IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received 24 December 2023, accepted 16 January 2024, date of publication 19 January 2024, date of current version 26 January 2024.

Digital Object Identifier 10.1109/ACCESS.2024.3356077

RESEARCH ARTICLE

Distributed Optimal Power Dispatch for Islanded DC Microgrids With Time Delays

MOHAMED ZAERY[1], (Member, IEEE), AND **MOHAMMAD A. ABIDO**[2,3,4], (Senior Member, IEEE)

[1]K. A. CARE Energy Research and Innovation Center, King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia
[2]Electrical Engineering Department, King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia
[3]Interdisciplinary Research Center for Sustainable Energy Systems, King Fahd University of Petroleum and Minerals (KFUPM), Dhahran 31261, Saudi Arabia
[4]SDAIA-KFUPM Joint Research Center for Artificial Intelligence, KFUPM, Dhahran 31261, Saudi Arabia

Corresponding author: Mohammad A. Abido (mabido@kfupm.edu.sa)

This work is supported by Renewable Energy Technical Incubator (RETI) under Interdisciplinary Research Center for Sustainable Energy Systems (IRC-SES), KFUPM, through Project No. CREP2522. The authors extend their appreciation to King Abdullah City for Atomic and Renewable Energy (K.A. CARE), KFUPM for their financial support in the completion of this work. Dr. Abido expresses gratitude for the support provided by the Saudi Data and AI Authority (SDAIA) and King Fahd University of Petroleum & Minerals (KFUPM), through the SDAIA-KFUPM Joint Research Center for Artificial Intelligence, under grant number JRC-AI-RFP-09.

ABSTRACT This paper proposes a fully distributed fixed-time control approach fulfilling the economic operation of DC microgrids (MGs) and considering time delays. A distributed cost optimizer is developed for maintaining MG's economical operation through equalizing DGs' incremental costs (ICs) while addressing DGs' capacity limits within a fixed settling time unrelated to the initial values. Besides, a fixed-time voltage regulator is presented to restore MG's average voltage for preserving generations-demands power balance. For further improvement of the system stability to oppose time delays, Artstein's reduction technique is employed for transforming the delayed system to a delay-free one. Accordingly, the proposed controller has an improved dynamic performance and lower integral squared error (ISE) than the existing fixed-time controller, especially with high time delays. Comprehensive convergence analysis confirms that the proposed controller is fixed-time stable regardless of the initial conditions. Extensive simulation case studies are carried out to verify the superiority of the developed controller.

INDEX TERMS DC microgrids, economic dispatch, fixed-time control strategy, Artstein's transformation.

I. INTRODUCTION

Recently, with the increasing installation of distributed generators (DGs) and electronic loads, the concept of microgrids has been developed to guarantee efficient and robust electric grids [1]. Generally, MGs in the distribution system are categorized into AC and DC MGs in terms of the connected DGs and loads natures [2]. Since DC MG is invulnerable to the natural barriers of AC MG, such as the inrush currents, reactive power control, and frequency synchronization, it has high efficiency and simple controllability over the AC ones [3]. Furthermore, it has the capability to be in grid-connected or islanded modes [4]. In islanding operating mode, MG's controller allocates DGs' output powers to guarantee a balanced generations and demands with the minimal total generation cost (TGC), named economic dispatch (ED) problem [5].

Generally, the ED is an optimization problem that could be executed in a centralized, decentralized, or distributed way. Countless the conventional techniques such as dynamic programming [6], lambda iteration [7], particle swarm optimization [8], and evolutionary algorithms [9] have been employed in a centralized manner. In which, a central controller is vital to gathering all DGs' information, defining the optimal decisions, and sending the proper commands to the controllable DGs. Accordingly, it is susceptible to single-point-of-failure with a complicated cyber network [10]. Therefore, it is no longer appropriate for large-scale MGs with many DGs. Accordingly, the distributed control structure has been developed as a promising strategy to get

The associate editor coordinating the review of this manuscript and approving it for publication was Yonghao Gui.

rid of the centralized controller's bottlenecks, as it maintains improved reliability and scalability with a peer-to-peer cyber network [11]. In this regard, each DG is connected only with the nearest two neighboring DGs on the cyber network [12]. Hence, solving the ED problem of DC MGs in a distributed manner has gotten significant research concerns.

A distributed control strategy based on the combined equal incremental cost (IC) and sub gradient algorithm is presented in [13] to restore MG's average voltage with the lowest TGC. In [14], a unified distributed controller is developed to decrease MG's TGC respecting ED's equality (generations-demands equilibrium) and inequality (DGs' capacity bounds) constraints. The work in [15] presents an adaptive droop control strategy to solve the ED problem for islanded DC MG in a fully distributed fashion. In [16], a distributed controller has been presented for the economic allocations of DGs' output powers; however, ED's inequality constraint is neglected. A multi-agent supervisory controller is developed in [17] for maintaining the optimal power management for islanded DC MGs. Furthermore, for guaranteeing an optimal load power dispatching of islanded DC MG having renewable and nonrenewable DGs, a fully distributed economic power management respecting ED's equality and in-equality constraints has been proposed in [18]. A consensus-based fully distributed dual-layer control system is implemented in [19] to accomplish the optimal operation of islanded DC MGs with a restored average voltage. The work in [20] introduces a distributed hierarchical control technique to minimalize the operating cost in a droop-based DC MGs respecting DGs' power limits. In [21], the hierarchy of secondary and tertiary controller is broken, and A fully distributed secondary controller is illustrated not only to regulate DC MG's average voltage but also minimize its TGC. However, the aforementioned works employ the linear consensus protocols having an asymptotic convergence, which might be improper for islanded MGs with fast-changing operating conditions due to intermittency of RES and uncertainty of demands.

Recently, the finite-time consensus protocol is presented in [22] for accelerating the convergence speed. Furthermore, it is distinguished with the disturbance rejection property and uncertainties' resiliency [23]. Accordingly, a distributed finite-time ED scheme is provided in [24] to optimally allocate loads among DGs in a MG within accelerated convergence time while addressing both generations-demands equality and generation capacity inequality constraints. Additionally, [25] presents a fully distributed finite-time ED control algorithm for islanded DC MGs to minimize MG's TGC within a prescribed settling time. A finite-time second-order cooperative control is developed in [26] for maintaining the optimal operation of islanded DC MGs within fast convergence time. However, the dependency of the finite-time protocols' settling time on the system's initial values restricts its employment on large-scale MGs.

To get rid of this obstacle, the fixed-time consensus protocol is introduced in [27], so the convergence time is unrelated to the initial values [28], [29]. Accordingly, the fixed-time control has been implemented in DC MGs for ensuring proportional load sharing among DGs [30] within an accelerated convergence time. However, few works deal with the optimal power dispatch problem for DC MGs. In [31], a fixed time terminal sliding-mode controller is developed for precise DC bus voltage regulation. The work in [32] proposes a dynamic average consensus-based distributed fixed-time controller for accurate current sharing and efficient voltage stabilization. In [33], a fixed-time control method is introduced for DC multi-microgrids to maintain proportional load sharing within accelerated convergence time, addressing the issue of convergence time dependence on initial conditions. However, few work consider the fixed-time based optimal dispatch for DC MGs. A distributed fixed-time based control is implemented in [34] for optimally allocating loads between different DGs in DC MGs in a preassigned fast convergence time unrelated to the initial values. Additionally, in [35], a fixed-time secondary controller consists of voltage regulator and power optimizer for elimination of voltage deviations and preserving the optimal power allocation in a fixed time manner, respectively. However, the capability of this control strategy with cyber and physical failures as well as cyber delays has not been verified. In [36], optimal power dispatch for interconnected DC MGs based on the distributed fixed-time control protocol is introduced to preserve the lowest global total generation cost for the entire cluster while meeting the ED problem's equality and in equality constraints.

Notably, cyber-based control systems inevitably suffer from time delays. Wherein, the delays of cyber links could be formulated as an input delay in the consensus control schemes [37]. Unfortunately, the delays expand the convergence time or even weaken the controller's stability; accordingly, it should be considered during the design of the controller. The aforementioned works can be classified as ignoring the delays [15], [18], [19], [21], [24], considering the delay effect after controller design with determining the relation between the delay boundary and control factors [13], [14], [17], [20], [25], or adopting delay dependent control strategies [16]. For delayed multi-agent systems, linear matrix inequality, Lyapunov-Krasovskii functions, and Lyapunov-Razumikhin functionals guarantee the convergence and system's stability. In [38], [39], Artstein's reduction technique utilizes a state predictor based on an integral operator for reducing the delayed systems to delay-free ones for designing the proposed controller. Accordingly, the control system which stabilizes the reduced system also efficiently guarantees the original system's stability [37]. This transformation has been expanded to the finite-time control [40], and the fixed-time control [41], [42], [43]. A fully distributed finite-time control mechanism that effectively accommodate time delay is proposed in [44] for preserving balanced batteries' state of charges and stabilized voltage in a DC MG. The work in [45] develops a fixed-time

TABLE 1. Existing control strategies.

Perceptions	Items	[13]	[14]	[15]	[16]	[17]	[18]	[19]	[20]	[21]	[24]	[25]	[26]	[34]	[35]	[45]	Proposed
Main objectives	Fully distributed control	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
	Fixed settling time												√	√	√	√	√
	Rapid convergence										√	√	√	√	√	√	√
	Economic operation	√	√	√	√	√	√	√	√	√	√	√	√	√	√		√
	Equality constraint	√	√	√	√	√	√	√	√	√	√	√		√	√	√	√
	DG's capacity limits	√	√			√	√	√		√	√	√		√			√
	Cyber delays				√											√	√
Features	Plug-n-Play capability	√	√	√					√		√	√	√	√	√	√	√
	Link-failure resiliency	√	√	√								√					√

control for DC MGs considering time delays to guarantee the proportional load sharing among DGs. To the best of authors' knowledge, none of the existing works consider the distributed fixed-time ED control system while respecting cyber delays as summarized in TABLE 1.

