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RESEARCH ARTICLE

Finite-Time Output Feedback Stabilization for High-Order Stochastic Nonlinear Systems With Unknown Output Function

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ABSTRACT This study investigates the problem of finite-time output feedback stabilization in probability for a class of stochastic high-order nonlinear systems with unknown output function. By combining the homogeneous domination techniques, backstepping method and sign function, we obtain the output feedback controller that includes state controller and recursive reduced-order observer. Subsequently, we present an analytical method to deal with the unknown output function within the stochastic high-order nonlinear system such that the designed output feedback controller can make the closed-loop system globally finite-time stabilization in probability when the unknown output function belongs to the maximum open sector Ω. The numerical simulation results show that the proposed scheme is feasible.

INDEX TERMS Stochastic high-order nonlinear systems, finite-time output feedback stabilization, unknown output function.

I. INTRODUCTION

Stochastic nonlinear systems have been extensively used to model uncertain natural and social systems in various fields, such as finance [1], [2], engineering [3], [4] and agriculture [5]. However, the inherent properties of stochastic systems, such as uncertainty and complexity, bring about many challenges. Initially, researchers focused on the global state feedback control problems such as [6], [7] and [8]. As the stochastic system theory undergone rapid advancement and widespread popularity of the integral backstepping method. Through the novel integration of these two methods, [9] pioneered the development of a global output feedback stabilization method specifically for nonlinear stochastic systems. Building upon their work, further research has extended the output feedback problems to other special stochastic systems. References [10] and [11] solved the tracking problem of nonlinear stochastic systems with undetectable velocities and nonlinear stochastic systems with unstable linearization, respectively. Focusing on stochastic high-order nonlinear systems, [12] explored the adaptive fuzzy problem for p-normal order systems. Reference [13] proposed the stochastic homogeneous domination method, which solved the output feedback problem for more common systems and relaxed nonlinear growth conditions. It was the first time that [14] refined the stability analysis of nonlinear stochastic systems by introducing sign function as a tool. By adding novel power integrators and mappings, [15] considered stochastic planar nonlinear systems in which the output is constrained. The prescribed-time output-feedback stabilization method for high-order nonlinear systems introduced by [16] enhanced the controllability of the stabilization time.

In many practical applications, typically, there are requirements regarding the time for required system stabilization. Compared to asymptotic stability, finite-time stability often exhibits distinct characteristics, such as, higher precision. [17] discussed the issues of finite-time tracking control for nonlinear systems with unmodeled dynamics. Finite-time stability is also applied to general dynamical systems under

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an event-triggered control mechanism and time-varying systems [18], [19]. Reference [20] investigated the finite-time stabilization of impulsive systems, which have inevitable disturbance impulses. During recent decades, there have been many significant results in the field of stochastic nonlinear systems. Reference [21] first introduced the notion of finite-time stabilization into stochastic nonlinear systems, then proposed Lyapunov criteria for it, upon which the theory of finite-time stability for stochastic nonlinear systems rapidly developed [22], [23], [24]. Reference [25] proposed the most general finite-time Lyapunov criteria for stochastic nonlinear systems by relaxing differential operator constraints. In the works of [26], [27], and [28], the researches focused on the finite-time stability issue of stochastic high-order nonlinear systems with inverse dynamics, high-order and low-order nonlinearities, and output constraints, respectively. Reference [29] considered stochastic low-order nonlinear systems. In recent research, [30] investigated the adaptive finite-time tracking problem for stochastic nonlinear systems, while [31] focused on finite-time stability optimization issues. Reference [32] introduced the notion of finite-time stochastic integral input-to-state stability(FT-SiISS) to further complete the finite-time stability theorem.

It is noteworthy to emphasize that all of the above works assume that the specific information of the output. However, practical systems in specific applications often exhibit nonlinear, uncertain, and variable relationships between the transducers output (e.g., DC Output) and the system physical value (e.g., displacement, angle, and temperature). The infrared distance sensor studied in [33], the single-link robot system investigated in [34], and the DC-DC buck power converter discussed in [35] exhibited deviations from the simplest output function $y = z_1$. Therefore, investigating systems with unknown output function holds significant importance. During the recent period, a series of relevant results have been obtained. Reference [36] studied the output feedback problem in cascade systems. Several researchers have concentrated on achieving global adaptive output feedback stabilization in nonlinear systems with unknown output function [37], [38]. The adaptive output feedback control of nonlinear systems was achieved by [38], it does not require prior known information of known output function (e.g., upper and lower bounds, second-order derivative). References [39], [40], and [41] considered stochastic nonlinear systems with time-delay. Yet, these relevant findings only focused on first-order nonlinear systems. There are currently no related results available for the commonly encountered stochastic high-order nonlinear systems in practical applications.

Building upon the aforementioned observations, an important and intriguing question arises: how can we address the output feedback finite-time stabilization problem for high-order stochastic high-order nonlinear systems with unknown output function? Motivated by these insights, this paper addresses the above questions. The principal findings of this work can be outlined as:

(1) This study focuses on the output feedback finite-time stabilization of stochastic high-order nonlinear systems with more general output than simple $y = z_1$. The proposed control and analysis approach achieves system stabilization when the unknown output falls within the maximum open sector.

(2) Unlike [14], [23], and [42], this study not only considers the issue of output feedback finite-time stabilization for stochastic high-order nonlinear systems but also considers the unknown output function in more general. Compared with the first-order systems considered in [39], [40], and [41], we focus on stochastic high-order nonlinear systems for the first time, relaxing the constraints on the system order.

(3) The requirement of the output function to be differentiable is highly restrictive. The previous studies [43], [44], and [45] tend to specify that $\hbar(\cdot)$ to be continuously differentiable and bounded, while [46] relaxed the requirement for the $\hbar(\cdot)$ to be Lipschitz continuous. In contrast, in this paper, we relax the restriction on $\hbar(\cdot)$ to require that $\hbar(\cdot)$ is only continuous.

This paper is organized as follows. Section II gives some preliminaries and assumptions. In Section III, the design and analysis of finite-time output controller is presented, following a simulation example in Section IV. Section V provides the conclusion for this paper.

II. PRELIMINARIES KNOWLEDGE AND PROBLEM DESCRIPTION

A. PRELIMINARIES

Notations: R^+ stands for the set of all the non-negative real numbers. X denotes a given vector, its transpose denoted by $X^\top$, If X is a square matrix, then $\mathrm{Tr}\{X\}$ represents the trace of X. The Euclidean and Frobenius norms of vector X are represented by $\|X\|$ and $\|X\|_F$, respectively. Additionally, $\|X\|_F \triangleq (\mathrm{Tr}(X^\top X))^{\frac{1}{2}}$. The set denoting all functions with continuous i-th partial derivatives is represented as $\mathcal{C}^i$. For convenience, we use $\mathcal{X}$ to denote $\mathcal{X}(t)$ at a given time t. Sign function $sgn(x)$ is defined as: $sgn(x) = 1$ if $x > 0$, $sgn(x) = 0$ if $x = 0$, $sgn(x) = -1$ if $x < 0$.

For general stochastic nonlinear system:

$$\mathrm{d}z(t) = f(t, z(t))\mathrm{d}t + g^\top(t, z(t))\mathrm{d}\omega(t), \tag{1}$$

for any $0 \leq t$, which f and g are continuous functions and $f(t, 0) = 0$, $g(t, 0) = 0$.

Definition 1 [47]: For any $\mathcal{C}^2$ Lyapunov function $V(x(t))$ for system (1), $\mathcal{L}V$ denoted by $\mathcal{L}V = \frac{\partial V}{\partial x}f + \frac{1}{2}\mathrm{Tr}\{g\frac{\partial^2 V}{\partial x^2}g^\top\}$, $\mathcal{L}V$ is called differential operator, $\frac{1}{2}\mathrm{Tr}\{g\frac{\partial^2 V}{\partial x^2}g^\top\}$ is referred to as the Hessian term.

Definition 2 [48]: For a function $\hbar$: $R \to R$, if there exist two positive values ρ_1 and ρ_2, where $\rho_1 < \rho_2$, such that $0 \geq (\hbar(s) - \rho_1 s)(\hbar(s) - \rho_2 s)$, it is called included in $[\rho_1, \rho_2]$. The sector is denoted as (ρ_1, ρ_2), when the inequality is strict.

Definition 3 [22]: For any initial date $x_0 \in R^n$, when system (1) has a solution, the trivial solution of (1) is called to be finite-time attractive in probability, indicated by $x(t; x_0)$. Then, we denote stochastic settling time $\pi_0 = inf t : x(t; t_0) = 0$. Obviously, π_0 is finite and $P(\pi_0 < \infty) = 1$.

When the trivial solution of (1) satisfies the condition that there is a $\varsigma = (\varepsilon, \alpha) > 0$, such that $P(\|x(t; x_0)\| < \alpha$ for all $t \geq 0) \geq 1 - \varepsilon$, for every combination of $\varepsilon \in (0, 1)$ and $r > 0$, whenever $\|x_0\| \leq \varsigma$, it is referred to as stable in probability.

The trivial solution of (1) is referred to as finite-time stable, which implies that it satisfies the above two conditions.

Definition 4 [49]: For fixed coordinates $(x_1, \ldots, x_n)^\top \in R^n$ and real numbers $r_i > 0 : i = 1, \ldots, n$, the dilation $\Delta_\varepsilon(x) = (\varepsilon^{r_1} x_1, \ldots, \varepsilon^{r_n} x_n, \forall \varepsilon > 0$,with r_i being called the weights of the coordinates, we define dilation weight $\Delta = (r_1, \ldots, r_n)$; a function $V \in \mathcal{D}(R^n, R)$ is said to be homogeneous of degree τ if there is a real number $\tau \in R$ such that $\forall x \in R\{0\}$, $\varepsilon > 0$, $V(\Delta_\varepsilon(x)) = \varepsilon^{\tau + r_i} f_i(x_1, \ldots, x_n)$; a homogeneous p-norm is defined as $\||x\||_{\Delta,p} = (\sum_{i=1}^m |x_i|^{\frac{p}{r_i}})^{\frac{1}{p}}$, $\forall x \in R^n$, for a constant $p \geq 1$.

The lemmas required for this paper are as follows.

Lemma 1 [22]: For the stochastic nonlinear system (1), a $\mathcal{C}^2$ Lyapunov function $V(x(t))$ exists if we can find two class $\mathcal{K}_\infty$ functions α_1, α_2, $1 > \theta > 0$ and $\beta > 0$, for all $x \in R^n$, such that

$$\alpha_2(|x(t)|) \geq V(x(t)) \geq \alpha_1(|x(t)|)$$
$$-\beta V^\theta(x(t)) \geq \mathcal{L}V(x(t)),$$

Lemma 2 [47]: Assuming that the stochastic nonlinear system (1) is an autonomous system, the Lyapunov function V is a radially unbounded non-negative function, which means $\lim_{\|x\|\to\infty} V(x) = \infty$. For any initial data, a solution exists when $\mathcal{L}(x) \leq 0$, $\forall x \in R^n$.

Lemma 3 [49]: There exists two real numbers $a, b > 0$, and a positive real valued function $\tau(x, y)$, such that $|x^a y^b| \leq \frac{a}{a+b}\tau(x, y)|x|^{a+b} + \frac{b}{a+b}\tau^{-\frac{a}{b}}(x, y)|y|^{a+b}$.

Lemma 4 [50]: Consider a function $f(a, b)$ that are continuous, smooth functions $\alpha_1(a)$, $\beta_1(b) \geq 0$, $\alpha_2(a)$, $\beta_2(b) \geq 1$ can be find, such that $|f(a, b)| \leq \alpha_1(a) + \beta_1(b)$, $|f(a, b)| \leq \alpha_2(a)\beta_2(b)$, where $a \in R^m$, $b \in R^n$.

