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RESEARCH ARTICLE

Magnetic Field Analysis of Electric Current in Coplanar Coils Using Equivalent Permanent Magnetization and Magnetic Scalar Potential

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ABSTRACT Use of magnetic scalar potential instead of magnetic vector potential can dramatically reduce the computational cost for 3D magnetic field analysis, but for most methods, the use of scalar potential was limited to current-free conditions. However, for coplanar coils characterized by homogeneous current density, the current density can be transformed into an equivalent permanent magnetization to satisfy the current-free condition. By utilizing this equivalent permanent magnetization as a source, a variational formulation for the magnetic scalar potential can be derived, and finite element analysis becomes feasible for complex systems that analytical method cannot be used. However, the equation for the equivalent permanent magnetization was previously presented only for rectangular coils, and its detailed derivation process, variational scalar potential formulation, and application method for finite element analysis were not introduced, limiting the practicality of the method. To generalize the method to practical applications, this study introduces a generalized equation for equivalent permanent magnetization applicable to arbitrary cross-sectional coplanar coils, its variational formulation using the magnetic scalar potential, and a method for its application in finite element analysis. The results of the magnetic field analysis using the scalar potential formulation were compared with those of the traditional vector potential formulation to demonstrate the feasibility and usefulness of the formulation.

INDEX TERMS Arbitrary shape, magnetic charge, magnetic field, coplanar winding, permanent magnetization.

I. INTRODUCTION

The utilization of magnetic scalar potential has been proven as a cost-effective approach for analyzing the magnetic field of a magnetostatic system [1], and various formulations were developed to utilize magnetic scalar potential. Namely, there are reduced magnetic scalar potential, total magnetic scalar potential, mixed scalar potential, and combined magnetic vector and scalar potential formulations [2], [3].

Among the formulations, total magnetic scalar potential provides the most accurate and fast computations; the reduced scalar potential is less accurate in regions with high permeability [2] and requires pre-computation of field T_0 [2], [3], [4]; the magnetic vector potential has a high computational cost. However, in scenarios involving steady current sources, the comprehensive use of total scalar potential is impossible. This is because the formulation of total magnetic scalar potential is based on assumptions of current-free conditions [2], [3], [5].

Consequently, various combined methods were introduced to utilize the magnetic scalar potential in more general cases. The combined methods partition the field analysis region into distinct areas, distinguishing between current-free and current-source regions, enabling the combined use of magnetic scalar potential and magnetic vector potential, respectively [6]. To ensure the consistency of the solution between the regions, boundary conditions or cut surfaces with potential jumps are introduced [1], [2], [3], [4], [5], [6],

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[7]. The most recent method utilizes cut surfaces assigned with potential jumps whose value is equal to the fraction of the total current; then, magnetic fields in simply connected regions enclosed by currents become a single-valued function. Thus, with adequate choice of cut surfaces, computation of current-free regions using total magnetic scalar potential formulation is possible, and this results in significant reduction in computational cost [1], [7].

Nonetheless, by employing equivalent permanent magnetization for the current, it becomes feasible to conduct a comprehensive total magnetic scalar potential field analysis for the entire region by eliminating the current-source region. While the equation for equivalent permanent magnetization of a coplanar rectangular coil has been established [8], [9], a generalized equation for arbitrary cross-sectional coils, its detailed derivation process, magnetic scalar potential formulation, variational formulation, and the methodology for its implementation in finite element method (FEM), remain unaddressed.

This study aims to provide a field analysis method applicable to arbitrary cross-sectional coplanar coils in the absence of magnetic materials. First, a generalized equation for equivalent permanent magnetization is derived with a detailed description of its premises and scope of application. This equation serves as the foundation for developing a total magnetic scalar formulation. Secondly, a variational formulation that has the equivalent permanent magnetization as a source is derived. This formulation is the fundamental equation used in FEM, and being able to directly use equivalent permanent magnetization instead of pre-calculated magnetic charge improves the feasibility of the method in complex geometries where analytical pre-calculation is impossible. Lastly, we present a straightforward method for its implementation in finite element analysis (FEA), benefiting from its alignment with electrostatic formulations, which simplifies the process.

Then, numerical examples are presented to demonstrate the feasibility and usefulness of the proposed method. Specifically, by employing FEM, we performed magnetic field analyses of three-dimensional (3D) coplanar coils using both magnetic scalar potential and magnetic vector potential. The results include a comparison of relative errors and computational costs between the two approaches.

II. DEFINITION OF THE PROBLEM

The problem under consideration pertains to the magnetic scalar potential field analysis method for closed-loop of uniform cross-sectional coplanar coils in air. Previous works on this subject were limited to rectangular cross-sectional coils. For practical uses, expansion of the method to arbitrary cross-sectional coils is required. To do so, three stages of the method should be developed.

First, the generalized equation for permanent magnetization of 3D arbitrary coils needs to be derived with clarification on the scope of its possible application. The derivation process of equivalent permanent magnetization in 2D rectangular coils is limited to a pair of antisymmetric current sources, each perpendicular to the cross-sectional plane. This prerequisite strictly limits the equation's transformation from 2D to 3D where no geometries meet such a requirement. Thus, the transformation process for 2D to 3D geometries needs to be clarified. Moreover, the derivation of equivalent permanent magnetization in rectangular coils assumes equal current segment heights. For arbitrary coils, such an assumption is not met, and the derivation process for such a scenario also needs to be introduced. These will be presented in sections III and IV.

Second, the derivation of the magnetic scalar potential state equation using the equivalent permanent magnetization and its variational formulation needs to be derived. The variational formulation is a fundamental equation used to calculate the magnetic scalar potential and magnetic field for a given system. Especially for arbitrary cross-sectional coils, the distribution of magnetic charge is complex and cannot be calculated analytically. In such cases, numerical analysis is mandatory, and being able to use the equivalent permanent magnetization directly as a source adds feasibility to the method. These will be presented in section V.

