Received 25 November 2023, accepted 31 December 2023, date of publication 9 January 2024,
date of current version 25 January 2024.

Digital Object Identifier 10.1109/ACCESS.2024.3351728

RESEARCH ARTICLE

Partition Dimension of Generalized Hexagonal Cellular Networks and Its Application

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ABSTRACT The notion of partition dimension was initially introduced in the field of graph theory, primarily to examine distances between vertices. The local partition dimension extends this idea by incorporating specific conditions into how vertices are represented. In graph theory, it is customary to represent the partition dimension of a graph as $pd(G)$. Network localization, on the other hand, is the process of precisely determining the position of nodes within a network concerning a selected subset of nodes, called the locating set. The smallest size of its locating set is used to represent the locating number of a network. The generalized hexagonal cellular network provides an innovative framework for network planning and analysis. In our study, we investigate the partition dimension of a generalized hexagonal cellular network and provide a rigorous proof of its exact partition dimension. Hence, our approach ensures the distinct recognition of each node within a generalized hexagonal cellular network. Additionally, we explore the utilization of the metric dimension in flood relief camping by the National Disaster Management Authority (NDMA) Pakistan during floods in 2022. NDMA established relief camps and relief centers with unique codes to rescue humans and animals.

INDEX TERMS Vertex-based metric resolvability, resolving set, metric dimension, vertex-based partition resolvability, partition resolving set, partition dimension.

I. INTRODUCTION

The localization of a network refers to the method used to determine the exact position of a vertex or node within the network. An illustrative example of this concept is when a computer in an office environment sends a printing command. Localization is of utmost importance in various tasks, including but not limited to determining the closest printer, identifying malfunctioning nodes, spotting network intrusions, finding damaged equipment, recognizing illicit or misapplied connections, and even tracing the whereabouts of a mobile robot [1]. However, it's important to note that network localization is a challenging, expensive, time-consuming, and labor-intensive process.

To achieve network localization, the strategy involves the careful selection of different vertices or nodes. These nodes are selected in such a manner as to facilitate the determination of the position of the target vertex. This determination relies on the target vertex's distinct characteristics, which may include its label, orientation, or location in relation to the chosen nodes. The main goal is to reduce the count of chosen vertices, ultimately enhancing the efficiency of this approach. Central to this procedure is the locating set, which, in a strictly theoretical context, is referred to as the metric basis and consists of the chosen vertices. The key metric in this context is the locating number, which represents the minimum feasible set of selected vertices. Calculating the locating number for a graph is a complex computational task, falling into the category of non-deterministic polynomial-time hard (NP) problems, and the search for an algorithmic solution continues to be elusive [3], [5].

The notion of the locating set was initially introduced in scientific research by reference [6], marking the commencement of investigations aimed at determining a graph's locating number. This locating number signifies the smallest possible size of a resolving set. As time has progressed, various names have been used for this concept; for example, in reference [7], it is termed metric dimension. Moreover,

The associate editor coordinating the review of this manuscript and approving it for publication was Yilun Shang.

in reference [8], the terms metric basis and resolving set have been employed to describe the same concept. In recent years, an advanced definition of the locating set, known as the fault-tolerant locating set, has emerged, providing a broader interpretation of the original concept [10]. This extended perspective of the locating set has found practical applications in a range of diverse fields [11], [13], [14].

From a more extensive viewpoint, the idea of a locating set has found applications across diverse domains, such as pharmaceutical chemistry [5], image processing [6], intricate gaming and robotic navigation [15], combinatorial programming [16], and network theory [11]. Additionally, it has been employed to explore polyphenyl structures, particularly within the polymer industry [13], and recent advancements have expanded its utility into the realm of electronics [14].

The extensive body of literature on the metric dimension, which is the concept of the locating set, covers a wide range of applications across various fields. However, in this study, we will focus on recent and more broadly applicable findings. In [17], the authors delve into a diverse range of graphs, including a generic graph of a kayak paddle and several related to cycles. In [18], researchers examined a cellulose network and established upper bounds for its structural properties. In [19], they presented a graph metric reminiscent of a coronoid shape. Exploring the concept of locating numbers, a class of hydrocarbon-based structures was investigated in [20], leading to the discovery of various related versions. By employing the definition of the locating set outlined in [21], researchers explored the generalized class of the Harary family. Furthermore, [22] provides insights into multi-graphs and generalized Peterson graphs, along with the concept of a metric basis. In the context of Cayley graphs, [23] identifies and acknowledges the academics who calculated the locating number for this generalized class and more detailed studies can be found in the references [24] and [25].

There is an extensive body of literature available regarding this subject, which includes recent research studies conducted by [27] and [29]. The authors of [30] described the fault-tolerant metric dimension of circulant graphs and literature about locating number of biswapped networks in [31] They delve into the examination of the internet graph and its robust topology in [32]. Furthermore, [33] employs the concept of a fault-tolerant locating number to explore a quartz structure and the network connections related to computers. References [34] and [35] explore the investigation of convex polytopes with a focus on fault-tolerant locating sets, aiming to determine the precise fault-tolerant locating numbers associated with them.

A hexagonal grid pertains to the orderly placement of shapes, particularly polygons, in a manner that eliminates any vacant spaces. It entails the generation of unending geometric designs through the utilization of hexagonal configurations, as originally suggested in [36]. These patterns have found applications in various fields, including network design, as discussed in the recent advancement, and in the context of cellular networks [37]. Hexagonal cellular networks are extensively used in network planning and analysis and are explored in academic settings, as detailed in the book [38]. For industrial applications, relevant studies can be found in [39], [40], and [41], with a focus on the topology of hexagonal cellular networks discussed in [42].

The partition dimension is a specific branch of research within graph theory, as discussed by [43]. It is closely related to the development of the metric dimension. To clarify, a set referred to as a locating set is often termed a resolving set, as explained by [44]. Additionally, the concept of a discriminating partition bears a resemblance to the resolving set in a graph. This involves grouping the vertices in a graph G into various partition classes and computing the distances of each vertex in G to all partition classes in order to represent each vertex in the graph, as described in [45], [46], and [47].

To lay the groundwork for our forthcoming conversation, we will commence by presenting fundamental ideas that will prove crucial in shaping our eventual outcomes.

Definition 1: Suppose $G(V(G), E(G))$ is an undirected graph of chemical structure with $V(G)$ is called a vertex set of G and $E(G)$ is called the edge set of G. Distance between two vertices $\lambda_1, \lambda_2 \in V(G)$, denoted as $d(\lambda_1, \lambda_2)$ is the minimum count of edges between λ_1 to λ_2 path.

Definition 2: Assume $R \subset V(G)$ is a subset of the vertex set and is specified as $R = \{\lambda_1, \lambda_2, \ldots, \lambda_s\}$, and consider a vertex$\lambda \in V(G)$. The identification $r(\lambda|R)$ of a vertex with regard to R is actually an s-ordered distance $(d(\lambda, \lambda_1), d(\lambda, \lambda_2), \ldots, d(\lambda, \lambda_s))$. If each vertex in $V(G)$ has a unique location according to the ordered subset R, then this subset is termed the metric basis of structure G.

