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THEORY

Consensus Control of Multi-Agent Systems by Intermittent Brownian Noise Stabilization Scheme

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ABSTRACT This study explores the consensus of multi-agent systems in the presence of ambient noise using intermittent Brownian noise stabilization. Firstly, the research provides a mathematical explanation of multi-agent systems and intermittent stochastic noise, and establishes a sufficient condition for multi-agent systems to achieve consensus using intermittent stochastic noise control input. Secondly, a consensus criterion is proposed for a class of multi-agent systems that are affected by ambient noise. Finally, the simulation results demonstrate that the intermittent stochastic noise stabilization technique can facilitate the establishment of consensus in multi-agent systems.

INDEX TERMS Consensus, intermittent Brownian noise, multi-agent systems, stochastic stabilization.

I. INTRODUCTION

The natural phenomenon of coordinated flight observed in creatures such as ants initially inspired humans to invent multi-agent systems. Today, this concept is believed to have numerous practical applications. Coordinated control of multi-agent systems has become a popular topic in control science and engineering, owing to advancements in computer, complex network, and communication technologies. These systems are frequently utilized in traffic networks, UAV formation, and communication systems. Using a distributed controller, multi-agent control enables numerous agents to maintain consensus while following the lead agent. The achievement of consensus in multi-agent systems implies that the state of each participant remains consistent with that of the leader throughout the movement process. The consensus problem is the most important research object in multi-agent system research.

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Vicsek published the first paper on multi-agent systems consensus in 1995 [1]. He created a model to mimic the consensus phenomenon, which occurs when a collection of particles all move in the same direction. The theoretical basis for multi-agent dynamic network consensus is established in [2]. In some early studies, most of them focused on deterministic systems [3], [4], [5], [6], [7], [8]. Among them, the distributed consensus problem of multi-agent systems on directed networks has been studied [7]. The composite rotational consensus problem for second-order multi agent systems with leader and non-uniform time delays is studied [8]. However, because there are several types of noises in reality, it is important to examine the dynamic characteristics of multi-agent systems when random noises are present. Itô introduced the idea of stochastic differential equations in 1951, which was the first time a mathematical model including random noise was formed, and the stochastic analysis theory has significantly grown since then. Numerous studies on multi-agent systems with stochastic noise have been published, such as multi-agent systems with multiplicative noises, which were investigated in stochastic consensus [9], [10]. Besides that, a novel consensus stability condition of stochastic multi-agent systems with communication noise was provided in the instance that the consensus gain function did not meet the robustness condition of the consensus stability [11]; the consensus requirements of stochastic multi-agent systems with noise and communication delays were presented [12].

Noises are often regarded as a type of interference in control systems. Random noise, on the other hand, has been found in certain studies to be helpful to system stability. Khasminskii used two white noises to stabilize a specific system [13], and Mao advocated, for example, using the Brownian motion to stabilize and destabilize the system [14]. In addition, a great number of results on stochastic stabilization have emerged one after another [15], [16], [17], [18], [19], [20], [21], [22]. On the other hand, intermittent control research has progressed to a point where significant accomplishments have been made [23], [24], [25], [26], [27], [28]. Inspired by the foregoing conclusions, an efficient approach for reducing the cost of control systems, namely intermittent stochastic noise stabilization, was discussed, where the stochastic noise is discontinuous and disappears intermittently. For instance, Zhang et al. studied the stabilization and destabilization of nonlinear systems using intermittent stochastic noise [29], as well as the stochastic stability of nonlinear systems using the intermittent Brownian motion under random disturbance [30]. Using the stochastic comparison principle, the Itô's formula, and the Borel-Cantelli lemma, Liu et al. investigated the stochastic intermittent stabilization issue based on discrete-time or delay time feedback [31], as well as stochastic stabilization by use of hybrid control strategies [32]. However, with the exception of [33], there have been few studies on multi-agent stochastic intermittent stabilization. Wu et al. explored the exponential consensus of multi-agent systems using aperiodically intermittent discrete-time state observation noise in [33].

Motivated by the above discussion, this paper proposes a stochastic intermittent noise stabilization technique to address the consensus problem of multi-agent systems affected by stochastic ambient noise. Initially, the multi-agent systems with environmental noise are transformed into a general stochastic differential system that requires stabilization. Next, a discontinuous Brownian motion control input is designed to stabilize the system in the presence of continuous stochastic ambient noise disturbances, which is a unique approach compared to previous studies. The noise intensity can be altered by maintaining a fixed control period, thereby modifying the lower limit of the selection range of noise width. The contributions of this work are: the application of intermittent stochastic noise to the stabilization of multi-agent systems with environmental noise; the method is convenient and flexible to select an appropriate control time for intermittent control.

The remainder of the study is organized as follows. Section II provides a comprehensive introduction to the mathematical representations, definitions, lemmas, and notations utilized in this study. In Section III, the mathematical model for multi-agent systems is presented, and the consensus of generic multi-agent systems through intermittent stochastic noise, including the associated consensus requirements, is discussed. In Section IV, the consensus of multi-agent systems impacted by environmental noise through intermittent stochastic noise is addressed, and the corresponding consensus conditions are proposed. Section V presents simulation examples for different scenarios, and concluding remarks are provided in the final section.

II. PRELIMINARIES

First, we'll go through some of the notations that will be utilized throughout this paper. $\mathbb{N}$ stands for the set of natural numbers; $C^{1,2}([t_0,\infty)\times\mathbb{R}^n;\mathbb{R}^+)$ represents the family of all nonnegative functions $V(t,x)$ on $[t_0,\infty)\times\mathbb{R}^n$ is continuous and differentiable first with respect to t, second with respect to x. The notation $|a|$ is the module of vector a. Let $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\le t_0},\mathbb{P})$ be a complete probability space with a growing and right continuous filtration $\{\mathcal{F}_t\}_{t\le t_0}$, and $\mathcal{F}_{t_0}$ contains all $\mathbb{P}$-null sets. The norm of A matrix is expressed as $\|A\|=\sqrt{\lambda_{\max}(A^{\mathrm{T}}A)}$. The Kronecker product of two matrices A and C is $A\otimes C$, $|\lambda(A)|_{\min}$ represents the minimum absolute value of the non-zero eigenvalue of A. All functions are assumed to meet the Lipschitz condition in this work.

