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RESEARCH ARTICLE

Application of a Multidimensional Approach in the Compensation of Industrial Loads

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ABSTRACT The instantaneous reactive power theory (IRPT) has long been utilized for load compensation through active power line conditioners (APLCs). IRPT effectively divides the current vector into two components: the instantaneous power current and the instantaneous reactive current. Among these components, only the former is necessary to transfer instantaneous real power to the load. However, the commonly adopted approach faces challenges when extended to multi-phase systems and their compensation strategy design. This challenge is particularly pertinent today due to the consensus that treating a three-phase system with a neutral wire as a four-conductor system is more appropriate. This paper introduces a formulation of IRPT tailored for multi-phase systems within the framework of geometric algebra (GA). GA is a mathematical structure that defines a single power variable, the instantaneous power multivector, encompassing both instantaneous real power and instantaneous reactive power within a unified mathematical entity. The current components can be directly derived from this power multivector. Additionally, this paper establishes a connection with the original p-q formulations and lays the foundations for time-instantaneous compensation (TIC) and time-average compensation (TAC). Finally, to validate the proposed model, simulation and experimental results from a three-phase four-wire industrial system using a novel approach are presented.

INDEX TERMS Geometric algebra, instantaneous power multivector, instantaneous reactive power theory, multi-phase systems, load compensation.

I. INTRODUCTION

Currently, active power line conditioners (APLCs) are widely used to mitigate the poor quality of electrical waveforms, and different applications of APLCs in power systems can be found in [1]. In the early 80s, Akagi et al. published the instantaneous reactive power theory (IRPT), the original theory, with the aim of providing a method for the design of APLC control. Thus, in [2] and [3], they formally introduced the concepts of instantaneous real power and instantaneous imaginary power in the framework of the $\alpha\beta 0$ coordinates. From these power variables, the instantaneous active and instantaneous reactive currents in $\alpha\beta$ coordinates were obtained, as the current component in the 0 coordinate was treated independently. This formulation has been widely used in the control of APLCs. Since the publication of the original IRPT, several new formulations have been proposed. All these can be considered alternative approaches within the IRPT framework. Here, we briefly review the suggested developments since the introduction of the original theory. Thus, the following proposals can be made: the modified instantaneous reactive power formulation, *d-q* transformation (and i_d-i_q, alternative formulation in a rotating frame), *p-q-r* coordinates, and the vectorial approach. These studies describe the energy transfer between the source and load through the instantaneous power $p(t)$ and other instantaneous reactive power variables $q_j(t)$ according to a particular development. In fact, in [4], a modified instantaneous reactive power formulation was introduced, and the instantaneous reactive power vector was defined by the cross-product

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of the voltage and current vectors; however, only two of its three components were independent. The current vector is obtained from the double cross-product of the voltage vector and the instantaneous reactive power vector (cross-product between the voltage and current vectors). Similarly, in [5], an analysis is made of the possible meaning of the power/current terms and the compensation process. Finally, in [6], the most frequently used development in the literature is given. Nevertheless, this same development in [6] can be carried out in $\alpha\beta 0$ coordinates as shown in [7] where the conceptual differences between the original and the modified theory are also established.

In contrast, in [8], the voltage and current signals were transformed from a stationary frame in *abc* phase coordinates to a rotating frame *dq0* with a speed equal to the fundamental pulsation of the grid. The latter requires the use of a phase-locked loop (PLL), which is the main drawback in determining reference signals in an APLC. To overcome this drawback, the i_d-i_q method is proposed in [9], where the direction of one of the rotating axes is adopted with the voltage vector in $\alpha\beta$ coordinates, although this procedure does not prevent the appearance of third harmonics in conditions of grid unbalance. The so-called *pqr* formulation [10] extends the i_d-i_q method by a double rotation that allows anchoring one of the axes with the voltage vector so that the rotational speed of the coordinate system will not be influenced by the imbalance and distortion of the power grid. These new coordinates define three power terms: a real power term and two instantaneous reactive power terms. The main weakness of this method is the requirement of several transformations. Finally, in [11], an approach for generating reference signals in the control of an APLC for different formulations was established, and a comparative analysis of their performances was presented. In any case, it should be said as a culmination of this review, that it is the emergence of instantaneous reactive power variables that differentiates the IRPT from other theories historically proposed on electrical power. Some of the properties of the power variables and current components introduced by the IRPT are discussed in [12].

This issue is still topical because the implementation of deregulation and distributed generation processes that require APLCs and other devices based on power converters remains a challenge. An example of this is shown in [13], where the i_d-i_q method is applied to the control of an APLC for load compensation under different conditions of asymmetry and voltage distortion. The performances of the control strategies for the *p-q* theory, symmetrical component method, and *d-q* formulation were compared. Moreover, in [14], a *p-q* formulation was proposed as the basis of a decentralized control strategy for voltage-source inverters in a microgrid. In the same line of thought, an interesting extension of the IRPT is presented in [15]. The efficient control design of power inverters acting as an interface between the renewable source and power grid requires a higher level of selectivity in the decomposition of voltage and current signals. The authors proposed the enhanced instantaneous power theory (EIPT) for unbalanced and nonlinear three-phase systems, in which the natural evolution of the IRPT can be considered. In the same context, in [16], a series active power filter (SAPF) is used as an interface between an electric vehicle and the grid during its charging process. SAPF control was implemented using an original method for estimating the *dq* components of the voltage from a Lyapunov filter.

To date, most published formulations have been limited to three-phase systems with three or four wires. In addition, all of these present difficulties in their generalization to systems with more than three phases. One of the first works that did not consider the limitation of the number of phases was [5]. The current vector is decomposed into two orthogonal components. The first is obtained by projecting the current vector onto the voltage vector, and the remaining difference constitutes the instantaneous reactive current. The instantaneous reactive power was determined using the latter. Unfortunately, [5] does not indicate any way to obtain the magnitudes of these components independently. Other contributions to the IRPT in multi-phase systems were subsequently published. In [17], the objective was to decompose the current vector over an arbitrary set of basis vectors subjected to a Schmidt orthonormalisation process. However, these selected vectors do not seem to be associated with any meaningful power term. Similarly, in [18], a purely formal proposal was presented because no operative expressions were established for these terms except in the case of three phases. In [19] and [20] this issue was overcome by introducing the outer product of the voltage and current. Using the dyadic product of the two vectors, an operating procedure can be employed to obtain the power and current terms for systems with more than three phases.

Despite the recent advances mentioned above, there is no general IRPT for obtaining the current from a single power variable valid for 1-phase to n-phase systems. In contrast, geometric algebra (GA) has recently been widely applied to various engineering problems. Sufficient evidence for this can be found in [21]. This is an alternative mathematical environment derived from Clifford algebra. The inner and outer products in the GA are handled in a unified manner through the so-called geometric products. Furthermore, the results can be extended naturally to an n-dimensional space. Thus, a multidimensional environment that is particularly suitable for developing instantaneous power analysis under general conditions is available [22]. To date, the application of GA in electrical power has been limited to single-phase systems in the frequency domain. The application of GA to electrical power analysis was first reported in a regular journal [23]. The distorted voltage and current signals are decomposed into orthonormal vectors associated with each sine and cosine term of the Fourier series expansion. From this, the geometric product between the voltage and current vectors is obtained, and the multivector power is defined for the first time. Each multivector component is associated with different apparent power values. This methodology is applicable only to single-phase systems and is difficult to extend to systems with a higher number of phases. In [24], the procedure introduced in [23] was continued by extending the concept of GA to that of generalized complex geometric algebra, where each coefficient of the multivector is represented by a phasor. The new generalized geometric product achieves a multivector representation of power that allows the identification of different powers reported in the technical literature on electrical power in non-sinusoidal regimes. However, it is limited to frequency-domain power analysis environments and does not support extensions to systems with more than one phase. In the same line, [25], represents the harmonics of the signal by k-vectors which are used to process the rms, phase angle, and pulsation of a sine wave. In [26], the same authors applied their proposed method to unveil the power phenomena under non-sinusoidal conditions. This approach, although original, suffers from the same limitations as in [24]. Recently, a generalized coordinate transformation valid for multi-phase systems was proposed [27]. The introduced transformation has a natural visualization framework in the GA environment which allows for a geometrical interpretation of classical transformations such as Clarke's or Park's transformations.

The IRPT formulation was first addressed in the GA framework in [28], and later, in a regular journal [29], the issue was formalized mathematically. An application of the proposed formulation for compensation using a series of hybrid active filters can be found in [30]. In these works, [28], [29], the formulation was subordinated from its origin to an orthogonal coordinate system, and there was no identification with the formulations established up to that moment. Similarly, its application from an experimental perspective has not been accredited.

