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RESEARCH ARTICLE

# An ExpTODIM-MACONT Based Multiple-Attribute Group Decision-Making Technique for Smart Classroom Teaching Evaluation of Basic English Under Interval-Valued Pythagorean Fuzzy Circumstances

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**ABSTRACT** With the rapid development of information technology, the university teaching model has also begun to break free from constraints from two dimensions of time and space, becoming a new teaching model - smart classroom. Supported by the theory of smart classrooms, the reform of basic English curriculum teaching in China has received much attention. Currently, the reform of basic English curriculum teaching mainly faces three problems: incomplete resource allocation, limited development conditions, and poor practicality. Based on this, utilizing the theory of smart classrooms, we can solve the problem of incomplete resource allocation by building an ‘‘educational community’’, innovate teaching models, enhance students' English autonomous learning and practical application abilities, and achieve strategic analysis of basic English curriculum teaching reform. The smart classroom teaching evaluation of basic English is a multiple-attribute group decision-making (MAGDM) problem. Recently, the Exponential TODIM (an acronym in Portuguese of interactive and multicriteria decision making) technique and Mixed Aggregation by Comprehensive Normalization Technique (MACONT) has been used to cope with MAGDM issues. The interval-valued Pythagorean fuzzy sets (IVPFSs) are used as a tool for characterizing uncertain information during the smart classroom teaching evaluation of basic English. In this manuscript, the interval-valued Pythagorean fuzzy Exponential TODIM- MACONT (IVPF-ExpTODIM-MACONT) technique is built to solve the MAGDM under IVPFSs. In the end, a numerical case study for smart classroom teaching evaluation of basic English is implemented to validate the proposed technique. The main contributions of this paper are outlined: 1) the ExpTODIM and MACONT technique has been extended to IVPFSs; 2) Information Entropy technique is employed to manage the weight values under IVPFSs; 3) IVPF-ExpTODIM-MACONT technique is founded to implement the MAGDM under IVPFSs; 4) numerical case study for smart classroom teaching evaluation of basic English and some comparative analysis is supplied to verify the IVPF-ExpTODIM-MACONT technique.

**INDEX TERMS** Multiple attribute group decision making (MAGDM), interval-valued Pythagorean fuzzy sets (IVPFSs), Exponential TODIM (ExpTODIM) technique, MACONT technique, smart classroom teaching evaluation.

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## I. INTRODUCTION

The construction of smart classroom has become the main development direction of basic English teaching in the era of ‘‘Internet plus’’, which requires a relatively long process to actively innovate classroom teaching forms [1], [2]. In the teaching of basic English courses, we generally emphasize the importance of putting practice first. The so-called practice is not only about the country vigorously practicing and finding suitable teaching forms, but also about universities and teachers constantly practicing, innovating teaching models, stimulating the motivation of teachers and students to learn [3], [4], [5]. It is also about cultivating students' awareness of self-directed and continuous learning. This internal motivation is the key to truly promoting the reform of basic English course education. To establish an autonomous learning model, it is necessary to work together from multiple directions, utilize the form of smart classrooms, and create more discussion opportunities for students during the teaching process. For example, large class teaching and small class discussions can be adopted to centralize learning time, allowing students to find the meaning and fun of basic English courses in the process of autonomous learning and communication. Only based on fun can students better stimulate their learning enthusiasm [6], [7], [8]. At the same time, in a learning atmosphere of mutual communication and complementarity, the speed of knowledge dissemination can be accelerated, and the problem of insufficient effectiveness of basic English courses can be solved [9], [10]. By setting up "thinking points", the teaching content can also be centralized, allowing students to improve learning efficiency and abilities during the process of self-reflection. This can cultivate students' thinking leaps, better open up their thinking and knowledge, and cultivate their autonomous learning ability. The importance of basic English curriculum education is already self-evident, and it is of great significance for the cultivation of versatile talents and the future talent planning of the country [11]. Especially in the context of the rise of smart classrooms, the reform of basic English curriculum education has encountered unprecedented development opportunities. We should start the reform with targeted measures, promote the smooth progress of the reform, construct reform strategies, and accelerate the further implementation of basic English curriculum education. Firstly, in response to the difficulty in implementing the educational philosophy of basic English courses, it can be solved by establishing basic curriculum units [12], [13]. A successful example of the implementation of the concept of basic course units is Fudan University. Some University is one of the most successful universities in China in implementing English course education. It is by adding a course group between courses and modules that the structure of university English courses is adjusted more meticulously, making the connection between courses closer, and then turning it into a systematic course structure. Secondly, in response to issues such as the lack of concentration of teaching staff and difficulty in mobilizing teachers' enthusiasm, an "educational community" can be established to solve them. Teachers who teach basic English courses are generally professional teachers from various universities and colleges, and there are not many issues with their knowledge [14], [15]. However, under the traditional teacher selection system, professional teachers are generally trapped in the misconception of scientific research achievements and promotion. The inconsistent recognition of English education concepts makes it difficult for the teaching team to spontaneously form an educational consensus, Therefore, it is necessary to further regulate and promote teacher teaching behavior through relevant systems and measures [16], [17]. Finally, in response to the widespread lack of effectiveness in curriculum teaching in current basic English curriculum education, the focus of the reform can be to change the teaching mode from the traditional teacher-based teaching mode to the student independent learning mode. No matter how well the curriculum is set, without effective teaching from teachers and students' self-directed learning as the foundation, it will be ineffective in terms of teaching effectiveness [18], [19]. This point is still extremely important in the current reform of basic English curriculum education. By analyzing the problems in the teaching of basic English courses, based on smart classrooms, the information-based support environment consists of digital learning resources and smart teaching platforms. The fundamental task is to support teaching interaction and self-directed learning, develop students' intelligent abilities, promote students' intelligent behavior, construct teaching reform strategies for basic English courses, cultivate students' interest in English learning, and improve the efficiency of students' English learning [20], [21].

MADM, as a commonly used decision-making method [22], [23], [24], is often difficult to evaluate things with accurate values due to limitations in decision-makers' knowledge level and the fuzziness of things themselves [25], [26]. In response to this issue, Zadeh [27] first proposed fuzzy sets in 1965 to characterize the uncertainty of information. Atanassov [28] extended it and proposed intuitionistic fuzzy-sets (IFS) that include membership, non-membership and hesitancy, with a total of 1. But when the membership degree and non-membership degree are independently implemented by the decision-maker, due to the fuzziness of the decision itself and the varying degrees of detail in describing things, there may be situations where their total is greater than 1. Therefore, Yager [29] proposed the Pythagorean fuzzy sets (PFSs), where the sum of the squares of membership and non-membership is not greater than 1, expanding its spatial range [30], [31], [32]. As the complexity and uncertainty of decision-making continue to increase, decision-makers may find it difficult to accurately give membership or non-membership degrees with a specific value. In response to this situation, combined with the relevant knowledge of interval numbers [33], [34], the concept of interval valued PFSs (IVPFSs) [35] has been proposed, which can represent membership or non-membership in the form of intervals. In the field of MADM, Haktanir and Kahraman [36] measured and prioritized product design requirements and customer needs based on IVPFS, and combines quality function deployment (QFD) to conduct evaluation decisions in the development and application process of solar photovoltaic power generation. Tang and Yang [37] implemented the concept of interval valued Pythagorean fuzzy preference relationship (IVPFPR) for the selection of shared electric bicycle recycling suppliers, and proposes a Pythagorean fuzzy decision-making method that considers trust level. Fu et al. [38] proposed a product ranking method that combines feature opinion IVPFS to effectively analyze and extract relevant opinions from online reviews, introducing IVPFS to represent interval values; References [39], [40], and [41] utilized the relevant measures of membership and non-membership in IVPFS to conduct decision analysis and research on risk assessment of autonomous vehicles, sustainable supplier selection, and high-tech project portfolio evaluation.