In this paper, a fully distributed fixed-time control approach of islanded DC MGs is exhibited for preserving the economical load allocation while considering cyber time delays. It consists of fixed-time cost optimizer and voltage regulator to equalize all DG's ICs realizing MG's economic operation meeting DGs' capacity limits and regulate MG's average voltage in a fixed settling time irrelevant to initial states, respectively. The significant novelties of the proposed control scheme are:

1) Unlike the existing control strategies, the proposed controller preserves MG's economic operation within accelerated preassigned fixed settling time without dependency on the initial conditions while meeting the ED problem' equality and inequality constraints.
2) The developed controller can tolerate nonuniform and unbounded time delays thanks to employing Artstein's reduction method over the existing fixed-time optimal dispatch control strategies.
3) The fixed-time stability of the proposed controller is proved through rigorous stability analysis.
4) The superiority of the developed controller is exhibited under variant loadings, cyber failures, DG's plug-and-play, a wide range of time delays, and comparative studies with the existing strategies.

The rest of this manuscript is as follow: Section II explains the ED problem for islanded MG and some principal preliminaries for the fixed-time control and Artstein's transformation. The proposed secondary controller is described in Section III. Verification of the developed control under different case studies is proved in section IV. Lastly, section V presents the manuscript's conclusion.

II. PROBLEM DESCRIPTION

A. MG MODELING AND ECONOMIC DISPATCH PROBLEM

Figure1 illustrates the schematic diagram of islanded DC MG involving dispatchable and non-dispatchable DGs, which

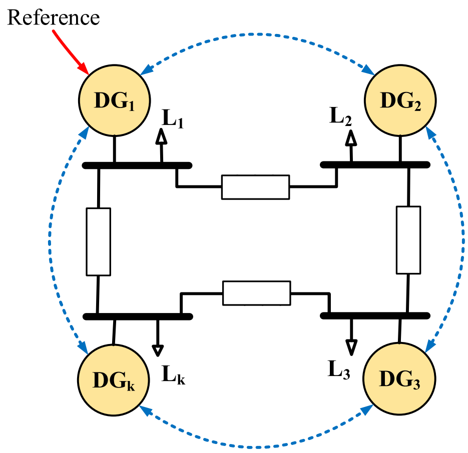

FIGURE 1. The modeled islanded DC MG.

are interconnected through transmission lines for supplying the local demands. Wherein, the non-dispatchable RES (PV, Wind turbines etc.) should be controlled by the MPPT techniques to deliver their maximum available power for increasing their utilization. Furthermore, the output powers of the dispatchable DGs (diesel, microturbine, fuel-cell, etc.) should be scheduled to supply economically the remaining demands according to their generation costs. Generally, dispatchable-DGs' generation costs are formulated in a quadratic convex formula in terms of the produced powers (P_i) as in (1) [5].

$$C_i\left(P_i\right) = a_i P_i^2 + b_i P_i + c_i \tag{1}$$

where $C_i\left(P_i\right)$ denotes DG_i's generation cost, and a_i, b_i, and C_i are its monetary factors. In General, the ED problem solving methods optimally allocate DGs' output powers for maintaining the total load with the minimal generation cost respecting the equality and in-equality, as expressed in (2) [14].

$$min\left(\sum_{i=1}^{K} C_i\left(P_i\right)\right) \tag{2a}$$

$$\sum_{i=1}^{K} P_i = P_L - P_{res} = P_D \tag{2b}$$

$$\underline{P_i} \leq P_i \leq \overline{P}_i \tag{2c}$$

where K, P_L, P_{res}, and P_D signify the number of MG's DGs, MG's demands, RES's generated powers, and the net loads, respectively. $\underline{P_i}$ and $\overline{P}_i$ indicate the minimum and maximum power boundaries of the dispatchable $\mathrm{DG_i}$.

Ignoring the inequality constraints, the Lagrange multiplier approach is employed to execute the optimization problem. Accordingly, the Lagrange function is expressed as

$$\ell\left(P_i, \lambda\right) = \sum_{i=1}^{N} C_i\left(P_i\right) + \lambda\left(P_D - \sum_{i=1}^{K} P_i\right) \tag{3}$$

where λ is the IC (the Lagrangian multiplier related to the equality condition). Optimality conditions of the Lagrange function can be attained with the differentiation of (3) with respect to P_i and λ as:

$$\begin{aligned} \frac{\partial \ell}{\partial P_i} &= \frac{\partial C_i\left(P_i\right)}{\partial P_i} - \lambda = 0 \\ \frac{\partial \ell}{\partial \lambda} &= P_D - \sum_{i=1}^{N} P_i = 0 \end{aligned} \tag{4}$$

Accordingly, for achieving the minimal TGC without considering DGs' capacity limits, the ICs of all DGs have to be equalized at the optimum value, λ^*, and the related DGs' generated powers can be determined as in (5).

$$P_i = \frac{\lambda^* - b_i}{2a_i} \tag{5}$$

Then, when DG's capacity limits are considered, the optimal operation's conditions might be modified slightly as below:

$$\begin{cases} \dfrac{\partial C_i\left(P_i\right)}{\partial P_i} \geq \lambda, & \text{for } P_i = \underline{P_i} \\ \dfrac{\partial C_i\left(P_i\right)}{\partial P_i} = \lambda, & \text{for } \underline{P_i} \leq P_i \leq \overline{P}_i \\ \dfrac{\partial C_i\left(P_i\right)}{\partial P_i} \leq \lambda, & \text{for } P_i = \bar{P}_i \end{cases} \tag{6}$$

To be concluded, for the economic operation of the MG, all DGs with non-active power limits should have equalized ICs, while others operating at either the lower or upper limits have either $\underline{\lambda_i}$ or $\overline{\lambda}_i$ associated to the lower power $\left(\underline{P_i}\right)$ or upper power limits $\left(\overline{P}_i\right)$, respectively.

B. PRIMARY CONTROL LAYER

The controller of each DG involves primary and secondary control layers. Wherein, the primary control includes the droop control and inner dual voltage and current control loops. Herein, the droop controller is widely utilized for tunning the voltage reference of the inner voltage control to accommodate load allocation among DGs as in

$$v_i = v_i^{nom} - r_i * P_i \tag{7}$$

where v_i and r_i denote the output voltage and droop gain of $\mathrm{DG_i}$, and v_i^{nom} is MG's nominal voltage, that is defined by the secondary controller. Then, substituting (5) into (7) gives

$$v_i = v_i^{nom} - \frac{r_i}{2a_i} * \lambda_i \tag{8}$$

Applying the feedback linearization procedure via differentiating (8) to attain the auxiliary control inputs with heterogeneous delays as

$$\dot{v}_i^{nom} = \dot{v}_i + \frac{r_i}{2a_i} * \dot{\lambda}_i = u_i^v\left(t - h_i\right) + u_i^c(t - h_i) \tag{9}$$

where $u_i^v\left(t - h_i\right) = \dot{v}_i$ and $u_i^c\left(t - h_i\right) = \frac{r_i}{2a_i} * \dot{\lambda}_i$ are the auxiliary control inputs of the voltage restoration and cost optimization with heterogeneous time delays (h_i), respectively. Accordingly, the control input $v_i^{nom}(t - h_i)$ with non-uniform delay can be determined as

$$v_i^{nom}\left(t - h_i\right) = \int \left(u_i^v\left(t - h_i\right) + u_i^c(t - h_i)\right) dt \tag{10}$$

Accordingly, the secondary controller determines the control inputs of voltage regulation and TGC's reduction. Artstein's model reduction strategy is exhibited for transforming an input-delayed system into a delay-free one. Wherein the stability problem could be investigated by the reduced system [38]. Therefore, for a first-order integrator system of DGs' ICs agreement and voltage regulation, the reduced systems λ_i^y, and v_i^y respectively can be attained as

$$\lambda_i^y = \lambda_i + \int_{t-h_i}^{t} u_i^c\left(s\right) ds, \ \dot{\lambda}_i^y = u_i^c\left(t\right) \tag{11}$$

$$v_i^y = v_i + \int_{t-h_i}^{t} u_i^v\left(s\right) ds, \ \dot{v}_i^y = u_i^v\left(t\right) \tag{12}$$

where λ_i^y, and v_i^y represent the reduced control

Remark 1: If the designed controller can stabilize the transformed systems (11) and (12) then it also will stabilize the original system (9). The complete proofs and analysis of the Artstein's reduction is presented in [38].

C. GRAPH THEORY AND FIXED-TIME CONTROLLER

Figure 1 shows the cyber network of the modeled DC MG to share the required information among its DGs, which can be characterized via the digraph $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$. Wherein, the set of nodes $\mathcal{V} = \{\mathcal{V}_1, \mathcal{V}_2, \cdots, \mathcal{V}_k\}$ signifies the k interconnected DGs of the MG, and the edges' set $\mathcal{E} = \{\mathcal{E}_1, \mathcal{E}_2, \cdots, \mathcal{E}_k\} \subset \mathcal{V} \times \mathcal{V}$ indicate the communication links among DGs. Moreover, $\mathcal{A} = \left[a_{ij}\right]_{k\times k}$ represents the adjacency matrix containing the cyber links' weights, where $a_{ij} > 0$ if there is a cyber-link among DG_i and DG_j, otherwise $a_{ij} = 0$. Let $\mathcal{L} = \left[l_{ij}\right]_{k\times k}$ denotes the Laplacian matrix of $\mathcal{G}$, and $l_{ij} = -a_{ij}$ for $i \neq j$ and $l_{ii} = \sum_{n=1,n\neq i}^{k} a_{ik}$. Besides, $\mathcal{B} = diag\{b_1, \ldots, b_k\}$ is the pinning matrix having pinning gains b_i as $b_i > 0$ only if $\mathrm{DG_i}$ is pinned to receive the nominal values, otherwise $b_i = 0$.