Third, a detailed and feasible application method for the formulation needs to be introduced. In arbitrary 3D cross-sectional coils, analytical methods cannot be used to calculate the equivalent magnetization or magnetic charge. The mandatory process for numerical calculation of the equivalent permanent magnetization is introduced by defining the reference wall and using the wall distance function, as shown in section IV. Then, a separate PDE module is needed to employ variational formulation and conduct numerical analysis. However, the formulation's similarity with that of electrostatics allows the use of a commercial electrostatics module to calculate the magnetic scalar potential with simple substitution of sources and conversion of units, as introduced in section V.

III. EQUIVALENT PERMANENT MAGNETIZATION OF 2D/2D-AXISYMMETRIC ARBITRARY CROSS-SECTIONAL COILS

It is widely known that the magnetic field of permanent magnets can be computed using the magnetic charge or surface current density. Conversely, the surface current density can also be modeled using permanent magnetization. Therefore, if the distribution of current can be represented by the superposition of segments of surface current density, then a rectangular coil can be represented by equivalent permanent magnetization [8]. However, for arbitrary cross-sectional coils with varying heights of current segments, the current must be further subdivided into infinitesimal volume segments, each characterized by its own volume current density.

A. Equivalent Magnetic Charge and Surface Current Density of Permanent Magnetization

The governing equation for the magnetostatic system with a current source is:

$$\nabla \times \nabla \times \mathbf{A} = \mu_0 \mathbf{J}_s, \tag{1}$$

where $\mathbf{J_s}$ and $\mathbf{A}$ are source-current density and magnetic vector potential, respectively.

For permanent magnets, the source can be modeled using the equivalent magnetization current density. The governing equations are as follows:

$$\nabla \times \nabla \times \mathbf{A} = \mu_0 \mathbf{J}_m, \tag{2}$$

$$\mathbf{J}_m = \nabla \times \mathbf{M_p}, \tag{3}$$

where $\mathbf{J_m}$ and $\mathbf{M_p}$ are the magnetization current density and permanent magnetization, respectively.

When $\mathbf{M}_\mathrm{p}$ is uniform, only magnetization surface current density, defined as

$$\mathbf{K}_m = \int \mathbf{J_m} dL = \mathbf{J_m} L, \tag{4}$$

where L is the length of a magnet, appears at the boundaries of the permanent magnet (as depicted in Fig. 1(b)) and can be calculated as:

$$\mathbf{K}_m = \mathbf{M_p} \times \mathbf{n}. \tag{5}$$

From this equation, it is evident that a pair of antisymmetric, normal to permanent magnetization vector, surface current density replaces homogeneous permanent magnetization.

Permanent magnetization can also be modeled using the magnetic charge density, defined as:

$$\rho_m = -\nabla \cdot \mathbf{M_p}, \tag{6}$$

where ρ_m is magnetic charge.

When $\mathbf{M}_\mathrm{p}$ is uniform, only the surface magnetic charge density appears at the boundaries (as shown in Fig. 1(c) and has the value:

$$\sigma_m = \mathbf{M_p} \cdot \mathbf{n}. \tag{7}$$

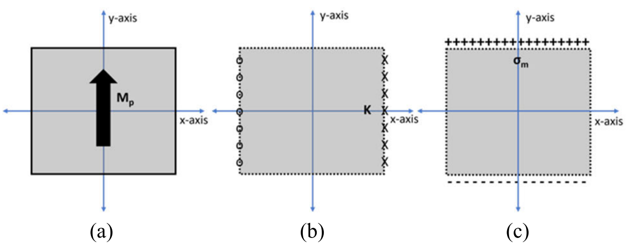


FIGURE 1. (a) Uniform permanent magnetization, (b) equivalent surface current density, and (c) equivalent magnetic charge.

From (1) and (2), it is evident that both $\mathbf{J_s}$ and $\mathbf{J_m}$ act as sources of $\mathbf{A}$, and substituting $\mathbf{J_s}$ with the equivalent $\mathbf{J_m}$ results in the same vector potential $\mathbf{A}$. Therefore, by determining the equivalent permanent magnetization $\mathbf{M}_\mathrm{p}$ or the equivalent magnetic charge density ρ_m for $\mathbf{J_m}$, it becomes possible to compute the magnetic field using the magnetic scalar potential.

B. Derivation of Equivalent Permanent Magnetization

To determine the equivalent magnetization for the rectangular coil, the volume current density region was separated into fragments of sheet currents, each having a length of L, width w, and surface current density $\mathbf{K}$. Then, these layers of equivalent magnetization from each current sheet were superposed.

For the rectangular coils, the magnetization increases linearly to $\mathbf{M_{max}}$ towards the center, as depicted in Fig. 2(b). In the equivalent model, only surface magnetic charge density σ_m is present, as shown in Fig. 3(c) [8].

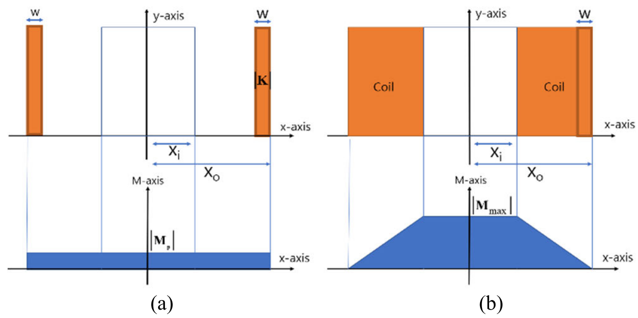


FIGURE 2. Equivalent magnetization of (a) sheet current and (b) volume current.

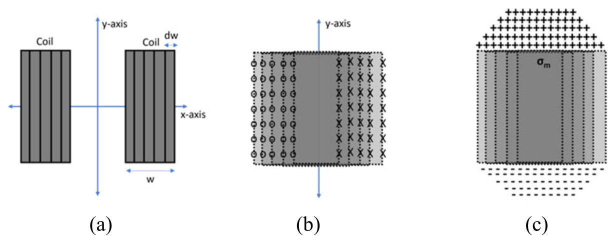


FIGURE 3. (a) Decomposition of volume current to sheet current segments, (b) accumulation of sheet current density, and (c) accumulation of surface magnetic charge.

However, for arbitrary cross-sectional coils with varying heights of current segments, the volume current density region must be further separated into fragments of an infinitesimal area of width dw and length dL, as shown in Fig. 4.