In this study, a new analysis of IRPT, applicable to any number of phases, was conducted. The original contributions of this study are as follows:

1. The IRPT was developed from a systemic perspective. It has been approached in a general (global) manner based on a single power variable. This approach defines an instantaneous power multivector that allows for the analysis of multi-phase systems following a model analogous to that of single-phase systems. This approach adheres to the standard set by the IRPT, defining a power variable and deriving the current components from it.
2. It has been demonstrated that GA is a valid mathematical tool for conducting an IRPT formulation for systems with any number of phases, irrespective of the coordinate system in use. Three-phase systems can be considered a specific case derived from multidimensional development.
3. The proposed methodology can identify various published IRPT formulations, even when they have been developed using different types of phase coordinates (e.g., $0\alpha\beta$ coordinates). Hence, this methodology is applicable in an environment employing any type of coordinate system.
4. The concepts of time-instantaneous compensation (TIC) and time-averaged compensation (TAC) have been applied to an industrial three-phase system under general conditions of imbalance and distortion, viewed from the perspective of a four-conductor system.

This paper is structured as follows: it begins with a multivector analysis of three-phase systems and subsequently extends its scope to encompass multi-phase systems. Section II covers the concepts related to the formulation of the IRPT under the framework of GA for both three-phase and multi-phase systems. In Section III, we establish connections between the developments carried out within GA and the conventional formulations in $0\alpha\beta$ coordinates or phase coordinates. Section IV delves into the multivector analysis of TIC and TAC for multi-phase systems. To illustrate the application of these concepts, Section V presents an industrial three-phase system that has been compensated by an APLC, along with the implementation of a simulation model and an experimental prototype. Waveforms showcasing the system's performance are provided. Section VI engages in a discussion of the introduced IRPT topics, and finally, in Section VII, we present the conclusions.

II. INSTANTANEOUS POWER MULTIVECTOR / INSTANTANEOUS CURRENT COMPONENTS

The voltages of each phase and the line currents of a power system with any number of phases can be represented by the voltage vectors **u** and current **i** corresponding to the Euclidean space. In the GA framework, it is well known that it is possible to define a multiplication rule between both vectors, called geometric products. The geometric products of vectors **u** and **i** have been used extensively in technical studies to identify different power terms, [22], [23], [24], [25], [26]. The instantaneous power multivector is defined as

$$\mathbf{s}(t) = \mathbf{i}\mathbf{u} = \mathbf{i} \cdot \mathbf{u} + \mathbf{i} \wedge \mathbf{u} \quad (1)$$

where $\mathbf{i}\cdot\mathbf{u}$ corresponds to the inner product and $\mathbf{i} \wedge \mathbf{u}$ to the outer product, respectively, between both vectors, in the sense proposed by Grassmann, and whose explicit expressions according to the chosen coordinate system will be obtained later. The result of (1) is a mathematical entity (multivector) consisting of a scalar part (0-vector) and a bivector part (2-vector). Reference [23] provides a quick overview of the basics of the GA.

The first term in **s** defines the instantaneous power $p(t)$, and the second term corresponds to the instantaneous reactive power bivector $\mathbf{q}$(t).

$$p(t) = \mathbf{i} \cdot \mathbf{u} \quad ; \quad \mathbf{q}(t) = \mathbf{i} \wedge \mathbf{u} \quad (2)$$

Here, we define multivector **s** according to (1), as opposed to another possible alternative definition in the form **ui**: This is not relevant; it is simply a matter of sign conventions. In a sinusoidal steady state situation, the magnitude value of $\mathbf{q} =$

$\mathbf{i} \wedge \mathbf{u}$ is related to the classical reactive power. By convention, this value is considered positive for inductive loads; thus, to respect this convention, it is more appropriate to define $\mathbf{s}$ according to (1).

From the instantaneous power multivector $\mathbf{s}$ it is possible to obtain the current vector by multiplying it by $\mathbf{u}^{-1} = \mathbf{u}/(\mathbf{uu})$. In fact,

$$\mathbf{i} = \frac{\mathbf{su}}{(\mathbf{uu})} = \frac{\mathbf{i}(\mathbf{uu})}{(\mathbf{uu})} = \frac{p\mathbf{u}}{(\mathbf{uu})} + \frac{\mathbf{qu}}{(\mathbf{uu})} \quad (3)$$

where the associative and distributive properties of the geometric product were used, [21]. As presented in (3), the time dependence of the different variables will be omitted to simplify the notation. The last relation, (3), shows the partition of the current vector into two components: the instantaneous power current $\mathbf{i_p}$ and the instantaneous reactive current $\mathbf{i_q}$:

$$\mathbf{i_p} = \frac{p\mathbf{u}}{(\mathbf{uu})} \quad ; \quad \mathbf{i_q} = \frac{\mathbf{qu}}{(\mathbf{uu})} \quad (4)$$

Both the current components are orthogonal as demonstrated in the appendix.

The instantaneous norm of any multivector can be defined using the concept of the 'reverse' of a multivector. The reversing process, which will be identified here with the symbol '†', is obtained from the original multivector by inverting the order of the vectors in all the k-parts that make it up. Thus, for example, the reverse of vector $\mathbf{i}$ will result in the vector itself, $\mathbf{i}^\dagger = \mathbf{i}$. The instantaneous norms of the voltage and current vectors are defined as:

$$u = \left(\mathbf{u}^\dagger \mathbf{u}\right)^{1/2} = (\mathbf{uu})^{1/2} \quad ; \quad i = \left(\mathbf{i}^\dagger \mathbf{i}\right)^{1/2} = (\mathbf{ii})^{1/2} \quad (5)$$

Preferably, the variables in italics indicate the instantaneous norms of the variable. Thus, given the orthogonality of the current components in (3),

$$i^2 = \left(\mathbf{i}_p + \mathbf{i}_q\right)\left(\mathbf{i}_p + \mathbf{i}_q\right) = i_p^2 + i_q^2 \quad (6)$$

because the vectors $\mathbf{i}_\mathrm{p}$ and $\mathbf{i}_\mathrm{q}$ are orthogonal and their geometric product is antisymmetric ($\mathbf{i}_\mathrm{p}\mathbf{i}_\mathrm{q}$ =-$\mathbf{i}_\mathrm{q}\mathbf{i}_\mathrm{p}$). In the case of the instantaneous power multivector, its reverse $\mathbf{s}^\dagger$ is

$$\mathbf{s}^\dagger = \mathbf{ui} = \mathbf{u} \cdot \mathbf{i} + \mathbf{u} \wedge \mathbf{i} = \mathbf{i} \cdot \mathbf{u} - \mathbf{i} \wedge \mathbf{u} = p - \mathbf{q} \quad (7)$$

and therefore, from (5), its instantaneous norm satisfies the relation,

$$s^2 = \mathbf{s}^\dagger \mathbf{s} = \mathbf{uiiu} = u^2 i^2 = u^2\left(i_p^2 + i_q^2\right) = p^2 + q^2 \quad (8)$$

Again, the associative property is applied to obtain (8). This last equation provides the partitioning of the instantaneous squared norm of multivector $\mathbf{s}$ (or instantaneous apparent power) with the instantaneous power squared and instantaneous reactive power squared.

Thus far, the relationships presented in (1)–(8) constitute a general development of the IRPT based on GA concepts, without the voltage and current vectors being referenced to any particular coordinate system. In the following section, an application for the analysis of three-phase and n-phase systems is presented. The objective is to obtain concrete operational expressions for the power and current terms in the GA framework. For this purpose, we assume that the Euclidean space supporting the GA has an orthonormal basis of vectors $\{e_i\}$.

A. THREE-PHASE SYSTEMS

In a three-phase system, the current and voltage vectors $\mathbf{u}$ and $\mathbf{i}$ are defined, respectively, by (9),

$$\mathbf{u} = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}^t \quad ; \quad \mathbf{i} = \begin{bmatrix} i_1 & i_2 & i_3 \end{bmatrix}^t \quad (9)$$

where the superscript 't' denotes a transpose. Both vectors consist of waveforms corresponding to the line-to-neutral voltage and line currents, respectively. Each voltage can be referenced to a neutral conductor or an artificial neutral point when it does not exist.