Definition 1: Consider the nonlinear autonomous system:

$$\dot{x}\left(t\right) = f\left(x\left(t\right)\right), x\left(0\right) = x_0 \tag{13}$$

where $x = [x_1, x_2, \cdots, x_N]^T \epsilon \mathbb{R}^N, f(x): \mathbb{R}^N \to \mathbb{R}^N$ is continuous on $\mathbb{R}^N$, and $f(0) = 0$. Assuming that the origin as the balance point of system (13). It can be stated that the origin is fixed-time stable when it is Lyapunov stable and fixed-time convergent having a convergence time less than a real number $T_{max} > 0$, which is irrelevant to the initial conditions.

Lemma 1: ([27]) Consider a continuous positive definite function of $V(x(t))$ such that:

$$\dot{V}(x(t)) \leq \left(-\alpha V(x(t))^p - \beta V(x(t))^q\right)^k \tag{14}$$

where α, β, p, q, and k are positive numbers while $pk < 1$, and $qk > 1$. Accordingly, the origin of system (13) is globally fixed-time stable with convergent time (T) upper-limited with a real number, which depends only in the control parameters as in (15).

$$T \leq T_{max} := \frac{1}{\alpha^k(1-pk)} + \frac{1}{\beta^k(qk-1)} \tag{15}$$

III. PROPOSED SECONDARY CONTROLLER

The proposed distributed secondary control scheme is developed to optimize the demands' allocation between DGs and regulate MG's average voltage within a prescribed upper-bounded fixed settling time with input delay. Figure 2 exhibits that the secondary controller involves the cost optimizer and voltage regulator. Wherein, the fixed-time cost optimizer ensures the equalization of DGs' ICs, and the voltage regulator is answerable for MG's average voltage restoration in a fixed-time manner as in (16a) and (16b), respectively through tunning the primary controller's nominal voltage.

$$\lim_{t \to t_c} \left| \lambda_j^y - \lambda_i^y \right| = 0, \forall t \geq t_c \tag{16a}$$

$$\lim_{t \to t_v} \left| \frac{1}{K} \sum_{i=1}^{k} v_i^y - V_{ref} \right| = 0, \forall t \geq t_v \tag{16b}$$

where t_c and t_v represent the upper-bounded fixed settling times for the convergence of DGs' ICs and regulation of MG's voltage, respectively.

A. DISTRIBUTED COST OPTIMIZER

Reducing MG's TGC can be realized through the optimal allocation of the required demand among the dispatchable DGs, which can be maintained by equalizing DGs' ICs at the optimal value. Therefore, inspired by [27], the designed fixed-time auxiliary control input associated with cost optimization is formulated in (17). In which, the transformed IC of each DG, λ_i^y, is compared with those of the neighboring DGs, λ_j^y, over the sparse cyber network.

$$\begin{aligned} u_i^c = {} & \alpha_c \sum_{j \epsilon N_i} a_{ij} sig\left(\lambda_j^y - \lambda_i^y\right)^{p_c} \\ & + \beta_c \sum_{j \epsilon N_i} a_{Mj} sig\left(\lambda_j^y - \lambda_i^y\right)^{q_c} \end{aligned} \tag{17}$$

where, α_c, β_c, p_c, and q_c are positive control gains while $p_c < 1$ and $q_c > 1$. $sig(\cdot)^\theta = |\cdot|^\theta\, sign(\cdot)$ and $sign(\cdot)$ represents the signum function. N_i is $\mathrm{DG_i}$'s neighbor set on the cyber graph.

Theorem 1: Assume that the graph $\mathcal{G}_v$ is connected and undirected, employing the fully distributed fixed-time control procedure (17), the balance between all DGs' ICs can be effectively realized in a fixed settling time undependable on the initial conditions.

Proof: Define the IC error $\delta_i^c = \lambda_i^y - \frac{1}{k}\sum_{i=1}^k \lambda_i^y$. Since $\frac{1}{k}\sum_{i=1}^k \dot{\lambda}_i^y = 0$ for an undirected and connected network, $\frac{1}{k}\sum_{i=1}^k \lambda_i^y$ is time invariant. Therefore, differentiating δ_i^c yields

$$\begin{aligned} \dot{\delta}_i^c &= \dot{\lambda}_i^y - \frac{1}{k}\sum_{i=1}^k \dot{\lambda}_i^y = u_i^c \\ &= \alpha_c \sum_{j \epsilon N_i} a_{ij} sig\left(\delta_j^c - \delta_i^c\right)^{p_c} \\ &\quad + \beta_c \sum_{j \epsilon N_i} a_{ij} sig\left(\delta_j^c - \delta_i^c\right)^{q_c} \end{aligned}$$

Lemma 2: ([46]) Let $\xi_1, \xi_2, \cdots, \xi_n \geq 0$, $0 < \rho \leq 1$ and $\sigma > 1$. Then

$$\sum_{i=1}^k \xi_i^\mu \geq \left(\sum_{i=1}^k \xi_i\right)^\rho \tag{18a}$$

$$\sum_{i=1}^k \xi_i^\nu \geq k^{1-\nu}\left(\sum_{i=1}^k \xi_i\right)^\sigma \tag{18b}$$

Lemma 3: ([47]) For an undirected graph $\mathcal{G}$, The Laplacian matrix ($\mathcal{L}$)'s properties are:

1) $x^T \mathcal{L} x = \frac{1}{2}\sum_{i,j=1}^k a_{ij}\left(x_j - x_i\right)^2$
2) Let $\Lambda_2(\mathcal{L})$ as the second smallest eigenvalue of $\mathcal{L}$, then $x^T \mathcal{L} x \geq \Lambda_2(\mathcal{L}) x^T x$.

Considering the following Lyapunov function:

$$V_1 = \frac{1}{2}\delta_c^T \delta_c = \frac{1}{2}\sum_{i=1}^k \left(\delta_i^c\right)^2 \tag{19}$$

where $\delta_c = \left[\delta_1^c, \delta_i^c, \ldots, \delta_k^c\right]^{\mathrm{T}}$ is the cost mismatch vector. Therefore, the time derivative of V_1 can be determined as follows:

$$\begin{aligned} \dot{V}_1 &= \sum_{i=1}^k \delta_i^c \dot{\delta}_i^c \\ &= \sum_{i=1}^k \delta_i^c \left[\alpha_c \sum_{j \epsilon N_i} a_{ij} sig\left(\delta_j^c - \delta_i^c\right)^{p_c}\right. \\ &\quad \left. + \beta_c \sum_{j \epsilon N_i} a_{ij} sig\left(\delta_j^c - \delta_i^c\right)^{q_c}\right] \\ &= -\frac{\alpha_c}{2}\sum_{i,j=1}^k \left(\left(a_{ij}\right)^{\frac{2}{1+p_c}}\left|\delta_j^c - \delta_i^c\right|^2\right)^{\frac{1+p_c}{2}} \\ &\quad - \frac{\beta_c}{2}\sum_{i,j=1}^k \left(\left(a_{ij}\right)^{\frac{2}{1+q_c}}\left|\delta_j^c - \delta_i^c\right|^2\right)^{\frac{1+q_c}{2}} \end{aligned} \tag{20}$$

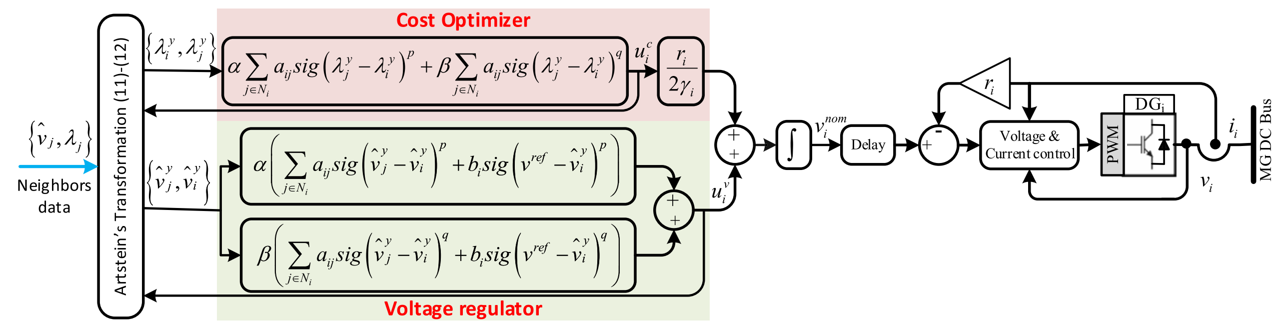

FIGURE 2. **Block diagram of the proposed fixed-time secondary controller.**

Using Lemma 2 and Lemma 3, one has that

$$\begin{aligned}
\dot{V}_1 &\le -\frac{\alpha_c}{2}\left[\sum_{i,j=1}^{k}\left(a_{ij}\right)^{\frac{2}{1+p_c}}\left|\delta_j^c-\delta_i^c\right|^2\right]^{\frac{1+p_c}{2}} \\
&\quad -\frac{\beta_c}{2}k^{\frac{1-q_c}{2}}\left[\sum_{i,j=1}^{k}\left(a_{ij}\right)^{\frac{2}{1+q_c}}\left|\delta_j^c-\delta_i^c\right|^2\right]^{\frac{1+q_c}{2}} \\
&= -\frac{\alpha_c}{2}\left[2\delta_c^T\left(\mathcal{L}^{p_c}\right)\delta_c\right]^{\frac{1+p_c}{2}}-\frac{\beta_c}{2}n^{\frac{1-q_c}{2}}\left[2\delta_c^T\left(\mathcal{L}^{q_c}\right)\delta_c\right]^{\frac{1+q_c}{2}} \\
&\le -\frac{\alpha_c}{2}\left[2\Lambda_2\left(\mathcal{L}^{p_c}\right)\delta_c^T\delta_c\right]^{\frac{1+p_c}{2}} \\
&\quad -\frac{\beta_c}{2}k^{\frac{1-q_c}{2}}\left[2\Lambda_2\left(\mathcal{L}^{q_c}\right)\delta_c^T\delta_c\right]^{\frac{1+q_c}{2}} \\
&\le -\frac{\alpha_c}{2}\left[4\Lambda_2\left(\mathcal{L}^{p_c}\right)V_1\right]^{\frac{1+p_c}{2}} \\
&\quad -\frac{\beta_c}{2}k^{\frac{1-q_c}{2}}\left[4\Lambda_2\left(\mathcal{L}^{q_c}\right)V_1\right]^{\frac{1+q_c}{2}}
\end{aligned} \tag{21}$$