The metric dimension of the graph G can be denoted as $dim(G)$ and it represents the smallest number of elements needed in the subset R to achieve this property.

Definition 3: Let R_p denote the set of l-ordered partitions, and $r(F|R_p) = (d(F, R_{p_1}), d(F, R_{p_2}), \ldots, d(F, R_{p_l}))$ represents the l-tuple distance values for a vertex F in relation to R_p. If the different ways of expressing F in relation to R_p are not the same, then B as the resolving partition set for the set of vertices in graph G. The minimum number of subsets in the resolving set of $V(G)$ is denoted as the partition dimension $(pd(G))$ of graph G.

II. CONSTRUCTION AND MAIN RESULTS

The arrangement of the network depicted in Figure 1 is versatile and takes the form of a hexagonal cellular network. This network is characterized by three variables or parameters that collectively establish its six-dimensional structure. We denote this as $HCN(\psi, \phi, \varrho)$, with the constraint that each parameter ψ, ϕ, ϱ must be greater than or equal to 2. Notably, these three parameters can take on distinct values or be identical. Furthermore, labels have been allocated to both the nodes and edges of this network in order to streamline the validation of our main discoveries, as demonstrated below $V(HCN(\psi, \phi, \varrho))$, as shown at the bottom of the next page.

The count of nodes and lines of this network are succinctly described as::

$$|V(HCN(\psi,\phi,\varrho))| = \begin{cases} \left(18\psi^2+30\psi+12\right), & \text{if } \psi=\phi=\varrho; \\ \left(18\phi^2+42\phi+22\right), & \text{if } \psi=\phi\neq\varrho; \\ \left(18\varrho^2+54\varrho+38\right), & \text{if } \psi\neq\phi=\varrho. \end{cases}$$

$$|E(HCN(\psi,\phi,\varrho))| = \begin{cases} (3(\psi+1)(3\psi+2)), & \text{if } \psi=\phi=\varrho; \\ (3\phi(3\phi+7)+11), & \text{if } \psi=\phi\neq\varrho; \\ (9\varrho(\varrho+3)+19), & \text{if } \psi\neq\phi=\varrho. \end{cases}$$

Figure 1 displays specific parameter values, where $\psi = 3 = \phi$, $\varrho = 4$. The labels in question are referenced within the proofs of the theorems. Furthermore, these labels, in conjunction with the illustrated edge and vertex sets, can be expanded to include all possible values of the parameters that are natural (ψ,ϕ,ϱ).

Note 4: Througout this article, the parameteric value of z *is 1 if the particular vertex is belongs to the subset* $R_{p_{k-1}}$, *and* 0 *otherwise.*

Theorem 5: Consider the graph $HCN(2,2,3)$, *which represents the hexagonal cellular network. In this context, the partition dimension of* $HCN(2,2,3)$ *is three.*

Proof: Let partition resolving set of $HCN(2,2,3) = \{R_{p_1}, R_{p_2}, R_{p_3}\}$ where $R_{p_1} = \{a_{1,1}\}, R_{p_2} = \{a_{1,7}\}, R_{p_3} = V(HCN(2,2,3)) \setminus \{a_{1,1}, a_{1,7}\}$, One can identify a suitable candidate for a partition-resolving set in a generalized hexagonal cellular network with parameters $HCN(2,2,3)$. To support our claim that this selected subset effectively functions as a partition-resolving set for $HCN(2,2,3)$, we will adhere to the procedure outlined in Definition 1 to compute the shortest paths from every node to the set of nodes represented by $\{a_{1,1}, a_{1,7}\}$. Subsequently, we will employ these calculated paths in the practical context of determining a location, as defined in Definition 3, which is presented as follows.

$$r(a_{\diamond,\Diamond}|R_p) = \begin{cases} (0,6,z), & \text{if } \diamond=1, \Diamond=1; \\ (2,4,z), & \text{if } \diamond=1, \Diamond=3; \\ (4,2,z), & \text{if } \diamond=1, \Diamond=5; \\ (6,0,z), & \text{if } \diamond=1, \Diamond=7; \\ (2,7,z), & \text{if } \diamond=2, \Diamond=2; \\ (3,5,z), & \text{if } \diamond=2, \Diamond=4; \\ (5,4,z), & \text{if } \diamond=2, \Diamond=6; \\ (7,1,z), & \text{if } \diamond=2, \Diamond=8; \\ (1,5,z), & \text{if } \diamond=1, \Diamond=2; \\ (3,3,z), & \text{if } \diamond=1, \Diamond=4; \\ (5,1,z), & \text{if } \diamond=1, \Diamond=6; \\ (2,8,z), & \text{if } \diamond=2, \Diamond=1; \\ (2,6,z), & \text{if } \diamond=2, \Diamond=3; \\ (4,4,z), & \text{if } \diamond=2, \Diamond=5; \\ (6,2,z), & \text{if } \diamond=2, \Diamond=7; \\ (8,2,z), & \text{if } \diamond=2, \Diamond=9; \end{cases}$$

$$r(b_{\diamond,\Diamond}|R_p) = \begin{cases} (3,9,z), & \text{if } \diamond=1, \Diamond=1; \\ (3,7,z), & \text{if } \diamond=1, \Diamond=3; \\ (5,5,z), & \text{if } \diamond=1, \Diamond=5; \\ (7,3,z), & \text{if } \diamond=1, \Diamond=7; \\ (9,3,z), & \text{if } \diamond=1, \Diamond=9; \\ (6,8,z), & \text{if } \diamond=2, \Diamond=2; \\ (6,6,z), & \text{if } \diamond=2, \Diamond=4; \\ (8,6,z), & \text{if } \diamond=2, \Diamond=6; \\ (4,8,z), & \text{if } \diamond=1, \Diamond=2; \\ (4,6,z), & \text{if } \diamond=1, \Diamond=4; \\ (6,4,z), & \text{if } \diamond=1, \Diamond=6; \\ (8,4,z), & \text{if } \diamond=1, \Diamond=8; \\ (5,9,z), & \text{if } \diamond=2, \Diamond=1; \\ (5,7,z), & \text{if } \diamond=2, \Diamond=3; \\ (7,5,z), & \text{if } \diamond=2, \Diamond=5; \\ (9,5,z), & \text{if } \diamond=2, \Diamond=7; \end{cases}$$