Vectors $\mathbf{u}$ and $\mathbf{i}$ can be considered as elements of a three-dimensional Euclidean space generated by an orthonormal basis e_j, with j = 1,2,3. Thus, the voltage and current vectors can be decomposed into basis vectors, as shown in (10).

$$\mathbf{u} = u_1 e_1 + u_2 e_2 + u_3 e_3 \quad ; \quad \mathbf{i} = i_1 e_1 + i_2 e_2 + i_3 e_3 \quad (10)$$

The geometric product of any two vectors of base $\{e_i\}$ is defined as follows:

$$e_i e_j = e_{ij} = e_i . e_j + e_i \wedge e_j = -e_j e_i = -e_{ji} \quad \forall i \neq j \quad (11)$$

constituted by an inner product and an outer product. In (11), the first term is null because the vectors e_i are orthonormal and the second term is a basis bivector. That is, the geometric product of two different basis vectors is a bivector or 2-vector (the outer product of e_i and e_j). Through the geometric product, the square of the two basis vectors is 1, and the square of the two basis bivectors is −1.

$$e_i e_i = 1; \; e_{ij} e_{ij} = e_i e_j e_i e_j = -e_j e_i e_i e_j = -1 \quad (12)$$

Fig. 1 has been included for improved visualization. The basis vectors and bivectors are shown, and the bivectors are represented by a directed area.

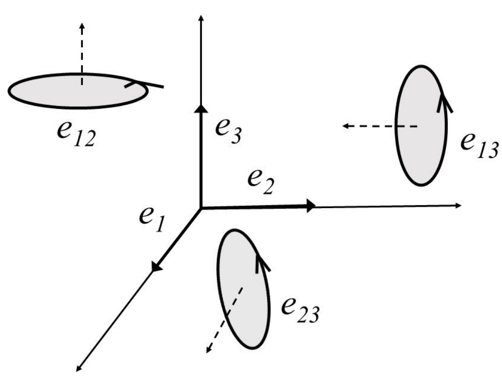

FIGURE 1. Basis vectors (e_1, e_2, e_3) and basis bivectors (e_{12}, e_{23}, e_{13}) in three-dimensional Euclidean space.

From (10) and (11), the following expression is obtained for multivector $\mathbf{s}$:

$$\mathbf{s} = \mathbf{iu} = p + q_{12} e_{12} + q_{13} e_{13} + q_{23} e_{23} = p + \mathbf{q} \quad (13)$$

The 2-vector part **q** is the three-phase instantaneous reactive power bivector; as indicated in (13), it consists of three components of instantaneous reactive power q_{12}, q_{13}, q_{23}, [1], [6],

$$\mathbf{q} = (i_1 u_2 - u_1 i_2)\, e_{12} + (i_1 u_3 - u_1 i_3)\, e_{13} + (i_2 u_3 - u_2 i_3)\, e_{23} = q_{12} e_{12} + q_{13} e_{13} + q_{23} e_{23} \quad (14)$$

From (8), (13), and (14), and the systematic application of rules (11) and (12), the square value of the instantaneous power multivector norm is obtained according to (15).

$$(s)^2 = (p)^2 + \left((q_{12})^2 + (q_{13})^2 + (q_{23})^2\right) = (p)^2 + (q)^2 \quad (15)$$

In contrast, the current components are obtained from the power variables defined in the previous section. The current was obtained from only an instantaneous power vector. From (4) and by application of the rules given by (11), we obtain each one of the components of the line currents (it should be noted that q_{ij}=-q_{ji}).

Fig. 2 shows a geometrical representation of the current vector decomposition in three-phase systems. Bivector **q** determines the plane in which the voltage and current vectors are arranged.

Each of the three component vectors, $\mathbf{i_p}$ and $\mathbf{i_q}$ correspond to the generic expressions given in (16):

$$i_{pj} = \frac{p u_j}{u^2}; \quad i_{qj} = \frac{\sum_{k=1, k\neq j}^{3} q_{jk} u_k}{u^2} \quad (16)$$

with j=1, 2, 3. Both are obtained from (5), considering (11) and (12).

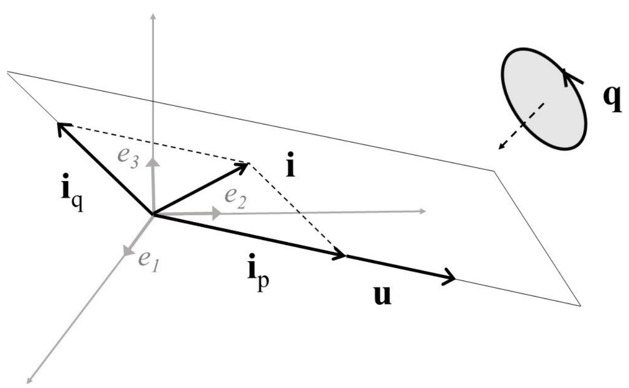

FIGURE 2. Instantaneous reactive power bivector q, and orthogonal decomposition of the current vector i.

B. MULTI-PHASE SYSTEMS

In multi-phase power systems, voltage and current vectors are defined as (17),

$$\mathbf{u} = [u_1 \ldots u_n]^t \quad ; \quad \mathbf{i} = [i_1 \ldots i_n]^t \quad (17)$$

which can be expressed as a linear combination of the orthonormal basis vectors $\{e_i\}$ of the n-dimensional Euclidean space in the form (18)

$$\mathbf{u} = u_1 e_1 + \ldots + u_n e_n \quad ; \quad \mathbf{i} = i_1 e_1 + \ldots + i_n e_n \quad (18)$$

The rules for operating with the basis vectors are established in (11) and (12).

Thus, the instantaneous power $p(t)$ is defined as the inner product of the voltage and current vectors (19).

$$p = \mathbf{i} \cdot \mathbf{u} = u_1 i_1 + \ldots + u_n i_n \quad (19)$$

and the outer product of voltage and current vectors, that is, the instantaneous reactive power bivector **q**, consists of n(n−1)/2 instantaneous reactive power components (20),

$$\begin{aligned}\mathbf{q}(t) &= (i_1 u_2 - u_1 i_2)\, e_{12} + \ldots + (i_1 u_n - u_1 i_n)\, e_{1n} + \ldots \\ &\quad + (i_2 u_n - u_2 i_n)\, e_{2n} + \ldots + (i_{n-1} u_n - u_{n-1} i_n)\, e_{(n-1)n} \\ &= q_{12} e_{12} + \ldots + q_{1n} e_{1n} + \ldots + q_{2n} e_{2n} \\ &\quad + \ldots + q_{(n-1)n} e_{(n-1)n} \end{aligned} \quad (20)$$

Each of the n(n-1)/2 components of the instantaneous reactive power bivector are given by,

$$q_{ij} = (i_i u_j - u_i i_j) \quad \forall i, j = 1, \ldots, n;\ i < j \quad (21)$$

The instantaneous reactive power terms in (21) verify (22).

$$\sum_{\substack{\forall i,j,k \\ i \neq j \neq k}} u_k q_{ij} = 0 \quad (22)$$

That is, only the (n(n-1)/2)−1 instantaneous reactive power variables were independent. Equation (22) emerges from the fact that vectors **u** and **q** are perpendicular; therefore, it is verified that

$$\mathbf{u} \cdot \mathbf{q} = \frac{1}{2}(\mathbf{uq} + \mathbf{qu}) = 0 \quad (23)$$

From (23) it follows that an energy system of n-phases is characterized by n(n-1)/2 variables; a variable p and (n(n-1)/2)-1 variables of q_{ij}.

Finally, similar to (16), each component of the vectors $\mathbf{i_p}$ and $\mathbf{i_q}$ corresponds to the generic expressions in (24).

$$i_{pj} = \frac{p u_j}{u^2}; \ i_{qj} = \frac{\sum_{k=1, k\neq j}^{n} q_{jk} u_k}{u^2} \quad (24)$$

with j=1, 2, …, n.

The model established using Eqs. (6) and (8) describes the power system in the IRPT domain according to the same pattern as that of the balanced system in a sinusoidal steady state. Thus, the squared norm of the instantaneous power multivector has an orthogonal decomposition in terms of the instantaneous and reactive powers [1], [6].