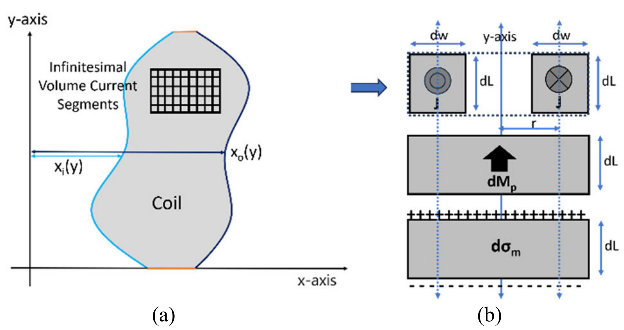


FIGURE 4. (a) Segmentation of 2D arbitrary coil model and (b) a pair of antisymmetric infinitesimal volume current density–equivalent permanent magnetization and magnetic charge density.

Subsequently, the equivalent permanent magnetization for each infinitesimal area must be defined and superposed. Since there is a difference in the magnitude of the magnetization in the y-direction, volume magnetic charge density ρ_m appears.

To define the equivalent permanent magnetization for an infinitesimal area of the volume current density **J**, the volume current density can be expressed using the surface current density **K** of infinitesimal length dL and width w:

$$\mathbf{J} = \frac{\mathbf{K}}{w} \tag{8}$$

The surface current density was defined to have a finite length L of a magnet, but we can assume a magnet segment of infinitesimal length dL to determine the equivalent infinitesimal surface current density and magnetization. This mathematical model aligns with the physical model and assumptions of continuum mechanics, where an object can be regarded as a collection of infinitesimal volume elements [11].

Given that **J** is constant, as $w \rightarrow dw$,

$$|\mathbf{K}| \rightarrow |d\mathbf{K}| \tag{9}$$

$$|\mathbf{M_p}| \rightarrow |d\mathbf{M_p}| \tag{10}$$

$$|d\mathbf{M_p}| = |d\mathbf{K}| = |\mathbf{J}|\, dw \tag{11}$$

Finally, the equation for permanent magnetization can be obtained by integrating the infinitesimal permanent magnetization of each horizontal segment at a given height y:

$$|\mathbf{M_p}| = \int |d\mathbf{M_p}| = \int |\mathbf{J}| dw = \mathbf{J}w, \tag{12}$$

where $w(x,y)$ represents the width of the volume-current density segment measured from the outer coil boundary. Thus, w can be expressed as:

$$\begin{aligned} w(x, y) &= (x_o(y) - x),\ for\ x > x_i \\ w(x, y) &= (x_o(y) - x_i(y)),\ for\ x < x_i \end{aligned} \tag{13}$$

Here, $x_o(y)$ and $x_i(y)$ represent the distances from the antisymmetric plane to the outer and inner coil boundaries, respectively.

Thus, for a 2D arbitrary cross-sectional coil, the equation for the magnetization is:

$$\mathbf{M_p}(x, y) = |\mathbf{J}|\,(x_o(y) - x) \cdot \mathbf{a}_y\,,\ \ for\ \ x\ > x_i(y), \tag{14}$$

$$\mathbf{M_p}(x, y) = |\mathbf{J}|\,(x_o(y) - x_i(y)) \cdot \mathbf{a}_y\,,\ \ for\ \ x\ < x_i(y), \tag{15}$$

where $\mathbf{a_y}$ denotes the unit vector. The magnetic charge density is expressed as follows:

$$\rho_m = -\nabla \cdot \mathbf{M_p} = -|\mathbf{J}|\,(\frac{dx_o(y)}{dy} - \frac{dx_i(y)}{dy}) \tag{16}$$

$$\sigma_m = \mathbf{M_p} \cdot \mathbf{n} \tag{17}$$

For an arbitrary 2D-axisymmetric model, magnetization is a function of r and z:

$$\mathbf{M_p}(r, z) = |\mathbf{J}|\,(r_o(z) - r) \cdot \mathbf{a}_z\,,\ \ for\ \ r\ > r_i(z), \tag{18}$$

$$\mathbf{M_p}(r, z) = |\mathbf{J}|\,(r_o(z) - r_i(z)) \cdot \mathbf{a}_z\,,\ \ for\ \ r\ < r_i(z), \tag{19}$$

where $\mathbf{a_r}$ is a unit vector. The equation is identical to that of the 2D model, with a change of variables to a cylindrical coordinate system.

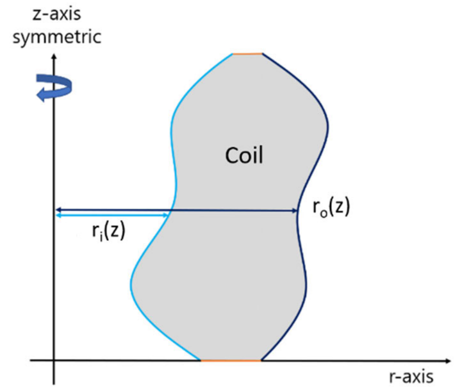


FIGURE 5. 2D-axisymmetric arbitrary coil model.

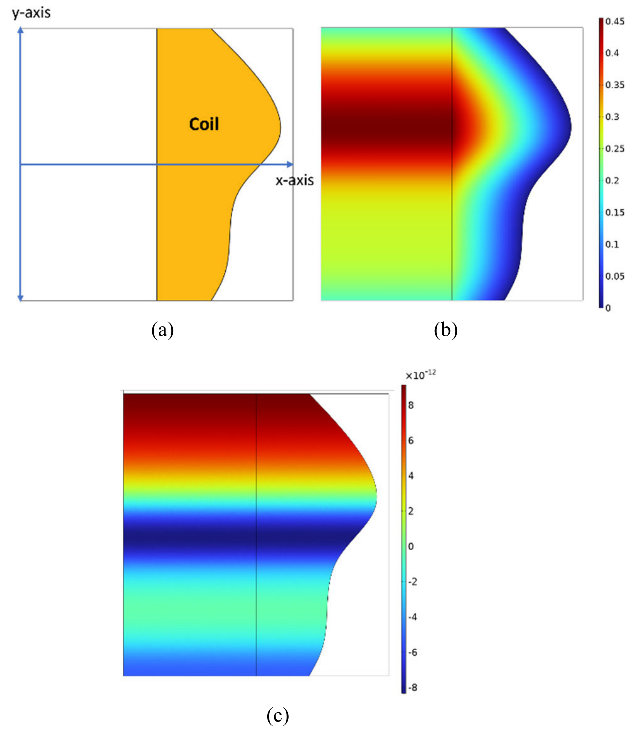


FIGURE 6. (a) 2D-arbitrary coil model, (b) permanent magnetization, and (c) magnetic charge density.

