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RESEARCH ARTICLE

Confidence Levels Bipolar Complex Fuzzy Aggregation Operators and Their Application in Decision Making Problem

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ABSTRACT A novel idea called bipolar complex fuzzy set makes it simple to represent difficult and ambiguous information in practical issues. Confidence levels, bipolar fuzzy set, and complex fuzzy set are three distinct theories that were combined to form the basic theory of the Confidence levels bipolar complex fuzzy set. Confidence levels bipolar complex fuzzy set (CLBCFS)is used as a method for resolving perplexing and suspect circumstances that occur in daily life. In this article, we define idea of bipolar complicated fuzzy collection of confidence levels. To do this, we developed a number of operational laws. Further, utilizing bipolar complex fuzzy operational laws, we diagnose the theories of confidence level bipolar complex fuzzy averaging and geometric operators. We reviewed some significant findings and some features of the developed operators in order to reduce the impact of this study. In addition, we computed a multiple-attribute decision-making rule while the initiated approaches were present and attempted to support it with specific examples. Finally, we contrasted our defined work with numerous prevalent methodologies and also detailed their geometrical representations in order to assess the influence and supremacy of the defined work.

INDEX TERMS Complex fuzzy set, bipolar fuzzy set, confidence levels, confidence levels bipolar complex aggregation operators.

I. INTRODUCTION

The expansion of information in culture has led to scientific and technical approach that has reduced the complexity of our daily lives. Nevertheless, despite advances in science that have made life simpler, some issues, such as decision making (DM), continue to be challenging. Multiple-criteria group decision making (MCGDM), in particular, has been enthusiastically embraced in a number of industries, where conventional techniques have recently fallen short. Information is frequently ambiguous, much like in real life, and information comes more complicated, more solutions are needed. Zadeh [34] developed the idea of a fuzzy set (FS) in 1965. A surprising performance with many applications is fuzzy set. A fuzzy set's membership degree (MD), which can take on any value between 0 and 1, defines it. In [1] Atanassov defined as the intuitionistic fuzzy set (IFS) and enlarged the fuzzy set.

Complex fuzzy sets (CFSs) ware defined in cites [26], [27], respectively. An improved version of traditional FS are used to handle fuzzy information in CFS. Since, a complex fuzzy value has two terms (phase term and an amplitude term), it may handle information in two different ways. Enhanced data storage is made possible by this CFS

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feature. The fundamental functions of CFS are described by Ramot et al. [27]. For instance, Merigo et al. [20] provide a challenging fuzzy generalised aggregate operator (AO) and show how it might be used in decision-making. Dai et al. extend CFSs with approximations of parallel and orthogonality relations [12], [13]. The idea of CFS equality is introduced by Zhang et al. [36]. Hu et al. in [8], and Alkouri and Salleh [3] have specified CFS distance measurements. New entropy measure types were provided in CFSs by Bi et al. [6]. Calculating the separation in CFSs, cross-entropy, and significance in diabetes is done by Liu et al. [18].

Rotational in-variance was extended to CFS operations by Dai [9]. Ma et al. [19] created the concept of CFS for issues with numerous periodic variables. A power aggregation operation for complex fuzzy information was proposed by Hu et al. For different intellectuals, including neighborhood operators, a number of CFS applications have been looked upon. See [4], [5], and [31] for more information about CFSs.

We are aware that the complex fuzzy set theory only discussed positive supporting grades and ignored negative supporting grades, which has led to a number of problems in many instances. Mahmood and Rehman [22] developed the theory of CFS to account for this and identified the mathematical form known as a bipolar complex fuzzy set (BCFS) using concept of positive degree and negative degree in the complex numbers form, have a real part and an imaginary part falling within unit intervals [0, 1] and [−1,0]. Following their examination, a small group of researchers created a large number of applications, including the BCFS-built Hamacher aggregate data that Mahmood et al. [23] evaluated. Furthermore, Mahmood and Rehman [24] offer the BCFS-based fundamental theory of Dombi operators.

Other than these surprise successes in a bipolar complex fuzzy environment, no other known initiatives make use of the degree of familiarity with the information fusion step.

In a multi-criteria decision making (MCDM) situation, experts assess alternatives exclusively in the light of the predetermined criteria; their familiarity (referred to as confidence levels) with the assessment objects is not taken into account. As a result, it is essential to include the observer's familiarity with the original information in a bipolar complicated fuzzy environment. By adding experts' confidence levels of their familiarity and experience with the analyzed alternatives in the bipolar complex fuzzy information fusion stage, this work aims to address this type of defect.

The structure of this theory as follows: Sec. II includes some current ideas such as bipolar complex fuzzy set, AOs, and some operation laws. In Sec. III, we used the theory of confidence levels bipolar complex fuzzy averaging/geometric AOs to diagnose the well-known theories such as CLBCFWA, CLBCFOWA, CLBCFHA, CLBCFWG, CLBCFOWG, and CLBCFHG operators, as well as evaluate their strategic properties and related findings. In Sec. IV, we suggest an approach using the mentioned operators when making decisions based on numerous criteria. The choice of a site for a waste disposal plant is then examined using a numerical example. To show the sustainability and capability of the suggested method, we compared the stated operators to existing methodologies in Section V of the paper. Sec. VI concludes by summarizing the results of the study.

II. PRELIMINARIES

The goal of this section is to convey the pre-existing basic definitions for CFS, BFS, and BCFS in a clear manner.

Definition 1 [26]: A complex fuzzy set C on Q (universal set) describes as;

$$C = \left\{ \langle \acute{n}, \mu_C(\acute{n}) \rangle \,|\acute{n} \in Q \right\}, \tag{1}$$

where $\mu_C : U \rightarrow \{z : z \in C, |z| \leq 1\}$ and $\mu_C(\acute{n}) = a + ib = \chi_C(\acute{n}).e^{2\pi i \Theta_C(\acute{n})}$. As, $\chi_C(\acute{n}) = \sqrt{a^2+b^2} \in R$ and $\chi_C(\acute{n}), \Theta_{C(\acute{n})} \in [0,1]$, where $i = \sqrt{-1}$.

Definition 2 [35]: A bipolar fuzzy set Υ on Q (universal set) described as;

$$\Upsilon = \left\{ \langle \acute{n}, \mu^+_\Upsilon(\acute{n}), \mu^-_\Upsilon(\acute{n}) \rangle \,|\acute{n} \in Q \right\}, \tag{2}$$

where, $\mu^+_\Upsilon : Q \rightarrow [0,1]$ and $\mu^-_\Upsilon : Q \rightarrow [-1,0]$.

III. BIPOLAR COMPLEX FUZZY SET

Definition 3 [24]: A bipolar complex fuzzy set Υ on Q (universal set) described as,

$$\Upsilon = \left\{ \langle \acute{n}, \left(\mu^+_\Upsilon(\acute{n}), \mu^-_\Upsilon(\acute{n})\right) \rangle \,|\acute{n} \in Q \right\}, \tag{3}$$

where, $\mu^+_\Upsilon : Q \rightarrow [0,1] + i[0,1]$, $\mu^-_\Upsilon : Q \rightarrow [-1,0] + i[-1,0]$ and $\upsilon^-_\Upsilon : Q \rightarrow [-1,0] + i[-1,0]$ is membership degree. $\mu^+_\Upsilon(\acute{n}) = a^+_\Upsilon(\acute{n}) + ib^+_\Upsilon(\acute{n})$, and $\mu^-_\Upsilon(\acute{n}) = a^-_\Upsilon(\acute{n}) + ib^-_\Upsilon(\acute{n})$ as $a^+_\Upsilon(\acute{n}), ib^+_\Upsilon(\acute{n}) \in [0,1]$ and $a^-_\Upsilon(\acute{n}), ib^-_\Upsilon(\acute{n}) \in [-1,0]$. BCF number is denoted as, $\Upsilon = \left(\left\langle\left(a^+_\Upsilon + ib^+_\Upsilon\right), \left(a^-_\Upsilon + ib^-_\Upsilon\right)\right\rangle\right)$.

Definition 4 [24]: For any two bipolar complex fuzzy numbers $\Upsilon_1 = \left(\left\langle a^+_{\Upsilon_1} + ib^+_{\Upsilon_1}, a^-_{\Upsilon_1} + ib^-_{\Upsilon_1}\right\rangle\right)$ and $\Upsilon_2 = \left(\left\langle a^+_{\Upsilon_2} + ib^+_{\Upsilon_2}, a^-_{\Upsilon_2} + ib^-_{\Upsilon_2}\right\rangle\right)$, and for any $\lambda > 0$. The subsequent operations are described as;

1) $\Upsilon_1 \oplus \Upsilon_2$

$$= \left\{ \left\langle \begin{array}{c} a^+_{\Upsilon_1} + a^+_{\Upsilon_2} - a^+_{\Upsilon_1}a^+_{\Upsilon_2} + \\ i\left(b^+_{\Upsilon_1} + b^+_{\Upsilon_2} - b^+_{\Upsilon_1}b^+_{\Upsilon_2}\right), -\left(a^-_{\Upsilon_1}a^-_{\Upsilon_2}\right) \\ +i\left(-\left(b^-_{\Upsilon_1}b^-_{\Upsilon_2}\right)\right) \end{array} \right\rangle \right\};$$

2) $\Upsilon_1 \otimes \Upsilon_2$

$$= \left\{ \left\langle \begin{array}{c} \left(a^+_{\Upsilon_1}a^+_{\Upsilon_2}\right) + i\left(b^+_{\Upsilon_1}b^+_{\Upsilon_2}\right), a^-_{\Upsilon_1} + a^-_{\Upsilon_2} \\ -a^-_{\Upsilon_1}a^-_{\Upsilon_2} + i\left(b^-_{\Upsilon_1} + b^-_{\Upsilon_2} - b^-_{\Upsilon_1}b^-_{\Upsilon_2}\right) \end{array} \right\rangle \right\};$$

3) $\lambda\Upsilon_1$

$$= \left\{ \left\langle \begin{array}{c} 1 - \left(1 - a^+_{\Upsilon_1}\right)^\lambda + i\left(1 - \left(1 - b^+_{\Upsilon_1}\right)^\lambda\right), \\ -\left(a^-_{\Upsilon_1}\right)^\lambda + i\left(-\left(b^-_{\Upsilon_1}\right)^\lambda\right) \end{array} \right\rangle \right\};$$

4) Υ_1^{λ}

$$=\left\{\left\langle \begin{array}{c}\left(a_{\Upsilon_1}^{+}\right)^{\lambda}+i\left(b_{\Upsilon_1}^{+}\right)^{\lambda},-1+\left(1+a_{\Upsilon_1}^{+}\right)^{\lambda}+\\ i\left(-1+\left(1+b_{\Upsilon_1}^{+}\right)^{\lambda}\right)\end{array}\right\rangle\right\};$$

Definition 5 [24]: Let $\Upsilon = \left(\left\langle a_{\Upsilon}^{+}+ib_{\Upsilon}^{+}, a_{\Upsilon}^{-}+ib_{\Upsilon}^{-}\right\rangle\right)$ be the bipolar complex fuzzy number. Then, the definition of the score function is;

$$S(\Upsilon)=\frac{1}{4}\left(2+a_{\Upsilon}^{+}+ib_{\Upsilon}^{+}+a_{\Upsilon}^{-}+ib_{\Upsilon}^{-}\right) \quad (4)$$

IV. CONFIDENCE LEVELS BIPOLAR COMPLEX FUZZY AVERAGING OPERATOR

All current efforts do not take into consideration experts' trust in themselves Acquaintance and grasp of the evaluated alternatives in the merging of BCFNs. A collection of geometric and confidence levels bipolar complex fuzzy averaging aggregation operators is produced by combining expert confidence levels with computed alternatives.