$$\begin{aligned}
&V(HCN(\psi,\phi,\varrho)) = \{a_{\diamond,\Diamond} : \Diamond = 1,2,\ldots,2(\varrho+\diamond)-1, \diamond = 1,2,\ldots,\psi\} \\
&\cup \{b_{1,\Diamond} : \Diamond = 1,2,\ldots,2(\varrho+\psi), \textit{if } \phi=\psi+1, \psi=\varrho \textit{ and } \phi=\psi+1, \phi=\varrho\} \cup \{b_{\diamond,\Diamond} : \\
&\Diamond = 1,2,\ldots,2(\varrho+\psi-\diamond)+3, \diamond=2,3,\ldots,\phi, \textit{if } \phi=\psi+1, \psi=\varrho \textit{ and } \phi=\psi+1, \\
&\phi+\varrho\} \cup \{b_{\diamond,\Diamond} : \Diamond = 1,2,\ldots 2(\varrho+\psi-\diamond)+1, \diamond=1,2,\ldots,\phi\}. \\
&E(HCN(\psi,\phi,\varrho)) = \{a_{\diamond,\Diamond}a_{\diamond,j+1} : \Diamond=1,2,\ldots,2(\varrho+\diamond-1), \diamond=1,2,\ldots,\psi\} \\
&\cup\{b_{1,\Diamond}b_{1,\Diamond+1} : \Diamond=1,2,\ldots,2(\varrho+\psi)-1, \textit{if } \phi=\psi+1, \psi=\varrho \textit{ and } k=\psi+1, \phi=\varrho\} \\
&\cup\{b_{\diamond,\Diamond}b_{\diamond,\Diamond+1} : \Diamond=1,2,\ldots,2(\varrho+\psi-\Diamond+1), \diamond=2,3,\ldots,\phi, \textit{if } \phi=\psi+1, \psi=\varrho \textit{ and } \phi=\psi+1, \phi=\varrho\} \\
&\cup\{b_{\diamond,\Diamond}b_{\diamond}, \Diamond+1 : \Diamond=1,2,\ldots,2(\varrho+\psi-\diamond), \diamond=1,2,\ldots,k\} \\
&\cup\{a_{\psi,\Diamond}b_{1,\Diamond+1} : \Diamond=1,3,\ldots,2(\varrho+\psi)-1\} \\
&\cup\{a_{\psi,\Diamond}b_{1,\Diamond+1} : \Diamond=1,3,\ldots,2(\varrho+\psi)-1, \textit{if } \phi=\psi+1, \psi=\varrho \textit{ and } \phi=\psi+1, \phi=\varrho\} \\
&\cup\{a_{\diamond,\Diamond}a_{\diamond+1,\Diamond+1} : \Diamond=1,3,\ldots,2(\varrho+\diamond)-1, \diamond=1,2,\ldots,\psi-1\} \\
&\cup\{b_{\diamond,\Diamond}b_{\diamond+1,\Diamond-1} : \Diamond=2,4,\ldots,2(\varrho+\psi-\diamond)+1, \diamond=1,2,\ldots,\phi-1\} \\
&\cup\{b_{1,\Diamond}b_{2,\Diamond} : \Diamond=1,3,\ldots,2(\varrho+\psi)-1, \textit{if } \phi=\psi+1, \psi=\varrho \textit{ and } \phi=\psi+1, \phi=\varrho\} \\
&\cup\{b_{\diamond,\Diamond}b_{\diamond+1,\Diamond-1} : \Diamond=2,4,\ldots,2(\varrho+\psi-\diamond), \diamond=1,2,\ldots\phi-1, \textit{if } \phi=\psi+1, \psi=\varrho \textit{ and } \phi=\psi+1, \phi=\varrho\}.
\end{aligned}$$

Examining the above tables provided in the positions of $HCN(2,2,3)$ nodes, it becomes evident that all of these positions are both unique and distinct. Consequently, it follows that $pd(HCN(2,2,3))$ equals 3, thus establishing the validity of the proof. □

Theorem 6: Let's consider the graph of the generalized hexagonal cellular network denoted as $GHCN(2,3,2)$. In this case, the partition dimension of $GHCN(2,3,2)$ is three.

Proof: Let a partition resolving set of $HCN(2,3,2) = \{R_{p_1}, R_{p_2}, R_{p_3}\}$ where $R_{p_1} = \{a_{1,2}\}, R_{p_2} = \{b_{2,1}\}, R_{p_3} = V(HCN(2,3,2)) \setminus \{a_{1,2}, b_{2,1}\}$, we must identify a suitable candidate for a partition resolving set within the context of the generalized hexagonal cellular network with parameter values $HCN(2,3,2)$. To validate our claim that this chosen subset functions as a partition resolving set for $HCN(2,3,2)$, We will adhere to Definition 1 in order to determine the shortest routes from every node to the set consisting of $\{a_{1,2}, b_{2,1}\}$. Afterward, we will utilize these routes as per the formal location definition specified in Definition 3, which is presented below.

$$r(a_{\diamond,\Diamond}|R_p) = \begin{cases} (1,5,z), & \text{if } \diamond = 1, \Diamond = 1; \\ (1,7,z), & \text{if } \diamond = 1, \Diamond = 3; \\ (3,9,z), & \text{if } \diamond = 1, \Diamond = 5; \\ (2,4,z), & \text{if } \diamond = 2, \Diamond = 2; \\ (2,6,z), & \text{if } \diamond = 2, \Diamond = 4; \\ (4,8,z), & \text{if } \diamond = 2, \Diamond = 6; \\ (0,6,z), & \text{if } \diamond = 1, \Diamond = 2; \\ (2,8,z), & \text{if } \diamond = 1, \Diamond = 4; \\ (3,3,z), & \text{if } \diamond = 2, \Diamond = 1; \\ (3,5,z), & \text{if } \diamond = 2, \Diamond = 3; \\ (3,7,z), & \text{if } \diamond = 2, \Diamond = 5; \\ (5,9,z), & \text{if } \diamond = 2, \Diamond = 7; \end{cases}$$

$$r(b_{\diamond,\Diamond}|R_p) = \begin{cases} (5,1,z), & \text{if } \diamond = 1, \Diamond = 1; \\ (5,3,z), & \text{if } \diamond = 1, \Diamond = 3; \\ (5,5,z), & \text{if } \diamond = 1, \Diamond = 5; \\ (5,7,z), & \text{if } \diamond = 1, \Diamond = 7; \\ (6,0,z), & \text{if } \diamond = 2, \Diamond = 1; \\ (6,2,z), & \text{if } \diamond = 2, \Diamond = 3; \\ (6,4,z), & \text{if } \diamond = 2, \Diamond = 5; \\ (6,6,z), & \text{if } \diamond = 2, \Diamond = 7; \\ (9,3,z), & \text{if } \diamond = 3, \Diamond = 2; \\ (9,5,z), & \text{if } \diamond = 3, \Diamond = 4; \\ (4,2,z), & \text{if } \diamond = 1, \Diamond = 2; \\ (4,4,z), & \text{if } \diamond = 1, \Diamond = 4; \\ (4,6,z), & \text{if } \diamond = 1, \Diamond = 6; \\ (6,8,z), & \text{if } \diamond = 1, \Diamond = 8; \\ (7,1,z), & \text{if } \diamond = 2, \Diamond = 2; \\ (7,3,z), & \text{if } \diamond = 2, \Diamond = 4; \\ (7,5,z), & \text{if } \diamond = 2, \Diamond = 6; \\ (8,2,z), & \text{if } \diamond = 3, \Diamond = 1; \\ (8,4,z), & \text{if } \diamond = 3, \Diamond = 3; \\ (8,6,z), & \text{if } \diamond = 3, \Diamond = 5; \end{cases}$$

Examining the above tables provided in the positions of $HCN(2,3,2)$ nodes, it becomes evident that all of these positions are both unique and distinct. Consequently, it follows that $pd(HCN(2,3,2)) = pd(HCN(2,2,3))$ equals 3, thus establishing the validity of the proof. □

Theorem 7: Consider the graph denoted as $HCN(3,3,3)$, which represents the generalized hexagonal cellular network. In this context, it's worth noting that the partition dimension of $HCN(3,3,3)$ equals three.