The multi-agent systems is as follows

$$\dot{x}_i(t)=Px(t)+u_i(t) \tag{1}$$

where $x_i(t)\in\mathbb{R}^n$ is the state of agent i, $P\in\mathbb{R}^{n\times n}$ is the system matrix, $u_i(t)\in\mathbb{R}^m$ is the consensus protocol, $i\in\mathcal{N}$, $\mathcal{N}=\{1,2,\cdots,N\}$, and N is the number of agents following the leader agent. $r(t)\in\mathbb{R}^n$ is the state of the leader, and its dynamic equation can be described as

$$\dot{r}(t)=Pr(t) \tag{2}$$

Denote $e_i(t)=x_i(t)-r(t)$, and $e(t)=[e_1^T(t),e_2^T(t),\cdots,e_N^T(t)]^T$. In this paper, the random noise is applied to design the consensus protocol as follows

$$u_i(t)=c\sum_{j=1}^{N}a_{ij}[x_j(t)-x_i(t)]+[K(t)(I_d\otimes e_i(t))]\xi_i(t)$$

where c is the gain constant, a_{ij} is the element of the adjacency matrix A, $a_{ij}=0$ or 1 for $i\neq j$, $a_{ii}=0$, $\xi_i(t)\in\mathbb{R}^d$ is the Gaussian white noises which satisfies $\int_0^t\xi_i(s)ds=B_i(t)$, $B_i(t)$ is the independent d-dimensional Brownian motion, and $K(t)\in\mathbb{R}^{n\times nd}$ is the coefficient matrix of the linear noise intensity function. Then, we can rewrite the error system to a compact form as follows

$$\begin{aligned}de(t)=&[(I_N\otimes P)-c(L\otimes I_n)]e(t)dt\\&+(I_N\otimes K(t))diag[I_d\otimes e_1(t),I_d\otimes e_2(t),\\&\cdots,I_d\otimes e_N(t)]dB(t)\end{aligned}$$

with the initial value $e_0 \in \mathbb{R}^{nN}$, where L is the Laplacian matrix that is symmetric, $B(t) = [B_1^T(t), B_2^T(t), \cdots, B_N^T(t)]^T$. Furthermore, the error system can be reduced to

$$de(t) = F(t, e(t))dt + G(t, e(t))dB(t) \tag{3}$$

where $F(t, e(t)) = [(I_N \otimes P) - c(L \otimes I_n)]e(t)$, and $G(t, e(t)) = (I_N \otimes K(t))\mathrm{diag}[I_d \otimes e_1(t), I_d \otimes e_2(t), \cdots, I_d \otimes e_N(t)]$.

Following that, a consensus definition of multi-agent systems is presented as well as a lemma that will be applied.

Definition 1: The multi-agent systems consensus problem is solved, namely, system (3) is almost surely (a.s.) exponentially stable, if for any $e_i(t_0) \in \mathbb{R}^n$

$$\lim_{t\to\infty} \sup \frac{1}{t} \log |e(t)| < 0 \qquad \text{a.s.}$$

Lemma 1 (Borel-Cantelli's Lemma): (a) If $A_k \in \mathcal{F}$ and $\sum_{k=1}^{\infty} \mathbb{P}(A_k) < \infty$, then $\mathbb{P}(\lim_{k\to\infty} \sup A_k) = 0$ that is there exist a set $\Omega_\theta \in \mathcal{F}$ with $\mathbb{P}(\Omega_\theta) = 1$ and an integer-valued random variable k_0 such that for every $W \in \Omega_0$ we have $W \notin A_k$ whenever $k \geq k_0(W)$. (b) If the sequence $A_k \in \mathcal{F}$ is independent and $\sum_{k=1}^{\infty} \mathbb{P}(A_k) = \infty$ then $\mathbb{P}(\lim_{k\to\infty} \sup A_k) = 1$ that is there exist a set $\Omega_0 \in \mathcal{F}$ with $\mathbb{P}(\Omega_\theta) = 1$ such that for every $W \in \Omega_\theta$. there exist a sub-sequence A_{ki} such that the W belongs to every A_{ki}.

Lemma 2 [34]: Under the conditions imposed above, the solution of system (3) obeys $\mathbb{P}\{e(t; t_0, e_0) \neq 0 \text{ on } t \geq 0\} = 1$ for all $e(t_0) \neq 0$.

Remark 1: For $V \in C^{1,2}([t_0, \infty] \times \mathbb{R}^n; \mathbb{R}^+)$, Define an integral operator $dV(t, x(t)) = \mathcal{L}V(t, x(t))dt + \mathcal{H}V(t, x(t))dB(t)$ where $\mathcal{L}V(t, x) = V_t + V_x f(t, x) + \frac{1}{2}\mathrm{Trace}[g^T(t, x)V_{xx}g(t, x)]$, $\mathcal{H}V(t, x) = V_x g(t, x)$, $V_t = \frac{\partial V}{\partial t}$, $V_x = (\frac{\partial V}{\partial x_1}, \frac{\partial V}{\partial x_2}, \cdots, \frac{\partial V}{\partial x_n})$,

$$V_{xx} = (\frac{\partial^2 V}{\partial x_i \partial x_j})_{n\times n} = \begin{pmatrix} \frac{\partial^2 V}{\partial x_1 \partial x_1} & \cdots & \frac{\partial^2 V}{\partial x_1 \partial x_n} \\ \vdots & \cdots & \vdots \\ \frac{\partial^2 V}{\partial x_n \partial x_1} & \cdots & \frac{\partial^2 V}{\partial x_n \partial x_n} \end{pmatrix}.$$

III. THE EFFECT OF STOCHASTIC INTERMITTENT NOISE ON GENERAL MULTI-AGENTS

In this section, we will use intermittent stochastic noise to design a suitable $u_i(t)$ to make system (3) get consensus. The noise control input u_i is designed as $u_i(t) = [K(t)(I_d \otimes e_i(t))]\xi_i(t)$, where

$$K(t) = \begin{cases} K', & t \in \Delta T_{1\ell}, \\ K'' = 0, & t \in \Delta T_{2\ell}. \end{cases}$$

where $\Delta T_{1\ell} = [\ell T, \ell T + \tau)$, $\Delta T_{2\ell} = [\ell T + \tau, (\ell + 1)T)$, $\ell \in N$, $T > 0$ is named the control period, and $\tau > 0$ denotes the noise width with $T > \tau$. Denote that

$$M^\eta = \big[0, \cdots, 0, \underset{\eta\text{-th}}{K'^T}, 0, \cdots, 0\big]^T$$

where η-th represents the η-th element, and $\eta = 1, 2, \cdots, d$. Next, we will introduce a important theorem which can solve the consensus problem of the multi-agent systems via imtermittent stochastic noise stabilization scheme.

