IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received 18 December 2023, accepted 6 January 2024, date of publication 15 January 2024, date of current version 23 January 2024.

Digital Object Identifier 10.1109/ACCESS.2024.3354030

RESEARCH ARTICLE

Integral Sliding Mode Control for a Class of Nonlinear Multi-Agent Systems With Multiple Time-Varying Delays

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This work was supported in part by the National Natural Science Foundation of China under Grant 62173136, in part by the Natural Science Foundation of Hunan Province under Grant 2020JJ2013, and in part by the Science Research Fund of Hunan Provincial Education Department under Grant 23A0425.

ABSTRACT This paper deals with the stability of nonlinear multi-agent systems (MASs) with multiple time delays. A class of multi-agent system (MAS) is constructed considering the effects of multiple time delays, nonlinearity, and uncertainty. An integral sliding mode surface is built based on the MAS, and the corresponding sliding mode dynamics are established. Using the Lyapunov method, sufficient conditions are obtained to ensure the sliding mode dynamic asymptotic stability with H_∞ performance. Consequently, the sliding mode controller parameter matrix is obtained. Furthermore, a sliding mode control law ensures that the considered system can be driven into the sliding mode surface. Finally, a numerical example is given to demonstrate the usefulness of the theoretical result.

INDEX TERMS Multi-agent systems, multiple time-delay, uncertainty, sliding mode control.

I. INTRODUCTION

Multi-agent systems (MASs) are a collection of multiple agents with interaction, coordination, and autonomy. Through communication and cooperation between agents, MASs can solve problems that single-agent systems cannot solve. With the development of network technology and artificial intelligence, the application of MASs has attracted considerable attention in intelligent tracking [1], [2], microgrid [3], [4], [5], environmental monitoring [6], [7]. Although MASs have been widely applied, there are still some problems that need to be solved.

Due to the large scale and complex communication of MASs, communication delays are inevitable when information is exchanged between agents. Communication delays may lead to incoherent information transmission of MASs and even ruin the system's stability, making it difficult to achieve the expected effect [8], [9], [10], [11], [12], [13], [14], [15], [16], [17]. Many literatures provide effective control schemes for systems with single time-delay, such as event-triggered control [18], [19], [20], [21], [22], adaptive control [23], [24], [25]. In addition, considering that there are information interactions between the agents in an MAS, communication delays may affect the system's performance. Few studies focus on MASs affected by multiple time delays. In [26], a delay-based double integral region partitioning method is proposed for MAs systems with multiple time delays, while the uncertainty of the system is not considered. Unlike the research above that considers the affected only by a single delay of MASs, this paper focuses on the MASs affected by multiple time delays, nonlinearities, and model uncertainties.

On the other hand, most of the mentioned achievements are designed for nominal MASs. In complex systems such as truck trailers and fluid mechanics, the influence of nonlinearities and model uncertainties on MASs cannot be ignored, reducing the control performance of the MASs [27], [28], [29], [30], [31], [32]. Therefore, designing an effective control scheme for these factors is necessary. Various control methods, such as T-S fuzzy control and composite nonlinear feedback (CNF), have been studied for nonlinearity and model uncertainty systems. T-S fuzzy control has a strong

The associate editor coordinating the review of this manuscript and approving it for publication was Qiang Li.

approximation ability for nonlinear systems and can be used to model nonlinear systems by the T-S fuzzy model approach. Based on the above characteristics, T-S fuzzy control was adopted in [33], [34], [35], and [36] to study nonlinear systems with uncertainty problems. CNF consists of two parts: linear feedback law and nonlinear feedback law. The linear feedback law is designed to produce a closed-loop system to achieve fast response, and the nonlinear feedback law is used to reduce the linear feedback overshoot. Due to the above characteristics, CNF is adopted in [37] and [38]. In addition to the above methods, the sliding mode control has recently attracted wide attention [39], [40], [41], [42] because it can counteract the effect of uncertainties and nonlinearities inevitable in most practical systems. In [42], the traditional sliding mode control is adopted to ensure the asymptotic stability of multiple time-delayed MASs with nonlinearities and model uncertainties. This method provides ideas for MASs with multiple time delays, nonlinearities, and model uncertainties. However, the time delay considered is constant while time delays in most practical systems are time-varying, which limits the application of the proposed methods. There are few research results on nonlinear MASs with multiple time-varying delays, nonlinearities, and model uncertainties, which motivates current research. Based on the above discussion, this paper aims to design a sliding mode controller for nonlinear MASs with multiple time-varying delays, nonlinearities and model uncertainties. The contributions of this paper are summarized as follows:

(1) A class of nonlinear MAS is established by considering disturbance, model uncertainties, and multiple delays.
(2) An integral sliding surface is constructed on the nonlinear MASs with multiple time-varying delays, nonlinearities and model uncertainties, and the corresponding sliding mode dynamics are derived.
(3) A sufficient condition is derived and presented in LMIs, which ensures that the sliding mode dynamics are asymptotically stable with H_∞ performance.
(4) A novel sliding mode control law is synthesized to guarantee the reachability condition.

The remaining sections are distributed as follows. The second section describes MASs and some necessary lemmas. The third section gives the principal results and proves the feasibility of the proposed sliding mode controller. In the fourth section, a simulation example is given to confirm the validity of the proposed approach. The fifth section concludes this paper.