III. IDENTIFICATION WITH THE FORMULATIONS OF THE P – Q THEORY

A. ORIGINAL/MODIFIED INSTANTANEOUS POWER THEORY IN 0 α β COORDINATES

A GA handles mathematical objects that are not associated with any particular coordinate system. This allows for the formulation of power in the time domain for any coordinate system in which the voltage and current vectors are decomposed. This quality is especially interesting for the identification of the formulation proposed here with the IRPT formulation originally developed in $0\alpha\beta$ coordinates. In the original p-q theory, the Clarke transformation was used to express the voltage and current variables. An interesting study on the handling of component transformations in a GA environment can be found in [27]. The vectors **u** and **i** are then expanded to a basis $\{e_0\ e_\alpha\ e_\beta\}$ in the form,

$$\begin{aligned}\mathbf{u} &= v_0 e_0 + v_\alpha e_\alpha + v_\beta e_\beta = v_0 e_0 + \mathbf{v}_{\alpha\beta}\\ \mathbf{i} &= i_0 e_0 + i_\alpha e_\alpha + i_\beta e_\beta = i_0 e_0 + \mathbf{i}_{\alpha\beta}\end{aligned} \tag{25}$$

The basis vectors $\{e_0\ e_\alpha\ e_\beta\}$ are governed by the same rules established in (11) and (12). The original formulation treats the zero-sequence component of voltage and current independently of the $\alpha\beta$ components. Thus, the instantaneous power multivector in the $\alpha\beta$ plane is defined by the following geometric product,

$$\begin{aligned}\mathrm{s}_{\alpha\beta} &= \mathbf{i}_{\alpha\beta}\mathbf{v}_{\alpha\beta} = \left(v_\alpha i_\alpha + v_\beta i_\beta\right) + \left(i_\alpha v_\beta - v_\alpha i_\beta\right) e_{\alpha\beta}\\ &= p_{\alpha\beta} + q_{\alpha\beta} e_{\alpha\beta}\end{aligned} \tag{26}$$

In (26) $p_{\alpha\beta}$ is the so-named instantaneous power in the $\alpha\beta$ plane, and $q_{\alpha\beta}$ is the instantaneous imaginary power in the $\alpha\beta$ plane that constitutes the only component of the instantaneous reactive power bivector. Similarly, the zero-sequence instantaneous power multivector is defined as

$$\mathrm{s}_0 = \mathbf{i}_0\mathbf{v}_0 = i_0 v_0 = p_0 \tag{27}$$

which only has a scalar part corresponding to the instantaneous zero-sequence power.

From the instantaneous power multivector (26) and (27) two current vectors are obtained: the current vector in the $\alpha\beta$ plane

$$\mathbf{i}_{\alpha\beta} = \mathrm{s}_{\alpha\beta}\frac{\mathbf{v}_{\alpha\beta}}{\mathbf{v}_{\alpha\beta}\mathbf{v}_{\alpha\beta}} = \frac{p_{\alpha\beta}\mathbf{v}_{\alpha\beta}}{v_{\alpha\beta}^2} + \frac{q_{\alpha\beta} e_{a\beta}\mathbf{v}_{\alpha\beta}}{v_{\alpha\beta}^2} = \mathbf{i}_{p\alpha\beta} + \mathbf{i}_{q\alpha\beta} \tag{28}$$

and the zero-sequence current vector,

$$\mathbf{i}_0 = \mathrm{s}_0\frac{\mathbf{v}_0}{\mathbf{v}_0\mathbf{v}_0} = \frac{p_0\mathbf{v}_0}{v_0^2} \tag{29}$$

The components of each of the vectors (28) and (29) are fully identified with the $0\alpha\beta$ components of the p-q theory.

An analogous development can be followed for identification using the modified p-q formulation. In effect, the instantaneous power multivector in the $0\alpha\beta$ plane is defined by the geometric product

$$\begin{aligned}\mathrm{s}_{0\alpha\beta} = \mathbf{iu} &= \left(v_0 i_0 + v_\alpha i_\alpha + v_\beta i_\beta\right) + \left(i_0 v_\alpha - v_0 i_\alpha\right) e_{0\alpha}\\ &\quad + \left(i_0 v_\beta - v_0 i_\beta\right) e_{0\beta} + \left(i_\alpha v_\beta - v_\alpha i_\beta\right) e_{\alpha\beta}\\ &= p_{0\alpha\beta} + \mathbf{q}_{0\alpha\beta}\end{aligned} \tag{30}$$

where the scalar part (instantaneous power) and bivector part are identified. The latter, on this occasion, includes three components corresponding to the three variables of instantaneous reactive power collected in the modified p-q theory. From the power multivector in (30), the current vector is obtained as

$$\mathbf{i} = \mathrm{s}_{0\alpha\beta}\frac{\mathbf{u}}{\mathbf{uu}} = p_{0\alpha\beta}\frac{\mathbf{u}}{v_{0\alpha\beta}^2} + \mathbf{q}_{0\alpha\beta}\frac{\mathbf{u}}{v_{0\alpha\beta}^2} = \mathbf{i}_{p0\alpha\beta} + \mathbf{i}_{q0\alpha\beta} \tag{31}$$

where the instantaneous norm of the voltage vector in coordinates $0\alpha\beta$ is

$$v_{0\alpha\beta}^2 = \mathbf{uu} = v_0^2 + v_\alpha^2 + v_\beta^2 \tag{32}$$

In (31), again, the vectors of instantaneous power current and instantaneous reactive current are identified in coordinates $0\alpha\beta$, [7].

B. RELATION BETWEEN THE GA APPROACH AND THE VECTORIAL FORMULATION

The modified p-q theory was initially developed from the cross-product of the voltage and current vectors (9) to define an instantaneous reactive power vector; therefore, it is also known as a vectorial formulation. In the vectorial IRPT, the vector **q** is,

$$\mathbf{q} = \mathbf{i} \times \mathbf{u} \tag{33}$$

The components of vector **q** obtained from the development of the cross-product in (33) are

$$\begin{aligned}\mathbf{q} &= \left(i_2 u_3 - u_2 i_3\right) e_1 + \left(i_3 u_1 - u_3 i_1\right) e_2 + \left(i_1 u_2 - u_1 i_2\right) e_3\\ &= q_{23} e_1 + q_{13}\left(-e_2\right) + q_{12} e_3\end{aligned} \tag{34}$$

The length of **q**, q, is the instantaneous reactive power, that is

$$q = \|\mathbf{q}\| = \|\mathbf{u} \times \mathbf{i}\| = \sqrt{q_{23}^2 + q_{31}^2 + q_{12}^2} \tag{35}$$

noting that $q_{31} = -q_{13}$.

However, the vector of the instantaneous reactive power in (34) matches the instantaneous reactive power bivector defined in (14). In fact, in (14) the 2-vector **q** is expressed in terms of the basis 2-vectors e_{ij} where the coincidence with the instantaneous reactive power components is appreciated.

However, this point deserves further investigation. In (14), the bivector **q** is expressed as a function of the basis bivectors e_{12}, e_{13}, e_{23}. In (34), vector **q** is expressed with the same components as a function of vectors e_1, $-e_2$, e_3. The basis bivectors are elements of the GA, defined as oriented planes whose perpendicular directions for a right-handed basis $\{e_i\}$ are shown in Fig. 1.

$$e_{23}^* = e_1 \quad ; e_{13}^* = -e_2 \quad ; e_{12}^* = e_3 \tag{36}$$

In (36), e_{ij}^* indicates the duals of each basis bivector. In the GA, the duality transformation applied to the basis bivectors determines the vectors perpendicular to their defined oriented planes. Mathematically, duality is formalized using the pseudoscalar **I** defined in three-dimensional space by the trivector **I**= $e_1e_2e_3$ [21], [23]. The geometric product of -**I** (the inverse of pseudoscalar **I**) with any bivector allows us to obtain the dual vector. Thus, the concept of duality in the GA can be used for the conversion between the representation of the bivector type and vector representation.

Finally, Equation (16) allows us to check the equivalence between the results obtained for the instantaneous reactive current by means of the geometric product of bivector **q** and vector **u**, with the cross-product between the instantaneous reactive power vector and the voltage vector of the modified p-q theory formulation given in the references. However, the development in the GA framework allows natural generalization to the case of multi-phase systems, as established in the previous section.

IV. COMPENSATION STRATEGIES IN A FOUR-WIRE THREE-PHASE POWER SYSTEM

This section discusses the topics reviewed for the compensation of an industrial load under the conditions of asymmetry and distortion. The convenience of considering a three-phase power system with a neutral conductor as a special type of four-conductor system has been well established in technical literature. That is, the measurement of the voltages of each line conductor, including the neutral conductor, refers to the virtual neutral point of a star formed by four resistors. The currents in each of the four conductors, including the neutral conductor, were considered in a similar manner. This methodology leads to a more convenient formulation of equations that describe the behavior of the system. An extensive discussion of this issue took place in the 1990s. Reference [31] summarizes this topic. In this respect, GA is a particularly appropriate tool because it allows the handling of four-component vectors in a natural form.

Here, the compensation of a non-sinusoidal unbalanced three-phase system is presented by considering it as a four-conductor system. Figure 3 shows a load consisting of three single-phase rectifiers connected to a star with an accessible neutral in parallel with a set of star RL branches. The two comprise a nonlinear unbalanced load supplied by a non-sinusoidal unbalanced power source. In addition, the possibility of varying the value of the resistance on the 'dc' side of each of the rectifiers has been considered. The connection of the shunt APLC enables the application of various types of compensation. Each line conductor, including the neutral conductor, had the same line resistance. As indicated above, the virtual neutral point N of a star formed by four resistors of the same value (a sufficiently high value was chosen) was used as a voltage reference. A generalization for the case of different line conductor resistance values can be easily established.