For a given coil shown in Fig. 6(a), the equivalent permanent magnetization and magnetic charge density are depicted. The magnitude of the permanent magnetization depends on the coil width, as indicated in (13), and the magnetic charge density depends on the slopes of the outer and inner coils, as presented in (16):

IV. EQUIVALENT PERMANENT MAGNETIZATION OF 3D ARBITRARY CROSS-SECTIONAL COILS

As depicted in Fig. 6, a 3D coplanar coil was constructed by sweeping the 2D cross-sectional geometry along the coplanar curve. Thus, it might seem possible to superpose layers of magnetization for the antisymmetric infinitesimal region of volume current density, as implemented in the 2D and 2D-axisymmetric cases. However, for nonsymmetric coils, it is impossible to identify the antisymmetric pair of the volume current density. Therefore, the superposition method is insufficient for fully deriving the equivalent magnetization in the 3D case.

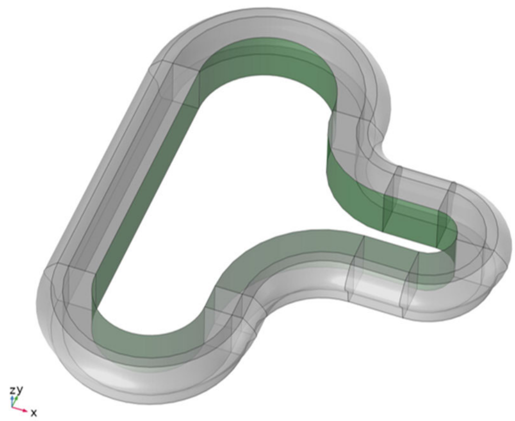


FIGURE 7. 3D-Axisymmetirc arbitrary coil model.

To solve this problem, Ampere's law for the magnetization current density can be utilized. Using Stokes' theorem, for any closed loop, (3) can be expressed as:

$$\iint \mathbf{J_m} \cdot \mathbf{ds} = \oint \mathbf{M_p} \cdot \mathbf{dl} \tag{20}$$

For coplanar coils, by taking this Ampere loop to contain an identical cross-sectional area of volume current density $\mathbf{J}$, we can demonstrate that permanent magnetization reaches its maximum value and remains constant for the inner domain surrounded by the coil, as indicated in Fig. 8. Consequently, for a closed loop of a coplanar coil, the assumption of an antisymmetric pair of current densities $\mathbf{J}$ can be considered negligible.

Therefore, for an arbitrary coil, the permanent magnetization can be calculated similarly to the 2D case. The equation for the permanent magnetization is equivalent to (12), where the coil width $w(x, y, z)$ represents the width of the volume current density segment measured from the outer coil boundary for a given coplanar height z:

$$\begin{aligned} w_{coil}(x, y, z) &= (d_o(x, y, z) - d(x, y, z)), \; for \; d > d_i \\ w_{inner}(z) &= (d_o(z) - d_i(z)), \; for \; d < d_i \end{aligned} \tag{21}$$

Here, w_{coil} and w_{inner} represent the widths inside the coil and inner regions, respectively, and d represents the distance measured from the reference axis, as shown in Fig. 8. The reference axis can be drawn with the coil cross-section and then swept together to create a reference wall. The distance from the reference axis was then obtained using the wall distance function. The reference axis can be any z-axis within the cross-sectional plane. However, for simplicity, it can be set to the leftmost point of the coil geometry.

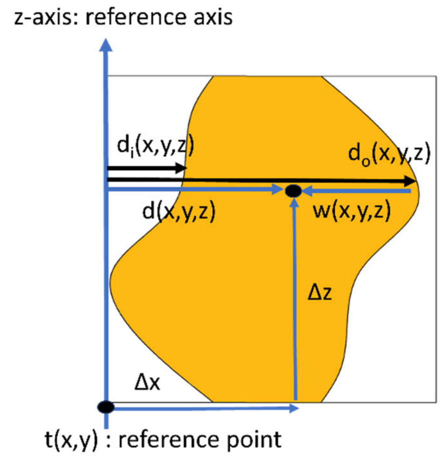


FIGURE 8. Reference axis, inner coil distance d_i, outer coil distance d_o, and distance from reference axis d, and coil width w.

The permanent magnetization distributions for the rectangular and arbitrary coils determined by the wall distance function are depicted in Fig. 9. For the rectangular coil, the equivalent permanent magnetization remains constant within the inner domain as in Fig.9(a), while for the arbitrary coil, it varies with height z, as illustrated in Fig. 9(b)(c).

V. WEAK FORMULATION OF THE EQUIVALENT MODEL USING THE MAGNETIC SCALAR POTENTIAL AND APPLICATION TO FEM

A. GOVERNING EQUATION AND WEAK FORMULATION OF THE EQUIVALENT PERMANENT MAGNETIZATION IN AIR

The governing equation for the equivalent permanent magnetization source in air can be derived from Maxwell's and the constitutive equations. Notably, the constitutive equation does not include a magnetization term because there is no magnetic material with relative permeability in the problem. It is essential to avoid simply adding a magnetization term to the problem, as this does not yield accurate field analysis results in the presence of magnetic materials. However, addressing such a problem is beyond the scope of this study.

