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RESEARCH ARTICLE

Improving Similarity Measures for Modeling Real-World Issues With Interval-Valued Intuitionistic Fuzzy Sets

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ABSTRACT The concept of interval-valued intuitionistic fuzzy sets (IVIFSs) presents a compelling and practical framework for modeling real-world problems. In various fields, such as pattern recognition and decision-making, the development of similarity measures tailored to this class holds significant importance. These measures play a pivotal role in the decision-making process involving IVIFSs, as they quantify the extent of similarity between two such sets. In this article, the shortcomings of the existing similarity measures within the framework of IVIFSs are highlighted, and an improved similarity measure is presented. A comparative study validates that this new similarity measure is better than the existing measures in the IVIF environment. This study systematically establishes several essential properties of the novel similarity measure and substantiates its effectiveness through numerical illustrations. Moreover, a comparative assessment is undertaken to validate the efficacy of the recently introduced measure in relation to established metrics, within the context of IVIFSs. To address the evaluation of software quality, a dedicated mechanism is devised, harnessing the proposed IVIFS similarity measure. Furthermore, an innovative production strategy is formulated utilizing the newly defined methodology to determine the optimal approach for the production of a specific product.

INDEX TERMS Decision making problem, interval valued intuitionistic fuzzy set, optimal production strategy, similarity measure, software quality model.

I. INTRODUCTION

Decision-making involves selecting a course of action from a variety of options, which entails choosing the best option from several choices. The process of multi-criteria decision-making (MCDM) involves finding the optimal solution to issues that fit specific criteria, which frequently arise in daily life. Due to the complexity of the accumulated social environment and the lack of accurate information, decisions are usually made by a group of experts rather than individual entities. Choosing the best option from multiple possibilities is a common task in MCDM. Over the last two decades, multi-criteria fuzzy decision-making (MCDM) has been rapidly adopted due to its practicality.

In 1965, Zadeh [1] introduced the concept of the fuzzy set (FS). According to Zadeh, one of the objectives of this theory is to simulate how the human mind processes information. It is observed that the human brain processes numerous hedges, such as good, very good, long, very long, tall, very tall, brilliant, and more brilliant, among an infinite list, more readily than numbers. One of the most crucial characteristics of human thought is the ability to condense information into FS that approximates the original material.

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In an FS, a membership function assigns a number from the closed unit interval to each component of the discourse universe, indicating its degree of belonging to the set under discussion. The degree of non-belongingness in FS is naturally the complement of the membership degree, which is 1. However, humans frequently fail to articulate a commensurate degree of non-membership as the complement to 1 while expressing the degree of membership of a particular element in a FS. The fact that linguistic negation does not necessarily correspond to logical negation highlights a well-known psychological reality. Therefore, Atanassov [2] introduced the idea of intuitionistic fuzzy sets (IFSs), which handle ambiguity more efficiently than FS. The degrees of membership and non-membership can be challenging to express using the IFS generalization, leading to reluctance in providing complete information and semantic representation. Many researchers consider it impractical to provide an expert opinion on a specific number and suggest giving a range of possible values to reflect the true nature of the situation. In 1975, Zadeh [3] proposed the idea of interval-valued fuzzy sets (IVFS). The concept of vague sets was introduced by Gau and Buehrer [4] in 1993, and later, Bustine and Burillo [5] observed in 1996 that the theories of vague sets and intuitionistic fuzzy sets are similar.

In 1999, Hong and Kim [6] introduced a new measure for IFS. Three novel similarity measures on IFSs were proposed by Liang and Shi [7] in 2003, and their use in pattern recognition was examined. In 2004, Szmidt and Kacprzyk [8] developed a new similarity measure on IFSs from a geometric viewpoint. In 2011, Ye [9] established a cosine similarity measure on IFSs. In 2014, Boran and Akay [10] determined a bi-parametric similarity measure on IFSs. In 2018, Ngan et al. [11] proposed a similarity measure on IFSs based on the maximum of the cross-evaluation factor. Dhivya and Sridevi [12] described a new similarity measure on IFS in 2019 that is based on using the midpoint of the right-angled triangle fuzzy number instead of the centroid point. In 2019, Jiang et al. [13] defined a similarity measure on IFS based on a distance measure. Dengfeng and Chuntian [14] introduced a similarity measure for IFSs by establishing an axiomatic definition for the similarity measure. Fan and Zhangyan [15] developed a similarity measure for the vague set. Haung and Yang [16] proposed a similarity measure for IFS based on Hausdorff distance. Wang and Xin [17] proposed a novel distance measure on IFS. Songs et al. [18] proposed a similarity measure on IFS based on the distance measure, or the relationship between membership and non-membership functions. Garg and Rani [19] suggested a similarity measure on IFS based on a transformed right-angled triangle in 2021. Gohain et al. [20] characterized two similarity measures on the IFS in 2022. Pan and Deng [21] developed a similarity measure for IFS in 2022. Chen and Liu [22] proposed an IF value similarity measure for IFS in 2022. Kumar [23] suggested a new similarity measure on IFS and applied it to pattern recognition and clustering in 2023. Ejegwa and Agbetayo [24] proposed the similarity-distance decision-making technique and applied it via intuitionistic fuzzy pairs in 2023. Distance and similarity measures using the intuitionistic fuzzy hypersoft set were characterized by Saqlain et al. [25] in 2023.

Due to the complexity and unpredictability inherent in physical problems, the challenge of researching MCDM has focused on problems with criteria values in the form of intervals, making it an attention-grabbing topic of study. Atanassov and Gargov [26] proposed the concept of IVIFSs, which are extensions not only of IFSs but also of IVFSs, with a subinterval of [0,1] representing the degree of membership, non-membership, and hesitancy. This allows for accurate portrayal of the dynamic nature of features. Because of the benefits that IVIFSs bring to real-world applications, various similarity measures based on IVIFS have been intensively researched by numerous scholars from various perspectives, and have been used in an extensive range of fields, including medical diagnosis and pattern recognition problems. Xu and Chen [27] developed important similarity measures of IFS in the framework of IVIFS. Wei et al. [28] suggested a measure of similarity for IVIFS and applied it to tackle issues with medical diagnostics and pattern recognition. Singh [29] developed a novel cosine similarity measure for IVIFS and applied it to pattern recognition. Khalaf [30] presented a new approach for medical diagnosis in the framework of the IVIFS setting. Dhivya and Sridevi [31] presented a new similarity measure for IVIFSs based on the midpoint of the transformed fuzzy numbers. Luo and Liang [32] developed a new similarity measure to solve MCDM problems in the IVIF environment. Ye and Du [33] suggested a similarity measure for interval-valued neutrosophic sets. In 2020, Jeevaraj [34] defined a new similarity measure for the class of interval-valued intuitionistic fuzzy numbers based on the non-hesitant score function and applied it to solve the pattern reorganization problem. Verma and Merigo [35] introduced a cosine similarity measure for IVIFSs based on weighted reduced IFSs, and also defined the quasi-ordered weighted IVIF cosine similarity (quasi-OWIVIFCS) measure. Tiwari and Gupta [36] developed metrics for distance, similarity, and entropy for IVIF soft sets. Rathnasabapathy and Palanisami [37] designed a cosine similarity measure for IVIFSs, applied in real-world decision problems like pattern recognition, medical diagnosis, and MCDM. Ohlan [38] proposed novel distance and entropy measures for IVIFSs to address multi-criteria group decision-making. Suo [39] established a knowledge measure function combining distance and the technique for order preference by similarity to the ideal solution. In 2023, Chen and Ke [40] proposed a cosine similarity measure to solve the MCDM problem by using IVIF knowledge. Nayagam et al. [41] suggested a similarity measure based on the accuracy score of conventional trapezoidal-valued intuitionistic fuzzy sets in 2023.

In 2019, Xiao and Ding [42] proposed a novel method for measuring the divergence between PFSs, referred to as the PFSJS distance. This method utilizes the Jensen-Shannon divergence. In 2020, Xiao [43] introduced a novel method for MCDM referred to as EFMCDM. This method combines the Dempster-Shafer theory with belief entropy to enhance the decision-making process. In this framework, each individual criterion can be conceptualized as a piece of evidence, while the collection of all options forms the frame of discernment within the context of Dempster-Shafer theory. In 2021, Xiao [44] developed a new distance measure based on the Jensen-Shannon divergence between IFSs. This new IFS distance measure not only satisfies the axiomatic definition of distance measure but also possesses nonlinear properties. Liu et al. [45] develop a novel hybrid multi-attribute group decision-making approach under interval-valued intuitionistic fuzzy sets (IVIFS) by integrating variable weight, correlation coefficient, and technique for order performance by similarity to an ideal solution in 2021. Chen [46] expanded grey relational analysis to single-valued neutrosophic sets to handle MAGDM with inadequate weight informatio.

A. RESEARCH GAP

It is imperative to note that the existing similarity measures utilized in IVIFSs possess inherent limitations, as they often produce inconsistent outcomes and inadequately characterize data when applied to situations involving multi-criteria decision-making problems. To provide an illustration, consider the following values: $A = \{[0.3, 0.4], [0.5, 0, 7]\}$, $B = \{[0.4, 0.5], [0, 5, 0.7]\}$ and $C = \{[0.4, 0.5], [0.4, 0, 6]\}$. If we applied the existing similarity measure formulas presented in Section III, we get $S(A, B) = S(A, C)$, whereas $B \neq C$, demonstrated the ineffectiveness of the existing similarity measures. This clearly illustrates the limitations of the existing similarity measures under IVIF environment. Consequently, existing similarity measures are unsuitable for distinguishing IVIFSs. Moreover, the mathematical properties and calculation methods of existing similarity measures fail to consider the minute variations in membership intervals. Therefore, it is necessary to create a new similarity measure that can address these drawbacks in the IVIF environment.