where $\mathcal{L}^{p_c}$ indicates the Laplacian matrix having an adjacency matrix $\mathcal{A}^{p_c}=[(a_{ij})^{\frac{2}{1+p_c}}]$, and $\mathcal{L}^{q_c}$ is the Laplacian matrix with an adjacency matrix $\mathcal{A}^{q_c}=[(a_{ij})^{\frac{2}{1+q_c}}]$. Let $K_1=\frac{\alpha_c}{2}\left[4\Lambda_2\left(\mathcal{L}^{p_c}\right)\right]^{\frac{1+p_c}{2}}$, and $K_2=\frac{\beta_c}{2}k^{\frac{1-q_c}{2}}\left[4\Lambda_2\left(\mathcal{L}^{q_c}\right)\right]^{\frac{1+q_c}{2}}$. Then we get

$$\dot{V}_1 \le -K_1\left(V_1\right)^{\frac{1+p_c}{2}}-K_2\left(V_1\right)^{\frac{1+q_c}{2}} \tag{22}$$

According to Lemma 1, $V_1 \rightarrow 0$ within an upper bounded fixed settling time, t_c.

$$t_c \le \frac{2}{\frac{\alpha_c}{2}\left[4\Lambda_2\left(\mathcal{L}^{p_c}\right)\right]^{\frac{1+p_c}{2}}\left(1-p_c\right)} + \frac{2}{\frac{\beta_c}{2}k^{\frac{1-q_c}{2}}\left[4\Lambda_2\left(\mathcal{L}^{q_c}\right)\right]^{\frac{1+q_c}{2}}\left(q_c-1\right)} \tag{23}$$

Accordingly, DGs' ICs mismatch becomes 0, and $\lambda_j^y=\lambda_i^y, \forall i,j$, within a fixed time t_c. The proof of Theorem 1 is completed.

Remark 2: respecting ED's inequality constraints, if one of the DGs reaches either its maximum or lower capacity limit, it should operate at this violated limit instead of the economic operated point as in (6). Therefore, the cost auxiliary control is updated to adapt its IC at the value of the violated power limit as in (24a) and (24b).

$$\begin{aligned}
u_i^c &= \alpha_c\sum_{j\epsilon N_i}a_{ij}sig\left(\underline{\lambda_i}-\lambda_i^y\right)^{p_c} \\
&\quad +\beta_c\sum_{j\epsilon N_i}a_{ij}sig\left(\underline{\lambda_i}-\lambda_i^y\right)^{q_c}
\end{aligned} \tag{24a}$$

$$\begin{aligned}
u_i^c &= \alpha_c\sum_{j\epsilon N_i}a_{ij}sig\left(\overline{\lambda}_i-\lambda_i^y\right)^{p_c} \\
&\quad +\beta_c\sum_{j\epsilon N_i}a_{ij}sig\left(\overline{\lambda}_i-\lambda_i^y\right)^{q_c}
\end{aligned} \tag{24b}$$

B. DISTRIBUTED VOLTAGE REGULATOR

For effective distributed restoration of MG's average voltage, each DG estimates the average voltage of the MG through employing the fixed-time consensus-based voltage observer [48]. Wherein, each DG just requires the states of nearest neighboring DGs to observe MG's average voltage in a fixed-time manner, as in (25). Accordingly, it estimates the average voltage in a predefined settling time despite the initial values.

$$\begin{aligned}
\hat{v}_i &= v_i+\int\Bigg(\alpha\sum_{j\epsilon N_i}a_{ij}sig\left(\hat{v}_j-\hat{v}_i\right)^p \\
&\quad +\beta\sum_{j\epsilon N_i}a_{ij}sig\left(\hat{v}_j-\hat{v}_i\right)^q\Bigg)dt
\end{aligned} \tag{25}$$

where v_i, and $\hat{v}_i$ denote DG$_i$'s output voltage and MG's average voltage estimation at DG$_i$, respectively. While the control factors α, $\beta>0$, $0<p<1$, and $q>1$.

Inspired by [27], a fully distributed fixed-time voltage regulator is developed for MG's average voltage restoration in a predetermined convergence time undependable on the initial values. Wherein, each DG acquires its local estimated value, and the ones of its neighboring DGs while at least one DG accesses the desired reference value. Accordingly, the controller of the pinned DGs (dominant DGs) compares their estimated value with the ones of the neighbors and the reference of value of the MG as in (26).

$$u_i^v = \alpha_v \left[\sum_{j\epsilon N_i} a_{ij} sig\left(\hat{v}_j^y - \hat{v}_i^y\right)^{p_v} + b_i sig\left(V_{ref} - \hat{v}_i^y\right)^{p_v} \right] + \beta_v \left[\sum_{j\epsilon N_i} a_{ij} sig\left(\hat{v}_j^y - \hat{v}_i^y\right)^{q_v} + b_i sig\left(V_{ref} - \hat{v}_i^y\right)^{q_v} \right] \tag{26}$$

where α_v, β_v, p_v, and q_v are positive control gains while $p_v < 1$ and $q_v > 1$. b_i illustrates the pinning gain of the dominant DGs, and $b_i > 0$ only if $\mathrm{DG_i}$ access the desired nominal value; otherwise $b_i = 0$.

Theorem 2: Assuming the cyber graph $\mathcal{G}_v$ is connected and undirected. Utilizing the proposed fully distributed fixed-time controller (26), MG's average voltage restoration is preserved within a fixed-time.

Proof: Define the local voltage restoration error $\delta_i^v = \hat{v}_i^y - V_{ref}$. Differentiating δ_i^v yields.

$$\begin{aligned} \dot{\delta}_i^v &= u_i^v \\ &= \alpha_v \left[\sum_{j\epsilon N_i} a_{ij} sig\left(\delta_j^v - \delta_i^v\right)^{p_v} - b_i sig\left(\delta_i^v\right)^{p_v} \right] \\ &\quad + \beta_v \left[\sum_{j\epsilon N_i} a_{ij} sig\left(\delta_j^v - \delta_i^v\right)^{q_v} - b_i sig\left(\delta_i^v\right)^{q_v} \right] \end{aligned}$$

Lemma 4: ([47])For an undirected graph $\mathcal{G}$ with a pining link to receive the reference, the Laplacian matrix $(\mathcal{L}+\mathcal{B})$ exhibits the following properties:

1) $x^T (\mathcal{L}+\mathcal{B}) x = \frac{1}{2} \sum_{i,j=1}^{k} a_{ij} \left(x_j - x_i\right)^2 + \sum_{i=1}^{N} b_i \left(x_i\right)^2$.
2) Let $\Lambda_2 (\mathcal{L}+\mathcal{B})$ is the smallest eigenvalue of $(\mathcal{L}+\mathcal{B})$, then $x^T (\mathcal{L}+\mathcal{B}) x \geq \Lambda_2 (\mathcal{L}+\mathcal{B}) x^T x$.

Consider the Lyapunov function

$$V_2 = \frac{1}{2} \delta_v^T \delta_v = \frac{1}{2} \sum_{i=1}^{n} \delta_i^{v2} \tag{27}$$

where $\delta_v = \left[\delta_1^v, \delta_i^v, \ldots, \delta_k^v\right]^{\mathrm{T}}$ is the disagreement vectors. Differentiating (27) gives

$$\begin{aligned} \dot{V}_2 &= \sum_{i=1}^{n} \delta_i^v \dot{\delta}_i^v \\ &= \sum_{i=1}^{n} \delta_i^v \left[\alpha_v \left(\sum_{j\epsilon N_i} a_{ij} sig\left(\delta_j^v - \delta_i^v\right)^{p_v} - b_i sig\left(\delta_i^v\right)^{p_v} \right) \right. \\ &\quad \left. + \beta_v \left(\sum_{j\epsilon N_i} a_{ij} sig\left(\delta_j^v - \delta_i^v\right)^{q_v} - b_i sig\left(\delta_i^v\right)^{q_v} \right) \right] \\ &= -\frac{\alpha_v}{2} \left[\sum_{i,j=1}^{k} \left(\left(a_{ij}\right)^{\frac{2}{1+p_v}} \left|\delta_j^v - \delta_i^v\right|^2 \right)^{\frac{1+p_v}{2}} \right. \\ &\quad \left. + 2 \sum_{i=1}^{k} \left(\left(b_i\right)^{\frac{2}{1+p_v}} \left|\delta_i^v\right|^2 \right)^{\frac{1+p_v}{2}} \right] \\ &\quad - \frac{\beta_v}{2} \left[\sum_{i,j=1}^{k} \left(\left(a_{ij}\right)^{\frac{2}{1+q_v}} \left|\delta_j^v - \delta_i^v\right|^2 \right)^{\frac{1+q_v}{2}} \right. \\ &\quad \left. + 2 \sum_{i=1}^{k} \left(\left(b_i\right)^{\frac{2}{1+q_v}} \left|\delta_i^v\right|^2 \right)^{\frac{1+q_v}{2}} \right] \end{aligned}$$

Define $\Gamma\left(\delta_v^{p_v}\right) = \frac{\alpha_v}{2} [\sum_{i,j=1}^{k} \left(\left(a_{ij}\right)^{\frac{2}{1+p_v}} \left|\delta_j^v - \delta_i^v\right|^2 \right)^{\frac{1+p_v}{2}} + 2 \sum_{i=1}^{k} \left(\left(b_i\right)^{\frac{2}{1+p_v}} \left|\delta_i^v\right|^2 \right)^{\frac{1+p_v}{2}}]$.

