\bar{z}_1\Gamma_1\dot{\varpi}\check{z}_1 \leq \bar{z}_1\Gamma_1\varpi(\frac{b_{12}\Gamma_1\dot{\varpi}^2\check{z}_1^3}{\beta_1}) + \frac{1}{4b_{12}} \tag{22}$$

$$\bar{z}_1\Gamma_1\varpi\tilde{\chi}_2 \leq \bar{z}_1\Gamma_1\varpi(b_{11}\bar{z}_1\Gamma_1\varpi) + \frac{\tilde{\chi}_2^2}{4b_{11}} \tag{23}$$

where $b_{1i} > 0$ $(i = 1, 2)$ are design parameters.

Substituting (22) and (23) into (21) leads to

$$\begin{aligned}\dot{V}_1 &\leq \bar{z}_1\Gamma_1\varpi\check{z}_2 + \bar{z}_1\Gamma_1\varpi(\frac{b_{12}\Gamma_1\dot{\varpi}^2\check{z}_1^3}{\beta_1} + b_{11}\bar{z}_1\Gamma_1\varpi \\ &\quad + \alpha_1 + \bar{\delta}_2 + \psi_1(y) - \dot{\bar{\delta}}_1 + \Phi_1(y)a - \dot{y}_d \\ &\quad - \frac{\Lambda_1}{\Gamma_1}\zeta_1\check{z}_1 - \frac{G_1}{\Gamma_1}\zeta_1^2\check{z}_1) - \frac{\tilde{a}^T\dot{\hat{a}}}{r_1} + \frac{\tilde{\chi}_2^2}{4b_{11}} \\ &\quad + \frac{1}{4b_{12}}\end{aligned} \tag{24}$$

Construct the virtual control law α_1 as

$$\begin{aligned}\alpha_1 &= \frac{-k_1\check{z}_1}{\Gamma_1\beta_1} - \frac{b_{12}\Gamma_1\dot{\varpi}^2\check{z}_1^3}{\beta_1} - \psi_1(y) + \dot{\bar{\delta}}_1 - \Phi_1(y)\hat{a} \\ &\quad + \dot{y}_d - \bar{\delta}_2 + \frac{\Lambda_1}{\Gamma_1}\zeta_1\check{z}_1 + \frac{G_1}{\Gamma_1}\zeta_1^2\check{z}_1 - b_{11}\bar{z}_1\Gamma_1\varpi\end{aligned} \tag{25}$$

Together with (24) and (25), the above inequality (24) can be further simplified as

$$\dot{V}_1 \le \bar{z}_1\Gamma_1\varpi\check{z}_2 - k_1\bar{z}_1^2 + \frac{\tilde{a}^T}{r_1}(r_1\pi_1 - \dot{\hat{a}}) + \frac{\tilde{\chi}_2^2}{4b_{11}} + \frac{1}{4b_{12}} \tag{26}$$

where $\pi_1 = \bar{z}_1\Gamma_1\varpi\Phi_1(y)$.

Step 2: Secondly, one designs the following Lyapunov function V_2 in a similar way,

$$V_2 = V_1 + \frac{\bar{z}_2^2}{2} \tag{27}$$

then, the derivative of V_2 can be calculated as

$$\begin{aligned}\dot{V}_2 = \dot{V}_1 + \bar{z}_2(\Gamma_2(\dot{\varpi}\check{z}_2 + \varpi(\hat{\chi}_3 + l_2\tilde{\chi}_1 + \psi_2(y)\\ - \dot{\alpha}_1 - \dot{\bar{\delta}}_2)) - \Lambda_2\zeta_2^2 - G_2\zeta_2^3)\end{aligned} \tag{28}$$

Substituting (1), (12), (14) and (16) into (28), one has

$$\begin{aligned}\dot{V}_2 \le{}& \bar{z}_1\Gamma_1\varpi\check{z}_2 - k_1\bar{z}_1^2 + \frac{\tilde{a}_1^T}{r_1}(r_1\pi_1 - \dot{\hat{a}}_1) + \frac{\tilde{\chi}_2^2}{4b_{11}}\\ &+ \frac{1}{4b_{12}} + \bar{z}_2\Gamma_2(\dot{\varpi}\check{z}_2 + \varpi\check{z}_3 + \varpi(\bar{\delta}_3 + \check{z}_3 + \alpha_2\\ &+ l_2\tilde{\chi}_1 + \psi_2(y) - \Xi_1 - \frac{\partial\alpha_1}{\partial y}\tilde{\chi}_2 - \frac{\partial\alpha_1}{\partial y}\Phi_1(y)a\\ &- \frac{\partial\alpha_1}{\partial\hat{a}_1}\dot{\hat{a}}_1 - \dot{\bar{\delta}}_2 - \frac{\Lambda_2}{\Gamma_2}\zeta_2\check{z}_2 - \frac{G_2}{\Gamma_2}\zeta_2^2\check{z}_2)\end{aligned} \tag{29}$$

where $\Xi_1 = \dot{\alpha}_1 - \frac{\partial\alpha_1}{\partial y}\tilde{\chi}_2 - \frac{\partial\alpha_1}{\partial y}\Phi_1(y)a - \frac{\partial\alpha_1}{\partial\hat{a}}\dot{\hat{a}}$.