B. MOTIVATION

The IFS paradigm limits membership and non-membership degrees to one real number, which cannot appropriately convey uncertainty. But IVIFS offers intervals for these degrees. This lets IVIFS handle a broader range of uncertainty. Interval representation in IVIFS improves imprecise or uncertain data handling. This paradigm depicts inherent uncertainty more adaptably and expressively, making it better for decision-making and problem-solving in uncertain circumstances. IVFS discusses intervals in terms of membership degrees, not non-membership degrees. IVFS does not properly reflect whole-scale ambiguity about inclusion and exclusion. IVIFS allows intervals for membership and non-membership, which may expand its uncertainty representation. Additional information on non-membership degrees can aid decision-making and modeling scenarios that need consideration of both membership and non-membership or uncertainty. In circumstances that need both types of uncertainty, Interval-Valued Intuitionistic Fuzzy Sets (IVIFS) can better describe uncertainty. IVIFS relies on similarity measurements to quantify their level of resemblance.

C. NOVELTY OF THE CURRENT RESEARCH WORK

The novelty of conducting this research is the formulation of new similarity measures in the framework of IVIF knowledge to solve MCDM problems. To highlight the drawbacks of the existing similarity measures under IVIF environment and to present a systematic mathematical mechanism to solve MCDM problems using newly proposed similarity measures under IVIF knowledge are other key features of conducting this research.

The excellence, performance, and stability of software are all indicators of its quality. It measures the extent to which the program satisfies its original objective and how well it meets the needs and expectations of its intended users. When developing and deploying software, considering the end user's perspective, the software's reception, and the credibility of the software vendor or developer is important. Prioritizing and investing in software quality is vital to ensuring that the program can deliver value and meet the expectations of consumers.

Prior to the International Organization for Standardization (ISO), there was no international standard for evaluating the quality and integrity of a software system by examining its underlying structure to identify serious structural flaws. Per the ISO guidelines, quality models serve as versatile tools for delineating and assessing software and software-intensive computer systems from multiple vantage points. These perspectives encompass the domains of acquisition, requirements, development, utilization, evaluation, support, maintenance, quality assurance, control, and audit, each corresponding to their respective specifications. In order to quantify these criteria for software assessment and ranking, the theory of IVIFS has proven to be a very helpful tool. In this paper, we propose a mechanism to address the issue of evaluating software quality using the proposed IVIFS similarity measure.

In addition, one of the primary goals of this study is to investigate the impact of the similarity measure on IVIFS in management sciences. Organizational problem-solving and decision-making by managers are fundamental aspects of management science. The goal of production management is to manage a business to create the goods and services that customers want. It comprises planning, implementing, and controlling procedures that convert inputs into final goods and services. Better customer happiness, consumer expectations and requirements, and long-term product design and manufacturing can promote company image and contribute to business success.

In this article, we investigate a strategic decision-making issue involving the identification of the best approach to manufacturing. The relationship between a company's revenues and costs is one that businesses regularly examine. Does it make financial sense to reduce the price of a product to boost sales, even if it results in an increase in the cost required to create each extra unit? Which product should a company produce? What is the demand from consumers? These questions are crucial for the business community. In this paper, we present a solution to this type of scenario by using a similarity measure in an interval-valued intuitionistic fuzzy environment.

D. OBJECTIVES

Our main research goals are:

- Analyze the limitations of the existing similarity measures and develop a novel similarity measure for IVIFS. It offers a novel mathematical approach to correctly assess IVIFS similarity.
- Identify key features of the new similarity measure. The recommended similarity measure is shown legitimate and effective by precise proofs and detailed mathematical analysis.
- Develop an algorithm using the novel IVIF similarity measure to address multi-criteria decision-making (MCDM) situations.
- Evaluate the efficacy of the novel measure through a comparative analysis with established methodologies.
- Demonstrate the proposed mathematical model numerically. MCDM challenges like software quality project selection and optimal production strategy study are solved using the new mathematical model.

E. SIGNIFICANCE AND CONTRIBUTIONS OF CONDUCTING THE CURRENT RESEARCH WORK

By utilizing similarity measures, the system is able to rank, compare, and classify IVIFS data. They facilitate in the detection of patterns, retrieval of information, and decision-making of recommendation systems. Similarity measures assist in resolving MCDM problems in IVIFS by addressing a multitude of crucial components. In order to assess, prioritize, and select alternatives according to intricate criterion interactions, ambiguity, coherence, and these metrics assist decision-makers. Their contribution is substantial to robust decision-making. Aggregation operators are dominated by similarity measures in decision-making, particularly in complex multi-criteria contexts. In contrast to aggregation operators, which amalgamate all criteria into a single functional value, similarity measurements assess the merits or qualities of each feature or characteristic in isolation when contrasting alternatives. For decision-making factors that are subjective, variable, or difficult to quantify, this mathematical procedure performs admirably. In the case of multidimensional preferences, similarity measures provide a more precise understanding of the relationships between choices. In clustering, pattern recognition, and recommendation systems, similarity measures are implemented. The objective of these strategies is to identify intricate connections and shared characteristics among multiple options in order to enhance decision-making that is well-informed and contextually conscious.

In this research article, we present a mathematical mechanism to solve MCDM problems by means of developing new similarity measures in the IVIF environment. Moreover, we implement the new defined strategy to solve the MCDM problem to demonstrate the effectiveness of IVIFSs using similarity measures.

F. LIMITATIONS

Although the suggested approach offers several advantages, but it also has specific limitations, especially in the context of MCDM. These limitations arise when the sum of the scores of membership and non-membership exceeds one or involves neutral membership. In order to counter these situations, we need some more generalized environments than the IVIF environment.

Furthermore, if we consider three IVIFS denoted as $A = \{[0.3, 0.5], [0.3, 0.5]\}$, $B = \{[0.2, 0.4], [0.2, 0.4]\}$ and $C = \{[0.4, 0.7], [0.4, 0.7]\}$, then in view of definition 6, we get $S(A, C) = S(B, C)$. The proposed similarity measure fails to handle situations where the membership degree and non-membership degree are equal. Consequently, there is potential for further refinement and improvement of this proposed similarity measure.

The following is a concise summary of the research presented in this article: In Section II, we define key terms associated with IVIFS. In Section III, we highlight the drawbacks of some existing similarity measures defined on IVIFS. In Section IV, we present a new similarity measure on IVIFS and conduct a comparative analysis to demonstrate its effectiveness. In Section IV-A, we apply this novel technique to analyze the quality of a software project and design an optimal production strategy in IVIFS environment. Finally, we address the paper findings in the concluding section.

II. PRELIMINARIES

In this section we briefly review the basic concept, which are necessary for further discussion.

Definition 1 [2]: Let $Y = \{\rho_1, \rho_2, \ldots, \rho_n\}$ be the universe of discourse. An intuitionistic fuzzy set (IFS) $\dot{A}$ on Y is defined as follows: $\dot{A} = \{\langle \rho, \dot{\zeta}_A(\rho), \dot{\eta}_A(\rho)\rangle \,|\, \rho \in Y\}$, where $\dot{\zeta}_A : Y \to [0, 1]$ and $\dot{\eta}_A : Y \to [0, 1]$ represent membership and non-membership degree, respectively, subject to the constraint that $0 \leq \dot{\zeta}_A(\rho) + \dot{\eta}_A(\rho) \leq 1$. Moreover, $\pi_A(\rho) = 1 - \dot{\zeta}_A(\rho) + \dot{\eta}_A(\rho)$ is the hesitation degree of IFS A.

Definition 2 [26]: Let $Y = \{\rho_1, \rho_2, \ldots, \rho_n\}$ be the universe of discourse. An IVIFS A on Y is defined as follows: $A = \{\langle \rho, \dot{\zeta}_A(\rho), \dot{\eta}_A(\rho)\rangle \,|\, \rho \in Y\}$, where $\dot{\zeta}_A(\rho) = [\dot{\zeta}_A^-(\rho), \dot{\zeta}_A^+(\rho)]$ and $\dot{\eta}_A(\rho) = [\dot{\eta}_A^-(\rho), \dot{\eta}_A^+(\rho)]$ defined as the membership and non-membership degrees of an element ρ of Y. Furthermore, the hesitance degree can be computed as $\pi_A(\rho) = [1 - \dot{\zeta}_A^+(\rho) - \dot{\eta}_A^+(\rho), 1 - \dot{\zeta}_A^-(\rho) - \dot{\eta}_A^-(\rho)]$.

Definition 3 [11]: The relation between any two IVIFS A and B of a universe Y is defined as follows:

1) $A \subseteq B \iff \dot{\zeta}_A^-(\rho) \le \dot{\zeta}_B^-(\rho), \dot{\zeta}_A^+(\rho) \le \dot{\zeta}_B^+(\rho), \dot{\eta}_A^-(\rho) \ge \dot{\eta}_B^-(\rho)$ and $\dot{\eta}_A^+(\rho) \ge \dot{\eta}_B^+(\rho) \forall \rho \in Y$.
2) $A = B \iff \dot{\zeta}_A^-(\rho) = \dot{\zeta}_B^-(\rho), \dot{\zeta}_A^+(\rho) = \dot{\zeta}_B^+(\rho), \dot{\eta}_A^-(\rho) = \dot{\eta}_B^-(\rho)$ and $\dot{\eta}_A^+(\rho) = \dot{\eta}_B^+(\rho) \forall \rho \in Y$.

Definition 4 [11]: Let $\mathcal{G}$ be the class of all IVIFSs defined on a finite universe Y. A mapping $S : \mathcal{G} \times \mathcal{G} \to [0, 1]$ is the similarity measure of IVIFSs of Y, if it satisfies the following conditions:

(S1) $0 \le S(A, B) \le 1$,
(S2) $S(A, B) = S(B, A)$,
(S3) $S(A, A) = 1$,
(S4) If $A \subseteq B \subseteq C$, then $S(A, C) \le \min\{S(A, B), S(B, C)\}$ for all $A, B, C \in$ IVIFS(X).

Definition 5 [20]: Let $G : I^4 \to \mathcal{R}, I = [0, 1]$ be a function defined as:

$$G(z_1, z_2, z_3, z_4) = \left[\frac{1}{12}\left\{\begin{array}{l}\left(|z_1 - z_3|^2 + |z_2 - z_4|^2\right) \\ (2 - |z_3 + z_4 - z_1 - z_2|)^2 \\ +2\left(\begin{array}{l}|\min\{z_1, z_4\} - \min\{z_3, z_2\}|^2 + \\ |\max\{z_1, z_4\} - \max\{z_3, z_2\}|^2\end{array}\right)\end{array}\right\}\right].$$

Then G satisfies the following axioms.