Based on Lemma 2 and Lemma 4 we get

$$\begin{aligned} \Gamma\left(\delta_v^{p_v}\right) &\geq \frac{\alpha_v}{2} \left[\sum_{i,j=1}^{n} \left(a_{ij}\right)^{\frac{2}{1+p_v}} \left|\delta_j^v - \delta_i^v\right|^2 + 2 \sum_{i=1}^{n} \left(b_i\right)^{\frac{2}{1+p_v}} \left|\delta_i^v\right|^2 \right]^{\frac{1+p_v}{2}} \\ &= \frac{\alpha_v}{2} \left[2\delta_v^T \left(\mathcal{L}^{p_v} + \mathcal{B}^{p_v}\right) \delta_v \right]^{\frac{1+p_v}{2}} \\ &\geq \frac{\alpha_v}{2} \left[2\Lambda_2 \left(\mathcal{L}^{p_v} + \mathcal{B}^{p_v}\right) \delta_M^T \delta_M \right]^{\frac{1+p_v}{2}} \\ &= \frac{\alpha_v}{2} \left[2\Lambda_2 \left(\mathcal{L}^{p_v} + \mathcal{B}^{p_v}\right) V_2 \right]^{\frac{1+p_v}{2}} \end{aligned} \tag{28}$$

where $\mathcal{B}^{p_v} = \mathrm{diag}\left\{\left(b_i\right)^{\frac{2}{1+p_v}}\right\}$ signifies the voltage pinning matrix. Define $\Gamma\left(\delta_v^{q_v}\right) = \frac{\beta_v}{2} [\sum_{i,j=1}^{k} \left(\left(a_{ij}\right)^{\frac{2}{1+q_v}} \left|\delta_j^v - \delta_i^v\right|^2 \right)^{\frac{1+q_v}{2}} + 2 \sum_{i=1}^{k} \left(\left(b_i\right)^{\frac{2}{1+q_v}} \left|\delta_i^v\right|^2 \right)^{\frac{1+q_v}{2}}]$, we get

$$\Gamma\left(\delta_v^{q_v}\right) \geq \frac{\beta_v}{2} k^{\frac{1-q_v}{2}} \left[2\Lambda_2 \left(\mathcal{L}^{q_v} + \mathcal{B}^{q_v}\right) V_2 \right]^{\frac{1+q_v}{2}} \tag{29}$$

where $\mathcal{B}^{q_v}$ indicates the voltage pinning matrix with the i^{th} diagonal element $\left(b_i\right)^{\frac{2}{1+q_v}}$. Combining (28) and (29), one has

$$\begin{aligned} \dot{V}_2 &\leq -\frac{\alpha_v}{2} \left[2\Lambda_2 \left(\mathcal{L}^{p_v} + \mathcal{B}^{p_v}\right) \right]^{\frac{p_v+1}{2}} \left[V_2\right]^{\frac{1+p_v}{2}} \\ &\quad - \frac{\beta_v}{2} k^{\frac{1-q}{2}} \left[2\Lambda_2 \left(\mathcal{L}^{q_v} + \mathcal{B}^{q_v}\right) \right]^{\frac{1+q_v}{2}} \left[V_2\right]^{\frac{1+q_v}{2}} \end{aligned}$$

Let $K_3 = \frac{\alpha_v}{2} \left[2\Lambda_2 \left(\mathcal{L}^{p_v} + \mathcal{B}^{p_v}\right) \right]^{\frac{1+p_v}{2}}$, and $K_4 = \frac{\beta_v}{2} k^{\frac{1-q_v}{2}} \left[2\Lambda_2 \left(\mathcal{L}^{q_v} + \mathcal{B}^{q_v}\right) \right]^{\frac{1+q_v}{2}}$. Then we have

$$\dot{V}_2 \leq -K_3 \left(V_2\right)^{\frac{1+p_v}{2}} - K_4 \left(V_2\right)^{\frac{1+q_v}{2}} \tag{30}$$

Thus, from Lemma 1, the MG's average voltage reaches the desired value within a fixed time bounded by

$$t_v \leq \frac{2}{\frac{\alpha_v}{2}\left[2\Lambda_2\left(\mathcal{L}^{p_v}+\mathcal{B}^{p_v}\right)\right]^{\frac{1+p_v}{2}}(1-p_v)} + \frac{2}{\frac{\beta_v}{2}k^{\frac{1-q_v}{2}}\left[2\Lambda_2\left(\mathcal{L}^{p_v}+\mathcal{B}^{p_v}\right)\right]^{\frac{1+q_v}{2}}(q_v-1)} \tag{31}$$

The proof of theorem 2 is completed.

Accordingly, the settling time of DGs' ICs agreement and the average voltage regulation is upper bounded by $t = max\{t_\lambda, t_v\}+max\{h_i\}$. It is only depending on the controller's coefficients, cyber graph, and time delays.

TABLE 2. Parameters of the modeled DC MG.

DGs generation costs					
DG	a (¢)	b(¢/W)	c (¢/W^2)	P_i^{max}	P_i^{min}
DG_1	95	0.64	0.013	350	55
DG_2	75	0.59	0.008	230	40
DG_3	85	0.62	0.011	450	65
DG_4	80	0.60	0.009	500	85
Transmission lines parameters					
parameter	R	L	C		
Value	0.5 Ω	50 μH	30 nF		
Secondary controller parameters					
α_c	β_c	p_c	q_c		
1	1	0.6	1.4		
α_v	β_v	p_v	q_v		
1	1	0.6	1.4		
r_1	r_2	r_3	r_4		
0.5	0.5	0.5	0.5		
h_1	h_3	h_3	h_4		
50ms	40ms	60ms	50ms		

IV. RESULTS

An islanded DC MG comprises four DGs, is modeled in PLECS simulation platform for the demonstration of proposed control. TABLE2summarizes the parameters of the DGs' generation costs, transmission lines and the developed controllers. The cyber network has an adjacency matrix of $\mathcal{A} = [0, 1, 0, 1; 1, 0, 1, 0; 0, 1, 0, 1; 1, 0, 1, 0]$, and only DG_1 receives MG's voltage reference so the diagonal pinning matrix is $\mathcal{B} = diag\{1, 0, 0, 0\}$.

First, the proposed controller's performance is demonstrated under different loadings with respecting ED's equality and inequality restrictions. Next, the robustness of the proposed controller with cyber link failures, plugging in/out DGs, under different time-delays, as well as comparison with the existing works have been developed.

A. CONTROLLER PERFORMANCE

Figure 3 depicts the operation of the proposed controller with variable loads. First, for $t < 1s$, the droop controller is activated, and the required demands are equally allocated among DGs based on their droop gains ($r_i = 0.5$). Furthermore, a reduction in MG's average voltage is illustrated in Figure 3(b) as an effect of the droop controller. Then, at $t = 1s$, the proposed secondary controller is activated; accordingly, the ICs of all DGs are precisely matched within a fixed-settling time at the optimal value to guarantee MG economic operation with the lowest TGC, as seen in Figure 3(a). Furthermore, DGs' generations are effectively economically scheduled to cover MG's demands, as shown in Figure3(c). MG's average voltage is reinstated at the nominal value, which ensures generations-demands power equilibrium, as in Figure 3(b). Next, with increasing the total loading, all DGs' generations are updated to optimally cover the additional demands in a fixed-time manner. Wherein, MG's average voltage regulation is successfully maintained within a fixed time. Finally, the added load is removed from the MG; therefore, DGs' output powers are rescheduled to their initial optimal values with accurate converged DGs' ICs and regulated MG's average voltage.

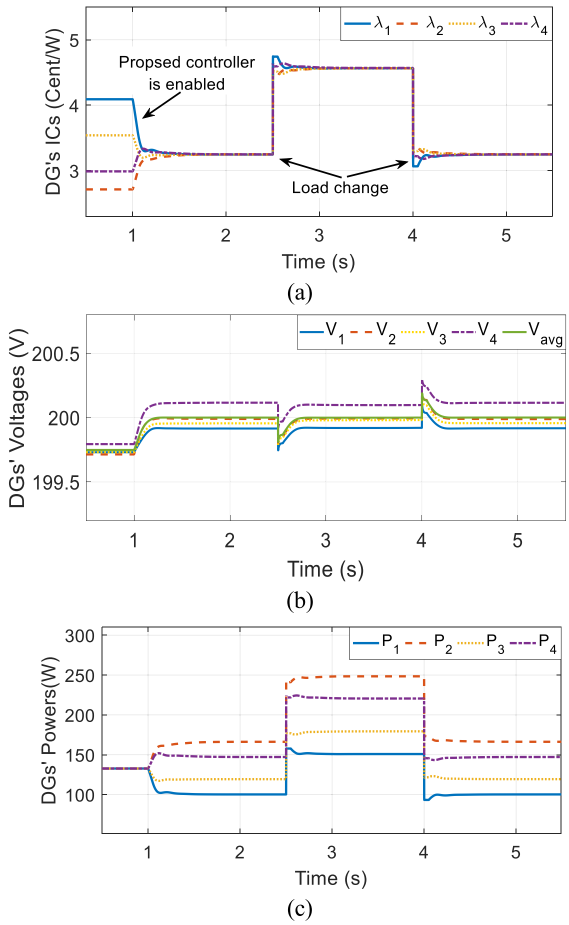

FIGURE 3. Proposed controller performance with different loadings: (a) DGs' ICs (b) DGs' voltages, (c) DGs' generations.