A. CONFIDENCE LEVELS BIPOLAR COMPLEX FUZZY WEIGHTED AVERAGING OPERATOR

Definition 6 : Let $\Upsilon_{\flat} = \left(\left\langle a_{\Upsilon_{\flat}}^{+}+ib_{\Upsilon_{\flat}}^{+}, a_{\Upsilon_{\flat}}^{-}+ib_{\Upsilon_{\flat}}^{-}\right\rangle\right)$ $(\flat = 1,\ldots,n)$ be a family of BCFNs and $l_{\flat}$ be the confidence levels of $\Upsilon_{\flat}$, weights are $\varrho = (\varrho_1,\ldots,\varrho_n)^T$, such that $\sum_{\flat=1}^{n}\varrho_{\flat}=1$ and $0\le\varrho_{\flat}\le 1$. Then, the CLBCFWA operator as;

$$CLBCFWA\left(\langle\Upsilon_1,l_1\rangle,\ldots,\langle\Upsilon_n,l_n\rangle\right)=\bigoplus_{\flat=1}^{n}\varrho_{\flat}\left(l_{\flat}\Upsilon_{\flat}\right), \quad (5)$$

based on definition (4), the aggregated value of CLBCFWA operator is presented in Theorem (1).

Theorem 1 Let $\Upsilon_i = \left(a_{\Upsilon_{\flat}}^{+}+ib_{\Upsilon_{\flat}}^{+}, a_{\Upsilon_{\flat}}^{-}+ib_{\Upsilon_{\flat}}^{-}\right)$ $(\flat = 1,\ldots,n)$ be the family of bipolar complex fuzzy numbers and $l_{\flat}$ be the confidence levels of $\Upsilon_{\flat}$, with $0\le l_{\flat}\le 1$ and weights $\varrho = (\varrho_1,\ldots,\varrho_n)^T$, such that $\sum_{\flat=1}^{n}\varrho_{\flat}=1$ and $0\le\varrho_{\flat}\le 1$. Then, CLBCFWA operator as;

$$\begin{aligned}&CLBCFWA\left(\langle\Upsilon_1,l_1\rangle,\ldots,\langle\Upsilon_n,l_n\rangle\right)\\&=\bigoplus_{\flat=1}^{n}\varrho_{\flat}\left(l_{\flat}\Upsilon_{\flat}\right)\\&=\left\{\left(\begin{array}{c}1-\prod_{i=1}^{n}\left(1-a_{\Upsilon_{\flat}}^{+}\right)^{l_{\flat}\varrho_{\flat}}\\ +i\left(1-\prod_{\flat=1}^{n}\left(1-b_{\Upsilon_{\flat}}^{+}\right)^{l_{\flat}\varrho_{\flat}}\right),\\ -\prod_{\flat=1}^{n}\left(a_{\Upsilon_{\flat}}^{-}\right)^{l_{\flat}\varrho_{\flat}}+i\left(-\prod_{\flat=1}^{n}\left(b_{\Upsilon_{\flat}}^{-}\right)^{l_{\flat}\varrho_{\flat}}\right)\end{array}\right)\right\}.\end{aligned} \quad (6)$$

Proof: We applied the mathematical induction concept to this theorem's proof. As we are know

$$\Upsilon_1\oplus\Upsilon_2=\bigoplus_{\flat=2}^{n}\varrho_{\flat}\left(l_{\flat}\Upsilon_{\flat}\right)$$

And

$$\varrho_1\Upsilon_1=\left\{\left(\begin{array}{c}1-\left(1-a_{\Upsilon_1}^{+}\right)^{l_1\varrho_1}+i\left(1-\left(1-b_{\Upsilon_1}^{+}\right)^{l_1\varrho_1}\right),\\ -\left(a_{\Upsilon_1}^{-}\right)^{l_1\varrho_1}+i\left(-\left(b_{\Upsilon_1}^{-}\right)^{l_1\varrho_1}\right)\end{array}\right)\right\}$$

Let Equ. (6) is true for $n = 2$. Then,

$$\begin{aligned}&CLBCFWA\left(\langle\Upsilon_1,l_1\rangle,\langle\Upsilon_2,l_2\rangle\right)=\bigoplus_{\flat=1}^{2}\varrho_{\flat}\left(l_{\flat}\Upsilon_{\flat}\right)\\&=\left\{\left(\begin{array}{c}1-\prod_{\flat=1}^{2}\left(1-a_{\Upsilon_{\flat}}^{+}\right)^{l_{\flat}\varrho_{\flat}}+\\ i\left(1-\prod_{\flat=1}^{2}\left(1-b_{\Upsilon_{\flat}}^{+}\right)^{l_{\flat}\varrho_{\flat}}\right),\\ -\prod_{\flat=1}^{2}\left(a_{\Upsilon_{\flat}}^{-}\right)^{l_{\flat}\varrho_{\flat}}+i\left(-\prod_{\flat=1}^{2}\left(b_{\Upsilon_{\flat}}^{-}\right)^{l_{\flat}\varrho_{\flat}}\right)\end{array}\right)\right\}\end{aligned}$$

The result true for $n = 2$.

Let Eq. (6) true for $n = \tau$. We have,

$$\begin{aligned}&CLBCFWA\left(\langle\Upsilon_1,l_1\rangle,\ldots,\langle\Upsilon_n,l_n\rangle\right)=\bigoplus_{\flat=1}^{n}\varrho_{\flat}\left(l_{\flat}\Upsilon_{\flat}\right)\\&=\left\{\left(\begin{array}{c}1-\prod_{\flat=1}^{\tau}\left(1-a_{\Upsilon_{\flat}}^{+}\right)^{l_{\flat}\varrho_{\flat}}+\\ i\left(1-\prod_{\flat=1}^{\tau}\left(1-b_{\Upsilon_{\flat}}^{+}\right)^{l_{\flat}\varrho_{\flat}}\right),\\ -\prod_{\flat=1}^{\tau}\left(a_{\Upsilon_{\flat}}^{-}\right)^{l_{\flat}\varrho_{\flat}}+i\left(-\prod_{\flat=1}^{\tau}\left(b_{\Upsilon_{\flat}}^{-}\right)^{l_{\flat}\varrho_{\flat}}\right)\end{array}\right)\right\}\end{aligned}$$

Next, let Equ. (6) hold for $n = \tau + 1$,

$$\begin{aligned}&CLBCFWA\left(\langle\Upsilon_1,l_1\rangle,\ldots,\langle\Upsilon_{\tau},l_{\tau}\rangle\oplus\langle\Upsilon_{\tau+1},l_{\tau+1}\rangle\right)\\&=\left(\bigoplus_{\flat=1}^{\tau}\varrho_{\flat}\left(l_{\flat}\Upsilon_{\flat}\right)\right)\oplus\varrho_{\tau+1}\left(l_{\tau+1}\Upsilon_{\tau+1}\right)\\&=\left\{\left(\begin{array}{c}1-\prod_{\flat=1}^{\tau}\left(1-a_{\Upsilon_{\flat}}^{+}\right)^{l_{\flat}\varrho_{\flat}}+\\ i\left(1-\prod_{\flat=1}^{\tau}\left(1-b_{\Upsilon_{\flat}}^{+}\right)^{l_{\flat}\varrho_{\flat}}\right),\\ -\prod_{\flat=1}^{\tau}\left(a_{\Upsilon_{\flat}}^{-}\right)^{l_{\flat}\varrho_{\flat}}+i\left(-\prod_{\flat=1}^{\tau}\left(b_{\Upsilon_{\flat}}^{-}\right)^{l_{\flat}\varrho_{\flat}}\right)\end{array}\right)\right\}\\&\oplus\left\{\left(\begin{array}{c}1-\left(1-a_{\Upsilon_{\tau+1}}^{+}\right)^{\varrho_{\tau+1}}+\\ i\left(1-\left(1-b_{\Upsilon_{\tau+1}}^{+}\right)^{\varrho_{\tau+1}}\right),\\ -\left(a_{\Upsilon_{\tau+1}}^{-}\right)^{\varrho_{\tau+1}}+i\left(-\left(b_{\Upsilon_{\tau+1}}^{-}\right)^{\varrho_{\tau+1}}\right)\end{array}\right)\right\}\end{aligned}$$

$$= \left\{ \left(\begin{array}{c} 1 - \prod_{\flat=1}^{\tau+1} \left(1 - a^{+}_{\Upsilon_\flat}\right)^{l_\flat \varrho_\flat} + \\ i\left(1 - \prod_{\flat=1}^{\tau+1} \left(1 - b^{+}_{\Upsilon_\flat}\right)^{l_\flat \varrho_\flat}\right), \\ -\prod_{\flat=1}^{\tau+1} \left(a^{-}_{\Upsilon_\flat}\right)^{l_\flat \varrho_\flat} + i\left(-\prod_{\flat=1}^{\tau+1} \left(b^{-}_{\Upsilon_\flat}\right)^{l_\flat \varrho_\flat}\right) \end{array} \right) \right\}$$

Which proof Eq. (6) hold for $n = \tau + 1$, and hold for $n \geq 1$.

The CLBCFWA operator fulfilled the properties listed below.