1) GOVERNING EQUATION OF THE EQUIVALENT PERMANENT MAGNETIZATION MODEL

The governing equation was obtained using Maxwell's equations and the constitutive equation given as:

$$\nabla \cdot \mathbf{B} = 0 \tag{22}$$

$$\nabla \times \mathbf{H} = 0 \tag{23}$$

$$\mathbf{B} = \mu_0 \mathbf{H_m} + \mu_0 \mathbf{M_p}, \tag{24}$$

where B, $\mathrm{H_m}$, $\mathrm{M_p}$ are magnetic flux density, magnetic field intensity, and permanent magnetization, respectively.

From (23), the magnetic scalar potential is defined as:

$$\mathbf{H_m} = -\nabla \phi_m \tag{25}$$

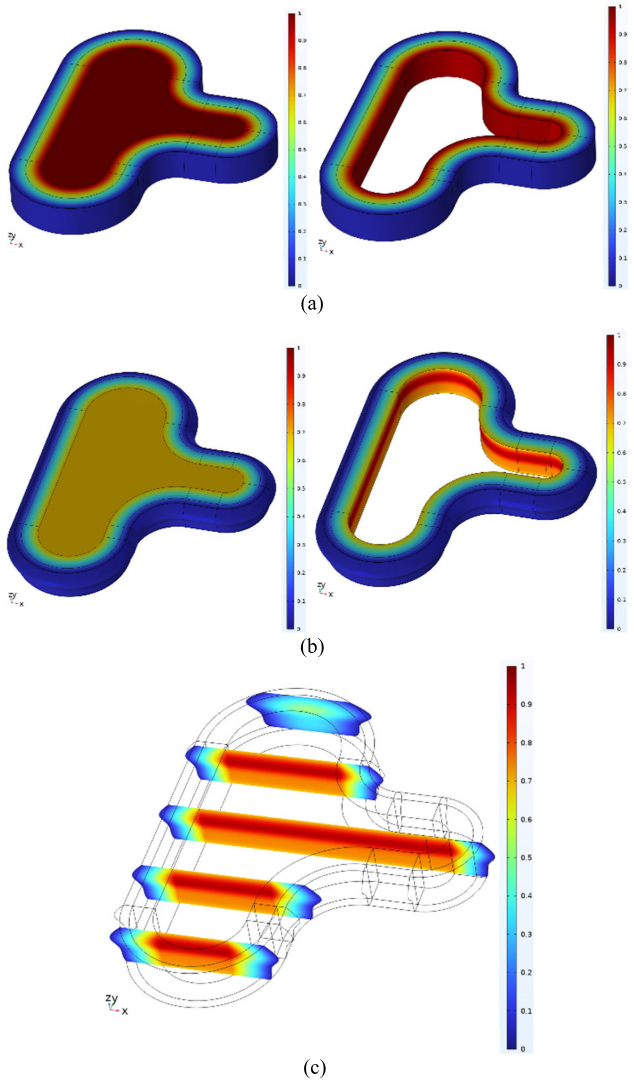


FIGURE 9. Equivalent permanent magnetization of (a) rectangular coil, (b) arbitrary cross-sectional coil, and (c) arbitrary coil cross-sectional view.

Taking divergence on both sides, (24) is rewritten as:

$$\nabla \cdot \mathbf{B} = \mu_0(\nabla \cdot \mathbf{H_m} + \nabla \cdot \mathbf{M_p}) \tag{26}$$

Substituting (22) and (25), (26) can be rewritten as:

$$0 = \mu_0(-\nabla^2\phi_m + \nabla \cdot \mathbf{M_p}) \tag{27}$$

Dividing μ_0 on both sides of (27), the governing equation is obtained as:

$$\nabla^2\phi_m = \nabla \cdot \mathbf{M_p} \tag{28}$$

In magnetic charge form, (28) is rewritten as:

$$\nabla^2\phi_m = -\rho_m \tag{29}$$

2) WEAK FORMULATION OF THE EQUIVALENT PERMANENT MAGNETIZATION MODEL

The derivation process for the variational formulation is similar to that used in other electromagnetic systems [12]. The governing equation (28) can be reduced to a variational state equation by multiplying both sides by an arbitrary virtual potential $\bar{\phi}$, as follows:

$$\int -\nabla \cdot (\nabla\phi_m - \mathbf{M_p})\,\bar{\phi}\,d\Omega = 0 \tag{30}$$

By employing the vector identity $\nabla \cdot (\nabla\psi\bar{\psi}) = (\nabla \cdot \nabla\psi)\bar{\psi} + \nabla\psi \cdot \nabla\bar{\psi}$, (30) can be rewritten as:

$$\int [(\nabla\phi_m - \mathbf{M_p})\,\nabla\bar{\phi} - \nabla \cdot \{(\nabla\phi_m - \mathbf{M_p})\bar{\phi}\}]d\Omega = 0 \tag{31}$$

Using the divergence theorem, (31) can be revised as:

$$\int (\nabla\phi_m - \mathbf{M_p})\,\nabla\bar{\phi}d\Omega = \iint (\nabla\phi_m - \mathbf{M_p}) \cdot \mathbf{n}\bar{\phi}d\Gamma, \tag{32}$$

$$\int (\nabla\phi_m - \mathbf{M_p})\,\nabla\bar{\phi}d\Omega = -\iint \frac{1}{\mu_0}(\mathbf{B_n}) \cdot \mathbf{n}\bar{\phi}d\Gamma, \tag{33}$$

where

$$\mathbf{B_n}(\phi_m) = (-\mu_0\nabla\phi_m + \mu_0\mathbf{M_p}) \cdot \mathbf{n} = \mu_0(-\frac{\partial\phi_m}{\partial n} + \mathbf{M_p} \cdot \mathbf{n}).$$

For the variational equation, the Dirichlet and homogeneous Neumann boundary conditions can be revised as:

$$\bar{\phi} = 0_{\mathrm{on}}\Gamma^1 \tag{34}$$

$$\mathbf{B}_n(\phi_m) = 0on\Gamma^1 \tag{35}$$

By imposing boundary conditions (34) and (35), the right-hand side of (33) becomes zero, resulting in the variational state equation:

$$\int \nabla\phi_m \cdot \nabla\bar{\phi}d\Omega \;= \int \mathbf{M_p} \cdot \nabla\bar{\phi}d\Omega \tag{36}$$

B. COMPARISON WITH ELECTROSTATICS

The governing equation and the variational formulation of the electrostatics are derived as:

$$\nabla^2\phi = \frac{-\rho + \nabla \cdot \mathbf{P_p}}{\varepsilon_0}, \tag{37}$$

$$\int \varepsilon_0\nabla\phi \cdot \nabla\bar{\phi}d\Omega = \int (\rho\phi + \mathbf{P_p} \cdot \nabla\bar{\phi})d\Omega, \tag{38}$$

where ρ is charge density and $\mathbf{P_p}$ is permanent polarization [12]. In (28) and (37), the divergence of permanent polarization and permanent magnetization are both sources of scalar potentials ϕ_m and ϕ, respectively. Furthermore, in (28), the divergence of $\mathbf{M_p}$ can be identified as the magnetic charge density ρ_{m}, and permeability μ_0 is not included in the governing system.