Notation: The notations used in this paper are standard. X^T, X^{-1} denote the transpose and inverse of a matrix X, respectively. $\|\cdot\|$ denotes the vector or matrix Euclidean norm. $P > 0$ ($P < 0$) denotes P as a positive-definite matrix (negative-definite matrix). The symbol * denotes the transposed element in the symmetric position of the matrix. *diag* denotes the diagonal matrices, *vec* denotes the row matrix, *col* denotes the column matrices. $diag_m\{N_i\}$ denotes the m-block-diagonal matrices $diag\{N_1, \cdots, N_m\}$. $vec_m\{N_i\}$ denotes the m-block-row matrix $vec\{N_1, \cdots, N_m\}$. $col_m\{N_i\}$ denotes the m-block-column matrices $col\{N_1, \cdots, N_m\}$.

II. PRELIMINARIES

A directed graph $G = (V, E)$ is used to characterize the communication between m agents, where $V = \{1, 2, \cdots, m\}$ is used to represent the set of agents, $E \subseteq V \times V$ stands for a set of edges. $(i, j) \in E$ represents that agent i is able to obtain information from the neighboring agent j. The adjacency matrix $A = \left[a_{ij}\right] \in R^{n\times n}$ of can also describe the digraph G. If any information flow exists between agents i and j, it can be represented by $a_{ij} > 0$, otherwise $a_{ij} = 0$. Considering that multiple agents are inevitably affected by communication delay, the MASs can be described by the following model:

$$\begin{aligned} \dot{x}(t) &= (A + \triangle A)x(t) + \sum_{i=1}^{m}(A_{di} + \triangle A_{di})x(t - \tau_i(t)) \\ &\quad + B(u(t) + g(t, x(t))) + Hw(t) \\ y(t) &= Cx(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $u(t) \in \mathbb{R}^m$ denotes system control input, $g(t, x(t))$ denotes continuous function of the system state, $y(t) \in \mathbb{R}^p$ represents measurement output, and $w(t) \in \mathbb{R}^q$ stands for external interference, which subject to $L_2[0, \infty)$, $A \in \mathbb{R}^{n\times n}$, $A_{di} \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$, $H \in \mathbb{R}^{n\times q}$ are constant matrices with appropriate dimensions; the communication delay, $\tau_i(t)$, are time-varying differentiable functions satisfying:

$$0 \le \tau_i(t) \le \tau_i^M, \dot{\tau}_i(t) \le \tau_{di}, \tag{2}$$

in which τ_i^M and τ_{di} are constants. Without loss of generality, it is assumed that $\tau_1^M \le \tau_2^M \le \cdots \le \tau_m^M$. For the sake of simplicity represent $\tau_i(t)$ with τ_i. $\triangle A$, $\triangle A_{di}$ are uncertain time-varying matrices, satisfying:

$$\left[\triangle A, \triangle A_{d1} \cdots \triangle A_{dm}\right] = MF(t)\left[N, N_{\tau 1} \cdots N_{\tau m}\right], \tag{3}$$

with $M \in \mathbb{R}^{n\times f_1}$, $N_{\tau i} \in \mathbb{R}^{f_2\times n}$ are constant matrix and $F(t) \in \mathbb{R}^{f_1\times f_2}$ is an unknown matrix satisfying $F^T(t)F(t) \le I$ without loss of generality.

Remark 1: Compared with [42], the model considered in this paper involves uncertainty and multiple time-varying delays, as well as the effects of measurement output, nonlinearity and external disturbances, which is more general.

The following assumptions are mentioned to facilitate the development of a sliding mode control strategy.

Assumption 1: If (A, B) is controllable, then there exists a matrix K such that $A - BK$ is also stable, where $rank(B) = m \le n$.

Assumption 2: For the nonlinear term $g(t, x(t))$, there exists a non-negative constant $\bar{g}$ satisfying $\|g(t, x(t))\| < \bar{g}$, and $\bar{g}$ is the upper bound of the nonlinear term $g(t, x(t))$.

Assumption 3: For the external interference $w(t)$ there exists a non-negative constant $\bar{w}$ satisfied $\|w(t)\| < \bar{w}$, where $\bar{w}$ is the upper bound of the external interference $w(t)$.

The above Assumptions 1, Assumptions 2 and Assumptions 3 are the same as those made in [42]. Next, we introduce the following lemmas essential in the subsequent derivation process.

Lemma 1: [43] For any constant matrix $M = M^T > 0$, $M \in \mathbb{R}^{n\times n}$, and assuming there are positive scalars $h > 0$ and vector function $\dot{x}(t)$:$[-h, 0] \longrightarrow \mathbb{R}^n$, then there are the following inequalities:

$$-h\int_{-h}^{0}\dot{x}(t+s)M\dot{x}(t+s)ds \leq \left[x^T(t)\ x^T(t-h)\right]\begin{bmatrix}-M & M\\ M & -M\end{bmatrix}\begin{bmatrix}x(t)\\ x(t-h)\end{bmatrix}. \quad (4)$$

Lemma 2: [44] Given matrices P, Q, and symmetric matrix Z,

$$Z + P\Phi(t)Q + Q^T\Phi^T(t)P^T < 0, \quad (5)$$

for $\Phi(t)$ with $\Phi(t)\Phi^T(t) \leq I$, if and only if there exists a scalar $\zeta > 0$ satisfied:

$$Z + \zeta PP^T + \zeta^{-1}Q^TQ < 0. \quad (6)$$

Definition 1: [45] For $\gamma > 0$, if the following condition should be satisfied:

$$\int_0^\infty y^T(t)y(t)dt < \gamma^2\int_0^\infty w^T(t)w(t)dt, \quad (7)$$

system (13) satisfies H_∞ performance, under zero initial conditions, all non-zero $w(t)$ holds.