1. If $z_i \in [0, 1], i = 1, 2, 3, 4$ and $(z_1 + z_2), (z_3 + z_4) \in [0, 1]$, then $0 \le G(z_1, z_2, z_3, z_4) \le 1$.
2. If $0 \le b_1 \le b_2 \le b_3 \le 1$ and $0 \le c_3 \le c_2 \le c_1 \le 1$,then $G(b_1, c_1, b_3, c_3) \ge G(b_i, c_i, b_{i+1}, c_{i+1})$ for $i = 1, 2$.

III. DRAWBACKE OF THE EXISTING SIMILARITY MEASURES OF IVIFS

In the following discussion, we will show that the similarity measures of IVIFS defined in [27], [28], [31], and [32] are ineffective.

Let $A = \left\{\left\langle \rho_i, \left[\dot{\zeta}_A^-(\rho_i), \dot{\zeta}_A^+(\rho_i)\right], \left[\dot{\eta}_A^-(\rho_i), \dot{\eta}_A^-(\rho_i)\right]\right\rangle \mid \rho \in Y\right\}$ and $B = \left\{\left\langle \rho_i, \left[\dot{\zeta}_B^-(\rho_i), \dot{\zeta}_B^+(\rho_i)\right], \left[\dot{\eta}_B^-(\rho_i), \dot{\eta}_B^-(\rho_i)\right]\right\rangle \mid \rho \in Y\right\}$ be two IVIFS on X.

The similarity measures S_1 and S_2 were developed by Xu and Chen's [27] and are expressed as follows:

1. $S_1(A, B) = 1 - \sqrt[p]{\frac{1}{4n}\sum_{i=1}^n \left(\begin{array}{l}\left|\dot{\zeta}_A^-(\rho_i) - \dot{\zeta}_B^-(\rho_i)\right|^p + \left|\dot{\zeta}_A^+(\rho_i) - \dot{\zeta}_B^+(\rho_i)\right|^p + \\ \left|\dot{\eta}_A^-(\rho_i) - \dot{\eta}_B^-(\rho_i)\right|^p + \left|\dot{\eta}_A^+(\rho_i) - \dot{\eta}_B^+(\rho_i)\right|^p\end{array}\right)}$,
2. $S_2(A, B) = 1 - \sqrt[p]{\frac{1}{n}\sum_{i=1}^n \max\left(\begin{array}{l}\left|\dot{\zeta}_A^-(\rho_i) - \dot{\zeta}_B^-(\rho_i)\right|^p + \left|\dot{\zeta}_A^+(\rho_i) - \dot{\zeta}_B^+(\rho_i)\right|^p + \\ \left|\dot{\eta}_A^-(\rho_i) - \dot{\eta}_B^-(\rho_i)\right|^p + \left|\dot{\eta}_A^+(\rho_i) - \dot{\eta}_B^+(\rho_i)\right|^p\end{array}\right)}$.

Examples 1 and 2 provide evidence of the ineffectiveness of S_1 and S_2.

Example 1: Consider $A = \{[0.3, 0.4], [0.5, 0.7]\}$, $B = \{[0.4, 0.5], [0, 5, 0.7]\}$ and $C = \{[0.4, 0.5], [0.4, 0.6]\}$ as the IVIFSs. We have $S_1(A, B) = S_1(A, C) = 0.9$, where $A \neq B$ and $A \neq C$. Thus, due to its indistinguishable characteristic the similarity measure S_1 is ineffective. The observed deficiency in discrimination may arise from the inherent attributes of S_1 and the specific characteristics of the IVIFSs being examined. It suggests that the current formulation of S_1 is not well-suited for discerning between IVIFSs with closely overlapping membership intervals. Consequently, this limitation becomes evident when applying the measure to IVIFSs with subtle distinctions that escape its grasp. The mathematical properties and computation approach of S_1 fall short in accounting for these minute fluctuations in membership intervals.

Example 2: Consider three IVIFSs $A = \{[0.3, 0.4], [0.5, 0.7]\}$, $B = \{[0.4, 0.5], [0.5, 0.7]\}$ and $C = \{[0.4, 0.5], [0.4, 0.6]\}$. We have $S_2(A, B) = S_2(A, C) = 0.9$, where $A \neq B$ and $A \neq C$. This shows that the similarity measure S_2 is ineffective due to its indistinguishable characteristic.

The similarity measure S_w was developed by Wei's et al. [28] and is expressed as follows:

$$S_w(A, B) = \frac{1}{n}\sum_{i=1}^n \frac{2 - \min\left(\dot{\zeta}_i^-, \dot{\eta}_i^-\right) - \min\left(\dot{\zeta}_i^+, \dot{\eta}_i^+\right)}{2 - \max\left(\dot{\zeta}_i^-, \dot{\eta}_i^-\right) - \max\left(\dot{\zeta}_i^+, \dot{\eta}_i^+\right)},$$

where,

$$\dot{\zeta}_i^- = \left|\dot{\zeta}_A^-(\rho_i) - \dot{\zeta}_B^-(\rho_i)\right|, \quad \dot{\zeta}_i^+ = \left|\dot{\zeta}_A^+(\rho_i) - \dot{\zeta}_B^+(\rho_i)\right|,$$

and

$$\dot{\eta}_i^- = \left|\dot{\eta}_A^-(\rho_i) - \dot{\eta}_B^-(\rho_i)\right|, \quad \dot{\eta}_i^+ = \left|\dot{\eta}_A^+(\rho_i) - \dot{\eta}_B^+(\rho_i)\right|.$$

Example 3: Let $A = \{[0.3, 0.4], [0.5, 0, 7]\}$, $B = \{[0.4, 0.5], [0, 5, 0.7]\}$ and $C = \{[0.4, 0.5], [0.4, 0, 6]\}$ be IVIFSs. We have $S_W(A, B) = S_W(A, C) = 0.21$, where $A \neq B$ and $A \neq C$. This finding leads to the conclusion that the Wei's similarity measure is ineffective due to its indistinguishable characteristic.

The similarity measure S_D was developed by Dhivya and Sridevi's [31] and is expressed as follows:

$$S_D(A, B) = 1 - \frac{1}{n}\sum_{i=1}^n \left(\begin{array}{l}\frac{1}{2}\left(\begin{array}{l}\left|\dot{\psi}_A^-(\rho_i) - \dot{\psi}_B^-(\rho_i)\right| + \\ \left|\dot{\psi}_A^+(\rho_i) - \dot{\psi}_B^+(\rho_i)\right|\end{array}\right). \\ \left(1 - \frac{\dot{\sigma}_A(\rho_i) + \dot{\sigma}_B(\rho_i)}{2}\right) + \\ \left|\dot{\sigma}_A(\rho_i) + \dot{\sigma}_B(\rho_i)\right|.\left(\frac{\dot{\sigma}_A(\rho_i) + \dot{\sigma}_B(\rho_i)}{2}\right)\end{array}\right),$$

where

$$\dot{\psi}_A^-(\rho_i) = \frac{\dot{\zeta}_A^-(\rho_i) + 1 - \dot{\eta}_A^-(\rho_i)}{2},$$

$$\dot{\psi}_A^+(\rho_i) = \frac{\dot{\zeta}_A^+(\rho_i) + 1 - \dot{\eta}_A^+(\rho_i)}{2},$$

$$\dot{\psi}_B^-(\rho_i) = \frac{\dot{\zeta}_B^-(\rho_i) + 1 - \dot{\eta}_B^-(\rho_i)}{2},$$

$$\dot{\psi}_A^+(\rho_i) = \frac{\dot{\zeta}_B^+(\rho_i) + 1 - \dot{\eta}_B^+(\rho_i)}{2},$$

$$\dot{\sigma}_A(\rho_i) = 1 - \frac{1}{2}\left(\dot{\zeta}_A^-(\rho_i) + \dot{\zeta}_A^+(\rho_i) + \dot{\eta}_A^-(\rho_i) + \dot{\eta}_A^+(\rho_i)\right)$$

and

$$\dot{\sigma}_B(\rho_i) = 1 - \frac{1}{2}\left(\dot{\zeta}_B^-(\rho_i) + \dot{\zeta}_B^+(\rho_i) + \dot{\eta}_B^-(\rho_i) + \dot{\eta}_B^+(\rho_i)\right).$$

Example 4: Let $A = \{[0.3, 0.4], [0.5, 0.7]\}$, $B = \{[0.4, 0.5], [0.5, 0.7]\}$ and $C = \{[0.4, 0.5], [0.4, 0, 6]\}$ be the IVIFs. We have $S_D(A, B) = S_D(A, C) = 1.00$, where $A \neq B$ and $A \neq C$. Therefore, the similarity measure S_D is ineffectual due to its indistinguishable characteristic.

The similarity measure S^p was developed by Luo and Liang's [32] and is expressed as follows:

$$S^p(A, B) = 1 - \left\{ \frac{1}{2n} \sum_{i=1}^{n} \left| \frac{t_1 \left[(\dot\zeta_A^-(\rho_i) - \dot\zeta_B^-(\rho_i)) + (\dot\zeta_A^+(\rho_i) - \dot\zeta_B^+(\rho_i)) \right] - \left[(\dot\eta_A^-(\rho_i) - \dot\eta_B^-(\rho_i)) + (\dot\eta_A^+(\rho_i) - \dot\eta_B^+(\rho_i)) \right]}{2(t_1 + 1)} \right|^p + \left| \frac{t_2 \left[(\dot\eta_A^-(\rho_i) - \dot\eta_B^-(\rho_i)) + (\dot\eta_A^+(\rho_i) - \dot\eta_B^+(\rho_i)) \right] - \left[(\dot\zeta_A^-(\rho_i) - \dot\zeta_B^-(\rho_i)) + (\dot\zeta_A^+(\rho_i) - \dot\zeta_B^+(\rho_i)) \right]}{2(t_2 + 1)} \right|^p \right\}^{\frac{1}{p}}.$$

Example 5: Consider $A = \{[0.3, 0.4], [0.5, 0, 7]\}$, $B = \{[0.4, 0.5], [0, 5, 0.7]\}$ and $C = \{[0.4, 0.5], [0.4, 0, 6]\}$ as the IVIFSs. We have $S^P(A, B) = S^P(A, C) = 0.93$, where $A \neq B$ and $A \neq C$. It shows that the similarity measure S^p is ineffectual due to its indistinguishable characteristic.