The smart classroom teaching evaluation of basic English is classical MAGDM. Recently, the Exponential TODIM (ExpTODIM) technique [42], [43] and MACONT technique [44], [45] has been used to cope with MAGDM issues. The IVPFSs [35] are used as a technique for characterizing uncertain information during the smart classroom teaching evaluation of basic English. Until now, there is not related research works to implement the ExpTODIM technique [42] and MACONT technique [44], [45], [46] under IVPFSs [35]. Thus, the main aim of this paper is to investigate the ExpTODIM-MACONT technique under IVPFSs and construct the corresponding MAGDM technique for smart classroom teaching evaluation of basic English. In this paper, the interval-valued Pythagorean fuzzy Exponential TODIM-MACONT (IVPF-ExpTODIM-MACONT) technique is built to solve the MAGDM under IVPFSs. The information Entropy is used to obtain the weight values based on the score value and accuracy value under IVPFSs. At last, a numerical case study for smart classroom teaching evaluation of basic English is implemented to validate the proposed technique. The main research aim and motivation of this paper is constructed: (1) the Exponential ExpTODIM-MACONT technique is extended to the IVPFSs; (2) the interval-valued Pythagorean fuzzy Exponential TODIM-MACONT (IVPF-ExpTODIM-MACONT) technique is built to solve the MAGDM under IVPFSs; (3) the information Entropy is used to obtain the weight values based on the score value and accuracy value under IVPFSs; (4) Finally, a numerical case study for smart classroom teaching evaluation of basic English is implemented to validate the proposed technique.

The framework of this paper is implemented below. In Section II, the IVPFSs is implemented. In Section III, IVPF-ExpTODIM-MACONT technique is implemented under IVPFSs with entropy. Section IV implements an illustrative case for smart classroom teaching evaluation of basic English and some comparative analysis. Some remarks are implemented in Section V.

## II. PRELIMINARIES

The PFSs and IVPFSs are implemented.

*Definition 1 ([47]):* Let $\Theta$ be fix set, the PFS is implemented:

$$AT = \{\langle\theta, (AM_{AT}(\theta), AM_{AT}(\theta))\rangle \,|\theta \in \Theta\} \tag{1}$$

where $AM_{AT}(\theta) \subseteq [0,1], AN_{AT}(\theta) \subseteq [0,1]$ denotes the membership non-membership of $\theta \in \Theta$ to $AT$, and, for $\theta \in \Theta$, it has the implemented condition

$$(AM_{AT}(\theta))^2 + (AN_{AT}(\theta))^2 \leq 1. \tag{2}$$

*Definition 2 ([47]):* Let $T_1 = (M_1, N_1)$, $AT_2 = (AM_2, AN_2)$, and $AT = (AM, AN)$ be three PFNs, and some operations are implemented.

$$(1)\ AT_1 \oplus AT_2 = \left(\sqrt{(AM_1)^2 + (AM_2)^2 - (AM_1)^2(AM_2)^2}, AN_1 AN_2\right);$$

$$(2)\ AT_1 \otimes AT_2 = \left(AM_1 AM_2, \sqrt{(AN_1)^2 + (AN_2)^2 - (AN_1)^2(AN_2)^2}\right);$$

$$(3)\ \lambda AT = \left(\sqrt{1-(1-AM^2)^{\lambda}}, AN^{\lambda}\right), \lambda > 0;$$

$$(4)\ (AT)^{\lambda} = \left(AM^{\lambda}, \sqrt{1-(1-AN^2)^{\lambda}}\right), \lambda > 0.$$

Liang, Zhang and Liu [35] implemented the interval-valued PFSs (IVPFSs).

*Definition 3 ([35]):* Let $\Theta$ be a fix set. An IVPFS is implemented:

$$XT = \{\langle\theta, (XM_{VT}(\theta), XN_{VT}(\theta))\rangle \,|\theta \in \Theta\} \tag{3}$$

where $XM_{VT}(\theta) \subseteq [0,1], XN_{VT}(\theta) \subseteq [0,1]$ are interval values, $XM_{XT}(\theta) = \left[XM_{VT}^L(\theta), XM_{VT}^R(\theta)\right]$, $XN_{XT}(\theta) = \left[XN_{XT}^L(\theta), XN_{XT}^R(\theta)\right]$, $0 \leq XM_{XT}^R(\theta) + XN_{XT}^R(\theta) \leq 1$, $\forall\theta \in \Theta$. $XM_{XT}(\theta), XN_{XT}(\theta)$ denote the membership and no-membership.

For convenience, $XT = \left(\left[XM^L, XM^R\right], \left[XN^L, XN^R\right]\right)$ is an interval-valued Pythagorean fuzzy number (IVPFN), and $\left[XM^L, XM^R\right], \left[XN^L, XN^R\right] \subset [0,1]$, $\left(XM^R\right)^2 + \left(XM^R\right)^2 \leq 1$.

*Definition 4 ([48]):* Let $XT_1 = \left(\left[XM_1^L, XM_1^R\right], \left[XN_1^L, XN_1^R\right]\right)$, $XT_2 = \left(\left[XM_2^L, XM_2^R\right], \left[XN_2^L, XN_2^R\right]\right)$, and $XT = \left(\left[XM^L, XM^R\right], \left[XN^L, XN^R\right]\right)$ be three IVPFNs, and some operations are implemented as shown in the equation at the bottom of the next page.

*Definition 5 ([48]):* Let

$$XT_1 = \left(\left[XM_1^L, XM_1^R\right], \left[XN_1^L, XN_1^R\right]\right),$$

$$XT_2 = \left(\left[XM_2^L, XM_2^R\right], \left[XN_2^L, XN_2^R\right]\right),$$

$$SV(XT_1) = \frac{1}{4}\left[\begin{array}{l}\left(1 + (XM_1^L)^2 - (XN_1^L)^2\right) \\ + \left(1 + (XM_1^R)^2 - (XN_1^R)^2\right)\end{array}\right]$$

and

$$SV(XT_2) = \frac{1}{4}\left[\begin{array}{l}\left(1 + (XM_2^L)^2 - (XN_2^L)^2\right) \\ + \left(1 + (XM_2^R)^2 - (XN_2^R)^2\right)\end{array}\right]$$

is the scores value of

$$XT_1 = \left(\left[XM_1^L, XM_1^R\right], \left[XN_1^L, XN_1^R\right]\right)$$

and

$$XT_2 = \left(\left[XM_2^L, XM_2^R\right], \left[XN_2^L, XN_2^R\right]\right),$$

$$AV\left(XT_1\right) = \frac{1}{2}\left[\begin{array}{l}\left(\left(XM_1^L\right)^2 + \left(XN_1^L\right)^2\right) \\ + \left(XM_1^R\right)^2 + \left(XN_1^R\right)^2\end{array}\right]$$

and

$$AV\left(XT_2\right) = \frac{1}{2}\left[\begin{array}{l}\left(\left(XM_2^L\right)^2 + \left(XN_2^L\right)^2\right) \\ + \left(XM_2^R\right)^2 + \left(XN_2^R\right)^2\end{array}\right]$$

is the accuracy value of

$$XT_1 = \left(\left[XM_1^L, XM_1^R\right], \left[XN_1^L, XN_1^R\right]\right)$$

and

$$XT_2 = \left(\left[XM_2^L, XM_2^R\right], \left[XN_2^L, XN_2^R\right]\right),$$

then if

$$SV\left(XT_1\right) < SV\left(XT_2\right), XT_1 < XT_2;$$

if $SV\left(XT_1\right) = SV\left(XT_2\right)$, (1) if $AV\left(XT_1\right) = AV\left(XT_2\right), XT_1 = XT_2$; (2) if $AV\left(XT_1\right) < AV\left(XT_2\right), XT_1 < XT_2$.