III. MAIN RESULTS

There are three main subsections in this section. Firstly, an appropriate integral sliding mode surface is selected to establish the corresponding dynamic model for the integral sliding mode. Then, the stability and H_∞ performance analysis of system (1) is discussed. Finally, a sliding mode control law is designed to ensure the system can be driven into the sliding mode surface.

A. INTEGRAL SLIDING SURFACE DESIGN

The integral sliding mode surface of system (1) is selected as:

$$s(t) = G(x(t) - x(0)) - G\int_0^t (A + BK)x(s)ds, \quad (8)$$

where $G\in \mathbb{R}^{m\times n}$ is a constant matrix satisfies $det(GB) \neq 0$, $K\in \mathbb{R}^{m\times n}$ is a constant matrix. According to the equation of system (1), it is easy to get:

$$\begin{aligned}x(t) = x(0) &+ \int_0^t ((A + \triangle A)x(s)\\ &+ \sum_{i=1}^{m}(A_{di} + \triangle A_{di})x(s - \tau_i(s))\\ &+ B(u(s) + g(s, x(s))) + Hw(s))ds. \quad (9)\end{aligned}$$

By substituting equation (9) into equation (8), we get:

$$\begin{aligned}s(t) = G\int_0^t ((\triangle A - BK)x(s) &+ \sum_{i=1}^{m}(A_{di} + \triangle A_{di})\\ &\times x(s - \tau_i(s)) + B(u(s) + g(s, x(s)))\\ &+ Hw(s))ds. \quad (10)\end{aligned}$$

According to the sliding mode theory, when sliding mode takes place $s(t) = 0$, $\dot{s}(t) = 0$, the derivation of equation (10) along system (1) is obtained:

$$\begin{aligned}\dot{s}(t) = G((&\triangle A - BK)x(t)\\ &+ \sum_{i=1}^{m}(A_{di} + \triangle A_{di})x(t - \tau_i(t))\\ &+ B(u(t) + g(t, x(t))) + Hw(t)). \quad (11)\end{aligned}$$

Therefore, the equivalent control law is as follows:

$$\begin{aligned}u_{eq} = -(GB)^{-1}G((&\triangle A - BK)x(t)\\ &+ \sum_{i=1}^{m}(A_{di} + \triangle A_{di})x(t - \tau_i(t))\\ &+ Bg(t, x(t)) + Hw(t)). \quad (12)\end{aligned}$$

Then the equivalent control law (12) is substituted into system (1) to obtain the sliding mode dynamics:

$$\begin{aligned}\dot{x}(t) = (A + BK + \bar{R}\triangle A)x(t) &+ \sum_{i=1}^{m}\bar{R}(A_{di} + \triangle A_{di})\\ &\times x(t - \tau_i(t)) + \bar{R}Hw(t), \quad (13)\end{aligned}$$

with $\bar{R} = I - B(GB)^{-1}G$.

Remark 2: By adjusting the initial value of the integrator, the initial state of the system is kept in the sliding mode, and the robustness of the system is ensured.

B. STABILITY AND H_∞ PERFORMANCE ANALYSIS

In this subsection, a sufficient condition of stability of sliding mode dynamic system (13) with given controller gain K is derived.

Theorem 1: System (13) with given control gain matrix K is robustly stable and satisfies H_∞ performance index $\gamma = \sqrt{\bar{\gamma}}$, if there exist real symmetric matrices $P > 0, R_i > 0$, $Q_{1i} \geq Q_{2i} > 0$ $(i = 1, \cdots, m)$, W, and scalars $\varepsilon > 0$ and $\bar{\gamma} > 0$, such that the linear matrix inequality (14) is satisfied and the optimal performance index can be obtained by solving the following condition,

$$\min_{P,W,R_i,Q_{1i},Q_{2i},\varepsilon} \bar{\gamma}$$
$$\begin{bmatrix}\Gamma + \varepsilon\tilde{N}^T\tilde{N} & \tilde{M}\\ * & -\varepsilon I\end{bmatrix} < 0, \quad (14)$$

where

$$\tilde{M} = \left[(\bar{R}M)^TW, 0_{n,2nm}, (\bar{R}M)^TW, 0_{n,mn}\right]^T$$
$$\tilde{N} = \left[N, N_1, \cdots, N_m, 0_{n,(2m+1)n}\right]$$
$$\Gamma = \begin{bmatrix}\Gamma_{11} & \Gamma_{12} & 0 & \Gamma_{14} & W^T\bar{R}H\\ * & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} & 0\\ * & * & \Gamma_{33} & 0 & 0\\ * & * & * & \Gamma_{44} & W^T\bar{R}H\\ * & * & * & * & \Gamma_{55}\end{bmatrix}$$

with

$$\begin{aligned}
\Gamma_{11} &= W^T(A+BK)+(A+BK)^TW+\textstyle\sum_{i=1}^{m}Q_{1i} \\
&\quad -\textstyle\sum_{i=1}^{m}R_i+C^TC \\
\Gamma_{12} &= vec_m\{W^T\bar{R}A_{di}+R_i\} \\
\Gamma_{14} &= P-W^T+(A+BK)^TW \\
\Gamma_{22} &= diag_m\{-(1-\tau_{di})(Q_{1i}-Q_{2i})-2R_i\} \\
\Gamma_{23} &= diag_m\{R_i\} \\
\Gamma_{24} &= col_m\{(\bar{R}A_{di})^TW\} \\
\Gamma_{33} &= diag_m\{-Q_{2i}-R_i\} \\
\Gamma_{44} &= -W-W^T+\textstyle\sum_{i=1}^{m}(\tau_i^M)^2R_i \\
\Gamma_{55} &= -\bar{\gamma}I.
\end{aligned}$$