Theorem 2 (Idempotency): Let $\Upsilon_\flat = \left(a^{+}_{\Upsilon_\flat} + ib^{+}_{\Upsilon_\flat}, a^{-}_{\Upsilon_\flat} + ib^{-}_{\Upsilon_\flat}\right)$ $(\flat = 1, \ldots, n)$ be the family of bipolar complex fuzzy numbers and $l_\flat$ be the confidence levels of $\Upsilon_\flat$, with weights $\varrho = (\varrho_1, \ldots, \varrho_n)^T$, such that $\sum_{\flat=1}^{n} \varrho_\flat = 1$ and $0 \leq \varrho_\flat \leq 1$. Then,

$$CLBCFWA\left(\langle \Upsilon_1, l_1\rangle, \ldots, \langle \Upsilon_n, l_n\rangle\right) = \langle \Upsilon, l\rangle. \tag{7}$$

Proof: As we know that;

$$CLBCFWA\left(\langle \Upsilon_1, l_1\rangle, \ldots, \langle \Upsilon_n, l_n\rangle\right) = \bigoplus_{\flat=1}^{n} \varrho_\flat \left(l_\flat \Upsilon_\flat\right)$$

$$= \left\{ \left(\begin{array}{c} 1 - \prod_{\flat=1}^{n} \left(1 - a^{+}_{\Upsilon_\flat}\right)^{l_\flat \varrho_\flat} + \\ i\left(1 - \prod_{\flat=1}^{n} \left(1 - b^{+}_{\Upsilon_\flat}\right)^{l_\flat \varrho_\flat}\right), \\ -\prod_{\flat=1}^{n} \left(a^{-}_{\Upsilon_\flat}\right)^{l_\flat \varrho_\flat} + i\left(-\prod_{\flat=1}^{n} \left(b^{-}_{\Upsilon_\flat}\right)^{l_\flat \varrho_\flat}\right) \end{array} \right) \right\}$$

$$\left\{ \left(\begin{array}{c} 1 - \left(1 - a^{+}_{\Upsilon}\right)^{\sum_{\flat=1}^{n} l_\flat \varrho_\flat} + \\ i\left(1 - \left(1 - b^{+}_{\Upsilon}\right)^{\sum_{\flat=1}^{n} l_\flat \varrho_\flat}\right), \\ -\left(a^{-}_{\Upsilon}\right)^{\sum_{\flat=1}^{n} l_\flat \varrho_\flat} + i\left(-\left(b^{-}_{\Upsilon}\right)^{\sum_{\flat=1}^{n} l_\flat \varrho_\flat}\right) \end{array} \right) \right\}$$

$$= \left(\left\langle a^{+}_{\Upsilon} + ib^{+}_{\Upsilon}, a^{-}_{\Upsilon} + ib^{-}_{\Upsilon}\right\rangle\right)$$
$$= \langle \Upsilon_1, l_1\rangle$$

Theorem 3 (Monotonicity): Let $\Upsilon_\flat = \left(\left\langle a^{+}_{\Upsilon_\flat} + ib^{+}_{\Upsilon_\flat}, a^{-}_{\Upsilon_\flat} + ib^{-}_{\Upsilon_\flat}\right\rangle\right)$ and $\Upsilon^{/}_{i} = \left(\left\langle a^{/+}_{\Upsilon_\flat} + ib^{/+}_{\Upsilon_\flat}, a^{/-}_{\Upsilon_\flat} + ib^{/-}_{\Upsilon_\flat}\right\rangle\right)$ $(\flat = 1, \ldots, n)$ be the family of BCFNs and $l_\flat$ be the confidence levels of $\Upsilon_\flat$ with weights $\varrho = (\varrho_1, \ldots, \varrho_n)^T$, such that $\sum_{\flat=1}^{n} \varrho_\flat = 1$ and $0 \leq \varrho_\flat \leq 1$, if $a^{+}_{\Upsilon_\flat} \geq a^{/+}_{\Upsilon_\flat}$, $ib^{+}_{\Upsilon_\flat} \geq ib^{/+}_{\Upsilon_\flat}$, $ib^{-}_{\Upsilon_\flat} \leq ib^{/-}_{\Upsilon_\flat}$, and $a^{-}_{\Upsilon_\flat} \leq a^{/-}_{\Upsilon_\flat}$. Then,

$$\begin{aligned} &CLBCFWA\left(\langle \Upsilon_1, l_1\rangle, \ldots, \langle \Upsilon_n, l_n\rangle\right) \\ &\quad \geq CLBCFWA\left(\left\langle \Upsilon^{/}_{1}, l_1\right\rangle, \ldots, \left\langle \Upsilon^{/}_{n}, l_n\right\rangle\right). \end{aligned} \tag{8}$$

Proof: As $a^{+}_{\Upsilon_\flat} \geq a^{/+}_{\Upsilon_\flat}$, $ib^{+}_{\Upsilon_\flat} \geq ib^{/+}_{\Upsilon_\flat}$, $ib^{-}_{\Upsilon_\flat} \leq ib^{/-}_{\Upsilon_\flat}$, and $a^{-}_{\Upsilon_\flat} \leq a^{/-}_{\Upsilon_\flat}$. Then,

$$\begin{aligned} &1 - a^{+}_{\Upsilon_\flat} \leq 1 - a^{/+}_{\Upsilon_\flat} \\ &\quad \Longrightarrow 1 - \prod_{\flat=1}^{n} \left(1 - a^{+}_{\Upsilon_\flat}\right)^{l_\flat \varrho_\flat} \geq 1 - \prod_{\flat=1}^{n} \left(1 - a^{/+}_{\Upsilon_\flat}\right)^{l_\flat \varrho_\flat}. \end{aligned}$$

and

$$\prod_{\flat=1}^{n} \left(a^{-}_{\Upsilon_\flat}\right)^{l_\flat \varrho_\flat} \leq \prod_{\flat=1}^{n} \left(a^{-}_{\Upsilon_\flat}\right)^{l_\flat \varrho_\flat}.$$

For imaginary part

$$\left(1 - \prod_{\flat=1}^{n} \left(1 - ib^{+}_{\Upsilon_\flat}\right)^{l_\flat \varrho_\flat}\right) \geq \left(1 - \prod_{\flat=1}^{n} \left(1 - ib^{+}_{\Upsilon_\flat}\right)^{l_\flat \varrho_\flat}\right).$$

And

$$\prod_{\flat=1}^{n} \left(ib^{-}_{\Upsilon_\flat}\right)^{l_\flat \varrho_\flat} \leq \prod_{\flat=1}^{n} \left(ib^{-}_{\Upsilon_\flat}\right)^{l_\flat \varrho_\flat}.$$

When real and imaginary components are combined, we get

$$= \left\{ \left(\begin{array}{c} 1 - \prod_{\flat=1}^{n} \left(1 - a^{+}_{\Upsilon_\flat}\right)^{l_\flat \varrho_\flat} \\ +i\left(1 - \prod_{\flat=1}^{n} \left(1 - b^{+}_{\Upsilon_\flat}\right)^{l_\flat \varrho_\flat}\right), \\ -\prod_{\flat=1}^{n} \left(a^{-}_{\Upsilon_\flat}\right)^{l_\flat \varrho_\flat} + i\left(-\prod_{\flat=1}^{n} \left(b^{-}_{\Upsilon_\flat}\right)^{l_\flat \varrho_\flat}\right) \end{array} \right) \right\}$$

$$\geq \left\{ \left(\begin{array}{c} 1 - \prod_{\flat=1}^{n} \left(1 - a^{/+}_{\Upsilon_\flat}\right)^{l_\flat \varrho_\flat} + \\ i\left(1 - \prod_{\flat=1}^{n} \left(1 - b^{/+}_{\Upsilon_\flat}\right)^{l_\flat \varrho_\flat}\right), \\ -\prod_{\flat=1}^{n} \left(a^{/-}_{\Upsilon_\flat}\right)^{l_\flat \varrho_\flat} + i\left(-\prod_{\flat=1}^{n} \left(b^{/-}_{\Upsilon_\flat}\right)^{l_\flat \varrho_\flat}\right) \end{array} \right) \right\}$$

Let, $CLBCFWA\left(\langle \Upsilon_1, l_1\rangle, \ldots, \langle \Upsilon_n, l_n\rangle\right) = \Upsilon_1$ and $CLBCFWA\left(\left\langle \Upsilon^{/}_{1}, l_1\right\rangle, \ldots, \left\langle \Upsilon^{/}_{n}, l_n\right\rangle\right) = \Upsilon^{/}_{1}$. So, utilizing Eq. (4), we get

$$S(\Upsilon_1, l_1) \geq S(\Upsilon^{/}_{1}, l_1)$$

Next, there are two possibilities:

1). When, $S(\Upsilon_1, l_1) \geq S(\Upsilon^{/}_{1}, l_1)$, then by Equ. (4), we get

$$\begin{aligned} &CLBCFWA\left(\langle \Upsilon_1, l_1\rangle, \ldots, \langle \Upsilon_n, l_n\rangle\right) \\ &\quad \geq CLBCFWA\left(\left\langle \Upsilon^{/}_{1}, l_1\right\rangle, \ldots, \left\langle \Upsilon^{/}_{n}, l_n\right\rangle\right) \end{aligned}$$

2). When, $S(\Upsilon_1, l_1) = S(\Upsilon^{/}_{1}, l_1)$, we get

$$= \left\{ \left(\begin{array}{c} 1 - \prod_{\flat=1}^{n} \left(1 - a^{+}_{\Upsilon_\flat}\right)^{l_\flat \varrho_\flat} + \\ i\left(1 - \prod_{\flat=1}^{n} \left(1 - b^{+}_{\Upsilon_\flat}\right)^{l_\flat \varrho_\flat}\right), \\ -\prod_{\flat=1}^{n} \left(a^{-}_{\Upsilon_\flat}\right)^{l_\flat \varrho_\flat} + i\left(-\prod_{\flat=1}^{n} \left(b^{-}_{\Upsilon_\flat}\right)^{l_\flat \varrho_\flat}\right) \end{array} \right) \right\}$$

$$\geq \left\{ \left(\begin{array}{c} 1 - \prod_{\flat=1}^{n} \left(1 - a_{\Upsilon_\flat}^{/+}\right)^{l_\flat \varrho_\flat} + \\ i\left(1 - \prod_{i=1}^{n} \left(1 - b_{\Upsilon_\flat}^{/+}\right)^{l_\flat \varrho_\flat}\right), \\ - \prod_{\flat=1}^{n} \left(a_{\Upsilon_\flat}^{/-}\right)^{l_\flat \varrho_\flat} + i\left(- \prod_{\flat=1}^{n} \left(b_{\Upsilon_\flat}^{/-}\right)^{l_\flat \varrho_\flat}\right) \end{array} \right) \right\}$$

Because the score functions are equivalent, we used the accuracy function.

$$= \left\{ \left(\begin{array}{c} 1 - \prod_{\flat=1}^{n} \left(1 - a_{\Upsilon_\flat}^{+}\right)^{l_\flat \varrho_\flat} + \\ i\left(1 - \prod_{\flat=1}^{n} \left(1 - b_{\Upsilon_\flat}^{+}\right)^{l_\flat \varrho_\flat}\right), \\ - \prod_{\flat=1}^{n} \left(a_{\Upsilon_\flat}^{-}\right)^{l_\flat \varrho_\flat} + i\left(- \prod_{\flat=1}^{n} \left(b_{\Upsilon_\flat}^{-}\right)^{l_\flat \varrho_\flat}\right) \end{array} \right) \right\}$$

$$\geq \left\{ \left(\begin{array}{c} 1 - \prod_{i=1}^{n} \left(1 - a_{\Upsilon_\flat}^{/+}\right)^{l_\flat \varrho_\flat} + \\ i\left(1 - \prod_{\flat=1}^{n} \left(1 - b_{\Upsilon_\flat}^{/+}\right)^{l_\flat \varrho_\flat}\right), \\ - \prod_{i=1}^{n} \left(a_{\Upsilon_\flat}^{/-}\right)^{l_\flat \varrho_\flat} + i\left(- \prod_{\flat=1}^{n} \left(b_{\Upsilon_\flat}^{/-}\right)^{l_\flat \varrho_\flat}\right) \end{array} \right) \right\}$$

$$= \frac{1}{4} \left\{ \left(\begin{array}{c} 1 - \prod_{\flat=1}^{n} \left(1 - a_{\Upsilon_\flat}^{+}\right)^{l_\flat \varrho_\flat} + \\ i\left(1 - \prod_{\flat=1}^{n} \left(1 - b_{\Upsilon_\flat}^{+}\right)^{l_\flat \varrho_\flat}\right), \\ - \prod_{\flat=1}^{n} \left(a_{\Upsilon_\flat}^{-}\right)^{l_\flat \varrho_\flat} + \\ i\left(- \prod_{\flat=1}^{n} \left(b_{\Upsilon_\flat}^{-}\right)^{l_\flat \varrho_\flat}\right) \end{array} \right) \right\}$$

$S(\Upsilon_1, l_1) = S(\Upsilon_1^{/}, l_1)$

From case (1) and case (2), we det the proof.