C. APPLICATION TO FEM

The structural similarity in the formulation between the equivalent permanent magnetization and electrostatics simplifies the application to numerical analysis. By ensuring the source terms in the differential equations are equivalent, it becomes possible to find the solution for the magnetic scalar potential ϕ_m. Thus, the equivalent permanent magnetization method can be applied using an electrostatic module based on the scalar potential formulation.

1) FEM USING PERMANENT POLARIZATION

From (37) and (38), by replacing permanent polarization $\mathbf{P_p}$ with the corresponding value of permanent magnetization $\mathbf{M_p}$ and charge density ρ with 0, we can acquire the equivalent differential equation, as represented in (28). Thus, the two equivalence conditions are defined as:

$$\mathbf{P_p} = \varepsilon_0 \mathbf{M_p} \tag{39}$$

$$\rho = 0 \tag{40}$$

Substituting (39) and (40), (37) becomes identical to (28), as follows:

$$\nabla^2 \phi = \frac{-\rho + \nabla \cdot \mathbf{P_p}}{\varepsilon_0} = \nabla \cdot \mathbf{M_p} = \nabla^2 \phi_m \tag{41}$$

From (41), the equivalence conditions ensure that the electrostatic potential ϕ and magnetic scalar potential ϕ_m are equal in magnitude. Thus, $\mathbf{H_m}$ can be obtained by determining the equivalent $\mathbf{E_{eqv}}$, as follows:

$$-\nabla \phi = \mathbf{E_{eqv}} = -\nabla \phi_m = \mathbf{H_m} \tag{42}$$

Finally, $\mathbf{B}$ can be computed via the constitutive relation as:

$$\mathbf{B} = u_0 \mathbf{H_m} + u_0 \mathbf{M_p} = u_0 \mathbf{E_{eqv}} + u_0 \mathbf{M_p} \tag{43}$$

Also, comparing the constitutive equation for current source(42) with (41),

$$\mathbf{B} = u_0 \mathbf{H}, \tag{44}$$

$$\mathbf{B} = u_0 \mathbf{H_m} + u_0 \mathbf{M_p} = u_0(\mathbf{H_m} + \mathbf{M_p}), \tag{45}$$

it is evident that the $\mathbf{H}$ is equivalent to:

$$\mathbf{H} = \mathbf{H_m} + \mathbf{M_p} \tag{46}$$

Here, $\mathbf{H}$ is the magnetic field intensity of the current source.

2) FEM USING CHARGE DENSITY

Similarly, the equivalent differential equation, that is (29), can be obtained using two equivalence conditions defined as follows:

$$\mathbf{P_p} = 0 \tag{47}$$

$$\rho = \varepsilon_0 \rho_m \tag{48}$$

Substituting (47) and (48), (37) becomes identical to (29), as follows:

$$\nabla^2 \phi = \frac{-\rho + \nabla \cdot \mathbf{P_p}}{\varepsilon_0} = -\rho_m = \nabla^2 \phi_m \tag{49}$$

The process for determining the $\mathbf{H}$ and $\mathbf{B}$ is the same as that described in the previous section.

VI. NUMERICAL EXAMPLE AND COMPARISON OF THE RESULTS

To verify the accuracy and effectiveness of the magnetic scalar potential method in reducing the computational cost, the magnetic flux density $\mathbf{B}$ and magnetic field intensity $\mathbf{H}$ were calculated using (a) equivalent magnetic charge/permanent magnetization and (b) external current as a source, using magnetic scalar potential and magnetic vector potential formulation, respectively. Subsequently, their relative errors, degrees of freedom (DOF), and computational times are presented for comparison.

A. 2D / 2D AXISYMMETRIC ARBITRARY CROSS-SECTIONAL COILS

For 2D/2D axisymmetric coil models, the magnetic scalar potential method does not reduce the computational costs, nor is it important because the computational costs are low. Therefore, in this section, only the accuracy of this method is discussed.

1) ARBITRARY 2D MODEL

The resultant magnetic fields ($\mathbf{B}$) calculated from the three sources are presented in Fig. 10. The resultant fields were identical, except for minor differences in certain regions, which decreased as the mesh size decreased, as shown in Fig. 11. The change in the average relative error was calculated between the magnetic scalar potential and vector potential method with respect to the mesh size factor (k), as depicted in Fig. 12. The mesh size factor (k) is defined as follows:

$$k = \frac{max\ coil\ width}{max\ element\ size}, \tag{50}$$

where k represents the minimum number of mesh elements per unit length of coil.

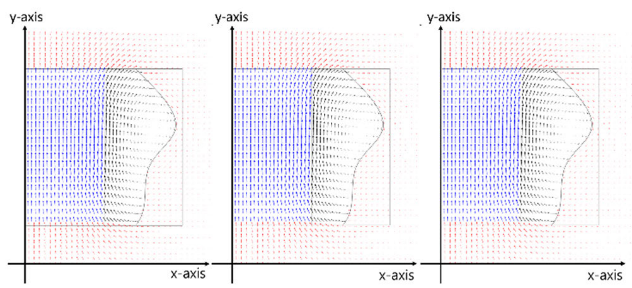


FIGURE 10. Magnetic field (B) calculated using (a) magnetic charge, (b) external current density, and (c) permanent magnetization.