Voltage and current vectors **u** and **i** for a four-conductor system are defined as follows:

$$\mathbf{u} = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 \end{bmatrix}^t \quad ; \quad \mathbf{i} = \begin{bmatrix} i_1 & i_2 & i_3 & i_4 \end{bmatrix}^t \tag{37}$$

with the particularity, in this case, that

$$\mathbf{1}^t \cdot \mathbf{u} = 0 \quad ; \quad \mathbf{1}^t \cdot \mathbf{i} = 0 \tag{38}$$

where

$$\mathbf{1} = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^t \tag{39}$$

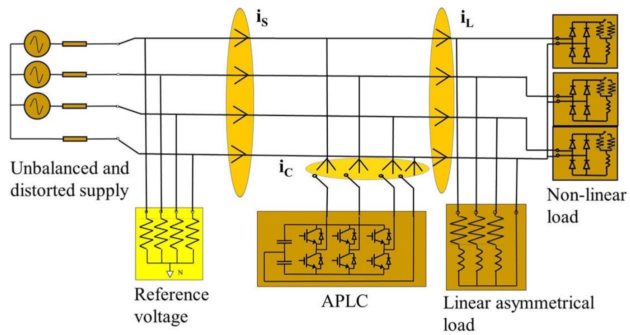

FIGURE 3. Shunt compensation with an active power line conditioner in a four-wire three-phase power system.

Table 1 presents the results of the geometric product of the vectors (37). In the main diagonal are the terms that constitute the instantaneous power, and the terms outside the main diagonal are those that constitute the instantaneous reactive power bivector.

TABLE 1. Terms of the instantaneous power multivector.

iu	u_1e_1	u_2e_2	u_3e_3	u_4e_4
i_1e_1	i_1u_1	$i_1u_2e_{12}$	$i_1u_3e_{13}$	$i_1u_4\,e_{14}$
i_2e_2	$-i_2u_1e_{12}$	i_2u_2	$i_2u_3e_{23}$	$i_2u_4e_{24}$
i_3e_3	$-i_3u_1e_{13}$	$-i_3u_2e_{23}$	i_3u_3	$i_3u_4e_{34}$
i_4e_4	$-i_4u_1e_{14}$	$-i_4u_2e_{24}$	$-i_4u_3e_{34}$	i_4u_4

In contrast, from (24), Table 2 includes the explicit current components of the four conductors as a function of instantaneous power and the instantaneous reactive power bivector terms.

TABLE 2. Current components of i.

i	i_{pj}	i_{qj}
i_1	$\frac{pu_1}{u^2}$	$\frac{\sum_{\forall j\neq 1} q_{1j}u_j}{u^2}$
i_2	$\frac{pu_2}{u^2}$	$\frac{\sum_{\forall j\neq 2} q_{2j}u_j}{u^2}$
i_3	$\frac{pu_3}{u^2}$	$\frac{\sum_{\forall j\neq 3} q_{3j}u_j}{u^2}$
i_4	$\frac{pu_4}{u^2}$	$\frac{\sum_{\forall j\neq 4} q_{4j}u_j}{u^2}$

The current vectors referring to the supply, load, and compensator are henceforth characterized by the subscripts 'S', 'L', and 'C', respectively, as shown in Fig. 3. Similarly, instantaneous power multivectors are distinguished at supply terminal $\mathbf{s_S}$, load terminal $\mathbf{s_L}$, and compensator terminal $\mathbf{s_C}$.

$$\mathbf{s_S} = \mathbf{i_S u} \quad ; \mathbf{s_L} = \mathbf{i_L u} \quad ; \mathbf{s_C} = \mathbf{i_C u} \tag{40}$$

with

$$\mathbf{s_C} = \mathbf{s_L} - \mathbf{s_s} \tag{41}$$

according to the references in Fig 3.

A. TIC

In the TIC [1], the objective is that the multivector instantaneous power of the supply only includes the scalar part:

$$\mathbf{s_S} = p_L \tag{42}$$

with

$$p_L = i_{L1}u_1 + \ldots + i_{L4}u_4 \tag{43}$$

and therefore,

$$\mathbf{i}_S = \frac{\mathbf{s_S u}}{(\mathbf{uu})} = \frac{p_L}{u^2}\mathbf{u} \tag{44}$$

It is a vector of four components corresponding to the currents through each line conductor that transfers the instantaneous power of the load with the minimum instantaneous power value of the transport losses. Other source current vectors are possible, but always correspond to vectors with a higher instantaneous norm value for the same purpose. The compensation current vector is then

$$\mathbf{i}_C = \frac{\mathbf{s_C u}}{(\mathrm{uu})} = \frac{\mathbf{q_L u}}{u^2} \tag{45}$$

which includes the instantaneous reactive power bivector given by

$$\mathbf{q_L} = \sum_{\substack{\forall i,j=1 \\ i<j}}^{4} \left(i_{Li}u_j - u_i i_{Lj}\right) e_{ij} \tag{46}$$

The previous relations correspond to concrete operational expressions that allow determination of the compensation current (45).

B. TAC

Two compensation strategies were considered: constant power compensation, TAC_P, and unity power factor compensation, TAC_PF.

1) CONSTANT POWER COMPENSATION, TAC_P

In this case, the compensation objective is

$$\mathbf{s_s} = P_L \tag{47}$$

where P_L corresponds to the average power absorbed by the load (and eventually, the compensator's power self-loss). Thus, for the references in Fig. 3, the multivector of the instantaneous power transferred by the compensator is

$$\mathbf{s_C} = \mathbf{s_L} - \mathbf{s_S} = p_L - P_L + \mathbf{q_L} = \tilde{p}_L + \mathbf{q_L} \tag{48}$$

which includes the oscillatory component of the load instantaneous power as a scalar part and the load instantaneous reactive power as a bivector part. Consequently, the vector of the current generated by the compensator was

$$\mathbf{i}_C = \frac{\mathbf{s_C u}}{(\mathbf{uu})} = \frac{\mathbf{u}\tilde{p}_L}{u^2} + \frac{\mathbf{q_L u}}{u^2} \tag{49}$$

2) UNITY POWER FACTOR COMPENSATION, TAC_PF

In this compensation strategy, a unity power factor, as observed from the power supply terminals, is sought. This assumes that the source current is given by:

$$\mathbf{i}_S = \frac{P_L}{U^2}\mathbf{u} \tag{50}$$

where

$$U^2 = \frac{1}{T}\int_0^T \sum_{j=1}^{4} u_j^2 dt \tag{51}$$

which represents the square of the RMS value in one period T of the voltage (37) (voltages of each of the conductors to virtual neutral). Therefore, the instantaneous supply power multivector is

$$\mathbf{s_S} = \frac{P_L u^2}{U^2} \tag{52}$$

The instantaneous supply power multivector (52) contains only a scalar component. The instantaneous power multivector of the compensator is obtained from (41).

$$\mathrm{s_C} = \left(p_L - \frac{P_L u^2}{U^2}\right) + \mathbf{q_L} \tag{53}$$

Thus, the compensation current is

$$\mathbf{i}_C = \frac{\mathrm{s_C}\mathbf{u}}{(\mathbf{uu})} = \left(\frac{p_L}{u^2} - \frac{P_L}{U^2}\right)\mathbf{u} + \frac{\mathbf{q_L u}}{u^2} \tag{54}$$

The compensation currents for the different compensation strategies are shown in Fig. 4. The diagram shows a schematic of the proposed methods; TAC needs the aid of a low-pass filter to obtain the average power and voltage RMS value.

C. MODEL COMPENSATOR

The inferred compensation currents were generated by the APLC. A four-leg inverter was used for this purpose. Each branch of the IGBTs was triggered using hysteresis band control. In other words, an error signal is generated by the difference between the compensation current signal (as a reference signal obtained by controlling (45), (49), or (54)) and the compensation current signal measured at the output of the converter. This error signal acted as an input to the hysteresis band comparator. Fig. 5a shows a schematic of an equivalent circuit in which the power inverter is modelled using voltage-controlled sources. However, the source located in branch 4 could be shifted to the remaining three branches, as shown in Fig. 5b. From a practical perspective, this simplification enables the use of a three-leg inverter according to the control technique illustrated in Fig. 5. In the following subsection, a collection of selected waveforms corresponding to different types of compensation is presented.