Theorem 1: For the error system (3), assume that there exist constants $k_1 \in \mathbb{R}$, $k_2 > 0$, $k_3 \geq 0$, such that for $t \geq 0$

(i) $\frac{\Theta^T + \Theta}{2} \leq k_1 I_{Nn}$,

(ii) $\|K'\| \leq \frac{k_3}{\sqrt{z}}$,

(iii) $\hat{\lambda} \geq 2k_2\sqrt{\frac{N}{d}}$,

where $\Theta = (I_N \otimes P) - c(L \otimes I_n)$, $\hat{\lambda} = \min_{1\leq\eta\leq d}\{|\lambda(M^{\eta T} + M^\eta)|_{\min}\}$, and z is the number of non-zero eigenvalues of $(K')^T K'$. Then, the following formula

$$\lim_{t\to\infty} \sup \frac{1}{t} \log |e(t)| \leq -\big[(k_2^2 - 0.5k_3^2)\nu - k_1\big] \text{ a.s.} \tag{4}$$

holds for all $e_0 \in \mathbb{R}^{nN}$, where $\nu = \frac{\tau}{T}$. In particular, if $(k_2^2 - 0.5k_3^2)\nu - k_1 > 0$, i.e.

(a) $2k_2^2 > k_3^2$, $\nu \in \left(\frac{k_1}{k_2^2 - 0.5k_3^2}, 1\right) \cap (0, 1)$,

(b) $2k_2^2 = k_3^2$, $k_1 < 0$ and ν is an arbitrary number belonging to $(0, 1)$, or

(c) $2k_2^2 < k_3^2$, $\nu \in \left(0, \frac{k_1}{k_2^2 - 0.5k_3^2}\right) \cap (0, 1)$.

Then we say that system (3) solves the consensus problem.

Proof: By Lemma 2, for any $e(0) \neq 0$, we have $e(t) \neq 0$ for all $t \geq 0$ almost surely. Hence, taking $V(t, e) = |e|^2$, we can apply Itô's formula for $t \geq 0$ to obtain

$$\log |e(t)|^2 = \log |e_0|^2 + \int_0^t O(s, e(s))ds - \frac{1}{2}\int_0^t Y(s, e(s))ds + U(t) \tag{5}$$

where $O(t, e(t)) = \frac{2e^T(t)F(t,e(t)) + \mathrm{Tr}[G^T(t,e(t))G(t,e(t))]}{|e(t)|^2}$, $Y(t, e(t)) = \frac{4|e^T(t)G(t,e(t))|^2}{|e(t)|^4}$, $U(t) = 2\int_0^t \frac{e^T(s)G(s,e(s))}{|e(s)|^2}dB(s)$ is a continuous martingale with $U(0) = 0$. The quadratic variation of this martingale is given by $\langle U(t), U(t)\rangle = \int_0^t Y(s, e(s))ds$. For an arbitrary $\sigma \in (0, 1)$, letting $\ell = 0, 1, 2, \cdots$, by the exponential martingale inequality, we have

$$P\left\{\sup_{0\leq t\leq(\ell+1)T}\left[U(t) - \frac{\sigma}{2}\langle U(t), U(t)\rangle\right] > \frac{2}{\sigma}\log(\ell + 1)\right\} \leq \frac{1}{(\ell + 1)^2} \tag{6}$$

By Lemma 1, we get that for almost all $\omega \in \Omega$, there exists an integer $\ell_0 = \ell_0(\omega)$ such that if $\ell > \ell_0$, $U(t) \leq \frac{2}{\sigma}\log(\ell + 1) + \frac{\sigma}{2}\langle U(t), U(t)\rangle$ holds for all $0 \leq t < (\ell + 1)T$. Therefore, we acquire

$$\log |e(t)|^2 \leq \log |e_0|^2 + \int_0^t O(s, e(s))ds - \frac{1}{2}(1 - \sigma)\langle U(t), U(t)\rangle + \frac{2}{\sigma}\log(\ell + 1)$$

Further more, for almost all $\omega \in \Omega$, when $\ell T \leq t < \ell T + \tau$ and $\ell > \ell_0$

$$\log \|e(t)\|^2 \leq \log \|e_0\|^2 + \int_0^\tau O(s, e(s))ds$$

$$+\int_{\tau}^{T} O(s,e(s))\mathrm{d}s+\int_{T}^{T+\tau} O(s,e(s))\mathrm{d}s + \int_{T+\tau}^{2T} O(s,e(s))\mathrm{d}s+\cdots+\int_{\ell T}^{t} O(s,e(s))\mathrm{d}s - \frac{1}{2}(1-\sigma)\Big[\int_{0}^{\tau} Y(s,e(s))\mathrm{d}s + \int_{\tau}^{T} Y(s,e(s))\mathrm{d}s+\int_{T}^{T+\tau} Y(s,e(s))\mathrm{d}s + \cdots+\int_{\ell T}^{t} Y(s,e(s))\mathrm{d}s\Big]+\frac{2}{\sigma}\log(\ell+1)$$

On the other hand, it follows that

$$\begin{aligned} e^T(s)F(t,e(s)) &= e^T(s)[(I_N\otimes P)-c(L\otimes I_n)]e(s)\\ &= e^T(s)\left(\frac{\Theta^T+\Theta}{2}\right)e(s)\\ &\le k_1|e(s)|^2 \end{aligned} \tag{7}$$

When $hT\le s<hT+\tau$, $h=0,1,\cdots,\ell$, it is derived that

$$\begin{aligned} &Tr\big[G^T(s,e(s))G(s,e(s))\big]\\ &=\lambda_1\big[G^T(s,e(s))G(s,e(s))\big]+\cdots\\ &\quad+\lambda_{dN}\big[G^T(s,e(s))G(s,e(s))\big]\\ &\le z\lambda_{\max}\big[G^T(s,e(s))G(s,e(s))\big]\\ &= z\|G(s,e(s))\|^2\\ &\le z\|K'\|^2|e(s)|^2 \end{aligned} \tag{8}$$