Proof: Choose the following Lyapunov functional:

$$\begin{aligned}
V(t) &= x^T(t)Px(t)+\sum_{i=1}^{m}\int_{t-\tau_i(t)}^{t}x^T(s)Q_{1i}x(s)ds \\
&\quad +\sum_{i=1}^{m}\int_{t-\tau_i^M}^{t-\tau_i(t)}x^T(s)Q_{2i}x(s)ds \\
&\quad +\tau_i^M\sum_{i=1}^{m}\int_{-\tau_i^M}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)R_i\dot{x}(s)ds,
\end{aligned} \tag{15}$$

Taking the derivative along the trajectory of system (13) yields

$$\begin{aligned}
\dot{V}(t) &= 2x^T(t)P\dot{x}(t)+(\tau_i^M)^2\sum_{i=1}^{m}\dot{x}^T(t)R_i\dot{x}(t) \\
&\quad +\sum_{i=1}^{m}(x^T(t)Q_{1i}x(t)-(1-\dot{\tau}_i(t)) \\
&\quad \times x^T(t-\tau_i(t))(Q_{1i}-Q_{2i})x(t-\tau_i(t)) \\
&\quad -x^T(t-\tau_i^M)Q_{2i}x(t-\tau_i^M)) \\
&\quad -\tau_i^M\sum_{i=1}^{m}\int_{t-\tau_i^M}^{t}\dot{x}^T(s)R_i\dot{x}(s)ds \\
&\le 2x^T(t)P\dot{x}(t)+(\tau_i^M)^2\sum_{i=1}^{m}\dot{x}^T(t)R_i\dot{x}(t) \\
&\quad +\sum_{i=1}^{m}(x^T(t)Q_{1i}x(t)-(1-\tau_{di}) \\
&\quad \times x^T(t-\tau_i(t))(Q_{1i}-Q_{2i})x(t-\tau_i(t)) \\
&\quad -x^T(t-\tau_i^M)Q_{2i}x(t-\tau_i^M)) \\
&\quad -\tau_i^M\sum_{i=1}^{m}\int_{t-\tau_i^M}^{t}\dot{x}^T(s)R_i\dot{x}(s)ds,
\end{aligned} \tag{16}$$

Considering that $0\le\tau_i(t)\le\tau_i^M$, we get:

$$\begin{aligned}
&-\tau_i^M\sum_{i=1}^{m}\int_{t-\tau_i^M}^{t}\dot{x}^T(s)R_i\dot{x}(s)ds \\
&= -\sum_{i=1}^{m}\tau_i^M(\int_{t-\tau_i^M}^{t-\tau_i(t)}\dot{x}^T(s)R_i\dot{x}(s)ds \\
&\quad +\int_{t-\tau_i(t)}^{t}\dot{x}^T(s)R_i\dot{x}(s)ds) \\
&\le J_1+J_2
\end{aligned} \tag{17}$$

with

$$\begin{aligned}
J_1 &= -\tau_i(t)\sum_{i=1}^{m}\int_{t-\tau_i(t)}^{t}\dot{x}^T(s)R_i\dot{x}(s)ds \\
J_2 &= -(\tau_i^M-\tau_i(t))\sum_{i=1}^{m}\int_{t-\tau_i^M}^{t-\tau_i(t)}\dot{x}^T(s)R_i\dot{x}(s)ds.
\end{aligned}$$

Define $\eta_1(t)=\left[x^T(t),\eta_{1a}^T(t),\eta_{1b}^T(t)\right]^T$ with

$$\begin{aligned}
\eta_{1a}(t) &= \left[x^T(t-\tau_1),\cdots,x^T(t-\tau_m)\right]^T \\
\eta_{1b}(t) &= \left[x^T(t-\tau_1^M),\cdots,x^T(t-\tau_m^M)\right]^T.
\end{aligned}$$

Applying Lemma 1 yields

$$J_1+J_2\le\eta_1^T(t)\Psi\eta_1(t), \tag{18}$$

where

$$\Psi=\begin{bmatrix}\Psi_{11} & \Psi_{12} & 0 \\ * & \Psi_{22} & \Psi_{23} \\ * & * & \Psi_{33}\end{bmatrix},$$

with

$$\begin{aligned}
&\Psi_{11}=-\textstyle\sum_{i=1}^{m}R_i,\ \Psi_{12}=vec_m\{R_i\}, \\
&\Psi_{22}=diag_m\{-2R_i\},\ \Psi_{23}=diag_m\{R_i\}, \\
&\Psi_{33}=diag_m\{-R_i\}.
\end{aligned}$$

It follows from (13) that the following zero equality is true.