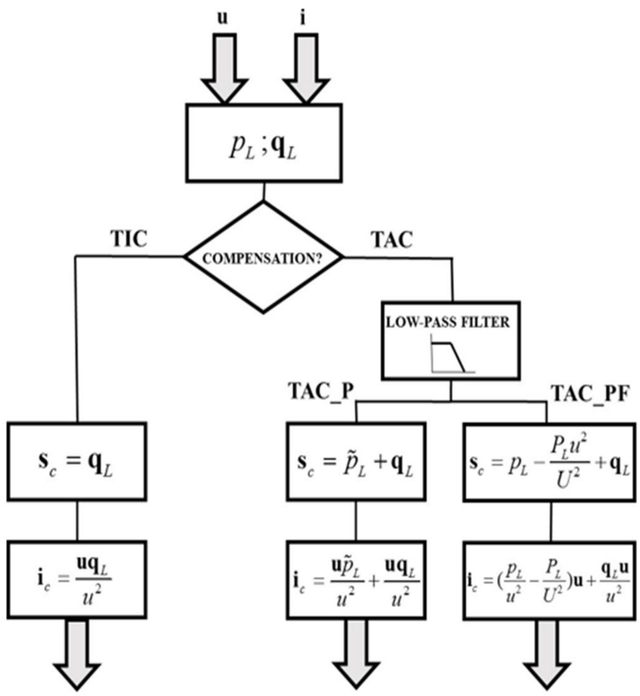

FIGURE 4. Flow chart for obtaining the compensation currents in TIC and TAC.

V. PRACTICAL APPLICATION

The system shown in Figure 3 was considered as a practical application. It is a non-sinusoidal unbalanced three-phase system with four conductors. Fig. 3 shows the star consisting of four 10 MΩ resistors whose neutral will act as a voltage reference. To apply the proposed methodology, a simulation environment was first used, and then an experimental prototype was implemented. The waveforms of interest are presented in the following subsections.

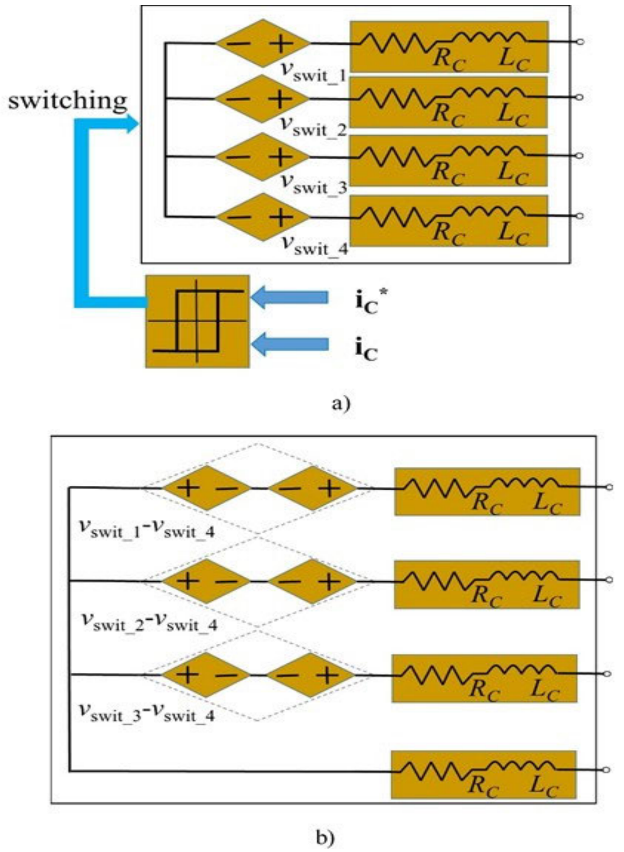

FIGURE 5. Modelling of an active power line conditioner using controlled sources: a) model of a four-leg converter, b) v_{swit_4} -shift through the common node.

A. SIMULATION RESULTS

A simulation model in the MATLAB-Simulink environment for the system shown in Fig. 3 was designed to validate the compensation strategies outlined in Section IV. This model permits clearer visualization of the waveforms and power variables for different compensation methodologies. For this purpose, we consider a voltage supply in the following form:

$$u_{j4} = \sqrt{2}230\, k_j \sin\left(2\pi 50\, t - \phi_j\right) + \sqrt{2}230\left(0.01\, k_j\right) \sin\left(2\pi 350\, t - 7\phi_j\right) \quad (55)$$

with j=1, 2, 3. The parameters k_j and φ_j of the voltages are given in Table 3.

TABLE 3. Voltage parameters.

k_1	k_2	k_3
1	1.05	0.95
φ_1	φ_2	φ_3
0	-2π/3	2π/3

Consider that the voltages of each conductor, referred to as artificial neutral N, are given by

$$u_j = u_{j4} + u_4 \quad (56)$$

Since the sum of the u_j is zero, with j=1, 2, 3, 4, it is verified that

$$u_4 = -\frac{u_{14} + u_{24} + u_{34}}{4} \tag{57}$$

Each of the single-phase rectifiers has two 20 Ω parallel resistors in series with an inductance of 0.1 H on the dc side branch. In contrast, the linear asymmetric load is made up of three RL branches in star with accessible neutral, R_1 =60 Ω, $R_2 = 0.9 \cdot R_1$, $R_3 = 1.1 \cdot R_1$; $L_1 = L_2 = L_3 = 0.01$ H.

Fig. 6 shows the voltage u_j and current i_j waveforms. A load variation at time t=0.16 s has been considered. In fact, on the 'dc' side of the rectifiers of the nonlinear load, a variation of the resistance value from R=20 Ω to R=10 Ω has been induced in order to verify the dynamic behavior of the system. Fig. 6a shows the four voltages of each of the four conductors, referred to as the virtual neutral N. The voltage waveform of conductor '1' is highlighted. Fig. 6b shows the four-conductor currents; similarly, the waveform of the current of conductor '1' is highlighted against the rest of the currents. Evidently, the current through conductor '4' corresponds to the neutral current.

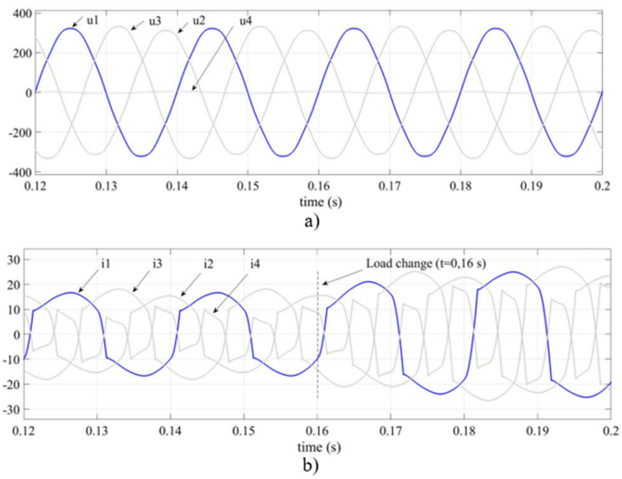

FIGURE 6. Voltages and currents of a four-wire three-phase system. a) Voltages (V) from each conductor to the virtual neutral point; b) Line currents (A) of each of the four conductors. Phase 1 has been highlighted.

The TIC and TAC were applied to the system, as shown in Fig. 3. For each compensation methodology, the following waveforms before and after compensation are shown in Fig. 7: conductor current $i_j(t)$, instantaneous power $p(t)$, instantaneous reactive power $q(t)$ (norm of the bivector **q**), and instantaneous apparent power $s(t)$ (norm of the multivector s).

Fig. 7a shows the current waveforms of each conductor before and after the TIC. After the TIC, at t=0.06 s, the currents transfer the instantaneous power with minimum instantaneous power losses. That condition is maintained after t=0.16 s when a load variation occurs. The current through conductor 4 (a neutral conductor) was practically zero. Fig. 7b shows the currents before and after TAC_P. In this case, from t=0.06 s the currents transport the average power absorbed by the load (and eventually the losses of the compensator) with minimum losses in the transport. Similarly, Fig. 7c shows the currents before and after the TAC_PF. After compensation, the currents were collinear with the voltage, obtaining a unity power factor with minimum possible transport losses.