IV. ROPERTIES OF THE NEW SIMILARITY MEASURE BETWEEN IVIFS'S

We have examined numerous similarity measures in the research field. Several of them are ineffective at facilitating actual decision-making. Despite the general consensus, many similarity measures do not correspond to the real world. In the existing method, the lack of discrimination observed can be due to the fundamental attributes and characteristics of the IVIFS under consideration. This indicates that the existing similarity measures, as currently formulated, are not optimized for distinguishing between IVIFSs that possess closely overlapping membership intervals. Consequently, this limitation becomes apparent when the measure is applied to IVIFSs, which exhibit small differences that are not adequately captured by the measure. The mathematical properties and calculation method of existing similarity measures fail to consider the minute variations in membership intervals. This section introduces a new IVIFS similarity measure and demonstrates its important characteristic.

Definition 6: Let A and B be any two IVIFS on the universe of discourse $Y = \{\rho_1, \rho_2, \rho_3, \ldots, \rho_n\}$. The similarity measure $\mathcal{S}(A, B)$ on IVIFS is defined as shown in the equation at the bottom of the next page.

Here, $\dot\pi$ denotes the hesitation function. To assess and measure similarities between two items or concepts, the hesitation function can capture the uncertainty of making such a comparison. The degree of reluctance is incorporated into the measurement procedure to create a more nuanced and adaptable similarity representation. By accounting for uncertainty, subjectivity, and individual preferences, the hesitation function in similarity measures makes similarity depictions more adaptive, sophisticated, and resilient. Similarity evaluations in pattern recognition, expert systems, information retrieval, and decision support become more reliable and useful. Consider the following example to demonstrate the suggested similarity measure's accuracy on IVIFS.

Definition 7: Consider the IVIFS $A = \{[0.3, 0.4], [0.5, 0, 7]\}$, $B = \{[0.4, 0.5], [0, 5, 0.7]\}$ and $C = \{[0.4, 0.5], [0.4, 0, 6]\}$. We have $\mathcal{S}(A, B) = 0.7$, and $\mathcal{S}(A, C) = 0, 8$, where $A \neq B$ and $A \neq C$. Thus, it's plausible to assume that this similarity measure is effective. By comparing Example 1 to Example 5 with Example 4.1.1, it is quite evident that the existing measure possesses the deficiency to solve MCDM problems under the IVIF environment, Whereas the recently defined similarity measure solves the problems quite effectively. Therefore, we draw the conclusion that the proposed measure provides a more suitable and effective strategy for decision analysis than the existing measures, which have proved to be inefficient.

Definition 8: For any two IVIFSs $A, B \in \mathcal{G}$, the following as shown in the equation at the bottom of the next page, is a similarity measure.

Proof Let $A = \{\langle \rho_i, [\dot\zeta_A^-(\rho_i), \dot\zeta_A^+(\rho_i)], [\dot\eta_A^-(\rho_i), \dot\eta_A^-(\rho_i)] \rangle : \rho \in Y\}$, $B = \{\langle \rho_i, [\dot\zeta_B^-(\rho_i), \dot\zeta_B^+(\rho_i)], [\dot\eta_B^-(\rho_i), \dot\eta_B^-(\rho_i)] \rangle : \rho \in Y\}$ and be the IVIFS on X.

(S1) Case I

Firstly, we solve this property for $\mathcal{S}^-$ which represents the lower membership and non-membership degrees of IVIFS.

Considering $\dot\zeta_A^-(\rho_i), \dot\zeta_B^-(\rho_i), \dot\eta_A^-(\rho_i), \dot\eta_B^-(\rho_i), \in [0, 1]$ as well as $\dot\zeta_A^-(\rho_i) + \dot\eta_A^-(\rho_i), \dot\zeta_B^-(\rho_i) + \dot\eta_B^-(\rho_i) \in [0, 1]$, we can apply Definition 5 to derive the following inequality:

$$0 \le G\left(\dot\zeta_A^-(\rho_i), \dot\zeta_B^-(\rho_i), \dot\eta_A^-(\rho_i), \dot\eta_B^-(\rho_i)\right) \le 1 \quad \forall i = 1, 2, 3, \ldots, n.$$

Now, consider the following inequality:

$$0 \le \left[\frac{1}{12} \left\{ \begin{array}{l} \left(\left|\dot\zeta_A^-(\rho_i) - \dot\zeta_B^-(\rho_i)\right|^2 + \left|\dot\eta_A^-(\rho_i) - \dot\eta_B^-(\rho_i)\right|^2 \right) \\ \left(2 - \left|\dot\pi_A^-(\rho_i) - \dot\pi_B^-(\rho_i)\right|^2\right) + \\ 2 \left(\left| \begin{array}{l} \min\{\dot\zeta_A^-(\rho_i), \dot\eta_B^-(\rho_i)\} - \\ \min\{\dot\zeta_B^-(\rho_i), \dot\eta_A^-(\rho_i)\} \end{array} \right|^2 + \left| \begin{array}{l} \max\{\dot\zeta_A^-(\rho_i), \dot\eta_B^-(\rho_i)\} - \\ \max\{\dot\zeta_B^-(\rho_i), \dot\eta_A^-(\rho_i)\} \end{array} \right|^2 \right) \end{array} \right\} \right] \le 1.$$

Taking the summation over all $i = 1, 2, \ldots, n$ and prefixing the factor $\frac{1}{n}$. We can conclude the following:

$$0 \le \left[\frac{1}{12n} \sum_{i=1}^{n} \left\{ \begin{array}{l} \left(\left|\dot\zeta_A^-(\rho_i) - \dot\zeta_B^-(\rho_i)\right|^2 + \left|\dot\eta_A^-(\rho_i) - \dot\eta_B^-(\rho_i)\right|^2 \right) \\ \left(2 - \left|\dot\pi_A^-(\rho_i) - \dot\pi_B^-(\rho_i)\right|^2\right) \\ +2 \left(\left| \begin{array}{l} \min\{\dot\zeta_A^-(\rho_i), \dot\eta_B^-(\rho_i)\} - \\ \min\{\dot\zeta_B^-(\rho_i), \dot\eta_A^-(\rho_i)\} \end{array} \right|^2 + \left| \begin{array}{l} \max\{\dot\zeta_A^-(\rho_i), \dot\eta_B^-(\rho_i)\} - \\ \max\{\dot\zeta_B^-(\rho_i), \dot\eta_A^-(\rho_i)\} \end{array} \right|^2 \right) \end{array} \right\} \right] \le 1.$$

Hence, $0 \le \mathcal{S}^-(A, B) \le 1$.

Case II

Similarly, one can establish the above inequality for $\mathcal{S}^{+}(A,B)$, where $\mathcal{S}^{+}$ denotes the upper membership and non-membership degrees of IVIFS.

Consequently, $0 \leq \mathcal{S}(A,B) \leq 1$.

Moreover, the condition (S2) and (S3) can be easily proved.

(S4) Case I

Let $A \subseteq B \subseteq C$, then

$$0 \leq \dot{\zeta}_A^-(\rho_i) \leq \dot{\zeta}_B^-(\rho_i) \leq \dot{\zeta}_C^-(\rho_i) \leq 1,$$

and

$$0 \leq \dot{\eta}_A^-(\rho_i) \leq \dot{\eta}_B^-(\rho_i) \leq \dot{\eta}_C^-(\rho_i) \leq 1.$$

In view of Definition 5, we obtain the following inequality.

$$G\left(\dot{\zeta}_A^-(\rho_i), \dot{\eta}_A^-(\rho_i), \dot{\zeta}_C^-(\rho_i), \dot{\eta}_C^-(\rho_i)\right) \geq G\left(\dot{\zeta}_A^-(\rho_i), \dot{\eta}_A^-(\rho_i), \dot{\zeta}_B^-(\rho_i), \dot{\eta}_B^-(\rho_i)\right) \quad (1)$$

$$G\left(\dot{\zeta}_A^-(\rho_i), \dot{\eta}_A^-(\rho_i), \dot{\zeta}_C^-(\rho_i), \dot{\eta}_C^-(\rho_i)\right) \geq G\left(\dot{\zeta}_B^-(\rho_i), \dot{\eta}_B^-(\rho_i), \dot{\zeta}_C^-(\rho_i), \dot{\eta}_C^-(\rho_i)\right) \quad (2)$$

Then from equation (1)

$$\left[\frac{1}{12}\left\{\begin{array}{l}\left(\begin{array}{l}\left|\dot{\zeta}_A^-(\rho_i)-\dot{\zeta}_C^-(\rho_i)\right|^2+\\ \left|\dot{\eta}_A^-(\rho_i)-\dot{\eta}_C^-(\rho_i)\right|^2\end{array}\right)\\ \left(2-\left|\dot{\pi}_A^-(\rho_i)-\dot{\pi}_C^-(\rho_i)\right|^2\right)\\ +2\left(\begin{array}{l}\left|\begin{array}{l}\min\left\{\dot{\zeta}_A^-(\rho_i),\dot{\eta}_C^-(\rho_i)\right\}-\\ \min\left\{\dot{\zeta}_C^-(\rho_i),\dot{\eta}_A^-(\rho_i)\right\}\end{array}\right|^2\\ +\left|\begin{array}{l}\max\left\{\dot{\zeta}_A^-(\rho_i),\dot{\eta}_C^-(\rho_i)\right\}-\\ \max\left\{\dot{\zeta}_C^-(\rho_i),\dot{\eta}_A^-(\rho_i)\right\}\end{array}\right|^2\end{array}\right)\end{array}\right\}\right]$$