Theorem 4 (Boundedness): Let $\Upsilon_\flat = \left(\left\langle a_{\Upsilon_\flat}^{+} + ib_{\Upsilon_\flat}^{+}, a_{\Upsilon_\flat}^{-} + ib_{\Upsilon_\flat}^{-}\right\rangle\right)$ $(\flat = 1, \ldots, n)$ be a family of BCFNs and $l_\flat$ be the confidence levels of $\Upsilon_\flat$ with the weights $\varrho = (\varrho_1, \ldots, \varrho_n)^T$, such that $\sum_{\flat=1}^{n} \varrho_i = 1$ and $0 \leq \varrho_\flat \leq 1$, if $\Upsilon_\flat^{+}$, $\Upsilon_\flat^{-}$ are the maximum and minimum BCFNs. Then,

$$\Upsilon_\flat^{+} \leq CLBCFWA\left(\langle \Upsilon_1, l_1\rangle, \ldots, \langle \Upsilon_n, l_n\rangle\right) \leq \Upsilon_\flat^{-}. \quad (9)$$

Proof: For membership degree, we looked at two scenarios (for the real and imagined components) separately.

For membership degree, we have

$$\left(1 - \prod_{\flat=1}^{n} \left(1 - \min_{1\leq\flat\leq n} a_{\Upsilon_\flat}^{+}\right)^{l_\flat \varrho_\flat}\right)$$
$$\leq \left(1 - \prod_{\flat=1}^{n} \left(1 - a_{\Upsilon_\flat}^{+}\right)^{l_\flat \varrho_\flat}\right)$$
$$\leq \left(1 - \prod_{\flat=1}^{n} \left(1 - \max_{1\leq\flat\leq n} a_{\Upsilon_\flat}^{+}\right)^{l_\flat \varrho_\flat}\right)$$
$$\Longrightarrow \left(1 - \left(1 - \min_{1\leq\flat\leq n} a_{\Upsilon_\flat}^{+}\right)^{\sum_{\flat=1}^{n} l_\flat \varrho_\flat}\right)$$
$$\leq \left(1 - \prod_{\flat=1}^{n} \left(1 - a_{\Upsilon_\flat}^{+}\right)^{l_\flat \varrho_\flat}\right)$$
$$\leq \left(1 - \left(1 - \max_{1\leq\flat\leq n} a_{\Upsilon_\flat}^{+}\right)^{\sum_{\flat=1}^{n} l_\flat \varrho_\flat}\right)$$

As $\sum_{\flat=1}^{n} l_\flat \varrho_\flat = 1$, so

$$\Longrightarrow \min_{1\leq\flat\leq n} a_{\Upsilon_\flat}^{+} \leq \left(1 - \prod_{\flat=1}^{n} \left(1 - a_{\Upsilon_\flat}^{+}\right)^{l_\flat \varrho_\flat}\right) \leq \max_{1\leq\flat\leq n} a_{\Upsilon_\flat}^{+}$$

Also, for $ib_{\Upsilon_\flat}^{+}$,

and

$$\prod_{\flat=1}^{n} \min_{1\leq\flat\leq n} \left(a_{\Upsilon_\flat}^{-}\right)^{l_\flat \varrho_\flat} \leq \prod_{i=1}^{n} \left(a_{\Upsilon_\flat}^{-}\right)^{l_\flat \varrho_\flat}$$
$$\leq \prod_{\flat=1}^{n} \max_{1\leq\flat\leq n} \left(a_{\Upsilon_\flat}^{-}\right)^{l_\flat \varrho_\flat}$$
$$\Longrightarrow \min_{1\leq\flat\leq n} \left(a_{\Upsilon_\flat}^{-}\right)^{\sum_{\flat=1}^{n} l_\flat \varrho_\flat} \leq \prod_{\flat=1}^{n} \left(a_{\Upsilon_\flat}^{-}\right)^{l_\flat \varrho_\flat}$$
$$\leq \max_{1\leq\flat\leq n} \left(a_{\Upsilon_\flat}^{-}\right)^{\sum_{\flat=1}^{n} l_\flat \varrho_\flat}$$

As $\sum_{\flat=1}^{n} l_\flat \varrho_\flat = 1$. Then,

$$\min_{1\leq\flat\leq n} a_{\Upsilon_\flat}^{-} \leq \prod_{\flat=1}^{n} \left(a_{\Upsilon_\flat}^{-}\right)^{l_\flat \varrho_\flat} \leq \max_{1\leq\flat\leq n} a_{\Upsilon_\flat}^{-}$$

We can also prove for $ib_{\Upsilon_i}^{-}$.

Then, based on score function, we get

$$Sc(\Upsilon_\flat^{+}) \leq Sc(\Upsilon_\flat) \leq Sc(\Upsilon_\flat^{-}).$$

In light of the aforementioned outcome and the score function's definition, we thus arrive at

$$\Upsilon^{+} \leq CLBCFWA\left(\langle \Upsilon_1, l_1\rangle, \ldots, \langle \Upsilon_n, l_n\rangle\right) \leq \Upsilon^{-}.$$

B. CONFIDENCE LEVELS BIPOLAR COMPLEX FUZZY ORDERED WEIGHTED AVERAGING OPERATOR

Definition 7: Let $\Upsilon_\flat = \left(\left\langle a_{\Upsilon_\flat}^{+} + ib_{\Upsilon_\flat}^{+}, a_{\Upsilon_\flat}^{-} + ib_{\Upsilon_\flat}^{-}\right\rangle\right)$ $(\flat = 1, \ldots, n)$ be the family of BCFNs and $l_\flat$ be the confidence levels of $\Upsilon_\flat$ with $0 \leq l_i \leq 1$ and weights are $\varrho = (\varrho_1, \ldots, \varrho_n)^T$, such that $\sum_{\flat=1}^{n} \varrho_\flat = 1$ and $0 \leq \varrho_\flat \leq 1$. Then, the CLBCFOWA operator as;

$$CLBCFOWA\left(\langle \Upsilon_1, l_1\rangle, \ldots, \langle \Upsilon_n, l_n\rangle\right)$$
$$= \bigoplus_{\flat=1}^{n} \varrho_\flat \left(l_{\sigma(\flat)} \Upsilon_{\sigma(\flat)}\right), \quad (10)$$

and for $\Upsilon_{\sigma(\flat-1)} \geq \Upsilon_{\sigma(\flat)}$ the permutation is $\sigma(1), \ldots, \sigma(n)$ for all $\flat = 1, \ldots, n$. Based on definition (7), aggregated value for CLBCFOWA operator is given in Theorem (5).

Theorem 5: Let $\Upsilon_\flat = \left(\left\langle a^{+}_{\Upsilon_\flat} + ib^{+}_{\Upsilon_\flat}, a^{-}_{\Upsilon_\flat} + ib^{-}_{\Upsilon_\flat}\right\rangle\right)$ $(\flat = 1, \ldots, n)$ be the family of BCFNs and $l_\flat$ be the confidence levels of $\Upsilon_\flat$ with $0 \leq l_i \leq 1$ and weights $\varrho = (\varrho_1, \ldots, \varrho_n)^T$, such that $\sum_{\flat=1}^{n} \varrho_\flat = 1$ and $0 \leq \varrho_\flat \leq 1$. Then, CLBCFOWA operator is obtained as;

$$\begin{aligned} &CLBCFOWA\left(\langle\Upsilon_1, l_1\rangle, \ldots, \langle\Upsilon_n, l_n\rangle\right) \\ &= \bigoplus_{\flat=1}^{n} \varrho_\flat \left(l_{\sigma(\flat)}\Upsilon_{\sigma(\flat)}\right) \\ &= \left\{\left(\begin{array}{c} 1 - \prod_{\flat=1}^{n}\left(1 - a^{+}_{\Upsilon_{\sigma(\flat)}}\right)^{l_\flat \varrho_\flat} \\ +i\left(1 - \prod_{\flat=1}^{n}\left(1 - b^{+}_{\Upsilon_{\sigma(\flat)}}\right)^{l_\flat \varrho_\flat}\right), \\ -\prod_{\flat=1}^{n}\left(a^{-}_{\Upsilon_{\sigma(\flat)}}\right)^{l_\flat \varrho_\flat} + \\ i\left(-\prod_{\flat=1}^{n}\left(b^{-}_{\Upsilon_{\sigma(\flat)}}\right)^{l_\flat \varrho_\flat}\right) \end{array}\right)\right\}, \end{aligned} \tag{11}$$

where $\sigma(1), \ldots, \sigma(n)$ is the permutation of $(\flat = 1, \ldots, n)$, for each $\Upsilon_{\sigma(\flat-1)} \geq \Upsilon_{\sigma(\flat)}$ for all $(\flat = 1, \ldots, n)$.

Proof: Proof is same as Theorem (1).

The CLBCFOWA operator fulfilled the properties listed below.

Theorem 6 (Idempotency): Let $\Upsilon_\flat = \left(\left\langle a^{+}_{\Upsilon_\flat} + ib^{+}_{\Upsilon_\flat}, a^{-}_{\Upsilon_\flat} + ib^{-}_{\Upsilon_\flat}\right\rangle\right)$ $(\flat = 1, \ldots, n)$ be the family of BCFNs and $l_\flat$ be the confidence levels of $\Upsilon_\flat$ with weights $\varrho = (\varrho_1, \ldots, \varrho_n)^T$, such that $\sum_{\flat=1}^{n} \varrho_\flat = 1$ and $0 \leq \varrho_\flat \leq 1$. Then,

$$CLBCFOWA\left(\langle\Upsilon_1, l_1\rangle, \ldots, \langle\Upsilon_n, l_n\rangle\right) = \Upsilon \tag{12}$$

Proof: Proof is same as Theorem (2).

Theorem 7 (Monotonicity): Let $\Upsilon_\flat = \left(\left\langle a^{+}_{\Upsilon_\flat} + ib^{+}_{\Upsilon_\flat}, a^{-}_{\Upsilon_\flat} + ib^{-}_{\Upsilon_\flat}\right\rangle\right)$ and $\Upsilon^{/}_\flat = \left(\left\langle a^{/+}_{\Upsilon_\flat} + ib^{/+}_{\Upsilon_\flat}, a^{/-}_{\Upsilon_\flat} + ib^{/-}_{\Upsilon_\flat}\right\rangle\right)$ $(\flat = 1, \ldots, n)$ be the family of BCFNs and $l_\flat$ be the confidence levels of $\Upsilon_\flat$ with weights $\varrho = (\varrho_1, \ldots, \varrho_n)^T$, such that $\sum_{\flat=1}^{n} \varrho_\flat = 1$ and $0 \leq \varrho_\flat \leq 1$, if $a^{+}_{\Upsilon_\flat} \geq a^{/+}_{\Upsilon_\flat}$, $ib^{+}_{\Upsilon_\flat} \geq ib^{/+}_{\Upsilon_\flat}$, $ib^{-}_{\Upsilon_\flat} \leq ib^{/-}_{\Upsilon_\flat}$, and $a^{-}_{\Upsilon_\flat} \leq a^{/-}_{\Upsilon_\flat}$. Then,

$$\begin{aligned} &CLBCFOWA\left(\langle\Upsilon_1, l_1\rangle, \ldots, \langle\Upsilon_n, l_n\rangle\right) \\ &\geq CLBCFOWA\left(\left\langle\Upsilon^{/}_1, l_1\right\rangle, \ldots, \left\langle\Upsilon^{/}_n, l_n\right\rangle\right). \end{aligned} \tag{13}$$

Proof: Proof is same as Theorem (3).