Based on (25), taking the derivative of α_1 yields

$$\begin{aligned}\dot{\alpha}_1 ={}& \sum_{i=0}^{1}\frac{\partial\alpha_1}{\partial y_d^{(i)}}y_d^{(i+1)} + \frac{\partial\alpha_1}{\partial y}(\hat{\chi}_2 + \tilde{\chi}_2 + \psi_1(y)\\ &+ \Phi_1(y)a - \dot{y}_d) + \sum_{i=0}^{1}\frac{\partial\alpha_1}{\partial\varpi^{(i)}}\varpi^{(i+1)} + \frac{\partial\alpha_1}{\partial\hat{a}}\dot{\hat{a}}\\ &+ \frac{\partial\alpha_1}{\partial\bar{\delta}_2}\dot{\bar{\delta}}_2 + \sum_{i=0}^{1}\frac{\partial\alpha_1}{\partial\bar{\delta}_1}\bar{\delta}_1^{(i+1)} + \sum_{i=0}^{1}\frac{\partial\alpha_1}{\partial\bar{F}_{1,1}^{(i)}}\bar{F}_{1,1}^{(i+1)}\\ &+ \sum_{i=0}^{1}\frac{\partial\alpha_1}{\partial\bar{F}_{1,2}^{(i)}}\bar{F}_{1,2}^{(i+1)}\end{aligned} \tag{30}$$

The following inequalities hold with Young's inequality:

$$-\bar{z}_2\Gamma_2\varpi\frac{\partial\alpha_1}{\partial y}\tilde{\chi}_2 \le b_{21}(\bar{z}_2\Gamma_2\varpi)^2(\frac{\partial\alpha_1}{\partial y})^2 + \frac{\tilde{\chi}_2^2}{4b_{21}} \tag{31}$$

$$\bar{z}_2\Gamma_2\dot{\varpi}\check{z}_2 \le \bar{z}_2\Gamma_2\varpi(\frac{b_{22}\Gamma_2\dot{\varpi}^2\check{z}_2^3}{\beta_2}) + \frac{1}{4b_{22}} \tag{32}$$

$$\bar{z}_1\Gamma_1\varpi\check{z}_2 \le \beta_2 b_{23}\bar{z}_1^2\Gamma_1^2\varpi\check{z}_2\bar{z}_2 + \frac{1}{4b_{23}} \tag{33}$$

where $b_{2i} > 0$ $(i = 1, 2, 3)$ are design parameters.

Substituting (30), (31), (32) and (33) into (29), it can be deduced that

$$\begin{aligned}\dot{V}_2 \le{}& -k_1\bar{z}_1^2 + \frac{\tilde{a}^T}{r_1}(r_1\pi_1 - \dot{\hat{a}}) + \bar{z}_2\Gamma_2\varpi(\check{z}_3 + \bar{\delta}_3 + \alpha_2\\ &+ l_2\tilde{\chi}_1 + \psi_2(y) - \frac{\partial\alpha_1}{\partial\hat{a}}\dot{\hat{a}} - \Xi_1 - \frac{\partial\alpha_1}{\partial y}\Phi_1(y)\,a\\ &+ \frac{b_{22}\Gamma_2\dot{\varpi}^2\check{z}_2^3}{\beta_2} + b_{21}\bar{z}_2\Gamma_2\varpi(\frac{\partial\alpha_1}{\partial y})^2 - \dot{\bar{\delta}}_2\\ &- \frac{\Lambda_2}{\Gamma_2}\zeta_2\check{z}_2 - \frac{G_2}{\Gamma_2}\zeta_2^2\check{z}_2 + \frac{\beta_2 b_{23}\bar{z}_1^2\Gamma_1^2\check{z}_2}{\Gamma_2})\\ &+ \sum_{i=1}^{2}\frac{\tilde{\chi}_2^2}{4b_{i1}} + D_2\end{aligned} \tag{34}$$

where $D_2 = \frac{1}{4b_{12}} + \sum_{i=2}^{3}\frac{1}{4b_{2i}}$.

Design the following virtual controller for the second subsystem,

$$\begin{aligned}\alpha_2 ={}& \frac{-k_2\check{z}_2}{\Gamma_2\beta_2} - \frac{\beta_2 b_{24}\bar{z}_1^2\Gamma_1^2\check{z}_2}{\Gamma_2} - \frac{b_{22}\Gamma_2\dot{\varpi}^2\check{z}_2^3}{\beta_2} + \frac{\partial\alpha_1}{\partial y}\Phi_1(y)\,\hat{a}\\ &+ \frac{\partial\alpha_1}{\partial\hat{a}}E_a - \psi_2(y) - \bar{\delta}_3 - b_{21}\bar{z}_2\Gamma_2\varpi(\frac{\partial\alpha_1}{\partial y})^2 + \dot{\bar{\delta}}_2(t)\\ &+ \Xi_1 + \frac{\Lambda_2}{\Gamma_2}\zeta_2\check{z}_2 + \frac{G_2}{\Gamma_2}\zeta_2^2\check{z}_2 + \frac{\partial\alpha_1}{\partial\hat{a}}\pi_2\end{aligned} \tag{35}$$

Substituting (35) into (34) results in

$$\begin{aligned}\dot{V}_2 \le{}& \bar{z}_2\Gamma_2\varpi\check{z}_3 - \sum_{i=1}^{2}k_i\bar{z}_i^2 + \frac{\tilde{a}^T}{r_1}(r_1\pi_2 - \dot{\hat{a}})\\ &+ \bar{z}_2\Gamma_2\varpi\frac{\partial\alpha_1}{\partial\hat{a}}(E_a - \dot{\hat{a}}) + D_2\end{aligned} \tag{36}$$

where $\pi_2 = \pi_1 - \bar{z}_2\Gamma_2\varpi\frac{\partial\alpha_1}{\partial y}\Phi_1(y)$.