*Definition 6 ([35]):* Let $XT_1 = (\left[XM_1^L, XM_1^R\right], [XN_1^L, XN_1^R]), XT_2 = (\left[XM_2^L, XM_2^R\right], \left[XN_2^L, XN_2^R\right])$, the normalized Hamming distance is:

$$HD\left(XT_1, XT_2\right) = \frac{1}{4}\left(\begin{array}{l}\left|\left(XM_1^L\right)^2 - \left(XM_2^L\right)^2\right| + \left|\left(XM_1^R\right)^2 - \left(XM_2^R\right)^2\right| \\ + \left|\left(XL_1^L\right)^2 - \left(XL_2^L\right)^2\right| + \left|\left(XN_1^R\right)^2 - \left(XN_2^R\right)^2\right|\end{array}\right) \quad (4)$$

## III. IVPF-ExpTODIM-MACONT TECHNIQUE FOR MAGDM WITH ENTROPY WEIGHT

The IVPF-ExpTODIM-MACONT technique is implemented for MAGDM. Let $XA = \{XA_1, XA_2, \cdots, XA_m\}$ be alternatives, and the attributes set $XG = \{XG_1, XG_2, \cdots, XG_n\}$ with weight $vw$, where $xw_j \in [0, 1], \sum_{j=1}^{n} xw_j = 1$ and a set of invited experts $DE = \left\{DE_1, DE_2, \cdots, DE_q\right\}$,let expert's weight be $\left\{dw_1, dw_2, \cdots, dw_q\right\}$ $dw_t \in [0, 1], \sum_{t=1}^{q} dw_t = 1$. Then, IVPF-ExpTODIM-MACONT technique is implemented for MAGDM (See Figure 1).

### A. STAGE 1: OBTAIN THE DECISION INFORMATION WITH IVPFSs

**Step 1.** Implement the IVPFN group decision matrix

$$XR^{(t)} = \left[XR_{ij}^{(t)}\right]_{m \times n} = \left(\left[XM_{ij}^{L(t)}, XM_{ij}^{R(t)}\right], \left[XN_{ij}^{L(t)}, XN_{ij}^{R(t)}\right]\right)_{m \times n}$$

and calculate the average matrix $XR = \left[XR_{ij}\right]_{m \times n} = \left(\left[XM_{ij}^L, XM_{ij}^R\right], \left[XN_{ij}^L, XN_{ij}^R\right]\right)_{m \times n}$:

$$XR^t = \left[XR_{ij}^t\right]_{m \times n} = \begin{array}{c} \\ XA_1 \\ XA_2 \\ \vdots \\ XA_m \end{array} \begin{array}{c} \begin{array}{cccc} XG_1 & XG_2 & \ldots & XG_n \end{array} \\ \left[\begin{array}{cccc} XR_{11}^t & XR_{12}^t & \ldots & XR_{1n}^t \\ XR\phi_{21}^t & XR_{22}^t & \ldots & XR_{2n}^t \\ \vdots & \vdots & \vdots & \vdots \\ XR_{m1}^t & XR_{m2}^t & \ldots & XR_{mn}^t \end{array}\right] \end{array} \quad (5)$$

$$XR = \left[XR_{ij}\right]_{m \times n} = \begin{array}{c} \\ XA_1 \\ XA_2 \\ \vdots \\ XA_m \end{array} \begin{array}{c} \begin{array}{cccc} XG_1 & XG_2 & \ldots & XG_n \end{array} \\ \left[\begin{array}{cccc} XR_{11} & XR_{12} & \ldots & XR_{1n} \\ XR_{21} & XR_{22} & \ldots & XR_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ XR_{m1} & XR_{m2} & \ldots & XR_{mn} \end{array}\right] \end{array} \quad (6)$$

In line with the IVPFWA technique, the $XR = \left[XR_{ij}\right]_{m \times n} = \left(\left[XM_{ij}^L, XM_{ij}^R\right], \left[XN_{ij}^L, XN_{ij}^R\right]\right)_{m \times n}$ is

$$(1) XT_1 \oplus XT_2 = \left(\left[\begin{array}{l}\sqrt{\left(XM_1^L\right)^2 + \left(XM_2^L\right)^2 - \left(XM_1^L\right)^2\left(XM_2^L\right)^2}, \\ \sqrt{\left(XM_1^R\right)^2 + \left(XM_2^R\right)^2 - \left(XM_1^R\right)^2\left(XM_2^R\right)^2}\end{array}\right], \left[XN_1^L XN_2^L, XN_1^R XN_2^R\right]\right);$$

$$(2) XT_1 \otimes XT_2 = \left(\left[XM_1^L XM_2^L, XM_1^R XM_2^R\right], \left[\begin{array}{l}\sqrt{\left(XN_1^L\right)^2 + \left(XN_2^L\right)^2 - \left(XN_1^L\right)^2\left(XN_2^L\right)^2}, \\ \sqrt{\left(XN_1^R\right)^2 + \left(XN_2^R\right)^2 - \left(XN_1^R\right)^2\left(XN_2^R\right)^2}\end{array}\right]\right);$$

$$(3)\ \lambda XT = \left(\left[\sqrt{1 - \left(1 - \left(XM^L\right)^2\right)^\lambda}, \sqrt{1 - \left(1 - \left(XM^R\right)^2\right)^\lambda}\right], \left[\left(XN^L\right)^\lambda, \left(XN^R\right)^\lambda\right]\right), \lambda > 0;$$

$$(4)\ (XT)^\lambda = \left(\left[\left(XM^L\right)^\lambda, \left(XM^R\right)^\lambda\right], \left[\sqrt{1 - \left(1 - \left(XN^L\right)^2\right)^\lambda}, \sqrt{1 - \left(1 - \left(XN^R\right)^2\right)^\lambda}\right]\right), \lambda > 0;$$