Proof: Let a partition resolving set: $HCN(3,3,3) = \{R_{p_1}, R_{p_2}, R_{p_3}\}$ where $R_{p_1} = \{a_{1,1}\}, R_{p_2} = \{b_{3,7}\}, R_{p_3} = V(HCN(3,3,3)) \setminus \{a_{1,1}, b_{3,7}\}$, it is crucial to find a suitable candidate that can function as a partition-resolving set in the context of $HCN(2,3,2)$ or for specific parameter in a hexagonal cellular network. To validate our claim that this chosen subset effectively acts as a partition-resolving set for $HCN(3,3,3)$, we will use Definition 1 to compute the shortest paths from every node to the designated destinations, namely $\{a_{1,1}, b_{3,7}\}$. Afterwards, we will apply these computed paths in accordance with the formal description of a location provided in Definition 3, as outlined below.

$$r(a_{\diamond,\Diamond}|R_p) = \begin{cases} (0,11,z), & \text{if } \diamond = 1, \Diamond = 1; \\ (2,9,z), & \text{if } \diamond = 1, \Diamond = 3; \\ (4,9,z), & \text{if } \diamond = 1, \Diamond = 5; \\ (6,9,z), & \text{if } \diamond = 1, \Diamond = 7; \\ (1,9,z), & \text{if } \diamond = 2, \Diamond = 2; \\ (3,8,z), & \text{if } \diamond = 2, \Diamond = 4; \\ (5,8,z), & \text{if } \diamond = 2, \Diamond = 6; \\ (7,8,z), & \text{if } \diamond = 2, \Diamond = 8; \\ (4,10,z), & \text{if } \diamond = 3, \Diamond = 1; \\ (4,8,z), & \text{if } \diamond = 3, \Diamond = 3; \\ (4,8,z), & \text{if } \diamond = 3, \Diamond = 5; \\ (6,5,z), & \text{if } \diamond = 3, \Diamond = 7; \\ (8,5,z), & \text{if } \diamond = 3, \Diamond = 9; \\ (10,5,z), & \text{if } \diamond = 3, \Diamond = 11; \\ (1,10,z), & \text{if } \diamond = 1, \Diamond = 2; \\ (3,10,z), & \text{if } \diamond = 1, \Diamond = 4; \\ (5,10,z), & \text{if } \diamond = 1, \Diamond = 6; \\ (2,10,z), & \text{if } \diamond = 2, \Diamond = 1; \\ (2,8,z), & \text{if } \diamond = 2, \Diamond = 3; \\ (4,7,z), & \text{if } \diamond = 2, \Diamond = 5; \\ (6,7,z), & \text{if } \diamond = 2, \Diamond = 7; \\ (8,7,z), & \text{if } \diamond = 2, \Diamond = 9; \\ (3,9,z), & \text{if } \diamond = 3, \Diamond = 2; \\ (3,7,z), & \text{if } \diamond = 3, \Diamond = 4; \\ (5,6,z), & \text{if } \diamond = 3, \Diamond = 6; \\ (7,6,z), & \text{if } \diamond = 3, \Diamond = 8; \\ (9,6,z), & \text{if } \diamond = 3, \Diamond = 10; \end{cases}$$

$$r(b_{\diamond,\Diamond}|R_p) = \begin{cases} (5, 9, z), & \text{if } \diamond = 1, \Diamond = 1; \\ (5, 7, z), & \text{if } \diamond = 1, \Diamond = 3; \\ (5, 5, z), & \text{if } \diamond = 1, \Diamond = 5; \\ (7, 4, z), & \text{if } \diamond = 1, \Diamond = 7; \\ (9, 4, z), & \text{if } \diamond = 1, \Diamond = 9; \\ (11, 4, z), & \text{if } \diamond = 1, \Diamond = 11; \\ (8, 6, z), & \text{if } \diamond = 2, \Diamond = 2; \\ (8, 4, z), & \text{if } \diamond = 2, \Diamond = 4; \\ (8, 2, z), & \text{if } \diamond = 2, \Diamond = 6; \\ (10, 1, z), & \text{if } \diamond = 2, \Diamond = 8; \\ (9, 5, z), & \text{if } \diamond = 3, \Diamond = 1; \\ (9, 3, z), & \text{if } \diamond = 3, \Diamond = 3; \\ (9, 1, z), & \text{if } \diamond = 3, \Diamond = 5; \\ (10, 0, z), & \text{if } \diamond = 3, \Diamond = 7; \\ (6, 8, z), & \text{if } \diamond = 1, \Diamond = 2; \\ (6, 6, z), & \text{if } \diamond = 1, \Diamond = 4; \\ (6, 4, z), & \text{if } \diamond = 1, \Diamond = 6; \\ (8, 3, z), & \text{if } \diamond = 1, \Diamond = 8; \\ (10, 3, z), & \text{if } \diamond = 1, \Diamond = 10; \\ (7, 7, z), & \text{if } \diamond = 2, \Diamond = 1; \\ (7, 5, z), & \text{if } \diamond = 2, \Diamond = 3; \\ (7, 3, z), & \text{if } \diamond = 2, \Diamond = 5; \\ (9, 2, z), & \text{if } \diamond = 2, \Diamond = 7; \\ (11, 2, z), & \text{if } \diamond = 2, \Diamond = 9; \\ (10, 4, z), & \text{if } \diamond = 3, \Diamond = 2; \\ (10, 2, z), & \text{if } \diamond = 3, \Diamond = 4; \\ (11, 1, z), & \text{if } \diamond = 3, \Diamond = 6; \end{cases}$$

Examining the above tables provided in the positions of $HCN(3, 3, 3)$ nodes, it becomes evident that all of these positions are both unique and distinct. Consequently, it follows that $pd(HCN(3, 3, 3)) = pd(HCN(2, 2, 3))$ equals 3, thus establishing the validity of the proof. □

Theorem 8: If we consider $HCN(\psi, \phi, \varrho)$ as the graph of the generalized hexagonal cellular network with $\psi < \phi$, it can be stated that the partition dimension of $HCN(\psi, \phi, \varrho)$ is four.