Step j $(3 \le j < \lambda)$: Consider the Lyapunov candidate function in the jth step,

$$V_j = V_{j-1} + \frac{\bar{z}_j^2}{2} \tag{37}$$

then, the derivative of V_j is computed as below,

$$\begin{aligned}\dot{V}_j ={}& \dot{V}_{j-1} + \bar{z}_j(\Gamma_j(\dot{\varpi}\check{z}_j + \varpi(z_{j+1} + \alpha_j + l_j\tilde{\chi}_1 + \psi_j(y)\\ &- \dot{\alpha}_{j-1} - \dot{\bar{\delta}}_j) - \Lambda_j\zeta_j^2 - G_j\zeta_j^3)\\ ={}& \dot{V}_{j-1} + \bar{z}_j\Gamma_j(\dot{\varpi}\check{z}_j + \varpi(z_{j+1} + \alpha_j + l_j\tilde{\chi}_1 + \psi_j(y)\\ &- \Xi_{j-1} - \frac{\partial\alpha_{j-1}}{\partial y}\tilde{\chi}_2 - \frac{\partial\alpha_{j-1}}{\partial y}\Phi_1(y)a - \frac{\partial\alpha_{j-1}}{\partial\hat{a}}\dot{\hat{a}}\\ &- \dot{\bar{\delta}}_j) - \frac{\Lambda_j}{\Gamma_j}\zeta_j^2 - \frac{G_j}{\Gamma_j}\zeta_j^3)\end{aligned} \tag{38}$$

where $\Xi_{j-1} = \dot{\alpha}_{j-1} - \frac{\partial\alpha_{j-1}}{\partial y}\tilde{\chi}_2 - \frac{\partial\alpha_{j-1}}{\partial y}\Phi_1(y)\,a - \frac{\partial\alpha_{j-1}}{\partial\hat{a}}\dot{\hat{a}}$.

Similar to (30), $\dot{\alpha}_{j-1}$ can be expressed as follows:

$$\begin{aligned}\dot{\alpha}_{j-1} = &\sum_{i=0}^{j-1} \frac{\partial \alpha_{j-1}}{\partial y_d^{(i)}} y_d^{(i+1)} + \frac{\partial \alpha_{j-1}}{\partial y}(\hat{\chi}_2 + \tilde{\chi}_2 \\ &+ \psi_1(y) + \Phi_1(y)a - \dot{y}_d) + \sum_{i=0}^{j-1} \frac{\partial \alpha_{j-1}}{\partial \varpi^{(i)}} \varpi^{(i+1)} \\ &+ \frac{\partial \alpha_{j-1}}{\partial a_1}\dot{a}_1 + \sum_{i=0}^{j-1} \frac{\partial \alpha_{j-1}}{\partial \bar{\delta}_1^{(i)}} \bar{\delta}_1^{(i+1)} \\ &+ \sum_{i=0}^{j-2} \frac{\partial \alpha_{j-1}}{\partial \bar{\delta}_2^{(i)}} \bar{\delta}_2^{(i+1)} + \ldots + \frac{\partial \alpha_{j-1}}{\partial \bar{\delta}_{j+1}} \dot{\bar{\delta}}_{j+1} \\ &+ \sum_{i=0}^{j-1} \frac{\partial \alpha_{j-1}}{\partial \bar{F}_{1,1}^{(i)}} \bar{F}_{1,1}^{(i+1)} + \sum_{i=0}^{j-1} \frac{\partial \alpha_{j-1}}{\partial \bar{F}_{1,2}^{(i)}} \bar{F}_{1,2}^{(i+1)} \\ &+ \sum_{i=0}^{j-2} \frac{\partial \alpha_{j-1}}{\partial \bar{F}_{2,1}^{(i)}} \bar{F}_{2,1}^{(i+1)} + \sum_{i=0}^{j-2} \frac{\partial \alpha_{j-1}}{\partial \bar{F}_{2,2}^{(i)}} \bar{F}_{2,2}^{(i+1)} \\ &+ \ldots + \sum_{i=0}^{1} \frac{\partial \alpha_{j-1}}{\partial \bar{F}_{j-1,1}^{(i)}} \bar{F}_{j-1,1}^{(i+1)} \\ &+ \sum_{i=0}^{1} \frac{\partial \alpha_{j-1}}{\partial \bar{F}_{j-1,2}^{(i)}} \bar{F}_{j-1,2}^{(i+1)} \end{aligned} \tag{39}$$