Lemma 5 [49]: If $p \geq 1$, $2^{\frac{p-1}{p}}(|\alpha| + |\beta|)^{\frac{1}{p}} \geq |\alpha|^{\frac{1}{p}} + |\beta|^{\frac{1}{p}} \geq |\alpha + \beta|^{\frac{1}{p}}$, $2^{p-1}|\alpha^p + \beta^p| \geq |\alpha + \beta|^p$. If odd integer $p \geq 1$, $2^{\frac{1}{p}}|\alpha - \beta|^{\frac{1}{p}} \geq |\alpha^{\frac{1}{p}} - \beta^{\frac{1}{p}}|$, $p|\alpha - \beta|(\alpha^{p-1} + \beta^{p-1}) \geq |\alpha^p - \beta^p|$, $c|\alpha - \beta||(\alpha - \beta)^{p-1} + \beta^{p-1}| \geq |\alpha^p - \beta^p|$ for any $\alpha, \beta \in \mathbb{R}$, where c is a positive constant.

Lemma 6 [49]: If $p \in R_{odd}^{\geq 1}$, consider any real numbers α, β, $-\frac{1}{2^{p-1}}(\alpha - \beta)^{p+1} \geq -(\alpha - \beta)(\alpha^p - \beta^p)$.

Lemma 7 [51]: For any $a,\ b \in R$, if $p = \frac{\alpha}{\beta} \in R_{odd}^{\geq 1}$, $\beta \geq 1$, then $2^{1-\frac{1}{\beta}}|sgn(a)|a|^\alpha - sgn(b)|b|^\alpha|^{\frac{1}{\beta}} \geq |a^p - b^p|$.

Lemma 8 [51]: If $f(a)$ is a sign function, we use $\lceil x \rceil$ denotes $f(a)$, where $\lceil a \rceil^\theta = sgn(a)|a|^\theta$ is C^1, $\dot{f}(a) = \theta|a|^{\theta-1}$, $\theta \geq 1$, $a \in R$.

B. PROBLEM ASSUMPTIONS

Throughout this study, the model of stochastic high-order nonlinear systems with unknown output function chosen for our examination is as follows:

$$\begin{aligned} \mathrm{d}z_i(t) &= (z_{i+1}^{p_i}(t) + \phi_i(z(t)))\mathrm{d}t + \psi_i^\top(z(t))\mathrm{d}\omega(t), \\ \mathrm{d}z_n(t) &= (u^{p_n}(t) + \phi_n(z(t)))\mathrm{d}t + \psi_n^\top(z(t))\mathrm{d}\omega(t), \\ y &= \hbar(z_1(t)), \end{aligned} \tag{2}$$

where $\bar{z}_i(t) = (z_1(t), \cdots, z_i(t))^\top \in R^n$, $i = 1, \cdots, n-1$, $y \in R$ and $u(t) \in R$ represents the state of system, control output and input, individually. For $i = 1, \cdots, n$, $p_i \in R_{\underset{\text{odd}}{\geq}} 1 \triangleq \{\frac{p}{q} \in R^+$: p and q are odd integers, $p \geq q\}$. System (2) is called as high-order system if there exists at least one $p_i > 1$. $\omega(t)$ represents an m-dimensional standard Wiener process defined within the framework of a complete probability space $(\Omega, \mathcal{F}, P)$, where Ω denotes the sample space, $\mathcal{F}$ signifies the filtration, and P stands for probability measure. The uncertain disturbances received by the system are denoted by the continuously differentiable $\phi_i : R^n \to R^n$ and $\psi_i : R^n \to R^n$, $i = 1, \cdots, n$, respectively. $\phi_i(0) = 0$, $\psi_i(0) = 0$. $\hbar(\cdot)$ represents an output function whose information is unknown to us.

Before proceeding with the controller design, we must make two assumptions about the system disturbance terms and the unknown output function. The following assumptions play a crucial role in the process of the controller design.

Assumption 1: There exists two positive constants c_1, c_2, $\tau \in (-\frac{1}{2\sum_{i=1}^{n} p_1 p_2 \cdots p_{i-1}}, 0)$ such that

$$|\phi_i(z(t))| \leq c_1 \sum_{l=1}^{i} |z_l(t)|^{\frac{r_i+\tau}{r_l}},$$
$$|\psi_i(z(t))| \leq c_2 \sum_{l=1}^{i} |z_l(t)|^{\frac{2r_i+\tau}{2r_l}}, \tag{3}$$

where $r_{i+1} = \frac{r_i+\tau}{p_i}$, $r_1 = \frac{1}{2}$ and $p_0 = 1$, $i = 1, \ \cdots, \ n$.

Assumption 2: The unknown output function $\hbar(\cdot)$ has the following properties (1) continuous; (2) the function value is zero at the origin.

Remark 1: Assumption 1 is a commonly used assumption condition as in [14] and [23], where $r_{i+1} = \frac{r_i+\tau}{p_i}$ is clearly the ratio of two odd numbers. Assumption 2 accounts for $\hbar(\cdot)$. In the context of this paper, $\hbar(\cdot)$ is considered to be only continuous, which is a weaker confidence. Compared to previous results, when the function $\hbar(\cdot) = z_1$, the system is not different from [14] and [23]. Therefore, the results of this study are also applicable to [14] and [23]. This paper differs from [52] in that it consider the stochastic terms of systems.

III. MAIN RESULT

A. DESIGN PROCESS OF THE OUTPUT FEEDBACK FOR SYSTEM (2)

Constructing the output feedback controller for system (2) involves three main components. Firstly,we transform a system with a gain with the help of a group of coordinate transformations. Secondly, for the nominal system, we are aim to construct state virtual controller and reduced-order observer. Thirdly, we examine the disturbance terms of stochastic nonlinear systems by some specific propositions and output feedback controller of system (2) is given.

Part I. Introduce the coordinates transformations

In order to convenience, we need to rewritten the system model (2) by the following coordinate transformation:

$$x_i(t)=\frac{z_i(t)}{L^{\theta_i}},\quad i=1,\cdots,n-1,\quad v(t)=\frac{u(t)}{L^{\theta_n}},\tag{4}$$

where $\theta_1=0$, $\theta_i=\frac{\theta_{i-1}+1}{p_{i-1}}$, $i=1,\cdots,n$, the gain $L\geq 1$ be selected latter. Therefore, system (2) becomes

$$\begin{aligned}\mathrm{d}x_i(t)&=(Lx_{i+1}^{p_i}(t)+f_i(x(t)))\mathrm{d}t+g_i^\top(x(t))\mathrm{d}\omega(t),\\ \mathrm{d}x_n(t)&=(Lv^{p_n}(t)+f_n(x(t)))\mathrm{d}t+g_n^\top(x(t))\mathrm{d}\omega(t),\\ y&=\hbar(x_1(t)).\end{aligned}\tag{5}$$

At the same time, with the help of coordinate transformation, Assumption 1 also can be rewritten as

$$\begin{aligned}|f_i(\cdot)|&\leq c_1L^{1-\nu}\sum_{l=1}^{i}|x_l(t)|^{\frac{r_i+\tau}{r_l}},\\ |g_i(\cdot)|&\leq c_2L^{1-2\bar\nu}\sum_{l=1}^{i}|x_l(t)|^{\frac{2r_i+\tau}{2r_l}},\end{aligned}\tag{6}$$

where $f_i=\frac{\phi_i}{L^{\theta_i}}$, $g_i=\frac{\psi_i}{L^{\theta_i}}$, $i=1,\cdots,n$. $\nu=\min\{0<\theta_i-\theta_j\frac{r_i+\tau}{r_j}+1\}$, $\bar\nu=\min\{0<\theta_i-\theta_j\frac{2r_i+\tau}{2r_j}+\frac{1}{2},\}$, $i\geq j\geq 1$, $n\geq i\geq 1$. Since $\phi_i(0)=0$, $\psi_i(0)=0$, it is obvious that $f_i(0)$, $g_i(0)$ have the same properties.

Part II. The design of state virtual controller and reduced-order observer for the nominal system (5)

Let's set aside the disturbance terms and focus on the designing of controller for the nominal system of (5).

$$\begin{aligned}\mathrm{d}x_i(t)&=Lx_{i+1}^{p_i}(t)\mathrm{d}t,\quad i=1,\cdots,n-1,\\ \mathrm{d}x_n(t)&=Lv^{p_n}(t)\mathrm{d}t,\\ y&=\hbar(x_1(t)).\end{aligned}\tag{7}$$

First Step: Taking $\xi_1=\lceil x_1\rceil^{\frac{1}{r_1}}$, $V_1(x_1)=\delta_1\frac{r_1}{4-\tau}|\xi_1|^{4-\tau}$ with $x_1^*=0$, $\delta_1>0$. By means of (9) and in accordance with Definition 1, we can get $\mathcal{L}V_1(x_1)=L\delta_1\lceil\xi_1\rceil^{4-r_2p_1}x_2^{p_1}$. The initial virtual controller $x_2^*=-(\frac{a_{1,1}}{\delta_1})^{p_1}\lceil\xi_1\rceil^{r_2}\triangleq-\beta_1^{r_2}\lceil\xi_1\rceil^{r_2}$ leads to $\mathcal{L}V_1(x_1)\leq -La_{1,1}\xi_1^4+L\delta_1\lceil\xi_1\rceil^{4-r_2p_1}(x_2^{p_1}-x_2^{*p_1})$, $a_{1,1}$ represents a constant that we will design in the subsequent part.

Inductive Step: Assume that at Step $i-1$, there is a $\mathcal{C}^2$, positive definite and radially unbounded Lyapunov function $V_{i-1}(\bar x_{i-1})$ and a series of virtual controllers $x_1^*,\cdots,x_i^*$ defined by

$$x_i^*=-\beta_{i-1}^{r_i}\lceil\xi_{i-1}\rceil^{r_i},\xi_i=\lceil x_i\rceil^{\frac{1}{r_i}}-\lceil x_i^*\rceil^{\frac{1}{r_i}},\ i=1,\cdots,n,\tag{8}$$

thus

$$\begin{aligned}\mathcal{L}V_{i-1}(\bar x_{i-1})\leq{}&-L\sum_{j=1}^{i-2}\Big(a_{j,j}-\sum_{l=j+1}^{i-1}\bar a_{l,j}-\tilde a_{j+1,j}\Big)\xi_j^4\\ &+L\delta_{i-1}\lceil\xi_{i-1}\rceil^{4-r_ip_{i-1}}(x_i^{p_{i-1}}-x_i^{*p_{i-1}})\\ &-La_{i-1,i-1}\xi_{i-1}^4,\end{aligned}\tag{9}$$

where $\bar a_{l,j}$, $\tilde a_{j+1,j}$ are nonnegative values, where $j=1,\cdots,i-2$, $l=j+1,\cdots,i-1$. $\bar x_{i-1}=(x_1,\cdots,x_{i-1})^\top$, $a_{1,1},\cdots,a_{i-1,i-1}>0$ be selected later, $\beta_1,\cdots,\beta_{i-1}>0$ are decided by $a_{1,1},\cdots,a_{i-1,i-1}$. Subsequently, we will proceed to verify the validity of inequality (9) in Step i.