It can be obtained that $Tr\big[G^T(s,e(s))G(s,e(s))\big]\le k_3^2|e(s)|^2$ (8) and condition (ii). So when $hT\le s<hT+\tau$, $h=0,1,\cdots,\ell$, we have

$$O(s,e(s))\le 2k_1+k_3^2 \tag{9}$$

On the other hand,

$$\begin{aligned} &e^T(s)G(s,e(s))\\ &=e^T(s)(I_N\otimes K')\\ &\quad diag[I_d\otimes e_1(s),I_d\otimes e_2(s),\cdots,I_d\otimes e_N(s)]\\ &=\big[e_1^T(s)K'(I_d\otimes e_1(s)),\cdots,e_N^T(s)K'(I_d\otimes e_N(s))\big]\\ &=\Big[\big[e_1^T(s)K'(e_1^T(s),0,\cdots,0)^T,\\ &\quad\cdots,e_1^T(s)K'(0,\cdots,0,e_1^T(s))^T\big],\\ &\quad\cdots,\\ &\quad\big[e_N^T(s)K'(e_N^T(s),0,\cdots,0)^T,\\ &\quad\cdots,e_N^T(s)K'(0,\cdots,0,e_N^T(s))^T\big]\Big] \end{aligned}$$

Let

$$e_i^\eta(s)=\big[0,\cdots,0,\underset{\eta\text{-th}}{e_i^T(s)},0,\cdots,0\big]^T$$

where $i\in\mathcal{N}$, η-th represents the η-th element, and $\eta=1,2,\cdots,d$.Then $e^T(s)G(s,e(s))$ can be rewritten as

$$\begin{aligned} e^T(s)G(s,e(s))=\Big[&e_1^{1T}(s)M^1e_1^1(s),\cdots,e_1^{dT}(s)M^de_1^d(s),\\ &e_2^{1T}(s)M^1e_2^1(s),\cdots,e_2^{dT}(s)M^de_2^d(s),\\ &\cdots,\\ &e_N^{1T}(s)M^1e_N^1(s),\cdots,e_N^{dT}(s)M^de_N^d(s)\Big] \end{aligned}$$

As a consequence, we have

$$\begin{aligned} |e^T(s)G(s,e(s))|^2 &=\sum_{i=1}^N\sum_{\eta=1}^d\Big[e_i^{\eta T}(s)W^\eta e_i^\eta(s)\Big]^2\\ &\ge\sum_{i=1}^N\sum_{\eta=1}^d\Big[\Big|\lambda\Big(\frac{W^{\eta T}+W^\eta}{2}\Big)\Big|_{\min}|e_i(s)|^2\Big]^2\\ &\ge\frac{d\hat\lambda^2}{4N}\Big[\sum_{i=1}^N|e_i(s)|^2\Big]^2\\ &=\frac{d\hat\lambda^2}{4N}|e(s)|^4 \end{aligned}$$

where $\hat\lambda=\min_{1\le\eta\le d}\{|\lambda(M^{\eta T}+M^\eta)|_{\min}\}$. In like wise, we attain by Theorem 1 that

$$|e^T(s)G(s,e(s))|^2\ge k_2^2|e(s)|^4$$

From condition (iii), we get $|e^{\mathrm{T}}(s)G(s,e(s))|\ge k_2|e(s)|^2$. So when $hT\le s<hT+\tau$, $h=0,1,\cdots,\ell$, it follows that

$$Y(s,e(s))\ge 4k_2^2 \tag{10}$$

When $hT+\tau\le s<(h+1)T+1$, $h=0,1,\cdots,\ell$, $\mathcal{G}(s,e(s))=0$, and therefore

$$O(s,e(s))\le 2k_1\qquad Y(s,e(s))=0 \tag{11}$$

From (9), (10) and (11), if $\ell T\le t<\ell T+\tau$, we get that

$$\begin{aligned} &\log|e(t)|^2\\ &\le\log|e_0|^2+(2k_1+k_3^2)\tau+2k_1(T-\tau)+\cdots\\ &\quad+(2k_1+k_3^2)(t-\ell T)-\frac{1}{2}(1-\sigma)\big[4\ell k_2^2\tau\\ &\quad+4k_2^2(t-\ell T)\big]+\frac{2}{\sigma}\log(\ell+1)\\ &=\log|e_0|^2+(2k_1+k_3^2)\ell\tau+(2k_1+k_3^2)(t-\ell T)\\ &\quad+2k_1(T-\tau)-2(1-\sigma)\big[k_2^2\ell\tau\\ &\quad+k_2^2(t-\ell T)\big]+\frac{2}{\sigma}\log(\ell+1)\\ &\le\log|e_0|^2+k_3^2(\ell+1)\tau+2k_1t-2(1-\sigma)k_2^2\ell\tau\\ &\quad+\frac{2}{\sigma}\log(\ell+1) \end{aligned}$$

By the same way, if $\ell T+\tau\le t<(\ell+1)T$ and $\ell>\ell_0$, we obtain

$$\begin{aligned} \log|e(t)|^2\le\log|e_0|^2&+\int_0^\tau O(s,e(s))\mathrm{d}s+\int_\tau^T O(s,e(s))\mathrm{d}s\\ &+\int_T^{T+\tau}O(s,e(s))\mathrm{d}s+\int_{T+\tau}^{2T}O(s,e(s))\mathrm{d}s+\cdots\\ &+\int_{\ell T+\tau}^{t}O(s,e(s))\mathrm{d}s-\frac{1}{2}(1-\sigma)\Big[\int_0^\tau Y(s,e(s))\mathrm{d}s \end{aligned}$$

$$+\int_{\tau}^{T} Y(s,e(s))\mathrm{d}s+\int_{T}^{T+\tau} Y(s,e(s))\mathrm{d}s+\cdots$$
$$+\int_{\ell T+\tau}^{t} Y(s,e(s))\mathrm{d}s\Big]+\frac{2}{\sigma}\log(\ell+1)$$

Then, we attain

$$\begin{aligned}\log|e(t)|^2 &\le \log|e_0|^2+(2k_1+k_3^2)(\ell+1)\tau+2k_1(T-\tau)(\ell\\&\quad+1)-2(1-\sigma)\ell k_2^2\tau+\frac{2}{\sigma}\log(\ell+1)\\&\le \log|e_0|^2+k_3^2(\ell+1)\tau+2k_1t-2(1-\sigma)k_2^2\ell\tau\\&\quad+\frac{2}{\sigma}\log(\ell+1)\end{aligned}$$

It follows that

$$\frac{1}{t}\log|e(t)|^2\le\frac{k_3^2(\ell+1)\tau}{\ell T}+2k_1-2\frac{(1-\sigma)k_2^2\ell\tau}{T(\ell+1)}+\frac{\log|e_0|^2+\frac{2}{\sigma}\log(\ell+1)}{\ell T}$$