$$(5)\ XT^c = \left(\left[XN^L, XN^R\right], \left[XM^L, XM^R\right]\right).$$

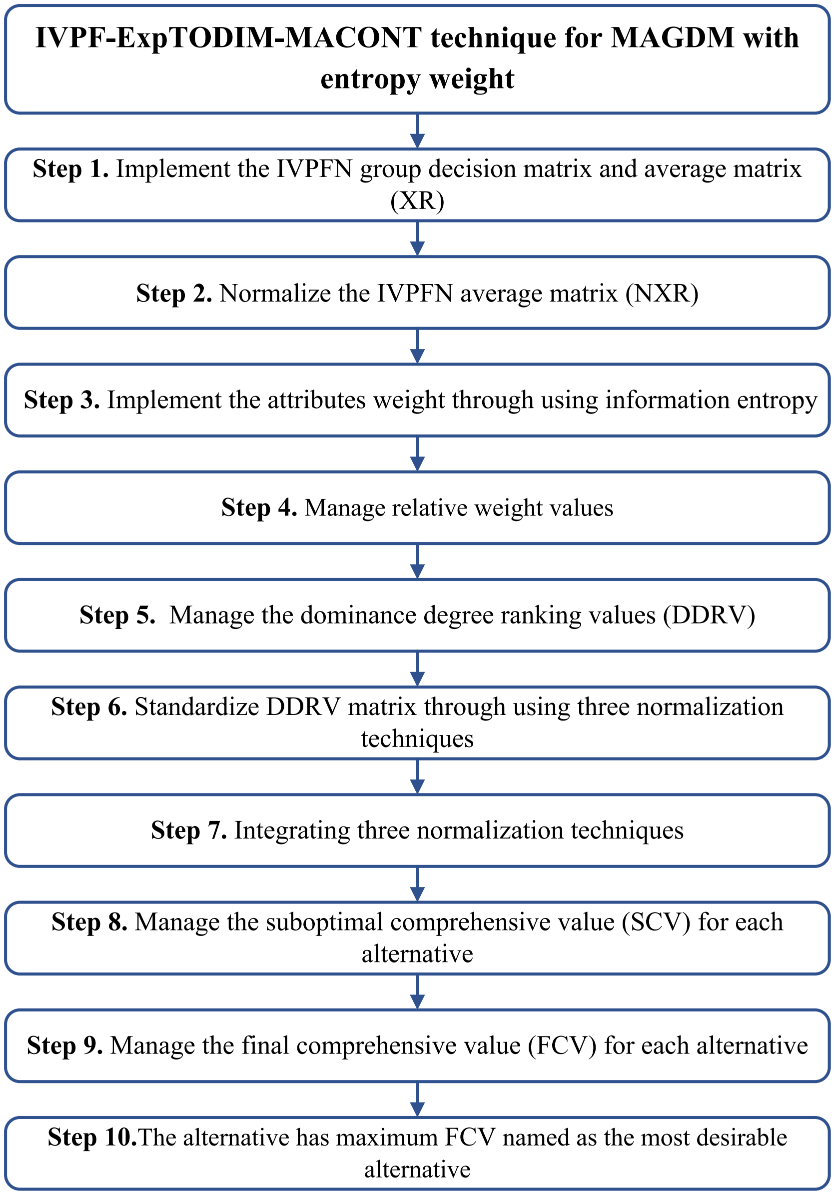


**FIGURE 1.** IVPF-ExpTODIM-MACONT technique for MAGDM with entropy weight.

implemented:

$$XR_{ij} = dw_1 XR_{ij}^1 \oplus dw_2 XR_{ij}^2 \oplus \cdots \oplus dw_q XR_{ij}^q$$
$$= \left( \begin{array}{c} \left[ \begin{array}{c} \sqrt{1 - \prod_{t=1}^{q} \left(1 - \left(XM_{ij}^{L(t)}\right)^2\right)^{dw_t}}, \\ \sqrt{1 - \prod_{t=1}^{q} \left(1 - \left(XM_{ij}^{R(h)}\right)^2\right)^{dw_t}} \end{array} \right], \\ \left[ \prod_{t=1}^{q} \left(XN_{ij}^{L(t)}\right)^{dw_t}, \prod_{t=1}^{q} \left(XN_{ij}^{R(t)}\right)^{dw_t} \right] \end{array} \right) \quad (7)$$

**Step 2.** Normalize the $XR = [XR_{ij}]_{m\times n}$ into $NXR = [NXR_{ij}]_{m\times n} = \left( \begin{array}{c} \left[NXM_{ij}^L, NXM_{ij}^R\right], \\ \left[NXN_{ij}^L, NXN_{ij}^R\right] \end{array} \right)_{m\times n}$.

For benefit attributes:

$$NXR_{ij} = \left(\left[NXM_{ij}^L, NXM_{ij}^R\right], \left[NXN_{ij}^L, NXN_{ij}^R\right]\right)$$
$$= XR_{ij} = \left(\left[XM_{ij}^L, XM_{ij}^R\right], \left[XN_{ij}^L, XN_{ij}^R\right]\right) \quad (8)$$

For cost attributes:

$$NXR_{ij} = \left(\left[NXM_{ij}^L, NXM_{ij}^R\right], \left[NXN_{ij}^L, NXN_{ij}^R\right]\right)$$
$$= \left(\left[XN_{ij}^L, XN_{ij}^R\right], \left[XM_{ij}^L, XM_{ij}^R\right]\right) \quad (9)$$

### B. STAGE 2: IMPLEMENT THE ATTRIBUTES WEIGHT THROUGH USING INFORMATION ENTROPY

**Step 3.Implement the attributes weight through using information entropy**

The weight is important for MAGDM [49], [50], [51]. Entropy [52] is a conventional technique to derive weight. Firstly, the normalized IVPF-matrix $XNN_{ij}$ is implemented:

$$XNN_{ij} = \frac{\dfrac{AV\left(\left[NXM_{ij}^{L}, NXM_{ij}^{R}\right], \left[NXN_{ij}^{L}, NXN_{ij}^{R}\right]\right)}{SV\left(\left[NXM_{ij}^{L}, NXM_{ij}^{R}\right], \left[NXN_{ij}^{L}, NXN_{ij}^{R}\right]\right)}}{\sum_{i=1}^{m} \dfrac{AV\left(\left[NXM_{ij}^{L}, NXM_{ij}^{R}\right], \left[NXN_{ij}^{L}, NXN_{ij}^{R}\right]\right)}{SV\left(\left[NXM_{ij}^{L}, NXM_{ij}^{R}\right], \left[NXN_{ij}^{L}, NXN_{ij}^{R}\right]\right)}}, \tag{10}$$

Then, the IVPFN Shannon information entropy (IVPFN-SIE) is implemented:

$$IVPFNSIE_j = -\frac{1}{\ln m} \sum_{i=1}^{m} XNN_{ij} \ln XNN_{ij} \tag{11}$$

and $XNN_{ij} \ln XNN_{ij} = 0$ if $XNN_{ij} = 0$.

Then, the weights $xw = (xw_1, xw_2, \cdots, xw_n)$ is implemented:

$$xw_j = \frac{1 - IVPFNSIE_j}{\sum_{j=1}^{n} \left(1 - IVPFNSIE_j\right)}, \quad j = 1, 2, \cdots, n. \tag{12}$$

### *C. STAGE 3: IVPF-ExpTODIM-MACONT TECHNIQUE FOR MAGDM*

The IVPF-ExpTODIM-MACONT technique is implemented for MAGDM.

**Step 4.** Manage relative weight of $XG_j$ as:

$$rxw_j = xw_j \Big/ \max_j xw_j, \tag{13}$$

**Step 5. Manage the dominance degree ranking values (DDRV).**

(1) The $DDRV_j(XA_i, XA_t)$ of $XA_i$ over $XA_t$ for $XG_j$ is managed (14), as shown at the bottom of the page, where $\pi$ is built in line with.Tversky and Kahneman [53] and $\pi \in [1, 5]$ [42].

(2) The $DDRV_j(XA_i)$ $(j = 1, 2, \cdots, n)$ with respect to $XG_j$ is implemented (15), as shown at the bottom of the page.

(3) The overall DDRV of $XA_i$ over other alternatives for $XG_j$ is managed:

$$DDRV_j(XA_i) = \sum_{t=1}^{m} DDRV_j(XA_i, XA_t) \tag{16}$$

The overall DDRV matrix is implemented (17), as shown at the bottom of the next page.