Suppose the i-th Lyapunov function

$$\begin{aligned}V_i(\bar x_i)&=V_{i-1}(\bar x_{i-1})+W_i(\bar x_i),\\ W_i(\bar x_i)&=\delta_i\int_{x_i^*}^{x_i}\lceil\lceil s\rceil^{\frac{1}{r_i}}-\lceil x_i^*\rceil^{\frac{1}{r_i}}\rceil^{4-r_i-\tau}ds,\end{aligned}\tag{10}$$

which $\delta_1,\delta_2,\cdots,\delta_i$ are some positive constants. Thus,

$$\begin{aligned}&\mathcal{L}V_i(\bar x_i)\\ &\leq -L\sum_{j=1}^{i-2}\Big(a_{j,j}-\sum_{l=j+1}^{i-1}\bar a_{l,j}-\tilde a_{j+1,j}\Big)\xi_j^4-La_{i-1,i-1}\xi_{i-1}^4\\ &\quad+L\delta_{i-1}\lceil\xi_{i-1}\rceil^{4-r_ip_{i-1}}(x_i^{p_{i-1}}-x_i^{*p_{i-1}})+L\delta_i\lceil\xi_i\rceil^{4-r_{i+1}p_i}x_{i+1}^{*p_i}\\ &\quad+L\sum_{j=1}^{i-1}\frac{\partial W_i}{\partial x_j}x_{j+1}^{p_j}+L\delta_{i-1}\lceil\xi_i\rceil^{4-r_{i+1}p_i}(x_{i+1}^{p_i}-x_{i+1}^{*p_i}).\end{aligned}\tag{11}$$

Following that, we reckon the components on the right trems of (11).

Proposition 1: For any $i=2,\cdots,n-1$,

$$L\delta_{i-1}\lceil\xi_{i-1}\rceil^{4-r_ip_{i-1}}(x_i^{p_{i-1}}-x_1^{*p_{i-1}})\leq L\tilde a_{i,i-1}\xi_{i-1}^4+L\varrho_{i,1}\xi_i^4,$$

where $\tilde a_{i,i-1}$ and $\varrho_{i,1}$ are two positive constants, $\varrho_{i,1}$ is decided by the former.

Proposition 2: For any $i=2,\cdots,n-1$,

$$L\sum_{j=1}^{i-1}\frac{\partial W_i}{\partial x_j}x_{j+1}^{p_j}\leq L\sum_{j=1}^{i-1}\bar a_{i,j}\xi_j^4+L\varrho_{i,2}\xi_i^4,$$

where $\sum_{j=1}^{i-1}\bar a_{i,j}$ and $\varrho_{i,2}$ are positive constants, $\varrho_{i,2}$ is decided by $\sum_{j=1}^{i-1}\bar a_{i,j}$.

With the help of Propositions 1-2, Lemma 3, Lemma 8, choosing

$$x_{i+1}^*=-\left(\frac{a_{i,i}+\varrho_{i,1}+\varrho_{i,2}}{\delta_i}\right)^{\frac{1}{p_i}}\lceil\xi_i\rceil^{r_{i+1}}\triangleq-\beta_i^{r_{i+1}}\lceil\xi_i\rceil^{r_{i+1}},\tag{12}$$

so that

$$\begin{aligned}\mathcal{L}V_i(\bar x_i)\leq{}&-L\sum_{j=1}^{i-1}\Big(a_{j,j}-\sum_{l=j+1}^{i}\bar a_{l,j}-\tilde a_{j+1,j}\Big)\xi_j^4-La_{i,i}\xi_i^4\\ &+L\delta_i\lceil\xi_i\rceil^{4-r_i-\tau}(x_{i+1}^{p_i}-x_{i+1}^{*p_i}),\end{aligned}\tag{13}$$

where $a_{i,i}>0$ to be selected latter.

Therefore, at the last Step n, the existence of a virtual controller

$$x_{n+1}^* = -\beta_n^{r_{n+1}} \lceil \xi_n \rceil^{r_{n+1}}, \tag{14}$$

such that $V_n(\bar{x}_n) = V_{n-1}(\bar{x}_{n-1}) + W_n(\bar{x}_n)$ satisfies

$$\begin{aligned}\mathcal{L}V_n(x) \le & -L\sum_{j=1}^{n-1}\Big(a_{j,j} - \sum_{l=j+1}^{n}\bar{a}_{l,i} - \tilde{a}_{j+1,j}\Big)\xi_j^4 - La_{n,n}\xi_n^4 \\ & + L\delta_n\lceil\xi_n\rceil^{4-r_n-\tau}(v^{p_n} - x_{n+1}^{*p_n}). \end{aligned}\tag{15}$$

Subsequently, the reduced-order observer needs to be designed and introduced,

$$\begin{aligned} d\eta_i &= -Ll_{i-1}\hat{x}_{i-1}^{p_{i-1}}dt, \\ \hat{x}_i &= \lceil \eta_i + l_{i-1}\hat{x}_{i-1}\rceil^{\frac{r_i}{r_{i-1}}},\ i = 2, \cdots, n, \end{aligned}\tag{16}$$

where $l_1, \cdots, l_{n-1}$ represents the gains to be decided latter and $l_1, \cdots, l_{n-1}$ are positive constants. $\hat{x}_i$ represents the estimated value and $\hat{x}_1 = y$. With the help of (14) and reduced-order observer (16), the output feedback controller can be given:

$$\begin{aligned} v &= -\beta_n^{r_{n+1}}\lceil\hat{\xi}_n\rceil^{r_{n+1}}, \quad \hat{\xi}_i = \lceil\hat{x}_i\rceil^{\frac{1}{r_i}} - \lceil\hat{x}_i^*\rceil^{\frac{1}{r_i}}, \\ \hat{x}_i^* &= -\beta_{i-1}^{r_i}\lceil\hat{\xi}_{i-1}\rceil^{r_i}, \quad i = 2, \cdots, n. \end{aligned}\tag{17}$$

Define the observation error $e_i = (x_i^{p_{i-1}} - \hat{x}_i^{p_{i-1}})^{\frac{1}{r_ip_{i-1}}}$ $(i = 2, \cdots, n)$, choose the following Lyapunov function V

$$\begin{aligned} V &= V_n + U, \quad U = \sum_{i=2}^{n} U_i, \\ U_i &= \bar{m}_i \int_{\lceil\lambda_i\rceil^{\frac{4-\tau-r_{i-1}}{r_{i-1}}}}^{\lceil x_i\rceil^{\frac{4-\tau-r_{i-1}}{r_i}}} (\lceil s\rceil^{\frac{r_{i-1}}{4-\tau-r_{i-1}}} - \lambda_i)ds \end{aligned}\tag{18}$$

where $\lambda_i = \eta_i + l_{i-1}x_{i-1}$ and $\bar{m}_2, \cdots, \bar{m}_n > 0$. Evidently, V conforms to our previous requirements.

$$\begin{aligned} &\mathcal{L}V \\ &= \mathcal{L}V_n + L\sum_{i=2}^{n}\Big(\frac{\partial U_i}{\partial x_i}x_{i+1}^{p_i} + \frac{\partial U_i}{\partial x_{i-1}}x_i^{p_{i-1}} - \frac{\partial U_i}{\partial \eta_i}l_{i-1}\hat{x}_i^{p_{i-1}}\Big) \\ &\le -L\sum_{i=1}^{n-1}\Big(a_{i,i} - \sum_{l=i+1}^{n}\bar{a}_{l,i} - \tilde{a}_{i+1,i}\Big)\xi_i^4 - La_{n,n}\xi_n^4 \\ &\quad + L\sum_{i=2}^{n}\Big(\bar{m}_i\frac{4-\tau-r_{i-1}}{r_i}|x_i|^{\frac{4-\tau-r_{i-1}-r_i}{r_i}}\Big(\lceil x_i\rceil^{\frac{r_{i-1}}{r_i}} - \lambda_i\Big)x_{i+1}^{p_i} \\ &\quad - \bar{m}_il_{i-1}(x_i^{p_{i-1}} - \hat{x}_i^{p_{i-1}})\Big(\lceil x_i\rceil^{\frac{4-\tau-r_{i-1}}{r_i}} - \lceil\hat{x}_i\rceil^{\frac{4-\tau-r_{i-1}}{r_i}}\Big) \\ &\quad - \bar{m}_il_{i-1}(x_i^{p_{i-1}} - \hat{x}_i^{p_{i-1}})\Big(\lceil\hat{x}_i\rceil^{\frac{4-\tau-r_{i-1}}{r_i}} - \lceil\lambda_i\rceil^{\frac{4-\tau-r_{i-1}}{r_{i-1}}}\Big)\Big) \\ &\quad + L\delta_n\lceil\xi_n\rceil^{4-r_n-\tau}(v - x_{n+1}^{*p_n}). \end{aligned}\tag{19}$$

Proposition 3: For any $i = 2, \cdots, n-1$,

$$\begin{aligned} &\bar{m}_i\frac{4-\tau-r_{i-1}}{r_i}|x_i|^{\frac{4-\tau-r_{i-1}-r_i}{r_i}}(\lceil x_i\rceil^{\frac{r_{i-1}}{r_i}} - \lambda_i)x_{i+1}^{p_i} \\ &\le \sum_{j=i-1}^{i+1}c_{i,j,1}\xi_j^4 + o_{i,2}e_i^4 + \hbar_{i,1}(l_{i-1})e_{i-1}^4, \end{aligned}$$

where $\sum_{j=i-1}^{i+1}c_{i,j,1}$, $o_{i,2}$, $\hbar_{i,1}(l_{i-1})$ are some positive constants, $\hbar_{i,1}(l_{i-1})$ depends on l_{i-1}.

Proposition 4: For any $i = 3, \cdots, n$,

$$\begin{aligned} &-\bar{m}_il_{i-1}e_i^{r_ip_{i-1}}(\lceil\hat{x}_i\rceil^{\frac{4-\tau-r_{i-1}}{r_i}} - \lceil\lambda_i\rceil^{\frac{4-\tau-r_{i-1}}{r_{i-1}}}) \\ &\le \sum_{j=i-1}^{i}c_{i,j,2}\xi_j^4 + o_{i,1}e_i^4 + \hbar_{i,2}(l_{i-1})e_{i-1}^4, \end{aligned}$$

where $\sum_{j=i-1}^{i+1}c_{i,j,2}$, $o_{i,1}$, $\hbar_{i,2}(l_{i-1})$ are some positive constants, $\hbar_{i,2}(l_{i-1})$ depends on l_{i-1}.

Proposition 5: For any $i = 2, \cdots, n$,

$$\begin{aligned} &-\bar{m}_il_{i-1}e_i^{r_ip_{i-1}}(\lceil x_i\rceil^{\frac{4-\tau-r_{i-1}}{r_i}} - \lceil\hat{x}_i\rceil^{\frac{4-\tau-r_{i-1}}{r_i}}) \\ &\le -o_il_{i-1}e_i^4, \end{aligned}$$

where o_i is a positive constant.

Proposition 6:

$$\begin{aligned} &\bar{m}_n\frac{4-\tau-r_{n-1}}{r_n}|x_n|^{\frac{4-\tau-r_{n-1}-r_n}{r_n}}(\lceil x_n\rceil^{\frac{r_{n-1}}{r_n}} - \lambda_n)v^{p_n} \\ &\le \sum_{i=1}^{n}c_{i,n,1}\xi_i^4 + \bar{o}_i\sum_{i=2}^{n}e_i^4 + \hbar_{n,1}(l_{n-1})e_{n-1}^4 + \bar{o}_1(x_1-y)^4, \end{aligned}$$

where $\sum_{i=1}^{n}c_{i,n,1}$, $\bar{o}_i$, $\hbar_{n,1}(l_{n-1})$, $\bar{o}_1$ are some positive constants, $\hbar_{n,1}(l_{n-1})$ depends on l_{n-1}.

Proposition 7:

$$\delta_n\lceil\xi_n\rceil^{4-r_n-\tau}(v^{p_n} - x_{n+1}^{*p_n}) \le \sum_{i=1}^{n}\bar{c}_i\xi_i^4 + \tilde{c}_i\sum_{i=2}^{n}e_i^4,$$

where $\sum_{i=1}^{n}\bar{c}_i$, $\tilde{c}_i$ are several positive values.