We can deduce that

$$\lim_{t\to\infty}\sup\frac{1}{t}\log|e(t)|^2\le\big[k_3^2-2(1-\sigma)k_2^2\big]\nu+2k_1\quad\text{a.s.}$$

As a result, we can get

$$\lim_{t\to\infty}\sup\frac{1}{t}\log|e(t)|\le\Big[\frac{1}{2}k_3^2-(1-\sigma)k_2^2\Big]\nu+k_1\quad\text{a.s.}$$

Because $\sigma>0$ is arbitrary, we further achieve that

$$\lim_{t\to\infty}\sup\frac{1}{t}\log|e(t)|\le-\big[(k_2^2-0.5k_3^2)\nu-k_1\big]\quad\text{a.s.}$$

If $(k_2^2-0.5k_3^2)\nu-k_1>0$, then it is apparent that

$$\lim_{t\to\infty}\sup\frac{1}{t}\log|e(t)|<0\quad\text{a.s.}$$

According to Definition 1, system (3) is almost surely exponentially stable, that is, the multi-agent systems achieves consensus. ■

Remark 2: It can be noted here that if the parameter k_1 is nonnegative, it means that the system without controller is unstable. To prove that the theorem can make the unstable system stable, it is necessary to ensure $(k_2^2-0.5k_3^2)\nu>k_1>0$. It can be seen that the larger the parameter k_1 and k_3, the greater the resistance to overcome, and the larger the parameter k_2, the stronger the control effect. Therefore, when the number of nodes controlled N is larger, the Gaussian white noise dimension d in the controller must also be increased to ensure that a larger k_2 makes $k_2^2-0.5k_3^2$ positive and as large as possible.

IV. THE EFFECT OF STOCHASTIC INTERMITTENT NOISE ON MULTI-AGENTS WITH ENVIRONMENTAL NOISE

Environmental noise, such as electromagnetic interference in the environment, may be seen in real life. As a result, the multi-agent systems with ambient random noise must be worth considering. Then system (1) can be gneralized into

$$\mathrm{d}x_i(t)=Px_i(t)\mathrm{d}t+u_i(t)\mathrm{d}t+\sum_{j=1}^{N}a_{ij}\alpha_{ij}x_i(t)\mathrm{d}W_i(t)$$

where α_{ij} is the noise interference density coefficient, J is the noise interference matrix, and α_{ij} is the element in the matrix J. Therefore, the error system evolves into

$$\mathrm{d}e(t)=F(t,e(t))\mathrm{d}t+g(t)\mathrm{d}W(t)+G(t,e(t))\mathrm{d}B(t)\qquad(12)$$

where $G(t,e(t))$ and $F(t,e(t))$ are defined by the same way in Section III. $\alpha_{ij}\in\mathbb{R}$, $W_i(t)\in\mathbb{R}$, $W(t)=[W_1(t),W_2(t),\cdots,W_i(t)]^{\mathrm{T}}$, $g(t)=\mathrm{diag}[g_1(t),g_2(t),\cdots,g_i(t)]$, and $g_i(t)=\sum_{j=1}^{N}a_{ij}\alpha_{ij}e_i(t)$. $a_{ij}\alpha_{ij}$ is each element of the environmental noise interference matrix J. $B_i(t)$ and $W_i(t)$ are two different Brownian motions.

Theorem 2: For the error system (12), assume that there exist constants $k_1\in\mathbb{R}$, $k_2>0$, $k_3\ge0$, $k_4\ge0$, such that

(i) $\frac{\Theta^{\mathrm{T}}+\Theta}{2}\le k_1I_{Nn}$,

(ii) $\|K'\|\le\frac{k_3}{\sqrt{z}}$,

(iii)$\hat{\lambda}\ge2k_2\sqrt{\frac{N}{d}}$,

(iv) $\max_{i\in N}\sum_{j=1}^{N}a_{ij}\alpha_{ij}\le k_4$,

for $t\ge0$, where Θ, $\hat{\lambda}$, M and z are the same definition as Theorem 1. Then, the following formula

$$\lim_{t\to\infty}\sup\frac{1}{t}\log|e(t)|\le k_1+(0.5k_3^2-k_2^2)\nu+0.5k_4^2\quad\text{a.s.}$$

holds for all $e_0\in\mathbb{R}^{nN}$, where $\nu=\frac{\tau}{T}$. In particular, if $(k_2^2-0.5k_3^2)\nu-0.5k_4^2-k_1>0$, i.e.

(a) $2k_2^2>k_3^2$, $\nu\in\Big(\frac{2k_1+k_4^2}{2k_2^2-k_3^2},1\Big)\cap(0,1)$,

(b) $2k_2^2=k_3^2$, $k_1<0$, and ν is an arbitrary number belonging to $(0,1)$, or (c) $2k_2^2<k_3^2$, $\nu\in\Big(0,\frac{2k_1+k_4^2}{2k_2^2-k_3^2}\Big)\cap(0,1)$.

Then we say that system (12) solves the consensus problem.

Proof: By Lemma 2, for any $e_0\ne0$, it can be deduced that $e(t)\ne0$ for all $t\ge0$ a.s.. Hence, taking $V(t,e)=|e|^2$, we may utilize Itô's formula for $t\ge0$ to obtain

$$\begin{aligned}\log|e(t)|^2&=\log|e_0|^2+\int_0^t O_1(s,e(s))\mathrm{d}s\\&\quad-\frac{1}{2}\int_0^t Y_1(s,e(s))\mathrm{d}s+\int_0^t O_2(s,e(s))\mathrm{d}s\\&\quad-\frac{1}{2}\int_0^t Y_2(s,e(s))\mathrm{d}s+U(t)+I(t)\end{aligned}\qquad(13)$$

where $O_1(t,e(t))=\frac{e^{\mathrm{T}}(t)F(t,e(t))+\mathrm{Tr}[G^{\mathrm{T}}(t,e(t))G(t,e(t))]}{|e(t)|^2}$, $Y_1(t,e(t))=\frac{4|e^{\mathrm{T}}(t)G(t,e(t))|^2}{|e(t)|^4}$, $Y_2(t,e(t))=\frac{4|e^{\mathrm{T}}(t)g(t)|^2}{|e(t)|^4}$, $O_2(t,e(t))=\frac{e^{\mathrm{T}}(t)F(t,e(t))+|g(t)|^2}{|e(t)|^2}$, $U(t)=2\int_0^t\frac{e^{\mathrm{T}}(s)G(s,e(s))}{|e(s)|^2}\mathrm{d}B(s)$, $I(t)=2\int_0^t\frac{e^{\mathrm{T}}(s)g(s)}{|e(s)|^2}\mathrm{d}W(s)$ is a continuous martingale with $U(0)=0$, $I(0)=0$.