Theorem 8 (Boundedness): Let $\Upsilon_\flat = \left(\left\langle a^{+}_{\Upsilon_\flat} + ib^{+}_{\Upsilon_\flat}, a^{-}_{\Upsilon_\flat} + ib^{-}_{\Upsilon_\flat}\right\rangle\right)$ $(\flat = 1, \ldots, n)$ be the family of BCFNs and $l_\flat$ be the confidence levels of $\Upsilon_\flat$ with weight are $\varrho = (\varrho_1, \ldots, \varrho_n)^T$, such as $\sum_{\flat=1}^{n} \varrho_\flat = 1$ and $0 \leq \varrho_\flat \leq 1$, if $\Upsilon^{+}_\flat$, $\Upsilon^{-}_\flat$ are the maximum and minimum BCFNs. Then,

$$\Upsilon^{+}_\flat \leq CLBCFOWA(\langle\Upsilon_1, l_1\rangle, \ldots, \langle\Upsilon_n, l_n\rangle) \leq \Upsilon^{-}_\flat \tag{14}$$

Proof: Proof is same as Theorem (4).

C. CONFIDENCE LEVELS BIPOLAR COMPLEX FUZZY HYBRID AVERAGING OPERATOR

Definition 8: Let $\Upsilon_\flat = \left(\left\langle a^{+}_{\Upsilon_\flat} + ib^{+}_{\Upsilon_\flat}, a^{-}_{\Upsilon_\flat} + ib^{-}_{\Upsilon_\flat}\right\rangle\right)$ $(\flat = 1, \ldots, n)$ be the family of BCFNs and $l_\flat$ be the confidence levels of $\Upsilon_\flat$ with $l_\flat \in [0, 1]$ and weights $\varrho = (\varrho_1, \ldots, \varrho_n)^T$, such as $\sum_{\flat=1}^{n} \varrho_\flat = 1$ and $0 \leq \varrho_\flat \leq 1$, and $F = (F_1, \ldots, F_n)^T$, such that $\sum_{\flat=1}^{n} F_\flat = 1$ and $0 \leq F_\flat \leq 1$ are associated weights of BCFNs. Then, the CLBCFHA operator as;

$$\begin{aligned} &CLBCFHA\left(\langle\Upsilon_1, l_1\rangle, \ldots, \langle\Upsilon_n, l_n\rangle\right) \\ &= \bigoplus_{\flat=1}^{n} \varrho_\flat \left(l_{\sigma(\flat)}\Upsilon^{*}_{\sigma(\flat)}\right), \end{aligned} \tag{15}$$

for $\Upsilon_{\sigma(\flat-1)} \geq \Upsilon_{\sigma(\flat)}$ is the permutation is $\sigma(1), \ldots, \sigma(n)$ for all $(\flat = 1, \ldots, n)$. Based on definition (8), aggregated value for CLBCFHA operator is given in Theorem (9).

Theorem 9: Let $\Upsilon_\flat = \left(\left\langle a^{+}_{\Upsilon_\flat} + ib^{+}_{\Upsilon_\flat}, a^{-}_{\Upsilon_\flat} + ib^{-}_{\Upsilon_\flat}\right\rangle\right)$ $(\flat = 1, \ldots, n)$ be the family of BCFNs and $l_\flat$ be the confidence levels of $\Upsilon_\flat$ with weights $\varrho = (\varrho_1, \ldots, \varrho_n)^T$, such as $\sum_{\flat=1}^{n} \varrho_\flat = 1$ and $0 \leq \varrho_\flat \leq 1$, and $F = (F_1, \ldots, F_n)^T$, such as $\sum_{\flat=1}^{n} F_\flat = 1$ and $0 \leq F_\flat \leq 1$ are associated weights of BCFNs. The, CLBCFHA operator as;

$$\begin{aligned} &CLBCFHA\left(\langle\Upsilon_1, l_1\rangle, \ldots, \langle\Upsilon_n, l_n\rangle\right) \\ &= \bigoplus_{\flat=1}^{n} \varrho_\flat \left(l_{\sigma(\flat)}\Upsilon^{*}_{\sigma(\flat)}\right) \\ &= \left\{\left(\begin{array}{c} 1 - \prod_{\flat=1}^{n}\left(1 - a^{*+}_{\Upsilon_{\sigma(\flat)}}\right)^{l_\flat \varrho_\flat} + \\ i\left(1 - \prod_{\flat=1}^{n}\left(1 - b^{*+}_{\Upsilon_{\sigma(\flat)}}\right)^{l_\flat \varrho_\flat}\right), \\ -\prod_{\flat=1}^{n}\left(a^{*-}_{\Upsilon_{\sigma(\flat)}}\right)^{l_\flat \varrho_\flat} + \\ i\left(-\prod_{\flat=1}^{n}\left(b^{*-}_{\Upsilon_{\sigma(\flat)}}\right)^{l_\flat \varrho_\flat}\right) \end{array}\right)\right\}. \end{aligned} \tag{16}$$

where $\sigma(1), \ldots, \sigma(n)$ are permutation of $(\flat = 1, \ldots, n)$, for each $\Upsilon_{\sigma(\flat-1)} \geq \Upsilon_{\sigma(\flat)}$ for all $(\flat = 1, \ldots, n)$. Biggest permutation value of BCFNs is given by $\Upsilon^{*}_{\sigma(\flat)} = nF_\flat\Upsilon_\flat$, and n is balancing coefficient.

Proof: Proof is same as Theorem (1).

V. CONFIDENCE LEVELS BIPOLAR COMPLEX FUZZY GEOMETRIC OPERATOR

Here, we defined some AOs such as, CLBCFWG, CLBCFOWG, and CLBCFHG operators.

A. CONFIDENCE LEVELS BIPOLAR COMPLEX FUZZY WEIGHTED GEOMETRIC OPERATOR

Definition 9: Let $\Upsilon_\flat = \left(\left\langle a^{+}_{\Upsilon_\flat} + ib^{+}_{\Upsilon_\flat}, a^{-}_{\Upsilon_\flat} + ib^{-}_{\Upsilon_\flat}\right\rangle\right)$ $(\flat = 1, \ldots, n)$ be a family of BCFNs and $l_\flat$ be the confidence levels of $\Upsilon_\flat$ with $l_\flat \in [0, 1]$ with weights $\varrho = (\varrho_1, \ldots, \varrho_n)^T$, such that $\sum_{\flat=1}^{n}\varrho_\flat = 1$ and $0 \leq \varrho_\flat \leq 1$. Then, the CLBCFWG operator as;

$$CLBCFWG\left(\langle\Upsilon_1, l_1\rangle, \ldots, \langle\Upsilon_n, l_n\rangle\right) = \bigotimes_{\flat=1}^{n}\left(\Upsilon_\flat^{l_\flat}\right)^{\varrho_\flat}, \tag{17}$$

Based on definition (9), aggregated value for CLBCFWG operator is shown in Theorem (10).

Theorem 10: Let $\Upsilon_\flat = \left(a^+_{\Upsilon_\flat} + ib^+_{\Upsilon_\flat}, a^-_{\Upsilon_\flat} + ib^-_{\Upsilon_\flat}\right)$ $(\flat = 1, \ldots, n)$ be the family of BCFNs and $l_\flat$ be the confidence levels of $\Upsilon_\flat$ with $l_\flat$ with weights $\varrho = (\varrho_1, \ldots, \varrho_n)^T$, such that $\sum_{\flat=1}^{n}\varrho_\flat = 1$ and $0 \leq \varrho_\flat \leq 1$. Then, CLBCFWG operator as;

$$CLBCFWG\left(\langle\Upsilon_1, l_1\rangle, \ldots, \langle\Upsilon_n, l_n\rangle\right) = \bigotimes_{\flat=1}^{n}\left(\Upsilon_\flat^{l_\flat}\right)^{\varrho_\flat}, = \left\{\left(\begin{array}{c}\prod_{\flat=1}^{n}\left(a^+_{\Upsilon_\flat}\right)^{l_\flat\varrho_\flat} + i\left(\prod_{\flat=1}^{n}\left(b^+_{\Upsilon_\flat}\right)^{l_\flat\varrho_\flat}\right), \\ -1 + \prod_{\flat=1}^{n}\left(1 + a^-_{\Upsilon_\flat}\right)^{l_\flat\varrho_\flat} + \\ i\left(-1 + \prod_{\flat=1}^{n}\left(1 + b^-_{\Upsilon_\flat}\right)^{l_\flat\varrho_\flat}\right)\end{array}\right)\right\}. \tag{18}$$

Proof: Proof is same as Theorem (1)

The following properties were fulfilled by the CLBCFWG operator.

Theorem 11 (Idempotency): Let $\Upsilon_\flat = \left(\left\langle a^+_{\Upsilon_\flat} + ib^+_{\Upsilon_\flat}, a^-_{\Upsilon_\flat} + ib^-_{\Upsilon_\flat}\right\rangle\right)$ $(\flat = 1, \ldots, n)$ be the family of BCFNs and $l_\flat$ be the confidence levels of $\Upsilon_\flat$ with $l_\flat$ with weights $\varrho = (\varrho_1, \ldots, \varrho_n)^T$, such that $\sum_{\flat=1}^{n}\varrho_\flat = 1$ and $0 \leq \varrho_\flat \leq 1$. Then,

$$CLBCFWG\left(\langle\Upsilon_1, l_1\rangle, \ldots, \langle\Upsilon_n, l_n\rangle\right) = \Upsilon. \tag{19}$$

Proof: Proof is same as (2).

Theorem 12 (Monotonicity): Let $\Upsilon_\flat = \left(\left\langle a^+_{\Upsilon_\flat} + ib^+_{\Upsilon_\flat}, a^-_{\Upsilon_\flat} + ib^-_{\Upsilon_\flat}\right\rangle\right)$ and $\Upsilon'_\flat = \left(\left\langle a'^+_{\Upsilon_\flat} + ib'^+_{\Upsilon_\flat}, a'^-_{\Upsilon_\flat} + ib'^-_{\Upsilon_\flat}\right\rangle\right)$ $(\flat = 1, \ldots, n)$ be the family of BCFNs and $l_\flat$ be the confidence levels of $\Upsilon_\flat$ with weights $\varrho = (\varrho_1, \ldots, \varrho_n)^T$, such that $\sum_{\flat=1}^{n}\varrho_\flat = 1$ and $0 \leq \varrho_\flat \leq 1$, if $a^+_{\Upsilon_\flat} \geq a'^+_{\Upsilon_\flat}$, $ib^+_{\Upsilon_\flat} \geq ib'^+_{\Upsilon_\flat}$, $ib^-_{\Upsilon_\flat} \leq ib'^-_{\Upsilon_\flat}$, and $a^-_{\Upsilon_\flat} \leq a'^-_{\Upsilon_\flat}$. Then,

$$CLBCFWG\left(\langle\Upsilon_1, l_1\rangle, \ldots, \langle\Upsilon_n, l_n\rangle\right) \geq CLBCFWG\left(\langle\Upsilon_1, l_1\rangle, \ldots, \langle\Upsilon_n, l_n\rangle\right). \tag{20}$$

Proof: Proof is same as (3).