B. CONSIDERING ED'S INEQUALITY CONSTRAINTS

Performance of the proposed control approach while DGs' capacity limits are taken into consideration is shown in

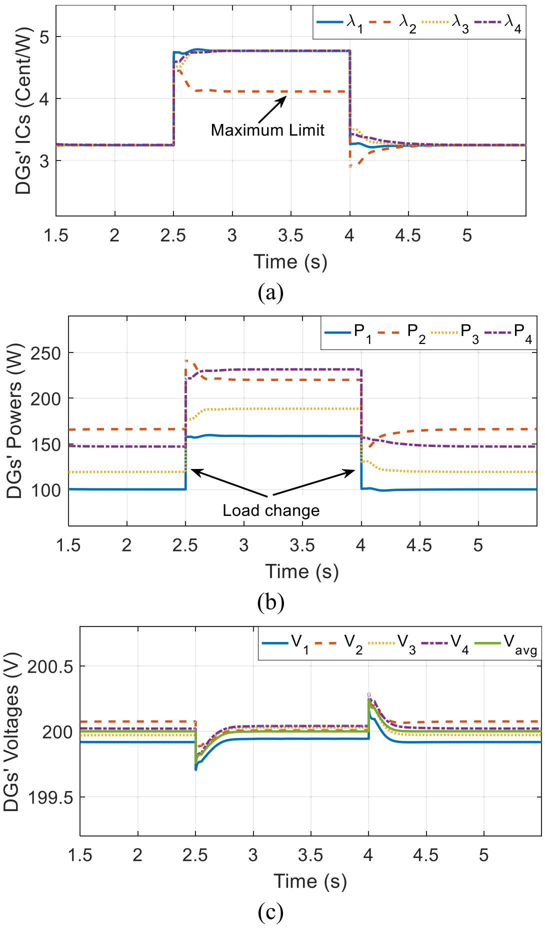

FIGURE 4. **Considering the inequality constraint: (a) DGs' ICs, (b) DGs' generations, (c) DGs' voltages.**

Figure 4. Initially, all DGs produce their optimal powers since their ICs are effectively equalized, as in Figure 4(a). At $t = 2.5s$, the total demand is increased, and DGs' generations are rescheduled to optimally allocate the required demand and preserve the generations- demands power balance. However, P_2 cannot be more than its maximum capacity limit, and it is adjusted at 230W (maximum bound), as in Fig 4(b). Accordingly, the ICs of all the other DGs are converged while λ_2 is amended at the value of its the maximum power limit ($\overline{\lambda}_2$). Figure 4(c) exhibits that MG's average voltage is regulated at 200V ensuring ED's equality constraint. Afterward, the total demand is reduced to the original value; hence, all DGs adjust their generations to supply the new demand and DG_2 returns to the ED mode. Accordingly, the equilibrium of all DGs' ICs is successfully accomplished at the optimum value.

C. PLUG-AND-PLAY CAPABILITY

The plug-and-play facility of the developed controller is proved by disconnecting/connecting one DG from/to the MG. At first, all DGs operate normally at the optimum point with equalized ICs. Then, at $t = 1.5s$, DG_1 is plugged-out from the MG, accordingly the remaining DGs readjust their generations to optimally cover the loads, as in Figure 5(b). Consequently, the ICs of the active DGs are effectively equalized at the new operating value, as depicted in Figure5(a). Next, at $t = 3s$, DG_1 is plugged into the MG to participate in supplying the demands. Therefore, all DGs economically cover the required loads respecting the ED's equality and inequality constraints.

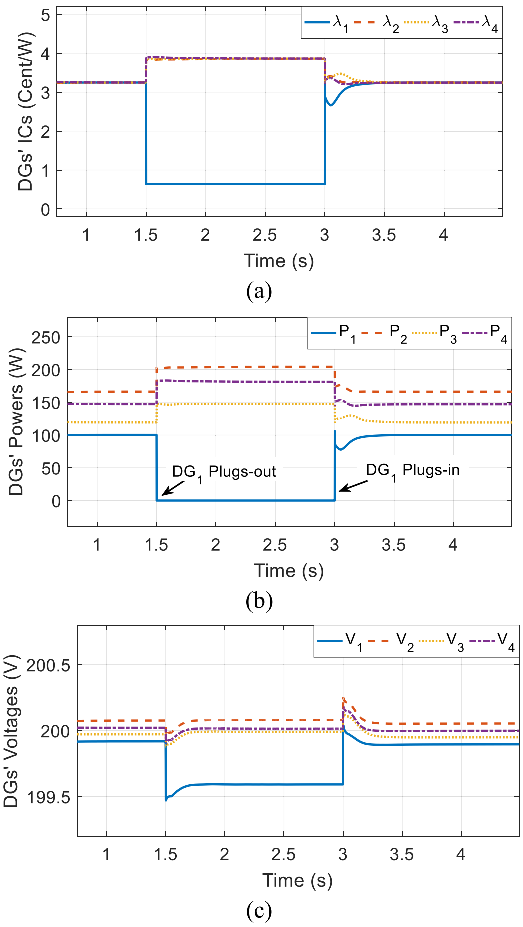

FIGURE 5. **Proposed controller's Plug-and-Play ability: (a) DGs' ICs (b) DGs' generations, (c) DGs' voltages.**

D. CYBER LINK FAILURES

The proposed controller's strength against losing cyber link is demonstrated in Figure 6. Firstly, MG's DGs are operating normally to guarantee the minimal TGC of the MG. Then, at $t = 1s$, the cyber link connects DG_1 and DG_4 has been failed. Figure6 reveals that even with losing one link on the cyber graph, DGs generations are still adjusted optimally. Furthermore, load fluctuations have been developed to reveal the proposed controller's superiority. The developed controller effectively realizes MG's economical operation respecting generations-demands balance and DGs' capacity limits. It is observed from Figure 6 that the convergence rate

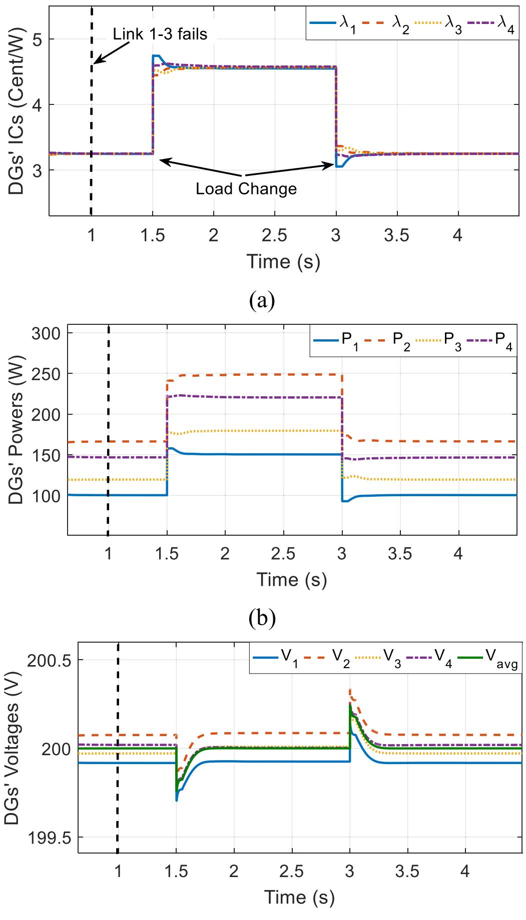

FIGURE 6. Proposed controller's resiliency with link failures: (a) DGs' ICs, (b) DGs' generations, (c) DGs' voltages.

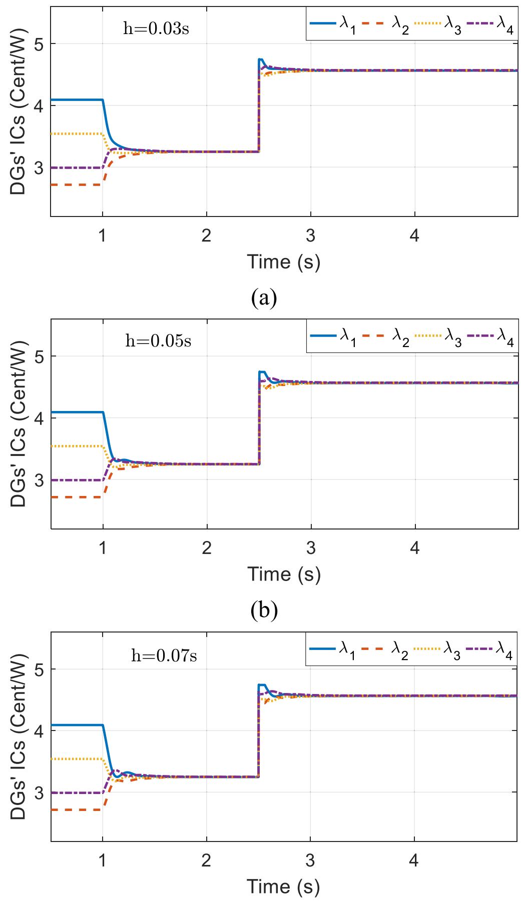

FIGURE 7. Controller's Performance under different time-delays: (a) h= 0.03s, (b) h= 0.05s, (c) h= 0.07s.

becomes a little slower as the link failures affect the algebraic connectivity of cyber graph.