If $\ell T\le t<\ell T+\tau$, we achieve that

$$\begin{aligned}&\log|e(t)|^2\\&\le\log|e_0|^2+\int_0^t O_1(s,e(s))\mathrm{d}s-\frac{1}{2}\int_0^t Y_1(s,e(s))\mathrm{d}s\\&\quad+\int_0^t O_2(s,e(s))\mathrm{d}s+U(t)+I(t)\end{aligned}$$

$$
\begin{aligned}
&= \log|e_0|^2 + \int_0^{\tau} O_1(s,e(s))\mathrm{d}s + \int_{\tau}^{T} O_1(s,e(s))\mathrm{d}s \\
&\quad + \int_T^{T+\tau} O_1(s,e(s))\mathrm{d}s + \int_{T+\tau}^{2T} O_1(s,e(s))\mathrm{d}s + \cdots \\
&\quad + \int_{\ell T}^{t} O_1(s,e(s))\mathrm{d}s - \frac{1}{2}\Big[\int_0^{\tau} Y_1(s,e(s))\mathrm{d}s \\
&\quad + \int_{\tau}^{T} Y_1(s,e(s))\mathrm{d}s + \int_T^{T+\tau} Y_1(s,e(s))\mathrm{d}s + \cdots \\
&\quad + \int_{\ell T}^{t} Y_1(s,e(s))\mathrm{d}s\Big] + \int_0^t O_2(s,e(s))\mathrm{d}s \\
&\quad + U(t) + I(t)
\end{aligned}
$$

Using the method in Theorem 1, when $hT \leq s < hT + \tau$, and $h = 0, 1, \cdots, \ell$, we have

$$
\begin{aligned}
&O_1(s,e(s)) \leq 2k_1 + k_3^2 \\
&Y_1(s,e(s)) \geq 4k_2^2 \\
&O_2(s,e(s)) \leq 2k_1 + k_4^2
\end{aligned} \tag{14}
$$

When $hT + \tau \leq s < (h+1)T + 1$, and $h = 0, 1, \cdots, \ell$, one has $G(s,e(s)) = 0$. Then it can be acquired that

$$
\begin{aligned}
&O_1(s,e(s)) \leq 2k_1 \\
&Y_1(s,e(s)) = 0 \\
&O_2(s,e(s)) \leq 2k_1 + k_4^2
\end{aligned} \tag{15}
$$

From (14) and (15), if $\ell T \leq t < \ell T + \tau$, we get that

$$
\begin{aligned}
&\log|e(t)|^2 \\
&\leq \log|e_0|^2 + (k_1 + k_3^2)\tau + k_1(T-\tau) \\
&\quad + (k_1 + k_3^2)\tau + k_1(T-\tau) + \cdots + (k_1 + k_3^2)(t - \ell T) \\
&\quad - 2k_2\ell\tau + (k_1 + k_4^2)t + U(t) + I(t) \\
&\leq \log|e_0|^2 + 2k_1 t + k_3^2(\ell+1)\tau + k_4^2 t - 2k_2^2\ell\tau \\
&\quad + U(t) + I(t)
\end{aligned}
$$

By the same way, if $\ell T + \tau \leq t < (\ell+1)T$, we also get the same conclusion. Then it follows that

$$
\begin{aligned}
\log|e(t)|^2 \leq \log|e_0|^2 + 2k_1 t + k_3^2(\ell+1)\tau \\
+ k_4^2 t - 2k_2^2\ell\tau + U(t) + I(t)
\end{aligned}
$$

So, it is easy to attain

$$
\begin{aligned}
\frac{1}{t}\log|e(t)|^2 \leq \frac{\log|e_0|^2}{t} + 2k_1 + \frac{k_3^2(\ell+1)\tau}{\ell T} \\
+ k_4^2 - \frac{2k_2^2\ell\tau}{(\ell+1)T} + \frac{U(t)}{t} + \frac{I(t)}{t}
\end{aligned}
$$

By the conditions (ii) and (iii)

$$
\begin{aligned}
\lim_{t\to\infty} \frac{\langle U(t), U(t)\rangle}{t} \leq 4k_3^2 \quad \text{a.s.} \\
\lim_{t\to\infty} \frac{\langle I(t), I(t)\rangle}{t} \leq 4k_4^2 \quad \text{a.s.}
\end{aligned}
$$

According to the strong law of large numbers ([34]), we can get

$$
\lim_{t\to\infty} \frac{U(t)}{t} = 0, \quad \lim_{t\to\infty} \frac{I(t)}{t} = 0 \quad \text{a.s.}
$$

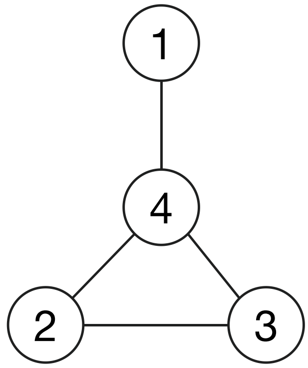

FIGURE 1. The topology of multi-agents.

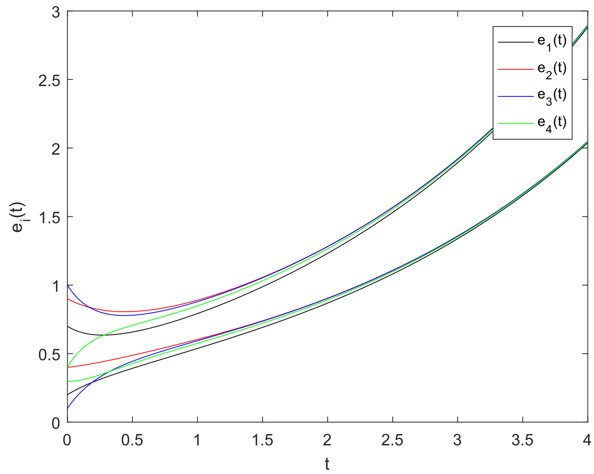

FIGURE 2. Systems without control input and environmental noise.