The following results are obtained using Young's inequality:

$$\begin{aligned} -\bar{z}_j\Gamma_j\varpi \frac{\partial \alpha_{j-1}}{\partial y}\tilde{\chi}_2 \leq \bar{z}_j\Gamma_j\varpi b_{j1}\bar{z}_j\Gamma_j\varpi(\frac{\partial \alpha_{j-1}}{\partial y})^2 \\ + \frac{\tilde{\chi}_2^2}{4b_{j1}} \end{aligned} \tag{40}$$

$$\bar{z}_j\Gamma_j\dot{\varpi}\check{z}_j \leq \bar{z}_j\Gamma_j\varpi(\frac{b_{j2}\Gamma_j\dot{\varpi}^2\check{z}_j^3}{\beta_j}) + \frac{1}{4b_{j2}} \tag{41}$$

$$\bar{z}_{j-1}\Gamma_{j-1}\varpi\check{z}_j \leq \beta_j b_{j3}\bar{z}_{j-1}^2\Gamma_{j-1}^2\varpi\check{z}_j\bar{z}_j + \frac{1}{4b_{j3}} \tag{42}$$

where $b_{j1} > 0$, $b_{j2} > 0$, $b_{j3} > 0$ are design parameters.

Substituting (39), (40), (41) and (42) into (38), it is then derived that

$$\begin{aligned}\dot{V}_j \leq &-\sum_{i=1}^{j-1} k_i\bar{z}_i^2 + \frac{\tilde{a}^T}{r_1}(r_1\pi_j - \dot{\hat{a}}) + \bar{z}_{j-1}\Gamma_{j-1}\varpi \\ &\times \frac{\partial \alpha_{j-2}}{\partial \hat{a}}(E_a - \dot{\hat{a}}) + \bar{z}_j\Gamma_j\varpi(\check{z}_{j+1} + \bar{\delta}_{j+1} + \alpha_j \\ &+ l_j\tilde{\chi}_1 + \psi_j(y) - \frac{\partial \alpha_{j-1}}{\partial \hat{a}}\dot{\hat{a}} + \frac{b_{j2}\Gamma_j\dot{\varpi}^2\check{z}_j^3}{\beta_j} \\ &- \frac{\partial \alpha_{j-1}}{\partial y}\Phi_1(y)\,a - \Xi_{j-1} + (\frac{\partial \alpha_{j-1}}{\partial y})^2 \\ &\times b_{j1}\bar{z}_j\Gamma_j\varpi - \dot{\bar{\delta}}_j - \frac{\Lambda_j}{\Gamma_j}\zeta_j\check{z}_j - \frac{G_j}{\Gamma_j}\zeta_j^2\check{z}_j \\ &+ \frac{\beta_j b_{j5}\bar{z}_{j-1}^2\Gamma_{j-1}^2\check{z}_j}{\Gamma_j}) + \sum_{i=1}^{j} \frac{\tilde{\chi}_2^2}{4b_{i1}} + D_j \end{aligned} \tag{43}$$

where $D_j = D_{j-1} + \sum_{i=2}^{3} \frac{1}{4b_{ji}}$, $\pi_j = \pi_{j-1} - \bar{z}_j\Gamma_j\varpi\frac{\partial \alpha_{j-1}}{\partial y}\Phi_1(y)$.

According to (43), let construct the virtual controller α_j as

$$\begin{aligned}\alpha_j = &\frac{-k_j\check{z}_j}{\Gamma_j\beta_j} - \frac{\beta_j b_{j4}\bar{z}_{j-1}^2\Gamma_{j-1}^2\check{z}_j}{\Gamma_j} - \frac{b_{j2}\Gamma_j\dot{\varpi}^2\check{z}_j^3}{\beta_j} \\ &+ \frac{\partial \alpha_{j-1}}{\partial y}\Phi_1(y)\hat{a} + \frac{\partial \alpha_{j-1}}{\partial \hat{a}}E_a - \bar{\delta}_{j+1} \\ &- \psi_j(y) + \Xi_{j-1} - b_{j1}\bar{z}_j\Gamma_j\varpi(\frac{\partial \alpha_{j-1}}{\partial y})^2 + \dot{\bar{\delta}}_j(t)) \\ &+ \frac{\Lambda_j}{\Gamma_j}\zeta_j\check{z}_j + \frac{G_j}{\Gamma_j}\zeta_j^2\check{z}_j - \frac{\partial \alpha_{j-1}}{\partial \hat{a}}\pi_j \end{aligned} \tag{44}$$

where $\pi_j = \pi_{j-1} - \frac{\partial \alpha_{j-1}}{\partial y}\bar{z}_j\Gamma_j\varpi\Phi_1(y)$.