With the help of Propositions 3-7, and (19) yields

$$\begin{aligned} \mathcal{L}V \le & -L\sum_{i=1}^{n-1}\Big(a_{i,i} - \sum_{l=i+1}^{n}\bar{a}_{l,i} - \tilde{a}_{i+1,i} - \sum_{j=i-1}^{i+1}c_{j,i,1} - \bar{c}_i \\ & - \sum_{j=i}^{i+1}c_{j,i,2} - c_{i,n,1}\Big)\xi_i^4 - L(a_{n,n} - c_{n,n,1} - \bar{c}_n)\xi_n^4 \\ & - L(o_nl_{n-1} - o_{n,1} - \bar{o}_n - \tilde{c}_n)e_n^4 - L\sum_{i=2}^{n-1}(o_il_{i-1} \\ & - o_{i,1} - o_{i,2} - \bar{o}_i - \hbar_{i+1,2}(l_i) - \hbar_{i+1,1}(l_i) - \tilde{c}_i)e_i^4 \\ & + L(\hbar_{2,1}(l_1) + \hbar_{2,2}(l_1) + \bar{o}_1)(x_1 - y)^4. \end{aligned}\tag{20}$$

For few positive constants $d_0, \tilde{d}_1, \cdots, \tilde{d}_n, \tilde{o}_{n,2}, \cdots, \tilde{o}_{n,n}$, via selecting

$$\begin{aligned} &a_{1,1} - \sum_{l=2}^{n}\bar{a}_{l,1} - \tilde{a}_{2,1} - \sum_{j=1}^{2}c_{j,1,1} - \sum_{j=1}^{2}c_{j,1,2} - c_{1,n,1} - \bar{c}_1 \\ &\ge \tilde{d}_1 + d_0, \end{aligned}$$

$$a_{i,i}-\sum_{l=i+1}^{n}\bar{a}_{l,i}-\tilde{a}_{i+1,i}-\sum_{j=i-1}^{i+1}c_{j,i,1}-\sum_{j=i}^{i+1}c_{j,i,2}-c_{i,n,1}-\bar{c}_i \geq \tilde{d}_i,\ i=2,\cdots,n-1,$$
$$a_{n,n}-c_{n,n,1}-\bar{c}_n \geq \tilde{d}_n, \tag{21}$$

and

$$l_{n-1} \geq \frac{\tilde{o}_{n-1}+o_{n,1}+\bar{o}_n+\tilde{c}_n}{o_n},$$
$$l_{i-1} \geq \frac{\tilde{o}_i+o_{i,1}+o_{i,2}+\bar{o}_i+\hbar_{i+1,1}(l_i)+\hbar_{i+1,2}(l_i)+\tilde{c}_i}{o_i},$$
$$l_1 \geq \frac{\tilde{o}_1+o_{2,1}+o_{2,2}+\bar{o}_1+\hbar_{3,1}(l_i)+\hbar_{3,2}(l_i)+\tilde{c}_1}{o_2}, \tag{22}$$

where $i=n-1,\cdots,3$, then, (20) can be written as

$$\mathcal{L}V \leq -Ld_0\xi_1^4-L\sum_{i=1}^{n}\tilde{d}_i\xi_i^4-L\sum_{i=2}^{n}\tilde{o}_ie_i^4 + L(\hbar_{2,1}(l_1)+\hbar_{2,2}(l_1)+\bar{o}_1)(x_1-y)^4. \tag{23}$$

Part III. Design of state virtual controller and reduced-order observer for the system (5)

Aim to (5), we construct controller and observer by using the similar constructions as (16), (17). Denote $\mathcal{X}=(x_1,\cdots,x_n,\eta_2,\cdots,\eta_n)^\top$, system (5), (16) and (17) can be represented by

$$\mathrm{d}\mathcal{X}=\Xi\mathrm{d}t+F\mathrm{d}t+G^\top\mathrm{d}\omega, \tag{24}$$

where $\Xi=(Lx_2^{p_1},\cdots,Lv^{p_n},-l_1L\hat{x}_2^{p_1},\cdots,-l_{n-1}L\hat{x}_n^{p_{n-1}})^\top$, $F=(f_1,\cdots,f_n,0,\cdots,0)^\top$, $G=(g_1,\cdots,g_n,0,\cdots,0)$. Construct the same Lyapunov function as V. In view of Definition 1, (23) and (24), as a result,

$$\begin{aligned}\mathcal{L}V \leq& -Ld_0\xi_1^4-L\sum_{i=1}^{n}\tilde{d}_i\xi_i^4-L\sum_{i=2}^{n}\tilde{o}_ie_i^4+L(\hbar_{2,1}(l_1)\\ &+\hbar_{2,2}(l_1)+\bar{o}_1)(x_1-y)^4+\frac{\partial V}{\partial\mathcal{X}}F+\frac{1}{2}\mathrm{Tr}\left\{G\frac{\partial^2V}{\partial\mathcal{X}^2}G^\top\right\}\\ \leq& -Ld_0\xi_1^4-L\sum_{i=1}^{n}\tilde{d}_i\xi_i^4-L\sum_{i=2}^{n}\tilde{o}_ie_i^4+L(\hbar_{2,1}(l_1)\\ &+\hbar_{2,2}(l_1)+\bar{o}_1)(x_1-y)^4+\sum_{i=1}^{n}\left|\frac{\partial V_n(\mathcal{X})}{\partial x_i}f_i\right|\\ &+\sum_{i=1}^{n}\left|\left(\frac{\partial U(\mathcal{X})}{\partial x_i}+\frac{\partial U(\mathcal{X})}{\partial\eta_i}\right)f_i\right|+\sum_{i=2}^{n}\left|\frac{\partial U(\mathcal{X})}{\partial x_{i-1}}f_{i-1}\right|\\ &+\frac{1}{2}\mathrm{Tr}\left\{G\frac{\partial^2V_n}{\partial\mathcal{X}^2}G^\top\right\}+\frac{1}{2}\mathrm{Tr}\left\{G\frac{\partial^2U}{\partial\mathcal{X}^2}G^\top\right\}\end{aligned} \tag{25}$$

We estimate (25) using some propositions and their detailed proofs process are provided in the Appendix.

Proposition 8:

$$\sum_{i=1}^{n}\left|\frac{\partial V_n(\mathcal{X})}{\partial x_i}f_i\right| \leq L^{1-\nu}\left(\sum_{i=1}^{n-1}\bar{a}_{n,i}\xi_i^4+\lambda_{n,1}\xi_n^4\right),$$

where $\bar{a}_{n,i},\lambda_{n,1}>0$.

Proposition 9:

$$\begin{aligned}\sum_{i=1}^{n}\left|\frac{\partial U(\mathcal{X})}{\partial x_{i-1}}f_{i-1}\right| \leq& L^{1-\nu}\Big(\sum_{i=1}^{n}\bar{\mu}_{i,1}\xi_i^4+\sum_{i=2}^{n}d_{i,1}e_i^4+\sum_{i=3}^{n}\rho_{i,1}(l_{i-1})\cdot e_{i-1}^4\Big)\\ &+L^{1-\nu}\rho_{2,1}(l_1)(x_1-y)^4\end{aligned}$$

where $\bar{\mu}_{i,1},d_{i,1}>0$.

Proposition 10:

$$\begin{aligned}\sum_{i=1}^{n}\left|\left(\frac{\partial U(\mathcal{X})}{\partial x_i}+\frac{\partial U(\mathcal{X})}{\partial\eta_i}\right)f_i\right| \leq& L^{1-\nu}\left(\sum_{i=1}^{n}\bar{\mu}_{i,2}\xi_i^4+\sum_{i=2}^{n}d_{i,2}e_i^4+\sum_{i=3}^{n}\rho_{i,2}(l_{i-1})e_{i-1}^4\right)\\ &+L^{1-\nu}\rho_{2,2}(l_1)(x_1-y)^4,\end{aligned}$$

where $\bar{\mu}_{i,2},d_{i,2}>0$.

Proposition 11: For any $i=1,\cdots,n$,

$$\frac{1}{2}\mathrm{Tr}\left\{G\frac{\partial^2V_n}{\partial\mathcal{X}^2}G^\top\right\} \leq L^{1-2\bar{\nu}}\left(\sum_{i=1}^{n-1}\bar{\mu}_{i,3}\xi_i^4+\lambda_{i,2}\xi_n^4\right),$$

where $\bar{\mu}_{i,3},\lambda_{i,2}$ are positive constants.

Proposition 12: For any $i=1,\cdots,n$,

$$\frac{1}{2}\mathrm{Tr}\left\{G\frac{\partial^2U}{\partial\mathcal{X}^2}G^\top\right\} \leq L^{1-2\bar{\nu}}\left(\sum_{i=1}^{n-1}\bar{\mu}_{i,4}\xi_i^4+\lambda_{i,4}\xi_n^4\right),$$

where $\bar{\mu}_{i,4},\lambda_{i,4}$ are the positive constants.

According to Propositions 8-12, we arrive at

$$\begin{aligned}\mathcal{L}V \leq& -Ld_0\xi_1^4-L\sum_{i=1}^{n}\tilde{d}_i\xi_i^4-L\sum_{i=2}^{n}\tilde{o}_ie_i^4\\ &+L^{1-\nu}\left(\sum_{i=1}^{n}\bar{c}_{i,1}\xi_i^4+\sum_{i=2}^{n}\bar{c}_{i,1}e_i^4\right)\\ &+L^{1-2\bar{\nu}}\sum_{i=1}^{n}\bar{c}_{i,2}\xi_i^4+L(\hbar_{2,1}(l_1)\\ &+\hbar_{2,2}(l_1)+\bar{o}_1+\rho_{2,1}(l_1)+\rho_{2,2}(l_1))(x_1-y)^4,\end{aligned} \tag{26}$$

where $\bar{c}_{i,1},\bar{c}_{i,2}$ and $\tilde{c}_{i,1}>0$.

Using (5), (17) and (18), the output feedback controller of system (2) can be sorted as follow:

$$\begin{aligned}&u=-L(\sum_{i=1}^{n}\bar{\beta}_i\lceil\hat{\xi}_i\rfloor^{\frac{1}{r_i}})^{r_{n+1}},\ \hat{\xi}_i=\lceil\hat{x}_i\rfloor^{\frac{1}{r_i}}-\lceil\hat{x}_i^*\rfloor^{\frac{1}{r_i}},\\ &\hat{x}_i^*=-\beta_{i-1}^{r_i}\lceil\hat{\xi}_{i-1}\rfloor^{r_i},\qquad \mathrm{d}\eta_i=-l_{i-1}L\hat{x}_i^{p_{i-1}}\mathrm{d}t,\\ &\hat{x}_i=\lceil\eta_i+l_{i-1}\hat{x}_{i-1}\rfloor^{\frac{r_i}{r_{i-1}}},\quad i=2,\cdots,n.\end{aligned} \tag{27}$$

B. UNKNOWN OUTPUT FUNCTION AND FINITE-TIME STABILITY ANALYSIS

In this section,by presenting the following theorem and its proof, we address the handling of the system unknown output function and conduct an analysis of the system finite-time stability.

Theorem 1: In the presence of the Assumptions 1-2, a maximal open sector Ω can be found, we define $\Omega = (1-\bar{\rho}, 1+\bar{\rho})$, $\bar{\rho} > 0$. If the unknown output function $\hbar(\cdot)$ lies within any closed sector contained in Ω, the output feedback controller (27) can make system(1) globally finite-time stability in probability.