**Step 6.** Standardize DDRV matrix through using three normalization techniques:

$$IVPFNPIS_j = \max_{j=1}^{n} DDRV_{ij}, \quad IVPFNNIS_j = \min_{j=1}^{n} DDRV_{ij} \tag{18}$$

$$NDDRV_{ij}^{(1)} = \begin{cases} \dfrac{DDRV_{ij} - IVPFNNIS_j}{IVPFNPIS_j - IVPFNNIS_j}, & \text{for benefit attribute} \\ \dfrac{DDRV_{ij} - IVPFNPIS_j}{IVPFNNIS_j - IVPFNPIS_j}, & \text{for } \cos t \text{ attribute} \end{cases} \tag{19}$$

$$NDDRV_{ij}^{(2)} = \begin{cases} \dfrac{e^{DDRV_{ij}}}{\sum_{i=1}^{m} e_{ij}^{DDRV_{ij}}}, & \text{for benefit attribute} \\ \dfrac{1/e^{DDRV_{ij}}}{\sum_{i=1}^{m} \left(1/e^{DDRV_{ij}}\right)}, & \text{for benefit attribute} \end{cases} \tag{20}$$

$$NDDRV_{ij}^{(3)} = \begin{cases} \dfrac{e^{DDRV_{ij}}}{\max_i \left(e^{DDRV_{ij}}\right)}, & \text{for benefit attribute} \\ \dfrac{\min_i \left(e^{DDRV_{ij}}\right)}{e^{DDRV_{ij}}}, & \text{for benefit attribute} \end{cases} \tag{21}$$

$$DDRV_j(XA_i, XA_t) = \begin{cases} \dfrac{rxw_j \times \left(1 - 10^{-\rho HD(NXR_{ij}, NXR_{tj})}\right)}{\sum_{j=1}^{n} rxw_j} & \text{if } SF(NXR_{ij}) > SF(NXR_{tj}) \\ 0 & \text{if } SF(NXR_{ij}) = SF(NXR_{tj}) \\ -\dfrac{1}{\pi} \dfrac{\sum_{j=1}^{n} rxw_j \times \left(1 - 10^{-\rho HD(NXR_{ij}, NXR_{tj})}\right)}{rxw_j} & \text{if } SF(NXR_{ij}) < SF(NXR_{tj}) \end{cases} \tag{14}$$

$$DDRV_j(XA_i) = \left[DDRV_j(XA_i, XA_t)\right]_{m \times m} = \begin{matrix} & XA_1 & XA_2 & \cdots & XA_m \\ XA_1 \\ XA_2 \\ \vdots \\ XA_m \end{matrix} \begin{bmatrix} 0 & DDRV_j(XA_1, XA_2) & \cdots & DDRV_j(XA_1, XA_m) \\ DDRV_j(XA_2, XA_1) & 0 & \cdots & DDRV_j(XA_2, XA_m) \\ \vdots & \vdots & \cdots & \vdots \\ DDRV_j(XA_m, XA_1) & DDRV_j(XA_m, XA_2) & \cdots & 0 \end{bmatrix} \tag{15}$$

**Step 7.** Integrating three normalization techniques:

$$NDDRV_{ij} = \left( \begin{array}{c} \lambda NDDRV_{ij}^{(1)} + \mu NDDRV_{ij}^{(2)} \\ + (1-\lambda-\mu)\, NDDRV_{ij}^{(3)} \end{array} \right) \tag{22}$$

Among them, $0 \leqslant \lambda, \mu \leqslant 1$, the values of these two equilibrium parameters are determined by experts. If the expert pays more attention to a particular option among all alternative options, the $\lambda$ value is higher; If experts want to highlight the best attributes of alternative solutions, the $\mu$ value is higher; If experts want to emphasize the gap between alternative solutions, that is, highlight the best and worst attributes of the alternative solutions, then the $\lambda$, $\mu$ value of and is smaller.

**Step 8.** Manage the suboptimal comprehensive value (SCV) for each alternative:

$$SCV_i^{(1)} = \delta \frac{XQ_i^{(1)}}{\sqrt{\sum_{i=1}^{m} \left(XQ_i^{(1)}\right)^2}} + (1-\delta) \frac{XQ_i^{(2)}}{\sqrt{\sum_{i=1}^{m} \left(XQ_i^{(2)}\right)^2}} \tag{23}$$

where,

$$XQ_i^{(1)} = \sum_{j=1}^{n} xw_j \left( NDDRV_{ij} - \frac{1}{m} \sum_{i=1}^{m} NDDRV_{ij} \right),$$

$$XQ_i^{(2)} = \frac{\prod_{x\gamma=1}^{n} \left( \frac{1}{m} \sum_{i=1}^{m} NDDRV_{ij} - NDDRV_{ij} \right)^{xw_j}}{\prod_{x\eta=1}^{n} \left( NDDRV_{ij} - \frac{1}{m} \sum_{i=1}^{m} NDDRV_{ij} \right)^{xw_j}},$$

$$SCV_i^{(2)} = \left( \begin{array}{l} \theta \max_j \left( xw_j \left( NDDRV_{ij} - \frac{1}{m} \sum_{i=1}^{m} NDDRV_{ij} \right) \right) \\ + (1-\theta) \min_j \left( xw_j \left( \begin{array}{c} NDDRV_{ij} - \\ \frac{1}{m} \sum_{i=1}^{m} NDDRV_{ij} \end{array} \right) \right) \end{array} \right) \tag{24}$$

where $x\gamma$ $(x\gamma = 1, 2, \cdots, n)$ represent the part of attributes that satisfy $\frac{1}{m} \sum_{i=1}^{m} NDDRV_{ij} \geq NDDRV_{ij}$, and $x\eta$ $(x\eta = 1, 2, \cdots, n)$ represent the part of attributes that satisfy $NDDRV_{ij} \geq \frac{1}{m} \sum_{i=1}^{m} NDDRV_{ij}$. In addition, $\delta$ and $\theta$ $(0 \leq \delta, \theta \leq 1)$ are preference parameters.

**Step 9.** Manage the final comprehensive value (FCV) for each alternative:

$$FCV_i = \frac{1}{2} \left( SCV_i^{(1)} + \frac{SCV_i^{(2)}}{\sqrt{\sum_{i=1}^{m} \left(SCV_i^{(2)}\right)^2}} \right) \tag{25}$$

**Step 10.** In line with FCV, the largest FCV is the optimal choice.

## IV. EMPIRICAL EXAMPLE AND COMPARATIVE ANALYSIS

### A. EMPIRICAL EXAMPLE FOR SMART CLASSROOM TEACHING EVALUATION OF BASIC ENGLISH

The teaching of basic English courses places great emphasis on the concept of effective teaching, which means that under the guidance of objective teaching laws, teachers spend the least time and energy to achieve the prescribed teaching objectives, achieve the teaching objectives, and meet the needs of students as much as possible. The process of effective teaching by teachers is essentially the process of disseminating the content of basic English courses to students. With the widespread dissemination of the concept of basic English curriculum, it is widely regarded by the academic community as a product of the combination of traditional education methods in China and modern educational concepts in the West. Chinese universities have had the concept of developing "generalist" education for a long time, but in the later stage, as national policies began to learn from the single talent education model of the Soviet Union, the education model mainly focused on cultural cultivation became the main form of university education in China. Over time, the drawbacks of a single talent cultivation model have gradually emerged, and the talents cultivated by universities cannot adapt to social needs. Under the long-term accumulation of social contradictions, after the reform and opening up, China began to realize that offering basic English courses and cultivating diverse talents can meet the development needs of adapting to the national economic system. In the late 20th century, the country began to