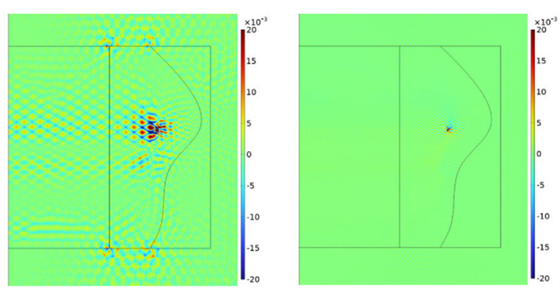


FIGURE 11. Relative error of H when (a) k=10 and (b) k=30.

The relative error for $\mathbf{B}$ is the same as that for $\mathbf{H}$ because $\mathbf{B}$ is calculated using $\mathbf{H}$, as in (44).

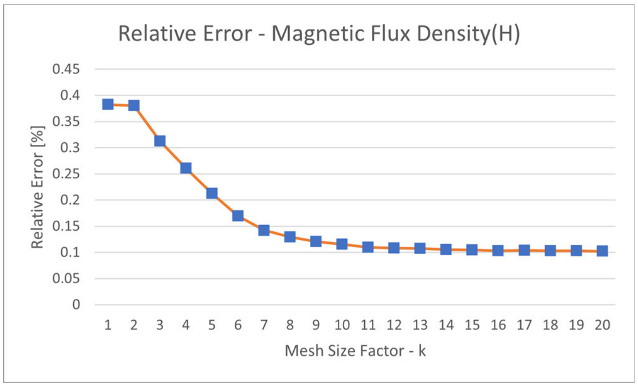


FIGURE 12. Change in relative error of H with respect to k–2D model.

2) ARBITRARY 2D-AXISYMMETRIC MODEL

This result is the same as that obtained for the 2D model. The resulting fields were almost identical for both scalar and vector potential methods, with minor relative errors in some regions. Figures for the 2D-axisymmetric model are omitted because of their high resemblance to the 2D model.

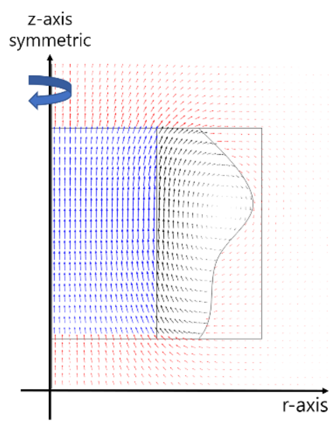


FIGURE 13. Magnetic field (B) calculated using equivalent magnetic charge – 2D-axismmetric model.

B. 3D ARBITRARY CROSS-SECTIONAL COILS

The resultant magnetic field of the 3D coil model calculated using the equivalent magnetization and magnetic scalar potential formulation is presented in Fig. 14. The resultant field appeared identical for both the magnetic scalar potential and magnetic vector potential methods.

The main source of error is the error in the wall distance function when finding equivalent permanent magnetization. This error is amplified during the process by which divergence of permanent magnetization is calculated in curved regions of the coil, as in Fig. 15. Thus, this error can be reduced by reducing the margin of error for the wall distance function and mesh size. However, for the efficiency of the method, reducing the mesh size is sufficient; the relative error of the magnetic field depended on the mesh size factor k,

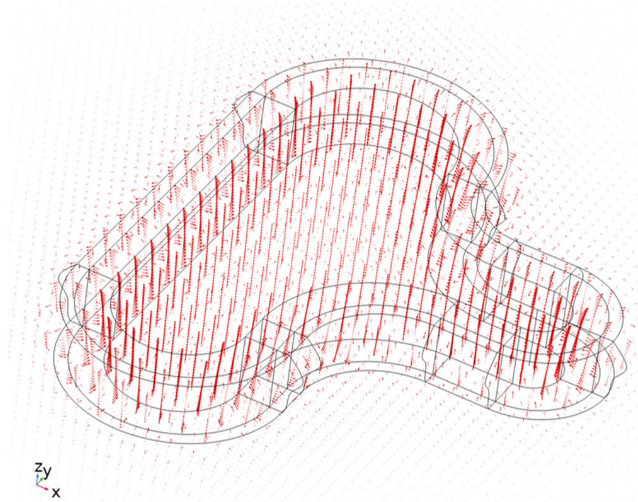
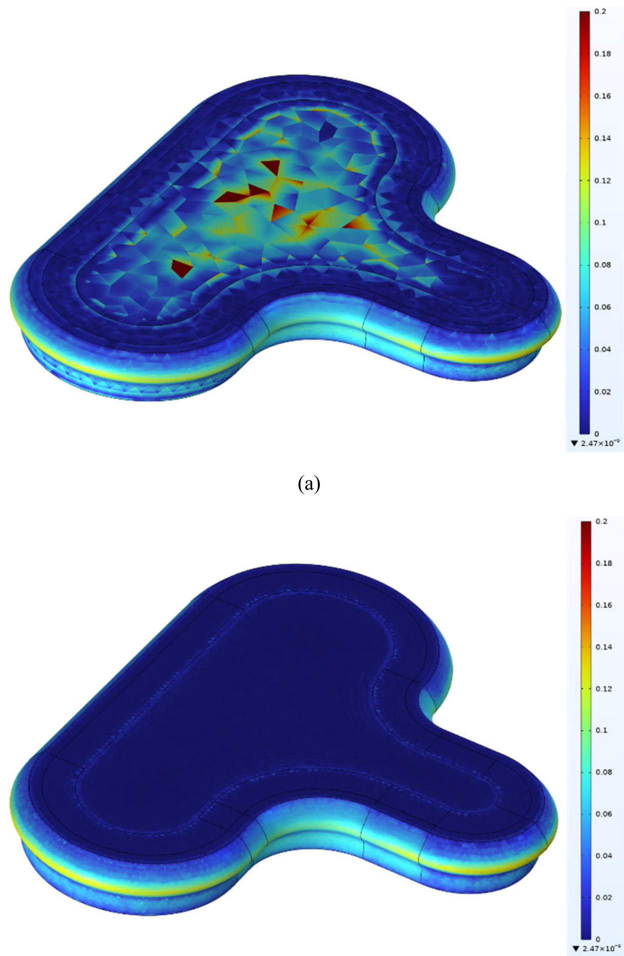


FIGURE 14. Magnetic field (B) calculated using equivalent magnetic charge– 3D Model.