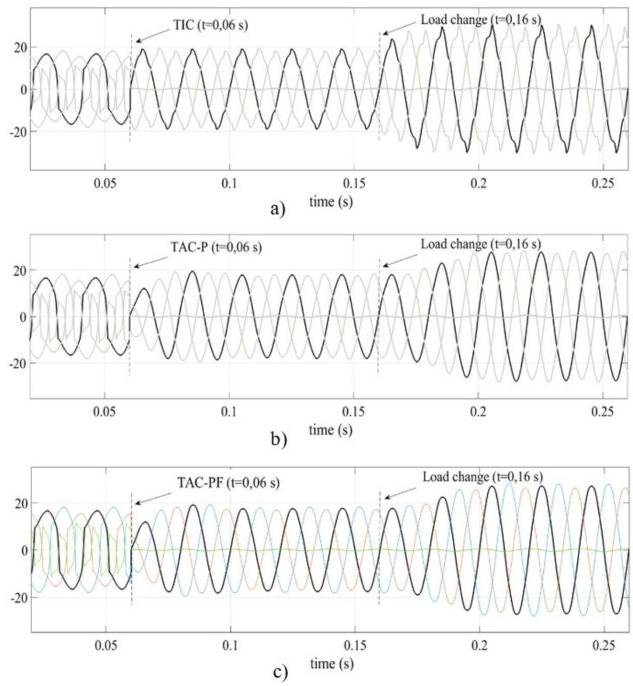

FIGURE 7. Current waveforms (A) of each of the conductors before/after compensation (t=0.06 s). The current of conductor 1 is highlighted. a) TIC; b) TAC_P; c) TAC_PF. At t=0.16 s there is a variation in the load.

Fig. 8a shows the instantaneous power $p(t)$ before and after the TIC; the instantaneous power variable is the same regardless of the TIC. Fig. 8b shows that the instantaneous reactive power $q(t)$ is zero after the TIC. The variation of the load at t = 0.16 s confirms this circumstance. However, Fig. 8c presents the instantaneous apparent power $s(t)$; here, it is observed that $s(t)$ includes only the instantaneous power after the TIC. However, TAC requires a low-pass filter (LPF) to determine the active power and rms value of the voltage when required. This is a characteristic of TAC versus TIC. As a compromise between settling time and speed of descent in the stopband a third order Butterworth filter has been used.

Fig. 9a shows $p(t)$ before/after TAC_P. After compensation, $p(t)$ becomes a constant value equal to the average power of the load (after the LPF settling time has elapsed): q(t) follows the same behavior presented in Fig. 8b because TAC_P completely eliminates $q(t)$. Fig. 9b shows $s(t)$; after compensation, $s(t)$ assumes a constant value that is identified with the active power drawn by the load. Similarly, for the TAC_PF methodology, Fig. 10a shows $p(t)$ before and after compensation; this time, it is no longer constant; it is the instantaneous active power whose average value is the load active power. Again, the graph corresponding to $q(t)$ is shown

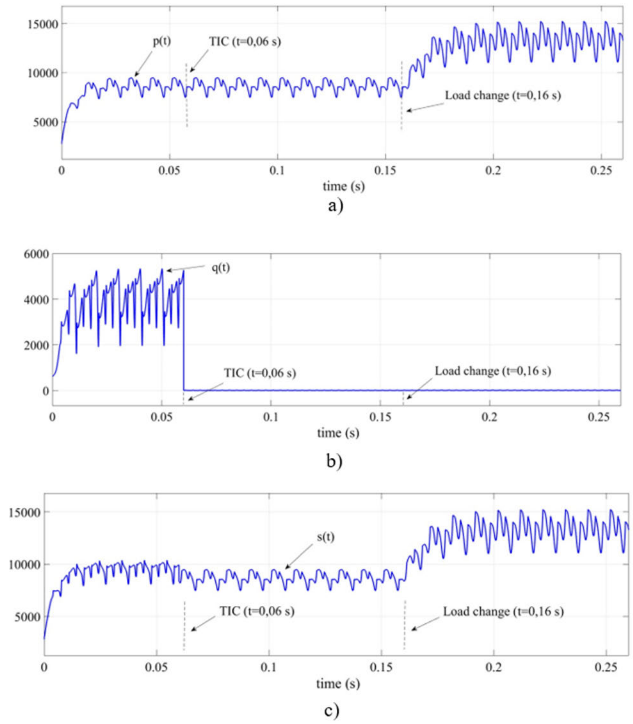

FIGURE 8. a) Instantaneous power (W) before/after TIC (t=0.06 s); b) Instantaneous reactive power (var) before/after TIC (t=0.06 s); c) Instantaneous apparent power (VA) before/after TIC (t=0.06 s). At t=0.16 s there is a change in the load.

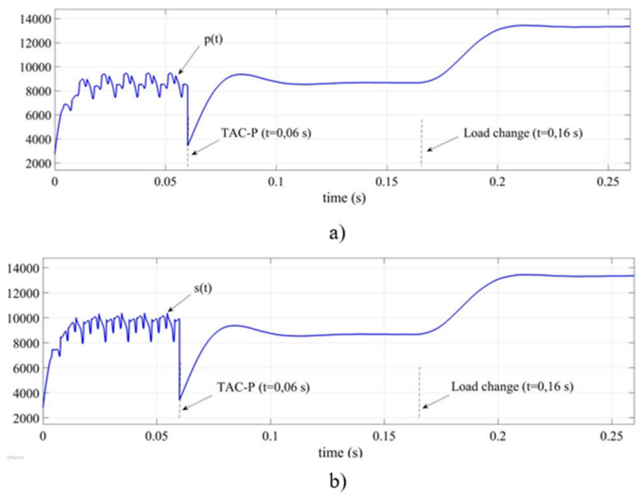

FIGURE 9. a) Instantaneous power (W) before/after TAC_P (t=0.06 s); b) Instantaneous apparent power (VA) before/after TAC_P (t=0.06 s). At t=0.16 s there is a change in the load.

in Fig. 8b, because $q(t)$ cancels out after compensation. Finally, $s(t)$ coincided with $p(t)$ after compensation, Fig. 10b.

B. EXPERIMENTAL RESULTS

The proposed formulation was experimentally validated using a laboratory prototype. The implemented model uses the same parameters for the elements and power supply of the system as those of the simulation platform described in the previous subsection. Thus, in the system shown in Fig. 3, the shunt APLC is connected. Using Semikron SKM50GB 123D modules consisting of two insulated-gate bipolar transistors (IGBTs) with their respective freewheeling diodes, a three-legged inverter was designed to form a three-phase converter. A set of two 2200 μF, 600 V, electrolytic capacitors in series were installed on the dc side of the converter. Both the compensation strategies (TIC and TAC) were implemented using the dSPACE acquisition and control system. A modular system that integrates a DS1005 PPC card that includes a PowerPC 750GX processor communicates via a PHS bus with specific input/output (I/O) cards. Specifically, DS2004 cards with 16 differential input channels and DS5101DWO cards with 16 TTL-type outputs were used. The real-time interface (RTI) tool allows a real-time interface between cards and MATLAB/Simulink blocks, which manages to execute designs previously developed in MATLAB-Simulink. Finally, from the RTW (real-time workshop) toolbox, C code was obtained from the Simulink models to control the firing of the IGBTs of the power converters. The current waveforms on the three-phase side before and after compensation were captured using the dSPACE ControlDesk tool, which allows real-time interaction with the power system [1]. The system was powered using a California Instruments 15003i/iX programmable power supply according to the parameters listed in Table 3.

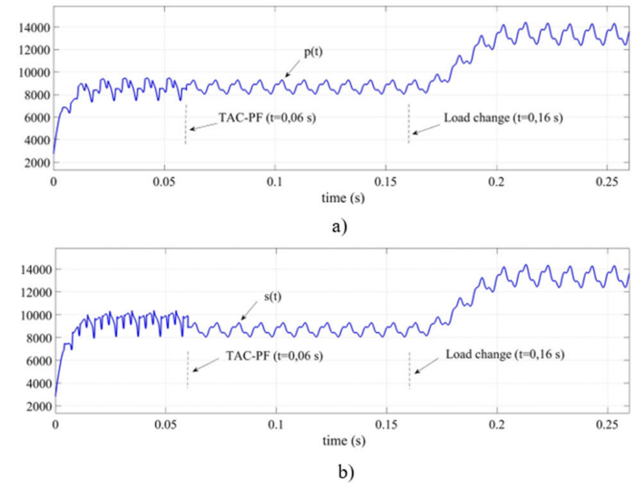

FIGURE 10. a) Instantaneous power (W) before/after TAC_PF (t=0.06 s); b) Instantaneous apparent power (VA) before/after TAC_PF (t=0.06 s). At t=0.16 s there is a change in the load.

Fig. 11 shows the waveforms of the supply voltage of conductor 1 versus the load current (the current was multiplied by 10 for plotting purposes).

Fig. 12a shows the source currents after the TIC. This includes the voltage and current waveforms (the current is scaled by 10) of conductor 1. After compensation, the source current transfers only instantaneous power, and the current waveform includes the ripple inherent to the hysteresis band control. Fig. 12b shows the source voltage and current after applying the constant-power compensation strategy TAC_P.