Theorem 13 (Boundedness): Let $\Upsilon_\flat = \left(\left\langle a^+_{\Upsilon_\flat} + ib^+_{\Upsilon_\flat}, a^-_{\Upsilon_\flat} + ib^-_{\Upsilon_\flat}\right\rangle\right)$ $(\flat = 1, \ldots, n)$ be the family of BCFNs and $l_\flat$ be the confidence levels of $\Upsilon_\flat$ with weights $\varrho = (\varrho_1, \ldots, \varrho_n)^T$, such that $\sum_{\flat=1}^{n}\varrho_\flat = 1$ and $0 \leq \varrho_\flat \leq 1$, if $\Upsilon^+_\flat$, $\Upsilon^-_\flat$ are the maximum and minimum BCFNs. Then,

$$\Upsilon^+_\flat \leq CLBCFWG\left(\langle\Upsilon_1, l_1\rangle, \ldots, \langle\Upsilon_n, l_n\rangle\right) \leq \Upsilon^-_\flat. \tag{21}$$

Proof: Proof is same as (4).

B. CONFIDENCE LEVELS BIPOLAR COMPLEX FUZZY ORDERED WEIGHTED GEOMETRIC OPERATOR

Definition 10: Let $\Upsilon_\flat = \left(\left\langle a^+_{\Upsilon_\flat} + ib^+_{\Upsilon_\flat}, a^-_{\Upsilon_\flat} + ib^-_{\Upsilon_\flat}\right\rangle\right)$ $(\flat = 1, \ldots, n)$ be the family of BCFNs with weight are $\varrho = (\varrho_1, \ldots, \varrho_n)^T$, such as $\sum_{\flat=1}^{n}\varrho_\flat = 1$ and $0 \leq \varrho_\flat \leq 1$. Then, the CLBCFOWG operator as;

$$CLBCFOWG\left(\langle\Upsilon_1, l_1\rangle, \ldots, \langle\Upsilon_n, l_n\rangle\right) = \bigotimes_{\flat=1}^{n}\left(\Upsilon_{\sigma(\flat)}^{l_{\sigma(\flat)}}\right)^{\varrho_\flat}, \tag{22}$$

and for $\Upsilon_{\sigma(\flat-1)} \geq \Upsilon_{\sigma(\flat)}$ the permutation is $\sigma(1), \ldots, \sigma(n)$ for all $(\flat = 1, \ldots, n)$. Based on definition (10), aggregated value for CLBCFOWG operator is presented in Theorem (14).

Theorem 14: Let $\Upsilon_\flat = \left(\left\langle a^+_{\Upsilon_\flat} + ib^+_{\Upsilon_\flat}, a^-_{\Upsilon_\flat} + ib^-_{\Upsilon_\flat}\right\rangle\right)$ $(\flat = 1, \ldots, n)$ be the family of BCFNs and $l_\flat$ be the confidence levels of $\Upsilon_\flat$ with $l_\flat \in [0, 1]$ with weights $\varrho = (\varrho_1, \ldots, \varrho_n)^T$, such that $\sum_{\flat=1}^{n}\varrho_\flat = 1$ and $0 \leq \varrho_\flat \leq 1$. Then, CLBCFOWG operator as;

$$CLBCFOWG\left(\langle\Upsilon_1, l_1\rangle, \ldots, \langle\Upsilon_n, l_n\rangle\right) = \bigotimes_{\flat=1}^{n}\left(\Upsilon_{\sigma(\flat)}^{l_{\sigma(\flat)}}\right)^{\varrho_\flat} = \left\{\left(\begin{array}{c}\prod_{\flat=1}^{n}\left(a^-_{\Upsilon_{\sigma(\flat)}}\right)^{l_\flat\varrho_\flat} + i\left(\prod_{\flat=1}^{n}\left(b^-_{\Upsilon_{\sigma(\flat)}}\right)^{l_\flat\varrho_\flat}\right), \\ -1 + \prod_{\flat=1}^{n}\left(1 + a^+_{\Upsilon_{\sigma(\flat)}}\right)^{l_\flat\varrho_\flat} + \\ i\left(-1 + \prod_{\flat=1}^{n}\left(1 + b^+_{\Upsilon_{\sigma(\flat)}}\right)^{l_\flat\varrho_\flat}\right)\end{array}\right)\right\}. \tag{23}$$

where $\sigma(1), \ldots, \sigma(n)$ be the permutation of the $(\flat = 1, \ldots, n)$, for each $\Upsilon_{\sigma(\flat-1)} \geq \Upsilon_{\sigma(\flat)}$ for all $(\flat = 1, \ldots, n)$.

Proof: Proof is same from Theorem (1).

The CLBCFOWG operator met the subsequent properties.

Theorem 15 (Idempotency): Let $\Upsilon_\flat = \left(\left\langle a^+_{\Upsilon_\flat} + ib^+_{\Upsilon_\flat}, a^-_{\Upsilon_\flat} + ib^-_{\Upsilon_\flat}\right\rangle\right)$ $(\flat = 1, \ldots, n)$ be the family of BCFNs and $l_\flat$ be the confidence levels of $\Upsilon_\flat$ with weights $\varrho = (\varrho_1, \ldots, \varrho_n)^T$, such that $\sum_{\flat=1}^{n}\varrho_\flat = 1$ and $0 \leq \varrho_\flat \leq 1$. Then,

$$CLBCFOWG\left(\langle\Upsilon_1, l_1\rangle, \ldots, \langle\Upsilon_n, l_n\rangle\right) = \Upsilon \tag{24}$$

Proof: Proof is same as Theorem (2).

Theorem 16 (Monotonicity): Let $\Upsilon_\flat = \left(\left\langle a^+_{\Upsilon_\flat} + ib^+_{\Upsilon_\flat}, a^-_{\Upsilon_\flat} + ib^-_{\Upsilon_\flat}\right\rangle\right)$ and $\Upsilon'_\flat = \left(\left\langle a'^+_{\Upsilon_\flat} + ib'^+_{\Upsilon_\flat}, a'^-_{\Upsilon_\flat} + ib'^-_{\Upsilon_\flat}\right\rangle\right)$ $(\flat = 1, \ldots, n)$ be the family of BCFNs and $l_\flat$ be the confidence levels of $\Upsilon_\flat$ with the weight vector $\varrho = (\varrho_1, \ldots, \varrho_n)^T$, such that $\sum_{\flat=1}^{n}\varrho_\flat = 1$ and $0 \leq \varrho_\flat \leq 1$, if $a^+_{\Upsilon_\flat} \geq a'^+_{\Upsilon_\flat}$, $ib^+_{\Upsilon_\flat} \geq ib'^+_{\Upsilon_\flat}$, $ib^-_{\Upsilon_\flat} \leq ib'^-_{\Upsilon_\flat}$, and $a^-_{\Upsilon_\flat} \leq a'^-_{\Upsilon_\flat}$.

Then,

$$
\begin{aligned}
&CLBCFOWG\left(\langle \Upsilon_1, l_1\rangle, \ldots, \langle \Upsilon_n, l_n\rangle\right) \\
&\geq CLBCFOWG\left(\left\langle \Upsilon_1^{/}, l_1\right\rangle, \ldots, \left\langle \Upsilon_n^{/}, l_n\right\rangle\right). \qquad (25)
\end{aligned}
$$

Proof: Proof is same as Theorem (3).

Theorem 17 (Boundedness): Let $\Upsilon_\flat = \left(\left\langle a^+_{\Upsilon_\flat} + ib^+_{\Upsilon_\flat}, a^-_{\Upsilon_\flat} + ib^-_{\Upsilon_\flat}\right\rangle\right)$ $(\flat = 1, \ldots, n)$ be the set of BCFNs and $l_\flat$ be the confidence levels of $\Upsilon_\flat$ with weight are $\varrho = (\varrho_1, \ldots, \varrho_n)^T$, such that $\sum_{\flat=1}^n \varrho_\flat = 1$ and $0 \leq \varrho_\flat \leq 1$, if $\Upsilon_\flat^+$, $\Upsilon_\flat^-$ are the maximum and minimum BCFNs. Then,

$$
\Upsilon_\flat^+ \leq CLBCFOWG\left(\langle \Upsilon_1, l_1\rangle, \ldots, \langle \Upsilon_n, l_n\rangle\right) \leq \Upsilon_\flat^- \qquad (26)
$$

Proof: Proof is same as Theorem (4).

C. CONFIDENCE LEVELS BIPOLAR COMPLEX FUZZY HYBRID GEOMETRIC OPERATOR

Definition 11: Let $\Upsilon_\flat = \left(\left\langle a^+_{\Upsilon_\flat} + ib^+_{\Upsilon_\flat}, a^-_{\Upsilon_\flat} + ib^-_{\Upsilon_\flat}\right\rangle\right)$ $(\flat = 1, \ldots, n)$ be the family of BCFNs and $l_\flat$ be the confidence levels of $\Upsilon_\flat$ with $l_\flat \in [0, 1]$ with weights $\varrho = (\varrho_1, \ldots, \varrho_n)^T$, such that $\sum_{\flat=1}^n \varrho_\flat = 1$ and $0 \leq \varrho_\flat \leq 1$, and $F = (F_1, \ldots, F_n)^T$, such as $\sum_{\flat=1}^n F_\flat = 1$ and $0 \leq F_\flat \leq 1$ be the associated weights of BCFNs. Then, the CLBCFHG operator as;

$$
\begin{aligned}
&CLBCFHG\left(\langle \Upsilon_1, l_1\rangle, \ldots, \langle \Upsilon_n, l_n\rangle\right) \\
&= \bigotimes_{\flat=1}^{n} \left(\Upsilon^{*l_{\sigma(\flat)}}_{\sigma(\flat)}\right)^{\varrho_\flat}, \qquad (27)
\end{aligned}
$$

for $\Upsilon_{\sigma(\flat-1)} \geq \Upsilon_{\sigma(\flat)}$ permutation is $\sigma(1), \ldots, \sigma(n)$ for all $(\flat = 1, \ldots, n)$. Based on definition (11), aggregated value for CLBCFHG operator is given in Theorem (18).