Proof.: We firstly select $a_{1,1}, \cdots, a_{n,n}, \beta_1, \cdots, \beta_n$. By the first step, for any given δ_1,$\bar{a}_{2,1}$, $\cdots$,$\bar{a}_{n,1}$, $\tilde{a}_{2,1}$, $c_{1,1,1}$,$c_{2,1,1}$, $c_{1,1,2}$,$c_{2,1,2}$,$c_{1,n,1}$,$\bar{c}_1$, d_0, $\tilde{d}_1$, by (21), $a_{1,1}$ can be chosen. By $\beta_1 = \frac{a_{1,1}}{\delta_1}$, β_1 is obtained. During the second step, δ_2 is an any constant, via Proposition 1, Proposition 2, we know $\varrho_{2,1}, \varrho_{2,2}$. For any given $\bar{a}_{3,2}$,$\cdots$,$\bar{a}_{n,2}$,$\tilde{a}_{3,2}$,$c_{1,2,1}$, $c_{2,2,1}$,$c_{3,2,1}$,$c_{2,2,2}$,$c_{3,2,2}$, $c_{2,n,1}$,$\bar{c}_2$ and $\tilde{d}_2$, according (21), $a_{2,2}$ can be selected and β_2 from (12). Step by step, until the nth step, for any given δ_n, from Proposition 1, Proposition 2, $\varrho_{n,1}, \varrho_{n,2}$ also can be calculated. For any $\delta_n, c_{n,n,1}, \bar{c}_n, \tilde{d}_n$, according to (21), we can select $a_{n,n}$. By (12), β_n can be obtained.

Secondly, our goal is to select $l_{n-1}, \cdots, l_1$. For the given $\delta_n, \bar{c}_n, c_{n,n,1}$ and any given $\bar{m}_n$, from Proposition 6 and Proposition 7, $\tilde{c}_{n,1}$ and $\bar{o}_n$ can be chosen. Thus, for any given o_n,$o_{n,1}$,$\tilde{o}_{n-1}$, by (22), l_{n-1} can be selected. For the given δ_n,$c_{n-2,n-1,1}$,$c_{n-1,n-1,1}$,$c_{n-,n-1,1}$,$c_{n-1,n-1,2}$,$c_{n,n-1,2}$, $c_{n-1,n,1}$,$\bar{c}_{n-1}$ and any given m_{n-1}, by Propositions 3-5, one can get $o_{n-1,1}$, $o_{n-1,2}$, $\bar{o}_i$, o_{n-1}, $\hbar_{n,1}(l_{n-1})$, $\hbar_{n,2}(l_{n-1})$, $\tilde{c}_{n-1,1}$. By (22) and any constant $\tilde{o}_{n-1}$, l_{n-2} can be chosen. Step by step, $l_{n-3}, \cdots, l_1$ can be listed respectively.

Lastly, we begin to find the Ω of $\hbar(\cdot)$ and to discuss the stability of system (2). Define $\hbar_2(l_1)$=$\hbar_{2,1}(l_1) + \hbar_{2,2}(l_1) + \bar{o}_1 + \tilde{c}_{1,1} + \rho_{2,2}(l_1) + \rho_{2,1}(l_1)$, $\rho = \rho(l_1) = \sqrt{d_0/\hbar_2(l_1)}$. The supremum of ρ can then be determined. Defining $U_l = \{l = (l_1, \cdots, l_{n-1}) | a_{1,1} - \sum_{l=2}^{n} \bar{a}_{l,1} - \tilde{a}_{2,1} - \sum_{j=1}^{2} c_{j,1,1} - \sum_{j=1}^{2} c_{j,1,2} - c_{1,n,1} - \bar{c}_1 \geq \tilde{d}_1 + d_0, a_{i,i} - \sum_{l=i+1}^{n} \bar{a}_{l,i} - \tilde{a}_{i+1,i} - \sum_{j=i-1}^{i+1} c_{j,i,1} - \sum_{j=i}^{i+1} c_{j,i,2} - c_{i,n,1} - \bar{c}_i \geq \tilde{d}_i, i = 2, \cdots, n-1, a_{n,n} - c_{n,n,1} - \bar{c}_n \geq \tilde{d}_n, k_{n-1} \geq \tilde{o}_{n-1} + o_{n,1} + \bar{o}_n + \tilde{c}_{n,1}/o_n, k_{i-1} \geq \tilde{o}_i + o_{i,1} + o_{i,2} + \bar{o}_i + \hbar_{i+1,1}(l_i) + \hbar_{i+1,2}(l_i) + \tilde{c}_{i,1}/o_i, i = n-1, \cdots, 3, k_1 \geq \tilde{o}_1 + o_{2,1} + o_{2,2} + \bar{o}_1 + \hbar_{3,1}(l_i) + \hbar_{3,2}(l_i) + \tilde{c}_{1,1}/o_2\}$, $\kappa = (\delta_1, \cdots, \delta_n, \tilde{a}_{2,1}, \cdots, \tilde{a}_{n,n-1}, \bar{a}_{2,1}, \bar{a}_{3,1}, \bar{a}_{3,2}, \cdots, \bar{a}_{n,1}, \cdots, \bar{a}_{n,n-1}, \bar{m}_2, \cdots, \bar{m}_n, c_{1,2,1}, c_{2,2,1}, \cdots, c_{n,n,1}, c_{2,2,2}, c_{3,2,2}, \cdots, c_{1,n,1}, \cdots, c_{n,n,1}, \bar{c}_1, \cdots, \bar{c}_n, o_{2,1}, o_{2,2}, \cdots o_{n,1}, \bar{o}_1, \cdots, \bar{o}_n, o_1, \cdots, o_n, \tilde{c}_{1,1}, \cdots, \tilde{c}_{n,1}, \tilde{d}_0, \tilde{d}_1, \cdots, \tilde{d}_n, \tilde{o}_1, \cdots, \tilde{o}_n)$, one can select constant

$$\bar{\rho} = \sup_{\kappa \in R^+, l \in U_l} \rho(l_1). \tag{28}$$

Label the supremum of ρ. Therefore, by Definition 2, $\Omega = (1-\bar{\rho}, 1+\bar{\rho})$. Suppose there exists ε, where $0 < \varepsilon < \bar{\rho}$. During $\hbar(x_1)$ of (5) included in $[1+\varepsilon-\bar{\rho}, 1-\varepsilon+\bar{\rho}] \subset \Omega$.

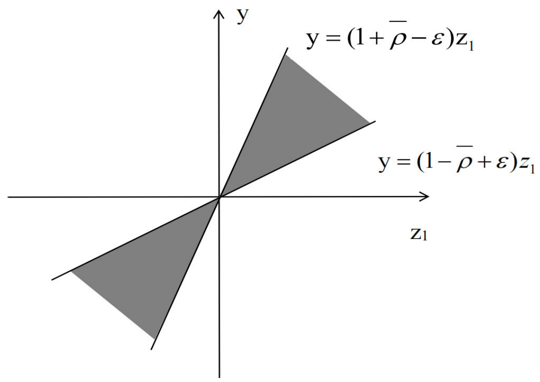

FIGURE 1. The sector of $\hbar(z_1)$.

According to Definition 2, we obtain

$$\begin{aligned} 0 &\geq (\hbar(x_1) - x_1(1+\bar{\rho}-\varepsilon))(\hbar(x_1) - x_1(1-\bar{\rho}+\varepsilon)) \\ &\Rightarrow x_1(\hbar(x_1) - (1-\bar{\rho}+\varepsilon)x_1) \geq 0 \geq x_1(\hbar(x_1) - (1+\bar{\rho}-\varepsilon)x_1) \\ &\Rightarrow x_1^2(1+\bar{\rho}-\varepsilon) \geq x_1\hbar(x_1) \geq x_1^2(1-\bar{\rho}+\varepsilon) \\ &\Rightarrow x_1^2(\bar{\rho}-\varepsilon) \geq x_1(\hbar(x_1) - x_1) \geq x_1^2(-\bar{\rho}+\varepsilon) \\ &\Rightarrow |x_1|(\bar{\rho}-\varepsilon) \geq |x_1 - \hbar(x_1)| \\ &\Rightarrow \xi_1^4(\bar{\rho}-\varepsilon)^4 \geq |x_1 - y|^4. \end{aligned} \tag{29}$$

From (28), (26) is reformulated as

$$\begin{aligned} \mathcal{L}V \leq &-L\sum_{i=1}^{n} \tilde{d}_i \xi_i^4 - L\sum_{i=2}^{n} \tilde{o}_i e_i^4 + L^{1-\nu}\Bigg(\sum_{i=1}^{n} \bar{c}_{i,1}\xi_i^4 \\ &+ \sum_{i=2}^{n} \tilde{c}_{i,1} e_i^4\Bigg) + L^{1-2\bar{\nu}} \sum_{i=1}^{n} \bar{c}_{i,2}\xi_i^4. \end{aligned} \tag{30}$$

By choosing

$$L \geq \max_{i=1,\cdots,n, j=2,\cdots,n} \left\{ \left(\frac{4c_{i,1}}{\tilde{d}_i}\right)^{\frac{1}{\nu}}, \left(\frac{4c_{i,2}}{\tilde{d}_i}\right)^{\frac{1}{2\bar{\nu}}}, \left(\frac{2\tilde{c}_{j,1}}{\tilde{o}_j}\right)^{\frac{1}{\nu}}, 1 \right\}. \tag{31}$$

Therefore, (30) becomes

$$\mathcal{L}V \leq -L\sum_{i=1}^{n} \frac{\tilde{d}_i}{2}\xi_i^4 - L\sum_{i=2}^{n} \frac{\tilde{o}_i}{2} e_i^4. \tag{32}$$

Based on the definition of r_i, we choose the dilation weight

$$\Delta = (\underbrace{r_1, r_2, \cdots, r_n}_{for\ z_1,\cdots,z_n}, \underbrace{r_1, r_2, \cdots, r_{n-1}}_{for\ \eta_1,\cdots,\eta_{n-1}}). \tag{33}$$

It is not difficult to see that system (24) is homogeneous of degree τ. Therefore, V and the right terms of (24) is homogeneous of degree $4-\tau$ and 4, respectively. So, it is easy to get that there exist two constants $\zeta_1 > 0, \zeta_2 > 0$ can make

$$V \leq \zeta_1 \|\mathcal{X}\|_{\Delta}^{4-\tau}, \quad \mathcal{L}V \leq -\zeta_2 \|\mathcal{X}\|_{\Delta}^{4}. \tag{34}$$

Hence, a constant $\zeta > 0$ can ensure

$$\mathcal{L}V \leq -\zeta \|V\|_{\Delta}^{\frac{4}{4-\tau}}. \tag{35}$$

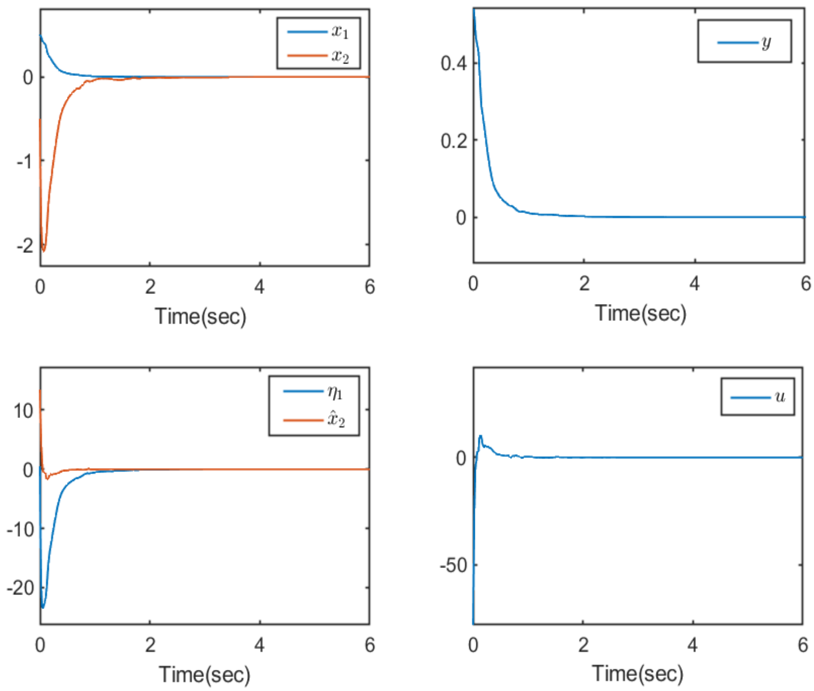

FIGURE 2. Response curves of state x_1, x_2,unknown output function y, state observer η_2, estimate value $\hat{x}_2$ and control input u.