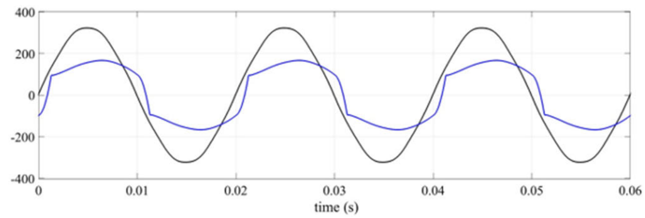

FIGURE 11. Voltage (V) and current (A) waveforms of the conductor 1 considered in the experimental setup. The current appears multiplied by 10.

The line currents transfer only the active power absorbed by the load (and possibly the specific losses of the compensator) after connecting the APLC. Fig. 12c shows the voltage and current after applying the unity power-factor compensation strategy TAC_PF. From the power supply, the currents of the conductors are in phase with the voltage of each conductor with respect to the virtual neutral.

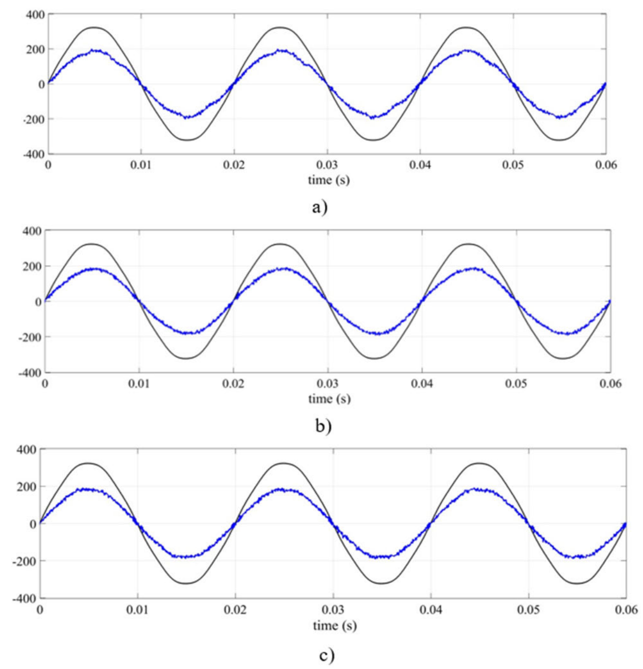

FIGURE 12. Voltage (V) and current (A) waveforms of conductor 1 after compensation. a) TIC; b) TAC_P; c) TAC_PF. The currents appear multiplied by 10.

VI. DISCUSSION

In this section, we outline the most noteworthy features of this study.

- The GA has proven to be an effective mathematical tool to formulate the IRPT for any n-phase power system in a general manner without the need to refer to any particular coordinate system. This implies having a multivector power variable that groups instantaneous power and instantaneous reactive power. From this, the two orthogonal current components of the IRPT are obtained and are not linked to the previously prefixed coordinates.
- The instantaneous power multivector $\mathbf{s}(t)$ is valid for a system with any number of phases. This extended the analysis from single-phase to n-phase systems. This characteristic distinguishes it from the IRPT analyses presented to date. In one-phase systems, if $\mathbf{u}= u e_1$ and $\mathbf{i}= i e_1$, then the algebraic subspace generated under the GA has a k-vector basis $\{1, e_1\}$. In this case the geometric product of $\mathbf{u}$ and $\mathbf{i}$ is $\mathbf{s}(t)=\mathbf{ui}=u \cdot i=p(t)$; that is, it consists of one component, the 0-vector whose value is the instantaneous real power. It follows that $\mathbf{i_p}$ matches $\mathbf{i}$ and the line losses cannot be reduced by the compensation equipment without energy storage elements. This fact is well known; however, it is of interest that it is included in such a theory, which is valid for multi-phase systems.
- The approach followed here has made it possible to find concrete operational expressions for the power terms and current components, both in phase coordinates and in other types of coordinates, such as the coordinates $0\alpha\beta$ of the original theory of Akagi et al. [2]. This made it possible to identify and compare the different formulations proposed within the IRPT framework.
- From a practical perspective, the multidimensional approach makes it possible to treat industrial three-phase systems by considering a neutral conductor as simply another line conductor. This facilitates an analysis and approach to the most current philosophy. This requires arranging the voltages and currents as vectors of the four components; the GA is an appropriate tool for this purpose.
- The concepts of instantaneous real power/instantaneous reactive power and instantaneous power/instantaneous reactive currents allow for the generalization of the two aspects of compensation introduced by the IRPT to multi-phase systems. Thus, it is possible to speak of the TIC and TAC for systems with any number of phases.

VII. CONCLUSION

In this study, a generalized formulation of the IRPT is presented. This new approach allows us to identify the power/current terms in both three-phase systems and systems with any number of phases. This methodology was developed in a GA environment. Thus, the instantaneous power multivector is defined as a new power variable that incorporates instantaneous power and instantaneous reactive power. The present power multivector directly follows the decomposition of the current into two perpendicular components: an instantaneous power current and an instantaneous reactive current. This approach was first presented for three-phase systems and was subsequently extended to multi-phase systems. The fundamentals of the compensation of balanced/unbalanced linear/nonlinear loads in the scope of the new methodology are discussed. Thus, the control strategies of TIC and TAC have been generalized to multi-phase systems. A discussion of the concepts of IRPT (power variables and current components) and their application to the compensation of nonlinear/unbalanced loads in the GA environment was established. Finally, the topics were applied to a practical power system compensated with an APLC. The results from the simulation model and experimental platform are presented. The GA methodology has proved useful for the formulation of the IRPT; in that sense, an interesting and still pending research would be its application in the formulation of electrical power for multi-phase systems in the frequency domain.

APPENDIX

In this section, the orthogonal relationship between $\mathbf{i_p}$ and $\mathbf{i_q}$ for an n-phase system is demonstrated. For system voltage $\mathbf{u}$, the current vector is split into two components: one parallel and one perpendicular to vector $\mathbf{u}$, that is

$$\mathbf{i} = \mathbf{i}_{\perp \mathrm{u}} + \mathbf{i}_{\llcorner \mathrm{u}} \tag{A1}$$

Both components, by hypothesis, satisfy the following two relationships,

$$\mathbf{u} \cdot \mathbf{i}_{\perp \mathrm{u}} = 0 \quad ; \quad \mathbf{u} \wedge \mathbf{i}_{\llcorner \mathbf{u}} = 0 \tag{A2}$$

From (A2), it follows that, for the geometric product of the first current component with vector $\mathbf{u}$,

$$\begin{aligned} \mathbf{u}\mathbf{i}_{\perp u} &= \mathbf{u} \cdot \mathbf{i}_{\perp u} + \mathbf{u} \wedge \mathbf{i}_{\perp u} = \mathbf{u} \wedge \mathbf{i}_{\perp \mathrm{u}} \\ &= \mathbf{u} \wedge \mathbf{i}_{\perp u} + \mathbf{u} \wedge \mathbf{i}_{\perp \mathrm{u}} = \mathbf{u} \wedge \mathbf{i} \end{aligned} \tag{A3}$$

By applying the geometric product of the voltage vector inverse by (A3), we obtain (A4).

$$\mathbf{i}_{\perp u} = \mathbf{u}^{-1}(\mathbf{u} \wedge \mathbf{i}) \equiv \mathbf{i}_q \tag{A4}$$

Likewise, it follows that the current component parallel to the voltage vector,

$$\begin{aligned} \mathbf{u}\mathbf{i}_{\square \mathrm{u}} &= \mathbf{u} \cdot \mathbf{i}_{\square \mathrm{u}} + \mathbf{u} \wedge \mathbf{i}_{\square \mathrm{u}} = \mathbf{u} \cdot \mathbf{i}_{\square \mathrm{u}} \\ &= \mathbf{u} \cdot \mathbf{i}_{\perp u} + \mathbf{u} \cdot \mathbf{i}_{\square \mathrm{u}} = \mathbf{u} \cdot \mathbf{i} \end{aligned} \tag{A5}$$

Finally, by applying the geometric product of the voltage-vector inverse by (A5), we obtain (A6).

$$\mathbf{i}_{\square \mathbf{u}} = \mathbf{u}^{-1}(\mathbf{u} \cdot \mathbf{i}) \equiv \mathbf{i}_p \tag{A6}$$

From (A4) and (A6), it follows that the $\mathbf{i_p}$ component is parallel to the voltage vector and the component $\mathbf{i_q}$ is perpendicular to the voltage vector; therefore, both are orthogonal to each other.

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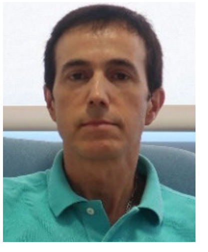

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