$$\begin{aligned}
0 &= 2(x^T(t)+\dot{x}^T(t))W^T((A+\bar{R}\triangle A+BK)x(t)+\bar{R} \\
&\quad \times\sum_{i=1}^{m}(A_{di}+\triangle A_{di})x(t-\tau_i(t))+\bar{R}Hw(t)-\dot{x}(t))
\end{aligned} \tag{19}$$

Let $\eta(t)=\left[\eta_1^T(t),\dot{x}^T(t)\right]^T$. Adding the right side of (19) into $\dot{V}(t)$ yields

$$\begin{aligned}
\dot{V}(t) &\le \eta^T(t)\Lambda\eta(t)+2x^T(t)W^T\bar{R}Hw(t) \\
&\quad +2\dot{x}^T(t)W^T\bar{R}Hw(t)
\end{aligned} \tag{20}$$

where

$$\Lambda=\begin{bmatrix}\Lambda_{11} & \Lambda_{12} & 0 & \Lambda_{14} \\ * & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} \\ * & * & \Lambda_{33} & 0 \\ * & * & * & \Lambda_{44}\end{bmatrix} \tag{21}$$

with

$$\begin{aligned}
\Lambda_{11} &= W^T(A+\bar{R}\triangle A+BK)+(A+\bar{R}\triangle A+BK)^TW \\
&\quad +\textstyle\sum_{i}^{m}(Q_{1i}-R_i) \\
\Lambda_{12} &= vec_m\{W^T\bar{R}(A_{di}+\triangle A_{di})+R_i\}
\end{aligned}$$

$$\Lambda_{14} = P - W^T + (A + \bar{R}\triangle A + BK)^T W$$
$$\Lambda_{22} = diag_m\{-(1-\tau_{di})(Q_{1i} - Q_{2i}) - R_i - R_i^T\}$$
$$\Lambda_{23} = diag_m\{R_i\}$$
$$\Lambda_{24} = col_m\{(\bar{R}A_{di} + \bar{R}\triangle A_{di})^T W\}$$
$$\Lambda_{33} = diag_m\{-Q_{2i} - R_i\}$$
$$\Lambda_{44} = -W - W^T + \textstyle\sum_{i=1}^m (\tau_i^M)^2 R_i.$$

Then analyze the H_∞ performance :

$$\int_0^\infty y^T(t)y(t)dt < \gamma^2 \int_0^\infty w^T(t)w(t)dt. \tag{22}$$

For the sliding mode dynamic (13) is admissible with the following specified H_∞ norm upper bound, the following performance index is introduced:

$$J = \dot{V}(t) + y^T(t)y(t) - \gamma^2 w^T(t)w(t). \tag{23}$$

According to the above discussion, it can be concluded that:

$$J \le \eta^T(t)\Lambda\eta(t) + x^T(t)C^T Cx(t) + 2x^T(t)W^T\bar{R}Hw(t) + 2\dot{x}^T(t)W^T\bar{R}Hw(t) - \gamma^2 w^T w(t). \tag{24}$$

By defining $\zeta(t) = \left[\eta^T(t), w^T(t)\right]^T$, the (24) can be represented as

$$J \le \zeta^T(t)\Theta\zeta(t), \tag{25}$$

where

$$\Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} & 0 & \Theta_{14} & W^T\bar{R}H \\ * & \Theta_{22} & \Theta_{23} & \Theta_{24} & 0 \\ * & * & \Theta_{33} & 0 & 0 \\ * & * & * & \Theta_{44} & W^T\bar{R}H \\ * & * & * & * & \Theta_{55} \end{bmatrix}$$

with

$$\Theta_{11} = W^T(A + \bar{R}\triangle A + BK) + (A + \bar{R}\triangle A + BK)^T W + \textstyle\sum_i^m (Q_{1i} - R_i) + C^T C$$
$$\Theta_{12} = vec_m\{W^T\bar{R}(A_{di} + \triangle A_{di}) + R_i\}$$
$$\Theta_{14} = P - W^T + (A + \bar{R}\triangle A + BK)^T W$$
$$\Theta_{22} = diag_m\{-(1-\tau_{di})(Q_{1i} - Q_{2i}) - R_i - R_i^T\}$$
$$\Theta_{23} = diag_m\{R_i\}$$
$$\Theta_{24} = col_m\{(\bar{R}A_{di} + \bar{R}\triangle A_{di})^T W\}$$
$$\Theta_{33} = diag_m\{-Q_{2i} - R_i\}$$
$$\Theta_{44} = -W - W^T + \textstyle\sum_{i=1}^m (\tau_i^M)^2 R_i$$
$$\Theta_{55} = -\gamma^2 I.$$

From the above inference, if $\Theta < 0$, we can obtain $J < 0$. Under initial state of zero, integrating both sides of $\dot{V}(t) < \gamma^2 w^T(t)w(t) - y^T(t)y(t)$ along 0 to ∞ to get

$$\int_0^\infty y^T(t)y(t)dt < \gamma^2 \int_0^\infty w^T(t)w(t)dt - V(\infty) \tag{26}$$