Subsequently, we will demonstrate following the first hitting timeπ_{X_0}, the solution $X(t+\pi_{X_0}) = 0$ for all $t \geq 0$ almost sure. The solution of the closed-loop system, comprising (1) and (17), is uniquely defined immediately after the first hitting time π_{X_0}.

Define the stopping time $\pi_n = \inf t \geq \pi_{X_0};\ |X(t;X_0)| \geq n$. It is evident that π_n is a growing sequence of stopping times. By the Itô formula, it can easy obtain:

$$\begin{aligned} &EV(X((t+\pi_{X_0}) \wedge \pi_n)) \\ &\quad = EV(X((\pi_{X_0} \wedge \pi_n)) + E\int_{\pi_{X_0}}^{(t+\pi_{X_0})\wedge\pi_n} \mathcal{L}V(X(s))ds \\ &\qquad + E\int_{\pi_{X_0}}^{(t+\pi_{X_0})\wedge\pi_n} \frac{\partial V(X(s))}{\partial X} G^\top(X(s))d\omega(s) \\ &\quad = E\int_{\pi_{X_0}}^{(t+\pi_{X_0})\wedge\pi_n} \mathcal{L}V(X(s))ds. \end{aligned} \tag{36}$$

Due to the positive definiteness of $V(X)$, we can get $EV(X((t+\pi_{X_0})\wedge\pi_n)) = 0$, which means $X((t+\pi_{X_0})\wedge\pi_n) = 0$ almost surely, for all $t \geq 0$. As n approaches ∞, we observe that $X((t+\pi_{X_0}) = 0$ for all $t \geq 0$ with almost sure certainty. According to Lemma 1, Lemma 2 and Definition 3, we can get the system (2) is finite-time stability in probability.

Remark 2: In previous studies on unknown output function, [45] employed an observer to estimate the system's state and subsequently design a controller based on the estimated state, while [34] by adjusting the controller parameters based on the error between the system output and the estimated output to adapt to the variations in the unknown output function. The assumptions regarding unknown output function differ slightly between the two approaches; [45] requires the unknown output function to be continuously differentiable and [34] requires the unknown output function to be Liphchitz continuity. However, for stochastic systems, the handling of differential operators often involves second-order derivatives. Therefore, the assumptions in the works of [34] and [45] may not be applicable when dealing with such systems. For system (2) satisfying Assumption 1, by finding the Ω of $\hbar(\cdot)$, $\hbar(\cdot)$ only needs to belong to any closed subinterval of Ω, the output feedback controller (27) is feasible for guaranteeing globally finite-time stability in probability of system (2).

What needs highlighting is that such output functions are universal. We use the shadow region of FIGURE 1 to show the sector of $\hbar(\cdot)$.

IV. ILLUSTRATIVE EXAMPLES

Take into account the second-order stochastic nonlinear system described below

$$\begin{aligned} dz_1 &= (z_2^{\frac{11}{9}} + 0.1\sin z_1)dt + 0.1\sin z_1 d\omega, \\ dz_2 &= (u + 0.025(z_1^{\frac{81}{110}} + z_2^{\frac{1620}{1719}}))dt, \\ y &= \hbar(z_1). \end{aligned} \tag{37}$$

At the beginning, choose $\tau = -\frac{9}{400} \in (-\frac{9}{40}, 0)$. It is not difficult to know $r_1 = \frac{1}{2}$, $r_2 = \frac{1719}{4400}$, $r_3 = \frac{81}{220}$, $\lambda_1 = 0$, $\lambda_2 = \frac{9}{11}$, $\lambda_3 = \frac{20}{11}$. Observing the above system, we can also get $p_1 = \frac{11}{9}$, $p_2 = 1$. Assumption 1 holds obviously.

Following the design procedure, one introduces the coordinate change $x_1 = z_1, x_2 = \frac{z_2}{L^{\frac{9}{11}}}, v = \frac{u}{L^{\frac{20}{11}}}$, then the above system (37) becomes

$$\begin{aligned} dx_1 &= (Lx_2^{\frac{11}{9}} + 0.1\sin x_1)dt + 0.1\sin x_1 d\omega, \\ dx_2 &= (Lv + 0.025(x_1^{\frac{19}{20}} + L^{-\frac{9}{11}}x_2^{\frac{1620}{1719}})dt, \\ y &= \hbar(x_1). \end{aligned} \tag{38}$$

Take $\kappa = (1, 0.2, 0.3, 1, 0.3, 0.1, 0.1, 0.2, 0.1, 0.2, 0.1, 0.4, 0.4, 0.4, 0.2, 0.5, 0.1, 0.3, 0.4, 0.3)$, $l_1 = 50$, $\beta_1 = 1$, $\beta_2 = 25$, the sector $\sup_{\kappa\in R^+, l\in U_l} \rho(l_1) = 0.053$ can be obtained. Choosing $L = 1.5$ and output function $\hbar(z_1) = z_1 + 0.1\sin z_1$, which is continuous and belongs to [0.947, 1.053]. It is worth mentioning that [0.947, 1.053] is included in Ω. Under the controller

$$\begin{aligned} u &= -L^{\frac{20}{11}}\beta_2^{\frac{81}{220}}\lceil \beta_1^{\frac{1719}{4400}}\lceil y\rfloor^2 + \lceil \hat{x}_2\rfloor^{\frac{4400}{1719}}\rfloor^{\frac{81}{220}}, \\ \hat{x}_2^{\frac{11}{9}} &= \lceil \eta_2 + l_1 y\rfloor^{\frac{191}{200}}, \quad d\eta_2 = -Ll_1\hat{x}_2^{\frac{11}{9}}dt, \end{aligned} \tag{39}$$

by selecting the initial conditions $(x_1(0), x_2(0), \eta_2(0)) = (0.5, -0.5, 0.5)$, FIGURE 2 demonstrates the feasibility of the controller designed in this study.

V. CONCLUSION

This work solves the issue of finite-time stability in probability by output feedback for a set of stochastic high-order nonlinear systems with unknown output function. Through the integration of the homogeneous domination techniques, backstepping method, sign function, selecting Lyapunov function that matches the characteristics of stochastic nonlinear systems, then the output-feedback controller can be obtained. Subsequently, Theorem 1 presents an analytical method to get maximum open sector Ω. By combining finite-time stability theory for stochastic nonlinear systems, the designed output feedback controller can make the closed-loop system globally finite-time stability in probability. Lastly, the feasibility of the proposed approach is shown by numerical simulation results.

APPENDIX

For the convenience of subsequent calculations, we estimate some important terms for the entire calculation by Lemma 3 and Lemma 5. For $i = 1, \cdots, n,\ \ j = 1, \cdots, i-1$,

$$\begin{aligned} &\frac{\partial(-\lceil x_i^*\rfloor^{\frac{1}{r_i}})}{\partial x_j} \\ &= \sum_{k=1}^{i-1}\frac{\partial\left(\prod_{l=k}^{i-1}\beta_l\right)}{\partial x_j}\lceil x_k\rfloor^{\frac{1}{r_k}} + \frac{\prod_{l=j}^{i-1}\beta_l}{r_j}|x_j|^{\frac{1}{r_j}-1} \\ &\le \sum_{k=1}^{i-1}(1+\beta_{k-1})(\prod_{l=k}^{i-1}\beta_l)|1+\xi_k^2+\xi_{k-1}^2|^{\frac{r_j}{2}}(|\xi_{k-1}|^{1-r_j}+|\xi_k|^{1-r_j}) \\ &\quad + \frac{\prod_{l=k}^{i-1}\beta_l(1+\beta_{j-1})^{1-r_j}}{r_j}(|\xi_{j-1}|^{1-r_j} + |\xi_j|^{1-r_j}) \\ &\le \gamma_{j,1}\sum_{k=1}^{i-1}|\xi_k|^{1-r_j}, \end{aligned} \tag{A.1}$$

where $\gamma_{j,1}$ are positive values.

$$\begin{aligned} &\frac{\partial^2(-\lceil x_i^*\rfloor^{\frac{1}{r_i}})}{\partial x_j^2} \\ &= \sum_{k=1}^{i-1}\frac{\partial^2\left(\prod_{l=k}^{i-1}\beta_l\right)}{\partial^2 x_j}\lceil x_k\rfloor^{\frac{1}{r_k}} + 2\frac{\partial\left(\prod_{l=j}^{i-1}\beta_l\right)}{\partial x_j r_j}|x_j|^{\frac{1}{r_j}-1} \\ &\quad + \frac{(1-r_j)\prod_{l=j}^{i-1}\beta_l}{r_j^2}\lceil x_j\rfloor^{\frac{1}{r_j}-2} \\ &\le \sum_{k=1}^{i-1}(1+\beta_{k-1})(\prod_{l=k}^{i-1}\beta_l)|1+\xi_{k-1}^2+\xi_k^2|^{r_j}(|\xi_{k-1}|^{1-2r_j}+|\xi_k|^{1-2r_j}) \\ &\quad + \frac{2(1+\beta_{j-1})^{1-r_j}}{r_j}(\prod_{l=j}^{i-1}\beta_l)|1+\xi_{k-1}^2+\xi_k^2|^{\frac{r_j}{2}}(|\xi_{j-1}|^{1-2r_j}+|\xi_j|^{1-2r_j}|) \\ &\quad + \frac{(1-r_j)\prod_{l=k}^{i-1}\beta_l(1+\beta_{j-1})^{1-2r_j}}{r_j^2}(|\xi_{j-1}|^{1-2r_j} + |\xi|^{1-2r_j}) \\ &\le \gamma_{j,2}\sum_{k=1}^{i-1}|\xi_k|^{1-2r_j}, \quad j = 1, \ldots, i-1, \end{aligned} \tag{A.2}$$

where $\gamma_{j,2}$ are positive values.

$$\begin{aligned} &\frac{\partial^2(-|x_i^*|^{\frac{1}{r_i}})}{\partial x_k\partial x_j} = \frac{\partial^2(-|x_i^*|^{\frac{1}{r_i}})}{\partial x_j\partial x_k} \\ &\le \sum_{k=1}^{i-1}(1+\beta_{l-1})(\prod_{l=k}^{i-1}\beta_l)|1+\xi_{l-1}^2+\xi_l^2|^{\frac{r_j+r_k}{2}}(|\xi_{l-1}|^{1-r_k-r_j} \\ &\quad + |\xi_l|^{1-r_k-r_j}) + \frac{\prod_{l=k}^{i-1}\beta_l(1+\beta_{j-1})^{1-r_j}}{r_j}|1+\xi_{j-1}^2+\xi_j^2|^{\frac{r_j}{2}} \\ &\quad \cdot(|\xi_{j-1}|^{1-r_k-r_j} + |\xi_j|^{1-r_k-r_j}) + \frac{\prod_{l=j}^{i-1}\beta_l(1+\beta_{k-1})^{1-r_j}}{r_j} \\ &\quad \cdot|1+\xi_{k-1}^2+\xi_k^2|^{\frac{r_k}{2}}(|\xi_{k-1}|^{1-r_j-r_k} + |\xi_k|^{1-r_j-r_k}) \\ &\le \gamma_{j,3}\sum_{k=1}^{i-1}|\xi_k|^{1-r_k-r_j}, \quad k = 1, \cdots, i-1, \end{aligned} \tag{A.3}$$

where $\gamma_{j,3}$ are positive values.