Proof: To confirm the correctness of this theorem under equality, we will investigate both conditions involving inequality. To ensure that the value of $pd(HCN(\psi, \phi, \varrho))$ does not exceed 4, we will analyze the partition resolving set of $HCN(\psi, \phi, \varrho) = \{R_{p_1}, R_{p_2}, R_{p_3}, R_{p_4}\}$ where $R_{p_1} = \{b_{1,1}\}, R_{p_2} = \{b_{2,1}\}, R_{p_3} = \{b_{1,2(\varrho+\psi)}\}, R_{p_4} = V(HCN(\psi, \phi, \varrho)) \setminus \{b_{1,1}, b_{2,1}, b_{1,2(\varrho+\psi)}\}$, A relevant candidate for a partition-resolving set of $HCN(\psi, \phi, \varrho)$ is sought, especially when considering the specific parameter values for a generalized hexagonal cellular network, with the condition that ψ is less than ϕ. To substantiate our assertion that the selected subset indeed serves as a partition-resolving set for $HCN(\psi, \phi, \varrho)$, we will follow Definition 1 for the purpose of calculating the shortest routes from all nodes to the set

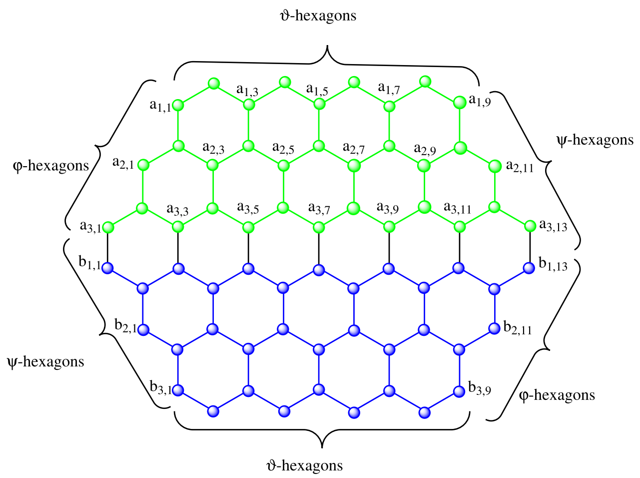

FIGURE 1. Generalized hexagonal cellular network with $\psi = 3 = \phi$, $\varrho = 4$.

of destinations, which includes $\{b_{1,1}, b_{2,1}$, and $b_{1,2(\varrho+\psi)}\}$. These calculated paths will then be utilized in the practical formulation of a location, as described in Definition 3 presented below.

The following explanation provides details about the locations, denoted as $r(a_{\diamond,\Diamond}|R_p)$, for the vertex subset $a_{\diamond,\Diamond}$ where $\Diamond \in \{1, 2, \ldots, 2(\varrho + \diamond) - 1\}$ and $\diamond \in \{1, 2, \ldots, \psi\}$.

$$r(a_{\diamond,\Diamond}|R_p) = (2(\psi - \diamond) + \Diamond + 1, 2(\psi - \diamond) + \Diamond, \times 2(\psi + \varrho) - \Diamond, z). \quad (1)$$

For $\Diamond \in \{1, 2, \ldots, 2(\varrho + \psi)\}$ with condition, if $\phi = \psi + 1$, $\psi = \varrho$ and $\phi = \psi + 1$, $\phi = \varrho$, the representations for the vertex subset denoted as $r(b_{1,\Diamond}|R_p)$ are explained as follows:

$$r(b_{1,\Diamond}|R_p) = (\Diamond - 1, \Diamond, 2(\psi - \varrho) - \Diamond, z). \quad (2)$$

For $\Diamond \in \{1, 2, \ldots, 2(\varrho + \psi - \diamond + 3)\}$ and $\diamond \in \{2, 3, \ldots, \phi\}$, with restriction that if $\phi = \psi + 1$, $\psi = \varrho$ and $\phi = \psi + 1$, $\phi = \varrho$, the locations denoted as $r(b_{\diamond,\Diamond}|R_p)$ for the vertex subset $b_{\diamond,\Diamond}$ are detailed in the following way:

$$r(b_{\diamond,\Diamond}|R_p) = (\Diamond + 2(\diamond - 2), \Diamond + 2\diamond - 5, 2(\psi + \varrho) - \Diamond + 1, z). \quad (3)$$

For $\Diamond \in \{1, 2, \ldots, 2(\varrho+\psi-\diamond+1)\}$ and $\diamond \in \{1, 2, 3, \ldots, \phi\}$, the locations denoted as $r(b_{\diamond,\Diamond}|R_p)$ for vertex subset $b_{\diamond,\Diamond}$ is characterized as follows:

$$r(b_{\diamond,\Diamond}|R_p) = \begin{cases} (\Diamond-1, \Diamond, \\ \quad 2(\psi+\varrho)-1-\Diamond, z), & \text{if } \diamond=1; \\ (2\diamond+\Diamond-3, 2\diamond+\Diamond-5, \\ \quad 2(\psi+\varrho-1-\Diamond), z), & \text{if } \diamond=2, 3, \ldots, \phi. \end{cases} \quad (4)$$

By examining the specific positions of the nodes in $HCN(\psi, \phi, \varrho)$ as defined in Equations 1-4, each of which is both unique and separate, so

$$pd(HCN(\psi, \phi, \varrho)) \leq 4.$$

we will investigate the converse inequality, which can be stated as $pd(HCN(\psi, \phi, \varrho)) \geq 4$. To rephrase it in a different form, we can represent it as either $|pd(HCN(\psi, \phi, \varrho))| < 4$ or $pd(HCN(\psi, \phi, \varrho)) = 3$.

As demonstrated earlier, all the selected samples for $pd(HCN(\psi, \phi, \varrho)) = 3$ lead to inconsistencies, leading to the conclusion that $pd(HCN(\psi, \phi, \varrho)) \neq 3$. This further supports the assertion that $pd(HCN(\psi, \phi, \varrho)) = 4$, thereby completing the proof. □

Theorem 9: If we consider the graph of the hexagonal cellular network as $HCN(\psi, \phi, \varrho)$, where ψ is greater than or equal to ϕ, then the partition dimension of $HCN(\psi, \phi, \varrho)$ is four.

Proof: To validate the credibility of this theorem based on equality, we will thoroughly examine both of the inequalities. To verify that $pd(HCN(\psi, \phi, \varrho))$ is less than or equal to 4, we consider partition resolving set of $HCN(\psi, \phi, \varrho) = \{R_{p_1}, R_{p_2}, R_{p_3}, R_{p_4}\}$ where $R_{p_1} = \{a_{\psi,1}\}, R_{p_2} = \{b_{1,1}\}, R_{p_3} = \{a_{\psi,2(\varrho+\psi)-1}\}, R_{p_4} = V(HCN(\psi, \phi, \varrho)) \setminus \{a_{\psi,1}, b_{1,1}, a_{\psi,2(\varrho+\psi)-1}\}$, a suitable candidate for a partition resolving set of a generalized hexagonal cellular network $HCN(\psi, \phi, \varrho)$, where ψ is greater than or equal to ϕ, can be selected. In order to confirm the legitimacy of our assertion that the selected subset serves as a partition resolving set for $HCN(\psi, \phi, \varrho)$, we will adhere to Definition 1 when determining the shortest routes from every node to the set $\{a_{\psi,1}, b_{1,1}, a_{\psi,2(\varrho+\psi)-1}\}$. These paths will then be utilized in the practical formulation of a location, as detailed in Definition 3, which is presented later in this document.