Substituting (44) into (43) yields

$$\begin{aligned}\dot{V}_j \leq &\bar{z}_j\Gamma_j\varpi\check{z}_{j+1} - \sum_{i=1}^{j} k_i\bar{z}_i^2 + \frac{\tilde{a}^T}{r_1}(r_1\pi_j - \dot{\hat{a}}) \\ &- \sum_{i=2}^{j} \bar{z}_i\Gamma_i\varpi\frac{\partial \alpha_{i-1}}{\partial \hat{a}}(\dot{\hat{a}} - E_a) + D_j \end{aligned} \tag{45}$$

Using the inductive manner, one can obtain

$$\begin{aligned}\dot{V}_{\lambda-1} \leq &\bar{z}_{\lambda-1}\Gamma_{\lambda-1}\varpi\check{z}_\lambda - \sum_{i=1}^{\lambda-1} k_i\bar{z}_i^2 + \frac{\tilde{a}^T}{r_1}(r_1\pi_{\lambda-1} - \dot{\hat{a}}) \\ &- \sum_{i=2}^{\lambda-1} \bar{z}_i\Gamma_i\varpi\frac{\partial \alpha_{i-1}}{\partial \hat{a}}(\dot{\hat{a}} - E_a) + D_{\lambda-1} \end{aligned} \tag{46}$$

Step λ **:** Select the Lyapunov candidate function as

$$V_\lambda = V_{\lambda-1} + \frac{\bar{z}_\lambda^2}{2} \tag{47}$$

Similar to the step j, the derivative of (47) is

$$\begin{aligned}\dot{V}_\lambda = &\dot{V}_{\lambda-1} + \bar{z}_\lambda\Gamma_\lambda(\dot{\varpi}\check{z}_\lambda + \varpi(\hat{\chi}_{\lambda+1} + l_\lambda\tilde{\chi}_1 \\ &+ \psi_\lambda(y) + b_{q_\lambda}\sigma(y)u - (\dot{\alpha}_{\lambda-1} - \frac{\partial \alpha_{\lambda-1}}{\partial y}\tilde{\chi}_2 \\ &+ \frac{\partial \alpha_{j-1}}{\partial y}\Phi_1(y)a + \frac{\partial \alpha_{j-1}}{\partial \hat{a}}\dot{\hat{a}}) - \dot{\bar{\delta}}_\lambda) - \frac{\Lambda_\lambda}{\Gamma_\lambda}\zeta_\lambda^2 \\ &- \frac{G_\lambda}{\Gamma_\lambda}\zeta_\lambda^3) \end{aligned} \tag{48}$$

With the help of Young's inequality, it follows that:

$$-\bar{z}_\lambda\Gamma_\lambda\varpi\frac{\partial \alpha_{\lambda-1}}{\partial y}\tilde{\chi}_2 \leq b_{\lambda1}(\bar{z}_\lambda\Gamma_\lambda\varpi)^2(\frac{\partial \alpha_{\lambda-1}}{\partial y})^2 + \frac{\tilde{\chi}_2^2}{4b_{\lambda1}} \tag{49}$$

$$\bar{z}_\lambda\Gamma_\lambda\dot{\varpi}\check{z}_\lambda \leq \bar{z}_\lambda\Gamma_\lambda\varpi(\frac{b_{\lambda2}\Gamma_\lambda\dot{\varpi}^2\check{z}_\lambda^3}{\beta_\lambda}) + \frac{1}{4b_{\lambda2}} \tag{50}$$

$$\bar{z}_{\lambda-1}\Gamma_{\lambda-1}\varpi\check{z}_\lambda \leq \beta_\lambda b_{\lambda3}\bar{z}_{\lambda-1}^2\Gamma_{\lambda-1}^2\varpi\check{z}_\lambda\bar{z}_\lambda + \frac{1}{4b_{\lambda3}} \tag{51}$$

where $b_{\lambda1}$, $b_{\lambda2}$, $b_{\lambda3}$ are constants designed.

From (49) − (51), one has

$$\dot{V}_\lambda \leq -\sum_{i=1}^{\lambda-1} k_i \bar{z}_i^2 + \frac{\tilde{a}^T}{r_1}(r_1\pi_{\lambda-1} - \dot{\hat{a}}) + \bar{z}_{\lambda-1}\Gamma_{\lambda-1}\varpi \times \frac{\partial\alpha_{\lambda-2}}{\partial\hat{a}}(E_a - \dot{\hat{a}}) + \bar{z}_\lambda\Gamma_\lambda\varpi(\hat{\chi}_{\lambda+1} + l_\lambda\tilde{\chi}_1 + \psi_\lambda(y) + b_{p_\lambda}\sigma(y)u + l_\lambda\tilde{\chi}_1 + \psi_{\lambda j}(y) - \frac{\partial\alpha_{\lambda-1}}{\partial\hat{a}}\dot{\hat{a}} - \Xi_{\lambda-1} + \frac{b_{\lambda 2}\Gamma_\lambda\dot{\varpi}^2\check{z}_\lambda^3}{\beta_\lambda} - \bar{z}_\lambda\Gamma_\lambda\varpi\frac{\partial\alpha_{\lambda-1}}{\partial y}\Phi_1(y)a + b_{\lambda 1}\bar{z}_\lambda\Gamma_\lambda\varpi(\frac{\partial\alpha_{\lambda-1}}{\partial y})^2 - \dot{\bar{\delta}}_\lambda - \frac{\Lambda_\lambda}{\Gamma_\lambda}\zeta_\lambda\check{z}_j - \frac{G_\lambda}{\Gamma_\lambda}\zeta_\lambda^2\check{z}_\lambda + \frac{\beta_\lambda b_{\lambda 3}\bar{z}_{\lambda-1}^2\Gamma_{\lambda-1}^2\check{z}_\lambda}{\Gamma_\lambda}) + \sum_{i=1}^{\lambda}\frac{\tilde{\chi}_2^2}{4b_{i1}} + D_\lambda \tag{52}$$