Considering $V(\infty) \ge 0$, we can get

$$\int_0^\infty y^T(t)y(t)dt < \gamma^2 \int_0^\infty w^T(t)w(t)dt. \tag{27}$$

It follows that system (13) is robustly stable with the attenuation level γ. Due to Θ contains uncertainty and nonlinearity, Θ converted into two parts:

$$\Theta = \Gamma + \triangle\Gamma, \tag{28}$$

where Γ is defined in Theorem 1 and

$$\triangle\Gamma = \begin{bmatrix} \triangle\Gamma_{11} & \triangle\Gamma_{12} & 0 & \triangle\Gamma_{14} & 0 \\ * & 0 & 0 & \triangle\Gamma_{24} & 0 \\ * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & 0 \end{bmatrix}$$

with

$$\triangle\Gamma_{11} = W^T\bar{R}\triangle A + (\bar{R}\triangle A)^T W$$
$$\triangle\Gamma_{12} = vec_m\{W^T\bar{R}\triangle A_i\}$$
$$\triangle\Gamma_{14} = (\bar{R}\triangle A)^T W$$
$$\triangle\Gamma_{24} = col_m\{(\bar{R}\triangle A_{di})^T W\}.$$

Then, the following equation can be obtained:

$$\triangle\Gamma = sym\{\tilde{M}F(t)\tilde{N}\}, \tag{29}$$

with $\tilde{M}$ and $\tilde{N}$ being defined in Theorem 1.

Considering $F^T(t)F(t) \le I$, one get from Lemma 2 that $\Theta < 0$ is equivalent to

$$\Gamma + \varepsilon\tilde{N}^T\tilde{N} + \varepsilon^{-1}\tilde{M}\tilde{M}^T < 0, \tag{30}$$

with ε being a positive constant. Through analysis, it can be seen that (14) is satisfied, which means $\Lambda < 0$ in (20), and according to Lyapunov's theorem, the system (13) is asymptotically stable with $w(t) = 0$. Furthermore, the optimal H_∞ performance index γ can be obtained by inequality (14). The proof is completed.

Remark 3: By adopting a delay-dependent method for dividing the double integral domain, we can derive a less conservative stability criterion through modification of $-\int_{t-\tau_i^M}^t \dot{x}^T(s)R_i\dot{x}(s)ds = -\int_{t-\tau_i^M}^{t-\tau_i(t)} \dot{x}^T(s)R_i\dot{x}(s)ds - \int_{t-\tau_i(t)}^t \dot{x}^T(s)R_i\dot{x}(s)ds$. The above integral terms are bounded by using the integral inequality given Lemma 1. Less conservative results can be obtained by using tighter inequalities in [46], [47], [48], [49], [50], [51], and [52].

C. SLIDING MODE CONTROLLER DESIGN

In what follows, Theorem 1 is extended to design the controller K for system (13).

Theorem 2: System (13) is robustly stable and satisfies the H_∞ performance index $\gamma = \sqrt{\bar{\gamma}}$, if there exist any matrices S with appropriate dimensions, real symmetric matrices $\hat{P} > 0, \hat{R}_i > 0, \hat{Q}_{1i} > \hat{Q}_{2i} > 0 (i = 1, \cdots, m), X$, scalars $\hat{\varepsilon} > 0$ and $\bar{\gamma} > 0$, such that the linear matrix inequality (31) is satisfied and the optimal performance index can be obtained by solving the following condition,

$$\min_{\hat{P},X,S,\hat{R}_i,\hat{Q}_{1i},\hat{Q}_{2i},\hat{\varepsilon}} \bar{\gamma}$$
$$\begin{bmatrix} \hat{\Gamma}+\hat{\varepsilon}\hat{M}\hat{M}^T & \hat{N}^T \\ * & -\hat{\varepsilon}I \end{bmatrix} < 0, \tag{31}$$

where

$$\hat{M} = \left[(\bar{R}M)^T, 0_{n,2nm}, (\bar{R}M)^T, 0_{n,mn}\right]^T$$
$$\hat{N} = \left[NX, N_1X, \cdots, N_mX, 0_{n,(2m+1)*n}\right]$$
$$\hat{\Gamma} = \begin{bmatrix} \hat{\Gamma}_{11} & \hat{\Gamma}_{12} & 0 & \hat{\Gamma}_{14} & \bar{R}HX & CX \\ * & \hat{\Gamma}_{22} & \hat{\Gamma}_{23} & \hat{\Gamma}_{24} & 0 & 0 \\ * & * & \hat{\Gamma}_{33} & 0 & 0 & 0 \\ * & * & * & \hat{\Gamma}_{44} & \bar{R}HX & 0 \\ * & * & * & * & \hat{\Gamma}_{55} & 0 \\ * & * & * & * & * & -I \end{bmatrix}$$

with

$$\hat{\Gamma}_{11} = (AX+BS)+(AX+BS)^T+\textstyle\sum_i^m(\hat{Q}_{1i}-\hat{R}_i)$$
$$\hat{\Gamma}_{12} = vec_m\{\bar{R}A_{di}X+\hat{R}_i\}$$
$$\hat{\Gamma}_{14} = \hat{P}-X+X^TA^T+S^TB^T$$
$$\hat{\Gamma}_{22} = diag_m\{-(1-\tau_{di})(\hat{Q}_{1i}-\hat{Q}_{2i})-\hat{R}_i-\hat{R}_i^T\}$$
$$\hat{\Gamma}_{23} = diag_m\{\hat{R}_i\}$$
$$\hat{\Gamma}_{24} = col_m\{(\bar{R}A_{di}X)^T\}$$
$$\hat{\Gamma}_{33} = diag_m\{-\hat{Q}_{2i}-\hat{R}_i\}$$
$$\hat{\Gamma}_{44} = -X-X^T+\textstyle\sum_{i=1}^m(\tau_i^M)^2\hat{R}_i$$
$$\hat{\Gamma}_{55} = -\bar{\gamma}I.$$

Furthermore, the controller matrix can be obtained by $K = SX^{-1}$.