The locations of the vertex subset $a_{\diamond,\Diamond}$ are explained as follows, where $\Diamond$ belongs to the set of values from 1 to $2(\varrho + \diamond) - 1$, and $\diamond$ ranges from 1 to ψ.

$$r(a_{\diamond,\Diamond}|R_p) = (2(\psi - \diamond) + \Diamond - 1, 2(\psi - \diamond) + \Diamond + 1, 2(\psi + \varrho) - \Diamond - 1, z). \tag{5}$$

For $\Diamond \in \{1, 2, \ldots, 2(\varrho + \psi)\}$ with restriction if $\phi = \psi + 1$, $\psi = \varrho$ and $\phi = \psi + 1$, $\phi = \varrho$, the locations of the vertex subset $b_{1,\Diamond}$ is described as follows:

$$r(b_{1,\Diamond}|R_p)=(2(\psi - \diamond) + \Diamond, \Diamond - 1, 2(\psi + \varrho) - \Diamond, z). \tag{6}$$

For $\Diamond \in \{1, 2, \ldots, 2(\varrho + \psi - \diamond + 3)\}$ and $\diamond \in \{2, 3, \ldots, \phi\}$, with restriction if $\phi = \psi + 1$, $\psi = \varrho$ and $\phi = \psi + 1$, $\phi = \varrho$, the locations associated with the vertex subset $b_{\diamond,\Diamond}$ are described as follows in the context of the function $r(b_{\diamond,\Diamond}|R_p)$:

$$r(b_{\diamond,\Diamond}|R_p)=(\Diamond + 2(\diamond - 2), \Diamond+2(\diamond - 2), 2(\psi+\varrho) - \Diamond, z). \tag{7}$$

For $\Diamond \in \{1, 2, \ldots, 2(\varrho + \psi - \diamond + 1)\}$ and $\diamond \in \{1, 2, 3, \ldots, \phi\}$, the locations of the vertex subset $b\diamond, \Diamond$, denoted as $r(b\diamond, \Diamond|R_p)$, are detailed as follows:

$$r(b_{\diamond,\Diamond}|R_p) = (2\diamond + \Diamond - 2, 2\diamond + \Diamond - 2, 2(\psi + \varrho) - \Diamond, z). \tag{8}$$

Observing the distinct and unique positions of the nodes in the provided equations (2.5)-(2.8) for $HCN(\psi, \phi, \varrho)$, so

$$pd(HCN(\psi, \phi, \varrho)) \leq 4.$$

Subsequently, we will confirm the inverse inequality, expressed as $pd(HCN(\psi, \phi, \varrho)) \geq 4$. By considering the negation, we can state that $|pd(HCN(\psi, \phi, \varrho))| < 4$ or specifically, $pd(HCN(\psi, \phi, \varrho)) = 3$. As demonstrated earlier, all the selected samples for which $pd(HCN(\psi, \phi, \varrho)) = 3$ result in contradictions. Consequently, it can be deduced that $pd(HCN(\psi, \phi, \varrho)) \neq 3$. This further supports the assertion that $pd(HCN(\psi, \phi, \varrho)) = 4$, completing the proof. □

Theorem 10: Consider the graph denoted as $HCN(\psi, \phi, \varrho)$, where $\psi < \phi$, representing a generalized hexagonal cellular network. In this context, the partition dimension of $HCN(\psi, \phi, \varrho)$ is established as five.

Proof: To confirm the theorem's equality, we will scrutinize both of the inequalities. To verify that $pd(HCN(\psi, \phi, \varrho)) \leq 5$, we consider partition resolving set of $HCN(\psi, \phi, \varrho) = \{R_{p_1}, R_{p_2}, R_{p_3}, R_{p_4}, R_{p_5}\}$ where $R_{p_1} = \{b_{1,1}\}, R_{p_2} = \{b_{2,1}\}, R_{p_3} = \{b_{1,2(\varrho+\psi)}, R_{p_4} = \{b_{2,2(\varrho+\psi)-1}\}, R_{p_5} = V(HCN(\psi, \phi, \varrho)) \setminus \{b_{1,1}, b_{2,1}, b_{1,2(\varrho+\psi)}, b_{2,2(\varrho+\psi)-1}\}$, to establish the suitability of a particular candidate as a partition resolving set for the generalized hexagonal cellular network with parameters $GHCN(\psi, \phi, \varrho)$, especially when the condition $\psi < \phi$ holds, we aim to substantiate this claim. We will follow the guidelines set out in Definition 1 to determine the shortest routes from every node to a group comprising $\{b_{1,1}, b_{2,1}, b_{1,2(\varrho+\psi)}, b_{2,2(\varrho+\psi)-1}\}$. These calculated paths will then be used in the precise explanation of a location, as described in Definition 3, which is presented in more detail below.

The locations of the vertex subset $a\diamond, \Diamond$, denoted as $r(a_{\diamond,\Diamond}|R_p)$, are explained as follows, where $\Diamond$ ranges from 1 to $2(\varrho + \diamond) - 1$ and $\diamond$ ranges from 1 to ψ.

$$r(a_{\diamond,\Diamond}|R_p) = (2(\psi - \diamond) + \Diamond + 1, 2(\psi - \diamond) + \Diamond, 2(\psi + \varrho) - \Diamond, 2(\varrho + \psi + 1) - \Diamond, z). \tag{9}$$

The locations of the vertex subset $r(b_{1,\Diamond}|R_p)$ where $\Diamond \in \{1, 2, \ldots, 2(\varrho + \psi)\}$, are explained as follows, under the condition that either $\phi = \psi + 1$ with $\psi = \varrho$ or $\phi = \psi + 1$ with $\phi = \varrho$.

$$r(b_{1,\Diamond}|R_p)=(\Diamond-1, \Diamond, 2(\psi-\varrho)-\Diamond, 2(\psi+\varrho+1)-\Diamond, z). \tag{10}$$

For $\Diamond \in \{1, 2, \ldots, 2(\varrho + \psi - \diamond + 3)\}$ and $\diamond \in \{2, 3, \ldots, \phi\}$, with restriction if $\phi = \psi + 1$, $\psi = \varrho$ and $\phi = \psi + 1$, $\phi = \varrho$, The representation of the vertex subset $b_{\diamond,\Diamond}$, denoted as $r(b_{\diamond,\Diamond}|R_p)$, are explained in the following way:

$$r(b_{\diamond,\Diamond}|R_p) = (\Diamond + 2(\diamond - 2), \Diamond + 2\diamond - 5, 2(\psi + \varrho) - \Diamond + 1, 1(\psi + \varrho) - 1 - \Diamond, z). \tag{11}$$