Proof of Proposition 1: According to Lemma 3,

$$\begin{aligned} &\delta_{i-1}\lceil\xi_{i-1}\rfloor^{4-r_ip_{i-1}}(x_i^{p_{i-1}} - x_i^{*p_{i-1}}) \\ &\le L\delta_{i-1}\lceil\xi_{i-1}\rfloor^{4-r_{i+1}-\tau}\left((x_i^{\frac{1}{r_i}})^{r_ip_{i-1}} - (x_i^{*\frac{1}{r_i}})^{r_ip_{i-1}}\right) \\ &\le L\, 2^{1-r_ip_{i-1}}\delta_{i-1}\lceil\xi_{i-1}\rfloor^{4-r_ip_{i-1}}\lceil\xi_i\rfloor^{r_ip_{i-1}} \\ &\le L\tilde{a}_{i,i-1}\xi_{i-1}^4 + L\varrho_{i,1}\xi_i^4. \end{aligned}$$

Proof of Proposition 2: Based on (8), (9), (A.1) and Lemma 3, Lemma 5 and Lemma 7,

$$L\sum_{j=1}^{i-1}\frac{\partial W_i}{\partial x_j}x_{j+1}^{p_j}$$

$$=L\sum_{j=1}^{i-1}(4-r_i-\tau)\left|\frac{\partial(-\lceil x_i^*\rceil^{\frac{1}{r_i}})}{\partial x_j}\right|\left|\int_{x_i^*}^{x_i}\left|\lceil s\rceil^{\frac{1}{r_i}}-\lceil x_i^*\rceil^{\frac{1}{r_i}}\right|^{3-r_i-\tau}ds\cdot x_{j+1}^{p_j}\right.$$
$$\leq L\sum_{j=1}^{i-1}(4-r_i-\tau)\gamma_{j,1}\sum_{k=1}^{i-1}|\xi_k|^{1-r_j}|\xi_i|^{3-\tau}|\xi_j|^{r_{j+1}p_j}$$
$$\leq L\sum_{j=1}^{i-1}\bar{a}_{i,j}\xi_j^4+L\varrho_{i,2}\xi_i^4.$$

Proof of Proposition 3: By using Lemma 3,

$$|x_{i-1}-\hat{x}_{i-1}|=|(x_{i-1})^{p_{i-2}}-(\hat{x}_{i-1})^{p_{i-2}}|\leq 2^{1-\frac{1}{p_{i-2}}}|e_{i-1}|^{r_{i-1}},$$
$$|\lceil x_i^{p_{i-1}}\rceil^{\frac{r_{i-1}}{r_ip_{i-1}}}-\lceil\hat{x}_i^{p_{i-1}}\rceil^{\frac{r_{i-1}}{r_ip_{i-1}}}|\leq d_i|e_i|^{r_{i-1}}+\hat{d}_i|x_i|^{\frac{r_{i-1}}{r_i}}. \tag{A.4}$$

Following Lemma 3 and Lemma 4, one has

$$\left|x_i^{\frac{4-\tau-r_{i-1}-r_i}{r_i}}x_{i+1}^{p_i}\right|$$
$$\leq c(|\xi_i|^{r_{i-1}}+|\xi_{i-1}|^{r_{i-1}})(|\xi_{i+1}|^{r_{i+1}p_i}+|\xi_i|^{r_{i+1}p_i}|)$$
$$\leq(|\xi_{i-1}|^{4-r_{i-1}}+|\xi_i|^{4-r_{i-1}}+|\xi_{i+1}|^{4-r_{i-1}}). \tag{A.5}$$

Using (A.4), (A.5), we can get

$$\bar{m}_i\frac{4-\tau-r_{i-1}}{r_i}|x_i|^{\frac{4-\tau-r_{i-1}-r_i}{r_i}}(\lceil x_i\rceil^{\frac{r_{i-1}}{r_i}}-\lambda_i)x_{i+1}^{p_i}$$
$$\leq\bar{m}_i\frac{4-\tau-r_{i-1}}{r_i}x_i^{\frac{4-\tau-r_{i-1}-r_i}{r_i}}(|x_i^{p_{i-1}}-\hat{x}_i^{p_{i-1}}|^{\frac{r_{i-1}}{r_ip_{i-1}}}$$
$$-k_{i-1}|x_{i-1}-\hat{x}_{i-1}|)x_{i+1}^{p_i}$$
$$\leq\sum_{j=i-1}^{i+1}c_{i,j,1}\xi_j^4+o_{i,2}e_i^4+\hbar_{i,1}(l_{i-1})e_{i-1}^4.$$

Proof of Proposition 4: By Lemma 3, Lemma 5 and (A.4),

$$-\bar{m}_il_{i-1}e_i^{r_ip_{i-1}}(\lceil\hat{x}_i\rceil^{\frac{4-\tau-r_{i-1}}{r_i}}-\lceil\lambda_i\rceil^{\frac{4-\tau-r_{i-1}}{r_{i-1}}})$$
$$=-\bar{m}_il_{i-1}|e_i|^{r_ip_{i-1}}\left|\lceil\eta_i+l_{i-1}\hat{x}_{i-1}\rceil^{\frac{4-\tau-r_{i-1}}{r_i}}-\lceil\eta_i+l_{i-1}x_{i-1}\rceil^{\frac{4-\tau-r_{i-1}}{r_{i-1}}}\right|$$
$$\leq-\bar{m}_il_{i-1}^2|e_i|^{r_ip_{i-1}}|e_{i-1}|^{r_{i-1}}(|\hat{x}_i|^{\frac{4-\tau-2r_{i-1}}{r_i}}+|l_{i-1}e_i|^{4-\tau-2r_{i-1}})$$
$$\leq\sum_{j=i-1}^{i}c_{i,j,2}\xi_j^4+o_{i,1}e_{i-1}^4+\hbar_{i,2}(l_{i-1})e_i^4.$$

Proof of Proposition 5: Following Lemma 6, (A.4) and definition of e_i,

$$-\bar{m}_il_{i-1}e_i^{r_ip_{i-1}}(\lceil x_i\rceil^{\frac{4-\tau-r_{i-1}}{r_i}}-\lceil\hat{x}_i\rceil^{\frac{4-\tau-r_{i-1}}{r_i}})$$
$$\leq-\bar{m}_il_{i-1}e_i^{r_ip_{i-1}}2^{1-\frac{4-\tau-r_{i-1}}{r_ip_{i-1}}}|x_i^{p_{i-1}}-\hat{x}_i^{p_{i-1}}|^{\frac{4-\tau-r_{i-1}+r_ip_{i-1}}{r_ip_{i-1}}}$$
$$\leq-o_il_{i-1}e_i^4.$$

Proof of Proposition 6: Following Lemma 3 and the homogeneity of v,

$$\bar{m}_n\frac{4-\tau-r_{n-1}}{r_n}|x_n|^{\frac{4-\tau-r_{n-1}-r_n}{r_n}}(\lceil x_n\rceil^{\frac{r_{n-1}}{r_n}}-\lambda_n)v^{p_n}$$
$$\leq(|\xi_n|^{4-\tau-r_{n-1}-r_n}+|\xi_{n-1}|^{4-\tau-r_{n-1}-r_n})(\sum_{i=1}^{n}|x_i|^{\frac{r_n+\tau}{r_i}}+\sum_{i=1}^{n}|e_i|^{r_n+\tau})$$
$$\cdot(c|e_i|^{r_{i-1}}+c(|\xi_i|^{r_{i-1}}+|\xi_{i-1}|^{r_{i-1}})+l_{i-1}2^{1-\frac{1}{p_{i-2}}}|e_{i-1}|^{r_{i-1}})$$
$$\leq\sum_{i=1}^{n}c_{i,n,1}\xi_i^4+\sum_{i=2}^{n}\bar{o}_ie_i^4+\hbar_{n,1}(l_{n-1})e_{n-1}^4+\bar{o}_1(x_1-y)^4.$$

Proof of Proposition 7: Based on (17) and Lemma 5,

$$\delta_n\lceil\xi_n\rceil^{4-r_n-\tau}(v^{p_n}-x_{n+1}^{*p_n})$$
$$\leq|\xi_n|^{4-r_n-\tau}\left(|\hat{x}_n^{\frac{1}{r_n}}-x_n^{\frac{1}{r_n}}|^{r_n+\tau}+|\hat{x}_{n-1}^{\frac{1}{r_{n-1}}}-x_{n-1}^{\frac{1}{r_{n-1}}}|^{r_n+\tau}\right.$$
$$\left.+\cdots+|\hat{x}_2^{\frac{1}{r_2}}-x_2^{\frac{1}{r_2}}|^{r_n+\tau}\right)$$
$$\leq c|\xi_n|^{4-r_n-\tau}\left(\sum_{i=2}^{n}|x_i^{\frac{1}{r_i}}-\hat{x}_i^{\frac{1}{r_i}}|\right)^{r_{n+1}p_n}$$
$$\leq c|\xi_n|^{4-r_n-\tau}\sum_{i=2}^{n}\left(|e_i|^{r_np_{n+1}}+|\xi_i|^{r_np_{n+1}}+|\xi_{i-1}|^{r_np_{n+1}}\right)$$
$$\leq\sum_{i=1}^{n}\bar{c}_i\xi_i^4+\sum_{i=2}^{n}\tilde{c}_ie_i^4.$$

Proof of Proposition 8: By using Lemmas 3-5, (A.1) and Integral mean value theorem,

$$\sum_{i=1}^{n}\left|\frac{\partial V_n(\mathcal{X})}{\partial x_i}f_i\right|$$
$$\leq\sum_{i=1}^{n-1}(4-r_{n+1}p_n)\int_{x_n^*}^{x_n}|\lceil s\rceil^{\frac{1}{r_n}}-|x_n^*|^{\frac{1}{r_n}}|^{3-r_{n+1}p_n}ds\left|\frac{\partial(-\lceil x_n^*\rceil^{\frac{1}{r_n}})}{\partial x_i}\right||f_i|$$
$$+|\xi_n|^{4-r_{n+1}p_n}|f_n|$$
$$\leq L^{1-\nu}\sum_{i=1}^{n-1}c_12^{1-r_n}(4-r_{n+1}p_n)\gamma_{i,1}|\xi_n|^{3-\tau}\sum_{j=1}^{i}|\xi_i|^{1-r_j}$$
$$\cdot\sum_{l=1}^{j+1}|\xi_l|^{r_{j+1}p_j}+c_1L^{1-\nu}|\xi_n|^{4-r_{n+1}p_n}\sum_{l=1}^{n}|\xi_l|^{r_{n+1}p_n}$$
$$\leq L^{1-\nu}\left(\sum_{i=1}^{n-1}\bar{a}_{n,i}\xi_i^4+\lambda_{n,1}\xi_n^4\right).$$