So we can get

$$
\lim_{t\to\infty} \sup \frac{1}{t}\log|e(t)|^2 \leq 2k_1 + k_3^2\nu + k_4^2 - 2k_2^2\nu \quad \text{a.s.}
$$

namely

$$
\lim_{t\to\infty} \sup \frac{1}{t}\log|e(t)| \leq k_1 + (0.5k_3^2 - k_2^2)\nu + 0.5k_4^2 \quad \text{a.s.}
$$

If $(k_2^2 - 0.5k_3^2)\nu - 0.5k_4^2 - k_1 > 0$, it is acquired that

$$
\lim_{t\to\infty} \sup \frac{1}{t}\log|e(t)| < 0 \quad \text{a.s.}
$$

According to Definition 1, system (12) is almost surely exponentially stable, that is, the multi-agent systems achieves consensus. ■

Remark 3: The environmental noise in this context differs significantly from the stochastic noise input of the noise controller. The latter is a control input intended to ensure consensus among multi-agent systems, while the former can negatively impact system stability and must be overcome. As a result, the estimated value k_4 of the ambient noise level occupies the same position in the stability criteria as the estimated value k_3 of a portion of the noise control input. This scenario is detrimental to system stability, and the value

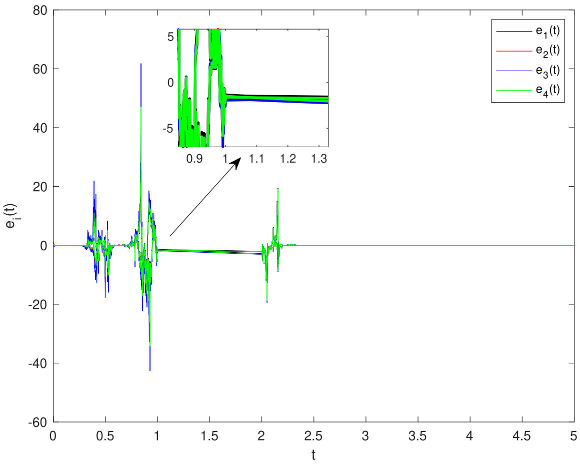

FIGURE 3. Systems with intermittent stochastic noise control input by $\|K'\| = 13.5204$.

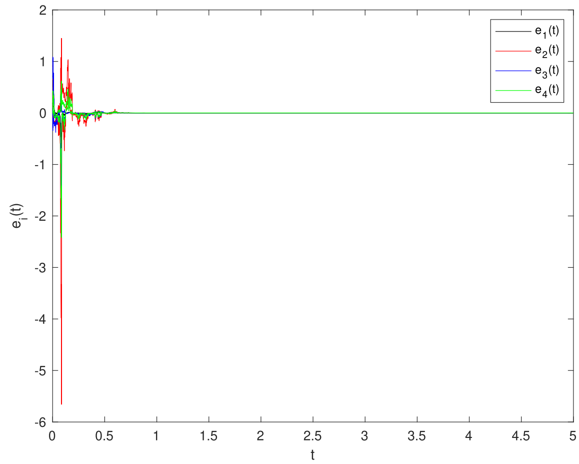

FIGURE 4. Systems with intermittent stochastic noise control input by $\|K'\| = 14.8198$.

of k_4 must be kept sufficiently low. Conversely, a positive relationship exists between the estimate of the controller noise input, k_2, and the system stability. Therefore, it must be set at a sufficiently high level to satisfy the equation $(k_2^2 - 0.5k_3^2)\nu - 0.5k_4^2 - k_1 > 0$ and ensure the consensus of the multi-agent systems.

V. THE SIMULATION RESULTS

Two examples are presented in this section to demonstrate the practicality of the algorithm designed in this study. Assume that the number of followers N is 4, $e_i(t)$ is a 2-dimensional vector, and the topological relationship between the followers is

$$A = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

and the topological relationship graph is shown as follows

The initial values are supposed as $e_1(0) = (0.7, 0.2)^{\mathrm{T}}$, $e_2(0) = (0.9, 0.4)^{\mathrm{T}}$, $e_3(0) = (1, 0.1)^{\mathrm{T}}$, $e_4(0) = (0.4, 0.3)^{\mathrm{T}}$, gain constant c is 1, and

$$P = \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix}$$

Thus the Laplacian matrix can be attained that

$$L = \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 2 & -1 & -1 \\ 0 & -1 & 2 & -1 \\ -1 & -1 & -1 & 3 \end{pmatrix}$$

The final errors will not approach zero in this case, indicating that multi-agent systems will be unable to attain group consensus without any control as illustrated in Fig. 2.

Example 1: We select the appropriate intermittent stochastic noise to solve the consensus problem. Let $k_1 = 3.1$, and we have $\frac{\Theta^{\mathrm{T}}+\Theta}{2} \leq 3.1 I_{Nn}$. Choose the dimension of Brownian motion $d = 10$.

$$K' = \begin{pmatrix} \Pi & \Pi & \Pi & \Pi & \Pi \end{pmatrix}$$

where

$$\Pi = \begin{pmatrix} 2.5 & -2.5 & 2.5 & -2.5 \\ 1.7 & -1.7 & 1.7 & -1.7 \end{pmatrix}$$

It is calculated that $\|K'\| = 13.5204$, $\hat{\lambda} = 12.7204$. Select $k_2 = 10.05$ and $k_3 = 13.521$ and the intermittent period $T = 2$. Therefore, we can get $0.65 < \tau < 2$ by Theorem 1, and take $\tau = 1$. Thus, the system is almost surely exponentially stable under the above parameter conditions, as shown in Fig. 3.

On the other hand, increase the noise dimension d to achieve greater noise intensity. Let

$$K' = \begin{pmatrix} \Pi & \Pi & \Pi & \Pi & \Pi & \Pi \end{pmatrix}$$

Then $\|K'\| = 14.8198$, and $\hat{\lambda} = 14.0108$. Take $k_1 = 3.1$, $k_2 = 12.13$, and $k_3 = 14.82$. It is calculated that $0.17 \leq \tau \leq 2$, and select $\tau = 1$. Apparently, the noise control intensity is greater than that in above situation. The simulation result is shown below in Fig. 4. It can be found that the error system is closed to zero in one intermittent period when the noise intensity increases. Obviously, the greater the noise strength is, the better the effect of the consensus is.

Example 2: The environmental noise interference matrix is

$$J = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}$$

and then the trajectories of the multi-agent systems with environmental noise is shown in Fig 5. It is easy to see that

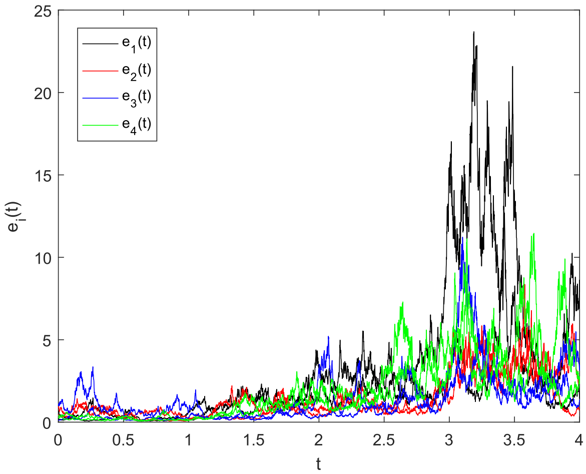

FIGURE 5. Systems with environmental noise (1).