For $\Diamond \in \{1, 2, \ldots, 2(\varrho+\psi-\diamond+1)\}$ and $\diamond \in \{1, 2, 3, \ldots, \phi\}$, the locations of the vertex subset $b_{\diamond,\Diamond}$ with respect to R_p are described as follows:

$$r(b_{\diamond,\Diamond}|R_p) = \begin{cases} (\Diamond - 1, \Diamond, 2(\psi+\varrho) - 1 - \Diamond, 2(\varrho+\psi+1) - \Diamond, z), \\ \text{if } \diamond = 1; \\ (2\diamond + \Diamond - 3, 2\diamond + \Diamond - 5, 2(\psi+\varrho-1-\Diamond), \\ 2(\varrho+\psi) - \Diamond - 1, z), \\ \text{if } \diamond = 2; \\ (2\diamond + \Diamond - 3, 2\diamond + \Diamond - 5, 2(\psi+\varrho-1-\Diamond), \\ 2(\varrho+\psi) - \Diamond - 1, z), \\ \text{if } \diamond = 3, 4, \ldots, \phi. \end{cases} \quad (12)$$

Examining the specific positions of the nodes in $HCN(\psi, \phi, \varrho)$ as outlined in Equations 9-12, each being both unique and separate, it follows that $pd(HCN(\psi, \phi, \varrho)) \leq 5$.

Next, we will verify the converse inequality, represented as $pd(HCN(\psi, \phi, \varrho)) \geq 5$. When we take the negation of this statement, it can be expressed as $|pd(HCN(\psi, \phi, \varrho))| < 5$ or $pd(HCN(\psi, \phi, \varrho)) = 4$. As demonstrated earlier, all the selected samples where $pd(HCN(\psi, \phi, \varrho)) = 3$ lead to contradictions, thereby ruling out the possibility of $pd(HCN(\psi, \phi, \varrho)) \neq 4$. This conclusion further supports the assertion that $pd(HCN(\psi, \phi, \varrho)) = 5$, thus finalizing the proof. □

Theorem 11: Consider the graph denoted as $HCN(\psi, \phi, \varrho)$, where $\psi \geq \phi$, representing the generalized hexagonal cellular network. It can be stated that the partition dimension of $HCN(\psi, \phi, \varrho)$ is five.

Proof: To confirm the theorem's validity when equality is considered, we will investigate both the inequalities. To verify that $pd(HCN(\psi, \phi, \varrho)) \leq 6$, we consider partition resolving set of $HCN(\psi, \phi, \varrho) = \{R_{p_1}, R_{p_2}, R_{p_3}, R_{p_4}, R_{p_5}\}$ where $R_{p_1} = \{a_{\psi,1}\}, R_{p_2} = \{b_{1,1}\}, R_{p_3} = \{a_{\psi,2(\varrho+\psi)-1}\}$, $R_{p_4} = \{b_{1,2(\varrho+\psi)-1}\}, R_{p_5} = V(GHCN(\psi, \phi, \varrho)) \setminus \{a_{\psi,1}, b_{1,1}, a_{\psi,2(\varrho+\psi)-1}, b_{1,2(n+m)-1}\}$, a suitable contestant for partition resolving set in the context $HCN(\psi, \phi, \varrho)$, particularly when ψ is greater than or equal to ϕ, will be identified. To support our claim that this selected subset effectively acts as a partition resolving set for $HCN(\psi, \phi, \varrho)$, we will adhere to Definition 1. By doing this, we will compute the most efficient routes from every node to a group encompassing $\{a_{\psi,1}, b_{1,1}, a_{m,2(\varrho+\psi)-1}, b_{1,2(\varrho+\psi)-1}\}$. Following this, the calculated routes will be employed in the official establishment of a geographical point, as outlined in Definition 3, which is presented subsequently.

For $\Diamond \in \{1, 2, \ldots, 2(\varrho+\diamond)-1\}$ and $\diamond \in \{1, 2, \ldots, \psi\}$, the locations of the vertex subset denoted as $a_{\diamond,\Diamond}$ are explained as follows with respect to R_p.

$$r(a_{\diamond,\Diamond}|R_p) = (2(\psi-\diamond)+\Diamond-1, 2(\psi-\diamond)+\Diamond+1, 2(\psi+\varrho) - \Diamond - 1, 2(\varrho+\psi) - \Diamond, z). \quad (13)$$

The locations of the vertex subset denoted as $b_{1,\Diamond}$ are explained as follows for values of $\Diamond \in \{1, 2, \ldots, 2(\varrho+\psi)\}$, under the specified conditions on the parameters. These conditions are either $\phi = \psi + 1$ when $\psi = \varrho$ or $\phi = \psi + 1$ when $\phi = \varrho$:

$$r(b_{1,\Diamond}|R_p) = (2(\psi - \diamond + \Diamond, \Diamond - 1, 2(\psi+\varrho) - \Diamond, 2(\psi + \varrho) - \Diamond - 1, z). \quad (14)$$

The locations of the vertex subset $b_{\diamond,\Diamond}$ are explained as follows for values of $\Diamond \in \{1, 2, \ldots, 2(\varrho+\psi-\diamond+3)\}$, with specific conditions on the parameters: $\phi = \psi + 1$, $\psi = \varrho$ and $\phi = \psi + 1$, $\phi = \varrho$.

$$r(b_{\diamond,\Diamond}|R_p) = (\Diamond + 2(\diamond - 2), \Diamond + 2(\diamond - 2), 2(\psi+\varrho) - \Diamond, 2(\psi+\varrho) - 1 - \Diamond, z). \quad (15)$$

For $\Diamond \in \{1, 2, \ldots, 2(\varrho+\psi-\diamond+1)\}$ and $\diamond \in \{1, 2, 3, \ldots, \phi\}$, the locations denoted as $r(b\diamond, \Diamond|Rp)$ for the subset of vertices $b_{\diamond,\Diamond}$ are detailed in the following way:

$$r(b_{\diamond,\Diamond}|R_p) = (\Diamond + 2(\diamond - 3), \Diamond + 2(\diamond - 3), 2(\psi+\varrho) - \Diamond, 2(\psi+\varrho) - 1 - \Diamond, z). \quad (16)$$

By examining the specified positions of the nodes within $HCN(\psi, \phi, \varrho)$ as outlined in Equations 13-16, each of which is both unique and clearly differentiated, so

$$pd(HCN(\psi, \phi, \varrho)) \leq 5.$$

Next, we will examine the reverse inequality, namely $pd(HCN(\psi, \phi, \varrho)) \geq 5$. By expressing it in negation, we can assert that $|pd(HCN(\psi, \phi, \varrho))| < 5$. Another possibility is that $pd(HCN(\psi, \phi, \varrho)) = 4$. As previously demonstrated, all the selected samples where $pd(HCN(\psi, \phi, \varrho)) = 4$ lead to contradictory outcomes, establishing that $pd(HCN(\psi, \phi, \varrho)) \neq 4$. This further confirms that the case where $pd(HCN(\psi, \phi, \varrho)) = 5$ is valid, thus concluding the proof. □