By taking the above inequalities, the following actual controller is

$$u = \frac{1}{b_{p_\lambda}\sigma(y)}(\frac{-k_\lambda\check{z}_\lambda}{\Gamma_\lambda\beta_\lambda} - \frac{\beta_\lambda b_{\lambda 3}\bar{z}_{\lambda-1}^2\Gamma_{\lambda-1}^2\check{z}_\lambda}{\Gamma_\lambda} - \frac{b_{\lambda 1}\Gamma_\lambda\dot{\varpi}^2\check{z}_\lambda^3}{\beta_\lambda} - \psi_\lambda(y) + \bar{z}_\lambda\Gamma_\lambda\varpi\frac{\partial\alpha_{\lambda-1}}{\partial y}\Phi_1(y)a + \Xi_{\lambda-1} - b_{\lambda 1}\bar{z}_\lambda\Gamma_\lambda\varpi(\frac{\partial\alpha_{\lambda-1}}{\partial y})^2 + \frac{\partial\alpha_{\lambda-1}}{\partial\hat{a}}E_a + \dot{\bar{\delta}}_\lambda(t)) + \frac{\Lambda_\lambda}{\Gamma_\lambda}\zeta_\lambda\check{z}_\lambda + \frac{G_\lambda}{\Gamma_\lambda}\zeta_\lambda^2\check{z}_\lambda - \frac{\partial\alpha_{\lambda-1}}{\partial\hat{a}}\pi_\lambda + \sum_{i=2}^{\lambda-1} r_1\bar{z}_i\tau_i\varpi\frac{\partial\alpha_{i-1}}{\partial\hat{a}}b_{\lambda 3}(\frac{\partial\alpha_{\lambda-1}}{\partial y})^2 \times \bar{z}_\lambda\Gamma_\lambda\varpi\Phi_1^2(y)), \tag{53}$$

where $\pi_\lambda = \pi_{\lambda-1} - \frac{\partial\alpha_{\lambda-1}}{\partial y}\bar{z}_\lambda\Gamma_\lambda\varpi\Phi_1(y)$.

Substituting (53) into (52), the inequality (52) can be rewritten as

$$\dot{V}_\lambda \leq -\sum_{i=1}^{\lambda} k_i\bar{z}_i^2 + \frac{\tilde{a}_1^T}{r_1}(r_1\pi_\lambda - \dot{\hat{a}}) - \sum_{i=2}^{\lambda}\bar{z}_i\Gamma_i\varpi\frac{\partial\alpha_{i-1}}{\partial\hat{a}}(\dot{\hat{a}} - E_a) + D_\lambda \tag{54}$$

The adaptive law for $\hat{a}$ is designed in the following form:

$$\dot{\hat{a}} = r_1\pi_\lambda \tag{55}$$

Let $E_a = r_1\pi_\lambda$, the following inequalities can be derived

$$\dot{V}_\lambda \leq -\sum_{i=1}^{\lambda} k_i\bar{z}_i^2 - \frac{\tilde{a}_1^T\tilde{a}_1}{2} + D_\lambda \tag{56}$$

where $D_\lambda = D_{\lambda-1} + \sum_{i=2}^{4}\frac{1}{4b_{\lambda i}} + \frac{1}{2}\max\|\tilde{a}\|^2$. Based on (19), (27), (37) and (47), the inequality (35) can be represented as

$$\dot{V}_\lambda \leq -a_\lambda V_\lambda + D_\lambda \tag{57}$$

where $a_\lambda = \min\{2k_i, r_1, i = 1, \cdots, \lambda\}$.

C. STABILITY ANALYSIS

Theorem 1: Assume that Assumptions 1 − 2 are fulfilled. Considering the strict feedback nonlinear system (21) with state constraints satisfying the virtual controllers (25), (35) , (44), the actual controller (53) and the adaptive law (55) are constructed. By properly selecting the design parameters L, Υ_1, b_{11}, b_{12}, $b_{ji}(j = 2, \cdots, \lambda, i = 1, 2, 3)$, all variables in the system can be guaranteed to be bounded and all states do not violate the desired constraints.

Proof: For proving the stability of systems in (21), the following Lyapunov candidate function is defined as

$$V = V_0 + V_\lambda \tag{58}$$

Based on (10) and (57), its derivative is

$$\dot{V}(t) \leq -aV(t) + D \tag{59}$$

where $a = \min\{a_j, \lambda_{min}(\bar{Q}), j = 1, \cdots, n\}$, $D = \sum_{j=1}^{\lambda} D_\lambda + \bar{P}$.

As described in [35], for a chosen Lyapuniv function $V(t)$, if the inequality $\dot{V}(t) \leq -aV(t) + c$ holds, then, it is easy to obtain the following inequality:

$$V(t) \leq e^{-at}V(0) + \frac{c}{a}(1 - e^{-at}) \tag{60}$$

with a and c being positive constants.