$$\geq\left[\frac{1}{12}\left\{\begin{array}{l}\left(\left|\dot{\zeta}_A^-(\rho_i)-\dot{\zeta}_B^-(\rho_i)\right|^2+\left|\dot{\eta}_A^-(\rho_i)-\dot{\eta}_B^-(\rho_i)\right|^2\right)\\ \left(2-\left|\dot{\pi}_A^-(\rho_i)-\dot{\pi}_B^-(\rho_i)\right|^2\right)+\\ 2\left(\begin{array}{l}\left|\begin{array}{l}\min\left\{\dot{\zeta}_A^-(\rho_i),\dot{\eta}_B^-(\rho_i)\right\}-\\ \min\left\{\dot{\zeta}_B^-(\rho_i),\dot{\eta}_A^-(\rho_i)\right\}\end{array}\right|^2+\\ \left|\begin{array}{l}\max\left\{\dot{\zeta}_A^-(\rho_i),\dot{\eta}_B^-(\rho_i)\right\}-\\ \max\left\{\dot{\zeta}_B^-(\rho_i),\dot{\eta}_A^-(\rho_i)\right\}\end{array}\right|^2\end{array}\right)\end{array}\right\}\right].$$

$$\mathcal{S}(A,B)=1-\left[\frac{1}{12n}\sum_{i=1}^{n}\left\{\begin{array}{l}\left(\begin{array}{l}\left|\dot{\zeta}_A^-(\rho_i)-\dot{\zeta}_B^-(\rho_i)+\dot{\zeta}_A^+(\rho_i)-\dot{\zeta}_B^+(\rho_i)\right|^2\\ +\left|\dot{\eta}_A^-(\rho_i)-\dot{\eta}_B^-(\rho_i)+\dot{\eta}_A^+(\rho_i)-\dot{\eta}_B^+(\rho_i)\right|^2\end{array}\right)\\ \left(2-\left|\begin{array}{l}\dot{\pi}_A^-(\rho_i)-\dot{\pi}_B^-(\rho_i)+\\ \dot{\pi}_A^+(\rho_i)-\dot{\pi}_B^+(\rho_i)\end{array}\right|\right)^2\\ +2\left(\begin{array}{l}\left|\begin{array}{l}\min\left\{\dot{\zeta}_A^-(\rho_i),\dot{\eta}_B^-(\rho_i)\right\}+\\ \min\left\{\dot{\zeta}_A^+(\rho_i),\dot{\eta}_B^+(\rho_i)\right\}-\\ \min\left\{\dot{\zeta}_B^-(\rho_i),\dot{\eta}_A^-(\rho_i)\right\}+\\ \min\left\{\dot{\zeta}_B^+(\rho_i),\dot{\eta}_A^+(\rho_i)\right\}\end{array}\right|^2\\ +\left|\begin{array}{l}\max\left\{\dot{\zeta}_A^-(\rho_i),\dot{\eta}_B^-(\rho_i)\right\}+\\ \max\left\{\dot{\zeta}_A^+(\rho_i),\dot{\eta}_B^+(\rho_i)\right\}\\ -\max\left\{\dot{\zeta}_B^-(\rho_i),\dot{\eta}_A^-(\rho_i)\right\}+\\ \max\left\{\dot{\zeta}_B^+(\rho_i),\dot{\eta}_A^+(\rho_i)\right\}\end{array}\right|^2\end{array}\right)\end{array}\right\}\right]^{\frac{1}{2}}$$

$$\mathcal{S}(A,B)=1-\left[\frac{1}{12n}\sum_{i=1}^{n}\left\{\begin{array}{l}\left(\begin{array}{l}\left|\dot{\zeta}_A^-(\rho_i)-\dot{\zeta}_B^-(\rho_i)+\dot{\zeta}_A^+(\rho_i)-\dot{\zeta}_B^+(\rho_i)\right|^2\\ +\left|\dot{\eta}_A^-(\rho_i)-\dot{\eta}_B^-(\rho_i)+\dot{\eta}_A^+(\rho_i)-\dot{\eta}_B^+(\rho_i)\right|^2\end{array}\right)\\ \left(2-\left|\begin{array}{l}\dot{\pi}_A^-(\rho_i)-\dot{\pi}_B^-(\rho_i)+\\ \dot{\pi}_A^+(\rho_i)-\dot{\pi}_B^+(\rho_i)\end{array}\right|\right)^2\\ +2\left(\begin{array}{l}\left|\begin{array}{l}\min\left\{\dot{\zeta}_A^-(\rho_i),\dot{\eta}_B^-(\rho_i)\right\}+\\ \min\left\{\dot{\zeta}_A^+(\rho_i),\dot{\eta}_B^+(\rho_i)\right\}-\\ \min\left\{\dot{\zeta}_B^-(\rho_i),\dot{\eta}_A^-(\rho_i)\right\}+\\ \min\left\{\dot{\zeta}_B^+(\rho_i),\dot{\eta}_A^+(\rho_i)\right\}\end{array}\right|^2\\ +\left|\begin{array}{l}\max\left\{\dot{\zeta}_A^-(\rho_i),\dot{\eta}_B^-(\rho_i)\right\}+\\ \max\left\{\dot{\zeta}_A^+(\rho_i),\dot{\eta}_B^+(\rho_i)\right\}\\ -\max\left\{\dot{\zeta}_B^-(\rho_i),\dot{\eta}_A^-(\rho_i)\right\}+\\ \max\left\{\dot{\zeta}_B^+(\rho_i),\dot{\eta}_A^+(\rho_i)\right\}\end{array}\right|^2\end{array}\right)\end{array}\right\}\right]^{\frac{1}{2}}$$

TABLE 1. Comparison of Similarity measures in the environment of IVIFS.

	Case I	Case II	Case III	Case IV
A_i	{[0.3,0.4], [0.5,0.7]}	{[0.3,0.4], [0.5,0.7]}	{[0.3,0.3], [0.5,0.5]}	{[0.3,0.3], [0.5,0.5]}
B_i	{[0.4,0.5], [0.5,0.7]}	{[0.4,0.5] , [0.4,0.6]}	{[0.4,0.5] , [0.5,0.7]}	{[0.4,0.5] , [0.5,0.6]}
S_1	0.9	0.9	0.92	0.92
S_2	0.9	0.9	0.9	0.9
S_W	0.21	0.21	0.11	0.11
S_D	1.00	1.00	0.98	0.98
S^P	0.93	0.93	0.91	0.91
$\mathcal{S}$	0.78	0.80	0.84	0.85

Taking the summation over all $i = 1, 2, \ldots, n$ and prefixing the factor $\frac{1}{n}$. It follows that as shown in the equation at the bottom of the next page. Hence, $\mathcal{S}^{-}(A, C) \leq \mathcal{S}^{-}(A, B)$.

Similarly, from equation (2), we obtain $\mathcal{S}^{-}(A, C) \leq \mathcal{S}^{-}(B, C)$.

Case II

The relations $\mathcal{S}^{+}(A, C) \leq \mathcal{S}^{+}(A, B)$ and $\mathcal{S}^{+}(A, C) \leq \mathcal{S}^{+}(B, C)$ can easily be established from (1) and (2) respectively.

Consequently,

$$\mathcal{S}(A, C) \leq \min\{\mathcal{S}(A, B), \mathcal{S}(B, C)\}.$$

Hence, we conclude that $\mathcal{S}(A, B)$ is a similarity measure.

Remark 1: The relations (S1), (S2), (S3) and (S4) defined in the above theorem are interpreted as follows:

1. (S1) represents the boundedness of the proposed similarity measure. This axiom ensures that the similarity measure will never present a negative value for similarity, which means that it yields a low or zero value when comparing two objects or concepts.

2. (S2) indicates symmetry, which states that the similarity between two objects does not depend on the order in which they are compared.

3. (S3) demonstrates reflexivity, which describes that an object should be perfectly similar to itself.

4. (S4) interprets triangular inequality, which states that the sum of similarities between any pair of objects should exceed that of any intermediate object.

Remark 2: It is important to note that the similarity measure defined in [29] is a special case of the proposed similarity measure in Definition 6, if $\dot{\zeta}_A^{-} = \dot{\zeta}_A^{+}, \dot{\zeta}_B^{-} = \dot{\zeta}_B^{+}, \dot{\eta}_A^{-} = \dot{\eta}_A^{+}, \dot{\eta}_B^{-} = \dot{\eta}_B^{+}$, as shown in the equation at the bottom of page 10, then, is a similarity measure between IFS A and B.

A. COMPERATIVE ANALYSIS

In the following discussion, we set up a comparison to demonstrate the validity and feasibility of the proposed similarity measure on IVIFS with the existing similarity measure in the IVIF environment.

In Table 1, upon comparing cases I and II of the existing similarity measures S_1, S_2, S_w and S^P, we observe that these similarity measures exhibit certain shortcomings, as they fail to adhere to the following relation: $S(A_1, B_1) = S(A_2, B_2)$, while $A_1 = A_2$ and $B_1 \neq B_2$. This non-conformance indicates that the current similarity measures are not reasonable for certain scenarios, indicating that the existing similarity measures are not reasonable.

Moreover, $S_D(A_1, B_1) = S_D(A_2, B_2) = 1.00$ while $A_1 = A_2$ and $B_1 \neq B_2$, indicating that this similarity measure S_D does not satisfy the property $(S3)$ as expressed in Definition 4.

Furthermore, in Table 1, upon comparing cases III and IV of S_1, S_2, S_w, S_D and S^P. This analysis highlights another shortcoming in these established measures, as they do not conform to the relationship $S(A_3, B_3) = S(A_4, B_4)$, while $A_3 = A_4$ and $B_3 \neq B_4$. This inconsistency underscores the inadequacy of the existing similarity measures.

In contrast, the newly proposed similarity measure S, handles these scenarios better, proving its superiority. In particular, this novel similarity measure uses the hesitance function to capture uncertainty, represent imprecise information, and support creative thinking and decision-making in challenging circumstances. Adding the hesitance function to the IVIFS framework broadens its decision-making applications. Thus, the proposed similarity measure for IVIFS solves MCDM issues more flexibly than current measures.

V. APPLICATION OF PROPOSED IVIF SIMILARITY MEASURE IN MCDM PRPBLEM

In this section, we illustrate the significance of the proposed similarity measure in the framework of the IVIF environment by presenting an algorithm for solving MCDM problems.