Proof: Define the following notations, $J = diag\{X, X, \cdots, X, X\}, X = W^{-1}, S = KX$, $\hat{Q}_{1i} = X^TQ_{1i}X, \hat{Q}_{2i} = X^TQ_{2i}X, \hat{R}_i = X^TR_iX$, $\hat{P} = X^TPX, \bar{\gamma} = \gamma^2, (i = 1, \cdots, m), \hat{\varepsilon} = \varepsilon^{-1}$. Pre- and post-multiplying (30) by J^T and J respectively, the following equation can be obtained:

$$\hat{\Gamma}+\hat{\varepsilon}^{-1}\hat{N}^T\hat{N}+\hat{\varepsilon}\hat{M}\hat{M}^T < 0, \tag{32}$$

according to Schur's complement, (32) is equivalent to (31), and if (31) has a feasible solution, system (13) robustly stable with a H_∞ performance index γ. Moreover, $K = SX^{-1}$ obtains the controller gain matrix. The proof is completed.

Next, it is proven that the obtained sliding mode control law can derive the system into the predefined sliding mode surface (8).

Theorem 3: For the controller gain matrix K obtained by Theorem 2, the trajectory of the system (1) can be driven to the sliding mode surface by the following sliding mode control law:

$$u(t) = Kx(t) - \sum_{i=1}^m (GB)^{-1}GA_{di}x(t-\tau_i(t))$$
$$-\rho\cdot sgn(s(t))(GB)^{-1}, \tag{33}$$

where

$$\rho = \lambda + \|GM\|\,\|N\|\,\bar{x} + \|GB\|\bar{g} + \|GH\|\,\bar{w}$$
$$+\sum_{i=1}^m \|GM\|\,\left\|N_{\tau_i}\right\|\bar{x}. \tag{34}$$

with λ being a positive constant.

Proof: If Theorem 2 holds, then $x(t)$ is bounded. So we can find a positive scalar $\bar{x}$ such that $sup_{0<t<\infty}\|x(t)\| \leq \bar{x}$.

Choose the Lyapunov functionals for $V_s(t) = \frac{1}{2}s^T(t)s(t)$, calculating the time-derivative of $V_s(t)$ yields:

$$\dot{V}_s(t) = s^T(t)\dot{s}(t) = s^T(t)G((\triangle A - BK)x(t)$$
$$+\sum_{i=1}^m (A_{di}+\triangle A_{di})x(t-\tau_i(t))$$
$$+B(u(t)+g(t,x(t)))+Hw(t)). \tag{35}$$

Substituting (33) into (35), one has

$$\dot{V}_s(t) = s^T(t)\dot{s}(t)$$
$$= s^T(t)G(\triangle Ax(t)+\sum_{i=1}^m\triangle A_{di}x(t-\tau_i(t))$$
$$+Bg(t,x(t))+Hw(t)-sgn(s(t))\rho)$$
$$\leq \|s(t)\|\|G\|(\|M\|\|N\|\bar{x}+\sum_{i=1}^m\|M\|\|N_{\tau_i}\|\bar{x}$$
$$+\|B\|\bar{g}+\|H\|\bar{w}-sgn(s(t))\rho)$$
$$= -\lambda\|s(t)\|. \tag{36}$$

Therefore, the reachability conditions are guaranteed, and the proof is completed.

IV. NUMERICAL EXAMPLES

In this section, a numerical example is used to verify the validity of the proposed theoretical method.

Consider system (1) with the following parameters

$$A = \begin{bmatrix} -2.2 & 1 \\ -1 & 0.9 \end{bmatrix}, M = \begin{bmatrix} 0.1 & 0.35 \\ 0 & 0.3 \end{bmatrix}, N = \begin{bmatrix} 0.35 & 0.5 \\ 0.2 & 2.7 \end{bmatrix},$$
$$N_{\tau 1} = N_{\tau 2} = \begin{bmatrix} 0.55 & 0.3 \\ 0.4 & 0.9 \end{bmatrix}, F(t) = \begin{bmatrix} 0.8sin(t) & 0 \\ 0 & 0.8sin(t) \end{bmatrix},$$
$$B = \begin{bmatrix} 0.01 \\ 3 \end{bmatrix}, A_{d1} = A_{d2} = \begin{bmatrix} -0.22 & 0.1 \\ -0.1 & 0 \end{bmatrix}, H = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, w(t) = \begin{bmatrix} 0.2sin(t) \\ 0.2cos(t) \end{bmatrix}, G = B^T,$$
$g(t, x(t)) = 0.11\sqrt{x_1^2+x_2^2}$, $\tau_1(t) = (0.3+0.1\sin(t))$, $\tau_2(t) = (0.2+0.1\cos(t))$.