Note that as $t \to \infty$, we have

$$V_\infty \leq \frac{D}{a} \tag{61}$$

Based on the analysis above, all variables ζ_j $(j = 1, \cdots, \lambda)$ are bounded. When $t > T$, $\zeta_j = \check{z}_j$, it can be concluded that all errors $\check{z}_j$ $(j = 1, \ldots, \lambda)$ satisfy the expected constraint requirements, that is $-\bar{F}_{j,2} \leq \check{z}_j \leq \bar{F}_{j,1}$. From $\check{z}_1 = \chi_1 - y_d - \bar{\delta}_1$ and $\underline{H}_{c_1} < y_d < \bar{H}_{c_1}$, we further obtain $\underline{H}_{c_1} - \bar{F}_{1,2} - F_{1,2} < \chi_1 < \bar{F}_{1,1} + \bar{H}_{c_1} + F_{1,1}$. Let $\bar{F}_{1,2} = \underline{H}_{c_1} - \underline{R}_{a_1} + F_{1,2}$ and $\bar{F}_{1,1} = \bar{R}_{a_1} - \bar{H}_{c_1} - F_{2,1}$, thus, the system output χ_1 is enforced to a desired constraint, that is, $\underline{R}_{a_1} < \chi_1 < \bar{R}_{a_1}$. Since the virtual control signal α_1 in (25) is continuous, there exist constants $\underline{R}_{b_1}$, and $\bar{R}_{b_1}$ such that $\underline{R}_{b_1} \leq \alpha_1 \leq \bar{R}_{b_1}$. According to $\check{z}_2 = \chi_2 - \alpha_1 - \bar{\delta}_2$, and $-\bar{F}_{2,2} < \check{z}_2 < \bar{F}_{2,1}$, we can obtain $\underline{R}_{b_1} - \bar{F}_{2,2} - F_{2,2} < \chi_2 < \bar{R}_{b_1} + \bar{F}_{2,1} + F_{2,1}$. Let $\bar{F}_{2,2} = \underline{R}_{b_1} - \underline{R}_{a_2}$ and $\bar{F}_{2,1} = \bar{R}_{a_2} - \bar{R}_{b_1}$, thus, $\underline{R}_{a_2} < \chi_2 < \bar{R}_{a_2}$. Similar to the same idea of χ_2, one has $\underline{R}_{a_j} < \chi_j < \bar{R}_{a_j}$ with $\bar{F}_{j,2} = \underline{R}_{b_{j-1}} - \underline{R}_{a_j} + F_{j,2}$ and $\bar{F}_{j,1} = \bar{R}_{a_j} - \bar{R}_{b_{j-1}} - F_{j,1}$, $j = (3, \ldots, n)$. Thus, we can get that all state are strictly within the constraint ranges when $t > T$, which concludes the proof. ■

IV. SIMULATION RESULTS

Starting with the previously described problem, we implement control strategy and allow the simulation to the following nonlinear system model:

$$\begin{cases} \dot{x}_1 = 0.5\sin(0.2x_1) + x_2 \\ \dot{x}_2 = x_1^2 + 0.05x_1 + (1 + x_1^2 u) \\ y = x_1 \end{cases} \tag{62}$$

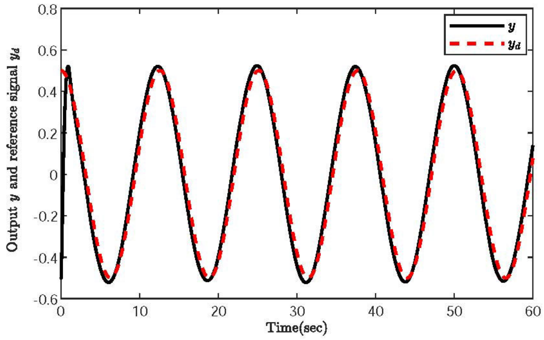

FIGURE 2. Output y and reference signal y_d.

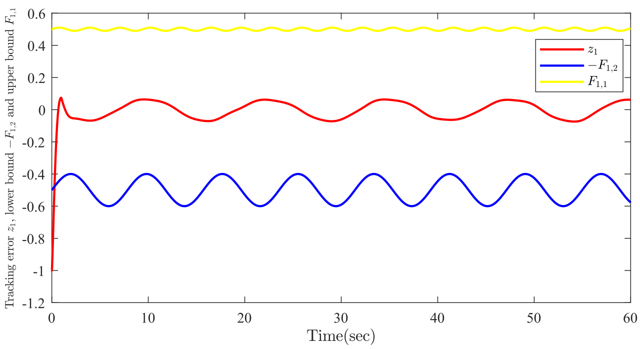

FIGURE 3. Tracking error z_1, and constraint functions $-F_{1,2}$ and $-F_{1,1}$.

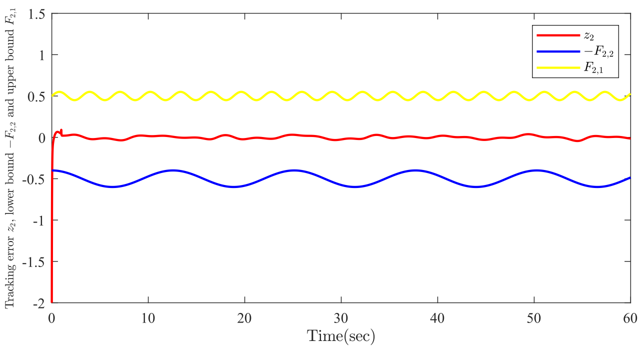

FIGURE 4. Tracking error z_2, and constraint functions $-F_{2,2}$ and $F_{2,1}$.