$$DDRV = \left(DDRV_{ij}\right)_{m\times n} = \begin{bmatrix} & XG_1 & XG_2 & \cdots & XG_n \\ XA_1 & \sum_{t=1}^{m} DDRV_1(XA_1, XA_t) & \sum_{t=1}^{m} DDRV_2(XA_1, XA_t) & \cdots & \sum_{t=1}^{m} DDRV_n(XA_1, XA_t) \\ XA_2 & \sum_{t=1}^{m} DDRV_1(XA_2, XA_t) & \sum_{t=1}^{m} DDRV_2(XA_2, XA_t) & \cdots & \sum_{t=1}^{m} DDRV_n(XA_2, XA_t) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ XA_m & \sum_{t=1}^{m} DDRV_1(XA_m, XA_t) & \sum_{t=1}^{m} DDRV_2(XA_m, XA_t) & \cdots & \sum_{t=1}^{m} DDRV_n(XA_m, XA_t) \end{bmatrix} \tag{17}$$

**TABLE 1.** Linguistic scale and IVPFNs [54].

| Linguistic Terms | IVPFNs |
|---|---|
| Exceedingly Terrible-XET | $\langle[0.00, 0.10], [0.85, 0.90]\rangle$ |
| Very Terrible-XVT | $\langle[0.00, 0.10], [0.70, 0.75]\rangle$ |
| Terrible-XT | $\langle[0.30, 0.40], [0.45, 0.50]\rangle$ |
| Medium-XM | $\langle[0.40, 0.50], [0.35, 0.40]\rangle$ |
| Well-XW | $\langle[0.60, 0.70], [0.15, 0.20]\rangle$ |
| Very Well-XVW | $\langle[0.70, 0.80], [0.05, 0.10]\rangle$ |
| Exceedingly Well-XEW | $\langle[1.00, 1.00], [0.00, 0.00]\rangle$ |

**TABLE 2.** Linguistic scale through $XE_1$.

| | $XG_1$ | $XG_2$ | $XG_3$ | $XG_4$ |
|---|---|---|---|---|
| $XA_1$ | XVW | XW | XVT | XM |
| $XA_2$ | XM | XW | XVT | XVW |
| $XA_3$ | XM | XT | XVW | XW |
| $XA_4$ | XVT | XM | XVW | XT |
| $XA_5$ | XT | XW | XM | XVW |

**TABLE 3.** Linguistic scale through $XE_2$.

| | $XG_1$ | $XG_2$ | $XG_3$ | $XG_4$ |
|---|---|---|---|---|
| $XA_1$ | XW | XM | XVT | XVW |
| $XA_2$ | XM | XT | XVW | XW |
| $XA_3$ | XW | XVW | XVT | XM |
| $XA_4$ | XVT | XVW | XM | XVT |
| $XA_5$ | XVW | XVT | XM | XT |

increase the social practice of college students, while solidly cultivating their professional foundation, reducing the variety of professional courses. This measure actively promoted the attention of various sectors of society to the development of basic English courses. However, the development of basic English courses still has a long way to go. At the end of

**TABLE 4.** Linguistic scale through $XE_3$.

| | $XG_1$ | $XG_2$ | $XG_3$ | $XG_4$ |
|---|---|---|---|---|
| $XA_1$ | XW | XVW | XT | XM |
| $XA_2$ | XVT | XVT | XM | XW |
| $XA_3$ | XM | XT | XVW | XW |
| $XA_4$ | XW | XVW | XVT | XT |
| $XA_5$ | XM | XW | XT | XM |

**TABLE 5.** The $XR = [XR_{ij}]_{5\times 4}$.

| Alternatives | $XG_2$ | $XG_2$ |
|---|---|---|
| $XA_1$ | ([0.1524,0.4531], [0.2678,0.5961]) | ([0.1647,0.3251], [0.2780,0.3806]) |
| $XA_2$ | ([01732,0.2105], [0.2208,0.3085]) | ([0.1316,0.3036], [0.1721,0.5098]) |
| $XA_3$ | ([0.1109,0.4514], [0.3580,0.5031]) | ([0.2345,0.5016], [0.1867,0.3084]) |
| $XA_4$ | ([0.3105,0.5026], [0.2157,0.6123]) | ([0.3031,0.6953], [0.1428,0.3125]) |
| $XA_5$ | ([0.2976,03595], [0.1456,0.2297]) | ([0.2789,0.6547], [0.1106,0.3572]) |

| Alternatives | $XG_3$ | $XG_4$ |
|---|---|---|
| $XA_1$ | ([0.1367,0.3478], [0.3652,0.6194]) | ([0.2845,0.5956], [0.1767,0.3778]) |
| $XA_2$ | ([0.2846,0.5953], [0.1785,0.5909]) | ([0.3456,0.7519], [0.1484,0.4136]) |
| $XA_3$ | ([0.3115,0.6947], [0.1215,0.2835]) | ([0.1439,0.4601], [0.1812,0.3338]) |
| $XA_4$ | ([0.1165,0.7145], [0.1875,0.2654]) | ([0.1216,0.3014], [0.1886,0.3872]) |
| $XA_5$ | ([0.4126,0.6434], [0.2446,0.3246]) | ([0.2337,0.3326], [0.2875,0.5894]) |

the 20th century, the concept of increasing students' cultural literacy education began to emerge in various universities. The concept of English education abroad truly played a practical role in China's university education, and in the later stage, a systematic concept including concept, form, practice, and system was formed, effectively solving the problems of one-sided education and one-sided talent cultivation in China's education field. In the later development process, some key universities have begun to actively explore the field of basic English courses, seeking suitable content for the current economic development model and social system needs of China, and actively promoting the standardized development of basic English course education. The 13th Five Year Plan officially proposed a new educational and training mechanism that combines English education with traditional educational concepts. Since then, the wave of reform in basic English education in China has officially started, and various modes of English education dissemination have emerged and achieved good development. The importance of basic English curriculum education is evident in the process of cultivating versatile talents. Looking back at the practical process of English education in universities, college students have gradually developed from the initial "What is English education, what is the significance of conducting basic English courses, and how to effectively carry out English education" to a systematic development

**TABLE 6.** The $NXR = \left[NXR_{ij}\right]_{5\times4}$.

| Alternatives | XG2 | XG2 |
|---|---|---|
| XA1 | ([0.1524,0.4531], [0.2678,0.5961]) | ([0.1647,0.3251], [0.2780,0.3806]) |
| XA2 | ([01732,0.2105], [0.2208,0.3085]) | ([0.1316,0.3036], [0.1721,0.5098]) |
| XA3 | ([0.1109,0.4514], [0.3580,0.5031]) | ([0.2345,0.5016], [0.1867,0.3084]) |
| XA4 | ([0.3105,0.5026], [0.2157,0.6123]) | ([0.3031,0.6953], [0.1428,0.3125]) |
| XA5 | ([0.2976,03595], [0.1456,0.2297]) | ([0.2789,0.6547], [0.1106,0.3572]) |

| Alternatives | XG3 | XG4 |
|---|---|---|
| XA1 | ([0.1367,0.3478], [0.3652,0.6194]) | ([0.2845,0.5956], [0.1767,0.3778]) |
| XA2 | ([0.2846,0.5953], [0.1785,0.5909]) | ([0.3456,0.7519], [0.1484,0.4136]) |
| XA3 | ([0.3115,0.6947], [0.1215,0.2835]) | ([0.1439,0.4601], [0.1812,0.3338]) |
| XA4 | ([0.1165,0.7145], [0.1875,0.2654]) | ([0.1216,0.3014], [0.1886,0.3872]) |
| XA5 | ([0.4126,0.6434], [0.2446,0.3246]) | ([0.2337,0.3326], [0.2875,0.5894]) |