Theorem 18: Let $\Upsilon_\flat = \left(\left\langle a^+_{\Upsilon_\flat} + ib^+_{\Upsilon_\flat}, a^-_{\Upsilon_\flat} + ib^-_{\Upsilon_\flat}\right\rangle\right)$ $(\flat = 1, \ldots, n)$ be the family of BCFNs and $l_\flat$ be the confidence levels of $\Upsilon_\flat$ with $l_\flat \in [0, 1]$ with weights $\varrho = (\varrho_1, \ldots, \varrho_n)^T$, such that $\sum_{\flat=1}^n \varrho_\flat = 1$ and $0 \leq \varrho_\flat \leq 1$, and $F = (F_1, \ldots, F_n)^T$, such that $\sum_{\flat=1}^n F_\flat = 1$ and $0 \leq F_\flat \leq 1$ are associated weights of BCFNs. The CLBCFHG operator as;

$$
\begin{aligned}
&CLBCFHG\left(\langle \Upsilon_1, l_1\rangle, \ldots, \langle \Upsilon_n, l_n\rangle\right) \\
&= \bigotimes_{\flat=1}^{n} \left(\Upsilon^{*l_{\sigma(\flat)}}_{\sigma(\flat)}\right)^{\varrho_\flat} \\
&= \left\{\left(\begin{array}{c}
\prod_{\flat=1}^{n} \left(a^{*-}_{\Upsilon_{\sigma(\flat)}}\right)^{l_\flat \varrho_\flat} + \\
i\left(\prod_{\flat=1}^{n} \left(b^{*-}_{\Upsilon_{\sigma(\flat)}}\right)^{l_\flat \varrho_\flat}\right), \\
-1 + \prod_{\flat=1}^{n} \left(1 + a^{*+}_{\Upsilon_{\sigma(\flat)}}\right)^{l_\flat \varrho_\flat} + \\
i\left(-1 + \prod_{\flat=1}^{n} \left(1 + b^{*+}_{\Upsilon_{\sigma(\flat)}}\right)^{l_\flat \varrho_\flat}\right)
\end{array}\right)\right\}. \qquad (28)
\end{aligned}
$$

where $\sigma(1), \ldots, \sigma(n)$ are permutation of $(\flat = 1, \ldots, n)$, for each $\Upsilon_{\sigma(\flat-1)} \geq \Upsilon_{\sigma(\flat)}$ for all $(\flat = 1, \ldots, n)$. Permutation value of BCFNs is denoted by $\Upsilon^*_{\sigma(\flat)} = (\Upsilon_\flat)^{nF_\flat}$, and balancing coefficient is n.

Proof: Proof is same from Theorem (1).

VI. APPROACH FOR MCDM BASED ON CONFIDENCE LEVELS BIPOLAR COMPLEX FUZZY INFORMATION

In this portion, we formulate an algorithm to solve MCDM problem using dveloped AOs. Let $\acute{E} = \left\{\acute{E}_1, \ldots, \acute{E}_n\right\}$ are a set of n criteria and $\wp = \{\wp_1, \ldots, \wp_m\}$ are set of m alternatives for aN MCDM problem. Weights of the criterion $\acute{E}_\flat$ as $\varrho = (\varrho_1, \ldots, \varrho_n)^T$, such that $\sum_{\flat=1}^n \varrho_\flat$ and $0 \leq \varrho_\flat$. The experts also indicate their familiarity with the analysis options and provide confidence levels l_{ij} $\left(0 \leq l_{ij} \leq 1\right)$ to include the the concept of confidence levels. The following are the key algorithmic steps:

Step 1: Define a decision matrix utilizing the evaluation information gathered with criteria for competent experts for each alternative;

$$
\mathbf{M} = \begin{bmatrix}
\Upsilon_{11} & \Upsilon_{12} & . & . & \Upsilon_{1n} \\
\Upsilon_{21} & \Upsilon_{22} & . & . & \Upsilon_{2n} \\
\Upsilon_{31} & \Upsilon_{32} & . & . & \Upsilon_{3n} \\
. & . & . & . & . \\
\Upsilon_{m1} & \Upsilon_{m2} & . & . & \Upsilon_{mn}
\end{bmatrix}
$$

Step 2: Evaluate the aggregated information with the help of CLBCFWA operator.

Step 3: Analyze the score value of alternatives.

Step 4: Give ranking to alternatives based on score value.

VII. EXAMPLE

Dealing with the risk of large accidents and disasters, involving government and other public organizations in the emergency response, prevention, disposal, and recovery process, and building an efficient response plan to take a variety of necessary procedures is all part of emergency management. Research, technology, planning, and management are all employed to ensure the security of public safety, health, and property related emergency operations, as well as to promote the society's harmonious and long term progress. Natural disasters have caused significant injury and damage to human lives and the global economy in recent years. To successfully limit the losses caused by major accidents and disasters, the Emergency Management Center (EMC) will build a variety of emergency choices based on the sorts of incidents and will ask experts from various disciplines to evaluate alternative emergency plans. Emergency preparedness includes alternative emergency assessment. The conventional decision making problem is at the heart of the subject, and it has received a great deal of attention. Therefore, in order to locate the finest EMC emergency solution, we'll apply the outlined technique to the stated situation.

Five better possibilities would be studied further after a series of screenings. There are four different sets, such as;

Expert take four needs for proper modeling of alternative qualities, which are given below; $\acute{E}_1$: preparation ability; $\acute{E}_2$: rescuing ability; $\acute{E}_3$: restore ability; $\acute{E}_4$: reaction capacity, and weights of the criteria are $\varrho = (0.16, 0.28, 0.36, 0.20)^T$. The information given by experts in Table 1.

Step 1: The entire information provided by experts for each alternative is given in Table 1.

TABLE 1. BCF information given by experts.

	$\acute{E}_1$	$\acute{E}_2$	$\acute{E}_3$	$\acute{E}_4$
$\wp_1$	$(\langle 0.3+i0.4,-0.5-i0.4\rangle, 0.92)$	$(\langle 0.5+i0.4,-0.7-i0.5\rangle, 0.99)$	$(\langle 0.6+i0.7,-0.6-i0.2\rangle, 0.96)$	$(\langle 0.3+i0.3,-0.6-i0.6\rangle, 0.56)$
$\wp_2$	$(\langle 0.3+i0.4,-0.5-i0.4\rangle, 0.74)$	$(\langle 0.1+i0.8,-0.1-i0.8\rangle, 0.73)$	$(\langle 0.5+i0.7,-0.4-i0.3\rangle, 0.81)$	$(\langle 0.7+i0.4,-0.6-i0.7\rangle, 0.92)$
$\wp_3$	$(\langle 0.8+i0.6,-0.2-i0.6\rangle, 0.82)$	$(\langle 0.2+i0.3,-0.8-i0.3\rangle, 0.59)$	$(\langle 0.4+i0.2,-0.3-i0.7\rangle, 0.84)$	$(\langle 0.4+i0.3,-0.7-i0.8\rangle, 0.71)$
$\wp_4$	$(\langle 0.2+i0.4,-0.3-i0.2\rangle, 0.68)$	$(\langle 0.2+i0.5,-0.3-i0.5\rangle, 0.83)$	$(\langle 0.6+i0.5,-0.3-i0.2\rangle, 0.66)$	$(\langle 0.8+i0.1,-0.6-i0.1\rangle, 0.75)$

Step 2: The outcomes of aggregating value utilizing expert information and the CLBCFWA operator are shown in Table 2.

Step 3: Examine the alternatives' score values.

Step 4: To decide which option is the best, use the score values listed in Table 3.

A. COMPARATIVE STUDY

The comparative study of determined techniques was discussed in this section, along with certain common operators based on accepted concept like as, BCFSs.

For this, we choose a few famous theories that are, Mahmood et al. [22], BCFSs and their applications in generalized similarity measures; Mahmood et al. [23] BCFS-based Hamacher aggregation information; Mahmood et al. [24], Dombi AOs under bipolar complex fuzzy information; Hayat et al. [16] group generalized q-rung orthopair fuzzy soft sets; Hayat et al. [17] New group-based generalized interval-valued q-rung orthopair fuzzy soft aggregation operators and their applications in sports decision-making problems; Yang et al. [33] aggregation and interaction aggregation soft operators on interval-valued q-rung orthopair fuzzy soft environment and application in automation company evaluation.

TABLE 2. Obtained values using CLBCFWA.

	$\acute{E}_1$	$\acute{E}_2$	$\acute{E}_3$	$\acute{E}_4$
$\wp_1$	$\langle 0.385+i0.221,-0.331-i0.194\rangle$	$\langle 0.276+i0.164,-0.337-i0.329\rangle$	$\langle 0.361+i0.217,-0.273-i0.385\rangle$	$\langle 0.183+i0.354,-0.280-i0.361\rangle$
$\wp_2$	$\langle 0.212+i0.436,-0.402-i0.309\rangle$	$\langle 0.361+i0.441,-0.185-i0.339\rangle$	$\langle 0.407+i0.332,-0.429-i0.329\rangle$	$\langle 0.390+i0.264,-0.297-i0.420\rangle$
$\wp_3$	$\langle 0.173+i0.485,-0.329-i0.280\rangle$	$\langle 0.343+i0.408,-0.207-i0.224\rangle$	$\langle 0.200+i0.308,-0.501-i0.210\rangle$	$\langle 0.319+i0.189,-0.339-i0.372\rangle$
$\wp_4$	$\langle 0.206+i0.264,-0.420-i0.318\rangle$	$\langle 0.226+i0.132,-0.375-i0.534\rangle$	$\langle 0.372+i0.385,-0.115-i0.273\rangle$	$\langle 0.286+i0.280,-0.398-i0.177\rangle$

TABLE 3. Score values of the alternatives.

Operators	$S(\acute{E}_1)$	$S(\acute{E}_2)$	$S(\acute{E}_3)$	$S(\acute{E}_4)$
CLBCFWA	0.482	0.431	0.406	0.577

TABLE 4. Alternative ranking.

CLBCFWA	$\acute{E}_4 > \acute{E}_1 > \acute{E}_2 > \acute{E}_3$

The method described in [16], [17], [22], [23], [24], and [33] contains bipolar fuzzy set details, but the given model cannot be solved using this method. Reviewing Table 5 reveals that the methods now in use lack basic information and are unable to solve or rank the case that has been provided. Compared to other methods already in use, the strategy suggested in this study is more capable and dependable. The main analysis of the identified and proposed hypotheses is presented in Table 5.

The results show that the BCFS and DM approaches' existing theories [16], [17], [22], [23], [24], [33] handle the data and yield the conclusion that $\acute{E}_4$ is the best option, but anyone can argue that this choice was unfairly made due to the lack of confidence levels that the experts gave to each attribute in his own decision. This demonstrates that the new AOs are more fair and effective than the previous ones. The outcomes further demonstrate that the DM process and the newly presented operators are suitable instruments for dealing with ambiguous and uncertain information in the context of the bipolar complex fuzzy set.

A BCFS is the result of combining BFS, which an expert needs to describe an item's positive and negative qualities, with CFS, which an expert needs to deal with

TABLE 5. Ranking of the existing methods.