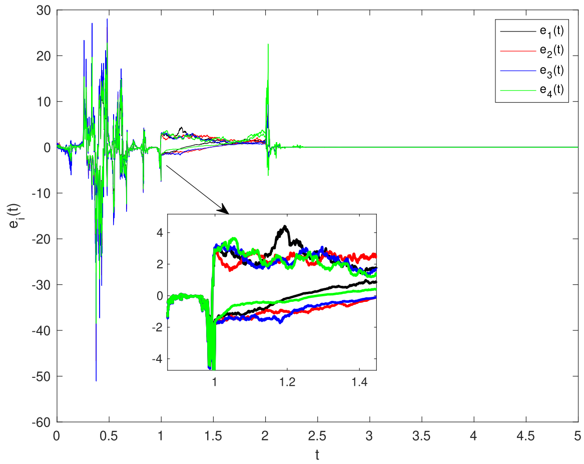

FIGURE 6. Systems with environmental noise under control input (1).

$\max_{i\in N}\sum_{j=1}^{N} a_{ij}\alpha_{ij} = \sum_{j=1}^{N} a_{1j}\alpha_{1j} = 1$. Select $k_4 = 1.1$, and the dimension of Brownian motion $d = 10$. Let

$$K' = \begin{pmatrix}\Pi & \Pi & \Pi & \Pi & \Pi\end{pmatrix}$$

Choose $k_1 = 3.1$, $k_2 = 10.05$, and $k_3 = 13.521$. Then it is obtained that $0.772 < \tau < 2$, and we therefore select $\tau = 1$. System (12) is almost surely exponentially stable under the above parameter conditions in accordance with Theorem 2, that is, the multi-agent systems approach consensus, as shown in Fig. 6. The simulation example clearly demonstrates that using adequate intermittent stochastic noise can address the multi-agent systems consensus problem. It can be seen that the fluctuation amplitude of error trajectories is relatively small in the time range of 1 to 2, because the system is only affected by environmental noise interference and not by the input of control noise during this period.

If environmental interference is enhanced, then the environmental noise interference matrix is assumed as follows,

$$J = \begin{pmatrix} 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 3 & 0 & 3 \\ 2 & 2 & 1 & 0 \end{pmatrix}$$

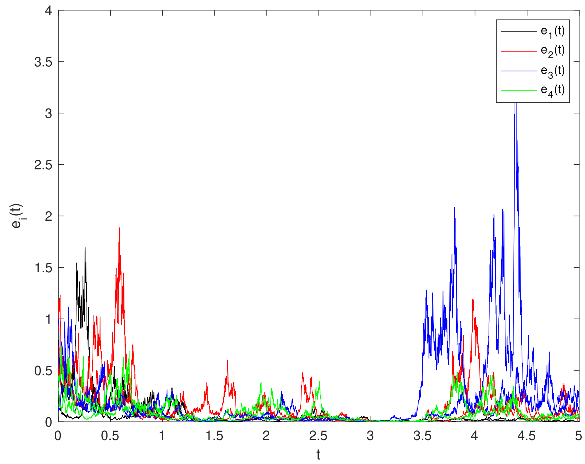

FIGURE 7. Systems with environmental noise (2).

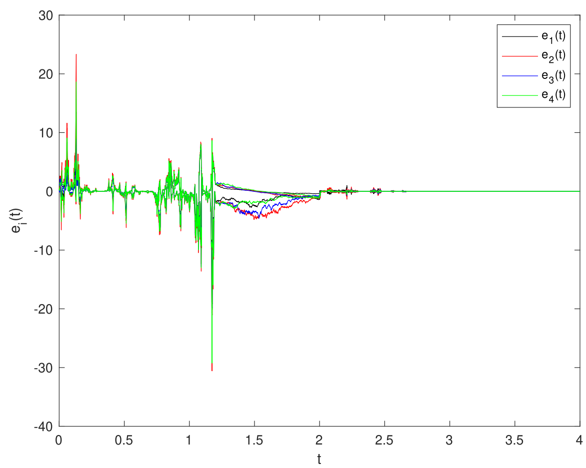

FIGURE 8. Systems with environmental noise under control input (2).

The trajectories of the multi-agent systems with environmental noise is shown in Fig. 7. At this point, it can be obtained that $\max_{i\in N}\sum_{j=1}^{N} a_{ij}\alpha_{ij} = \sum_{j=1}^{N} a_{1j}\alpha_{1j} = 6$, and take $k_4 = 6.1$. If other parameters remain unchanged, then there is no number $\nu \in (0, 1)$ such that

$$(10.05^2 - 0.5 \times 13.521^2)\nu > 0.5 \times 6.1^2 + 3.1,$$

At this time, the dimension of Brownian motion can be increased to 12, and let

$$K' = \begin{pmatrix}\Pi & \Pi & \Pi & \Pi & \Pi & \Pi\end{pmatrix}$$

It is easy to calculate that $\|K'\| = 14.8198$, Select $k_1 = 3.1$, $k_3 = 14.82$, $k_2 = 12.13$, and $k_4 = 6.1$. There is a $\nu \in (0, 1)$ that causes

$$(12.13^2 - 0.5 \times 14.82^2)\nu > 0.5 \times 6.1^2 + 3.1,$$

$37.3207\nu > 21.705$, and thus we attain $1.16 < \tau < 2$. Fix $\tau = 1.2$. The stabilization control effect is shown in Fig. 8. From the above simulation examples, it can be observed that due to the enhanced environmental noise interference, a larger control gain is required to ensure that the control time remains constant within a cycle; to control the gain unchanged, a longer control time is required.

VI. CONCLUSION

In this study, an intermittent stochastic noise stabilization technique is used to investigate the consensus of linear multi-agent systems with environmental noise. The intermittent stochastic noise is employed as a controller to address the consensus problem. Future research may focus on addressing other issues, such as multi-agent systems consensus with stochastic parameters or achieving multi-agent systems consensus using non-periodic intermittent stochastic noise control input.

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