**TABLE 7.** The attribute weight information.

| | XG1 | XG2 | XG3 | XG4 |
|---|---|---|---|---|
| *vw* | 0.2760 | 0.2959 | 0.1616 | 0.2665 |

**TABLE 8.** The relative weight information.

| | XG1 | XG2 | XG3 | XG4 |
|---|---|---|---|---|
| *rxw* | 0.9327 | 1.0000 | 0.5461 | 0.9006 |

model that combines ideas with practice, teachers and students work together, and students consciously learn. At present, the reform of basic English curriculum education has made preliminary achievements, but it is still far from enough in terms of the national talent demand and the current situation of talent cultivation. In the future, under the promotion of smart classroom theory, there is still a long way to go for the reform of basic English curriculum education. The smart classroom teaching evaluation of basic English is MAGDM issue. Therefore, the smart classroom teaching evaluation of basic English is presented to demonstrate the approach developed in this paper. There is a panel with five English colleges $XA_i$ $(i = 1, 2, 3, 4, 5)$ to choose. The experts select four attributes to assess the five English colleges: $XG_1$ is teachers' teaching ability; ②$XG_2$ is teaching cost; ③$XG_3$ is teaching content; ④$XG_4$ is subject knowledge. The $XG_2$ is cost attribute and others attributes are beneficial. The five possible English colleges $XA_i$ $(i = 1, 2, 3, 4, 5)$ are to be evaluated with IVPFNs with the four criteria by three experts $XE_t$ $(t = 1, 2, 3)$ (Suppose expert's weight is $(0.2595, 0.3897, 0.3508)$.

The IVPF-ExpTODIM-MACONT technique is used to solve the smart classroom teaching evaluation of basic English.

**Step 1.** Construct the $XR = \left[XR_{ij}^t\right]_{5\times4}$ through linguistic scale (See Table 2-4).

Then according to IVPFWA technique, the $XR = \left[XR_{ij}\right]_{5\times4}$ is obtained (See Table 5).

**TABLE 9.** The $DDRV = \left(DDRV_{ij}\right)_{5\times 4}$.

| | $XG_1$ | $XG_2$ | $XG_3$ | $XG_4$ |
|---|---|---|---|---|
| $XA_1$ | 3.4572 | -4.3251 | 1.7211 | -3.7494 |
| $XA_2$ | -2.0610 | 3.4310 | -5.9212 | 2.8724 |
| $XA_3$ | -2.4566 | -0.2111 | -0.6050 | 3.9416 |
| $XA_4$ | 1.4786 | 0.5687 | -1.5765 | 2.0549 |
| $XA_5$ | -1.9779 | -2.6519 | -3.0593 | -5.1291 |

**TABLE 10.** The $NDDRV_{ij}^{(1)}$.

| | $XG_1$ | $XG_2$ | $XG_3$ | $XG_4$ |
|---|---|---|---|---|
| $XA_1$ | 1.0000 | 0.0000 | 1.0000 | 0.0000 |
| $XA_2$ | 0.0669 | 1.0000 | 0.0000 | 0.9075 |
| $XA_3$ | 0.0000 | 0.5304 | 0.7905 | 1.0000 |
| $XA_4$ | 0.6859 | 0.6000 | 0.6979 | 0.6766 |
| $XA_5$ | 0.0535 | 0.3601 | 0.4229 | 0.1077 |

**TABLE 11.** The $NDDRV_{ij}^{(2)}$.

| | $XG_1$ | $XG_2$ | $XG_3$ | $XG_4$ |
|---|---|---|---|---|
| $XA_1$ | 0.7389 | 0.0003 | 0.6827 | 0.0087 |
| $XA_2$ | 0.0181 | 0.9349 | 0.0015 | 0.3435 |
| $XA_3$ | 0.0139 | 0.0216 | 0.1890 | 0.4992 |
| $XA_4$ | 0.2120 | 0.0377 | 0.1070 | 0.1350 |
| $XA_5$ | 0.0172 | 0.0055 | 0.0198 | 0.0135 |

**TABLE 12.** The $NDDRV_{ij}^{(3)}$.

| | $XG_1$ | $XG_2$ | $XG_3$ | $XG_4$ |
|---|---|---|---|---|
| $XA_1$ | 1.0000 | 0.0003 | 1.0000 | 0.0175 |
| $XA_2$ | 0.0245 | 1.0000 | 0.0022 | 0.6881 |
| $XA_3$ | 0.0188 | 0.0231 | 0.2768 | 1.0000 |
| $XA_4$ | 0.2868 | 0.0404 | 0.1568 | 0.2704 |
| $XA_5$ | 0.0232 | 0.0059 | 0.0290 | 0.0271 |

**TABLE 13.** The $NDDRV_{ij}$.

| | $XG_1$ | $XG_2$ | $XG_3$ | $XG_4$ |
|---|---|---|---|---|
| $XA_1$ | 0.9130 | 0.0002 | 0.8942 | 0.0088 |
| $XA_2$ | 0.0365 | 0.9783 | 0.0012 | 0.6464 |
| $XA_3$ | 0.0109 | 0.1917 | 0.4188 | 0.8331 |
| $XA_4$ | 0.3949 | 0.2260 | 0.3206 | 0.3607 |
| $XA_5$ | 0.0313 | 0.1238 | 0.1572 | 0.0494 |

**TABLE 14.** The $SCV_i^{(1)}$ and $SCV_i^{(2)}$.

| | $XQ_i^{(1)}$ | $XQ_i^{(2)}$ | $SCV_i^{(1)}$ | $SCV_i^{(2)}$ |
|---|---|---|---|---|
| $XA_1$ | -0.2364 | 0.8457 | -0.0687 | -0.0112 |
| $XA_2$ | -0.0089 | 0.8516 | 0.2115 | 0.0109 |
| $XA_3$ | 0.0800 | 0.1452 | 0.1290 | 0.0185 |
| $XA_4$ | 0.3177 | 1.1815 | 0.7012 | 0.0825 |
| $XA_5$ | -0.0683 | 0.8203 | 0.1303 | 0.0048 |

**Step 2.** Normalize the $XR = [XR_{ij}]_{5\times4}$ into $NXR = [NXR_{ij}]_{5\times4}$ (See Table 6).

**Step 3.** Implement the weight values (See Table 7):

**Step 4.** Produce the relative weight (See Table 8):

**Step 5.** Implement the $DDRV = (DDRV_{ij})_{5\times4}$ (See table 9):

**TABLE 15.** The FCV.

| Alternative | FCV | Order |
|---|---|---|
| $XA_1$ | -0.0994 | 5 |
| $XA_2$ | 0.1690 | 3 |
| $XA_3$ | 0.1719 | 2 |
| $XA_4$ | 0.8296 | 1 |
| $XA_5$ | 0.0930 | 4 |