E. EFFECT OF DIFFERENT TIME DELAYS

Figure 7 illustrates the strength of the proposed controller at diverse time delays (e.g., 0.03s, 0.05s, and 0.07s) with the same parameters as in the case study A. It is observed that the proposed control scheme stably retains the equilibrium of all DGs' ICs at the optimal value at different time delays. Although a slight and negligible dynamic appears in the transient period at higher time delays, the proposed controller can efficiently maintain the convergence within a fixed time.

TABLE 3. ISE values with different time delays.

Control strategy	Communication time delays		
	30ms	50ms	70ms
Proposed Control strategy	58.654	60.9175	63.0342
Existing Fixed-time control	60.1276	69.6707	148.0684

F. COMPARATIVE STUDY

A comparative study with the existing fixed-time control strategy in [34], which neglects the effect of time delays, is carried out under the same operating conditions of case study E, as depicted in Figure 8. It is well known that ignoring communication delays may deteriorate system stability, especially at high-time delays. Figure 8(a) illustrates the performance of the existing fixed-time control strategy with 30ms of delay. Although the controller [34] maintains the agreement of all DG's ICs at the optimal value, there is slightly higher dynamics than those of the proposed controller, as in Figure 7(a). In Figure 8(b), it is clearly noticed that increasing the communication delay reduces the convergence speed as well as increases the dynamics during the transient period. Furthermore, Figure 8(c) exhibits that the existing controller in [34] cannot guarantee DGs' ICs convergence at a 0.07s time delay with undamped oscillations leading to system's instability. Furthermore, TABLE 3 summarizes the integral squared error (ISE) of the step response for both the proposed and existing controllers with various time delays. It is observed that the proposed controller has a lower ISE than the existing control. Accordingly, with 70ms time delays, a 43% reduction in the ISE is achieved with the proposed controller compared to the conventional control strategy. Figure 7 and Figure 8 indicate that, at the same time

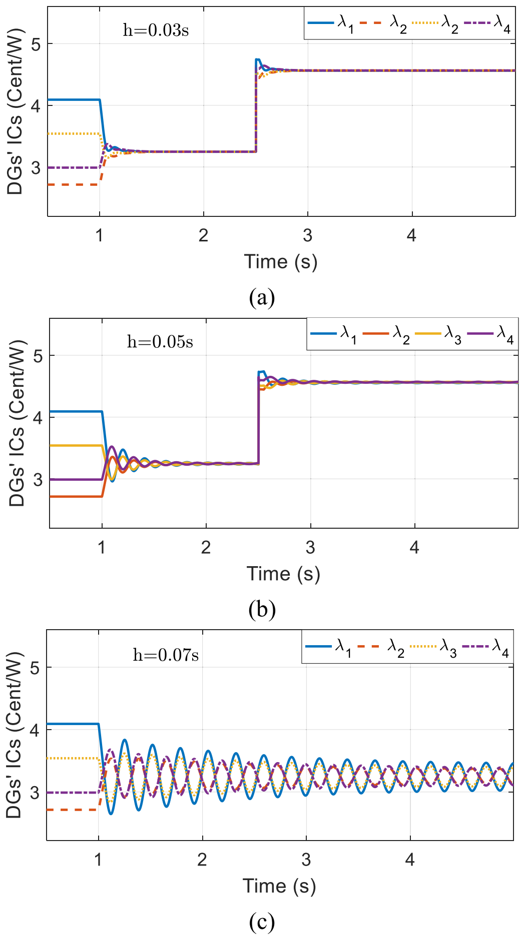

FIGURE 8. Existing controller's performance under different time-delays [34]: (a) $h = 0.03s$, (b) $h = 0.05s$, (c) $h = 0.07s$.

delay, thanks to the Artestein's transformation technique, the proposed controller has a significant performance compared to the existing fixed-time controller, especially with higher time delays.

V. CONCLUSION

In this article, a fully distributed fixed-time controller realizes the economic operation of islanded DC MGs respecting time delays. Artstein's transformation methodology has been employed to enhance the stability of the proposed controller with heterogeneous time delays through reducing the delayed system to a delay-free one. Accordingly, based on the reduced system, a fixed-time distributed cost optimizer is proposed to economically allocate demands among DGs while respecting ED's inequality constraint within a fixed settling time irrelevant to the initial conditions. Wherein realizing ED's equality constraint is maintained by the fixed-time voltage regulator through restoring MG's average voltage at the nominal value. Unlike the existing control strategies, the proposed controller attains a fast convergence rate regardless of the initial values with its capability of handling heterogenous time delays. The developed controller's efficacy is established theoretically and demonstrated via extensive simulation case studies under variable loads, cyber and physical failures, wide range of cyber delays, thereby underscoring the robustness and practical applicability of the proposed controller in real-world conditions.

REFERENCES

[1] R. H. Lasseter, "Smart distribution: Coupled microgrids," *Proc. IEEE*, vol. 99, no. 6, pp. 1074–1082, Jun. 2011.

[2] H. Lotfi and A. Khodaei, "AC versus DC microgrid planning," *IEEE Trans. Smart Grid*, vol. 8, no. 1, pp. 296–304, Jan. 2017.

[3] B. Modu, M. P. Abdullah, M. A. Sanusi, and M. F. Hamza, "DC-based microgrid: Topologies, control schemes, and implementations," *Alexandria Eng. J.*, vol. 70, pp. 61–92, May 2023.

[4] T. Dragicevic, X. Lu, J. C. Vasquez, and J. M. Guerrero, "DC microgrids—Part I: A review of control strategies and stabilization techniques," *IEEE Trans. Power Electron.*, vol. 31, no. 7, pp. 4876–4891, Jul. 2016.

[5] A. J. Wood and B. F. Wollenberg, *Power Generation, Operation, and Control*. New York, NY, USA: Wiley, 1996.

[6] J. Kim and K.-K.-K. Kim, "Dynamic programming for scalable just-in-time economic dispatch with non-convex constraints and anytime participation," *Int. J. Electr. Power Energy Syst.*, vol. 123, Dec. 2020, Art. no. 106217.

[7] J. P. Zhan, Q. H. Wu, C. X. Guo, and X. X. Zhou, "Fast λ-iteration method for economic dispatch with prohibited operating zones," *IEEE Trans. Power Syst.*, vol. 29, no. 2, pp. 990–991, Mar. 2014.

[8] M. A. Abido, "Optimal design of power-system stabilizers using particle swarm optimization," *IEEE Trans. Energy Convers.*, vol. 17, no. 3, pp. 406–413, Sep. 2002.

[9] M. Basu, "Fast convergence evolutionary programming for economic dispatch problems," *IET Gener., Transmiss. Distrib.*, vol. 11, no. 16, pp. 4009–4017, Nov. 2017.

[10] S. K. Sahoo, A. K. Sinha, and N. K. Kishore, "Control techniques in AC, DC, and hybrid AC–DC microgrid: A review," *IEEE J. Emerg. Sel. Topics Power Electron.*, vol. 6, no. 2, pp. 738–759, Jun. 2018.

[11] I. Alotaibi and M. Abido, "Distributed multi-agent consensus-based virtual inertia control of low inertial microgrids," in *Proc. IEEE Power Energy Soc. Gen. Meeting (PESGM)*, Orlando, FL, USA, Jul. 2023, pp. 1–5.

[12] Y. Han, K. Zhang, H. Li, E. A. A. Coelho, and J. M. Guerrero, "MAS-based distributed coordinated control and optimization in microgrid and microgrid clusters: A comprehensive overview," *IEEE Trans. Power Electron.*, vol. 33, no. 8, pp. 6488–6508, Aug. 2018.

[13] Z. Wang, W. Wu, and B. Zhang, "A distributed control method with minimum generation cost for DC microgrids," *IEEE Trans. Energy Convers.*, vol. 31, no. 4, pp. 1462–1470, Dec. 2016.

[14] S. Moayedi and A. Davoudi, "Unifying distributed dynamic optimization and control of islanded DC microgrids," in *IEEE Trans. Power Electron.*, vol. 32, no. 3, pp. 2329–2346, Mar. 2017.

[15] J. Hu, J. Duan, H. Ma, and M.-Y. Chow, "Distributed adaptive droop control for optimal power dispatch in DC microgrid," *IEEE Trans. Ind. Electron.*, vol. 65, no. 1, pp. 778–789, Jan. 2018.

[16] H. Han, H. Wang, Y. Sun, J. Yang, and Z. Liu, "Distributed control scheme on cost optimisation under communication delays for DC microgrids," *IET Gener., Transmiss. Distrib.*, vol. 11, no. 17, pp. 4193–4201, Nov. 2017.

[17] A. A. Hamad, M. A. Azzouz, and E. F. El-Saadany, "Multiagent supervisory control for power management in DC microgrids," *IEEE Trans. Smart Grid*, vol. 7, no. 2, pp. 1057–1068, Mar. 2016.

[18] A. A. Hamad and E. F. El-Saadany, "Multi-agent supervisory control for optimal economic dispatch in DC microgrids," *Sustain. Cities Soc.*, vol. 27, pp. 129–136, Nov. 2016.

[19] D. Liu, K. Jiang, L. Yan, X. Ji, K. Cao, and P. Xiong, "A fully distributed economic dispatch method in DC microgrid based on consensus algorithm," *IEEE Access*, vol. 10, pp. 119345–119356, 2022.