III. APPLICATION OF METRIC DIMENSION

The NDMA in Pakistan is the government agency responsible for disaster management and coordination of disaster-related activities in the country. It was established in 2007 under the national disaster management ordinance, with the primary purpose of formulating policies, plans, and strategies for disaster management at the national level. The NDMA works to mitigate the impact of disasters, respond to emergencies, and coordinate relief and recovery efforts in the event of natural or man-made disasters. The NDMA works in partnership with multiple government agencies, provincial disaster management authorities, and pertinent organizations to facilitate a unified and efficient reaction to calamities such as earthquakes, floods, cyclones, and other emergencies. Its objective is to enhance disaster preparedness in Pakistan and decrease the susceptibility of communities to diverse risks.

During the period from June to August 2022, Pakistan experienced extensive flooding that significantly affected various parts of the nation, including Sindh, Balochistan, Punjab, and other regions. The provinces of Balochistan and Sindh were hit the hardest, and the floods extended as far north as Kashmir. The flooding resulted in a tragic loss of

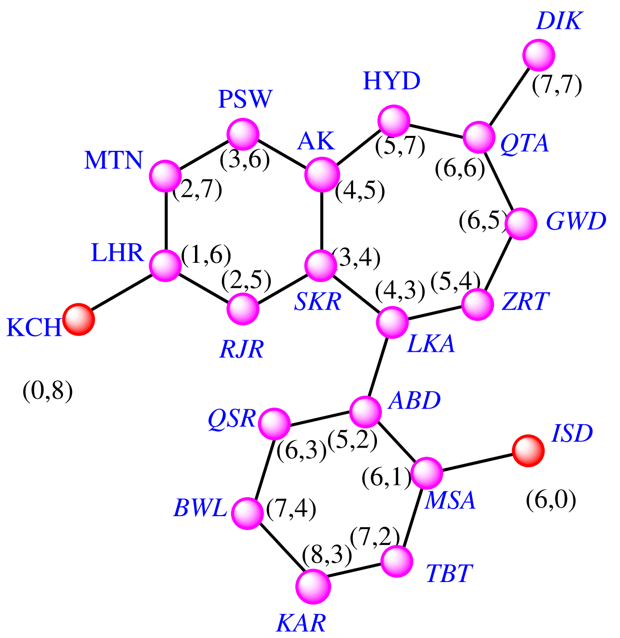

FIGURE 2. List of relief camps established by NDMA.

at least 1,500 lives and affected approximately 16 million children. Beyond the human toll and suffering caused by these floods, they also inflicted significant harm to agricultural activities, especially in Sindh. It was reported that the flood-related damage to crops in Sindh alone reached an estimated value of around Rs297 billion. This exacerbated the economic consequences of the floods and added to the challenges faced by the affected communities as they tried to rebuild their lives and livelihoods.

National Disaster Management Authority made 20 relief camps all over Pakistan to rescue people during floods in 2022, Karachi(KCH), Islamabad(ISD), Lahore(LHR), Multan(MTN), Peshawar(PSW), Azad Kashmir(AK), Hyderabad(HYD), Dera Ismaeel Khan(DIK), Quetta(QTA), Gwader(GWD), Ziarat(ZRT), Larkana(LKA), Sukhar(SKR), Rajanpur(RJR), Abbotabad(ABD), Mansehra(MSA), Turbat (TBT), Khairpur(KAR), Bahawalpur(BWL) and Qasur (QSR). In the context of network C representing multiple relief camps in Pakistan, we have identified and designated a select few relief centers with distinct codes based on their proximity to the relief camps shown in Figure 2. These unique codes have been assigned to ensure efficient access during transportation, and the objective is to keep the number of relief centers to a minimum, a measure referred to as the metric dimension. These specially chosen relief centers, forming the resolving set (denoted as R), play a crucial role in this context. These representations (r) are codes assigned to individual relief camps, taking into account their distances from the elements in set R. By implementing the concept of metric dimension, we achieve cost savings and time efficiency, as it enables swift and unambiguous access to the required relief camp, ultimately benefiting flood victims. To optimize cost-effectiveness, our goal is to maintain only two relief centers.

IV. CONCLUSION

The definition of the representation of vertex v in the context described above can be expressed as an l-vector, denoted as $r(v|R_p) = (d(v, R_{p_1}), d(v, R_{p_2}), \ldots, d(v, R_{p_l}))$, which consists of the distances between v and each vertex in the ordered l-partition $R_p = \{R_{p_1}, R_{p_2}, \ldots, R_{p_l}\}$ of the vertices in the graph G. This is applicable to any vertex v in G. When every distinct vertex in G has a unique representation with respect to the partition Rp, this division is referred to as a resolving partition for the graph G. The partition dimension of a graph G, denoted as $pd(G)$, is defined as the smallest value of l for which there exists a resolving l-partition of the vertex set V(G). To clarify further, a minimal resolving partition is a partition of the vertex set V(G) that both acts as a resolving partition and has exactly $pd(G)$ elements [48]. In this research, we have find the partition dimension of Generalized Hexagonal Cellular Networks

$$pd(HCN(2,2,3)) = 3$$
$$pd(HCN(2,3,2)) = 3$$
$$pd(HCN(3,3,3)) = 3$$
$$pd(HCN(\psi, \phi, \varrho)) \leq 4.$$

In this research, we have also given the application of metric dimension in flood relief camps by the NDMA Pakistan to rescue humans and animals during devastating floods in 2022. NDMA allowed unique codes to relief camps and relief centers to save money and time.

A. FUTURE DIRECTION

There are many other resolvability parameters like edge metric dimension, fault-tolerant version of metric and edge metric dimension. Local metric dimension is also a resolvability parameter. One can consider these parameters to study this novel structure of hexagonal cellular networks. Mixed metric dimension is also one of the recent parameters on resolvability. One can see the article for all the proper definitions of these parameters.

B. LIMITATION OF THE PROPOSED STUDY

Most of the theorems of this study concluded in terms of bounds. As the partition dimension is quite a complicated topic and to compute exact partitions is very complex. It is proven that the partition dimension is an NP-hard problem. So, we computed the bounds only. One can consider the exact results of these theorems.

DATA AVAILABILITY

The datasets used and/or analysed during the current study available from the corresponding author on reasonable request.

FUNDING STATEMENT

There is no funding for this manuscript.

AUTHOR CONTRIBUTIONS STATEMENT

All authors contributed equally for this manuscript.

CONFLICTS OF INTEREST

The authors declare that they have no conflicts of interest.

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