(a)

(b)

FIGURE 15. Relative error of B when (a) k=2 (b) k=10.

and the relative error converged from 4 to 0.7% for mesh size factors of 1–10, as depicted in Fig. 16.

Most importantly, the magnetic scalar potential method significantly reduced computational costs; the DOF and

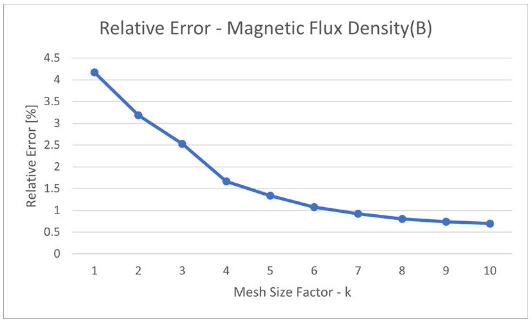


FIGURE 16. Change in the relative error of B with respect to k–3D model.

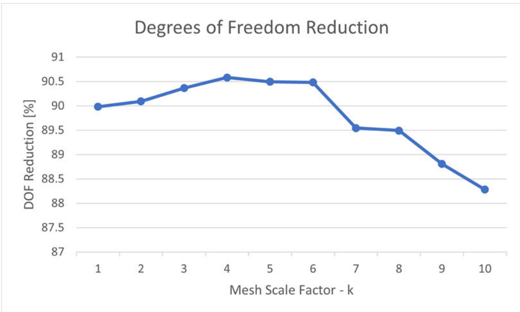


FIGURE 17. Change in DOF with respect to k–3D model.

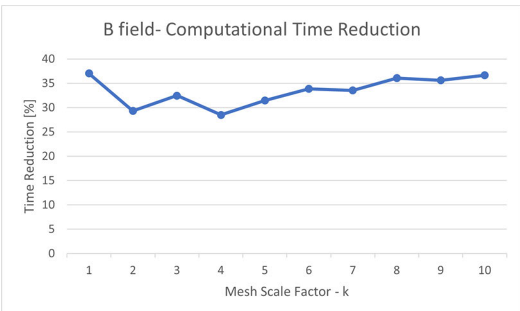


FIGURE 18. Change in computational time with respect to k–3D model.

computational time were both reduced by approximately 89.5 and 35%, respectively, as indicated in Figs. 17 and 18, respectively.

The amount of reduction in DOF is anticipated for the magnetic scalar potential method, and it seems much more plausible for the computational time to be reduced to a similar degree. However, unlike other methods suggested for rectangular coils which pre-calculate divergence of permanent magnetization or magnetic charge before solving for magnetic potential, pre-calculation is impossible for arbitrary 3D coils. This is because boundaries of arbitrary coils can have varying shapes, and the resultant equivalent permanent magnetization and distribution of volume magnetic charge density cannot be calculated analytically.

In this case, numerical calculation of divergence of permanent magnetization is mandatory, and it must be included in the computational time. Thus, a 35% reduction in computational time is reasonable, and it is still a significant reduction, considering the high accuracy of the method.

The relative error for B is the same as that for H because B is calculated using H, as in (44).

VII. CONCLUSION

This study proposes a magnetic scalar potential field analysis method for closed-loop of arbitrary cross-sectional coplanar coils in air. From the previous method on rectangular cross-sectional coils, we expand the method's applicability, practicality, and feasibility by suggesting the equations and steps required for finite element analysis.

First, a generalized equation for equivalent permanent magnetization for 3D arbitrary coplanar coils is derived. By providing a detailed derivation process for 2D/2D axisymmetric case and transformation from 2D to 3D case, we clarify that the method can be applied to non-symmetric and arbitrary cross-sectional cases. Justification for not meeting the antisymmetric volume current density requirement and introduction of infinitesimal volume current segment are provided during the process.

Secondly, a variational magnetic scalar potential formulation of the equivalent permanent magnetization is suggested. The variational formulation is necessary for finite element analysis, as the formulation defines the PDE solved for. The capability of directly using equivalent permanent magnetization as a source adds the feasibility of the method for general cases when pre-calculation of magnetic charge is impossible.

Finally, details required for the application of the method are introduced. To calculate equivalent permanent magnetization, a reference wall is defined and used in the wall distance function. Moreover, from the result of the formulation, we conclude that the parallel structure of the formulation with that of electrostatics allows a convenient method for application to FEM by defining the equivalence conditions. The simple substitution of the sources stated in the equivalence conditions makes the state variable equations identical, and magnetic scalar potential can be calculated using the electrostatics module.

To demonstrate the usefulness and accuracy of the proposed method, this study demonstrated a reduction in computational cost and relative error compared with the vector potential method. Overall, the DOF and the computational time were reduced by 89.5 and 35%, respectively. The relative error was dependent on the mesh size but converged to 0.7% for fine meshes, primarily attributable to the computational error of the wall distance function. The computational time for the method included the time required for the calculation of divergence of equivalent permanent magnetization, or magnetic charge, leading to a lower reduction in computational time compared to DOF. However, unlike other coils with simple geometry where pre-calculation of magnetic charge is possible, such a process is mandatory for arbitrary coils and 35% is still a meaningful reduction considering the high accuracy of the method.

Overall, the proposed method enables feasible and useful field computation of coplanar coils in air. By providing the magnetic scalar potential field analysis method for non-symmetric and arbitrary cross-sectional coils, the design of coil geometry in applications that utilize axial flux can be conducted with much greater design space, and its numerical field analysis results can be obtained with much lower computational cost. Furthermore, steps suggested for the derivation of the generalized equation and its variational magnetic scalar potential formulation provide the basis for further expansion of the method to cases with magnetic materials, where computational costs are much higher, and reduction in computational costs can be more valuable.

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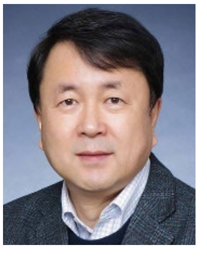

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