Let $\{A_1, A_2, A_3, \ldots, A_m\}$ be the set of distinct alternatives, and $\{\epsilon_1, \epsilon_2, \ldots, \epsilon_n\}$ be the set of attributes. Suppose $M = \left[\left(\left[\dot{\zeta}_{ij}^{-}, \dot{\zeta}_{ij}^{+}\right], \left[\dot{\eta}_{ij}^{-}, \dot{\eta}_{ij}^{+}\right]\right)\right]_{m\times n}$ is an $m\times n$ IVIF decision matrix, where $0 \leq \dot{\zeta}_{ij}^{-}(\epsilon) + \dot{\eta}_{ij}^{-}(\epsilon) \leq 1$ and $0 \leq \dot{\zeta}_{ij}^{+}(\epsilon) + \dot{\eta}_{ij}^{+}(\epsilon) \leq 1$.

Assume that

$$B = \left\{ \begin{array}{c} \left\langle \epsilon_1, \left[\dot{\zeta}_B^{-}(\epsilon_1), \dot{\zeta}_B^{+}(\epsilon_1)\right], \left[\dot{\eta}_B^{-}(\epsilon_1), \dot{\eta}_B^{+}(\epsilon_1)\right]\right\rangle, \ldots, \\ \left\langle \epsilon_n, \left[\dot{\zeta}_B^{-}(\epsilon_n), \dot{\zeta}_B^{+}(\epsilon_n)\right], \left[\dot{\eta}_B^{-}(\epsilon_n), \dot{\eta}_B^{+}(\epsilon_n)\right]\right\rangle : \epsilon_j \in X \end{array} \right\}$$

is an IVIFS classified as a test sample. The algorithm to solve the MCDM problem in the framework of IVIF is designed in the following way:

TABLE 2. IVIF decision matrix of software based on software quality criteria.

	ϵ_1	ϵ_2	ϵ_3	ϵ_4
P_1	([0.05,0.10], [0.25, 0.30])	([0.05, 0.10], [0.35, 0.40])	([0.25, 0.30], [0.15, 0.20])	([0.40, 0.45], [0.00, 0.05])
P_2	([0.25, 0.30], [0.15, 0.20])	([0.30, 0.35], [0.05, 0.10])	([0.50, 0.50], [0.00, 0.00])	([0.05, 0.10], [0.30, 0.35])
P_3	([0.20, 0.25], [0.15, 0.20])	([0.30, 0.35], [0.10, 0.15])	([0.45, 0.50], [0.00, 0.00])	([0.00, 0.05], [0.40, 0.45])
P_4	([0.50, 0.50], [0.00, 0.00])	([0.50, 0.50], [0.00, 0.00)	([0.40, 0.45], [0.00, 0.05])	([0.35, 0.40], [0.05, 0.10])
	ϵ_5	ϵ_6	ϵ_7	ϵ_8
P_1	([0.20, 0.25], [0.15, 0.20])	([0.00, 0.05], [0.40, 0.45])	([0.15, 0.20], [0.25, 0.30])	([0.50, 0.50], [0.00, 0.00])
P_2	([0.00, 0.05], [0.40, 0.45])	([0.30, 0.3.5], [0.10, 0.15])	([0.25, 0.30], [0.15, 0.20])	([0.30, 0.35], [0.10, 0.15])
P_3	([0.00, 0.05], [0.40, 0.45])	([0.30, 0.3.5], [0.10, 0.15])	([0.05, 0.10], [0.35, 0.40])	([0.10, 0.15], [0.30, 0.35])
P_4	([0.00, 0.05], [0.55, 0.45])	([0.00, 0.05], [0.40, 0.45])	([0.05, 0.10], [0.35, 0.40])	([0.05, 0.10], [0.35, 0.40])
	ϵ_9	ϵ_{10}	ϵ_{11}	ϵ_{12}
P_1	([0.10, 0.15], [0.3, 0.35])	([0.20, 0.25], [0.20, 0.25])	([0.00, 0.05], [0.40, 0.45])	([0.20, 0.25], [0.20, 0.25])
P_2	([0.50, 0.50], [0.00, 0.00])	([0.05, 0.10], [0.35, 0.40])	([0.00, 0.05], [0.40, 0.45])	([0.35, 0.40], [0.05, 0.10])
P_3	([0.25, 0.30], [0.10, 0.20])	([0.50, 0.50], [0.00, 0.00])	([0.15, 0.20], [0.20, 0.25])	([0.00, 0.05], [0.40, 0.45])
P_4	([0.20, 0.25], [0.15, 0.20])	[0.50, 0.50], [0.00, 0.00])	([0.15, 0.20], [0.20, 0.25])	([0.00, 0.05], [0.40, 0.45])

STEP I

Convert the decision matrix M into a normalized matrix $S = \left(s_{ij}\right)_{m\times n}$ (if necessary), where S_{ij} is computed by the following equation:

$$S_{ij} = \begin{cases} \left(\left[\dot{\zeta}_{ij}^{-}, \dot{\zeta}_{ij}^{+}\right], \left[\dot{\eta}_{ij}^{-}, \dot{\eta}_{ij}^{+}\right]\right); \text{for benefit type criteria} \\ \left(\left[\dot{\eta}_{ij}^{-}, \dot{\eta}_{ij}^{+}\right], \left[\dot{\zeta}_{ij}^{-}, \dot{\zeta}_{ij}^{+}\right]\right); \text{for loss type criteria} \end{cases}$$

STEP II

Calculate the similarity measures $\mathcal{S} = (A_i, B)$ between $A_i\,(i = 1, 2, 3, \ldots, m)$ and B as follows shown in the equation at the bottom of the next page.

STEP III

Rank all the alternatives and choose the maximum one.

TABLE 3. Computed results under different similarity measure.

	$S(P_1\ ,B)$	$S(P_2\ ,B)$	$S(P_3\ ,B)$	$S(P_4\ ,B)$
S_1	0.910	0.907	0.936	0.965
S_2	0.900	0.892	0.908	0.911
S_D	0.993	0.991	0.994	0.998
$\mathcal{S}$	0.901	0.896	0.916	0.920

A. SOLUTION OF REAL-WORLD PROBLEMS THROUGH PROPOSED SIMILARITY MEASURE

1) SOFTEARE QUALITY MODEL

Software not only has the potential to make a firm more efficient, but it also enables computer hardware to execute essential functions. The development of new software is

$$\left[\frac{1}{12\text{n}}\sum_{i=1}^{n}\left\{\begin{array}{l}\left(\left|\dot{\zeta}_A^{-}(\rho_i)-\dot{\zeta}_C^{-}(\rho_i)\right|^2+\left|\dot{\eta}_A^{-}(\rho_i)-\dot{\eta}_c^{-}(\rho_i)\right|^2\right)\\ \left(2-\left|\dot{\pi}_A^{-}(\rho_i)-\dot{\pi}_C^{-}(\rho_i)\right|^2\right)\\ +2\left(\begin{array}{l}\left|\begin{array}{l}\min\left\{\dot{\zeta}_A^{-}(\rho_i),\dot{\eta}_C^{-}(\rho_i)\right\}-\\ \min\left\{\dot{\zeta}_C^{-}(\rho_i),\dot{\eta}_A^{-}(\rho_i)\right\}\end{array}\right|^2\\ +\left|\begin{array}{l}\max\left\{\dot{\zeta}_A^{-}(\rho_i),\dot{\eta}_C^{-}(\rho_i)\right\}-\\ \max\left\{\dot{\zeta}_C^{-}(\rho_i),\dot{\eta}_A^{-}(\rho_i)\right\}\end{array}\right|^2\end{array}\right)\end{array}\right\}\right]$$

$$\geq\left[\frac{1}{12\text{n}}\sum_{i=1}^{n}\left\{\begin{array}{l}\left(\left|\dot{\zeta}_A^{-}(\rho_i)-\dot{\zeta}_B^{-}(\rho_i)\right|^2+\left|\dot{\eta}_A^{-}(\rho_i)-\dot{\eta}_B^{-}(\rho_i)\right|^2\right)\\ \left(2-\left|\dot{\pi}_A^{-}(\rho_i)-\dot{\pi}_B^{-}(\rho_i)\right|^2\right)\\ +2\left(\begin{array}{l}\left|\begin{array}{l}\min\left\{\dot{\zeta}_A^{-}(\rho_i),\dot{\eta}_B^{-}(\rho_i)\right\}-\\ \min\left\{\dot{\zeta}_B^{-}(\rho_i),\dot{\eta}_A^{-}(\rho_i)\right\}\end{array}\right|^2\\ +\left|\begin{array}{l}\max\left\{\dot{\zeta}_A^{-}(\rho_i),\dot{\eta}_B^{-}(\rho_i)\right\}-\\ \max\left\{\dot{\zeta}_B^{-}(\rho_i),\dot{\eta}_A^{-}(\rho_i)\right\}\end{array}\right|^2\end{array}\right)\end{array}\right\}\right].$$

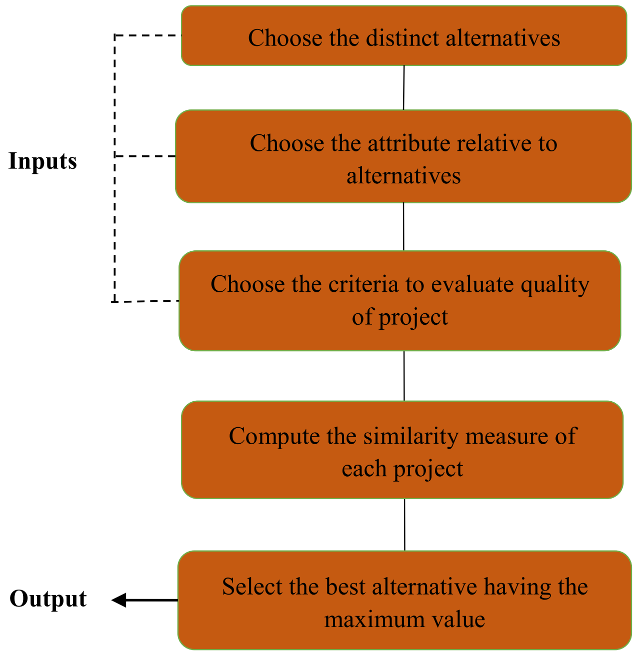

FIGURE 1. Step-by-step procedure for choosing the best software project.