**Step 6. Standardize DDRV matrix through using three normalization techniques**(See table 10-12)**:**

**Step 7. Integrating three normalization techniques** (See table 13)**:**

**TABLE 16.** Order of these different techniques.

| Methods | Order |
|---|---|
| IVPFWA technique[48] | $XA_4 \succ XA_3 \succ XA_2 \succ XA_5 \succ XA_1$ |
| IVPFWG technique[48] | $XA_4 \succ XA_3 \succ XA_5 \succ XA_2 \succ XA_1$ |
| WIVPFPA technique [56] | $XA_4 \succ XA_3 \succ XA_2 \succ XA_5 \succ XA_1$ |
| WIVPFPG technique [56] | $XA_4 \succ XA_3 \succ XA_5 \succ XA_2 \succ XA_1$ |
| IVPFAAWA technique [55] | $XA_4 \succ XA_3 \succ XA_2 \succ XA_5 \succ XA_1$ |
| IVPF-WASPAS technique [57] | $XA_4 \succ XA_3 \succ XA_2 \succ XA_5 \succ XA_1$ |
| IVPF-QUALIFLEX technique [58] | $XA_4 \succ XA_3 \succ XA_2 \succ XA_5 \succ XA_1$ |
| IVPF-TOPSIS technique [59] | $XA_4 \succ XA_3 \succ XA_2 \succ XA_5 \succ XA_1$ |
| IVPF-TODIM technique [60] | $XA_4 \succ XA_3 \succ XA_2 \succ XA_5 \succ XA_1$ |
| IVPF-ExpTODIM-MACONT technique | $XA_4 \succ XA_3 \succ XA_2 \succ XA_5 \succ XA_1$ |

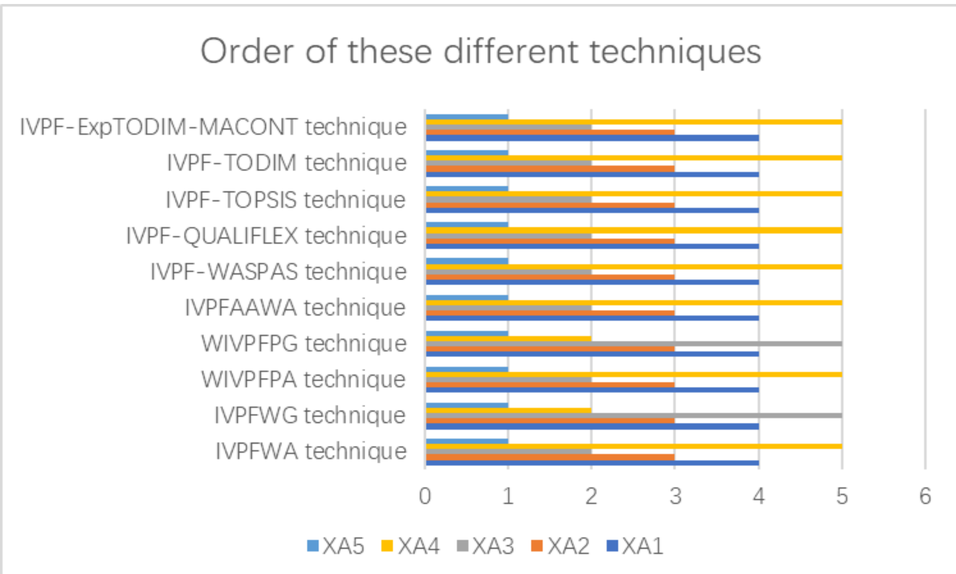


**FIGURE 2.** Order of these different techniques.

**Step 8. Manage the SCV for each alternative** (See table 14)**:**

**Step 9. Manage the FCV for each alternative** (See table 15)**:**

**Step 10.** Finally, the order is obtained: $XA_4 \succ XA_3 \succ XA_2 \succ XA_5 \succ XA_1$, and thus the optimal English college is $XA_4$.

### B. COMPARATIVE ANALYSIS

Then, the IVPF-ExpTODIM-MACONT technique is compared with IVPFWA technique [48], IVPFWG technique [48], interval-valued Pythagorean fuzzy Aczel-Alsina weighted average (IVPFAAWA) technique [55], weighted interval-valued Pythagorean fuzzy power average (WIVPFPA) technique [56], weighted interval-valued Pythagorean fuzzy power geometric (WIVPFPG) technique [56], interval-valued Pythagorean fuzzy WASPAS (IVPF-WASPAS) technique [57] and interval-valued Pythagorean fuzzy QUALIFLEX (IVPF-QUALIFLEX) technique [58], interval-valued Pythagorean fuzzy TOPSIS (IVPF-TOPSIS) technique [59] and IVPF-TODIM technique [60]. The comparative decision results are shown in Table 16 and Figure 2.

In accordance with WS coefficients [61], [62], the WS coefficient calculation between IVPFWA technique [48], IVPFWG technique [48], IVPFAAWA technique [55], WIVPFPA technique [56], WIVPFPG technique [56], IVPF-WASPAS technique [57], IVPF-QUALIFLEX technique [58], IVPF-TOPSIS technique [59], IVPF-TODIM technique [60] and the proposed IVPF-ExpTODIM-MACONT technique is 1.0000, 0.7266, 1.0000, 0.7266, 1.0000, 1.0000, 1.0000, 1.0000, 1.0000, respectively. The WS coefficient calculation shows that the order result of IVPFWA technique [48], IVPFAAWA technique [55], WIVPFPA technique [56], IVPF-WASPAS technique [57], IVPF-QUALIFLEX technique [58], IVPF-TOPSIS technique [59] and IVPF-TODIM technique [60] is same with the order result of the proposed IVPF-ExpTODIM-MACONT technique. The WS coefficient calculation shows that the order result of IVPFWG technique [48] and WIVPFPG technique [56] is slight different from the order result of the proposed IVPF-ExpTODIM-MACONT technique. However, these techniques have the same optimal English college and worst English college. This shows the IVPF-ExpTODIM-MACONT technique is reasonable and effective.

## V. CONCLUSION

The smart classroom teaching evaluation of basic English (IP) perspective is the MAGDM. Recently, the Exponential TODIM (ExpTODIM) technique and MACONT technique has been used to cope with MAGDM issues. The IVPFSs are used as a technique for characterizing uncertain information during the smart classroom teaching evaluation of basic English. In this manuscript, the IVPF-ExpTODIM-MACONT technique is built to solve the MAGDM under IVPFSs. In the end, a numerical case study for smart classroom teaching evaluation of basic English is implemented to validate the proposed technique. The main contributions of this paper are outlined: (1) the ExpTODIM and MACONT technique has been extended to IVPFSs; (2) Information Entropy technique is employed to manage the weight values under IVPFSs. (3) the IVPF-ExpTODIM-MACONT technique is founded to implement the MAGDM under IVPFSs; (4) a numerical case study for smart classroom teaching evaluation of basic English and some comparative analysis is supplied to verify the IVPF-ExpTODIM-MACONT technique.

There may be some possible limitations of this work, which could be further managed the smart classroom teaching evaluation of basic English: (1) It is a worthwhile research work to manage consensus [63], [64], [65] to smart classroom teaching evaluation of basic English under IVPFSs; (2) It is also worthwhile research work to manage regret theory to the smart classroom teaching evaluation of basic English under IVPFSs [66], [67], [68]; (3) In subsequent research works, the integration of TODIM techniques with other decision-making techniques employing different fuzzy extensions could be managed for smart classroom teaching evaluation of basic English [69], [70].

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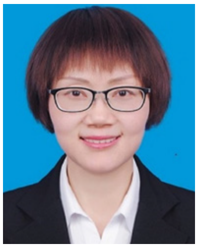

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