[20] Z. Lv, Z. Wu, X. Dou, M. Zhou, and W. Hu, "Distributed economic dispatch scheme for droop-based autonomous DC microgrid," *Energies*, vol. 13, no. 2, p. 404, Jan. 2020.

[21] Y. Dou, M. Chi, Z.-W. Liu, G. Wen, and Q. Sun, "Distributed secondary control for voltage regulation and optimal power sharing in DC microgrids," *IEEE Trans. Control Syst. Technol.*, vol. 30, no. 6, pp. 2561–2572, Nov. 2022.

[22] L. Wang and F. Xiao, "Finite-time consensus problems for networks of dynamic agents," *IEEE Trans. Autom. Control*, vol. 55, no. 4, pp. 950–955, Apr. 2010.
[23] S. P. Bhat and D. S. Bernstein, "Finite-time stability of continuous autonomous systems," *SIAM J. Control Optim.*, vol. 38, no. 3, pp. 751–766, Jan. 2000.
[24] G. Chen, J. Ren, and E. N. Feng, "Distributed finite-time economic dispatch of a network of energy resources," *IEEE Trans. Smart Grid*, vol. 8, no. 2, pp. 822–832, Mar. 2017.
[25] M. Zaery, P. Wang, R. Huang, W. Wang, and D. Xu, "Distributed economic dispatch for islanded DC microgrids based on finite-time consensus protocol," *IEEE Access*, vol. 8, pp. 192457–192468, 2020.
[26] M. M. Gomez and R. C. Dobson, "Finite-time second-order cooperative control for the economic dispatch in DC microgrids," in *Proc. 46th Annu. Conf. IEEE Ind. Electron. Soc.*, Oct. 2020, pp. 1596–1601.
[27] A. Polyakov, "Nonlinear feedback design for fixed-time stabilization of linear control systems," *IEEE Trans. Autom. Control*, vol. 57, no. 8, pp. 2106–2110, Aug. 2012.
[28] Z. Zuo, Q.-L. Han, B. Ning, X. Ge, and X.-M. Zhang, "An overview of recent advances in fixed-time cooperative control of multiagent systems," *IEEE Trans. Ind. Informat.*, vol. 14, no. 6, pp. 2322–2334, Jun. 2018.
[29] J. Dávila and A. Pisano, "On the fixed-time consensus problem for nonlinear uncertain multiagent systems under switching topology," *Int. J. Robust Nonlinear Control*, vol. 31, no. 9, pp. 3841–3858, Jun. 2021.
[30] P. Wang, R. Huang, M. Zaery, W. Wang, and D. Xu, "A fully distributed fixed-time secondary controller for DC microgrids," *IEEE Trans. Ind. Appl.*, vol. 56, no. 6, pp. 6586–6597, Nov. 2020.
[31] N. Sarrafan, J. Zarei, R. Razavi-Far, M. Saif, and M.-H. Khooban, "A novel on-board DC/DC converter controller feeding uncertain constant power loads," *IEEE J. Emerg. Sel. Topics Power Electron.*, vol. 9, no. 2, pp. 1233–1240, Apr. 2021.
[32] Q.-F. Yuan, Y.-W. Wang, X.-K. Liu, and Y. Lei, "Distributed fixed-time secondary control for DC microgrid via dynamic average consensus," *IEEE Trans. Sustain. Energy*, vol. 12, no. 4, pp. 2008–2018, Oct. 2021.
[33] S. Sahoo, S. Mishra, S. M. Fazeli, F. Li, and T. Dragicevic, "A distributed fixed-time secondary controller for DC microgrid clusters," *IEEE Trans. Energy Convers.*, vol. 34, no. 4, pp. 1997–2007, Dec. 2019.
[34] Z. Cheng, J. Liu, Z. Li, J. Si, and S. Xu, "Distributed fixed-time secondary control for voltage restoration and economic dispatch of DC microgrids," *Sustain. Energy, Grids Netw.*, vol. 34, Jun. 2023, Art. no. 101042.
[35] M. Zaery, P. Wang, X. Lu, R. Huang, W. Wang, and D. Xu, "Fully distributed fixed-time optimal dispatch for islanded DC microgrids," in *Proc. IEEE Appl. Power Electron. Conf. Expo. (APEC)*, Mar. 2020, pp. 603–608.
[36] M. Zaery, P. Wang, W. Wang, and D. Xu, "A novel fully distributed fixed-time optimal dispatch of DC multi-microgrids," *Int. J. Electr. Power Energy Syst.*, vol. 129, Jul. 2021, Art. no. 106792.
[37] Z. Zuo, C. Wang, and Z. Ding, "Robust consensus control of uncertain multi-agent systems with input delay: A model reduction method," *Int. J. Robust Nonlinear Control*, vol. 27, no. 11, pp. 1874–1894, Jul. 2017.
[38] Z. Artstein, "Linear systems with delayed controls: A reduction," *IEEE Trans. Autom. Control*, vol. AC-27, no. 4, pp. 869–879, Aug. 1982.
[39] W. Kwon and A. Pearson, "Feedback stabilization of linear systems with delayed control," *IEEE Trans. Autom. Control*, vol. AC-25, no. 2, pp. 266–269, Apr. 1980.
[40] E. Moulay, M. Dambrine, N. Yeganefar, and W. Perruquetti, "Finite-time stability and stabilization of time-delay systems," *Syst. Control Lett.*, vol. 57, no. 7, pp. 561–566, Jul. 2008.
[41] Z. Zuo, "Fixed-time stabilization of general linear systems with input delay," *J. Franklin Inst.*, vol. 356, no. 8, pp. 4467–4477, May 2019.
[42] J. Ni, L. Liu, C. Liu, and J. Liu, "Fixed-time leader-following consensus for second-order multiagent systems with input delay," *IEEE Trans. Ind. Electron.*, vol. 64, no. 11, pp. 8635–8646, Nov. 2017.
[43] X. Ai and L. Wang, "Distributed fixed-time event-triggered consensus of linear multi-agent systems with input delay," *Int. J. Robust Nonlinear Control*, vol. 31, no. 7, pp. 2526–2545, May 2021.
[44] R. Zhang and B. Hredzak, "Distributed finite-time multiagent control for DC microgrids with time delays," *IEEE Trans. Smart Grid*, vol. 10, no. 3, pp. 2692–2701, May 2019.
[45] Y. W. Feng and J. L. Liu, "Distributed fixed time control for DC microgrid with input delay," *Int. Trans. Electr. Energy Syst.*, vol. 2023, pp. 1–13, Feb. 2023.
[46] Z. Zuo and L. Tie, "Distributed robust finite-time nonlinear consensus protocols for multi-agent systems," *Int. J. Syst. Sci.*, vol. 47, no. 6, pp. 1366–1375, Apr. 2016.
[47] H. Zhang, F. L. Lewis, and Z. Qu, "Lyapunov, adaptive, and optimal design techniques for cooperative systems on directed communication graphs," *IEEE Trans. Ind. Electron.*, vol. 59, no. 7, pp. 3026–3041, Jul. 2012.
[48] S. E. Parsegov, A. E. Polyakov, and P. S. Shcherbakov, "Fixed-time consensus algorithm for multi-agent systems with integrator dynamics," *IFAC Proc. Volumes*, vol. 46, no. 27, pp. 110–115, 2013.

MOHAMED ZAERY (Member, IEEE) received the B.S. and M.S. degrees in electrical engineering from the Faculty of Engineering, Aswan University, Aswan, Egypt, in 2014 and 2017, respectively, and the Ph.D. degree in electrical engineering from the Harbin Institute of Technology, Harbin, China, in 2021.

In 2016, he was a Lecturer Assistant with the Department of Electrical Engineering, Faculty of Engineering, Aswan University, where he has been an Assistant Professor, since 2021. He is currently a K. A. CARE Energy Research and Innovation Center (ERIC) Postdoctoral Fellow with the King Fahd University of Petroleum and Minerals. His research interests include the distributed control systems and the optimal operation of microgrids.

MOHAMMAD A. ABIDO (Senior Member, IEEE) received the B.Sc. (Hons.) and M.Sc. degrees in electrical engineering from Menoufia University, Shebeen El-Kom, Egypt, in 1985 and 1989, respectively, and the Ph.D. degree in electrical engineering from the King Fahd University of Petroleum and Minerals (KFUPM), Dhahran, Saudi Arabia, in 1997.

He is currently a Distinguished University Professor with KFUPM. He is also a Senior Researcher with the K. A. CARE Energy Research and Innovation Center, Dhahran. He has authored or coauthored two books and more than 400 papers in reputable journals and international conferences. He has participated in more than 60 funded projects and supervised more than 70 M.S. and Ph.D. students. He accumulated more than 18700 citations with H-index of 61 and i10-index of 201 according to Google Scholar accessed in August 2022. His research interests include power system control and operation and renewable energy resources integration to power systems.

Dr. Abido was a recipient of the KFUPM Excellence in Research Award, in 2002, 2007, and 2012; the KFUPM Best Project Award, in 2007, 2010, and 2020; the First Prize Paper Award of the Industrial Automation and Control Committee of the IEEE Industry Applications Society, in 2003; the Abdel-Hamid Shoman Prize for Young Arab Researchers in Engineering Sciences, in 2005; the Best Applied Research Award of 15th GCC-CIGRE Conference, Abu Dhabi, United Arab Emirates, in 2006; and the Best Poster Award from the International Conference on Renewable Energies and Power Quality (ICREPQ'13), Bilbao, Spain, in 2013. He was also a recipient of the Almarai Prize for Scientific Innovation for 2017–2018; the Distinguished Scientist, Saudi Arabia, in 2018; the Khalifa Award for Education for 2017–2018; and the Higher Education, Distinguished University Professor in Scientific Research, Abu Dhabi, in 2018.