essential to the growth of computing technology. The right use of software has the potential to ease the way for novel methods of labor organization. With well-coordinated and efficient software, applications may more effectively address

$$\mathcal{S}(A,B) = 1 - \left[\frac{1}{12n}\sum_{i=1}^{n}\left\{\begin{array}{l}\left(\left|\dot{\zeta}_A(\rho_i) - \dot{\zeta}(\rho_i)\right|^2 + \left|\dot{\eta}_A(\rho_i) - \dot{\eta}_B(\rho_i)\right|^2\right)\\ \left(2 - \left|\dot{\pi}_A(\rho_i) - \dot{\pi}_B(\rho_i)\right|\right)^2\\ +2\left(\begin{array}{l}\left|\begin{array}{l}\min\left\{\dot{\zeta}_A(\rho_i), \dot{\eta}_B(\rho_i)\right\} + \\ \min\left\{\dot{\zeta}_B(\rho_i), \dot{\eta}_A(\rho_i)\right\}\end{array}\right|^2\\ +\left|\begin{array}{l}\max\left\{\dot{\zeta}_A(\rho_i), \dot{\eta}_B(\rho_i)\right\} + \\ \max\left\{\dot{\zeta}_B(\rho_i), \dot{\eta}_A(\rho_i)\right\}\end{array}\right|^2\end{array}\right)\end{array}\right\}\right]^{\frac{1}{2}}$$

$$\mathcal{S}(A_i,B) = 1 - \left[\frac{1}{12n}\sum_{j=1}^{n}\left\{\begin{array}{l}\left(\begin{array}{l}\left|\begin{array}{l}\dot{\zeta}_A^-(\epsilon_{ij}) - \dot{\zeta}_B^-(\epsilon_j) + \\ \dot{\zeta}_A^+(\epsilon_j) - \dot{\zeta}_B^+(\epsilon_j)\end{array}\right|^2\\ +\left|\begin{array}{l}\dot{\eta}_A^-(\epsilon_{ij}) - \dot{\eta}_B^-(\epsilon_j) + \\ \dot{\eta}_A^+(\epsilon_j) - \dot{\eta}_B^+(\epsilon_j)\end{array}\right|^2\end{array}\right)\\ \left(2 - \left|\begin{array}{l}\dot{\pi}_A^-(\epsilon_{ij}) - \dot{\pi}_B^-(\epsilon_j) + \\ \dot{\pi}_B^+(\epsilon_j) - \dot{\pi}_B^+(\epsilon_j)\end{array}\right|\right)^2\\ +2\left(\begin{array}{l}\left|\begin{array}{l}\min\left\{\dot{\zeta}_A^-(\epsilon_{ij}), \dot{\eta}_B^-(\epsilon_j)\right\} + \\ \min\left\{\dot{\zeta}_A^+(\epsilon_{ij}), \dot{\eta}_B^+(\epsilon_j)\right\}\\ -\min\left\{\dot{\zeta}_B^-(\epsilon_j), \dot{\eta}_A^-(\epsilon_{ij})\right\} + \\ \min\left\{\dot{\zeta}_B^+(\epsilon_j), \dot{\eta}_A^+(\epsilon_{ij})\right\}\end{array}\right|^2\\ +\left|\begin{array}{l}\max\left\{\dot{\zeta}_A^-(\epsilon_{ij}), \dot{\eta}_B^-(\epsilon_j)\right\} + \\ \max\left\{\dot{\zeta}_A^+(\epsilon_{ij}), \dot{\eta}_B^+(\epsilon_j)\right\}\\ -\max\left\{\dot{\zeta}_B^-(\epsilon_j), \dot{\eta}_A^-(\epsilon_{ij})\right\} + \\ \max\left\{\dot{\zeta}_B^+(\epsilon_j), \dot{\eta}_A^+(\epsilon_{ij})\right\}\end{array}\right|^2\end{array}\right)\end{array}\right\}\right]^{\frac{1}{2}}$$

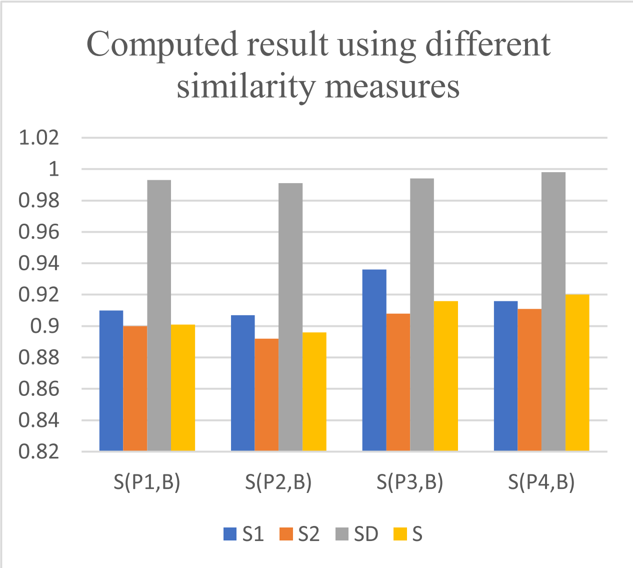

FIGURE 2. **Computed result using different similarity measures.**

complex problems in the real world. High-quality software that is able to meet deadlines and budget constraints is considered to be effective software. Because of its importance, a company's software should be carefully chosen to fulfill all necessary needs. Buying software is a significant financial commitment that requires justification because it is an intellectual product. For such expenditures to be worthwhile, it is crucial that the program fully address the needs of its end users. Before ISO, there was no universally accepted method of determining whether or not a software system was lacking in major structural problems by examining its source code. ISO 25010 is a new standard for evaluating the quality of systems and software that was released in 2011.

The IVIFS framework is superior at addressing problems of uncertainty. Thus, we developed the IVIFS software quality model, an MCDM framework for assessing programs according to the proposed IVIFS similarity measure. In order to assess the quality of software projects, we formulated an MCDM model. The following important quality identifier criterion was part of this standard, which was verified in IOS 2017.

1) ϵ_1 : Functional Suitability
2) ϵ_2 : Functional Correctness
3) ϵ_3 : Testability
4) ϵ_4 : Performance Efficiency
5) ϵ_5 : Compatibility
6) ϵ_6 : Usability
7) ϵ_7 : Appropriateness Recognizability
8) ϵ_8 : User Interface Aesthetics
9) ϵ_9 : Reliability
10) ϵ_{10} : Security
11) ϵ_{11} : Maintainability
12) ϵ_{12} : Modifiability

Software projects are measured against the highest standards using a predetermined list of criteria to determine how well they have been developed.

A decision-making problem using the newly proposed IVIFS similarity measure is analyzed to evaluate the software quality of four software projects P_1, P_2, P_3 and P_4. Let as shown in the equation at the bottom of the next page, be the criteria for evaluating the software project quality. The flowchart of the MCDM problem is illustrated in the Figure 1.

The software projects based on software quality criteria are represented in an IVIFS decision matrix.

STEP 1

The information provided by a decision maker for the above software quality criteria for these four projects is evaluated under the IVIF environment and summarized in Table 2.

STEP II

The similarity measure of each project corresponding to the above criteria is computed in the subsequent Table 3 by using Definition 6.

The graphical interpretation of the above outcomes are illustrated in Figure 2.

STEP III

From the above table we note that $S(P_4, B) > S(P_3, B) > S(P_1, B) > S(P_2, B)$. This means that $P_4 > P_3 > P_1 > P_2$. Consequently, in view of ranking order, P_4 is the best software project.

The applicability of the newly developed similarity measure in the context of software quality is a very important factor in ensuring the dependability and efficiency of software development processes. The validity of this technique in software quality is of the utmost importance to ensure that the choices made based on the technique are accurate, trustworthy, and in line with the intended objectives for improving software quality. The selection of the project demonstrates that software development teams and quality managers can make more informed decisions to improve the quality of software products.

2) OPTIMAL PRODUCTION STRATEGY

Business and other organizations can benefit from management science since it employs scientific methods to solve problems and make decisions. Industrial businesses must find ways to save expenses while still meeting the needs of their diverse customer base and turning a profit in order to stay competitive. Production systems get more complicated due to the presence of various uncertainties. Large and small businesses can set production priorities through the use of production strategies. As part of a comprehensive manufacturing plan, a company's leaders should carefully consider the company's central purpose. This option helps managers focus on crucial targets and the actions needed to achieve them. An optimal production strategic decision-making problem using the newly proposed IVIFS similarity measure is analyzed in the following discussion. Assume that a company is contemplating developing a new product and is trying to determine the best way to maximize profits. After market

TABLE 4. Decision matrix on production strategies.

	ϵ_1	ϵ_2	ϵ_3	ϵ_4
A_1	([0.40, 0.45], [0.00, 0.05])	([0.35, 0.40], [0.05, 0.10])	([0.25,0.30], [0.10, 0.15])	([0.30, 0.40], [0.05,0.10])
A_2	([0.25, 0.30], [0.05, 0.15])	([0.40, 0.45], [0.00, 0.05])	([0.30, 0.40], [0.05, 0.10])	([0.20, 0.30], [0.05, 0.10])
A_3	([0.35, 0.40], [0.05, 0.10])	([0.35, 0.45], [0.00, 0.05])	([0.20, 0.30], [0.10, 0.20])	([0.15, 0.25], [0.10, 0.20])
A_4	([0.40, 0.45], [0.00, 0.05])	([0.35, 0.40], [0.05, 0.10])	([0.35, 0.45], [0.00,0.05])	([0.40, 0.45], [0.00,0.05])

analysis, a market strategist considers the four possible strategies A_1, A_2, A_3 and A_4 where:

1. A_1 : Create a new product oriented to the rich customers: The short-term benefits of a new product for rich customers include increased revenue and a competitive advantage, while the mid-term benefits include sustained profitability, customer loyalty, and credibility. Strategic advantages, market expansion, and customer happiness provide long-term, affluent customer success. Skilled and motivated employees drive innovation, customer-centricity, and efficient product development. Affluent clients want high-quality, premium items, and enough financial conditions allow for research and development, marketing, and production.
2. A_2 : Create a new product oriented toward mid-level and low-level customers: Short-term benefits include higher sales volume and market penetration when creating a mid- or low-level product. Mid-term gains include consumer loyalty, market share, and competitive advantage. Customer relationship management and product enhancement provide long-term profitability, brand recognition, and market growth. Human resources ensure customer-centricity, efficiency, and cost-effective solutions to achieve these benefits. Market research, product development, marketing tactics, and distribution methods are needed to provide economical, high-quality items for mid- and low-income clients.
3. A_3 : Create a new product adapted to all the customers: When designing a product for all customers, consider short-term rewards like sales and market presence. Mid-term rewards include consumer loyalty, brand reputation, and market advantage. Innovation, customer service, and market adaptability drive long-term profitability, market expansion, and customer retention. Human resources' customer-centric culture, top talent recruitment, and cross-functional collaboration help achieve these benefits. Financial resources are needed to invest in marketing, and production to create adaptable products that fulfill all customers' needs.
4. A_4 : Do not encourage anyone to create any products: Short-term gains might inhibit the creation of sustainable and creative goods, missing market growth and customer satisfaction chances. Mid-term benefits might hinder strategic planning, competitiveness, and long-term success. Insufficient investment in human resources and financial conditions might hinder the expertise, motivation, and resources needed to create and launch successful products, slowing management science growth.