Proof of Proposition 9: With the help of Lemma 3 and (A.5),

$$\sum_{i=1}^{n}\left|\frac{\partial U(\mathcal{X})}{\partial x_{i-1}}f_{i-1}\right|$$
$$\leq\sum_{i=1}^{n}|\bar{m}_il_{i-1}(\lceil x_i\rceil^{\frac{4-\tau-r_{i-1}}{r_i}}-\lceil\lambda_i\rceil^{\frac{4-\tau-r_{i-1}}{r_{i-1}}}||f_{i-1}|$$
$$\leq L^{1-\nu}\sum_{i=1}^{n}|m_il_{i-1}c_1|e_i|^{4-\tau-r_{i-1}}\sum_{j=1}^{i-1}|\xi_j|^{r_{i-1}+\tau}|+L^{1-\nu}\sum_{i=1}^{n}|\bar{m}_i$$
$$\cdot l_{i-1}c_1|e_{i-1}|^{r_{i-1}}\sum_{j=1}^{i-1}|\xi_j|^{r_{i-1}+\tau}(|e_i|^{r_i}+|\xi_i|^{r_i}|\xi_{i-1}|^{r_i})^{\frac{4-\tau-2r_{i-1}}{r_i}}$$
$$+L^{1-\nu}\sum_{j=1}^{i-1}|\xi_j|^{r_{i-1}+\tau}\sum_{i=1}^{n}|m_il_{i-1}c_1\|l_{i-1}e_{i-1}|^{4-\tau-r_{i-1}}\sum_{j=1}^{i-1}|\xi_j|^{r_{i-1}+\tau}$$

$$\le L^{1-v}\Big(\sum_{i=1}^{n}\bar{\mu}_{i,1}\xi_i^4+\sum_{i=2}^{n}d_{i,1}e_i^4+\sum_{i=3}^{n}\rho_{i,1}(l_{i-1})e_{i-1}^4\Big)$$
$$+L^{1-v}\rho_{2,1}(l_1)(x_1-y)^4.$$

Proof of Proposition 10: With the help of Lemma 3. It is not difficult to get that $\frac{r_{i-1}}{r_ip_{i-1}}=\frac{r_{i-1}}{r_{i-1}+\tau}>1$. Based on (24) and Lemma 5,

$$\sum_{i=1}^{n}\left|\left(\frac{\partial U(\mathcal{X})}{\partial x_i}+\frac{\partial U(\mathcal{X})}{\partial \eta_i}\right)f_i\right|$$
$$\le\sum_{i=1}^{n}|\bar{m}_i\frac{4-\tau-r_{i-1}}{r_i}|x_i|^{\frac{4-\tau-r_{i-1}-r_i}{r_i}}(\lceil x_i\rceil^{\frac{r_{i-1}}{r_i}}-\lambda_i)||f_i|$$
$$\le\sum_{i=1}^{n}|m_i\frac{4-\tau-r_{i-1}}{r_i}|x_i|^{\frac{4-\tau-r_{i-1}-r_i}{r_i}}(|x_i^{p_{i-1}}-\hat{x}_i^{p_{i-1}}|^{\frac{r_{i-1}}{r_ip_{i-1}}}$$
$$-l_{i-1}|x_{i-1}-\hat{x}_{i-1}|)||f_i|$$
$$\le L^{1-v}\left(\sum_{i=1}^{n}\bar{\mu}_{i,2}\xi_i^4+\sum_{i=2}^{n}d_{i,2}e_i^4+\sum_{i=3}^{n}\rho_{i,2}(l_{i-1})e_{i-1}^4\right)$$
$$+L^{1-v}\rho_{2,2}(l_1)(x_1-y)^4.$$

Proof of Proposition 11: Using Lemmas 3-5, Integral mean value theorem, (A.1), (A.2) and (A.3), one get

$$\frac{1}{2}\mathrm{Tr}\left\{G\frac{\partial^2V_n}{\partial\mathcal{X}^2}G^\top\right\}$$
$$\le\frac{1}{2}\sum_{i=1}^{n}\Bigg(\sum_{k,j=1,k\neq j}^{i}\left|\frac{\partial^2W_i}{\partial x_k\partial x_j}\right||g_k||g_j|+\sum_{k=1}^{i-1}\left|\frac{\partial^2W_i}{\partial x_k^2}\right|||g_k||^2$$
$$+2\sum_{k=1}^{i-1}\left|\frac{\partial^2W_i}{\partial x_i\partial x_k}\right||g_i||g_k|+\left|\frac{\partial^2W_i}{\partial x_i^2}\right|\|g_i\|^2\Bigg)$$
$$\le\frac{1}{2}\sum_{i=1}^{n}\Bigg(\Big[\sum_{k=1,j=1,k\neq j}^{i-1}(4-r_{i+1}p_i)(3-r_{i+1}p_i)$$
$$\cdot\left|\int_{x_i^*}^{x_i}\left\lceil\lceil s\rceil^{\frac{1}{r_i}}-\lceil x_i^*\rceil^{\frac{1}{r_i}}\right\rceil^{2-r_{i+1}p_i}ds\right|\left|\frac{\partial\lceil x_i^*\rceil^{\frac{1}{r_i}}}{\partial x_k}\right|\left|\frac{\partial\lceil x_i^*\rceil^{\frac{1}{r_i}}}{\partial x_j}\right|$$
$$+(4-r_{i+1}p_i)\left|\int_{x_i^*}^{x_i}\left\lceil\lceil s\rceil^{\frac{1}{r_i}}-\lceil x_i^*\rceil^{\frac{1}{r_i}}\right\rceil^{3-r_{i+1}p_i}ds\right|\left|\frac{\partial^2\lceil x_i^*\rceil^{\frac{1}{r_i}}}{\partial x_k\partial x_j}\right|$$
$$\cdot|g_k||g_j|\Big]+\sum_{j=1}^{i}\Big[\Big((4-r_{i+1}p_i)(3-r_{i+1}p_i)\left|\frac{\partial\lceil x_i^*\rceil^{\frac{1}{r_i}}}{\partial x_k}\right|^2$$
$$\cdot\left|\int_{x_i^*}^{x_i}\left\lceil\lceil s\rceil^{\frac{1}{r_i}}-\lceil x_i^*\rceil^{\frac{1}{r_i}}\right\rceil^{2-r_{i+1}p_i}ds+(4-r_{i+1}p_i)\right|\int_{x_i^*}^{x_i}\left\lceil\lceil s\rceil^{\frac{1}{r_i}}\right.$$
$$\left.-\lceil x_i^*\rceil^{\frac{1}{r_i}}\right\rceil^{3-r_{i+1}p_i}ds\left|\left|\frac{\partial^2\lceil x_i^*\rceil^{\frac{1}{r_i}}}{\partial x_j^2}\right|\Big)||g_j||^2\Big]+2\sum_{j=1}^{i-1}(4-r_{i+1}p_i)$$
$$\cdot|\xi_i|^{3-r_{i+1}p_i}\left|\frac{\partial\lceil x_i^*\rceil^{\frac{1}{r_i}}}{\partial x_k}\right|\|g_i\|\|g_k\|+\frac{4-r_{i+1}p_i}{r_i}|\xi_i|^{3-r_{i+1}p_i}$$
$$\cdot|x_i|^{\frac{1}{r_i}-1}\|g_i\|^2\Bigg)$$

$$\le L^{1-2\bar{v}}\sum_{i=1}^{n}\Bigg(\alpha_1\sum_{k,j=1,k\neq j}^{i-1}\Big(|\xi_i|^{2-\tau}\sum_{l=1}^{i-1}|\xi_l|^{1-r_j}\sum_{l=1}^{i-1}|\xi_l|^{1-r_k}$$
$$+|\xi_i|^{3-\tau}\sum_{l=1}^{i-1}|\xi_l|^{1-r_k-r_j}\Big)\sum_{l=1}^{j}|\xi_l|^{\frac{2r_j+\tau}{2}}\sum_{l=1}^{k}|\xi_l|^{\frac{2r_k+\tau}{2}}$$
$$+\alpha_2\sum_{j=1}^{i-1}\Big(|\xi_i|^{2-\tau}\big(\sum_{l=1}^{i-1}|\xi_l|\big)^2+|\xi_i|^{3-\tau}\sum_{l=1}^{i-1}|\xi_l|^{1-2r_j}\Big)\big(\sum_{l=1}^{i}|\xi_l|^{\frac{2r_j+\tau}{2}}\big)^2$$
$$+\alpha_3|\xi_i|^{3-r_{i+1}p_i}\sum_{j=1}^{i-1}\sum_{l=1}^{i-1}|\xi_l|^{1-r_j}\sum_{l=1}^{i}|\xi_l|^{\frac{2r_i+\tau}{2}}\cdot\sum_{l=1}^{j}|\xi_l|^{\frac{2r_j+\tau}{2}}$$
$$+\alpha_4|\xi_i|^{3-r_{i+1}p_i}(|\xi_{i-1}|^{1-r_i}+|\xi_i|^{1-r_i})\big(\sum_{l=1}^{i}|\xi_l|^{\frac{2r_i+\tau}{2}}\big)^2\Bigg)$$
$$\le L^{1-2\bar{v}}\Big(\sum_{i=1}^{n-1}\bar{\mu}_{i,3}\xi_i^4+\lambda_{i,2}\xi_n^4\Big).$$

Proof of Proposition 12: With the help of Lemma 3, (8) and (18),

$$\frac{1}{2}\mathrm{Tr}\left\{G\frac{\partial^2U}{\partial\mathcal{X}^2}G^\top\right\}$$
$$\le\frac{1}{2}\sum_{i=1}^{n}\Bigg(\sum_{k,j=1,k\neq j}^{i}\left|\frac{\partial^2U_i}{\partial x_k\partial x_j}\right|\|g_k\|\|g_j\|+\sum_{j=1}^{i}\left|\frac{\partial^2U_i}{\partial x_j^2}\right|\|g_j\|^2$$
$$+\sum_{j=1}^{i}\left|\frac{\partial^2U_i}{\partial\eta_j^2}\right|\|g_j\|^2+\sum_{j=1}^{i}\left|\frac{\partial^2U_i}{\partial x_j\partial\eta_{j-1}}\right|\|g_j\|\|g_{j-1}\|$$
$$+2\sum_{j=1,k=2}^{i}\left|\frac{\partial^2U_i}{\partial x_j\partial\eta_k}\right|\|g_j\|\|g_k\|\Bigg)$$
$$\le\frac{1}{2}\sum_{i=1}^{n}\Bigg(\Big(\sum_{j=1}^{i}(\bar{m}_i\frac{4-\tau-r_{j-1}}{r_j}|x_j|^{\frac{4-\tau-r_{j-1}-2r_j}{r_j}}(|x_j|^{\frac{r_{j-1}}{r_j}}-\lambda_j)$$
$$+\bar{m}_i\frac{4-\tau-r_{j-1}}{r_j}|x_j|^{\frac{4-\tau-r_{j-1}-r_j}{r_j}}|x_j|^{\frac{r_{j-1}-r_j}{r_j}})||g_j||^2$$
$$+\sum_{j=1,k=j-1}^{i}(-\bar{m}_j\frac{4-\tau-r_{j-1}}{r_j}l_{i-1}|x_j|^{p_{j-1}})\|g_j\|\|g_{j+1}\|\Bigg)$$

$$\le\frac{1}{2}\sum_{i=1}^{n}L^{1-2\bar{v}}\Bigg(\alpha_5\sum_{j=1}^{i}|\xi_j|^{4-\tau-r_{j-1}-2r_j+r_{j-1}}\sum_{l=1}^{i}|\xi_l|^{2r_j+\tau}$$
$$+\alpha_6\sum_{j=1,k=j-1}^{i}|\xi_j|^{4-\tau-r_j-r_{j-1}}\sum_{l=1}^{j}|\xi_l|^{\frac{2r_j+\tau}{2}}\sum_{l=1}^{j-1}|\xi_l|^{\frac{2r_{j-1}+\tau}{2}}\Bigg)$$
$$\le L^{1-2\bar{v}}\Big(\sum_{i=1}^{n-1}\bar{\mu}_{i,4}\xi_i^4+\lambda_{i,4}\xi_n^4\Big).$$

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