Methods	Score value $É_1$	$É_2$	$É_3$	$É_4$	Ranking
Mahmood et al. [22]	0.702	0.712	0.636	0.748	$É_4 > É_1 > É_2 > É_3$
Mahmood et al. [23]	0.573	0.597	0.552	0.603	$É_4 > É_2 > É_1 > É_3$
Mahmood et al. [24]	0.264	0.229	0.207	0.284	$É_4 > É_1 > É_2 > É_3$
Hayat et al. [16]	0.175	0.143	0.130	0.189	$É_4 > É_1 > É_2 > É_3$
Hayat et al. [17]	0.441	0.343	0.401	0.484	$É_4 > É_1 > É_2 > É_3$
Yang et al. [33]	0.862	0.841	0.811	0.874	$É_4 > É_1 > É_2 > É_3$

two-dimensional data. One of the most sophisticated and extensively utilized structures is the BCFS structure, and these operators have not yet been specified within the BCFS environment. Biploar complex fuzzy set is extremely important in real-life DM, since it deals with two-dimensional information on an object as well as all of its properties and counter-properties, or positive and negative aspects. Take into consideration the BCFS's effective ability to handle ambiguity and inconsistent circumstances as well as to check the unclear and confusing information that arises in real-life situations.

VIII. CONCLUSION

The bipolar complex fuzzy set (BCFS) theory is a better tool for expressing information in an unpredictable environment in multi-criteria decision making situations. However, Mahmood et al. [23] shows that the BCFS is more general than the bipolar fuzzy set. Several writers proposed aggregation operators for adding bipolar complex fuzzy numbers. However, existing confidence level bipolar complex fuzzy AOs is based on the assumption that experts are completely familiar with the evaluated items, i.e., all experts offered their assessment of the distinct alternatives with the same level of confidence. This type of situation is partially satisfied in real-world problem modeling. The current study provides a series of confidence levels averaging and confidence levels geometric AOs for this purpose by adding expert confidence levels throughout the evaluation step in a bipolar complex fuzzy environment. Some of its key features are well established. These defined operators can more perceptibly explain real-life circumstances with the help of experts' confidence levels during evaluation and will resemble much more real-life situations in a bipolar complicated fuzzy environment. Lastly, a thorough debate was conducted to demonstrate the applicability and superiority of the presented strategy over the current ones.

Based on the larger acceptance of a bipolar complex fuzzy set, we will make efforts in the future to apply the concept of confidence levels bipolar complex fuzzy set to real-world challenges such as fuzzy cluster analysis, uncertain programming, and pattern recognition, among others. In addition, we will concentrate on the development of novel AOs for bipolar complex fuzzy numbers.

REFERENCES

[1] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets Syst.*, vol. 20, no. 1, pp. 87–96, Aug. 1986.

[2] P. Aragonés-Beltrán, J. P. Pastor-Ferrando, F. García-García, and A. Pascual-Agulló, "An analytic network process approach for siting a municipal solid waste plant in the metropolitan area of Valencia (Spain)," *J. Environ. Manage.*, vol. 91, no. 5, pp. 1071–1086, May 2010.

[3] A. U. M. Alkouri and A. R. Salleh, "Linguistic variable, hedges and several distances on complex fuzzy sets," *J. Intell. Fuzzy Syst.*, vol. 26, no. 5, pp. 2527–2535, 2014.

[4] H. Alolaiyan, H. A. Alshehri, M. H. Mateen, D. Pamucar, and M. Gulzar, "A novel algebraic structure of (a,b)-complex fuzzy subgroups," *Entropy*, vol. 23, no. 8, p. 992, Jul. 2021.

[5] M. Ahsan, M. Saeed, A. Mehmood, M. H. Saeed, and J. Asad, "The study of HIV diagnosis using complex fuzzy hypersoft mapping and proposing appropriate treatment," *IEEE Access*, vol. 9, pp. 104405–104417, 2021.

[6] L. Bi, Z. Zeng, B. Hu, and S. Dai, "Two classes of entropy measures for complex fuzzy sets," *Mathematics*, vol. 7, no. 1, p. 96, Jan. 2019.

[7] S. J. Chen and C. L. Hwang, "Fuzzy multiple attribute decision making methods," in *Fuzzy Multiple Attribute Decision Making*, 1992, pp. 289–486.

[8] S. Dai, L. Bi, and B. Hu, "Distance measures between the interval-valued complex fuzzy sets," *Mathematics*, vol. 7, no. 6, p. 549, Jun. 2019.

[9] S. Dai, "A generalization of rotational invariance for complex fuzzy operations," *IEEE Trans. Fuzzy Syst.*, vol. 29, no. 5, pp. 1152–1159, May 2021.

[10] S. Greenfield, F. Chiclana, and S. Dick, "Interval-valued complex fuzzy logic," in *Proc. IEEE Int. Conf. Fuzzy Syst. (FUZZ-IEEE)*, Jul. 2016, pp. 2014–2019.

[11] H. Garg and D. Rani, "New generalised Bonferroni mean aggregation operators of complex intuitionistic fuzzy information based on Archimedean t-norm and t-conorm," *J. Experim. Theor. Artif. Intell.*, vol. 32, no. 1, pp. 81–109, Jan. 2020.

[12] B. Hu, L. Bi, and S. Dai, "The orthogonality between complex fuzzy sets and its application to signal detection," *Symmetry*, vol. 9, no. 9, p. 175, Aug. 2017.

[13] B. Hu, L. Bi, S. Dai, and S. Li, "The approximate parallelity of complex fuzzy sets," *J. Intell. Fuzzy Syst.*, vol. 35, no. 6, pp. 6343–6351, Dec. 2018.

[14] B. Hu, L. Bi, S. Dai, and S. Li, "Distances of complex fuzzy sets and continuity of complex fuzzy operations," *J. Intell. Fuzzy Syst.*, vol. 35, no. 2, pp. 2247–2255, Aug. 2018.

[15] B. Hu, L. Bi, and S. Dai, "Complex fuzzy power aggregation operators," *Math. Problems Eng.*, vol. 2019, pp. 1–7, Dec. 2019.

[16] K. Hayat, R. A. Shamim, H. AlSalman, A. Gumaei, X.-P. Yang, and M. Azeem Akbar, "Group generalized Q-rung orthopair fuzzy soft sets: New aggregation operators and their applications," *Math. Problems Eng.*, vol. 2021, pp. 1–16, Dec. 2021.

[17] K. Hayat, M. S. Raja, E. Lughofer, and N. Yaqoob, "New group-based generalized interval-valued Q-rung orthopair fuzzy soft aggregation operators and their applications in sports decision-making problems," *Comput. Appl. Math.*, vol. 42, no. 1, p. 4, Feb. 2023.

[18] P. Liu, Z. Ali, and T. Mahmood, "The distance measures and cross-entropy based on complex fuzzy sets and their application in decision making," *J. Intell. Fuzzy Syst.*, vol. 39, no. 3, pp. 3351–3374, Oct. 2020.

[19] J. Ma, G. Zhang, and J. Lu, "A method for multiple periodic factor prediction problems using complex fuzzy sets," *IEEE Trans. Fuzzy Syst.*, vol. 20, no. 1, pp. 32–45, Feb. 2012.

[20] J. M. Merigó, A. M. Gil-Lafuente, D. Yu, and C. Llopis-Albert, "Fuzzy decision making in complex frameworks with generalized aggregation operators," *Appl. Soft Comput.*, vol. 68, pp. 314–321, Jul. 2018.

[21] T. Mahmood, Z. Ali, and A. Gumaei, "Interdependency of complex fuzzy neighborhood operators and derived complex fuzzy coverings," *IEEE Access*, vol. 9, pp. 73506–73521, 2021.
[22] T. Mahmood and U. Rehman, "A novel approach towards bipolar complex fuzzy sets and their applications in generalized similarity measures," *Int. J. Intell. Syst.*, vol. 37, no. 1, pp. 535–567, Jan. 2022.
[23] T. Mahmood, U. U. Rehman, J. Ahmmad, and G. Santos-García, "Bipolar complex fuzzy Hamacher aggregation operators and their applications in multi-attribute decision making," *Mathematics*, vol. 10, no. 1, p. 23, Dec. 2021.
[24] T. Mahmood and U. U. Rehman, "A method to multi-attribute decision making technique based on dombi aggregation operators under bipolar complex fuzzy information," *Comput. Appl. Math.*, vol. 41, no. 1, pp. 1–23, Feb. 2022.
[25] W. Peng, A. Pivato, F. Garbo, and T. Wang, "Stabilization of solid digestate and nitrogen removal from mature leachate in landfill simulation bioreactors packed with aged refuse," *J. Environ. Manage.*, vol. 232, pp. 957–963, Feb. 2019.
[26] D. Ramot, R. Milo, M. Friedman, and A. Kandel, "Complex fuzzy sets," *IEEE Trans. Fuzzy Syst.*, vol. 10, no. 2, pp. 171–186, Apr. 2002.
[27] D. Ramot, M. Friedman, G. Langholz, and A. Kandel, "Complex fuzzy logic," *IEEE Trans. Fuzzy Syst.*, vol. 11, no. 4, pp. 450–461, Aug. 2003.
[28] D. Rani and H. Garg, "Complex intuitionistic fuzzy power aggregation operators and their applications in multicriteria decision-making," *Expert Syst.*, vol. 35, no. 6, Dec. 2018, Art. no. 12325.
[29] M. Rezaee, V. N. Rajagopalan, and R. Q. Honaker, "Conceptual modifications of coal preparation plants to minimize potential environmental issues associated with coal waste disposal," *Fuel*, vol. 250, pp. 352–361, Aug. 2019.
[30] J. Song, Y. Sun, and L. Jin, "PESTEL analysis of the development of the waste-to-energy incineration industry in China," *Renew. Sustain. Energy Rev.*, vol. 80, pp. 276–289, Dec. 2017.
[31] H. Song, L. Bi, B. Hu, Y. Xu, and S. Dai, "New distance measures between the interval-valued complex fuzzy sets with applications to decision-making," *Math. Problems Eng.*, vol. 2021, pp. 1–9, Mar. 2021.
[32] Z. Tadros, "Some aspects of solid waste disposal site selection: The case of Wadi Madoneh, Jordan," *Int. J. Environ. Stud.*, vol. 66, no. 2, pp. 207–219, Apr. 2009.
[33] X. Yang, K. Hayat, M. S. Raja, N. Yaqoob, and C. Jana, "Aggregation and interaction aggregation soft operators on interval-valued Q-rung orthopair fuzzy soft environment and application in automation company evaluation," *IEEE Access*, vol. 10, pp. 91424–91444, 2022.
[34] C. V. Negoita, "Fuzzy sets," *Fuzzy Sets Syst.*, vol. 133, no. 2, p. 275, Jan. 2003.
[35] W.-R. Zhang, "Bipolar fuzzy sets and relations: A computational framework for cognitive modeling and multiagent decision analysis," in *Proc. NAFIPS/IFIS/NASA 1st Int. Joint Conf. North Amer. Fuzzy Inf. Process. Soc. Biannual Conf. Ind. Fuzzy Control Intell. Syst. Conf., NASA Joint Technol. Wo*, Dec. 1994, pp. 305–309.
[36] G. Zhang, T. S. Dillon, K. Y. Cai, J. Ma, and J. Lu, "Operation properties and $\delta-$equalities of complex fuzzy sets," *Int. J. Approx. reasoning*, vol. 50, no. 8, pp. 1227–1249, 2009.

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