Choose the initial value of system state is $x(0) = [-0.5, -0.03, 0.1, 0, 0.2]$. Suppose,the reference signal y_d is defined as $y_d = 0.5\cos(0.5t)$. To proceed with the design of control law and adaptive law, the main parameters are chosen as $b_{11} = 0.02$ $b_{12} = 2$ $b_{13} = 1$ $b_{15} = 1.5$ $b_{21} = 0.02$ $b_{22} = 0.01$ $b_{25} = 0.01$ $k_1 = 2$ $k_2 = 45$ $c = 1$ $l_1 = 5$ $l_2 = 12$ $r = 0.1$.

According to Figs.3-7, we can obtain the following simulation results. Fig.2 shows the trajectories of the output y and the reference signal y_d. It can be seen from this figure that the system has good tracking performance. Fig.3 and Fig.4 show the trajectories of errors z_1 and z_2 as well as their constraint functions, respectively. It shows that all errors are

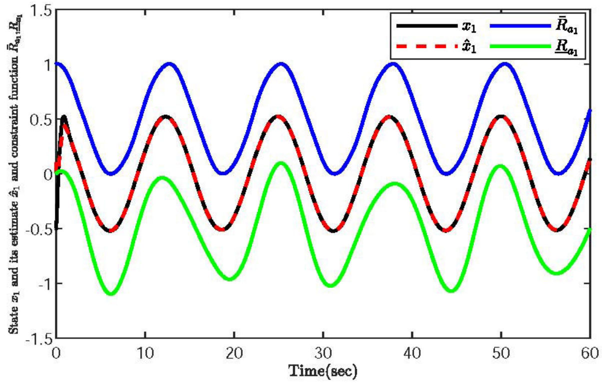

FIGURE 5. State x_1 and its estimated state $\hat{x}_1$ and constraint functions $\overline{R}_{a_1}$,$\underline{R}_{a_1}$.

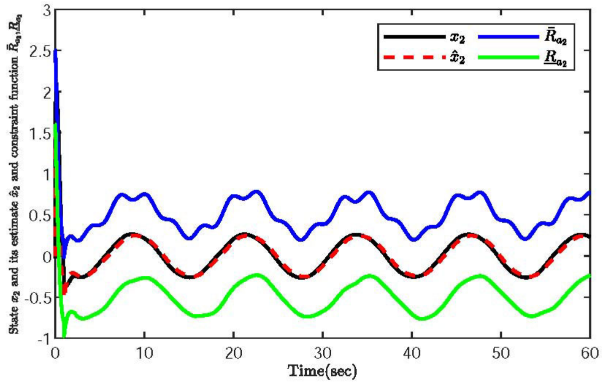

FIGURE 6. State x_2 and its estimated state $\hat{x}_2$ and constraint functions $\overline{R}_{a_2}$,$\underline{R}_{a_2}$.

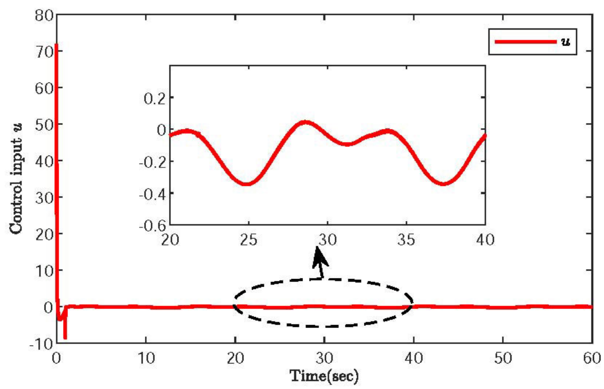

FIGURE 7. The control input u.

strictly within the range of constraints after a period of time. The responses of state x_1, x_2, their estimation $\hat{x}_1, \hat{x}_2$ and their constraint functions are given in Fig.5 and Fig.6, respectively. From them, we can know that the designed observer can accurately estimate all the system states, and all states do not exceed the range of constraints. Fig. 7 plots the response of control input u. Through the above simulation results, it is verified that the proposed control method can guarantee all errors and states are within their constraint range after a period of time. Meanwhile, other variables are bounded.

V. CONCLUSION

The paper developed an output feedback control for a class of nonlinear system with deferred constraints. The unmeasurable state of the considered system is perfectly estimated by introducing a linear observer. In order to make the constraint problem studied more general, a new coordinate transformation was performed on all errors. In addition, a new shifting function was introduced for achieving the control objective of deferred constraint on all states. Based on the backstepping method and Lyapunov theory, the whole design process for controller analyzed in detail. At last, the stability and performance with the proposed control scheme was affirmed by simulation results. Although the proposed method is effective for solving full state constraints, the problem of "explosion of complexity" still exists. In following work, inspired by the ideas in [36] and [37], we will try to propose a backstepping design method embedded with time-varying command filters to solve the consensus problem of multiagent systems.

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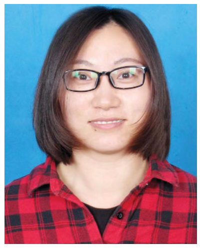

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