It is easy to check that the time-varying delay satisfy (2) with $\tau_1^M = 0.4, \tau_2^M = 0.3, \tau_{d1} = \tau_{d2} = 0.1$. For this system, using Theorem 1, the H_∞ performance index can be

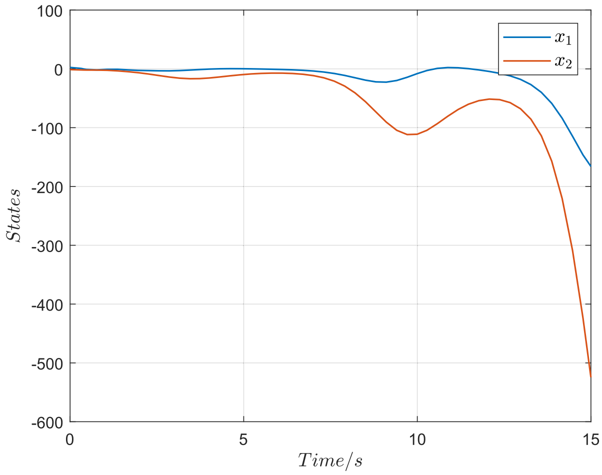

FIGURE 1. System's open loop response $x(t)$.

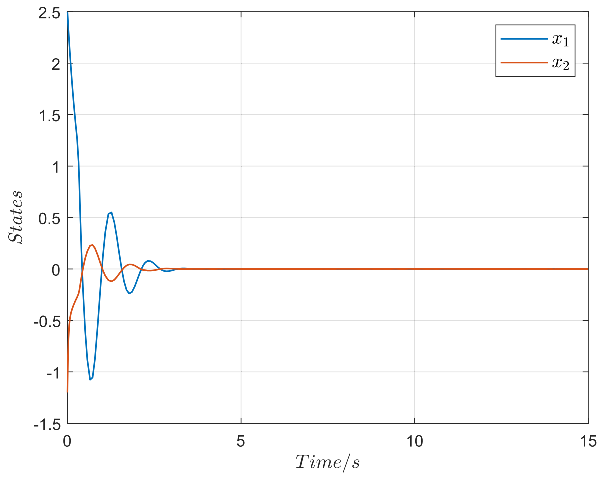

FIGURE 2. The response of system state $x(t)$.

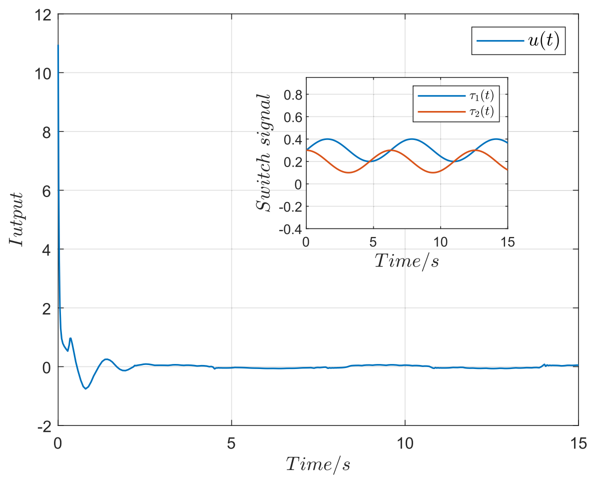

FIGURE 3. The response of control input $u(t)$.

obtained as $\gamma = 8.8513 \times 10^{-5}$. To verify the effect of the proposed method, it is assumed that the system initial value $x(t_0) = \begin{bmatrix} 2.5 & -1.2 \end{bmatrix}^T$. When the controller is not applied, the open-loop response of the system is shown in Figure 1. It is observed that the system state is divergent. Setting $\gamma =$

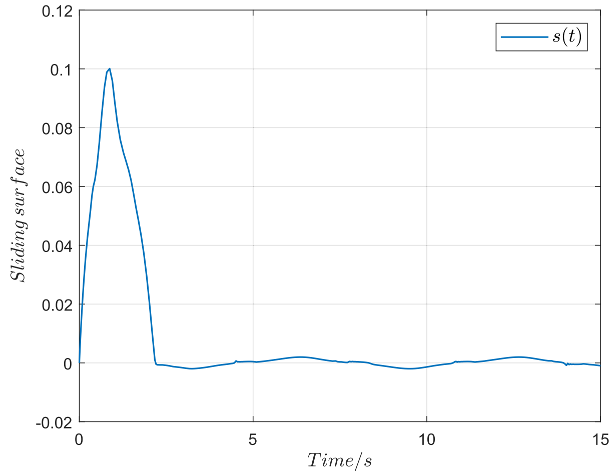

FIGURE 4. The response of sliding mode surface $s(t)$.

8.86×10^{-5}, the controller gain $K = \begin{bmatrix} -2.693 & -14.734 \end{bmatrix}$ can be obtained by using Theorem 2. The simulation results are provided in Figure 2-Figure 4. Figure 2 and Figure 3 depict, respectively, the responses of system state $x(t)$ and control input $u(t)$, which shows that the system state is gradually stabilized to 0 by the designed controller. Figure 4 plots the response of sliding mode surface $s(t)$ and internal subgraphs represent graphs of time-varying delays. It can be seen from Figure 1 that the open-loop system without controller K is unstable, while Figure 2 shows that the system is stable under the action of the designed controller K. The above simulations illustrate that the proposed integral sliding mode control strategy is effective.

V. CONCLUSION

In this paper, an integral sliding mode control method is proposed for nonlinear MASs with multiple time delays, external disturbances, and uncertainties. An integral sliding mode surface and the corresponding sliding mode dynamic model were established. Then, the Lyapunov method is used to obtain sufficient conditions that ensure the dynamic asymptotic stability of the sliding mode dynamics with H_∞ performance. In addition, the sliding mode controller parameter matrix is obtained. The sliding mode control law ensures that the system under consideration can be driven to the sliding mode surface. Finally, numerical examples and simulation results validate the effectiveness of the proposed method.

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