TABLE 5. Computed result using different similarity measures.

	$S(A_1,B)$	$S(A_2,B)$	$S(A_3,B)$	$S(A_4,B)$
S_1	0.863	0.893	0.882	0.872
S_2	0.834	0.877	0.85	0.834
S_D	0.983	0.991	0.986	0.984
S	0.833	0.857	0.850	0.816

Let $X = \{\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4\}$ be the factors that may affect the implementation of strategies.

1. ϵ_1 : Short term benefits: It refers to the direct benefits that will result from a company's choice of production tactics. Rather than accruing over a long period of time, these advantages tend to be experienced quickly, within a few weeks or months. Immediate advantages are the main focus of short-term benefits.
2. ϵ_2 : Mid-term benefits: Mid-term advantages are often seen over a little longer timescale, spanning from several months to a couple of years, as opposed to short-term benefits, which are instantaneous and sometimes reached within weeks or months. The advantages in the middle term can change with the type of business, the state of the market, and other variables.

$$B = \left\{ \begin{array}{l} \langle\epsilon_1, ([0.45, 0.50], [0.00, 0.00])\rangle, \langle\epsilon_2, ([0.45, 0.50], [0.00, 0.00])\rangle, \\ \langle\epsilon_3, ([0.35, 0.40], [0.05, 0.10])\rangle, \langle\epsilon_4, ([0.30, 0.35], [0.05, 0.10])\rangle, \\ \langle\epsilon_5, ([0.00, 0.05], [0.40, 0.45])\rangle, \langle\epsilon_6, ([0.05, 0.10], [0.35, 0.40])\rangle, \\ \langle\epsilon_7, ([0.05, 0.10], [0.35, 0.40])\rangle, \langle\epsilon_8, ([0.05, 0.10], [0.35, 0.40])\rangle, \\ \langle\epsilon_9, ([0.20, 0.25], [0.15, 0.20])\rangle, \langle\epsilon_{10}, ([0.50, 0.50], [0.00, 0.00])\rangle, \\ \langle\epsilon_{11}, ([0.15, 0.20], [0.20, 0.25])\rangle, \langle\epsilon_{12}, ([0.00, 0.05], [0.35, 0.45])\rangle \end{array} \right\}$$

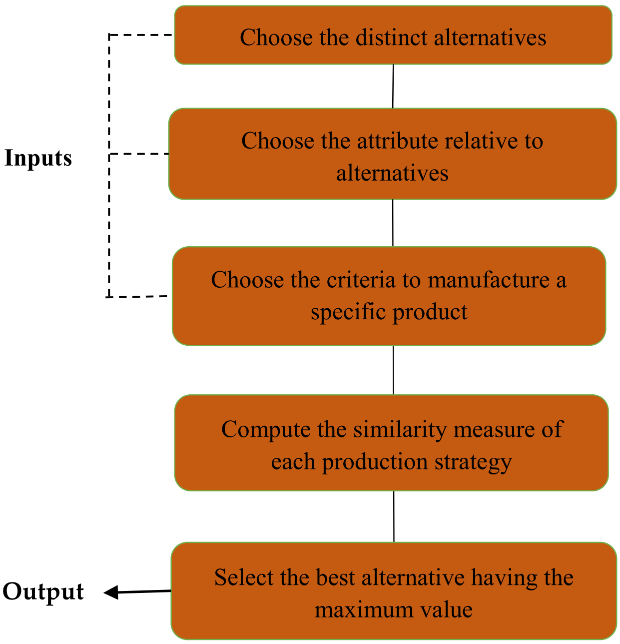

FIGURE 3. Step-by-step procedure for choosing the optimal strategy to manufacture a product.

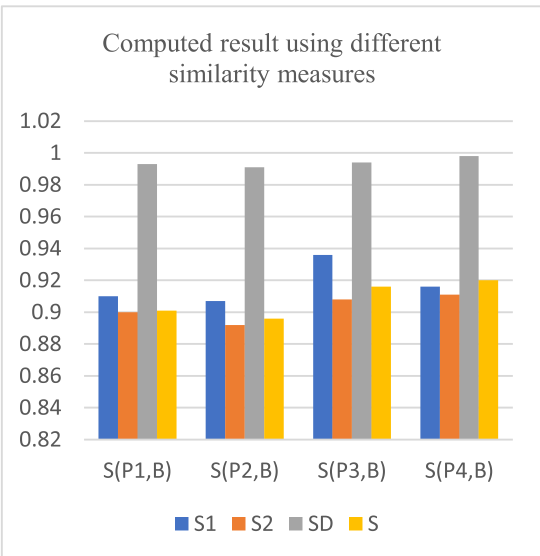

FIGURE 4. Computed result using different similarity measure.

3. ϵ_3 : Long term benefits: Due to long-term strategic decisions and activities, these benefits frequently appear years or decades later. They involve investing, partnering, and implementing strategies that increase the company's long-term competitiveness and profitability.
4. ϵ_4 : Human resources and financial conditions: These are two major considerations for crafting and carrying out an effective method of manufacturing. Talent acquisition, development, and engagement are all aspects of human resource management that contribute to an organization's preparedness to put its strategy into action. Equally important to the financial viability of the production strategy is the ability to allocate resources, control costs, invest in capital, and reduce risk. as shown in the equation at the top of the next page,

be the criteria for an optimal strategy to manufacture a specific product. The flowchart of the MCDM problem is illustrated in Figure 3.

The criteria for a specific product based on production strategy are represented in an IVIF decision matrix.

STEP 1

The information provided by the decision maker for the optimal production of these four strategies is evaluated under the IVIF environment and summarized in Table 4.

STEP II

The similarity measure of each product corresponds to the above criteria is computed in the subsequent Table 5 by using Definition 6.

The graphical interpretation of the above outcome are illustrated in Figure 4

STEP III

From the above table we note that, $S(A_2, B) > S(A_3, B) > S(A_1, B) > S(A_4, B)$. This means that $A_2 > A_3 > A_1 > A_4$. Consequently, in view of ranking order, A_2 is the best strategy for production.

In the context of determining the optimal production strategy, the validity of the proposed similarity measure in the framework of the IVIF environment is crucial for ensuring that the chosen strategy corresponds with the organization's objectives and constraints. After having applied this technique, we observe that the production strategy A_2 is optimal as it guarantees production efficiency, cost-effectiveness, and overall performance.

VI. CONCLUSION

The IVIFSs hold significant importance in numerous domains by simplifying the process of information acquisition, aiding in complex decision-making, and facilitating the management of uncertainty. In the context of IVIF knowledge, similarity measures assume paramount significance as they enable the quantification of the degree of similarity or dissimilarity between two IVIFS. In order to deal with the unpredictability of information systems, different similarity measures between IVIFSs have already been developed. Nonetheless, the majority of these measures have produced findings that are paradoxical. However, these strategies have some limitations to solve MCDM problems in the IVIF environment adequately. This fact has effectively been demonstrated in the present study by presenting various examples. In this article, a new similarity measure has been proposed to overcome the drawbacks of the existing similarity measures in the IVIF environment. In the present study, it has been demonstrated that this new similarity measure is better than the existing measures in the IVIF environment by conducting a comparative study presented in table 1. Additionally, an algorithm has been devised in order to handle the challenge of deploying this improved similarity measure in

$$B = \left\{ \begin{array}{c} \langle \epsilon_1, [0.2, 0.25], [0.05, 0.1] \rangle, \langle \epsilon_2, [0.35, 0.4], [0.05, 0.1] \rangle, \\ \langle \epsilon_3, [0.45, 0.45], [0.0.05] \rangle, \langle \epsilon_4, [0.15, 0.25], [0.1, 0.2] \rangle \end{array} \right\},$$

MCDM problems. In this research, a mechanism has been presented to handle the issues of evaluating software quality and selection of optimal production strategy using the proposed IVIF similarity measure.

The approach proposed in this paper offers several advantages, but it also has specific limitations, especially in the context of MCDM. These limitations arise when the sum of the scores of membership and non-membership exceeds one or involve neutral membership. In order to counter these situations, we need some more generalized environments than the IVIF environment. Moreover, consider $A = \{[0.3, 0.5], [0.3, 0.5]\}$, $B = \{[0.2, 0.4], [0.2, 0.4]\}$ and $C = \{[0.4, 0.7], [0.4, 0.7]\}$, be any three IVIFS, then in view of definition 6, we get $S(A, C) = S(B, C)$. The proposed similarity measure struggles to handle situations where the membership degree and non-membership degree are equal. This proposed similarity measure allocates equal weights to both the membership degree and non-membership degree. Consequently, there exists potential for further refinement and improvement of this proposed similarity measure.

Future work will focus on applying the suggested similarity measure to MCDM challenges in environmental protection, engineering, and other physical phenomena. We will also develop similarity measures on interval-valued PFSs since they allow decision-makers to examine. Our research will also incorporate interval-valued bipolar fuzzy sets. They help decision-makers with ambiguity and uncertainty, leading to better, more balanced choices.

CONFLICTS OF INTEREST

No conflicts of interest are disclosed by the authors.

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